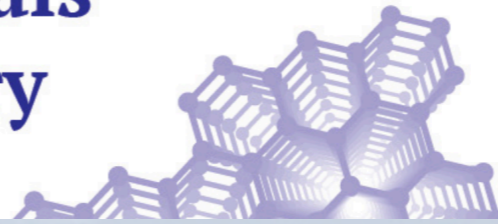




RUTGERS
THE STATE UNIVERSITY
OF NEW JERSEY

RUTGERS CENTER FOR
**Materials
Theory**



**FLATIRON
INSTITUTE**
Center for Computational
Quantum Physics

Lecture 4: Adaptive Quantum Circuits and Control Induced Phase Transitions

Jed Pixley

ICTS Lecture Series:

Quantum Dynamics in the pre-fault tolerant era

6/9/2026



LECTURE SERIES OVERVIEW

Quantum dynamics in the pre fault tolerant era

Lecture 1: From Classical and quantum chaos to thermalization in isolated quantum systems

Lecture 2: Analog and digital quantum simulators from unitary dynamics to midcircuit measurements.

Lecture 3: Monitored quantum dynamics in random quantum circuits

Lecture 4: Adaptive quantum circuits and control induced phase transitions

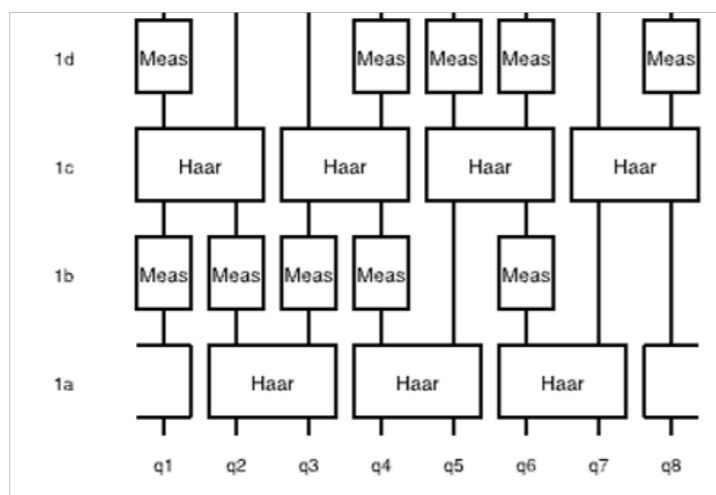
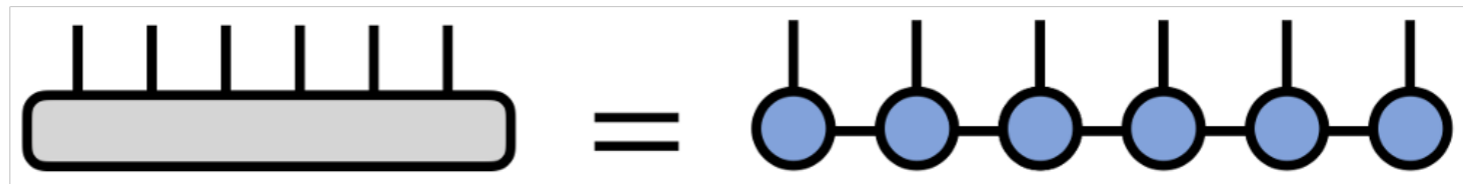
Lecture 5: Open quantum dynamics software tutorial

ADVERTISING LECTURE 5

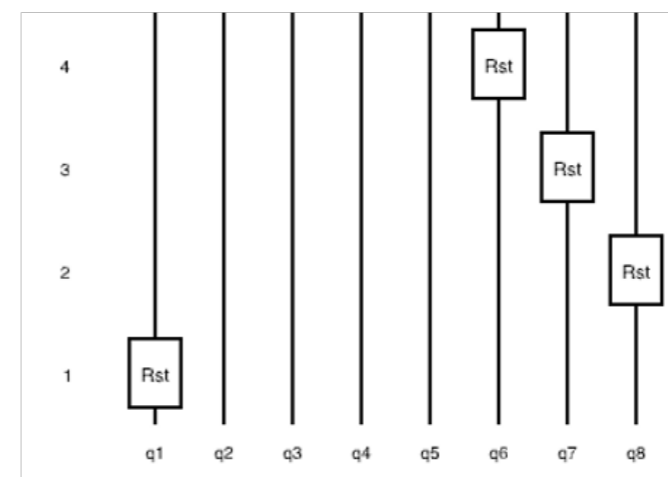
Lecture 5: Open quantum dynamics software tutorial

Classical simulations of quantum circuits in the Pre-fault-tolerant era

Matrix Product States:



Measurement-induced phase transition



Control-induced phase transition

<https://github.com/hainingpan/QuantumCircuitsMPS.jl>

LECTURE SERIES, LEARNING GOALS

- I. Lecture 1: Classical and Quantum Chaos, from single particle to many-body
- II. Lecture 2: Quantum platforms
- III. Lecture 3: Entanglement phases and phase transitions driven by interplay of unitary and projective evolution.
- IV. Lecture 4: **Controlling chaos in classical and quantum systems.**
- V. Lecture 5: Numerical approaches to adaptive quantum dynamics

LECTURE 4: LEARNING GOALS

Adaptive Quantum Dynamics

Understand how to control classical chaos. Apply to control several quintessential chaotic systems.

Quantize this procedure to find a route to controlling quantum dynamics.

We will understand the universality classes discovered, their interplay with the measurement induced phase transition, and the role of quantum fluctuations.

Experimental data on trapped ions and superconducting qubits will be presented.

OUTLINE

I. Motivation

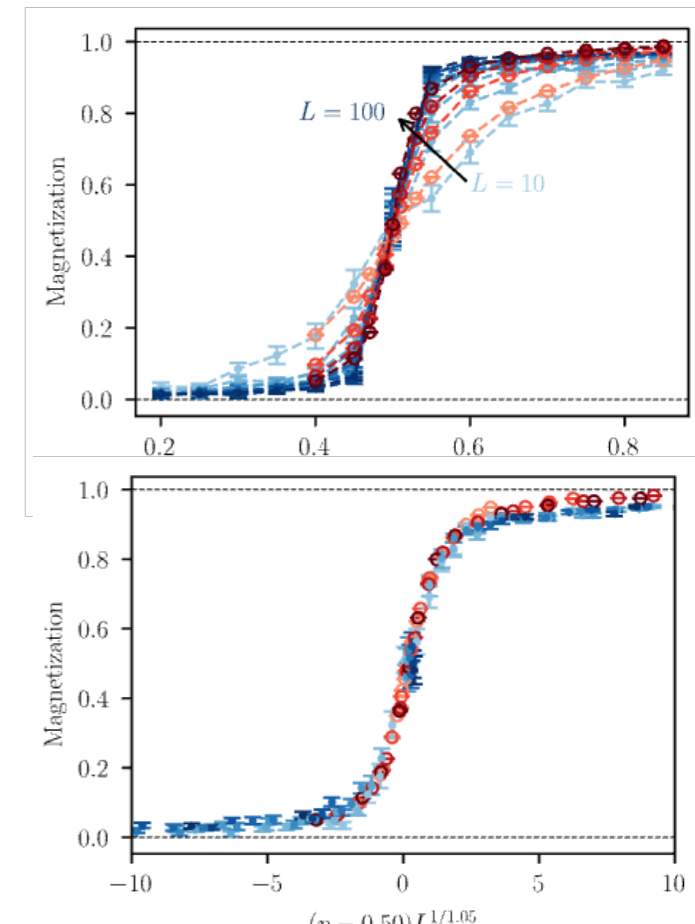
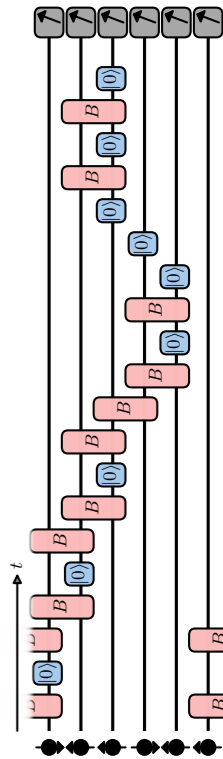
- Controlling classical chaos
- Examples of classical control transitions

II. Quantum Control Induced Transition

- Control with an Exact Dark State
- Controlling the quantum top
- Controlling the Bernoulli map

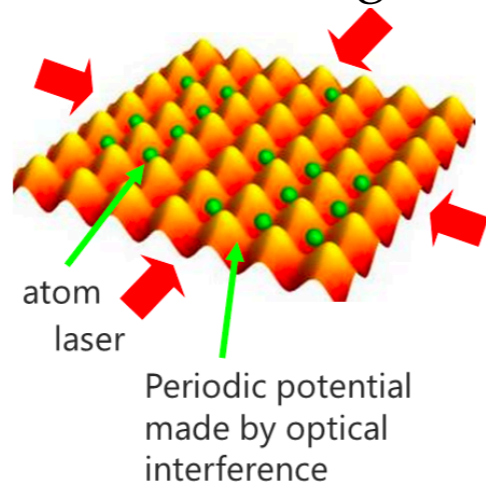
III. Realization across circuits

- IBM's quantum computer
- Accessing Quantum fluctuations



QUANTUM PLATFORMS OF TODAY

Cold atomic gases



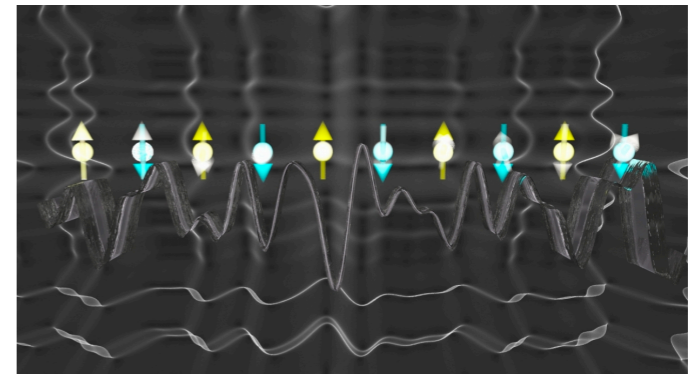
M. Kozuma group

Neutral atoms



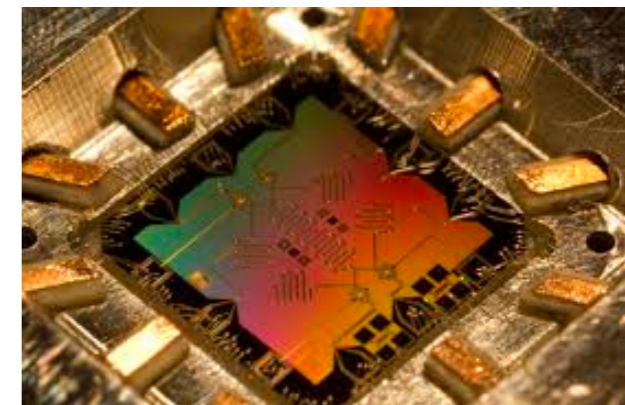
M. Endres group

Trapped ions



C. Monroe group

Superconducting qubits

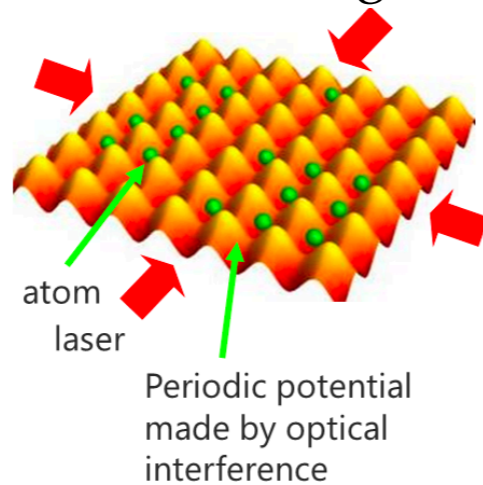


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Lecture 2: We reviewed each of these systems in detail.

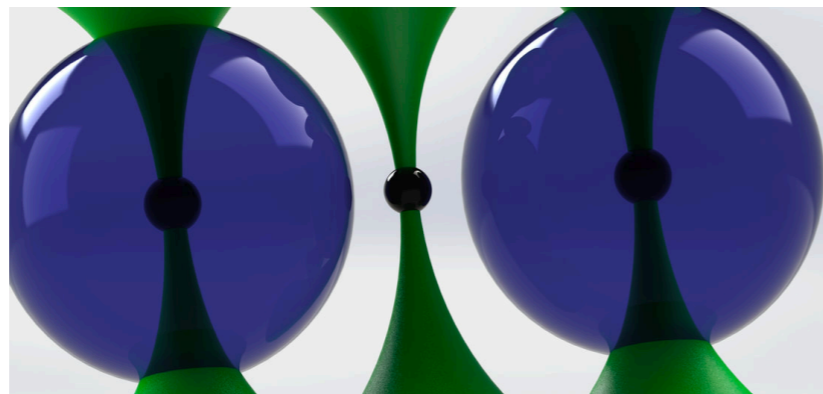
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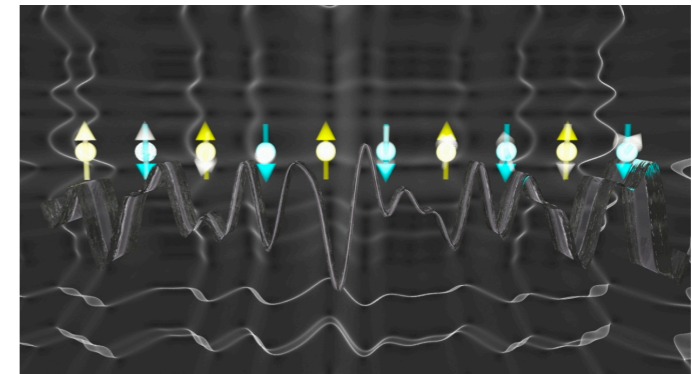
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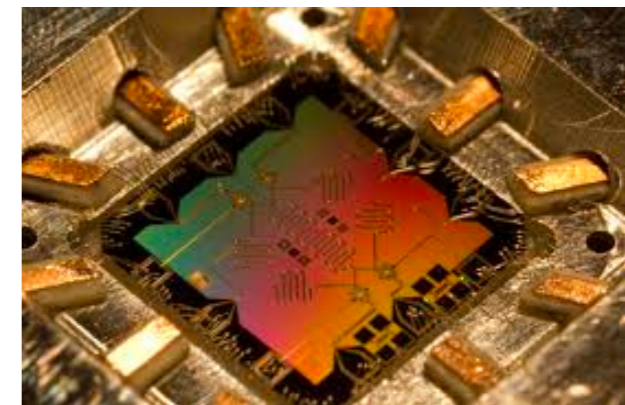
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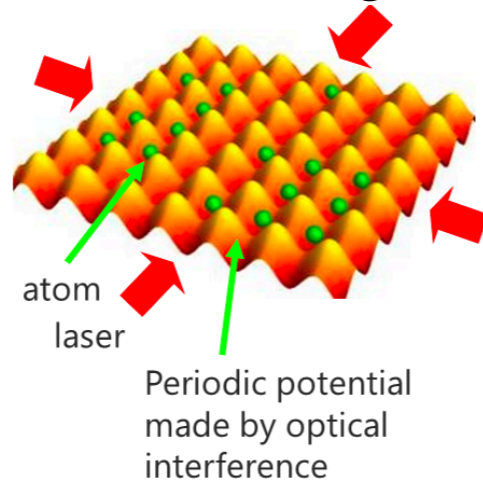
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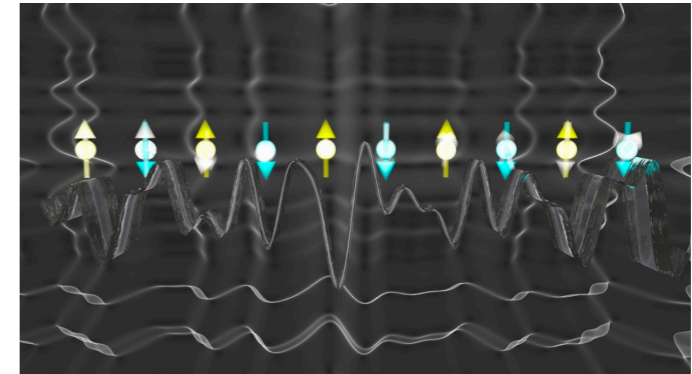
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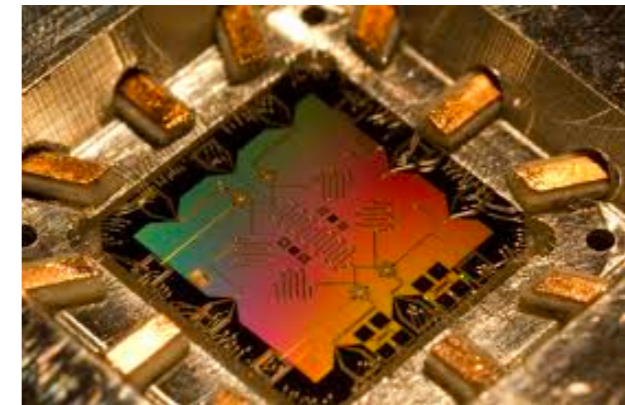
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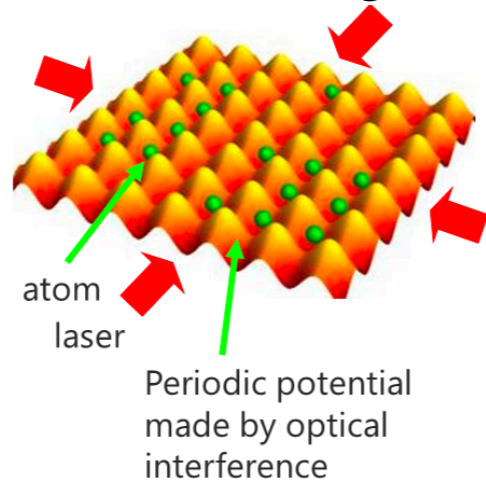
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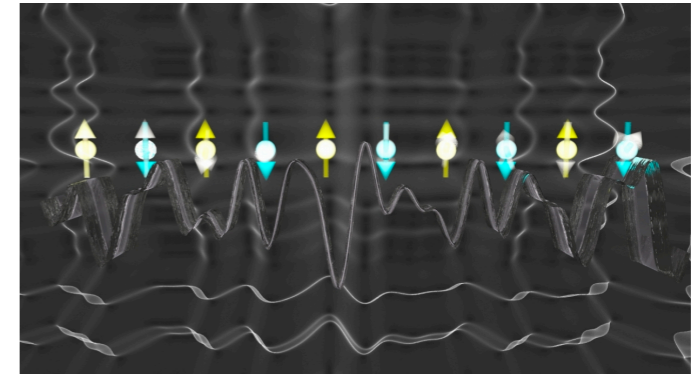
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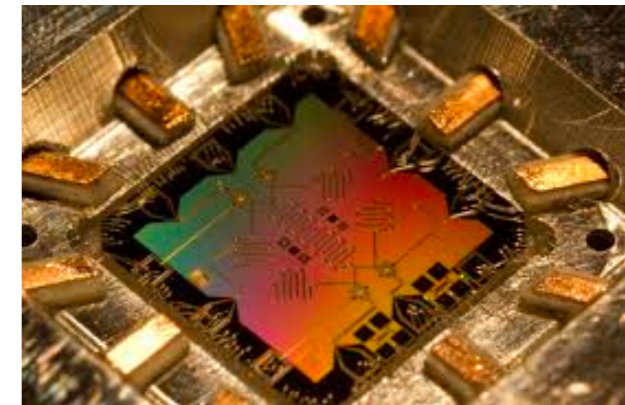
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Lets start by controlling classical systems!

UNPREDICTABILITY OF CHAOS

Classical Chaos: unpredictable deterministic evolution of the classical equations of motion.

In **lecture 1** we reviewed **chaotic dynamics** in maps and Hamiltonian dynamics

Single particle models

Logistic map

Bernoulli map

Kicked rotor

Kicked top

Many-body models

Heisenberg spin chain

d-dimensional hyper cubic spin models

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Lets control each of the models highlighted below!

Many-body models

Heisenberg spin chain

d-dimensional hyper cubic spin models

CANNOT GENERICALLY CONTROL CHAOS.....

We have to exploit structure to control the dynamics.

Generic quantum evolution cannot be controlled or error corrected (without exponential resources)

Exploit classical chaotic structure to control some quantum systems.

HOW DO WE CONTROL CHAOS?

To control classical chaos, have to exploit its underlying structure.

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Use monitoring and feedback to “push” the system onto a **chosen** unstable orbit

Controlling Chaos

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University of Maryland, College Park, Maryland 20742
(Received 22 December 1989)

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VOLUME 64, NUMBER 11

PHYSICAL REVIEW LETTERS

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Physics Letters A

Volume 180, Issues 1-2, 30 August 1993, Pages 99-102



Experimental control of chaos by
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K. Pyragas^{1,2}, A. Tamaševičius

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Computers & Mathematics with Applications
Volume 34, Issues 2-4, July-August 1997, Pages 373-389



Probabilistic control of Chaos: Chaotic maps under control ☆, ☆☆

I. Antoniou^{a b 1}, V. Basios^{c d}, F. Bosco^{e f g 1}

stochastic feedback

APPLICATIONS OF CONTROLLING CHAOS I.

Designing trajectories of space craft travel ("chaotic targeting")

Touring the moons of Jupiter

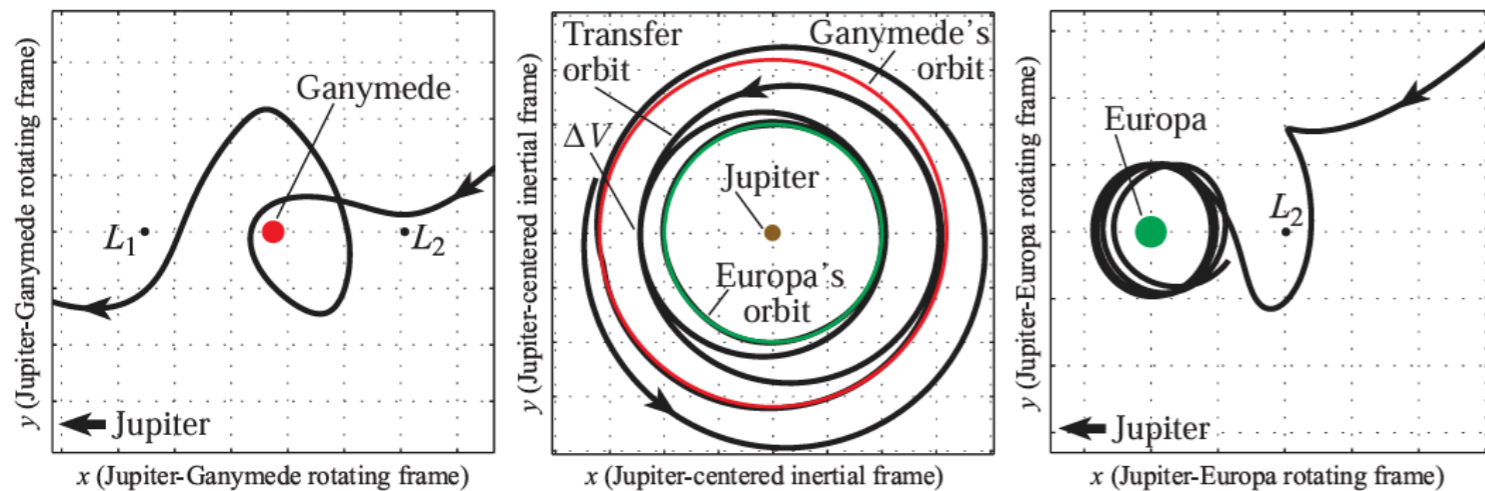


Figure 1.3: The "Petit Grand Tour" space mission concept for the Jovian moons. In our example, we show an orbit coming into the Jupiter system and (a) performing one loop around Ganymede (shown in the Jupiter-Ganymede rotating frame), (b) transferring from Ganymede to Europa using a single impulsive maneuver (shown in the Jupiter-centered inertial frame), and (c) getting captured by Europa (shown in the Jupiter-Europa rotating frame).

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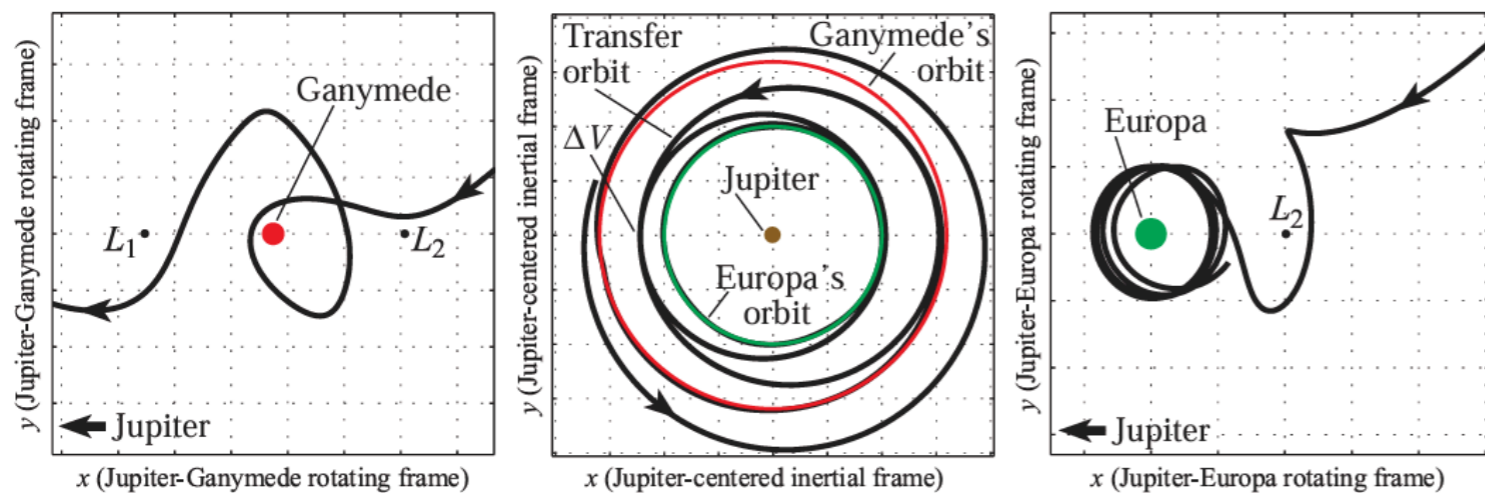
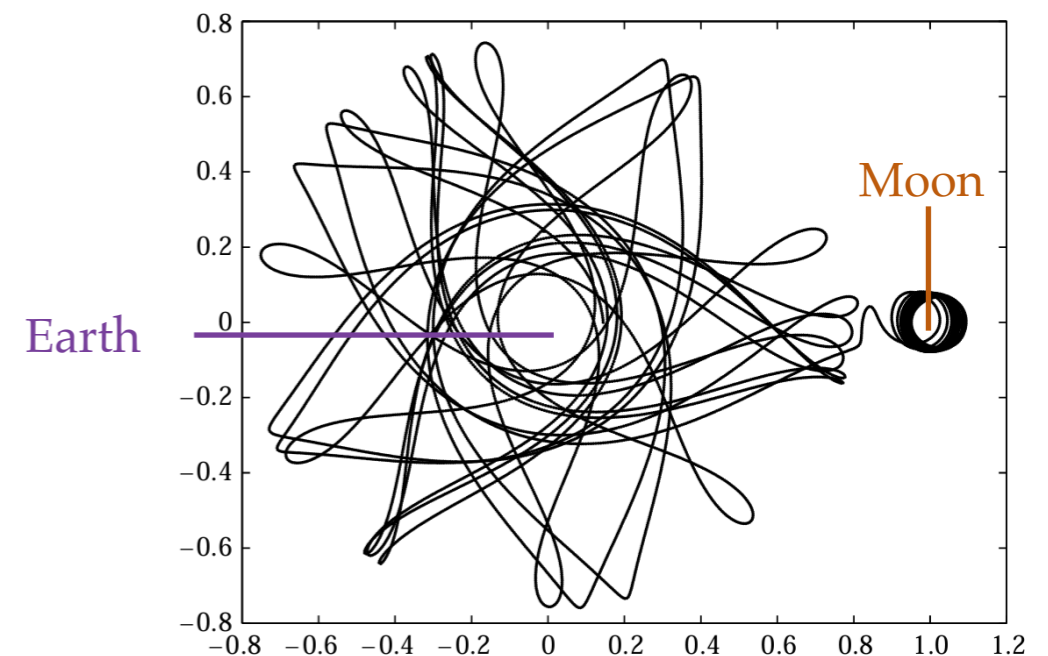


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From earth to the moon



Macau and Grebogi, Royal Society (2006)

Both trajectories minimize fuel usage along the trip

APPLICATIONS OF CONTROLLING CHAOS II.

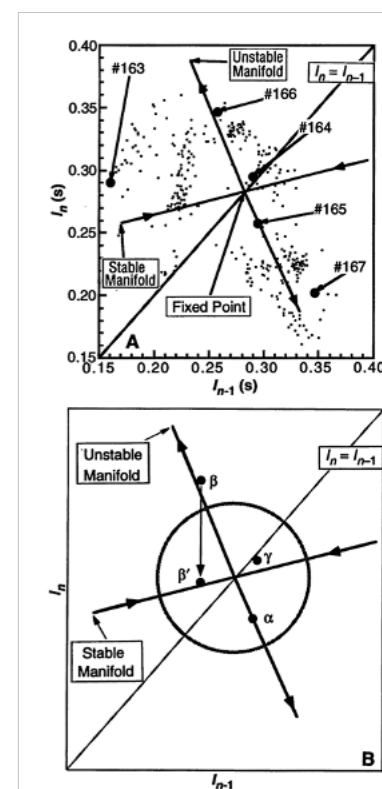
Overcoming heart arrhythmias

Controlling Cardiac Chaos

Alan Garfinkel, Mark L. Spano, William L. Ditto, James N. Weiss

The extreme sensitivity to initial conditions that chaotic systems display makes them unstable and unpredictable. Yet that same sensitivity also makes them highly susceptible to control, provided that the developing chaos can be analyzed in real time and that analysis is then used to make small control interventions. This strategy has been used here to stabilize cardiac arrhythmias induced by the drug ouabain in rabbit ventricle. By administering electrical stimuli to the heart at irregular times determined by chaos theory, the arrhythmia was converted to periodic beating.

SCIENCE • VOL. 257 • 28 AUGUST 1992



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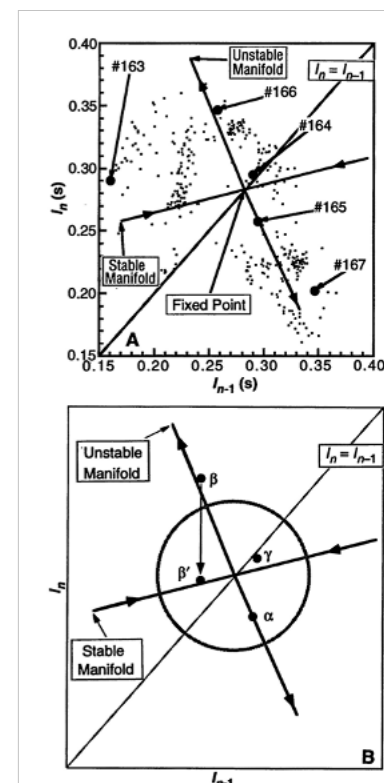
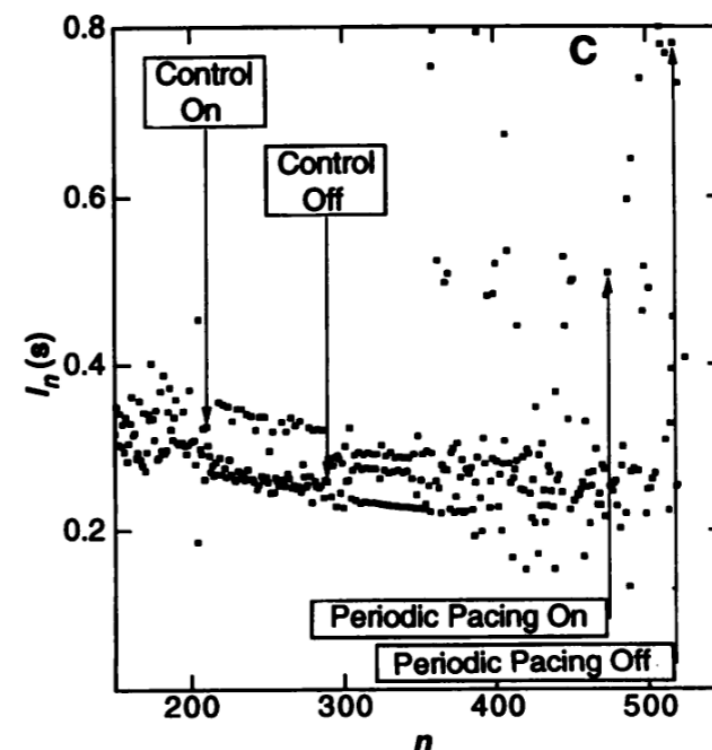
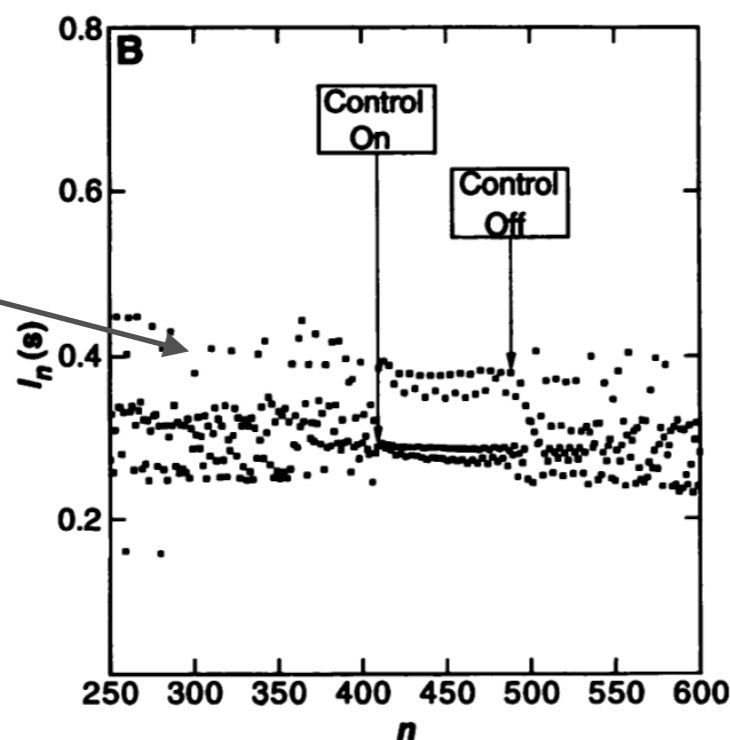
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Chaotic heartbeat



Interbeat interval I_n versus beat number n

CONTROLLING THE BERNOULLI (DOUBLING) MAP

$$x_{n+1} = 2x_n \bmod 1 \quad 0 < x < 1$$

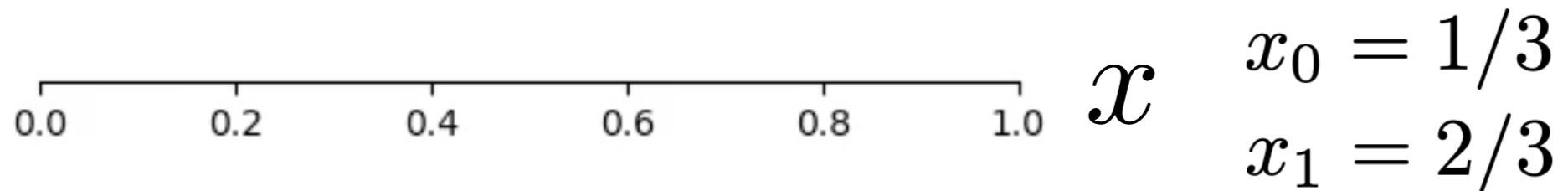
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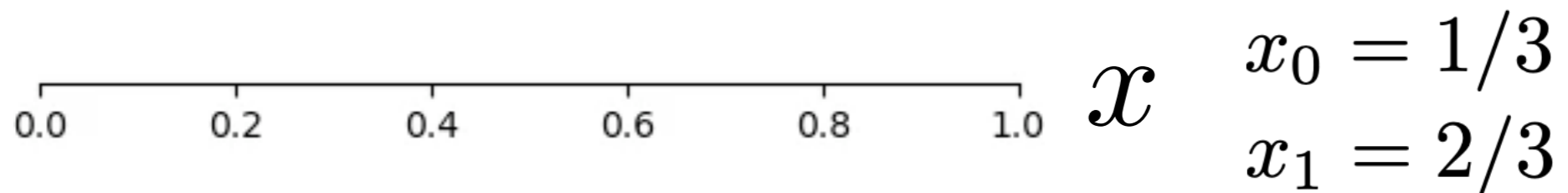


CONTROLLING THE BERNOULLI (DOUBLING) MAP

$$x_{n+1} = 2x_n \pmod{1} \quad 0 < x < 1$$

Starting from an **irrational number**, this map is **chaotic**

The rational numbers yield periodic orbits $x_0 = a/b$



But, the irrationals are dense on the number line:

hence, these sets of rationals are all **unstable orbits** of the map

CONTROLLING THE BERNOULLI (DOUBLING) MAP

Now let's design the control protocol:

define the **control map** C with strength a

$$x_{n+1} = (1 - a)x_j^{\text{orbit}} + ax_n \quad \text{if } x \in \Delta_j$$

Neighborhood of x_j^{orbit}



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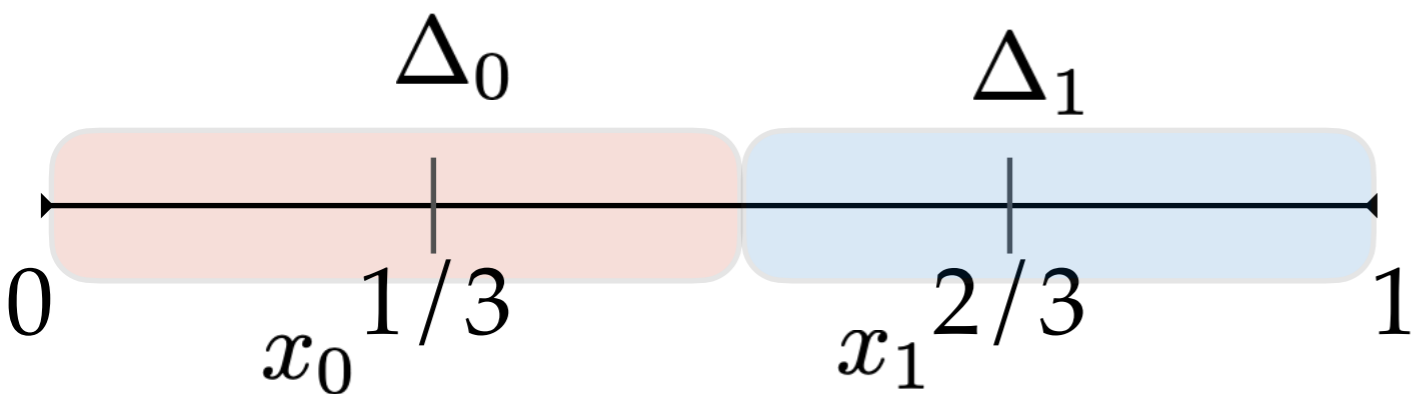
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Lets consider $x_j^{\text{orbit}} = \{1/3, 2/3\} = \{x_0, x_1\}$

$\Delta_0 : (0, 1/2)$ $\Delta_1 : (1/2, 1)$



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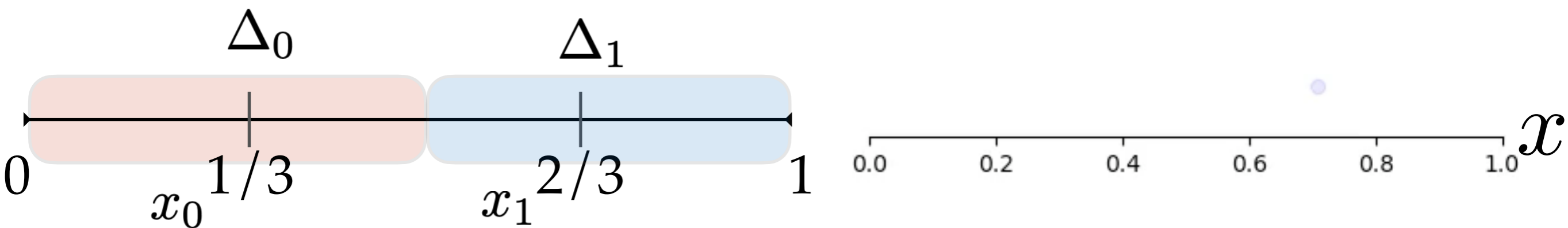
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B Bernoulli map

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Now put this together **stochastically** applying one or the other map

Apply **Bernoulli map** with probability $1 - p_{\text{ctrl}}$

Apply **Control map** with probability p_{ctrl}

CONTROLLING THE BERNOULLI (DOUBLING) MAP

To understand the phase diagram lets go through this stochastic process.

For simplicity control onto $x_j = 0$

Write the number x in binary representation: $x = \sum_{j=1}^{\infty} y_j 2^{-j}$

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$$B(x) = \sum_{j=1}^{\infty} y_j 2^{-(j-1)} = \sum_{j=1}^{\infty} y_{j+1} 2^{-j}$$

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The **control map** $C(x) = ax$ with $|a| < 1$ taking $a = 2^{-n}$ has the action

$$C(x) = \sum_{j=1}^{\infty} y_j 2^{-(j+n)} = \sum_{j=1+n}^{\infty} y_{j-n} 2^{-j}$$

CONTROLLING THE BERNOULLI (DOUBLING) MAP

The **Bernoulli map** has the action: $B(x) = \sum_{j=1}^{\infty} y_{j+1} 2^{-j}$

Control map has the action $C(x) = \sum_{j=1+n}^{\infty} y_{j-n} 2^{-j}$

$$x = \sum_{j=1}^{\infty} y_j 2^{-j}$$

To find when a bit string's chaos begins, consider **the first domain wall (FDW)**

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A periodic orbit will have a repeating decimal expansion $y_j = y_{j+N}$ for $N \geq 1$

When this pattern *first* breaks for an irrational number we call the first domain wall.

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The **Control map** acts with probability p and moves the domain wall to the **Right** by n as $y_N \rightarrow y_{N-n}$

CONTROLLING THE BERNOULLI (DOUBLING) MAP

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Control map has the action $C(x) = \sum_{j=1+n}^{\infty} y_{j-n} 2^{-j}$

$$x = \sum_{j=1}^{\infty} y_j 2^{-j}$$

The **Bernoulli map** acts with probability $1 - p$ and moves the domain wall to the **Left** by one $y_N \rightarrow y_{N+1}$

The **Control map** acts with probability p and moves the domain wall to the **Right** by n as $y_N \rightarrow y_{N-n}$

The velocity of the first domain wall $v_{dw} = np - (1 - p) = -\frac{\log(a)}{\log(2)}p - (1 - p)$

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DYNAMICAL PHASE DIAGRAM OF BERNOULLI+CONTROL

B Apply **Bernoulli map** with probability $1 - p_{\text{ctrl}}$

C Apply **Control map** with probability p_{ctrl}

Antoniou, V. Basios, and F. Bosco,
Int. J. Bifurcation Chaos 06, 1563 (1996) & (1998).

Find a transition when $\lambda = 0$

$$|x'_n - x_n| \sim e^{\lambda t}$$

$$\lambda = (1 - p_{\text{ctrl}}) \log(2) + p_{\text{ctrl}} \log(a)$$

$$\lambda = -\log(2)v_{dw}$$

$$p_c(a) = \frac{\log 2}{\log 2 - \log a}$$

DYNAMICAL PHASE DIAGRAM OF BERNOULLI+CONTROL

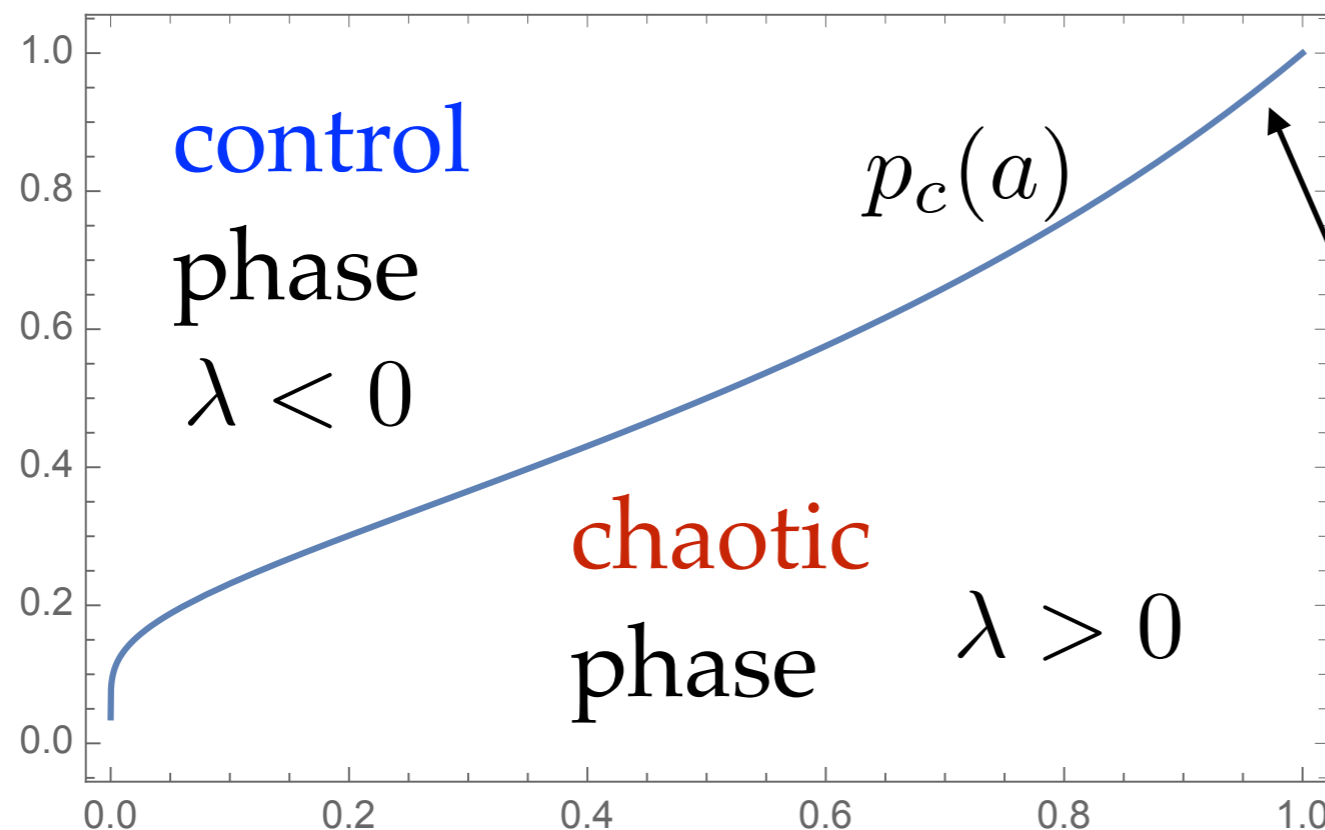
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a

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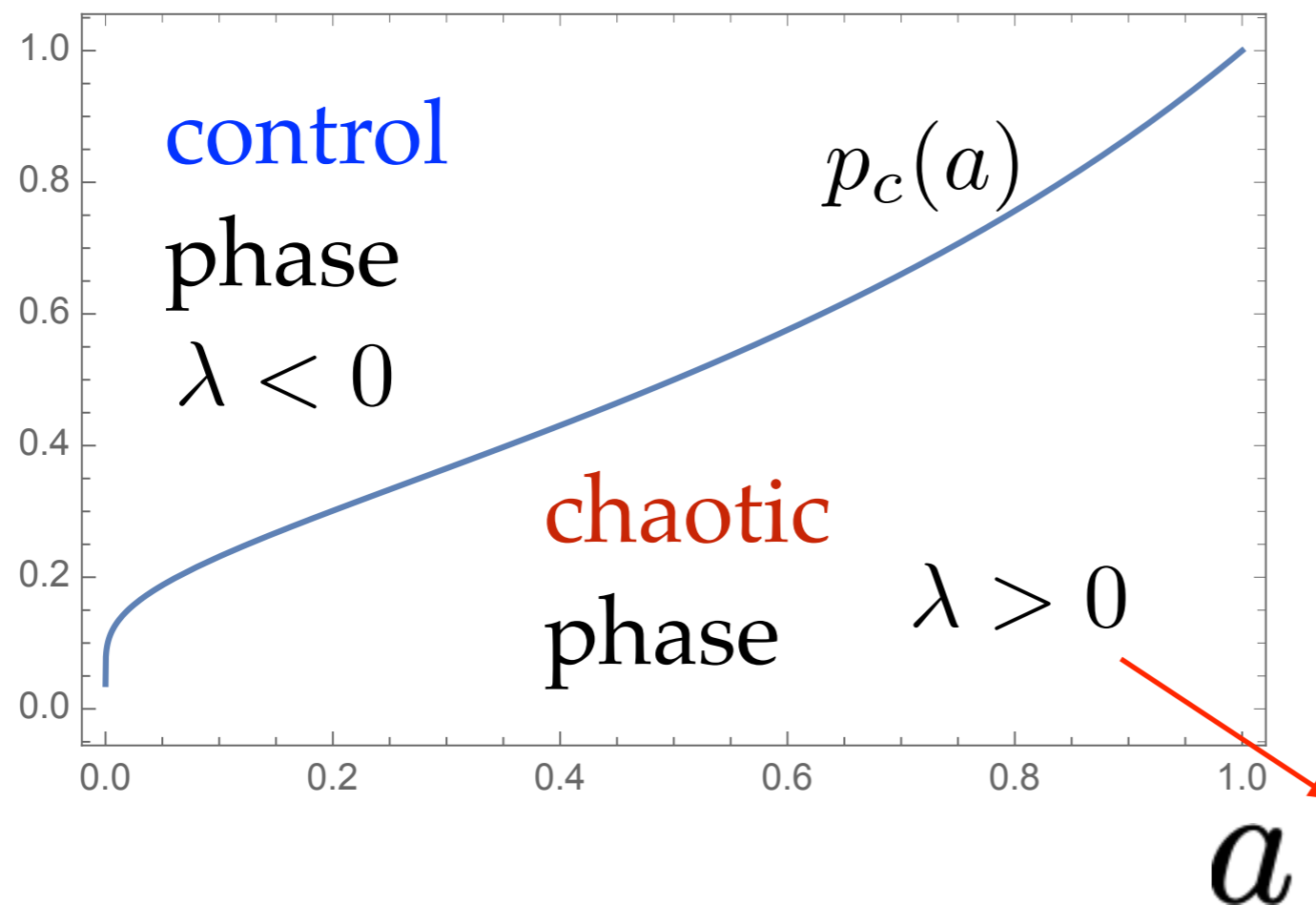
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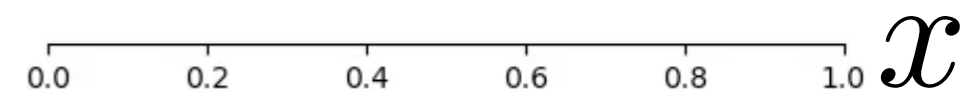


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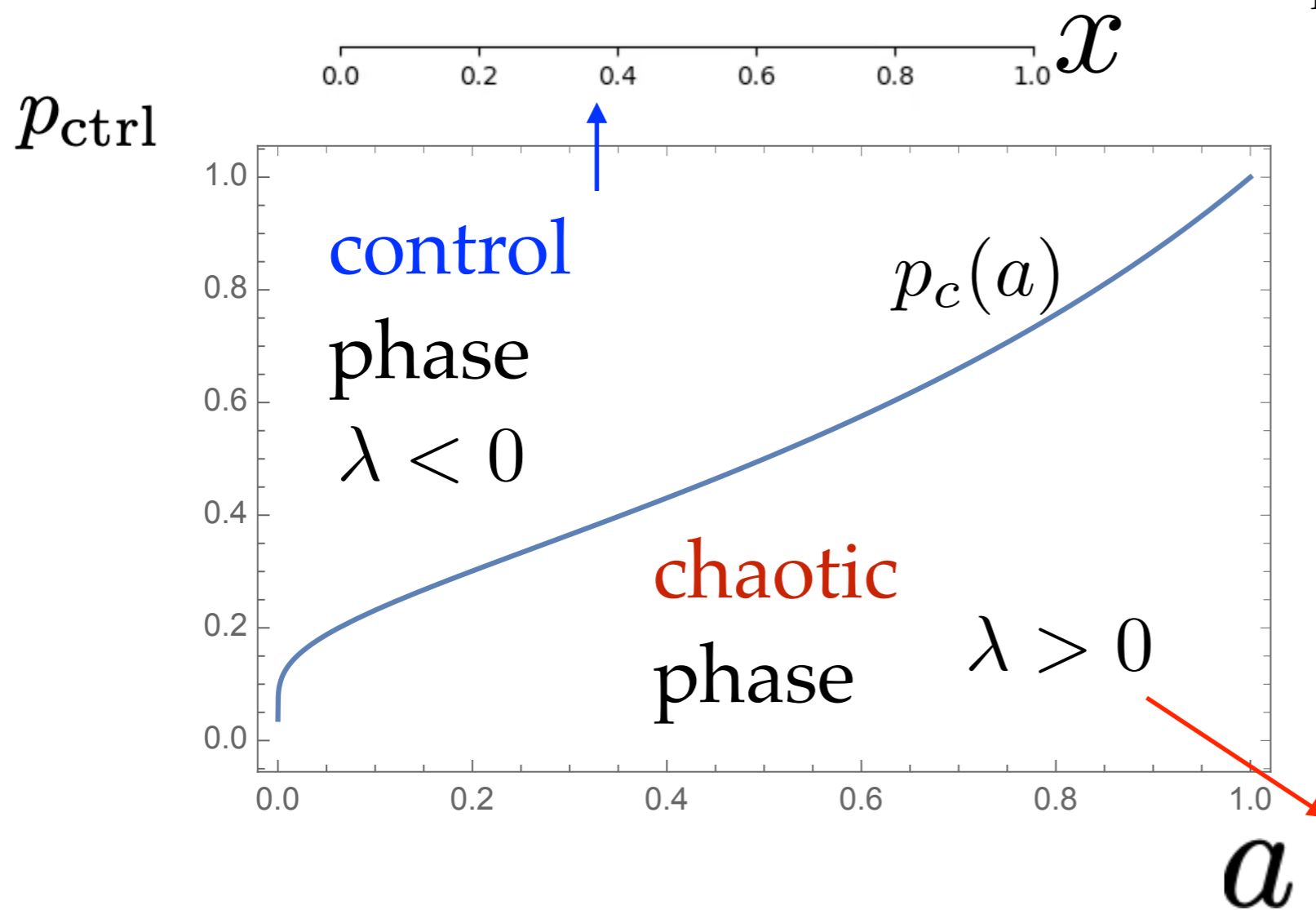
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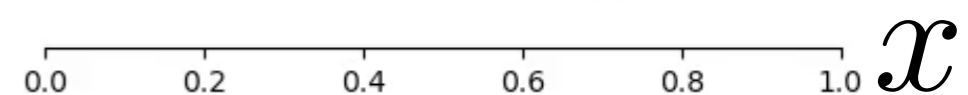


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CLASSICAL CONTROL TRANSITION: RECIPE

How do we control chaos?

Classical Chaos: exponential
Sensitivity to initial conditions

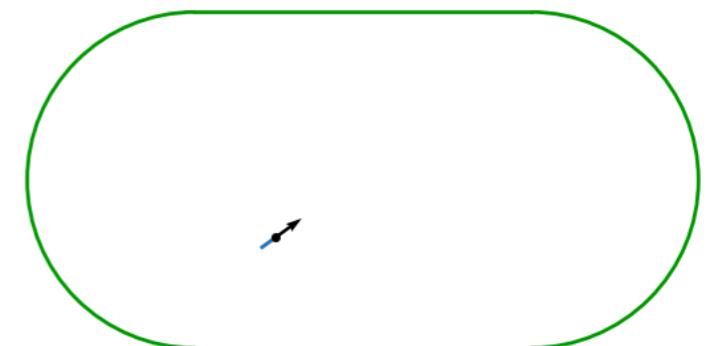
$$\|x(t) - x'(t)\| \sim e^{t\lambda_{\text{Lyapunov}}}$$



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Wikipedia

CLASSICAL CONTROL TRANSITION: RECIPE

How do we control chaos?

Take a chaotic dynamical system.

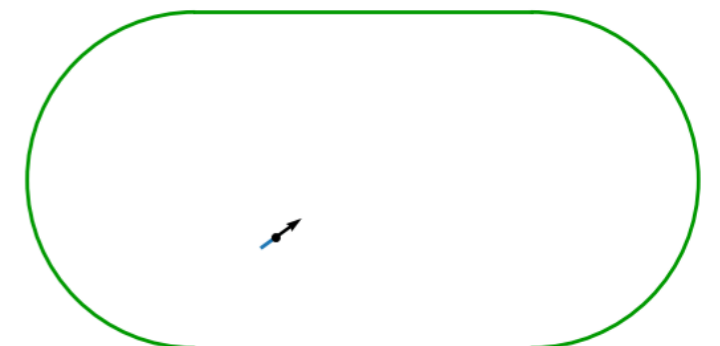
Identify an unstable orbit: an orbit that is stable under the chaotic dynamics but any infinitesimal perturbation will drive the system out of the orbit.



Google



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Wikipedia

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CLASSICAL CONTROL TRANSITION: RECIPE

How do we control chaos?

Take a chaotic dynamical system.

Identify an unstable orbit: an orbit that is stable under the chaotic dynamics but any infinitesimal perturbation will drive the system out of the orbit.

Design a control protocol:

Interleave operations that push the dynamical system onto a specified unstable orbit.



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Wikipedia

LETS NOW APPLY THIS TO THE KICKED TOP

Recall from Lecture 1

$$H_{\text{KT}} = \alpha J_y + \frac{k J_z^2}{2S} \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\alpha = \pi/(2T)$$

$$\{J_i, J_j\} = i\epsilon_{ijk} J_k$$

Conserved angular momentum

$$\{\mathbf{J}^2, H\} = 0$$

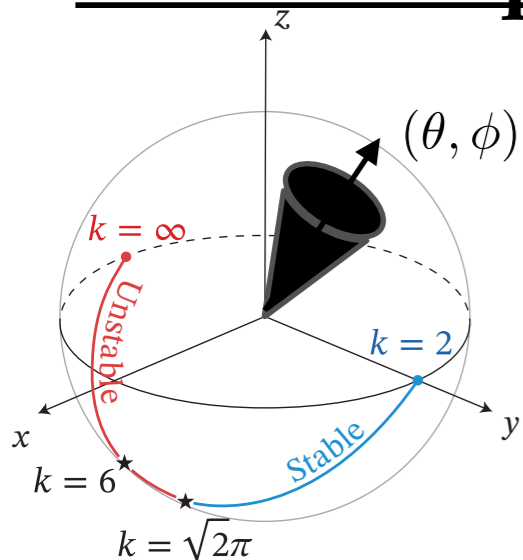
Equation of Motion

$$\frac{d\mathbf{J}}{dt} = \{\mathbf{J}, H\} = -\mathbf{J} \times \frac{\partial H}{\partial \mathbf{J}}$$

Classical limit $S \rightarrow \infty$

$$x = J_x/S, y = J_y/S, z = J_z/S$$

Kicked top



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$$x_{n+1} = z_n \cos(kx_n) + y_n \sin(kx_n)$$

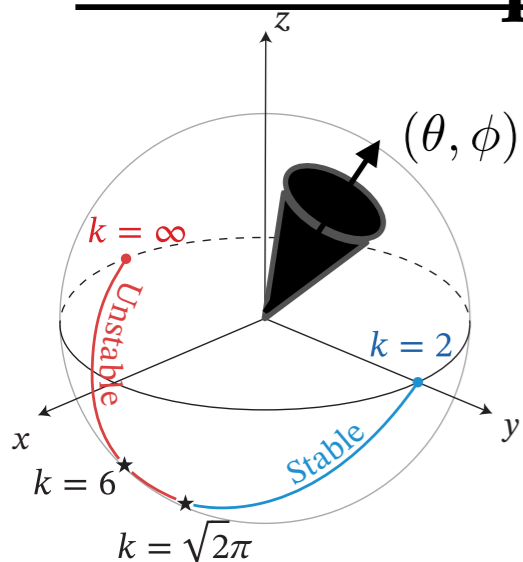
$$\mathbf{r}_n = (x_n, y_n, z_n)$$

$$y_{n+1} = -z_n \sin(kx_n) + y_n \cos(kx_n)$$

$$\mathbf{r}_{n+1} = f(\mathbf{r}_n)$$

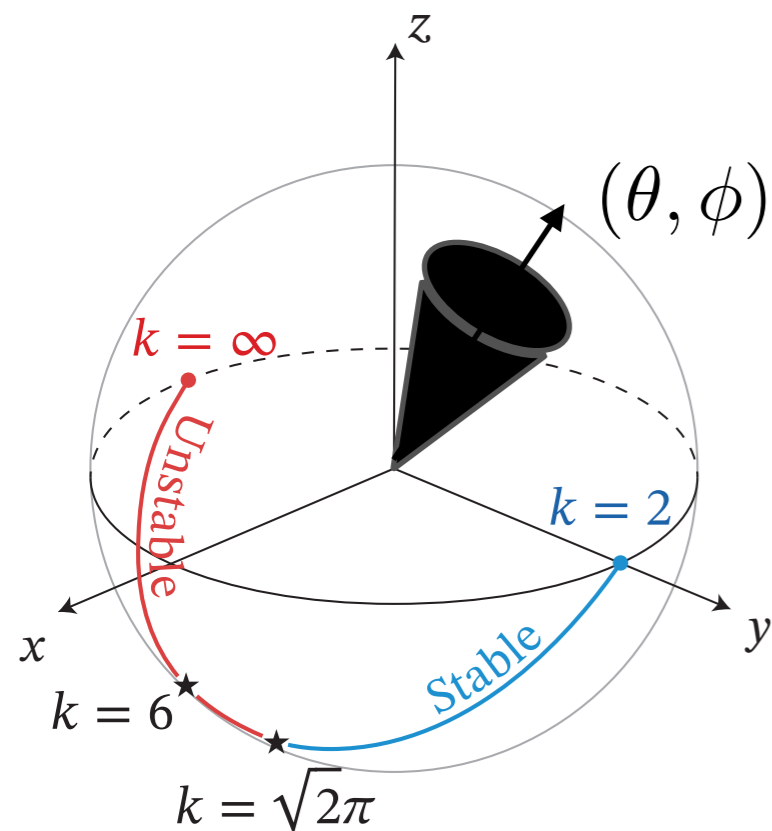
$$z_{n+1} = -x_n$$

Kicked top



FIRST FIND UNSTABLE FIXED POINTS OF THE KICKED TOP

Recall from Lecture 1



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$$\alpha = \pi / (2T)$$

fixed points of the map

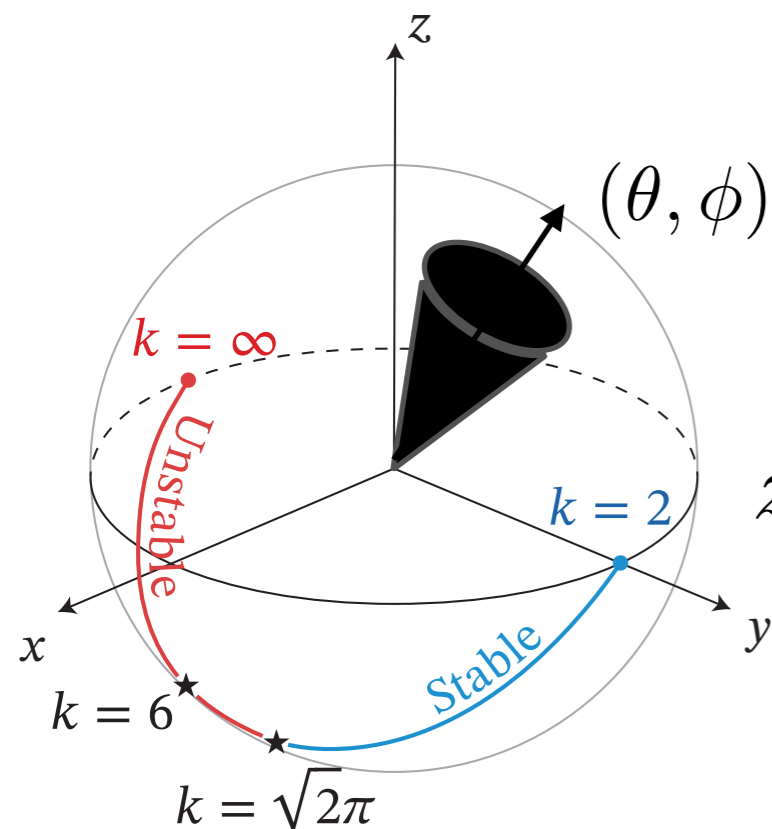
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$$x_{n+1} = x_n = x_0$$

$$z_0 = -x_0$$

$$y_0 = x_0 \sin(kx_0) + y_0 \cos(kx_0)$$

$$y_0/x_0 = \sin(kx_0)/(1 - \cos(kx_0)) = \cot(kx_0/2)$$

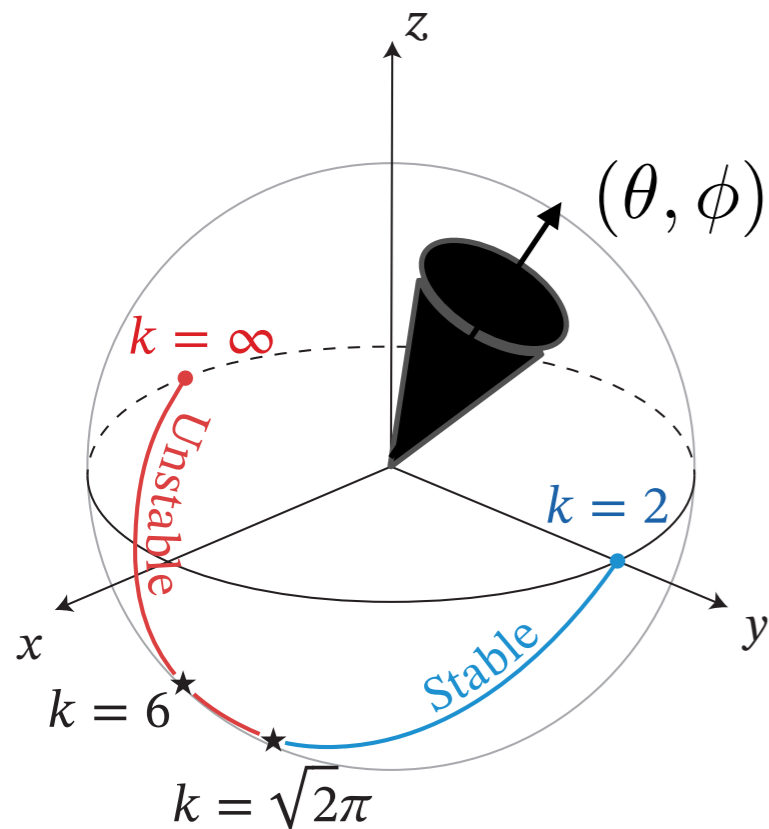
$$x_0^2 + y_0^2 + z_0^2 = 1$$

$$x_0^2 = \frac{\sin^2\left(\frac{kx_0}{2}\right)}{1 + \sin^2\left(\frac{kx_0}{2}\right)}$$

Linearize about x_0 to find the unstable fixed points

FIRST FIND UNSTABLE FIXED POINTS OF THE KICKED TOP

Recall from Lecture 1



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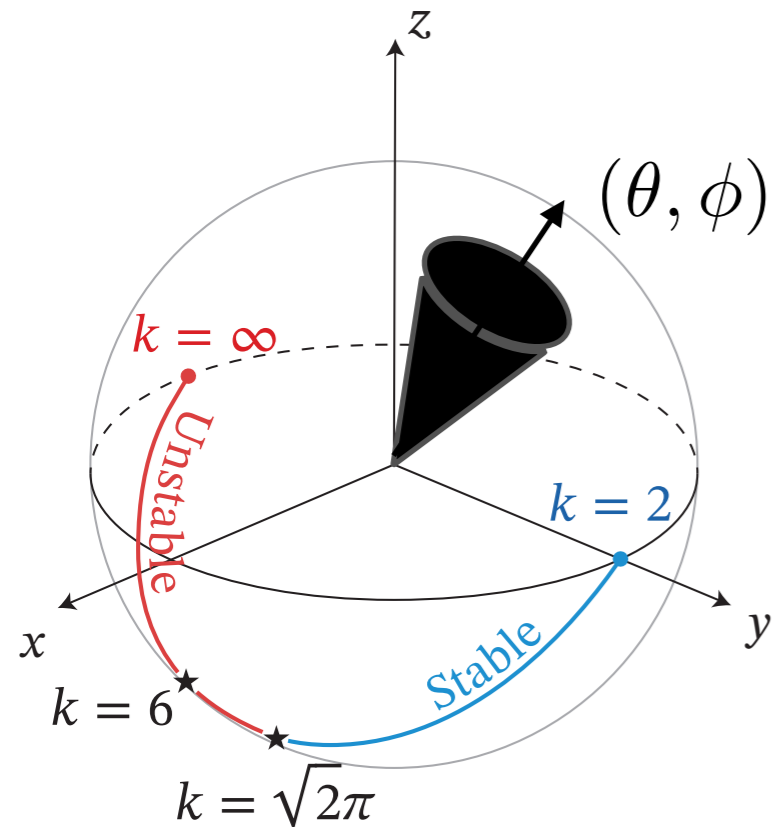
$$\begin{bmatrix} \delta x' \\ \delta y' \\ \delta z' \end{bmatrix} = \begin{bmatrix} kx_0 \cot(kx_0/2) & \sin(kx_0) & \cos(kx_0) \\ -kx_0 & \cos(kx_0) & -\sin(kx_0) \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Eigenvalues 1 & $\lambda_{\pm}(k) = -\left[h(k) \pm \sqrt{h(k)^2 - 1}\right]$

$$h(k) = \sin^2(kx_0/2) - \frac{kx_0}{2} \cot(kx_0/2)$$

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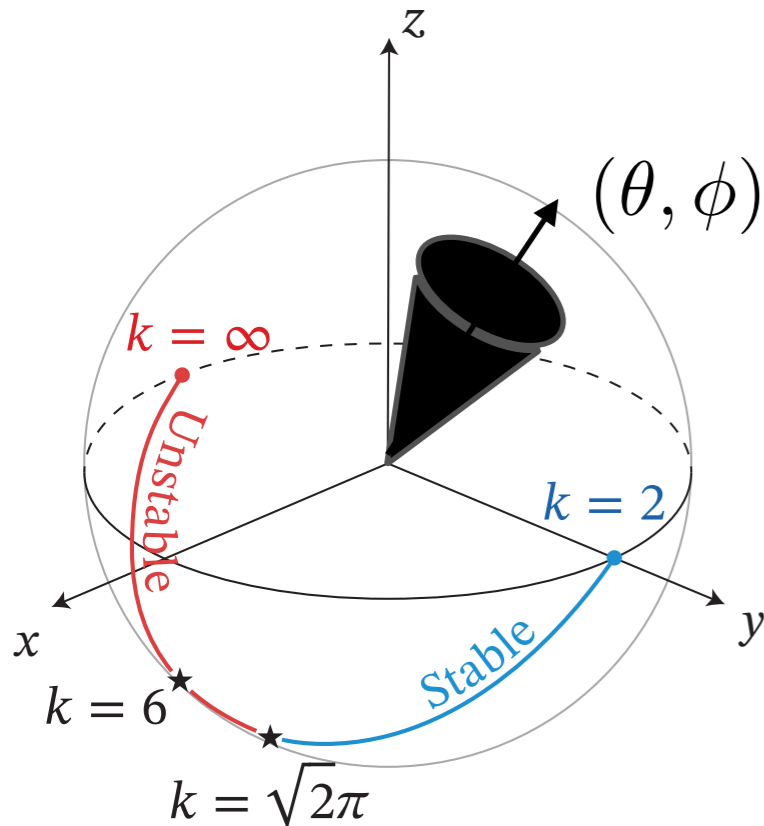
$$h(k) = \sin^2(kx_0/2) - \frac{kx_0}{2} \cot(kx_0/2)$$

Becomes unstable when $h(k) = 1$ $\sin(kx_0/2) = 1$

$$kx_0/2 = \pi/2 \quad x_0^2 = 1/2 \quad k_c = \sqrt{2}\pi \approx 4.44$$

For $k > k_c$ we have found that x_0 is unstable fixed point

DEFINE A CONTROL MAP FOR TO THE KICKED TOP



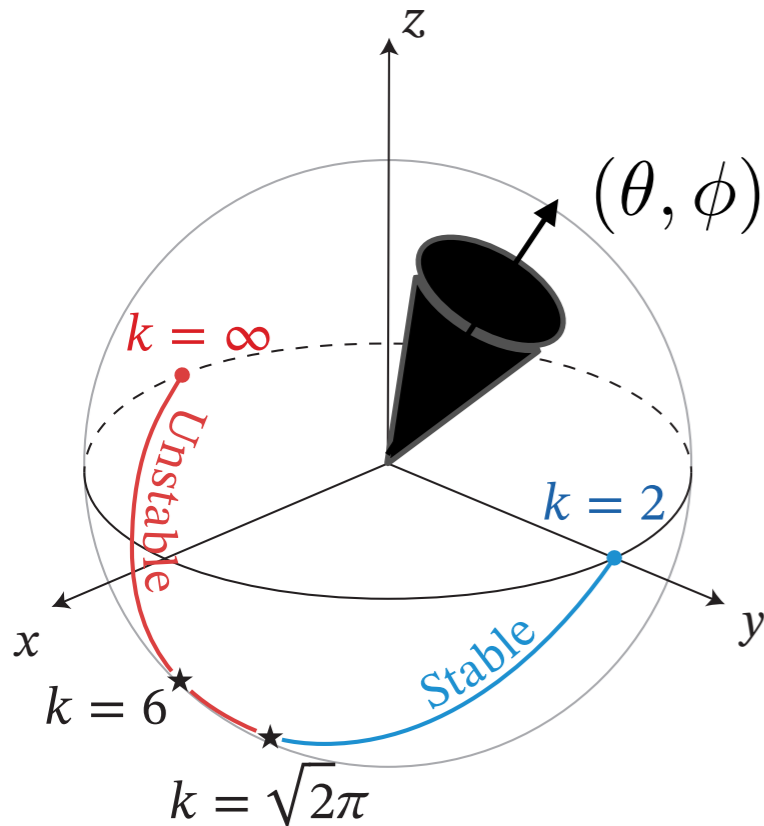
For $k > k_c$ we have found
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$$x_0^2 = \frac{\sin^2\left(\frac{kx_0}{2}\right)}{1 + \sin^2\left(\frac{kx_0}{2}\right)} \quad z_0 = -x_0 \quad y_0 = x_0 \sin(kx_0) + y_0 \cos(kx_0)$$

$$\mathbf{r}_0 = (x_0, y_0, z_0)$$

Lets define the control map to push the top
towards the unstable fixe point on the sphere

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$$\mathbf{r} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

$$\mathbf{r}(t+1) = \frac{a \mathbf{r}(t) + (1-a) \mathbf{r}_0}{|a \mathbf{r}(t) + (1-a) \mathbf{r}_0|}, \quad \mathbf{r}(t) \in \Omega$$

$$\mathbf{r}_0 \in \Omega$$

$$\theta(t+1) = a\theta(t) + (1-a)\theta_0$$

$$\phi(t+1) = a\phi(t) + (1-a)\phi_0$$

MODEL FOR CHAOS AND CONTROL IN THE KICKED TOP

Can now define the model

(Switch from n to t)

Apply the kicked top dynamics with probability $1 - p$ $f(\mathbf{r}(t)) = \mathbf{r}(t + 1)$

$$x_{n+1} = z_n \cos(kx_n) + y_n \sin(kx_n)$$

$$y_{n+1} = -z_n \sin(kx_n) + y_n \cos(kx_n)$$

$$z_{n+1} = -x_n$$

Apply the control map with probability p

$$g(\mathbf{r}(t)) = \mathbf{r}(t + 1)$$

$$\theta(t + 1) = a\theta(t) + (1 - a)\theta_0$$

$$\phi(t + 1) = a\phi(t) + (1 - a)\phi_0$$

$$\mathbf{r}_0 = (x_0, y_0, z_0)$$

$$\mathbf{r}(t + 1) = \begin{cases} \mathbf{f}(\mathbf{r}(t)), & \text{with probability } 1 - p, \\ \mathbf{g}(\mathbf{r}(t)), & \text{with probability } p. \end{cases}$$

PHASE DIAGRAM FOR CHAOS AND CONTROL IN THE KICKED TOP

Determine the phase diagram by computing the Lyapunov exponent

$$\mathbf{r}(t+1) = \begin{cases} \mathbf{f}(\mathbf{r}(t)), & \text{with probability } 1-p, \\ \mathbf{g}(\mathbf{r}(t)), & \text{with probability } p. \end{cases}$$

Linearize about the fixed point $\bar{\mathbf{r}}(t) = \bar{\mathbf{r}}_0 + \delta\mathbf{r}(t)$

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$$\delta\mathbf{r}(t) = u(t)\mathbf{v}_1 + w(t)\mathbf{v}_2$$


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Leading Lyapunov
of kicked top

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A given realization at time T , with N control maps and $T-N$ kicked top maps

$$|u_p(t)| = |u(0)|(1-p)^{T-N} |\lambda(k)|^{T-N} |a|^N p^N$$

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$$\mu(k) \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \left\langle \log \frac{|u(t)|}{|u(0)|} \right\rangle = \log |\lambda(k)|(1-p) + |a|p$$

PHASE DIAGRAM FOR CHAOS AND CONTROL IN THE KICKED TOP

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$$p_c = \frac{\log |\lambda(k)|}{\log |\lambda(k)| - \log(a)}$$

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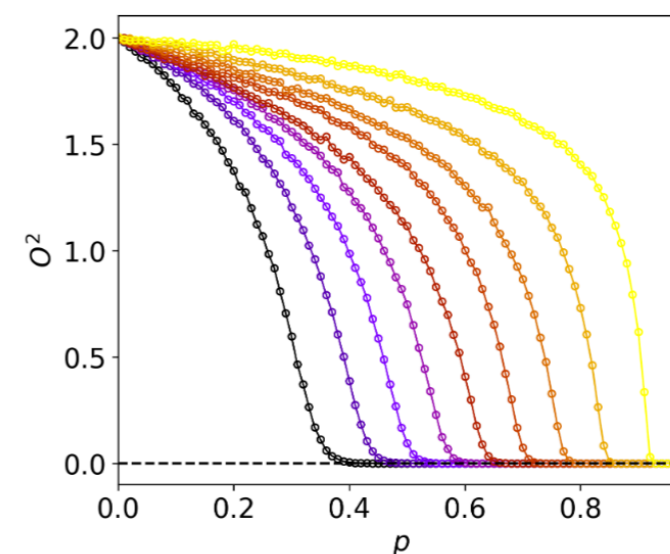
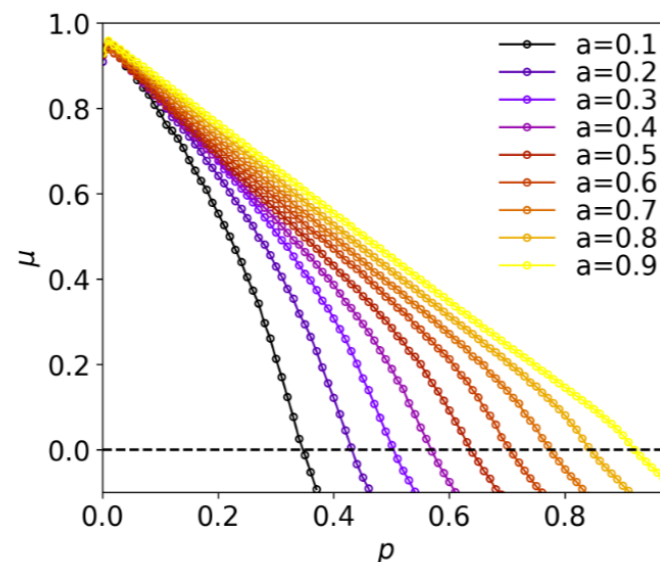
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Probe the transition via the Lyapunov and the deviation from the fixed point

$$\delta \mathbf{r}(t) = \mathbf{r}(t) - \mathbf{r}_0$$

$$O^2 = \overline{|\delta \mathbf{r}(t \rightarrow \infty)|^2},$$



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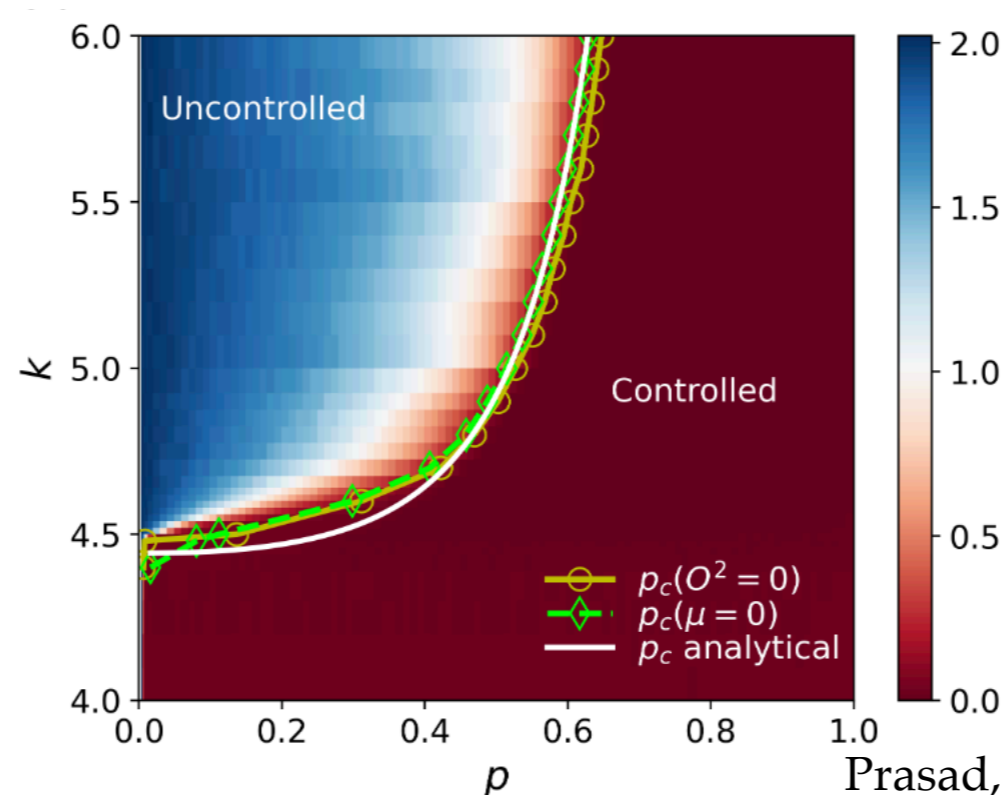
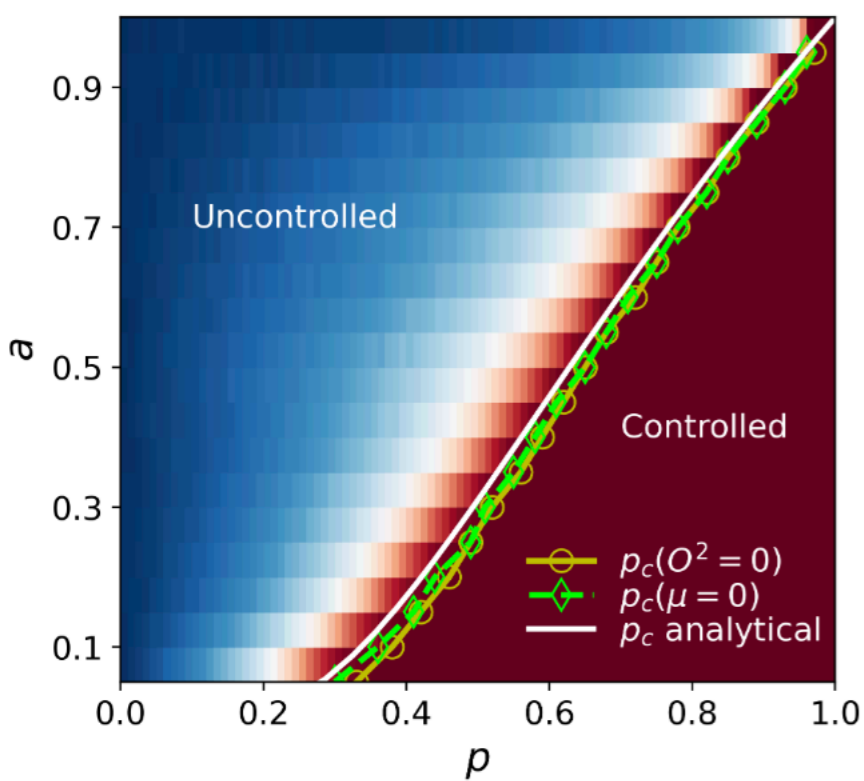
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$$k_c = \sqrt{2}\pi \approx 4.44$$



EXTEND THIS PARADIGM TO CONTROL MANY BODY CLASSICAL CHAOS

Lets control the Heisenberg spin chain

Heisenberg spin chain [O(3) vector. \mathbf{S}_i]

$$H = -J \sum_{i=1}^L (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + S_i^z S_{i+1}^z)$$

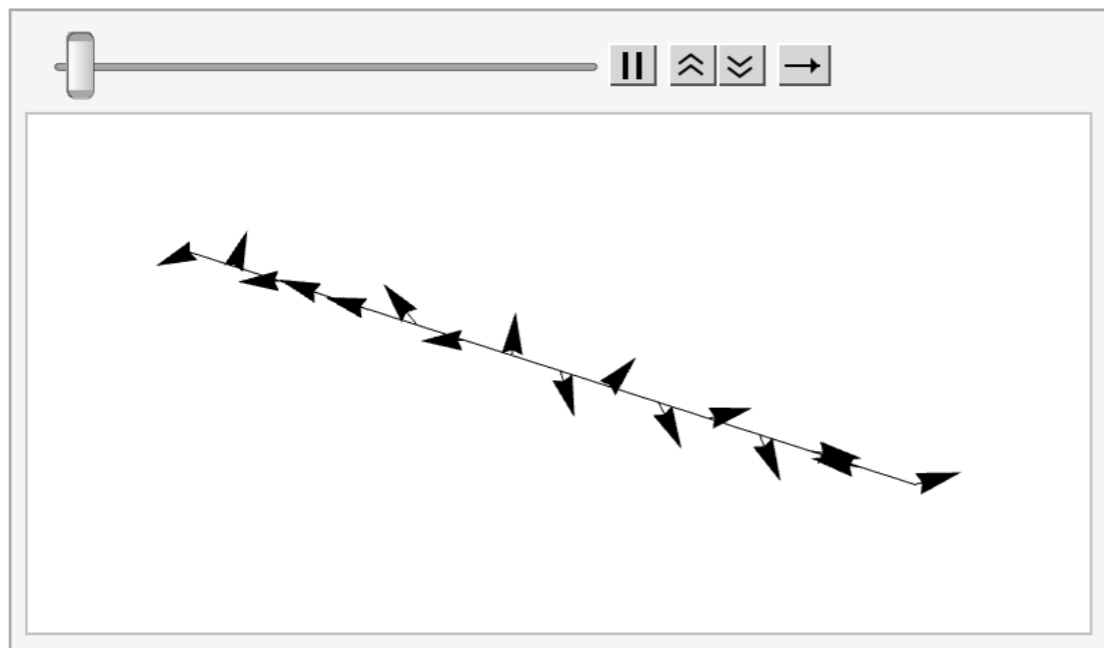
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Dynamics is chaotic $\frac{d\mathbf{S}_j}{d\tau} = -J(\mathbf{S}_{j-1} + \mathbf{S}_{j+1}) \times \mathbf{S}_j.$



EXTEND THIS PARADIGM TO CONTROL MANY BODY CLASSICAL CHAOS

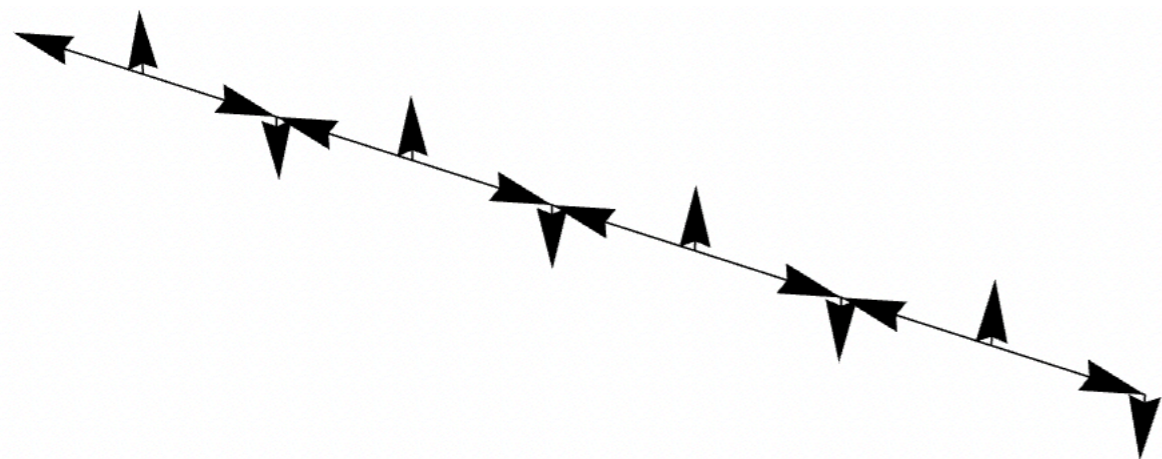
Lets control the Heisenberg spin chain

$$H = -J \sum_{i=1}^L (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + S_i^z S_{i+1}^z) \quad [\text{O}(3) \text{ vector. } \mathbf{S}_i]$$

$$\frac{d\mathbf{S}_j}{d\tau} = -J(\mathbf{S}_{j-1} + \mathbf{S}_{j+1}) \times \mathbf{S}_j.$$

Control the chaotic dynamics:
Find an **unstable fixed point** of the map

For the **spiral** $\frac{d\mathbf{S}_j^0}{d\tau} = 0$



$$\mathbf{S}_j^0 = (0, \cos(\pi j/2), \sin(\pi j/2))$$

Change coordinates

$$S_j^x = s_j, \quad S_j^y = f(s_j) \cos(\phi_j), \quad S_j^z = f(s_j) \sin(\phi_j)$$

$$\phi_j^0 = 2\pi j/N$$

$$s_j^0 = 0$$

Small deviations: $\phi_j = \phi_j^0 + \delta\phi_j$

$$s_j = s_j^0 + \delta s_j$$

EXTEND THIS PARADIGM TO CONTROL MANY BODY CLASSICAL CHAOS

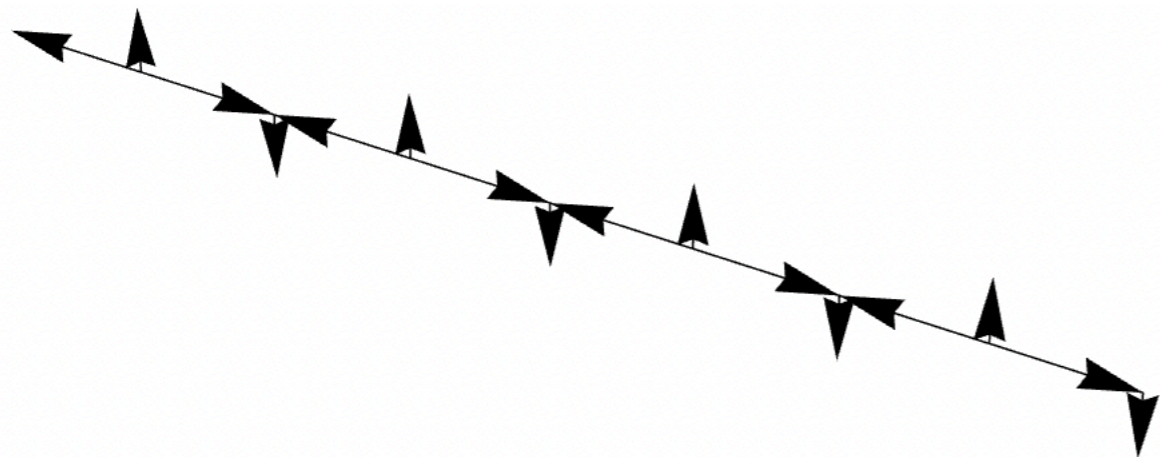
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Any spiral will do but $N=4$ is special

For the **spiral** $\phi_j^0 = 2\pi j/N$ $\frac{d\mathbf{S}_j^0}{d\tau} = 0$
 $s_j^0 = 0$

Small deviations: $\phi_j = \phi_j^0 + \delta\phi_j$
 $s_j = s_j^0 + \delta s_j$

$$\frac{d\delta\phi_j}{dt} = -J(\delta s_{j-1} + \delta s_{j+1} - 2 \cos(2\pi/N)\delta s_j)$$

$$\frac{d\delta s_j}{dt} = J \cos(2\pi/N) (\delta\phi_{j+1} + \delta\phi_{j-1} - 2\delta\phi_j)$$

Unstable to leading order (unless $N=2$ or 4)

PUT THEM TOGETHER: CHAOS AND CONTROL COMPETE

one time step = chaotic time evolution up to $1/J$ and then a control operation (pushes the system towards the fixed point)

Chaotic dynamics

Evolve the system following

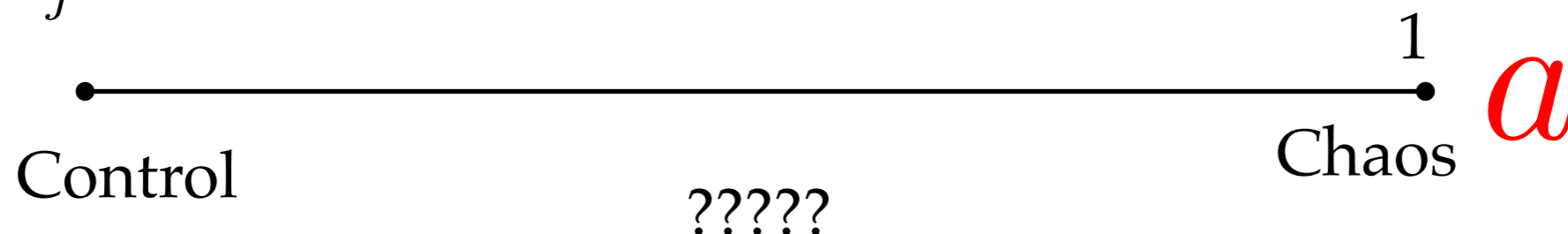
$$\frac{d\mathbf{S}_j}{d\tau} = -J(\mathbf{S}_{j-1} + \mathbf{S}_{j+1}) \times \mathbf{S}_j.$$

Up to a time $\tau_F = 1/J$

Control the chaotic dynamics with strength a :

Control onto the spiral at all sites at once (global control) $\mathbf{S}_j(\tau') \rightarrow \frac{(1-a)\mathbf{S}_j^0 + a\mathbf{S}_j(\tau_F)}{|(1-a)\mathbf{S}_j^0 + a\mathbf{S}_j(\tau_F)|}$

$$\phi_j^0 = \pi j/2 \quad s_j^0 = 0$$



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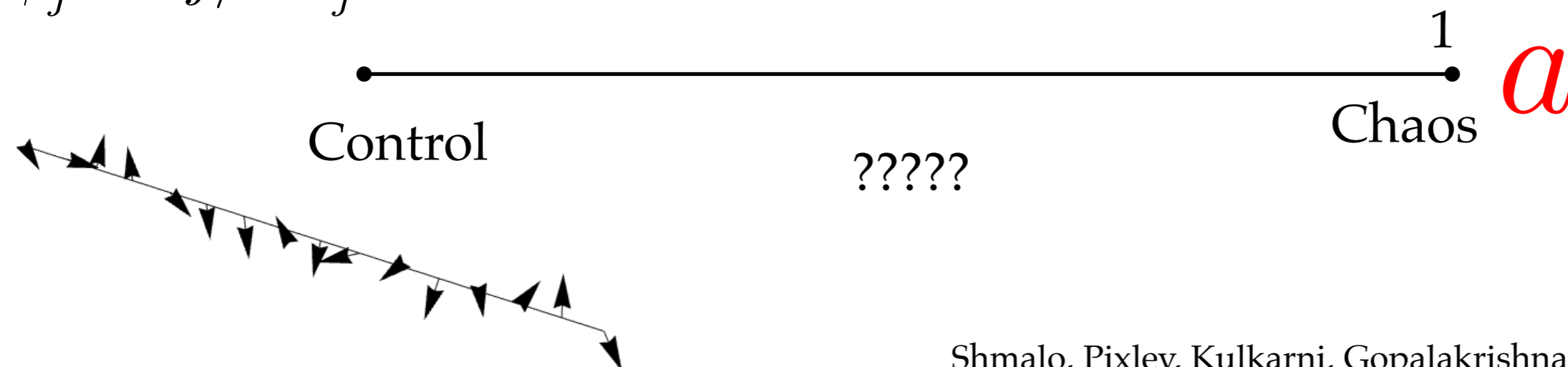
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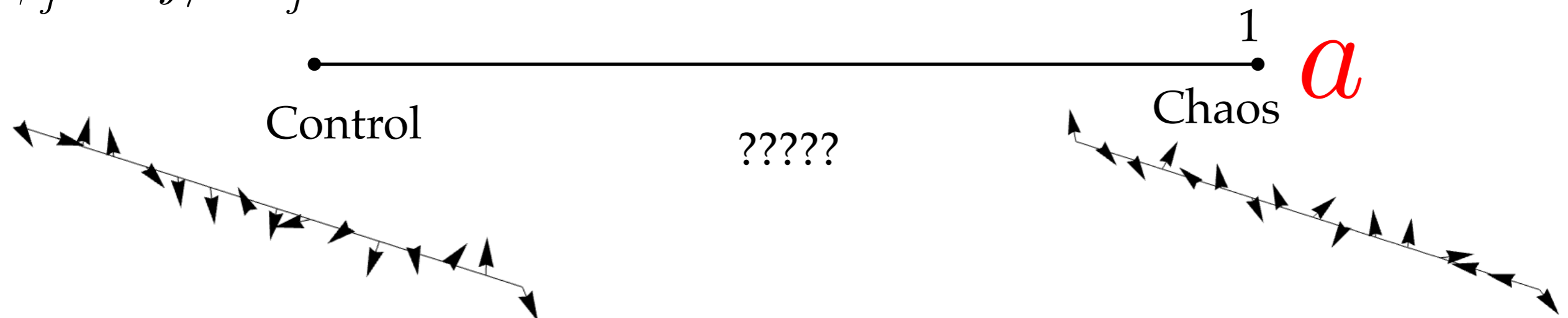
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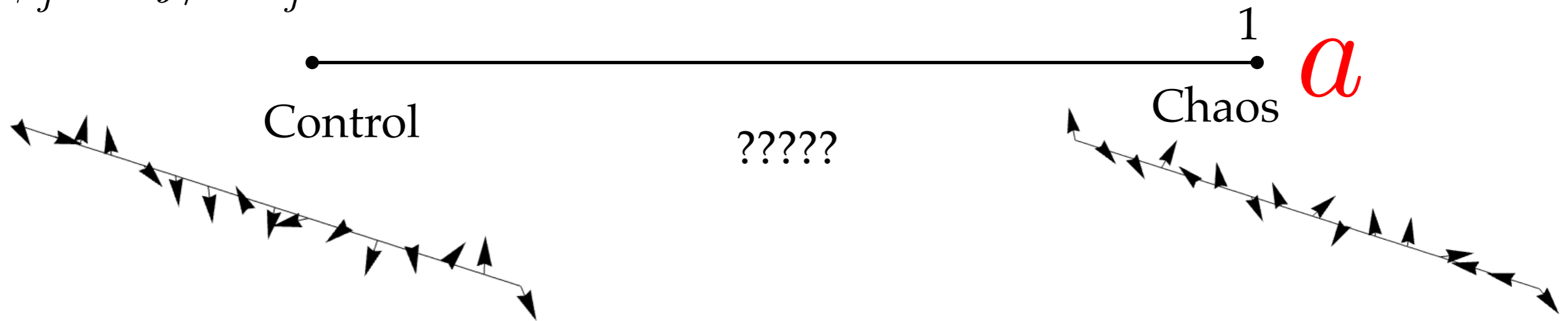
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$$\phi_j^0 = \pi j/2 \quad s_j^0 = 0$$



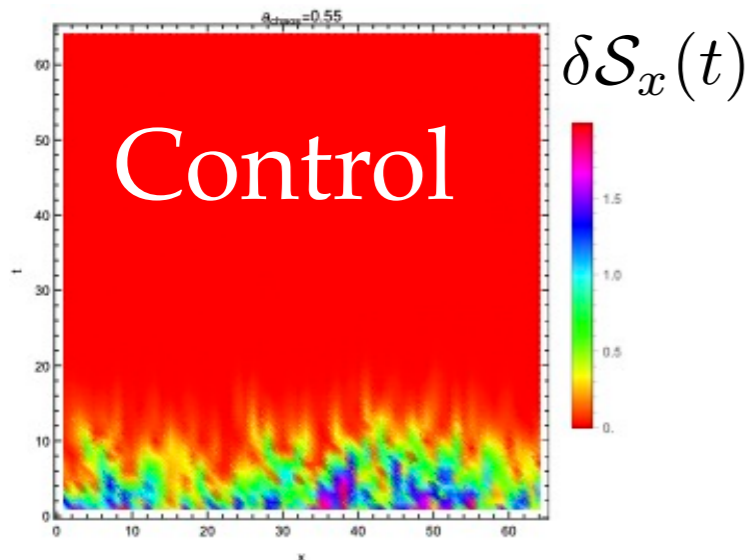
REMOVING ENERGY CONSERVATION AND SOLITONS

$$\phi_j^0 = \pi j/2 \quad s_j^0 = 0$$



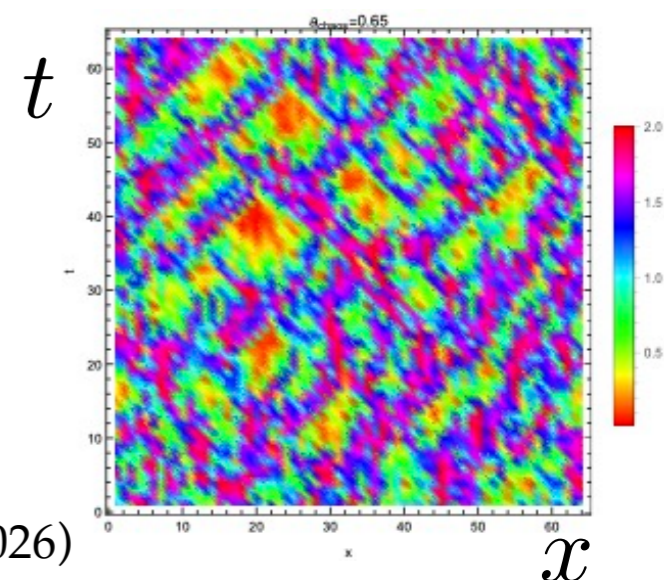
We can define the **activity** of the dynamics from its deviation from the target (unstable) fixed point.

$$S_{\text{diff}}(t) = \frac{1}{L} \sum_i |\mathbf{s}_i(t) - \mathbf{s}_i^0| \equiv \frac{1}{L} \sum_i \delta S_i(t)$$



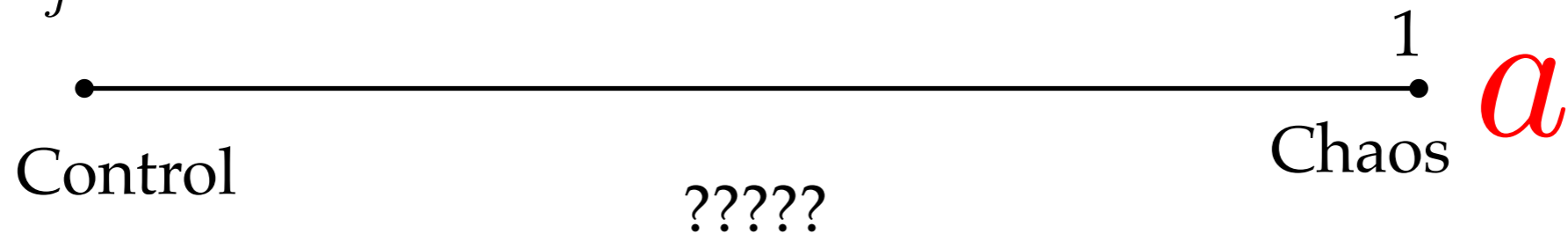
Chaos $S_{\text{diff}}(t) > 0$

Control $S_{\text{diff}}(t) = 0$

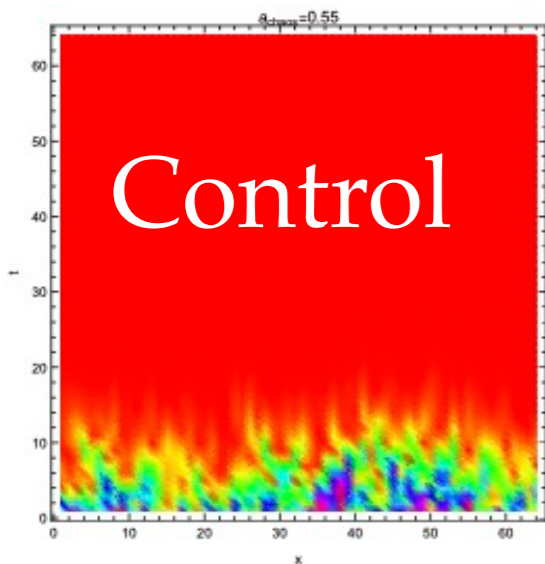


REMOVING ENERGY CONSERVATION AND SOLITONS

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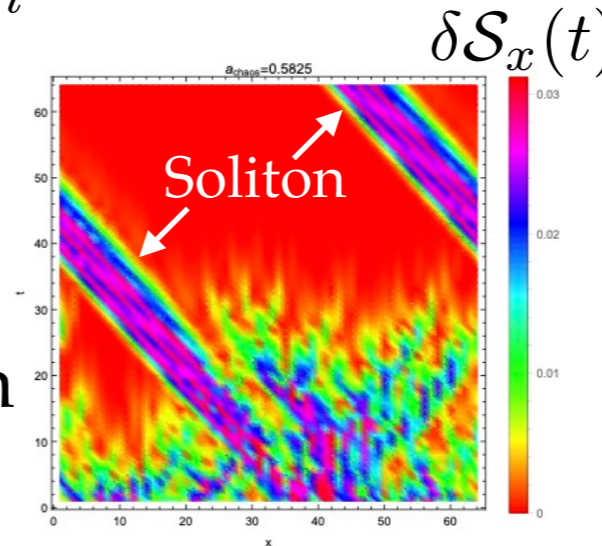
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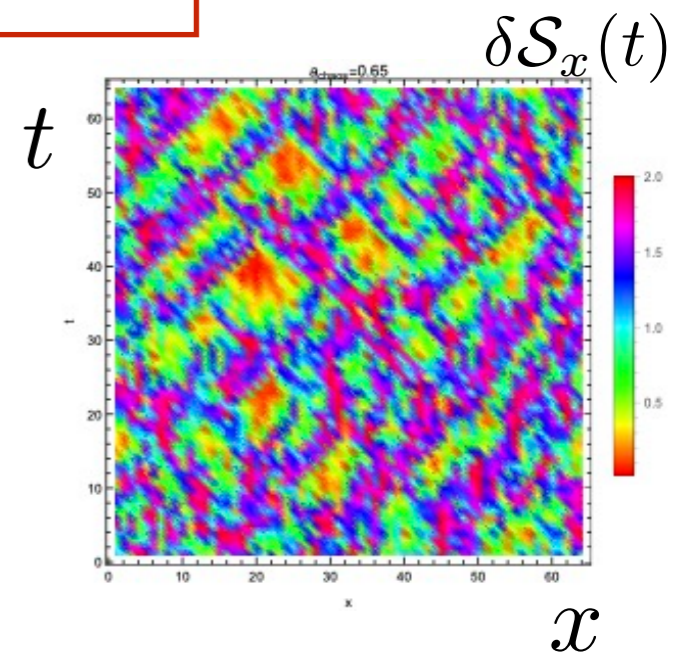
$\delta S_x(t)$

But near the transition
“soliton like”
solutions form

$$v_{\text{soliton}} \approx \pm 1$$



$\delta S_x(t)$

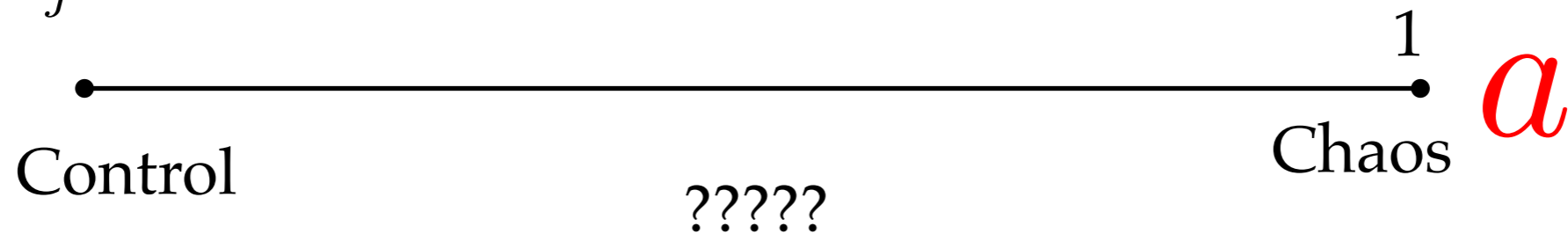


$\delta S_x(t)$

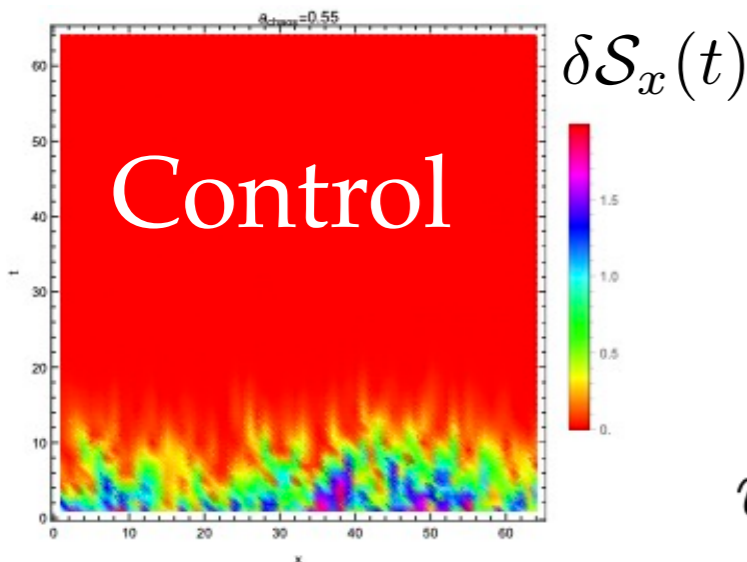
x

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$$\phi_j^0 = \pi j/2 \quad s_j^0 = 0$$

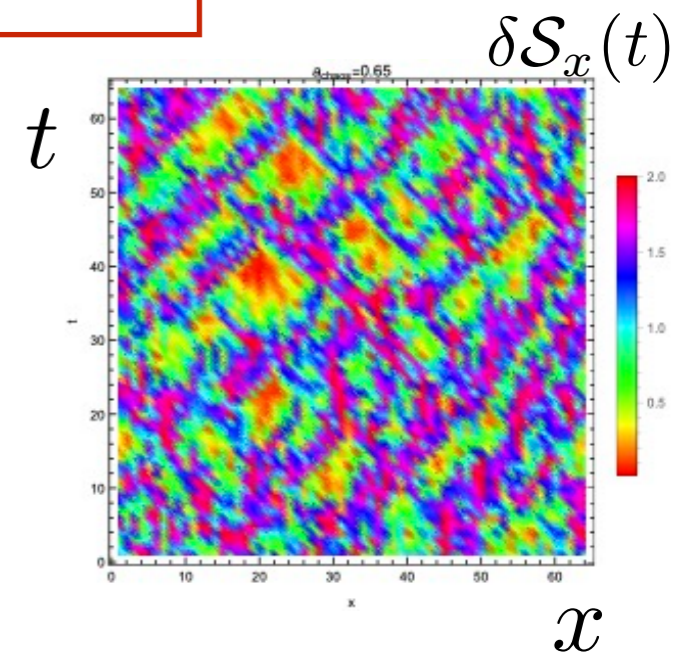
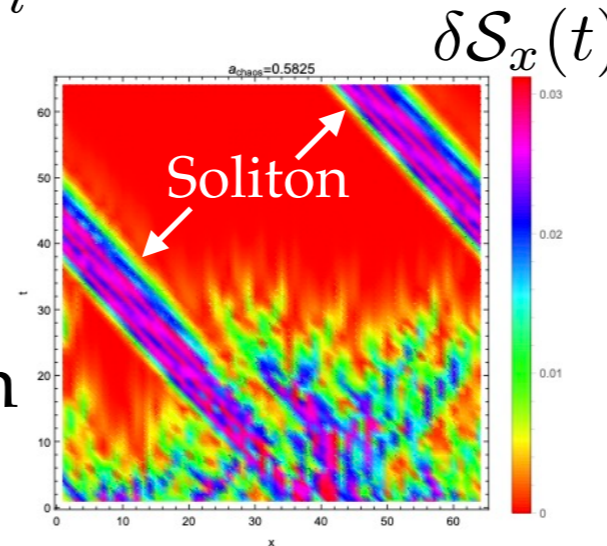


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Remove the solitons by randomizing the chaotic evolution without destroying the spiral fixed point:

$$H = - \sum_{i=1}^L (J_x(t) S_i^x S_{i+1}^x + J_y(t) S_i^y S_{i+1}^y + J_z S_i^z S_{i+1}^z)$$

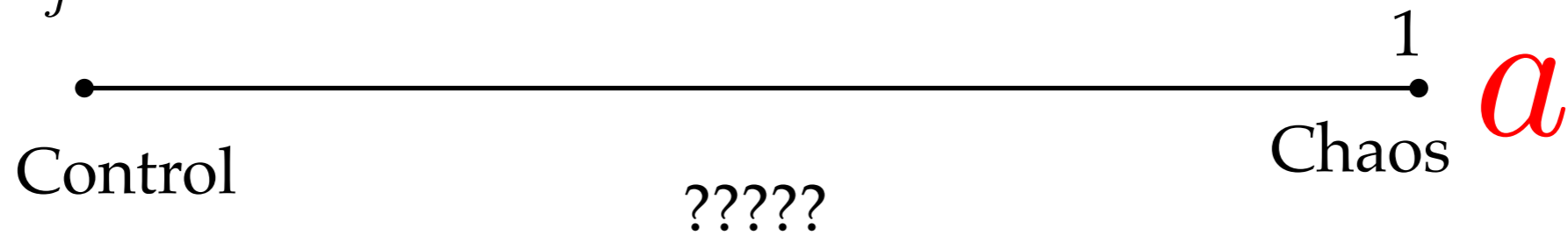
Each time step

$$J_x = \pm J_z$$

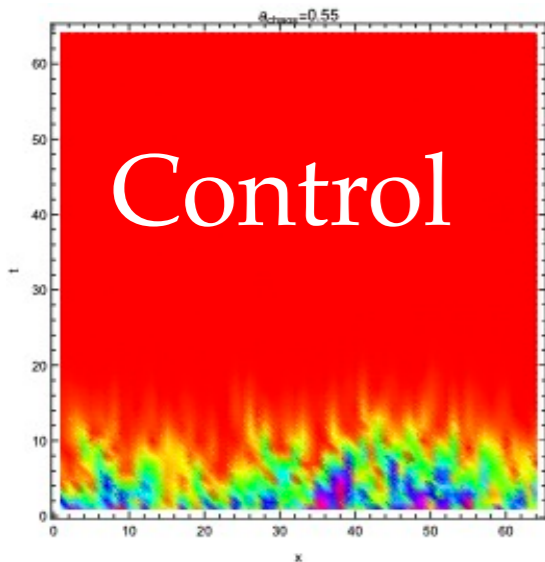
$$J_y = \pm J_z$$

REMOVING ENERGY CONSERVATION AND SOLITONS

$$\phi_j^0 = \pi j / 2 \quad s_j^0 = 0$$

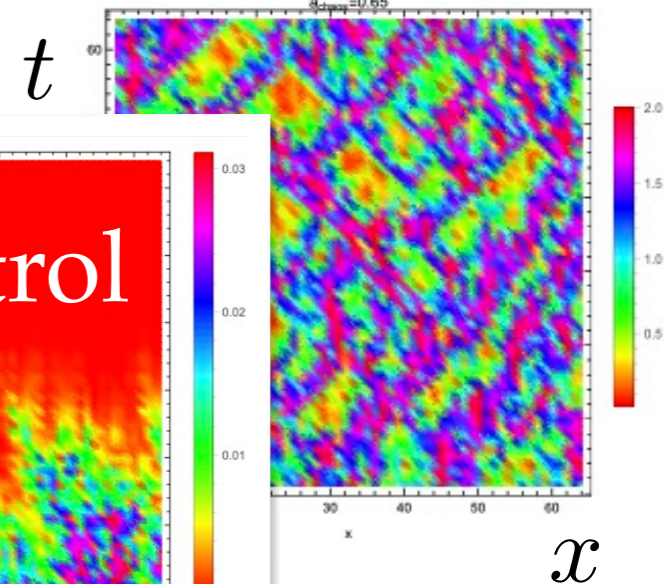
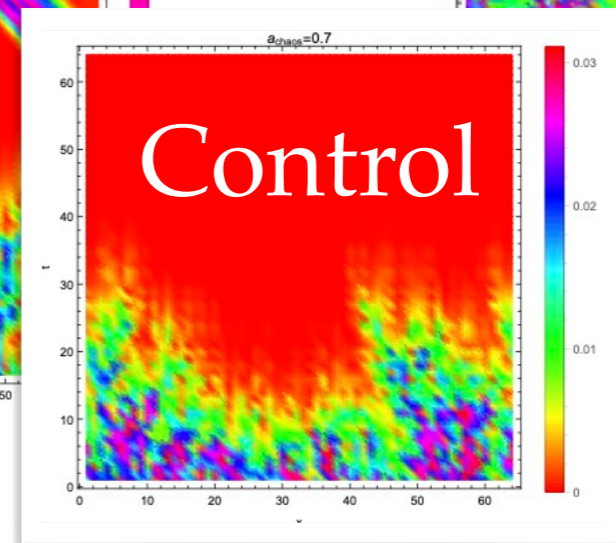
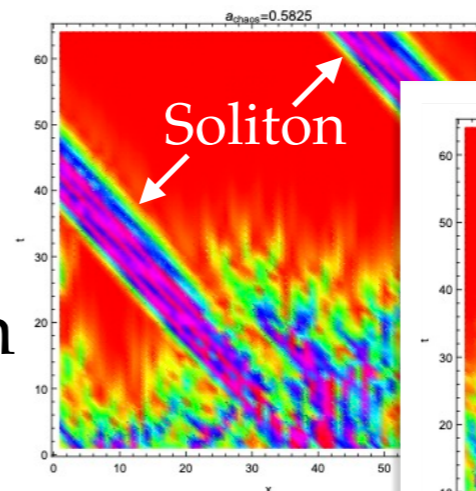


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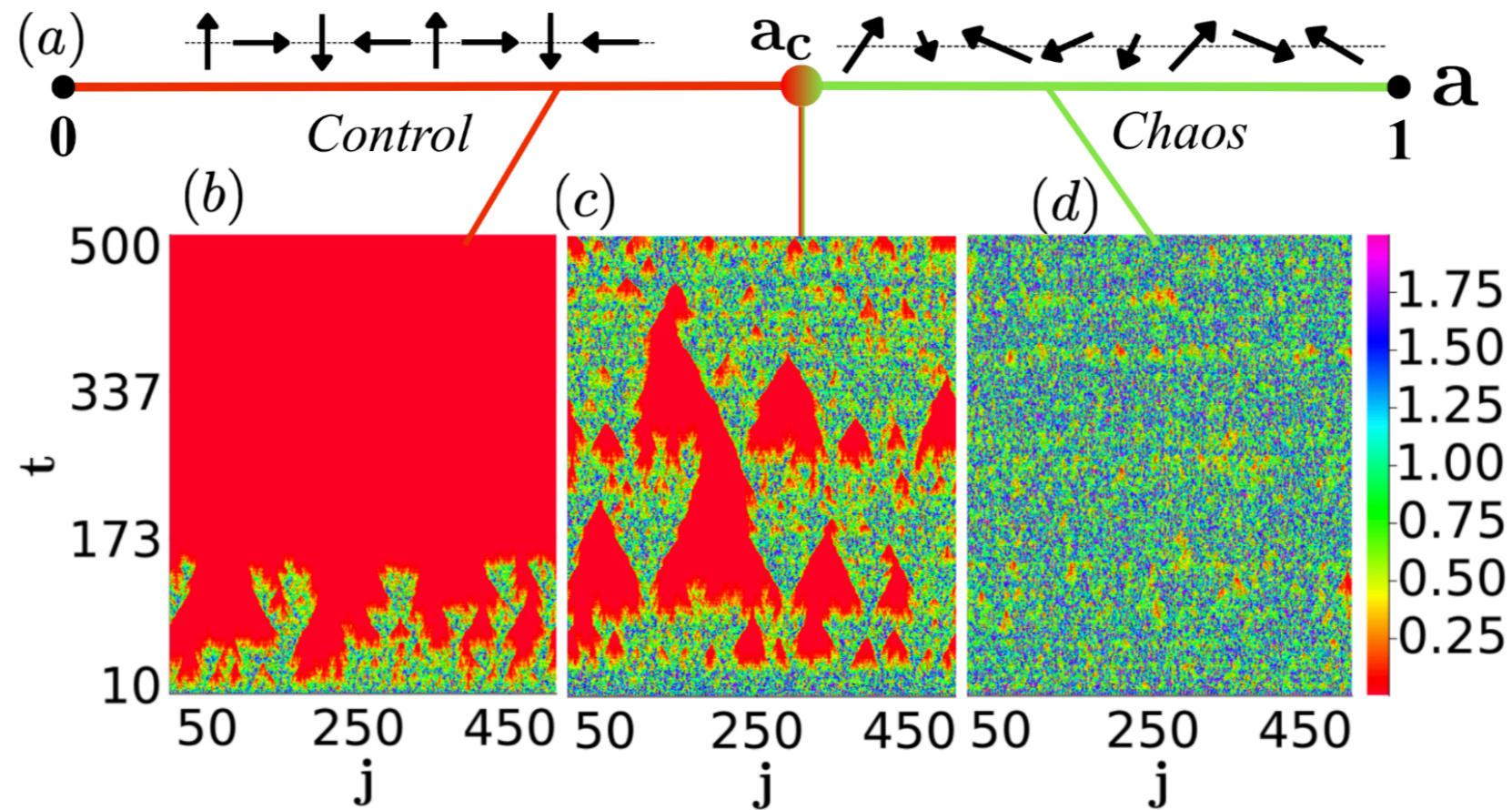
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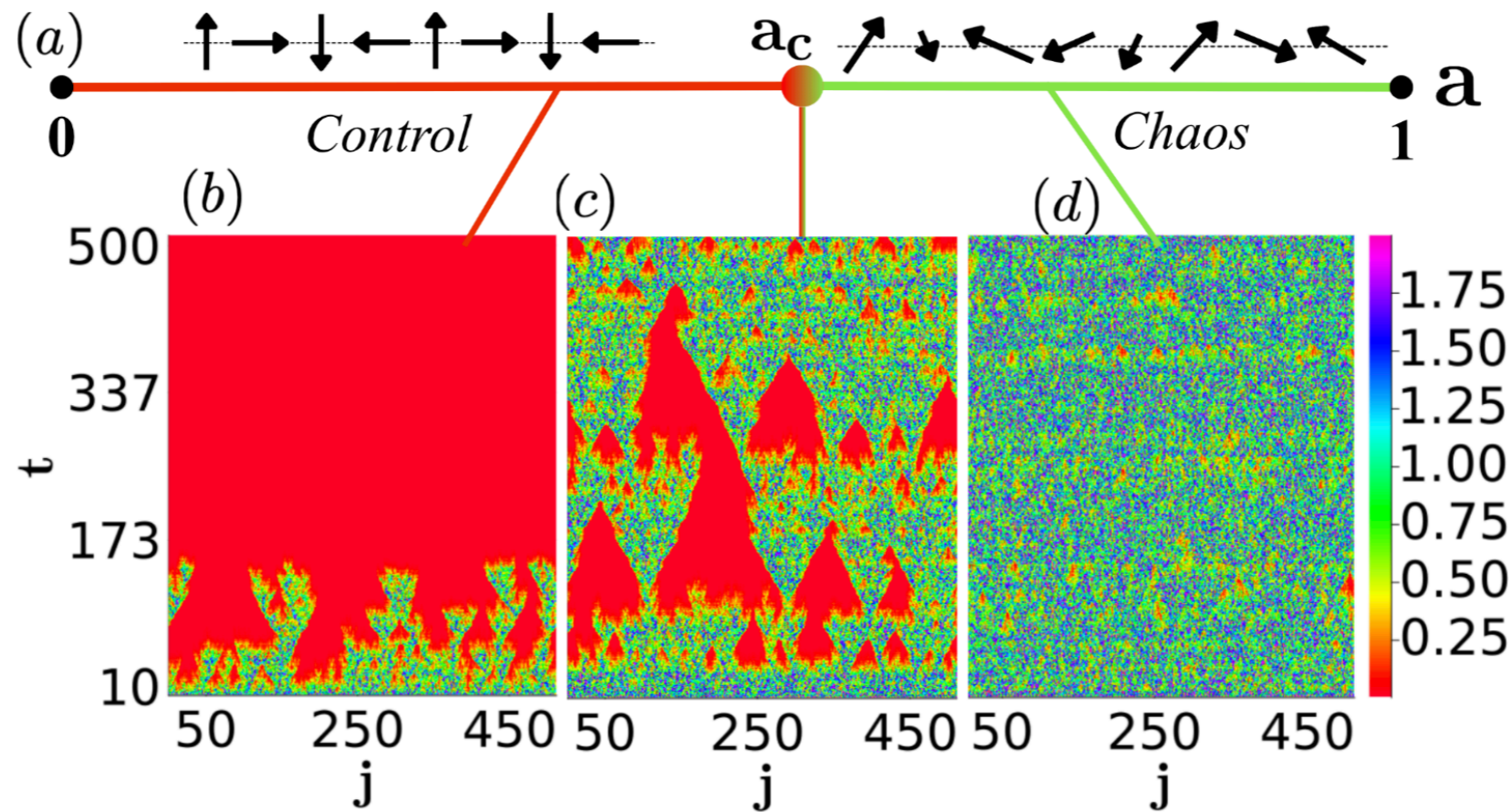
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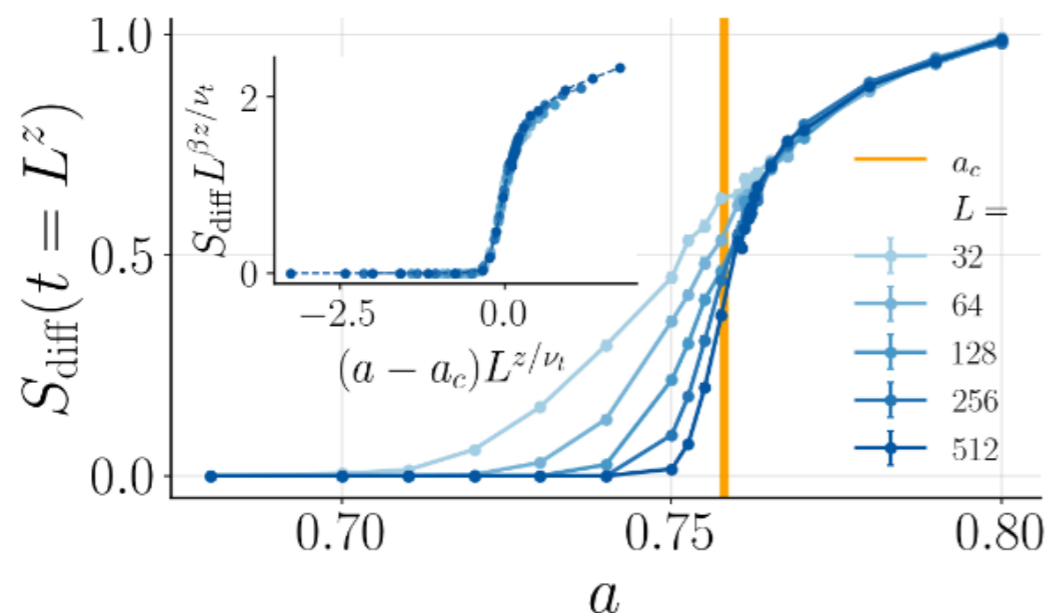
DIRECTED PERCOLATION 'LIKE' TRANSITION IN THE ACTIVITY



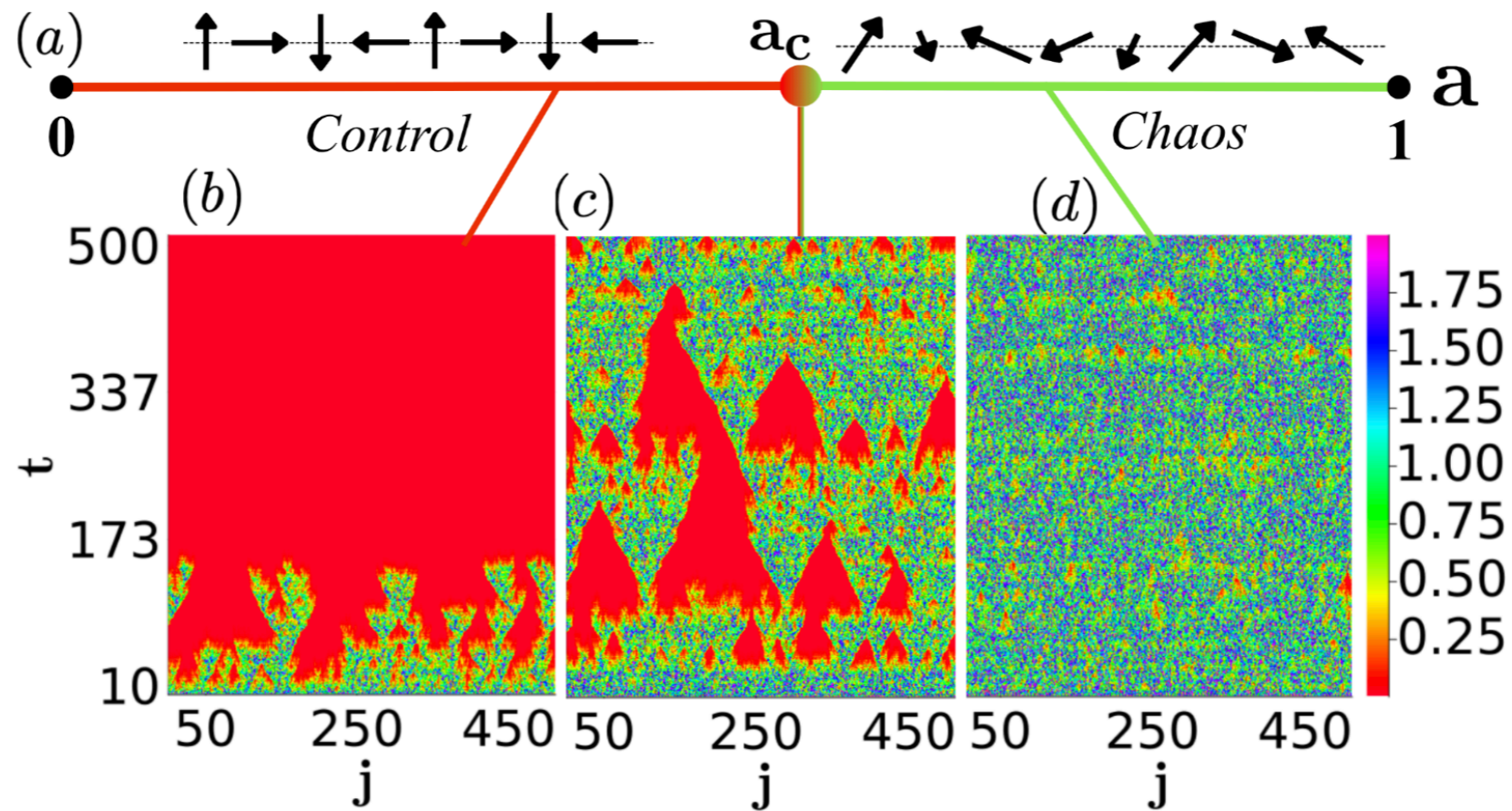
DIRECTED PERCOLATION 'LIKE' TRANSITION IN THE ACTIVITY



Activity undergoes a continuous transition

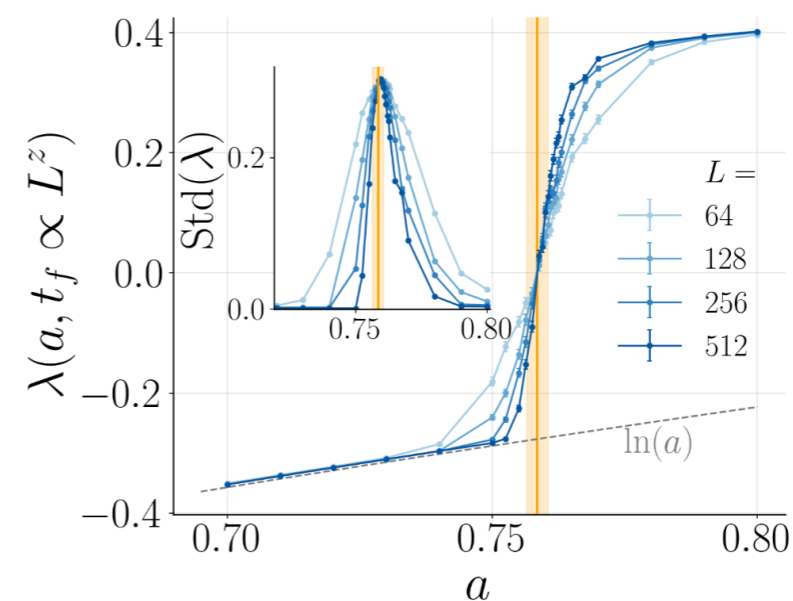
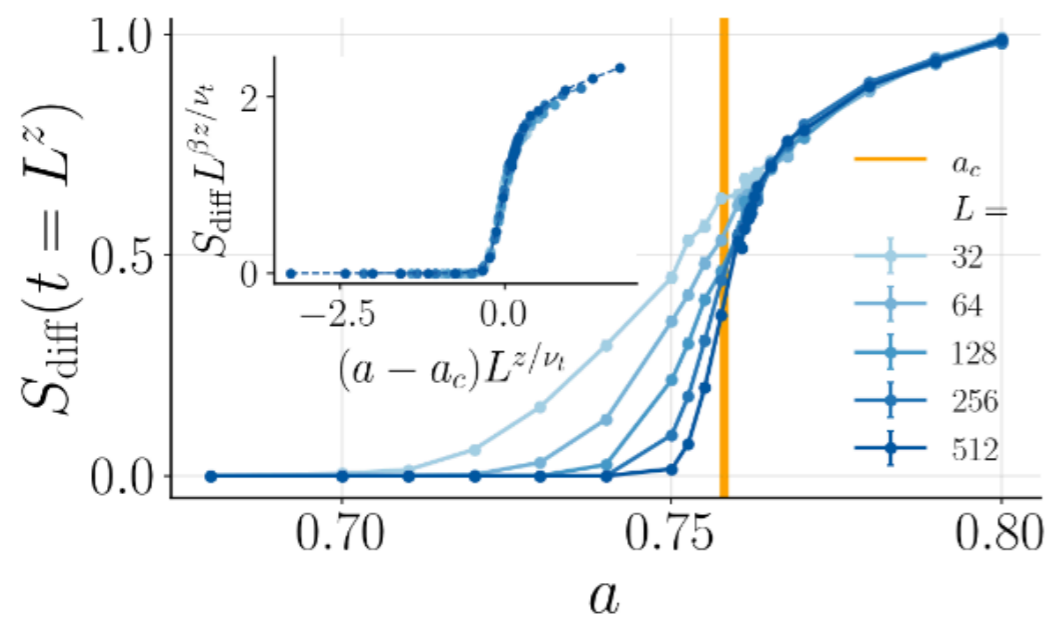


DIRECTED PERCOLATION 'LIKE' TRANSITION IN THE ACTIVITY



Activity undergoes a continuous transition

Lyapunov exponent jumps!

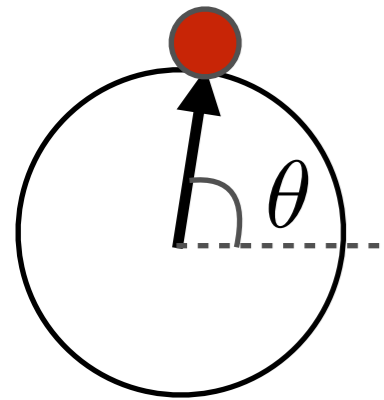


CLASSICAL HAMILTONIAN CONTROL

TRANSITION:SUMMARY

We have extended this idea to a wide array of classically chaotic systems

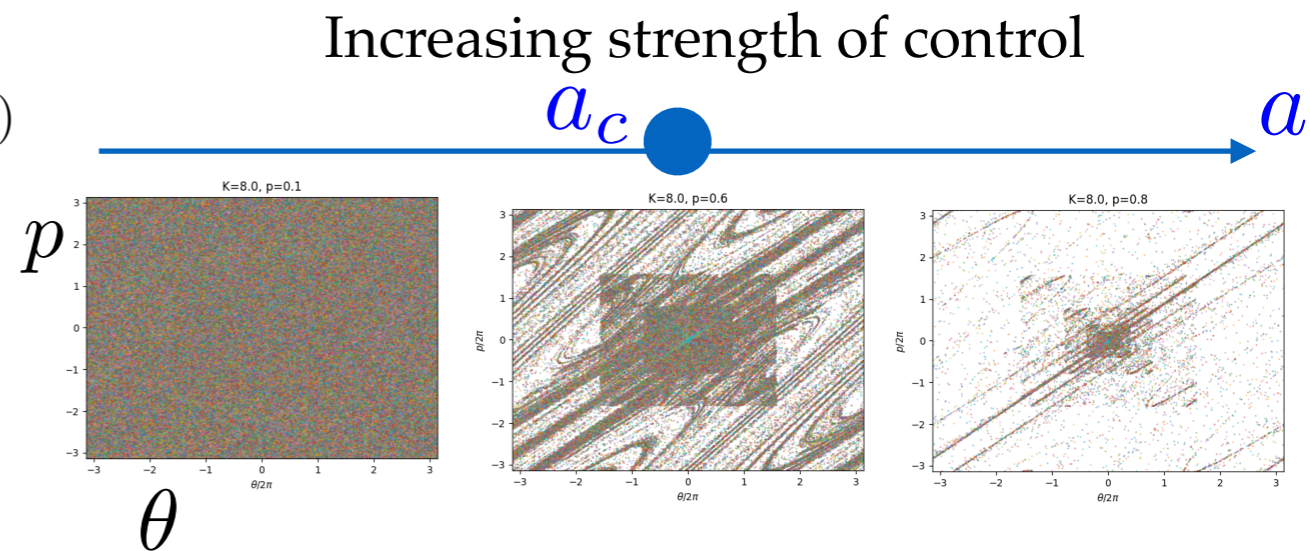
Kicked rotor $H_{KR} = \frac{p^2}{2} + k \cos(x) \sum_{n=-\infty}^{+\infty} \delta(t - nT)$



$$\theta \rightarrow (1 - a)\theta_j + a\theta$$

$$p \rightarrow (1 - a)p_j + ap$$

Poincare sections



Same procedure works for the kicked rotor!

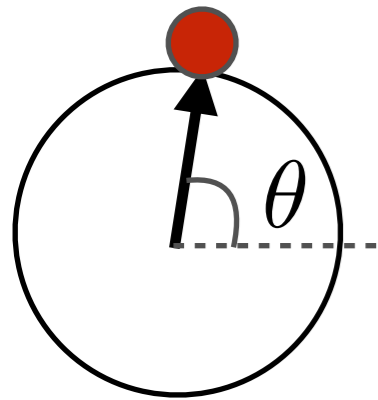
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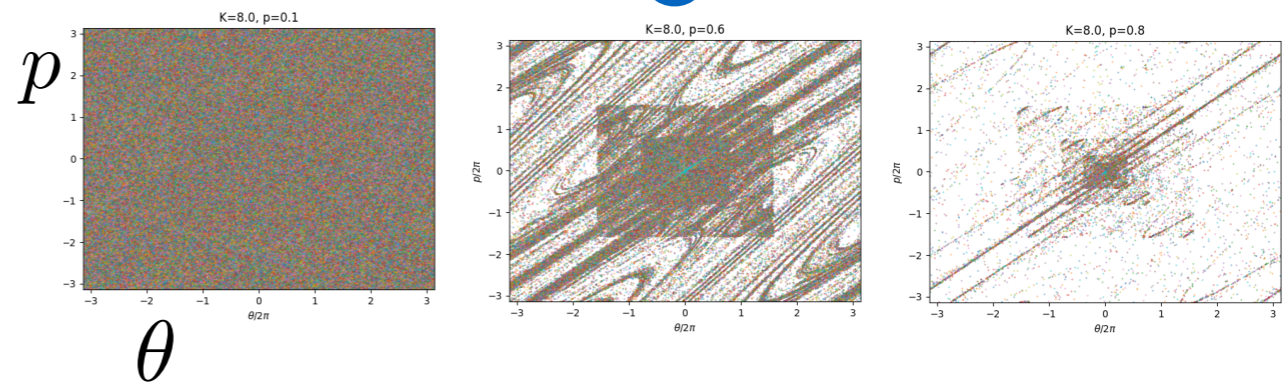


$$\begin{aligned} \theta &\rightarrow (1 - a)\theta_j + a\theta \\ p &\rightarrow (1 - a)p_j + ap \end{aligned}$$

Poincare sections

Increasing strength of control

a_c a

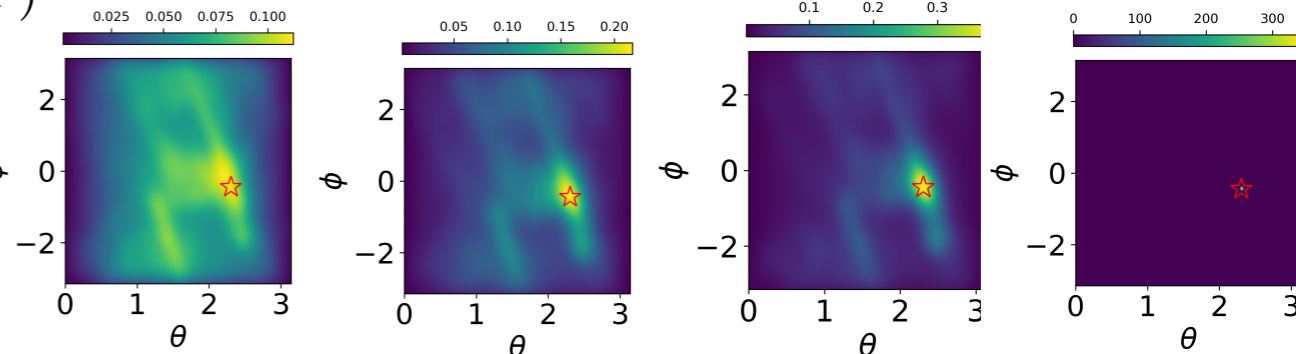
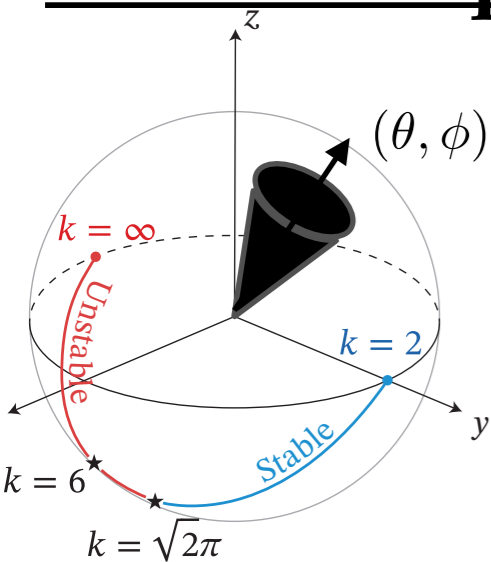


Kicked top

$$H_{\text{KT}} = \alpha J_y + \frac{k J_z^2}{2j} \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\begin{aligned} \phi &\rightarrow (1 - a)\phi_j + a\phi \\ \theta &\rightarrow (1 - a)\theta_j + a\theta \end{aligned}$$

Location on sphere



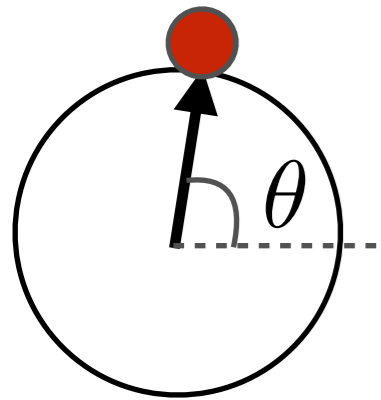
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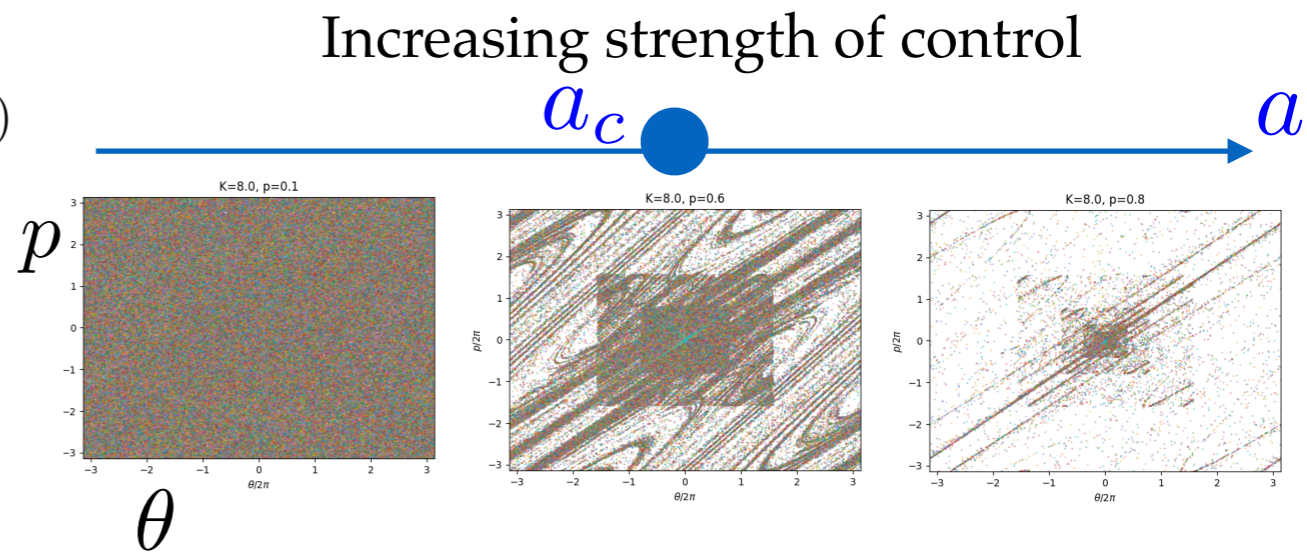
Kicked rotor

$$H_{\text{KR}} = \frac{p^2}{2} + k \cos(x) \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



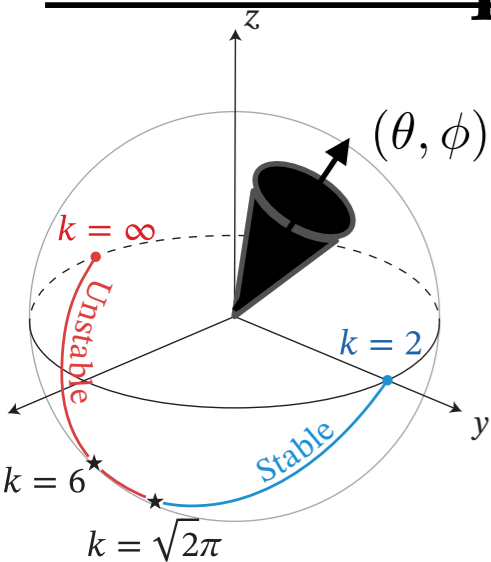
$$\begin{aligned} \theta &\rightarrow (1 - a)\theta_j + a\theta \\ p &\rightarrow (1 - a)p_j + ap \end{aligned}$$

Poincare sections



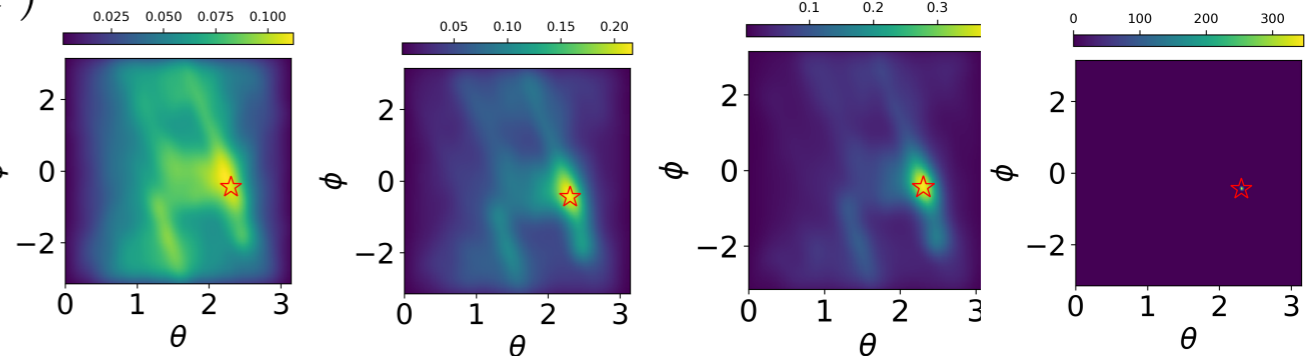
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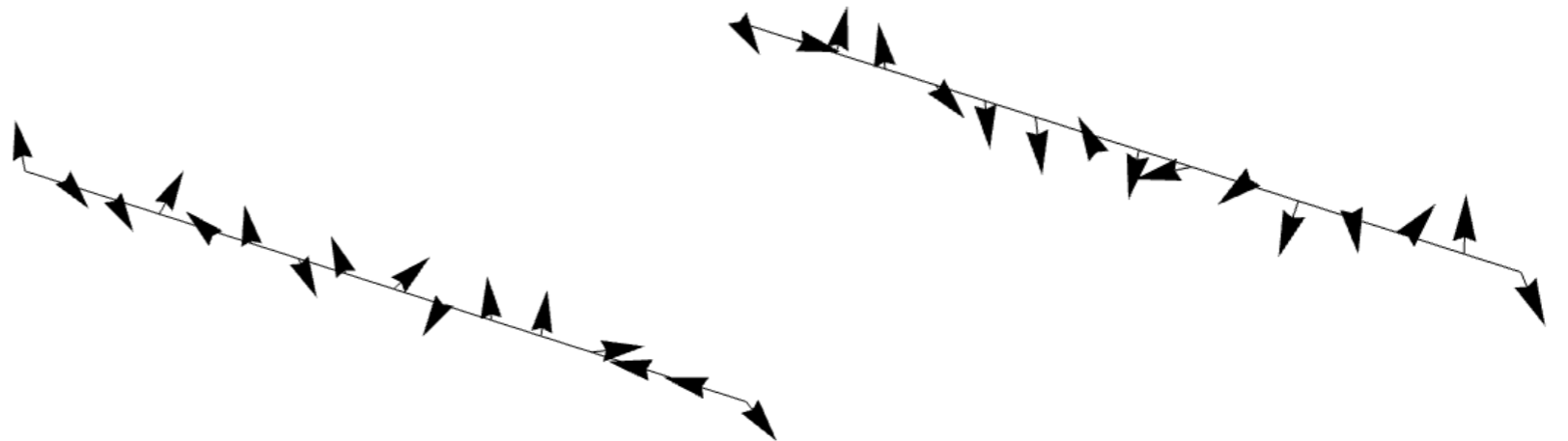
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Location on sphere



Interacting spin chain

$$H = -J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$



CAN THIS BE TRANSFERRED TO THE QUANTUM REGIME?

Clear demonstrations of successfully controlling classical chaos from the large to the small.

Central question of this talk:

Can we utilize classical control protocols (i.e. based on unstable orbits of classical dynamics) to control quantum systems?

Applications to: “solving” open quantum system dynamics, state preparation, quantum error correction

OUTLINE

I. Motivation

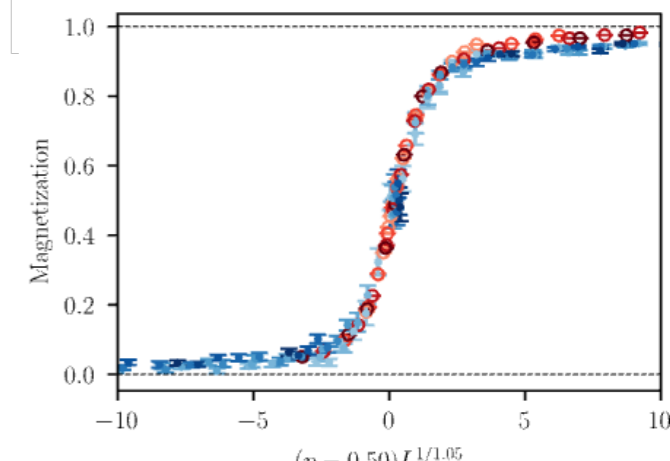
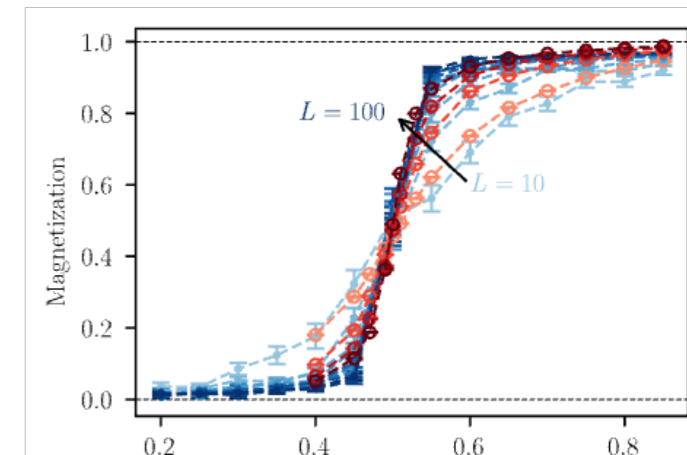
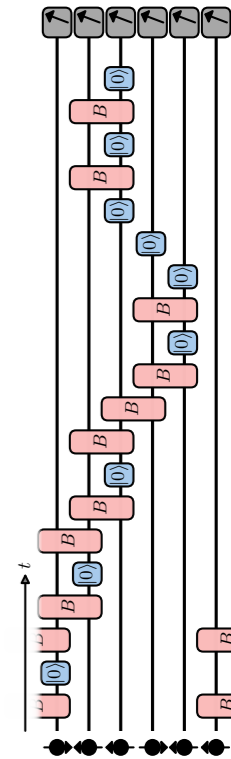
- Controlling classical chaos
- Examples of classical control transitions

II. Quantum Control Induced Transition

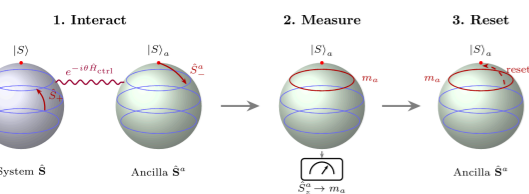
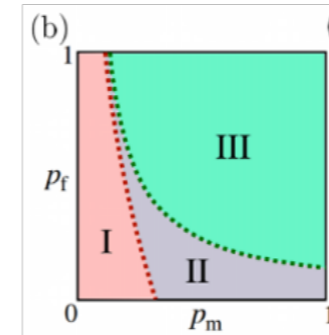
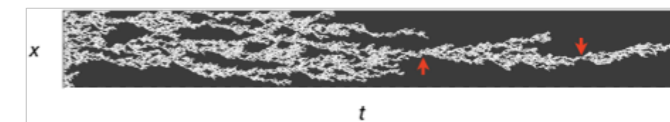
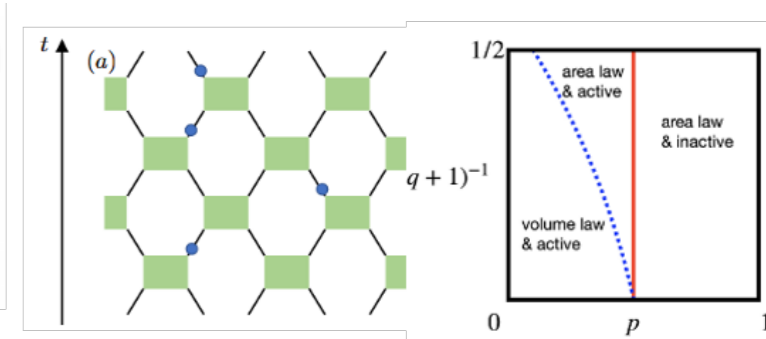
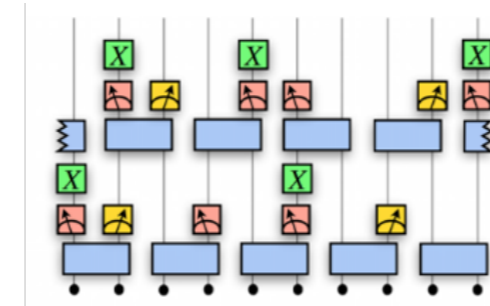
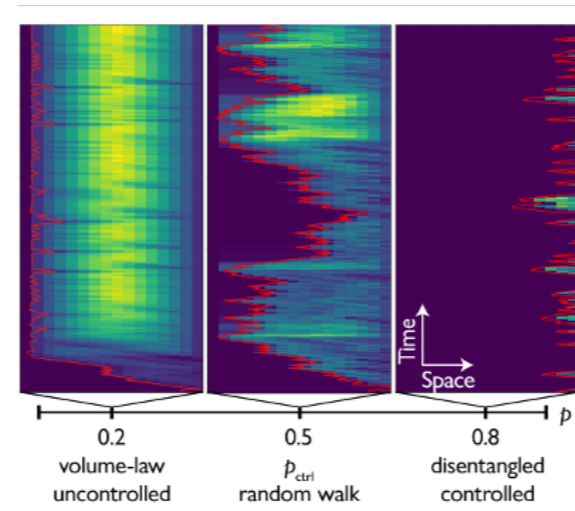
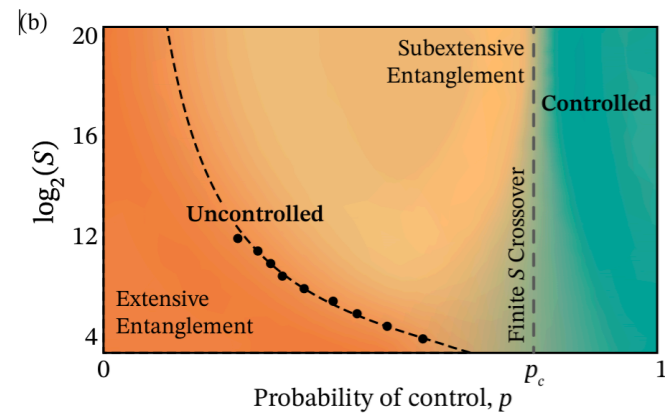
- Control with an Exact Dark State
- Controlling the quantum top
- Controlling the Bernoulli map

III. Realization across circuits

- IBM's quantum computer
- Accessing Quantum fluctuations



EXAMPLES OF QUANTUM CONTROL TRANSITIONS



Controlling the kicked quantum top

Control transition Random walk

Iadecola, Ganeshan, Pixley, Wilson PRL (2023)

Target a FM/cluster state Direct percolation (DP)

O'dea, Morningstar, Gopalakrishnan, Khemani, PRB (2024)

Clifford gates Direct percolation (DP)

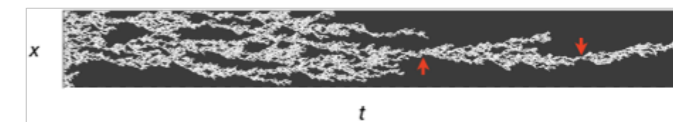
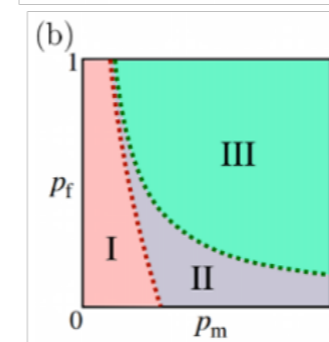
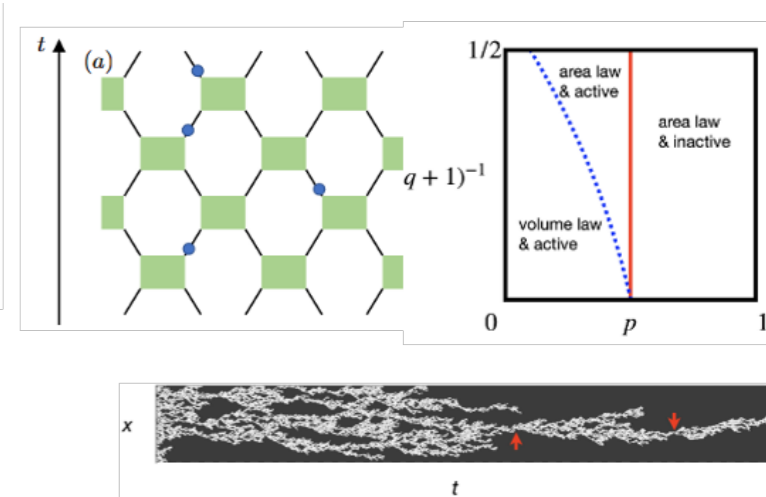
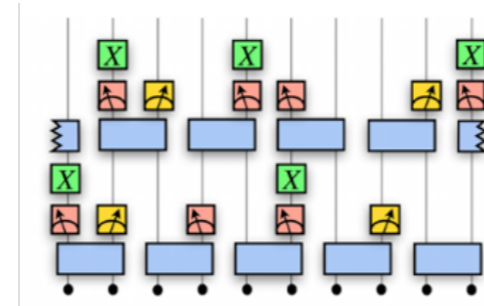
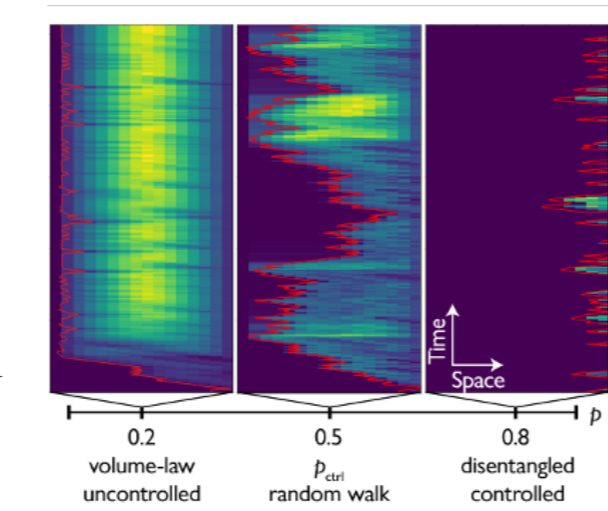
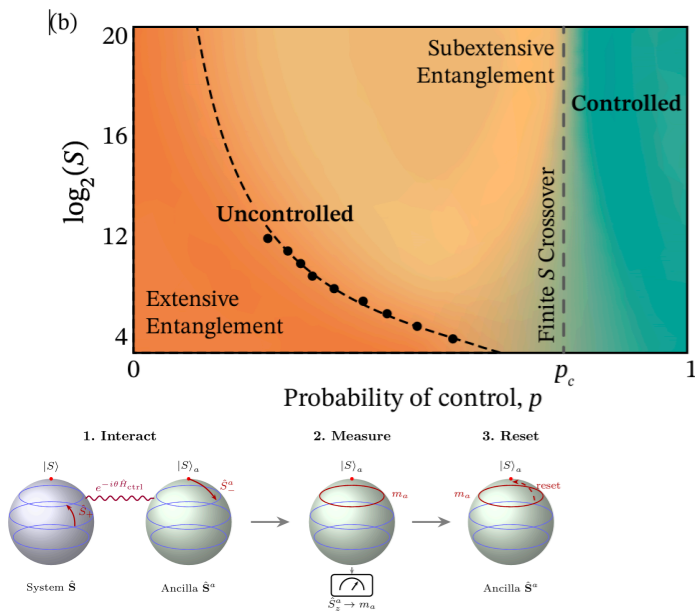
Piroli, Li, Vasseur, Nahum, PRB (2024)

Embed dark states **approximately** (via a classical construction) or **exactly** (after the quantum model is constructed) into the unitary dynamics.

All models require a combination of measurement and feedback.

Do these transitions retain quantum fluctuations?

EXAMPLES OF QUANTUM CONTROL TRANSITIONS



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Prasad, Chakraborty, Iadecola, Kulkarni, Pixley, Ganeshan, Wilson arXiv (2026)

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CONTROLLING QUANTUM MANY-BODY SYSTEMS

Controlling quantum systems in general is too difficult.

Take inspiration from the classical success:

Can we “embed” stochastic control into a quantum system?

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Step 1: First we choose quantum dynamics that have an **analog of an unstable fixed point**

Embed a **dark state into the dynamics**: Doesn't evolve under the unitary dynamics

$$\longrightarrow U|\Psi_{\text{Dark}}\rangle = |\Psi_{\text{Dark}}\rangle$$

Choose the generator of the unitary dynamics so that it has a dark state (either exactly or approximately from a classical limit)

CONTROLLING QUANTUM MANY-BODY SYSTEMS

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Step 2: We need a way to determine how “far” we are from the fixed point (locally) and then push the system towards the dark state.

We can determine where we are through a **measurement** and then can use **feed back** through a unitary operation to, push the system **onto the dark state**

QUANTUM CONTROL TRANSITIONS

First controlling single particle quantum chaotic systems.

Quantum Kicked Top

Prasad, Chakraborty, Iadecola, Kulkarni,
Pixley, Ganeshan, Wilson arXiv (2026)

We will then describe controlling the quantum Bernoulli model, which will become a quantum many-body system upon quantization.

Iadecola, Ganeshan, JHP, Wilson PRL (2023)

These are both examples of embedding classical chaos and control into a quantum system, have approximate dark states.

We first, briefly describe the approach to embedded dark states exactly.

QUANTUM CONTROL TRANSITIONS: EXACT DARK STATES

We first, briefly describe the approach to embedded dark states exactly.

Sierant and Thurkeshi, PRL (2023) — Clifford circuit

O’dea, Morningstar, Gopalakrishnan, Khemani PRB (2024)

$$U_{i,i+1} = 1 \oplus U_{3 \times 3} = \begin{matrix} & \begin{matrix} |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b^* & e & f \\ 0 & c^* & f^* & i \end{pmatrix} & |\psi_{\text{Dark}}\rangle = |\uparrow\uparrow\uparrow \dots\rangle \end{matrix}$$

Feedback: if down is measured correct to up with probability p_f , pushes the dynamics onto the dark state

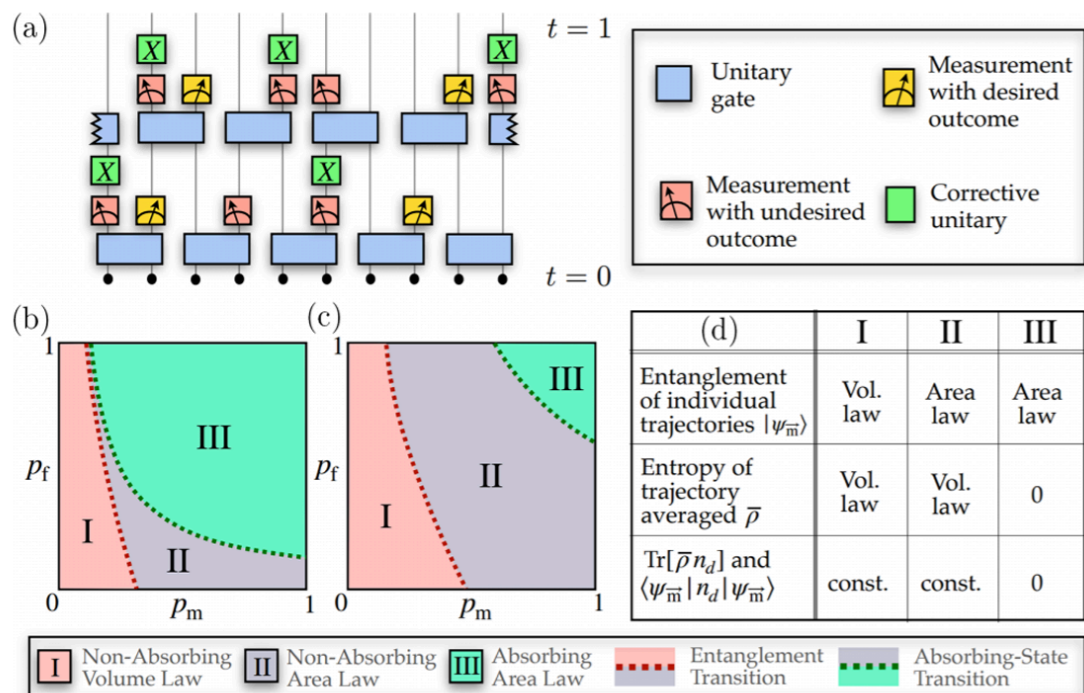
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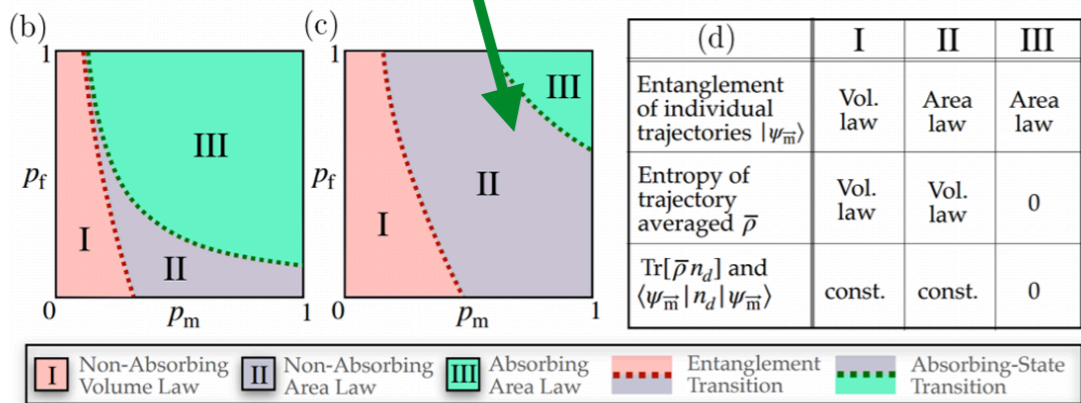
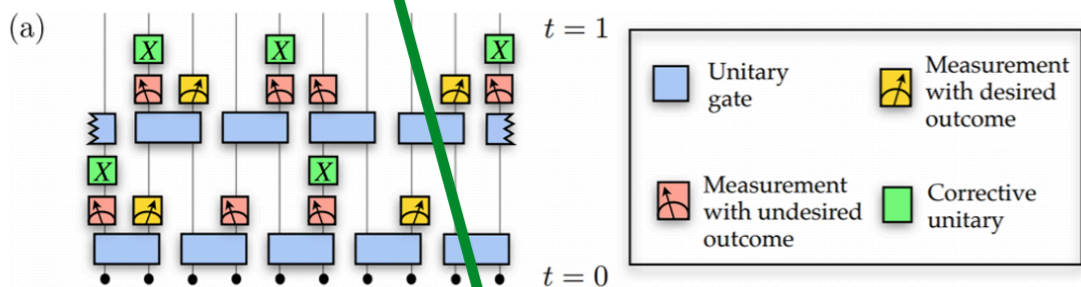
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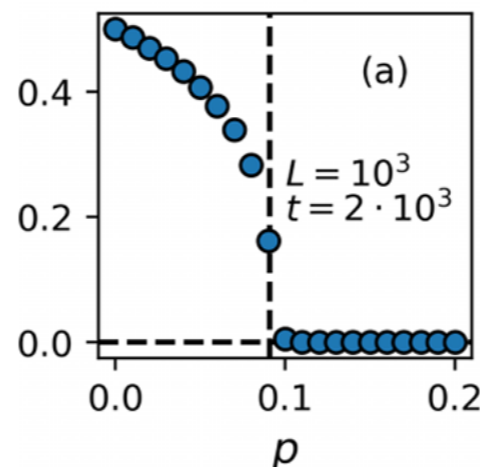
O’dea, Morningstar, Gopalakrishnan, Khemani PRB (2024)

Absorbing state transition separates from the entanglement transition.

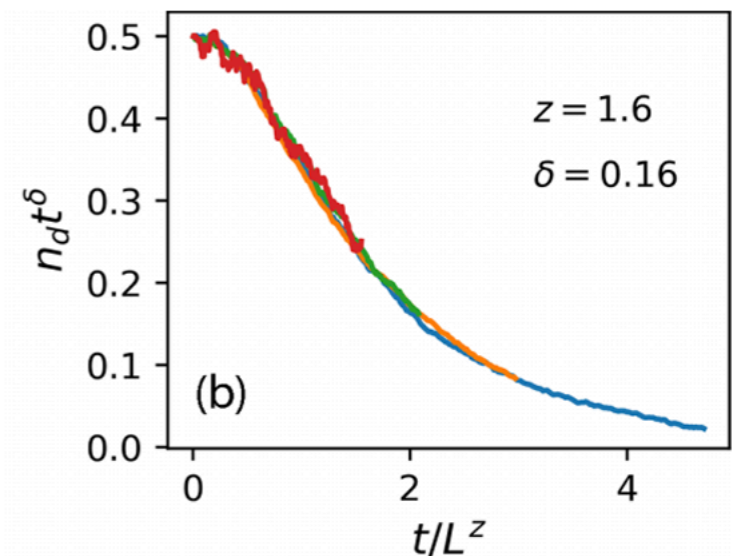
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$$n_d = \frac{1}{L} \sum_i \frac{1 - Z_i}{2}$$



Directed percolation



CONTROLLING THE QUANTUM KICKED TOP

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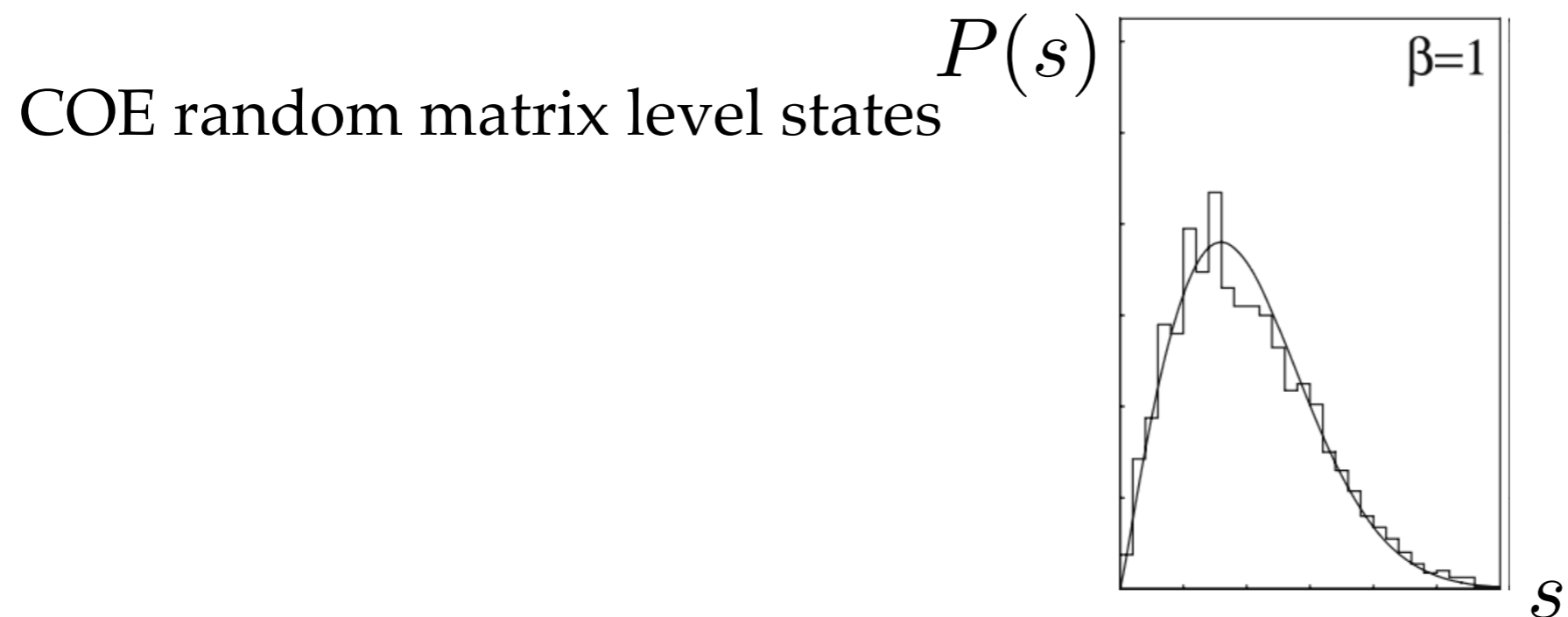
Quantum Kicked Top

Prasad, Chakraborty, Iadecola, Kulkarni,
Pixley, Ganeshan, Wilson arXiv (2026)

Lecture 1 $\hat{H}_{\text{KT}} = \alpha \hat{J}_y + \frac{k \hat{J}_z^2}{2S} \sum_{n=-\infty}^{\infty} \delta(t - nT)$

Quantum chaotic dynamics

$$\hat{U}_{\text{KT}} = e^{-i\alpha \hat{J}_y T} e^{-ik \hat{J}_z^2 / (2S)}$$



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Control Dynamics For $k > k_c$ we have found that x_0 is an unstable fixed point

$$\mathbf{r}_0 = (x_0, y_0, z_0)$$

$$x_0^2 = \frac{\sin^2\left(\frac{kx_0}{2}\right)}{1 + \sin^2\left(\frac{kx_0}{2}\right)}$$

$$y_0 = x_0 \sin(kx_0) + y_0 \cos(kx_0)$$

$$z_0 = -x_0$$

Convenient to rotate the
spin operators such that $\mathbf{r}_0 = \hat{z}$

$$\hat{\mathbf{S}} = \hat{R} \hat{\mathbf{J}} \hat{R}^\dagger = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$$

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Now introduce an ancillary quantum top $\hat{\mathbf{S}}_a$ and couple them and time evolve

Initial state,
maximal S
for the ancilla

$$|\psi\rangle \otimes |S\rangle_a$$

$$\hat{H}_{\text{ctrl}} = \frac{1}{i} (S + \hat{S}_z)^{-\frac{1}{2}} \hat{S}_+ \hat{S}_-^a (S + \hat{S}_z^a)^{-\frac{1}{2}} + \text{H.c.}$$

$$U_{\text{ctrl}}(\theta) = e^{-i\theta \hat{H}_{\text{ctrl}}}$$

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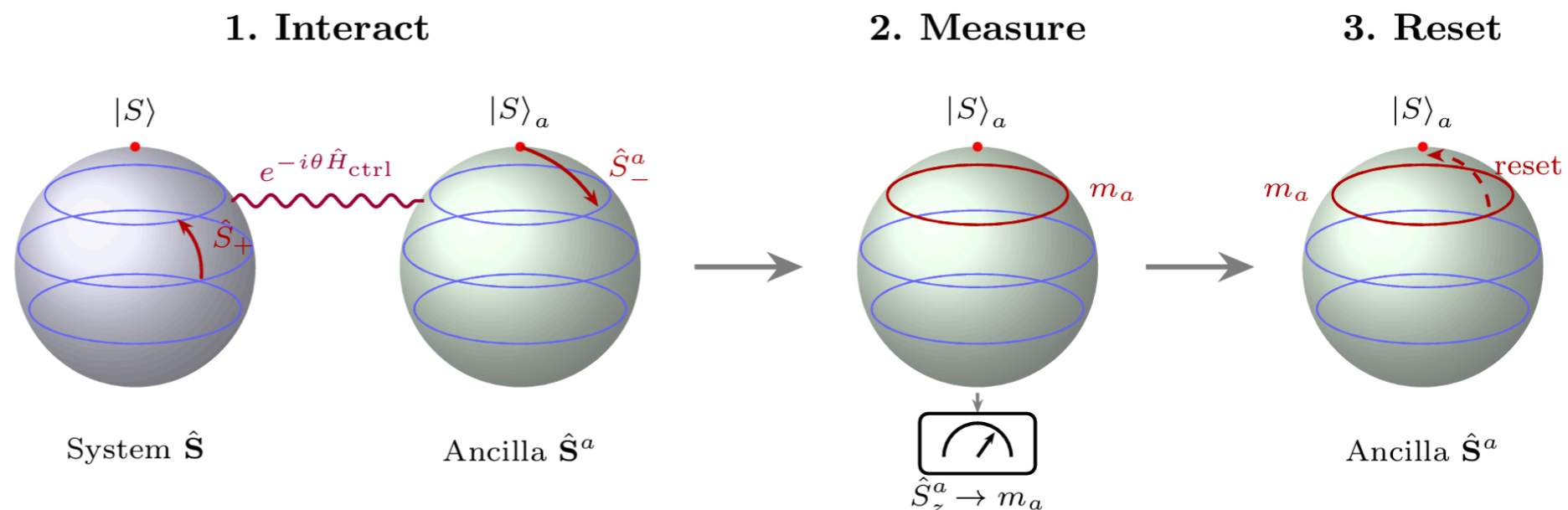
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We then measure
the ancilla, and reset
it to maximal spin S



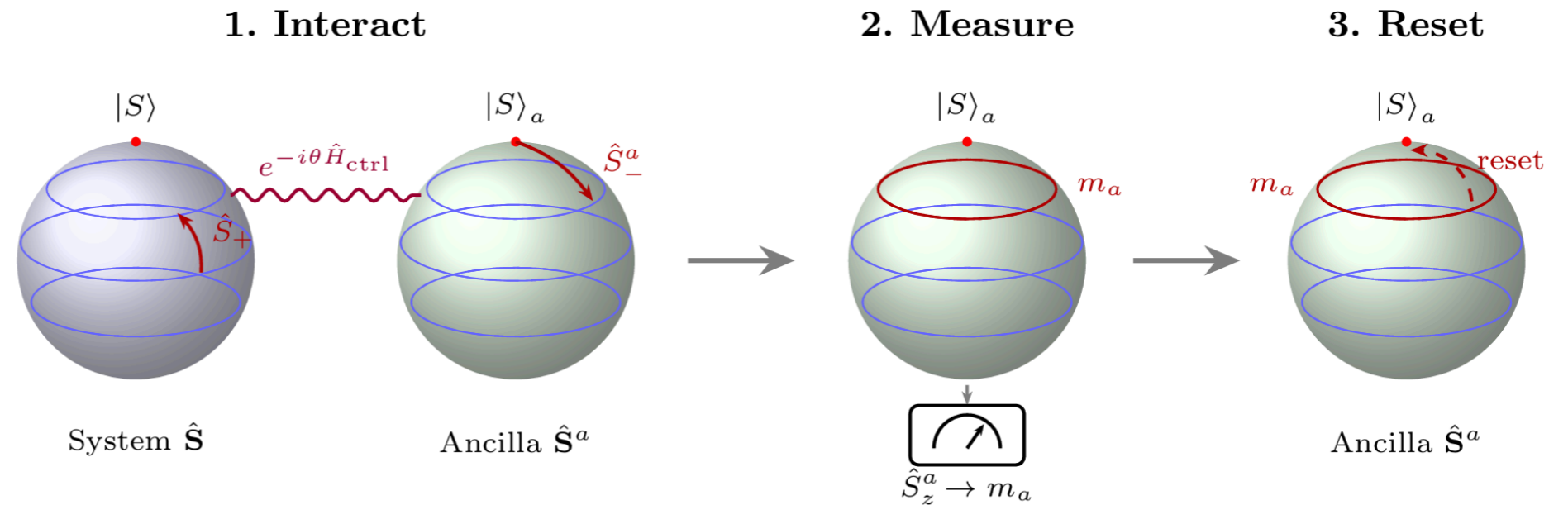
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Measurements are given by the Krauss operators after we trace out the ancilla

$$\hat{K}_{m_a} |\psi\rangle = \sum_{m=-S}^{m_a} \psi_m \sqrt{\binom{S-m}{S-m_a}} \left(\cos \frac{\theta}{2}\right)^{m_a-m} \times \left(\sin \frac{\theta}{2}\right)^{S-m_a} |S-m_a+m\rangle$$

$$|\psi\rangle \mapsto \frac{\hat{K}_{m_a}}{\sqrt{p(m_a)}} |\psi\rangle$$

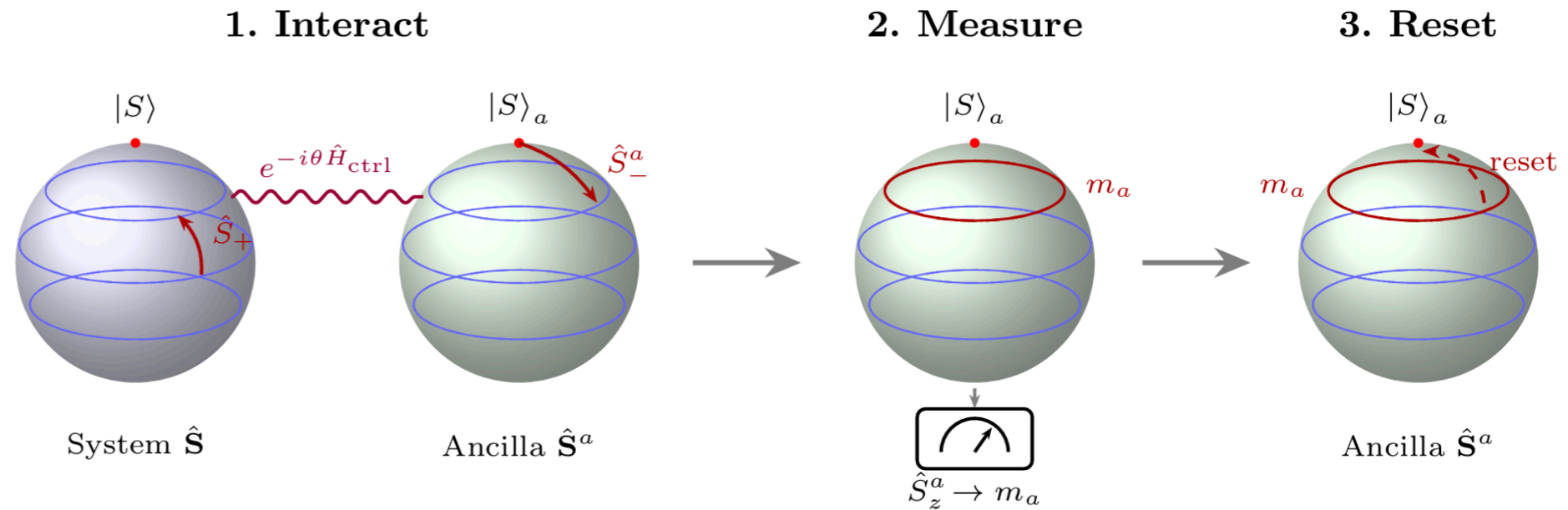
$$p(m_a) = \langle \psi | \hat{K}_{m_a}^\dagger \hat{K}_{m_a} | \psi \rangle$$

CONTROLLING THE QUANTUM KICKED TOP

Quantum chaotic dynamics
Apply with probability $1 - p$

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Control Dynamics
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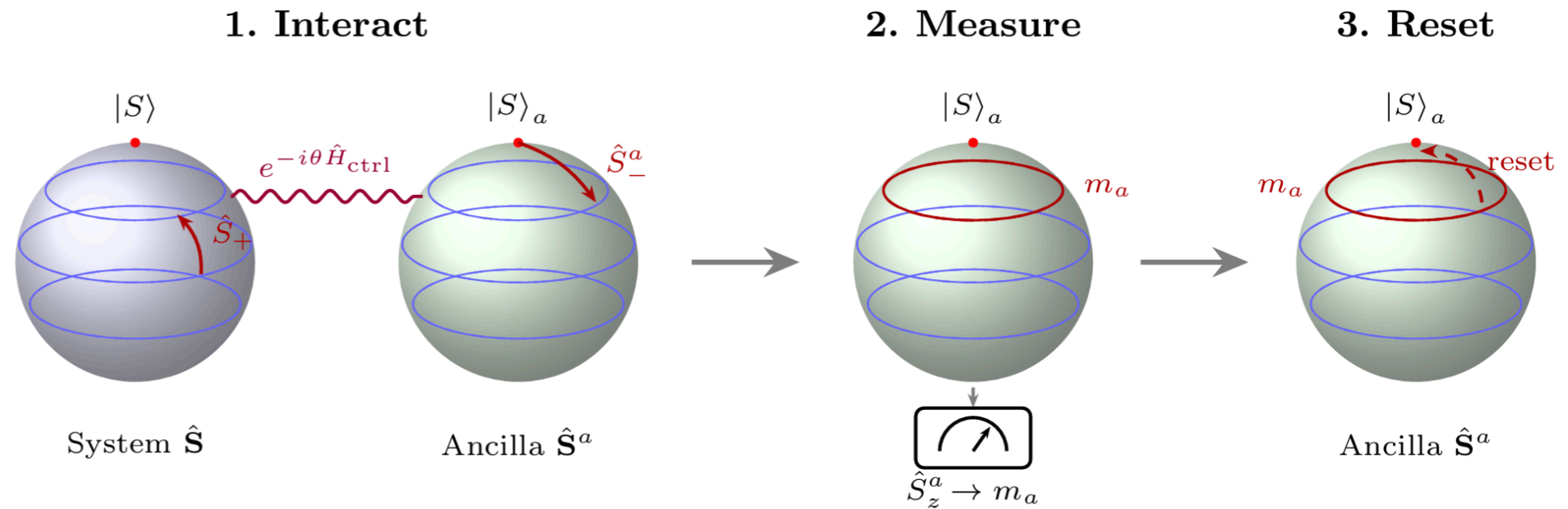


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Quantum activity

$$R^2 = \overline{|\langle \delta \hat{\mathbf{r}}(t \rightarrow \infty) \rangle_m|^2}$$

$$\langle \delta \hat{\mathbf{r}}(t) \rangle_m = \frac{1}{S} \langle \hat{\mathbf{S}}(t) \rangle_m - 1$$

Transverse quantum
fluctuations

$$s_{\perp}^2 = \frac{\langle \hat{S}_x^2 \rangle + \langle \hat{S}_y^2 \rangle}{S(S+1)}$$

Fidelity

$$F = \overline{|\langle \psi | S \rangle|^2}$$

Relevant
Quantities

CONTROLLING THE QUANTUM KICKED TOP

	Quantum activity	Transverse quantum fluctuations	Fidelity
Relevant Quantities	$R^2 = \overline{ \langle \delta \hat{\mathbf{r}}(t \rightarrow \infty) \rangle_m ^2}$ $\langle \delta \hat{\mathbf{r}}(t) \rangle_m = \frac{1}{S} \langle \hat{\mathbf{S}}(t) \rangle_m - 1$	$s_{\perp}^2 = \frac{\overline{\langle \hat{S}_x^2 \rangle + \langle \hat{S}_y^2 \rangle}}{S(S+1)}$	$F = \overline{ \langle \psi S \rangle ^2}$

CONTROLLING THE QUANTUM KICKED TOP

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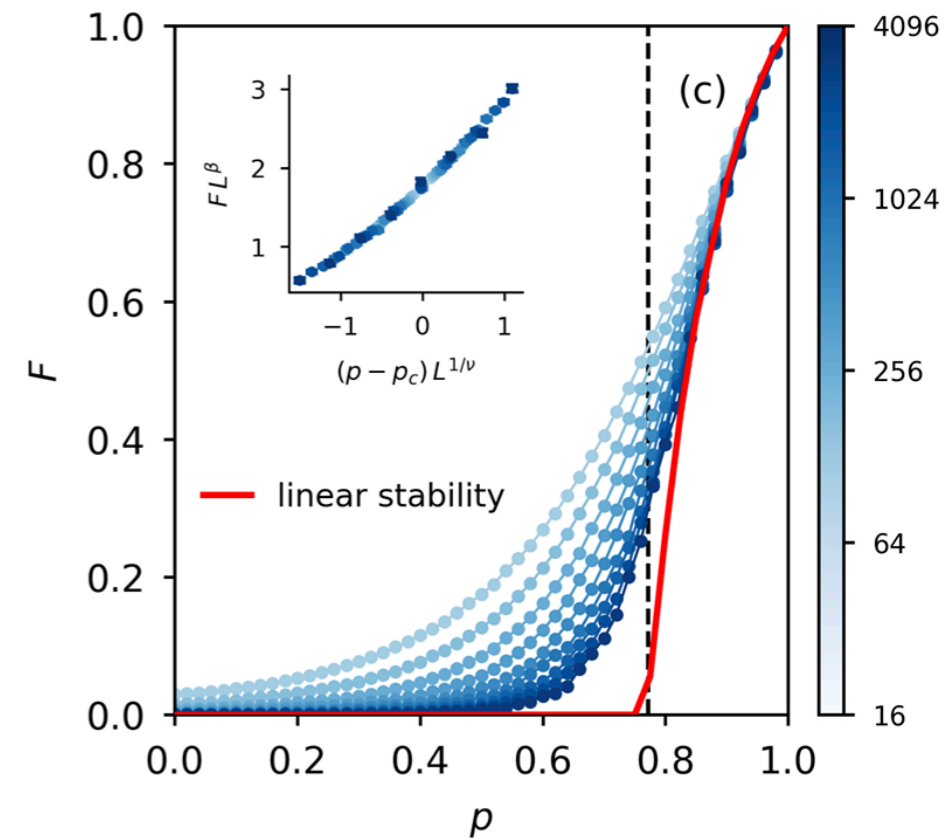
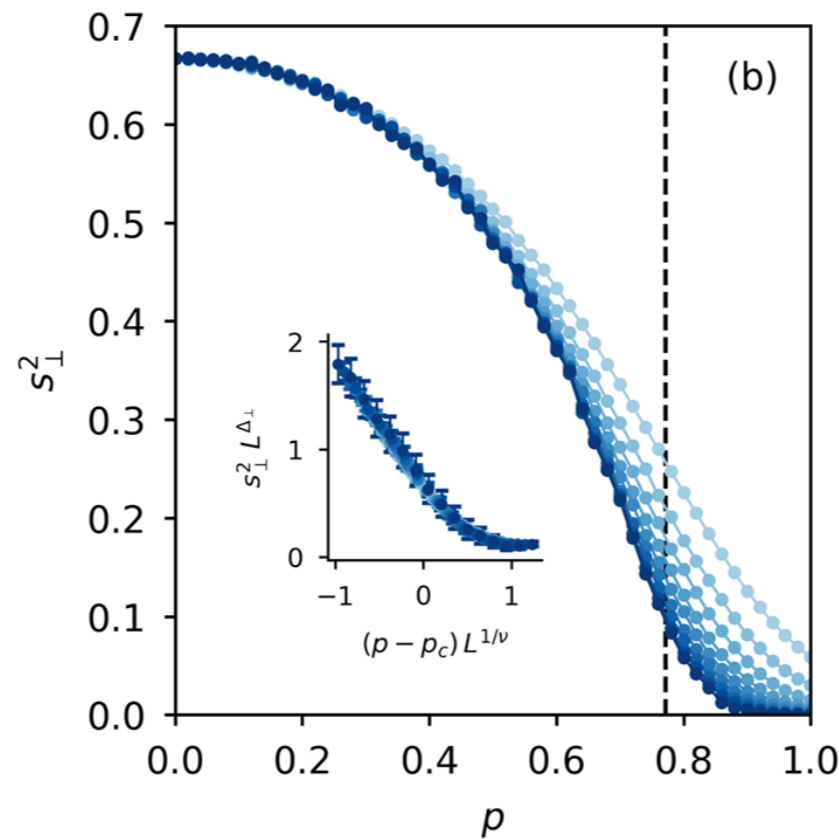
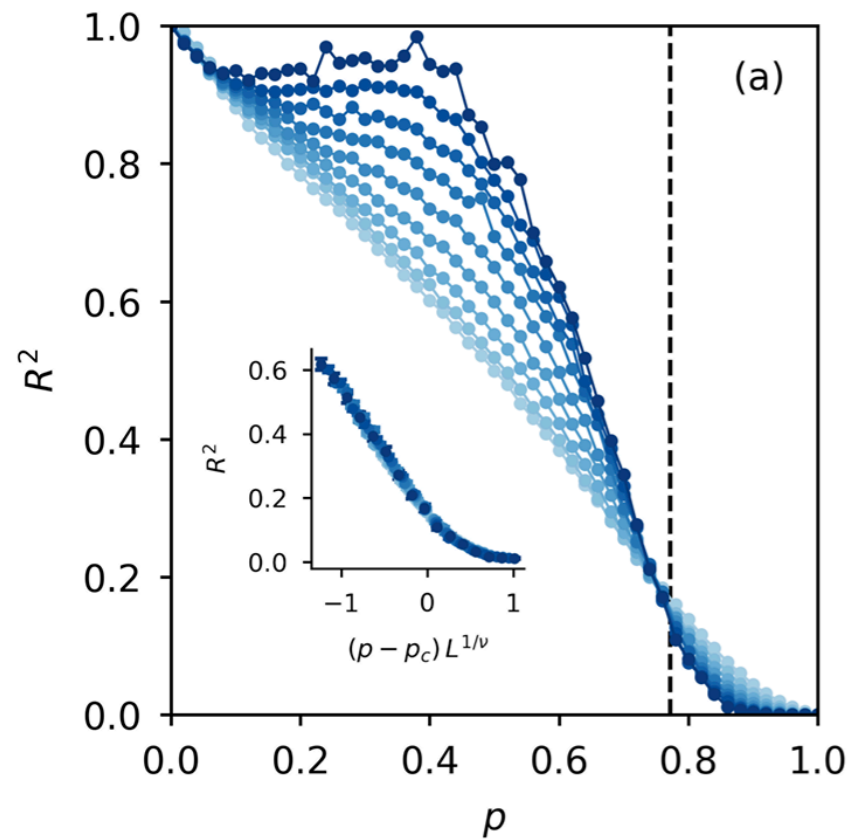
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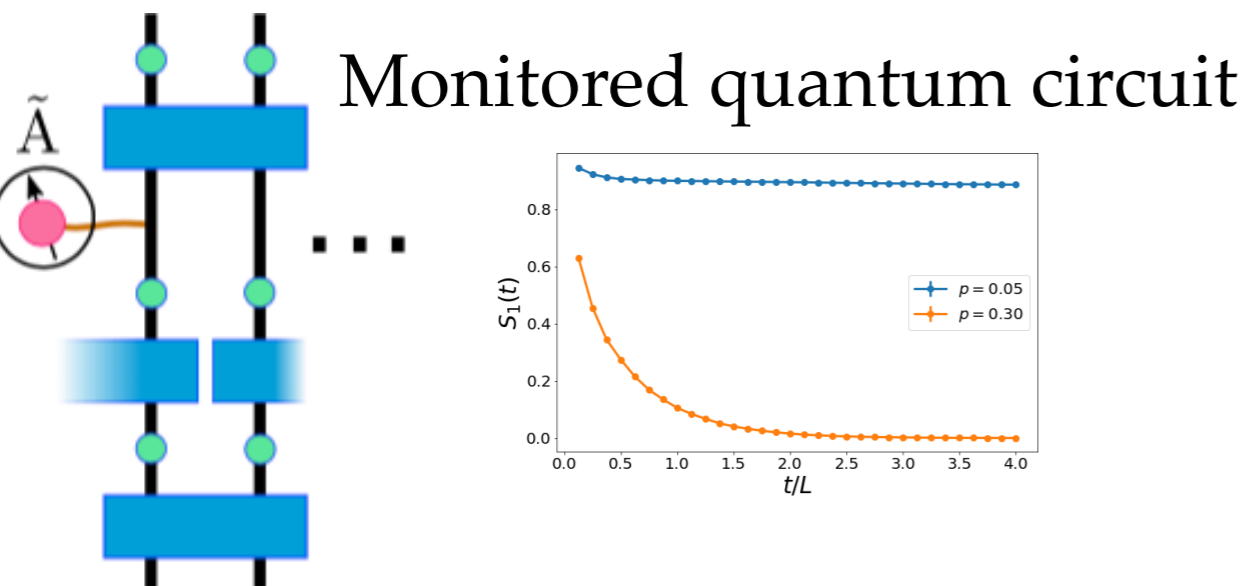


Observable	p_c	ν	Crit. Exp.
Fidelity F	0.781(16)	0.85(13)	$\beta = 0.58(13)$
R^2	0.76(5)	1.23(30)	$\Delta_R \approx 0$
s_{\perp}^2	0.73(7)	1.25(29)	$\Delta_{\perp} = 0.76(58)$

CONTROLLING THE QUANTUM KICKED TOP

Absence of an entangling phase

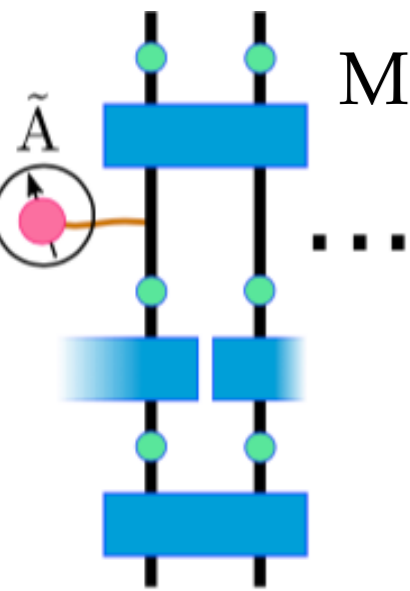
Recall the ancillary from Lecture 3



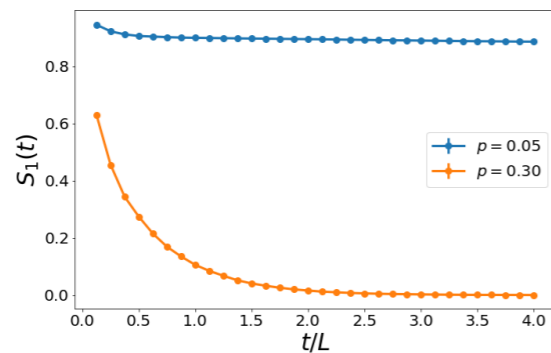
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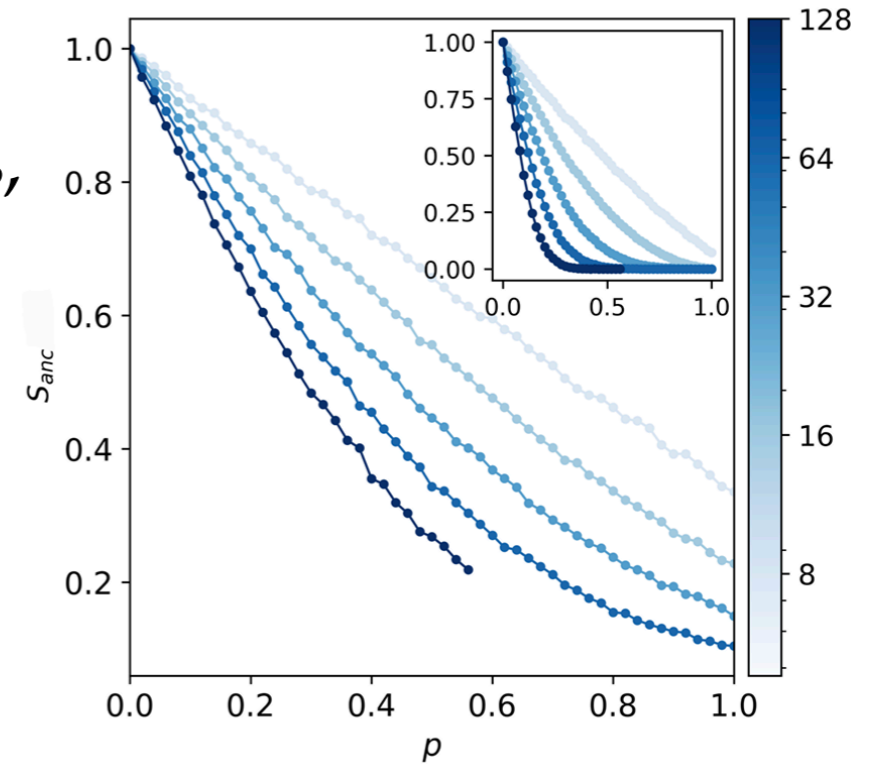
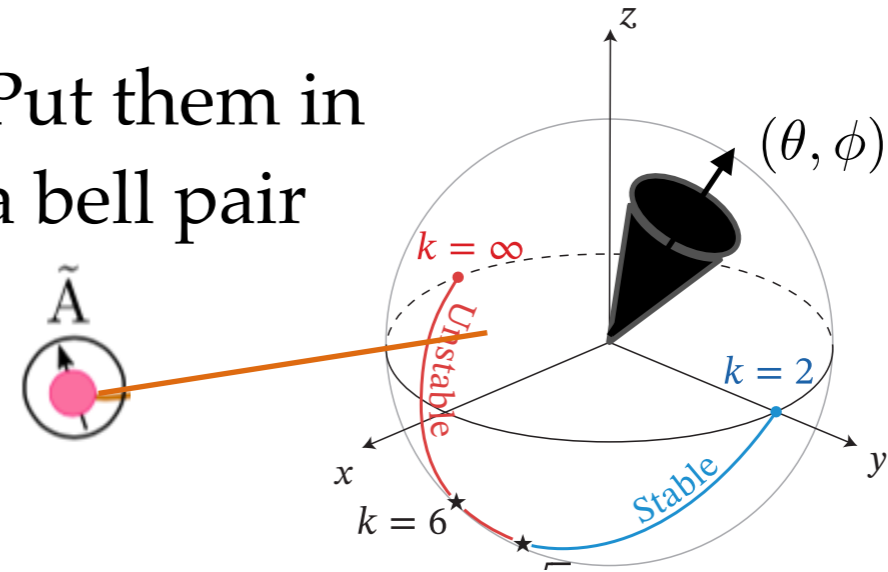
Monitored quantum circuit



Ancilla just purifies,
never entangles!

Now couple it to the kicked top

Put them in
a bell pair



CONTROLLING THE QUANTUM KICKED TOP

Absence of an entangling phase

We can also compute the entanglement of the quantum kicked top by decomposing the big S spin into several spin-1/2s

$$\hat{J}_\alpha = \frac{1}{2} \sum_{i=1}^N \hat{\sigma}_i^\alpha, \quad \alpha = x, y, z,$$

We can now make a partial trace in this space of states

$$|\psi\rangle = \sum_{m=-S}^S \psi_m |m\rangle$$

Basis of J_z eigenstates

$$\begin{aligned} |m\rangle &\equiv |D_{m+N/2}^N\rangle \\ &= \frac{1}{\sqrt{\binom{N}{m+N/2}}} \sum_{x \in \{0,1\}^N |w(x)=m+N/2} |x\rangle \end{aligned}$$

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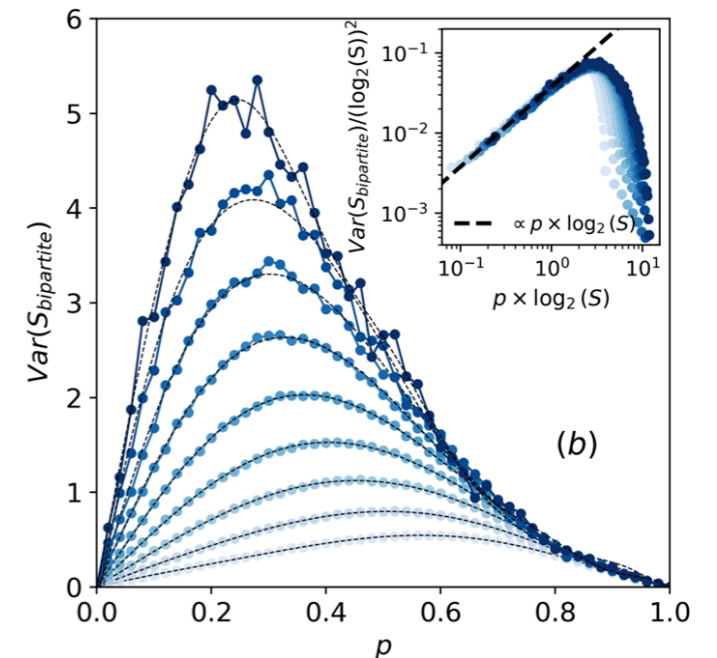
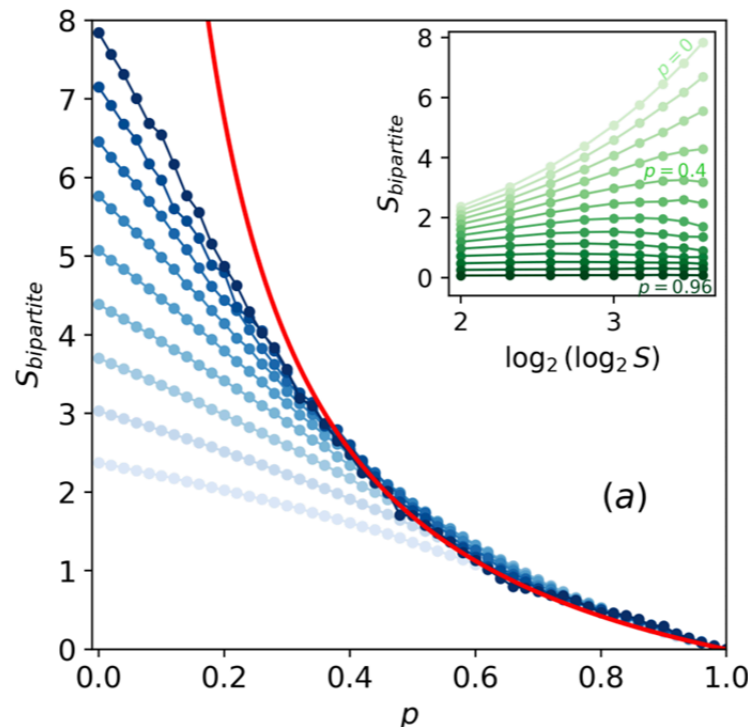
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SUMMARY: CONTROLLING THE QUANTUM KICKED TOP

Successfully embedded a control transition into the quantum kicked top.

Note, a consequence of our results is that the classical limit $S \rightarrow \infty$ and the long time limit $t \rightarrow \infty$ don't commute.

Found the absence of an entangling phase, cannot encode a qubit of information into the adaptive quantum kicked top.

OUTLINE

I. Motivation

- Controlling classical chaos
- Examples of classical control transitions

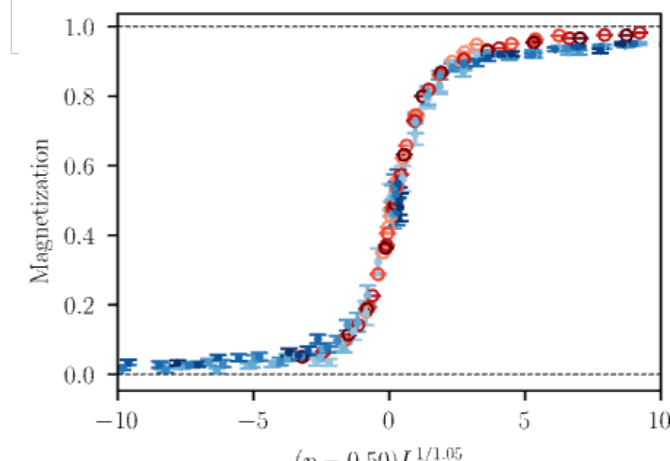
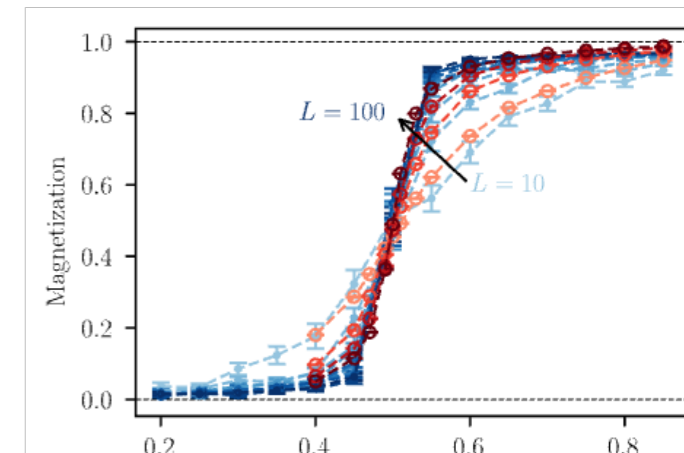
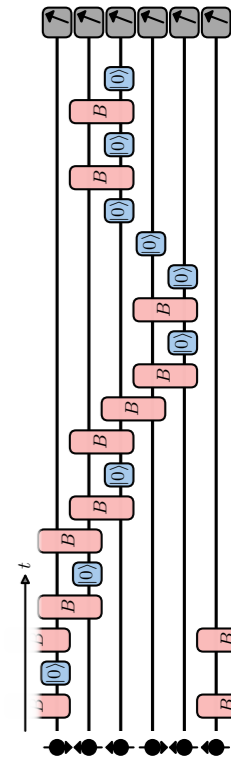
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- IBM's quantum computer
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Maybe time for a break?



QUANTUM CONTROL TRANSITION IN THE BERNOULLI MAP

Apply **Bernoulli map** $x_{n+1} = 2x_n \bmod 1$ with probability $1 - p_{\text{ctrl}}$

Apply **Control map** $x_{n+1} = (1 - a)x_j^{\text{orbit}} + ax_n$ with probability p_{ctrl}

Lets represent the
number using binary

$$x = 0.b_1b_2b_3 \dots \quad b_i \in \{0, 1\}$$

QUANTUM CONTROL TRANSITION IN THE BERNOULLI MAP

Apply **Bernoulli map** $x_{n+1} = 2x_n \bmod 1$ with probability $1 - p_{\text{ctrl}}$

Apply **Control map** $x_{n+1} = (1 - a)x_j^{\text{orbit}} + ax_n$ with probability p_{ctrl}

Lets represent the
number using binary $x = 0.b_1b_2b_3 \dots \quad b_i \in \{0, 1\}$

The basis of the quantum system can
then be represented with a series of qubits $|x\rangle = |b_1b_2b_3 \dots\rangle$

QUANTUM CONTROL TRANSITION IN THE BERNOULLI MAP

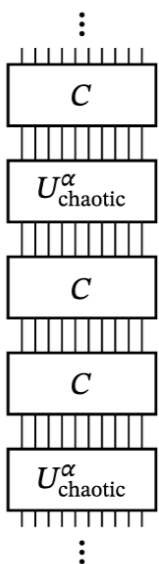
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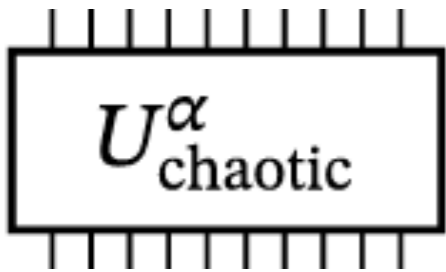
The model can be written as a quantum
circuit! The control map **quantization**
requires measurement and feedback

QUANTIZING THE BERNOULLI MAP

Consider the state of the quantum system $|x\rangle = |b_1 b_2 b_3 \dots\rangle$

$x \rightarrow 2x \pmod{1}$ on the infinite bit string $x = 0.b_1 b_2 b_3 \dots$

$$2x \pmod{1} = 0.b_2 b_3 b_4 \dots$$

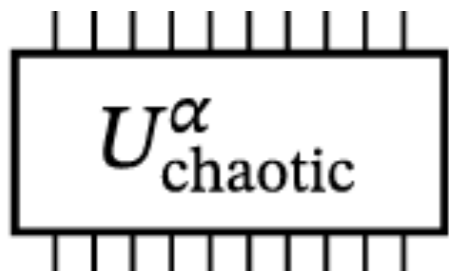


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$$U_{\text{chaotic}}^\alpha = S_\alpha T$$

Translation (with periodic bcs)

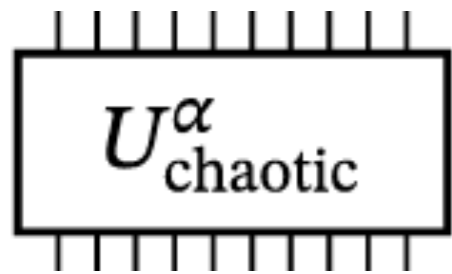
$$T |b_1 b_2 \dots b_L\rangle = |b_2 b_3 \dots b_L b_1\rangle$$

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“scrambling operation”

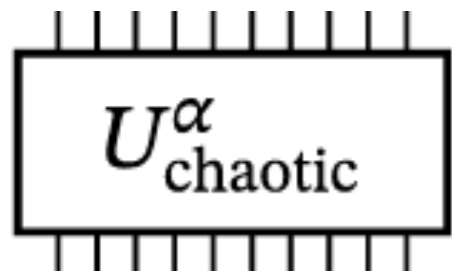
Classical model: 3-site random permutation on the last 3 qubits

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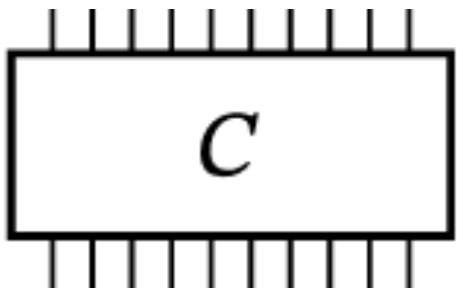
Classical model: 3-site random permutation on the last 3 qubits

Quantum model: 2-site Haar random unitary choose to act on the last 2 qubits

QUANTIZING THE CONTROL

MAP: NEEL STATE

Consider the state of the quantum system $|x\rangle = |b_1 b_2 b_3 \dots\rangle$



$$x \rightarrow \underline{x/2} + \underline{x_j/2} \quad \text{if } x \in \Delta_j \quad a = 1/2$$

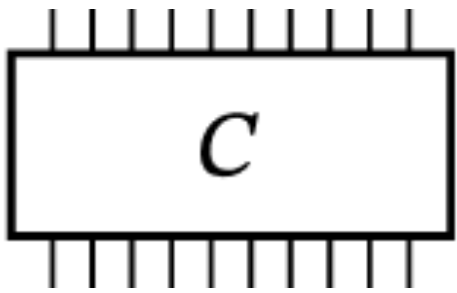
Lets start with controlling onto the two Neel states

$$|x_j\rangle = \{ |x_0\rangle = |1/3\rangle = |0101 \dots 01\rangle, \\ |x_1\rangle = |2/3\rangle = |1010 \dots 10\rangle \}$$

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Break into 2 steps

$$\underline{|x\rangle} \mapsto \underline{|2^{-1}x\rangle} \quad \underline{|2^{-1}x\rangle} \mapsto \underline{|2^{-1}x + 2^{-1}x_j\rangle}$$

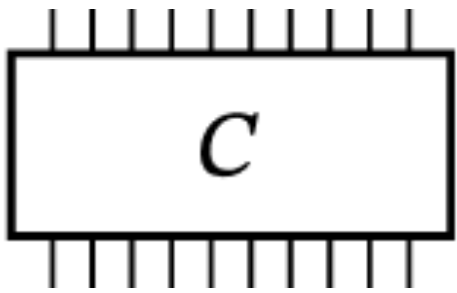
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$$C = \underline{A_{\text{ctrl}}} \underline{T^{-1}} R_L$$

measurement
+feedback!

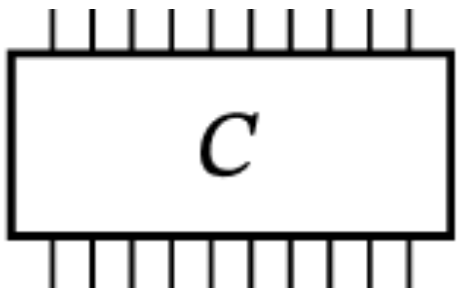
$$R_L |\psi\rangle = \begin{cases} \frac{P_L^0 |\psi\rangle}{\|P_L^0 |\psi\rangle\|} & \text{with probability } \|P_L^0 |\psi\rangle\|^2 \\ \frac{X_L P_L^1 |\psi\rangle}{\|P_L^1 |\psi\rangle\|} & \text{with probability } \|P_L^1 |\psi\rangle\|^2 \end{cases}$$

$$\underline{T^{-1} |b_1 b_2 \dots b_{L-1} 0\rangle} = |0 b_1 b_2 \dots b_{L-1}\rangle$$

QUANTIZING THE CONTROL

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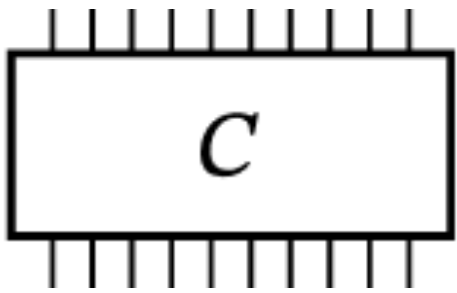
controlled “adder”

$$A_{\text{ctrl}} |x\rangle = \begin{cases} |x + 0.00101 \dots 011\rangle & \text{if } b_2 = 0 \\ |x + 0.01010 \dots 101\rangle & \text{if } b_2 = 1 \end{cases}$$

QUANTIZING THE CONTROL

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$$C = \underline{A_{\text{ctrl}}} \underline{T^{-1}} \underline{R_L}$$

Quantum realization of this adder is global!

$$A_{\text{ctrl}} |x\rangle = \begin{cases} |x + \underline{0.00101 \dots 011}\rangle & \text{if } b_2 = 0 \\ |x + \underline{0.01010 \dots 101}\rangle & \text{if } b_2 = 1 \end{cases} = \text{[Circuit Diagram]} |x + a\rangle \langle x|$$

has to act on all the qubits

QUANTUM CONTROL (ONTO THE NEEL STATE) TRANSITION

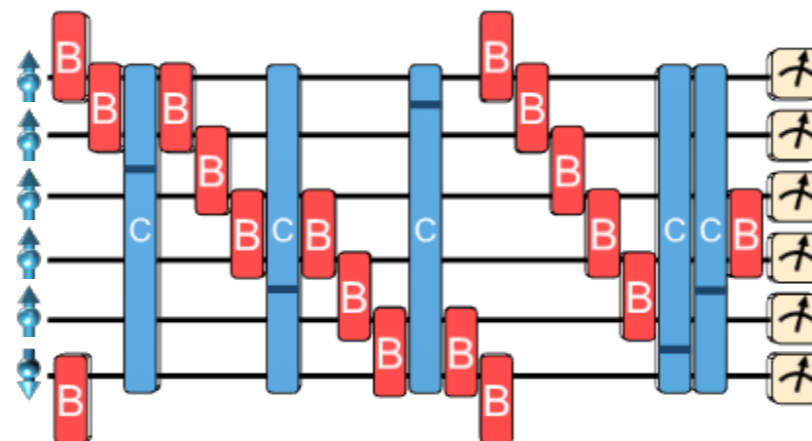
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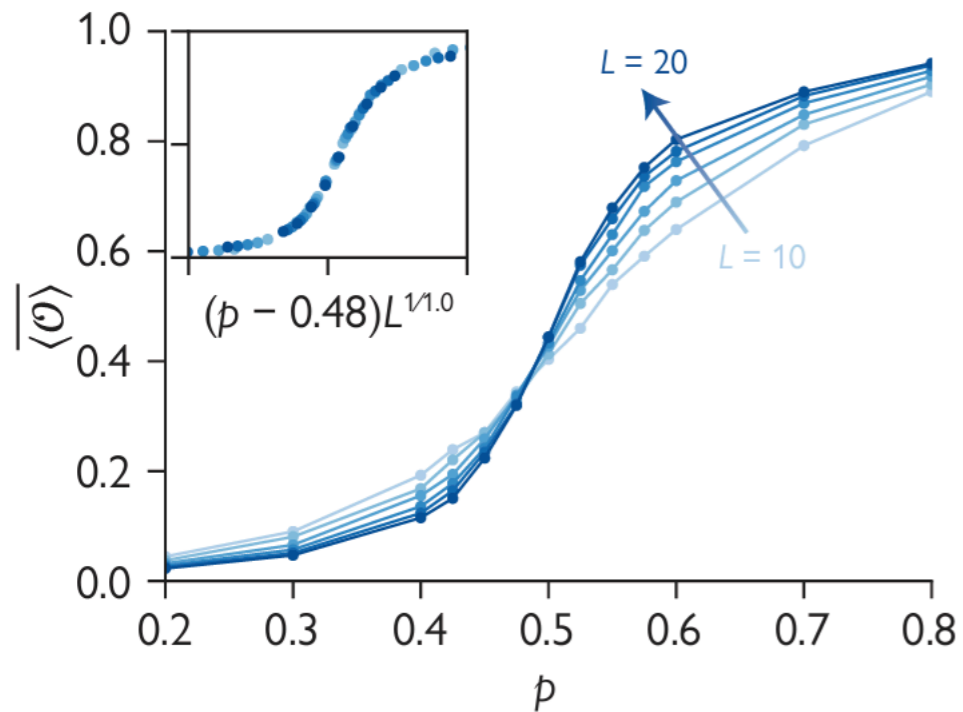


QUANTUM CONTROL (ONTO NEEL STATE) TRANSITION

Local order parameter

$$\mathcal{O} = -\frac{1}{L} \sum_{i=1}^L Z_i Z_{i+1}$$

$$\nu \approx 1$$

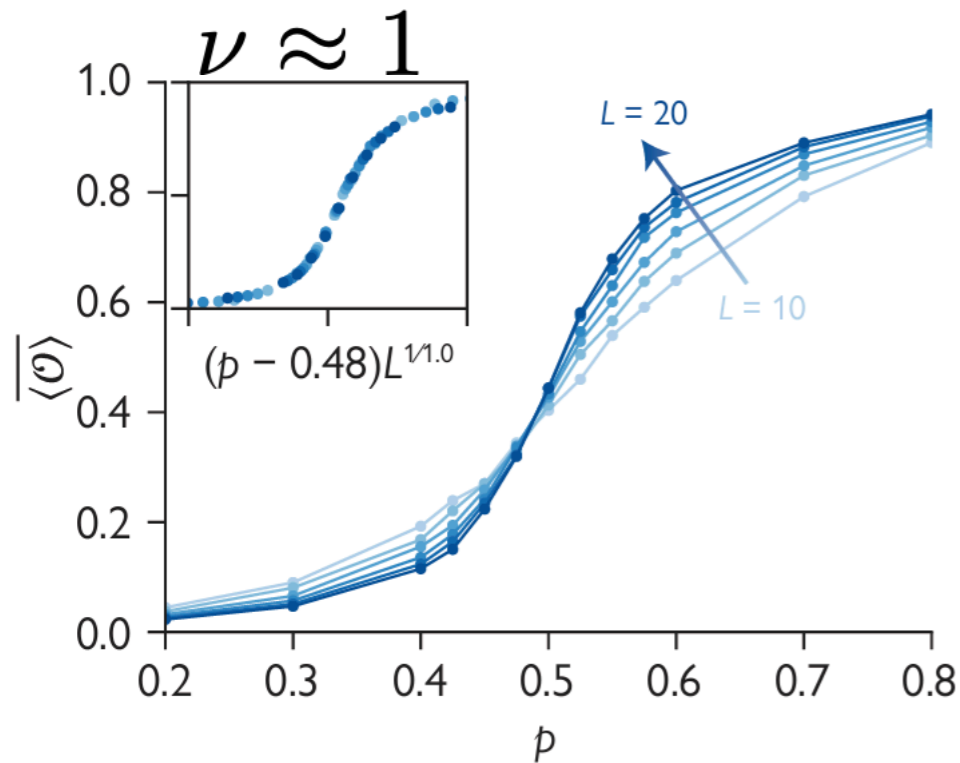


Clear signature of the transition!

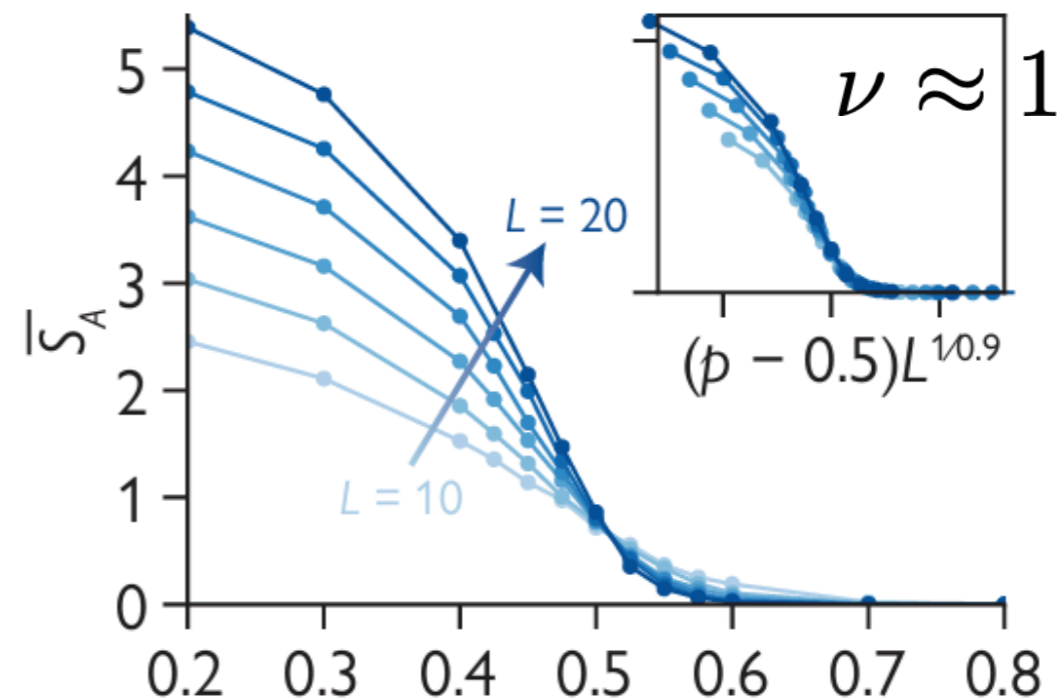
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Entanglement entropy



$$\bar{S}(t \gg \overset{p}{L}) \sim L^0$$

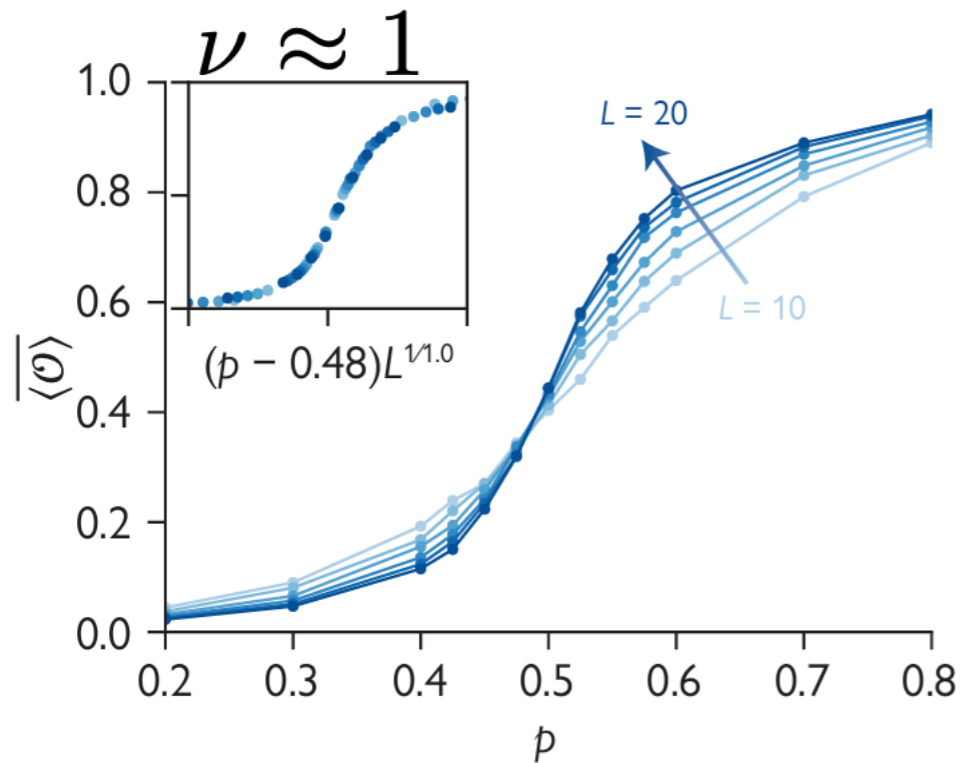
Area law at the transition!

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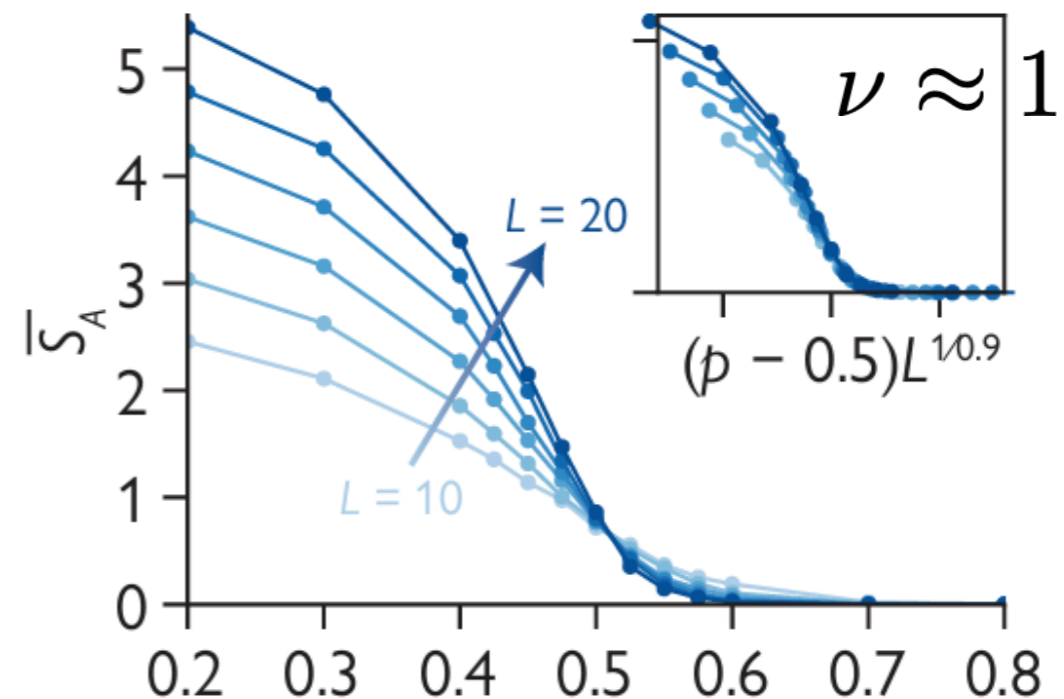
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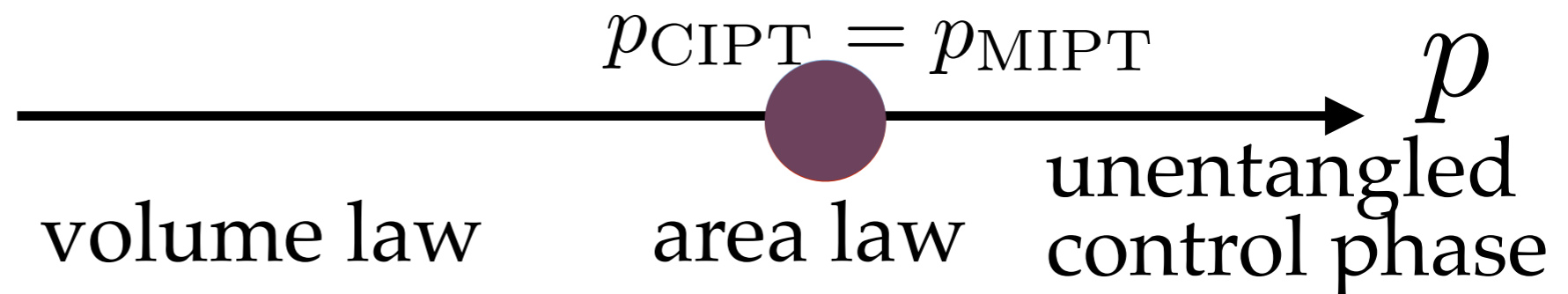
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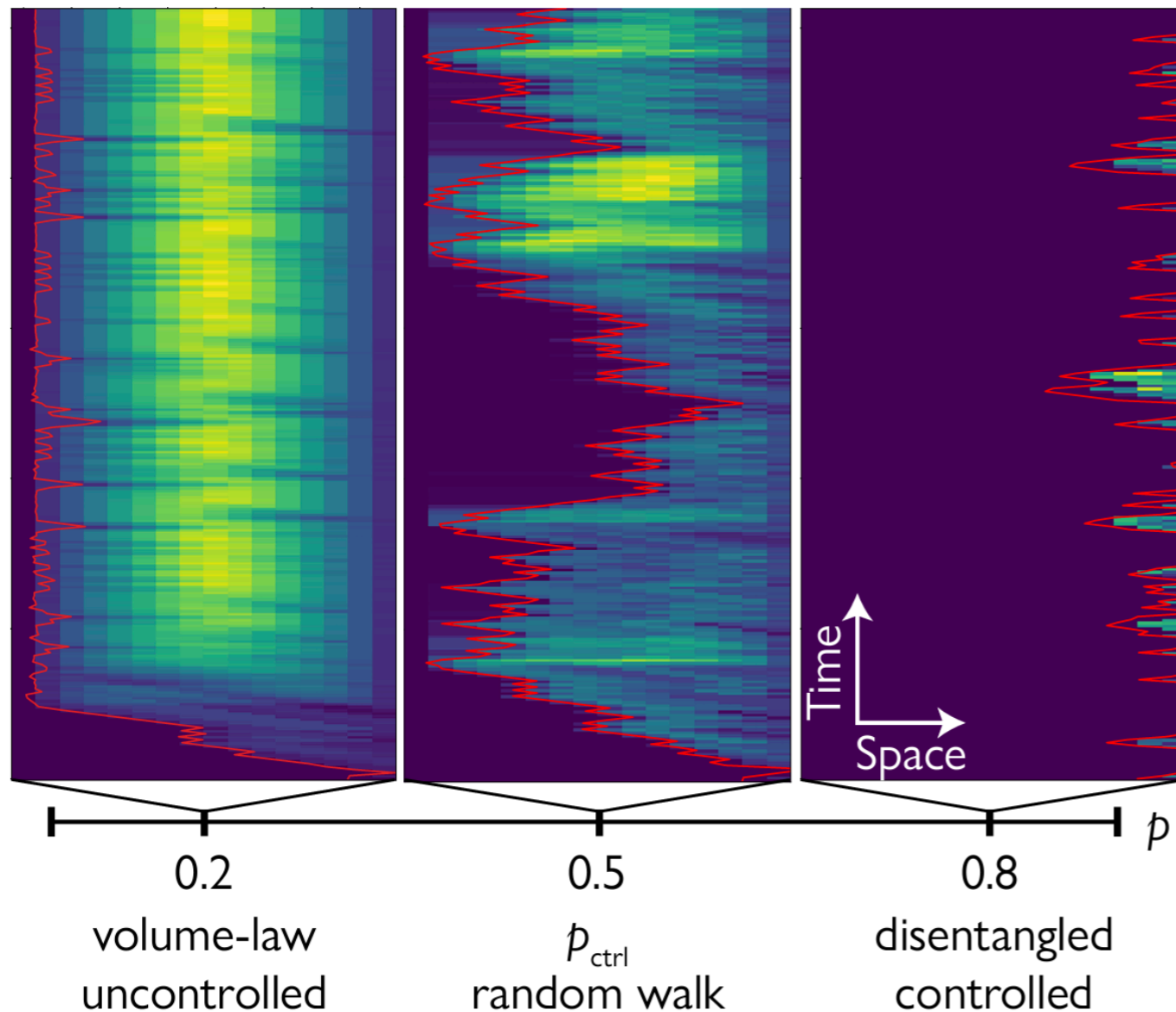


QUANTUM CONTROL (ONTO NEEL STATE) TRANSITION: DYNAMICS

Brighter color = larger entanglement entropy

Red line = avg. location of 1st domain wall

Single trajectories



First domain wall

$|010101011011100\dots\rangle$

Controlled

Uncontrolled

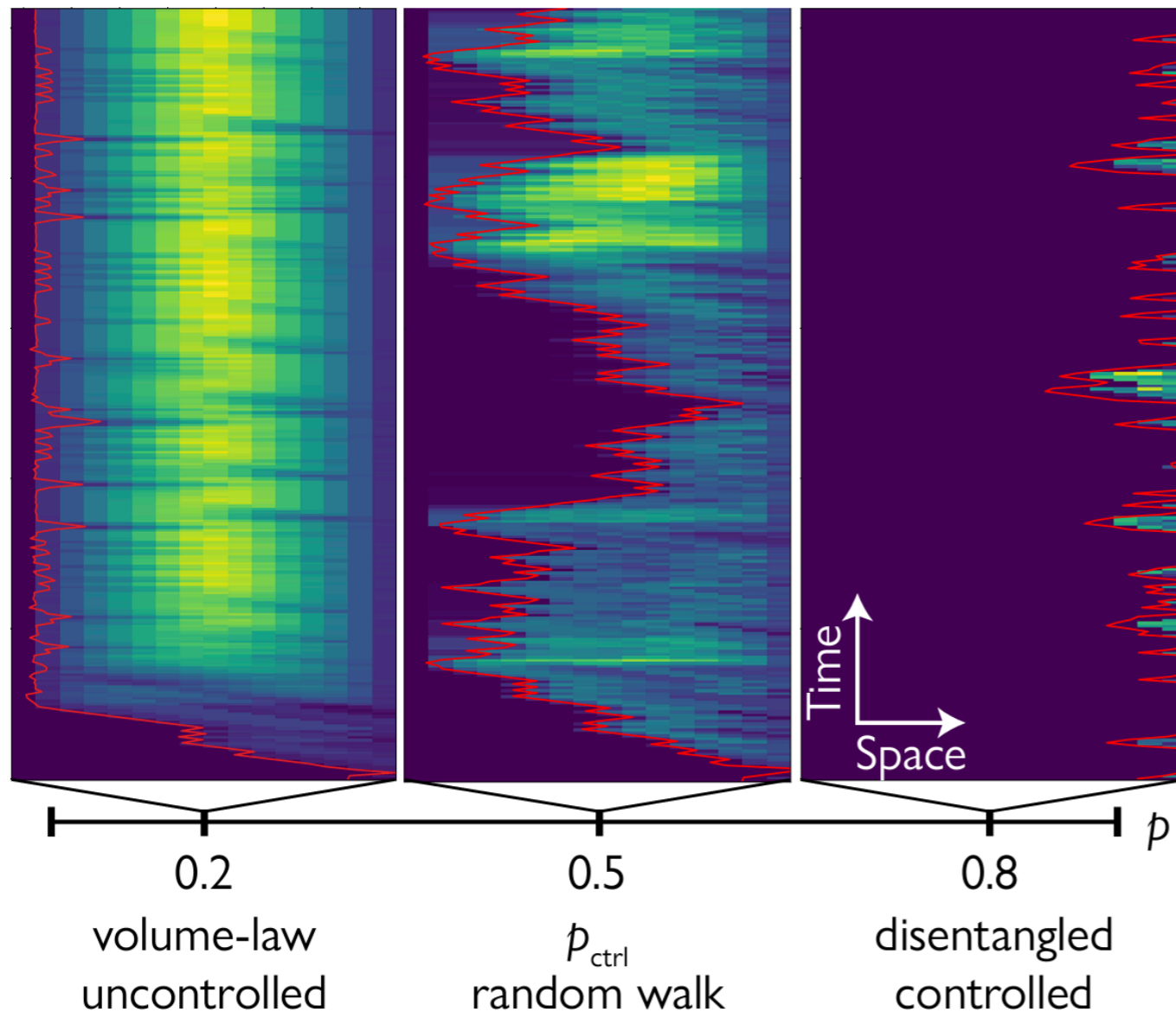
Becomes a wave packet
in the quantum limit

$$\langle r(t) \rangle = \sum_x |\langle x | \psi(t) \rangle|^2 r_x$$

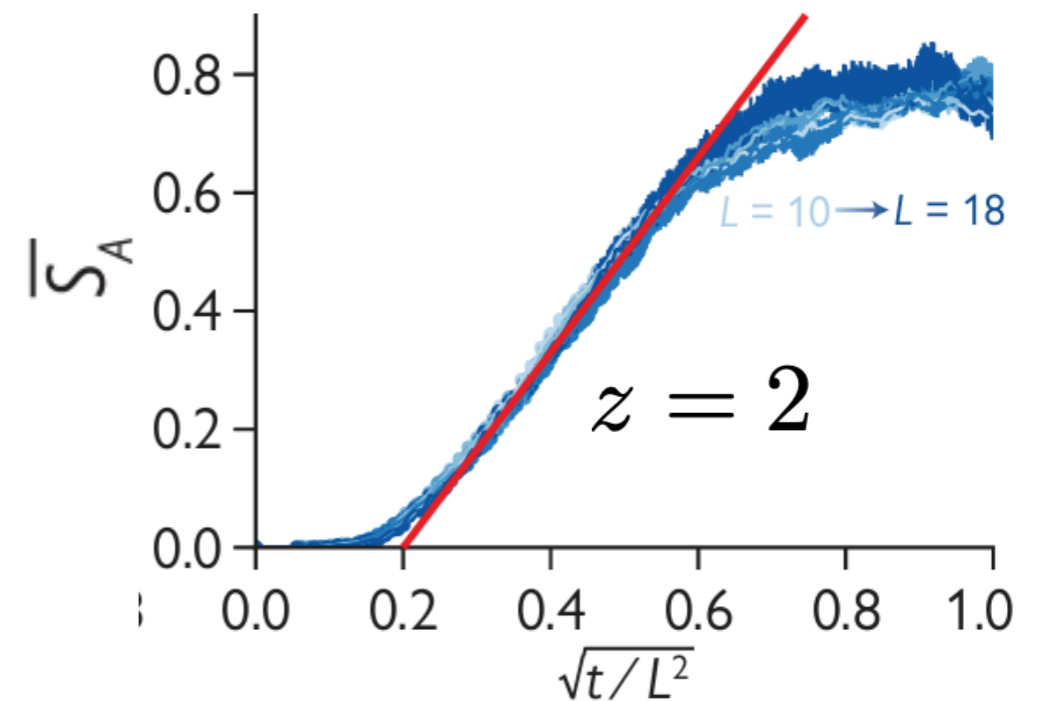
QUANTUM CONTROL (ONTO NEEL STATE) TRANSITION: DYNAMICS

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Single trajectories



Entanglement entropy dynamics at the transition



Diffusive growth

$$\bar{S}(t \ll L) \sim \sqrt{t/L^2}$$

The transition is diffusive,
 random walk universality class
 $\nu \approx 1 \quad z = 2$

STRUCTURE OF CLASSICAL CHAOS IN THE WAVEFUNCTION

Decompose the wavefunction in terms of the fixed point overlap

$$|\psi(t)\rangle = \sum_{j \in x_j} a_j |x_j\rangle + \sum_{x \neq x_j} b_x |x\rangle$$

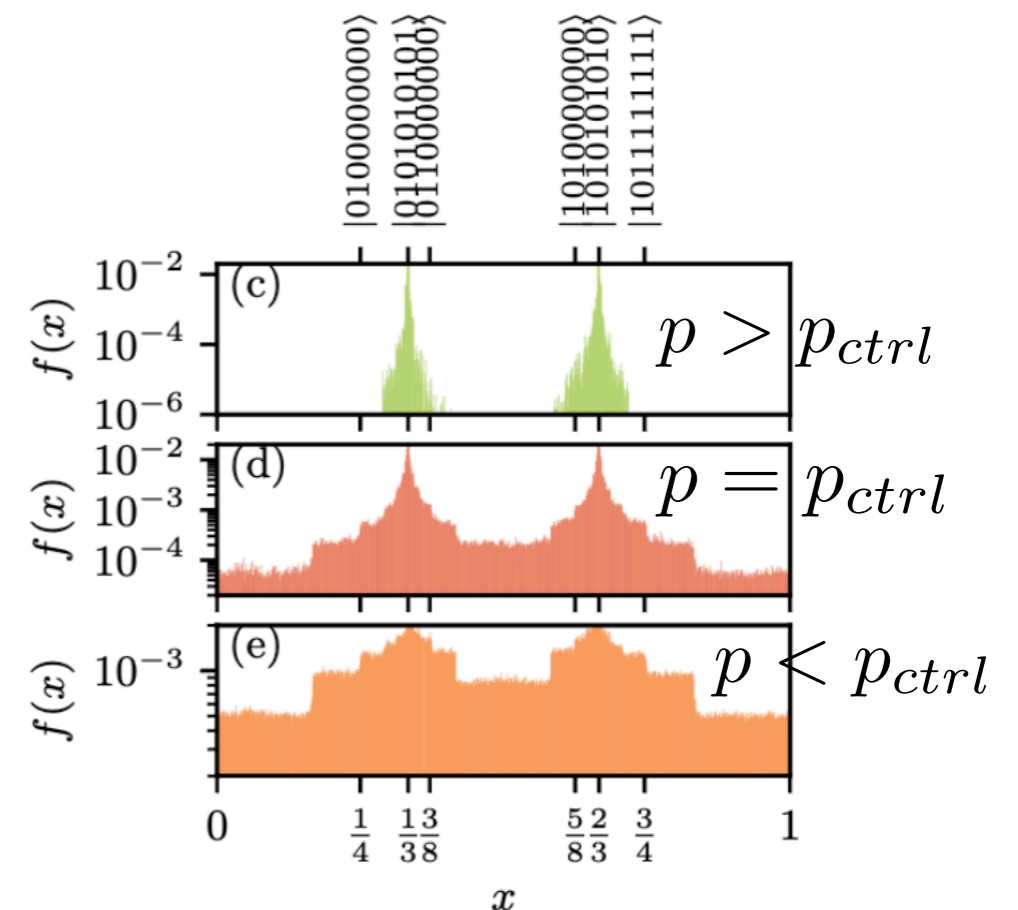
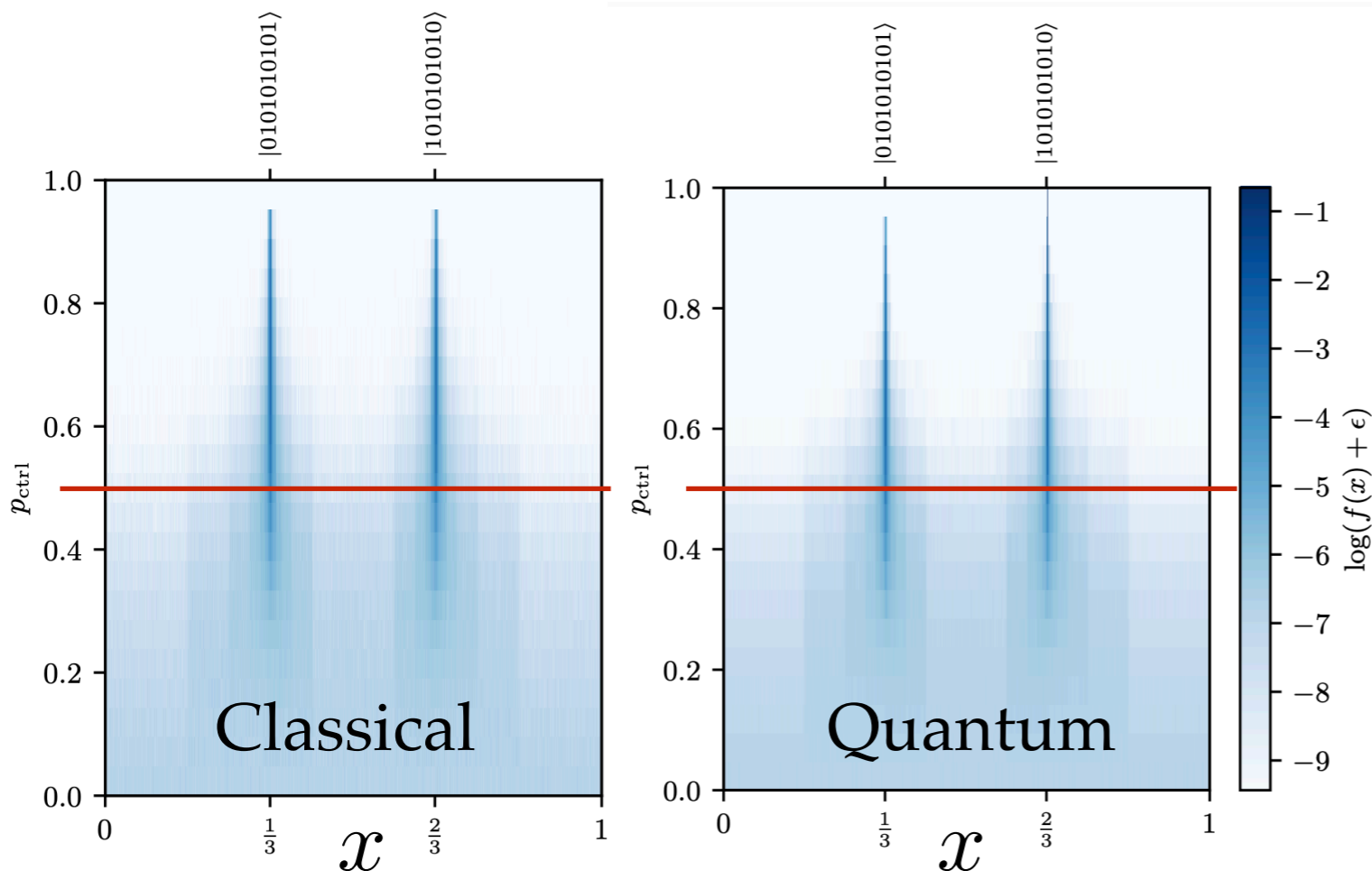
Bit string distribution in the steady state $f(x) = \lim_{t \rightarrow \infty} \overline{|\langle x | \psi(t) \rangle|^2}$

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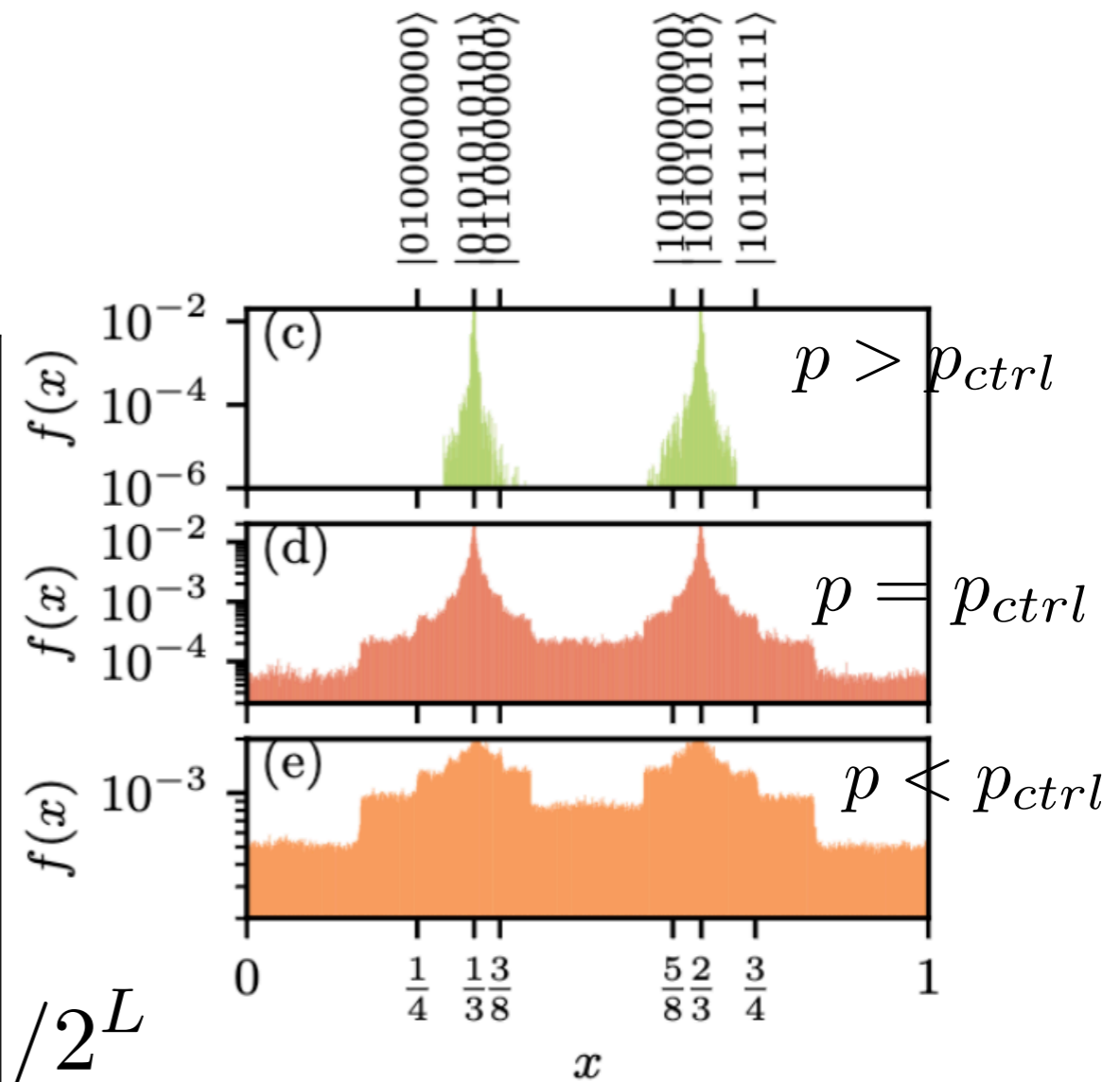
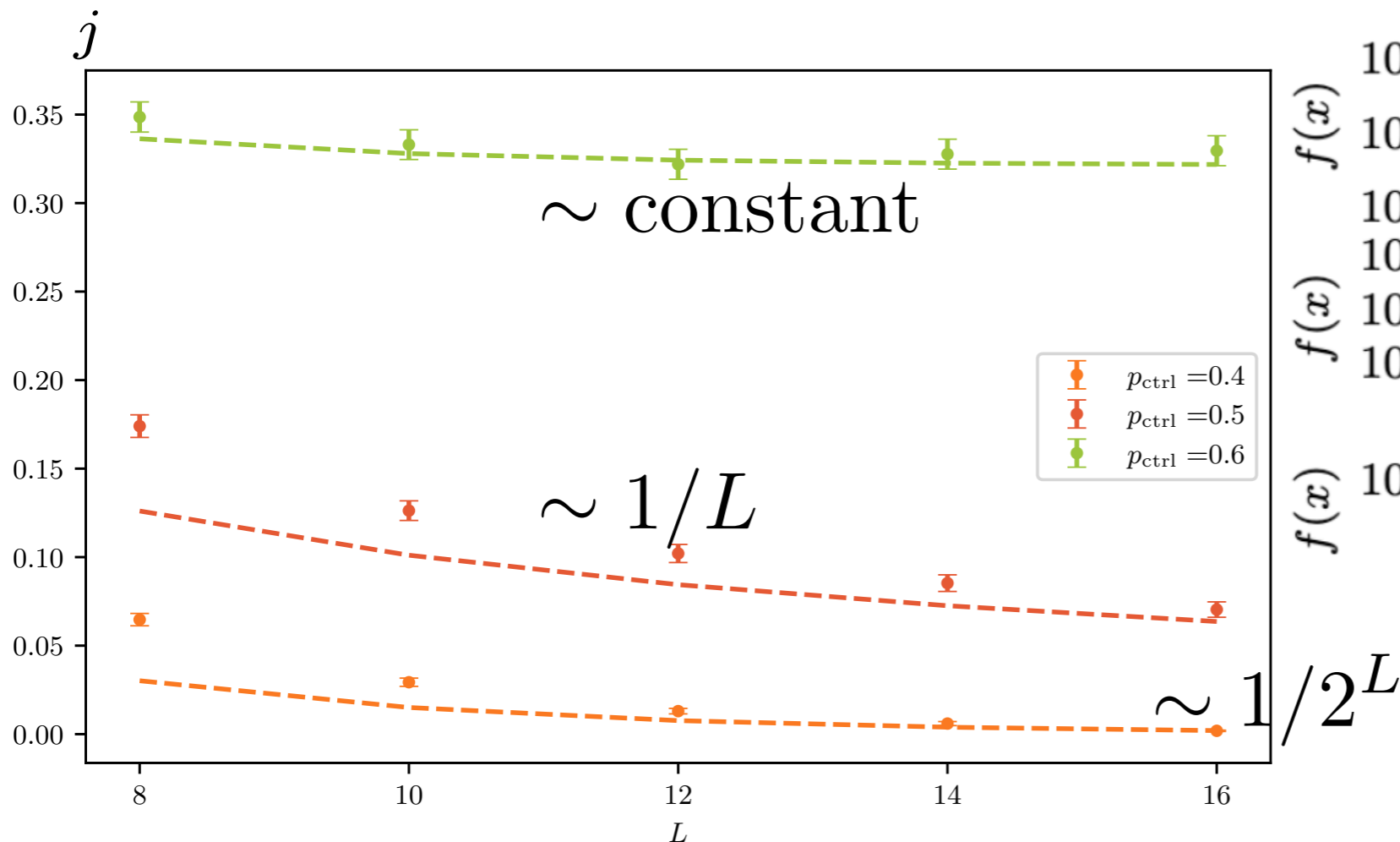


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$$\sum_j \overline{|a_j|^2}$$

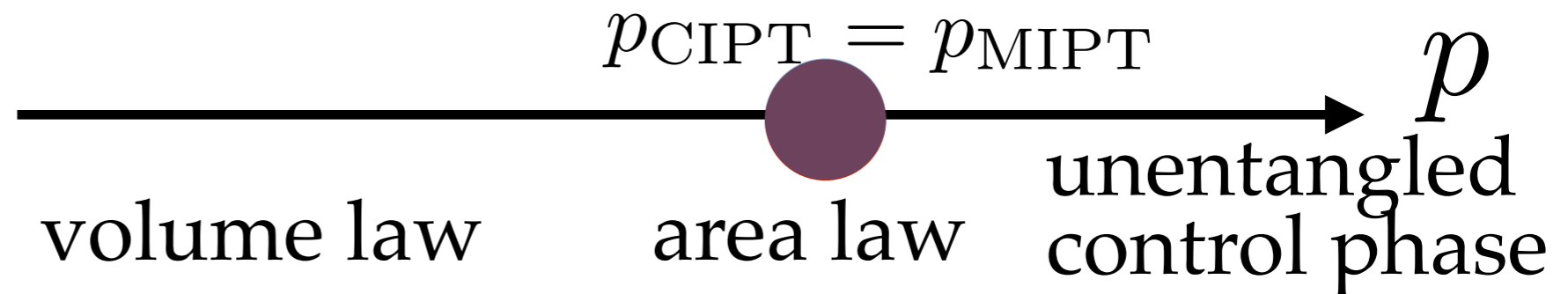
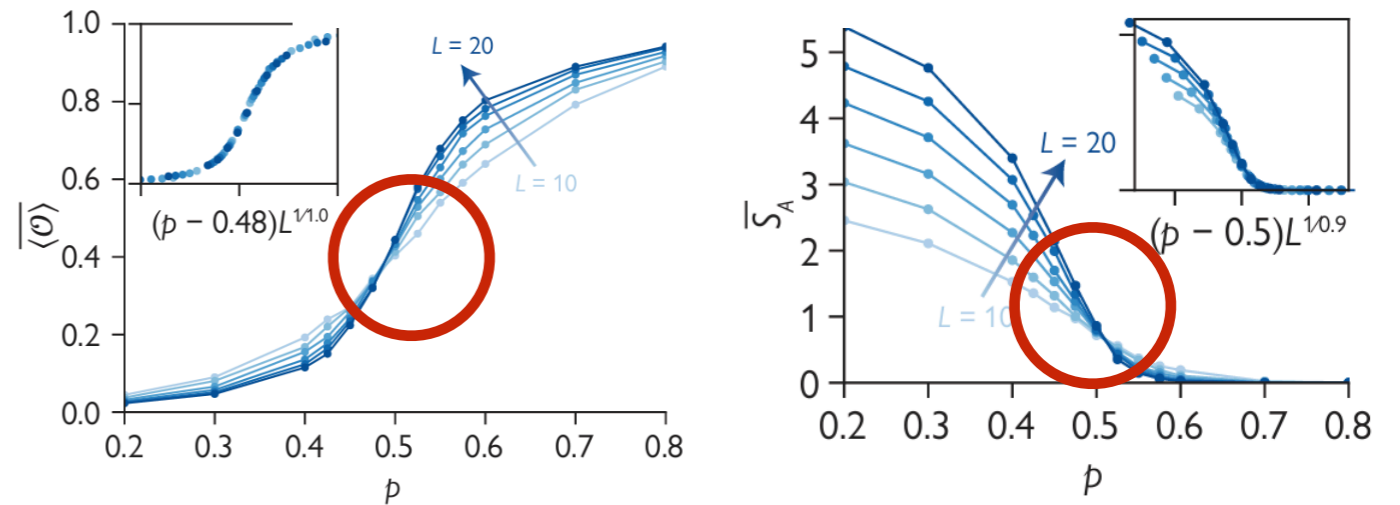


SUMMARY: QUANTUM CONTROL TRANSITION

Here, the control and entanglement transitions coincide.

Is this fundamental?

$$\mathcal{O} = -\frac{1}{L} \sum_{i=1}^L Z_i Z_{i+1}$$

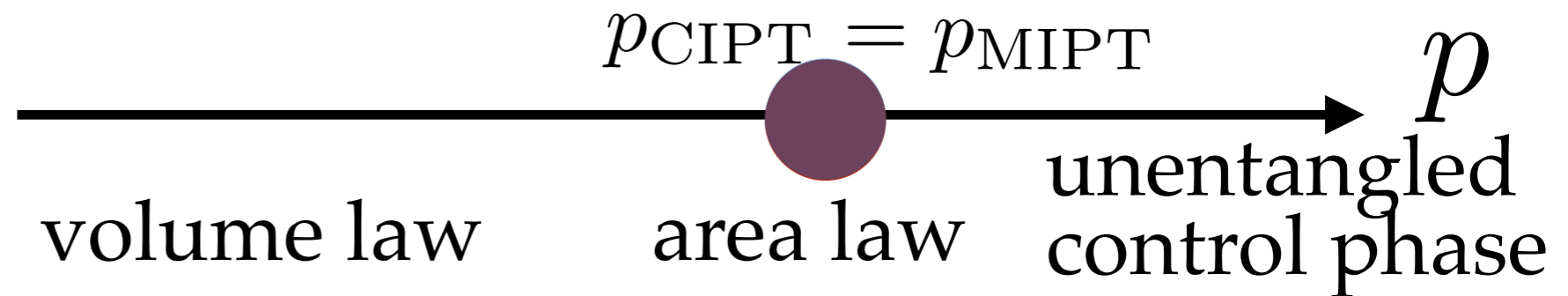
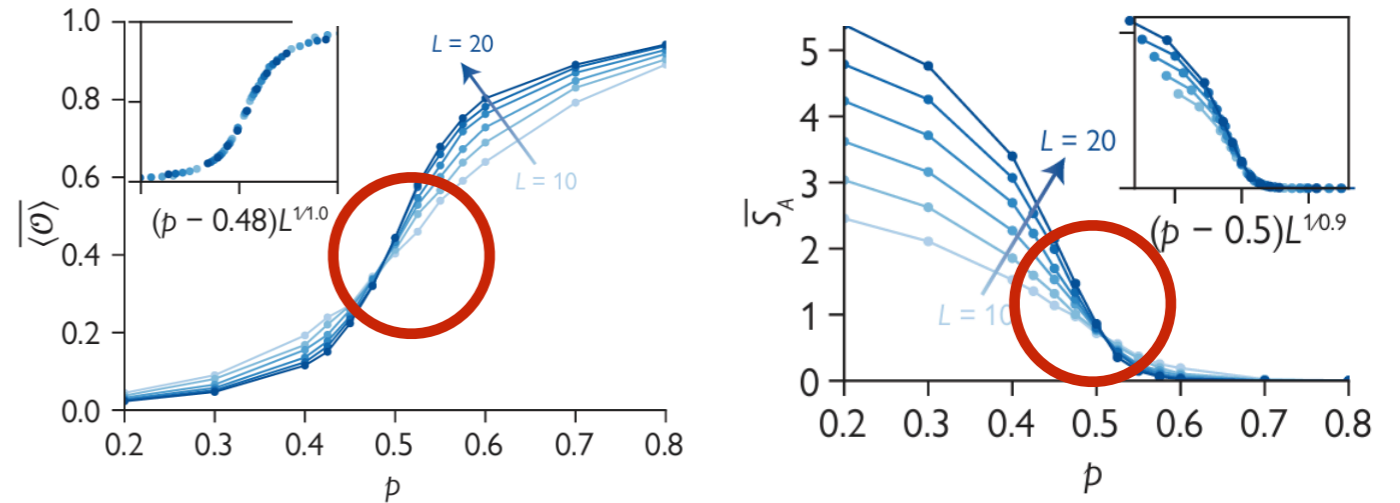


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$$\mathcal{O} = -\frac{1}{L} \sum_{i=1}^L Z_i Z_{i+1}$$



Can we “pull” them apart?

Can we construct stabilizer models to reach larger system sizes?

QUANTUM CONTROL (ONTO THE FERROMAGNET) TRANSITION

Lets represent the number using binary

$$x = 0.b_1b_2b_3 \dots \quad b_i \in \{0, 1\}$$

The basis of the quantum system can then be represented with a series of qubits

$$|x\rangle = |b_1b_2b_3 \dots\rangle$$

Now lets consider controlling onto the **ferromagnet**

$$|x_0\rangle = |0.0\rangle \approx |00 \dots 0_L\rangle \quad \text{With } L \text{ qubits}$$

Truncating the bit string

Pan, Iadecola, Ganeshan, JHP PRB (2024)

Iadecola, Ganeshan, JHP, Wilson PRL (2023)

QUANTUM CONTROL (ONTO THE FERROMAGNET) TRANSITION

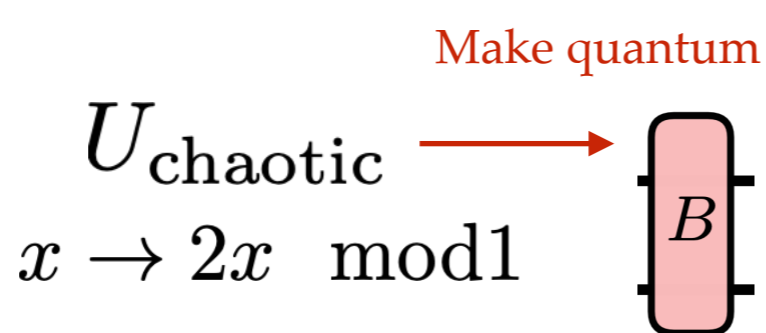
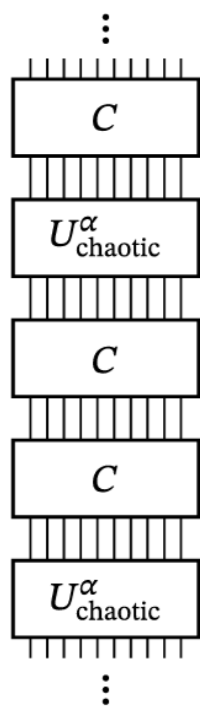
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$|b_1 \dots b_L\rangle \rightarrow |b_2 b_1 \dots b_L b_1\rangle$
 Then apply **Haar random quantum gate**
 Or random stabilizer gate on $b_L b_1$

Pan, Iadecola, Ganeshan, JHP PRB (2024)

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QUANTUM CONTROL (ONTO THE FERROMAGNET) TRANSITION

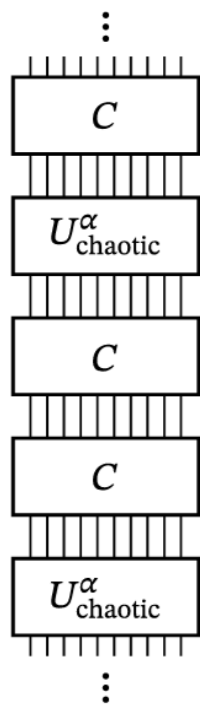
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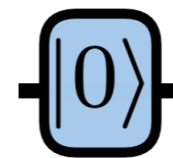
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$$C \xrightarrow{\text{Make quantum}} x \rightarrow x/2$$

$$x_j = 0 \quad a = 1/2$$



Measure b_L (L -th part of x)
 If we measure up,
flip the qubit down,
 then translate all by 1 digit

$$|b_1 \dots b_L\rangle \rightarrow |0b_1 \dots b_{L-1}\rangle$$

QUANTUM CONTROL (ONTO THE FERROMAGNET) TRANSITION

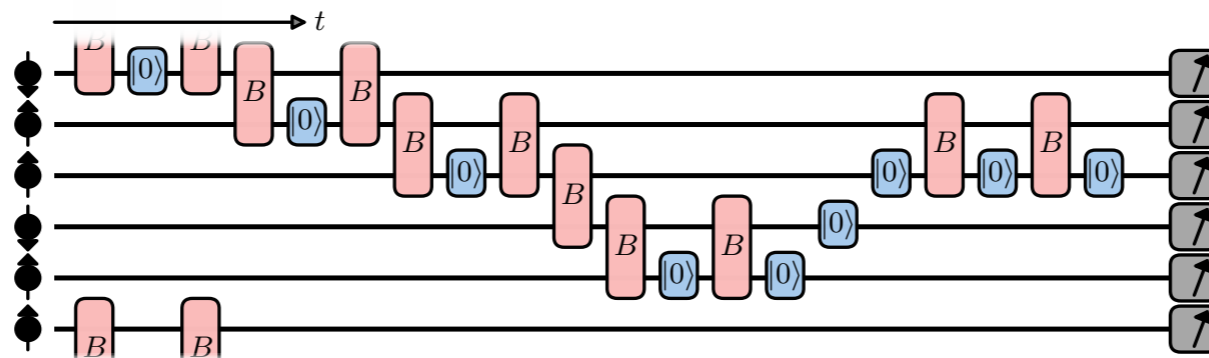
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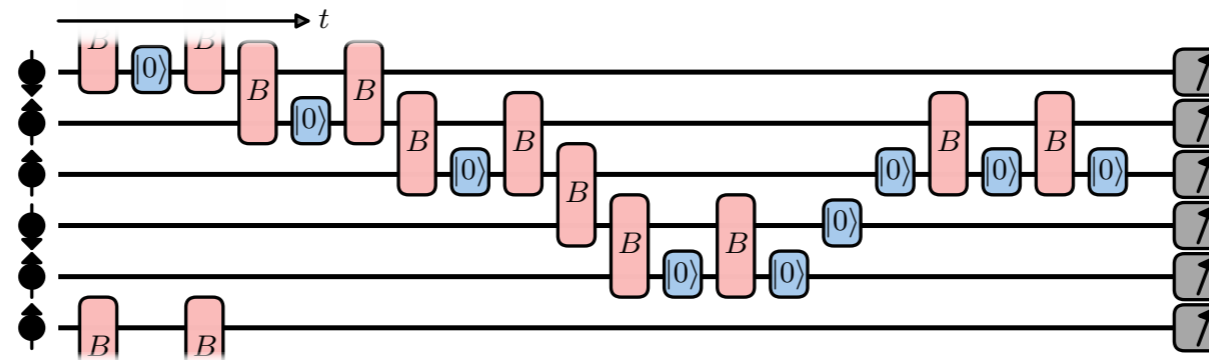
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The model can be written as an **adaptive quantum circuit!** The control map **quantization requires measurement and LOCAL feedback**

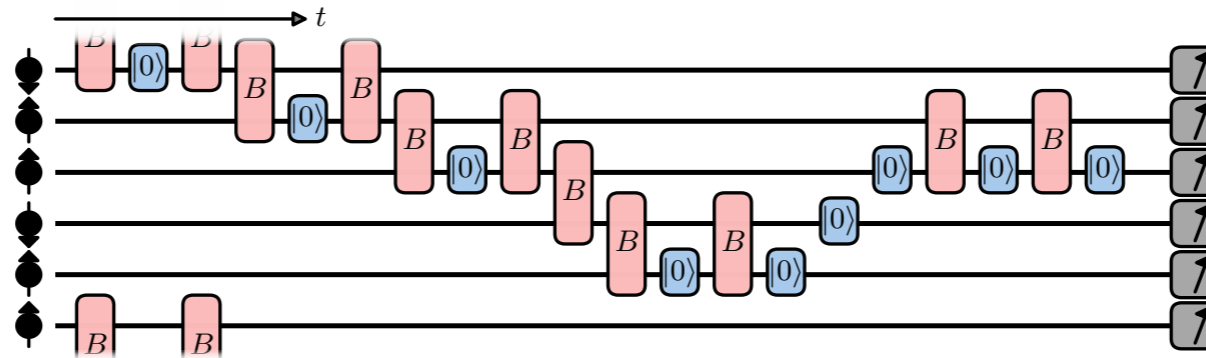


LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT



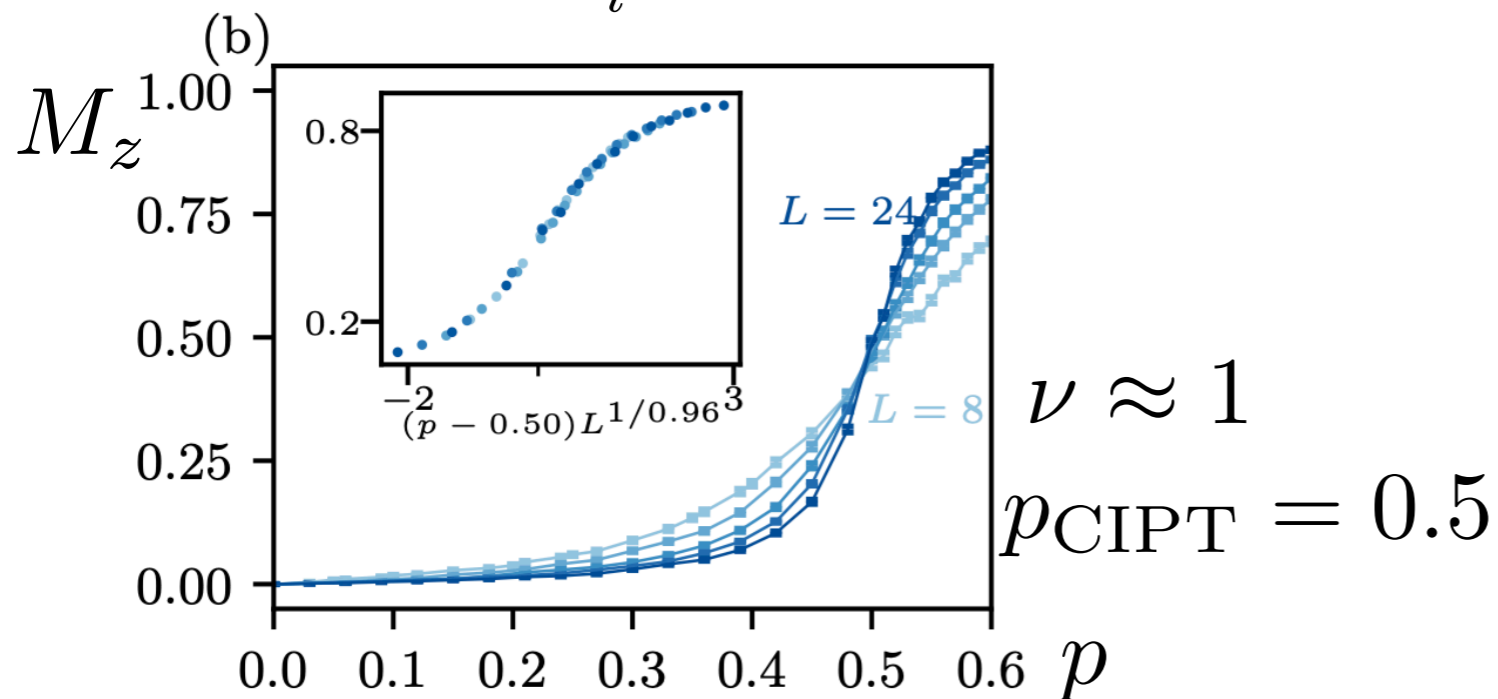
Now we are controlling onto the ferromagnetic state $|000 \dots 0\rangle$ **Control map is now local**

LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT

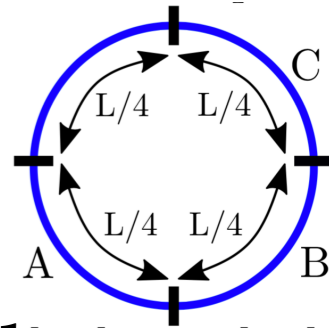
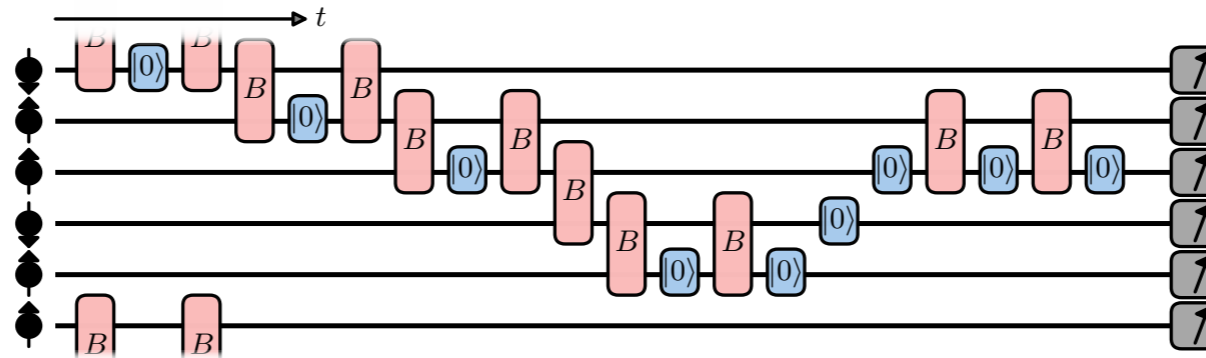


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$$M_z = \frac{1}{L} \sum_i \frac{1}{2} (1 - \sigma_i^z)$$



LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT

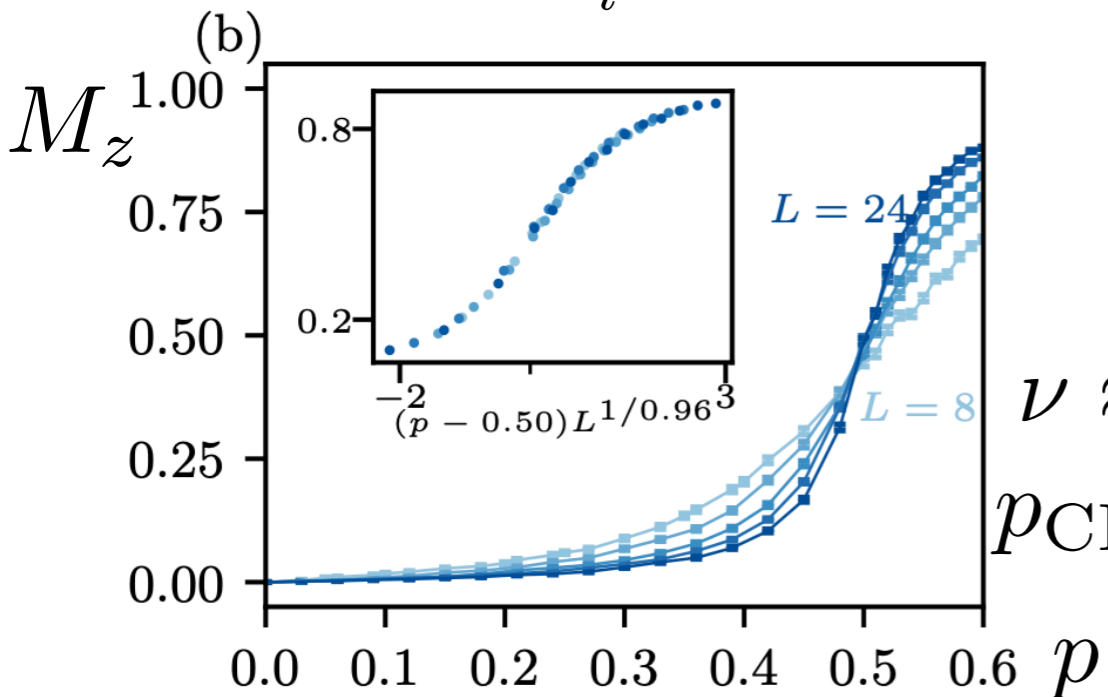


Now we are controlling onto the ferromagnetic state

$$|000 \dots 0\rangle$$

Control map is now local
Recall Lecture 3:

$$M_z = \frac{1}{L} \sum_i \frac{1}{2} (1 - \sigma_i^z) \quad \mathcal{I}_{3,n}(A, B, C) \equiv S_n(A) + S_n(B) + S_n(C) - S_n(A \cup B) - S_n(A \cup C) - S_n(B \cup C) + S_n(A \cup B \cup C). \quad (1)$$

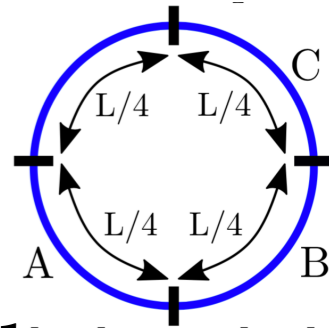
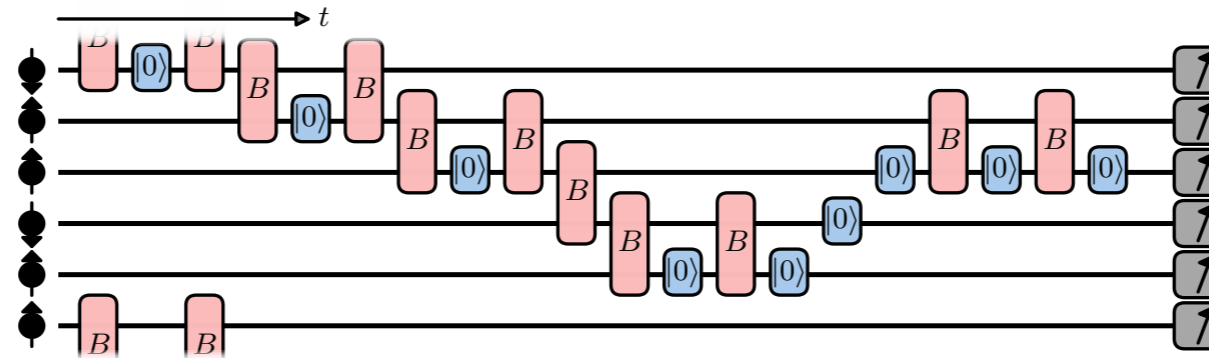


$\nu \approx 1$
 $p_{\text{CIPT}} = 0.5$

Tripartite mutual information is ideal to probe the **measurement induced phase transition**.

Data for different sizes (L) will cross.

LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT



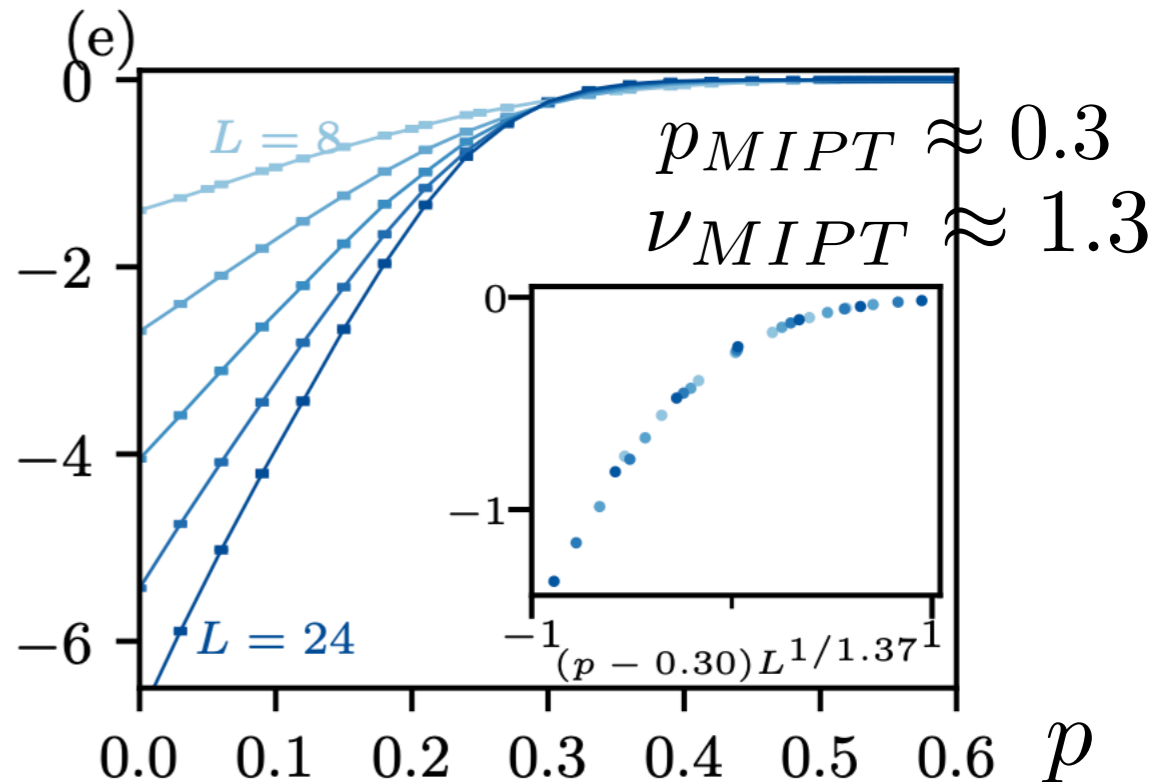
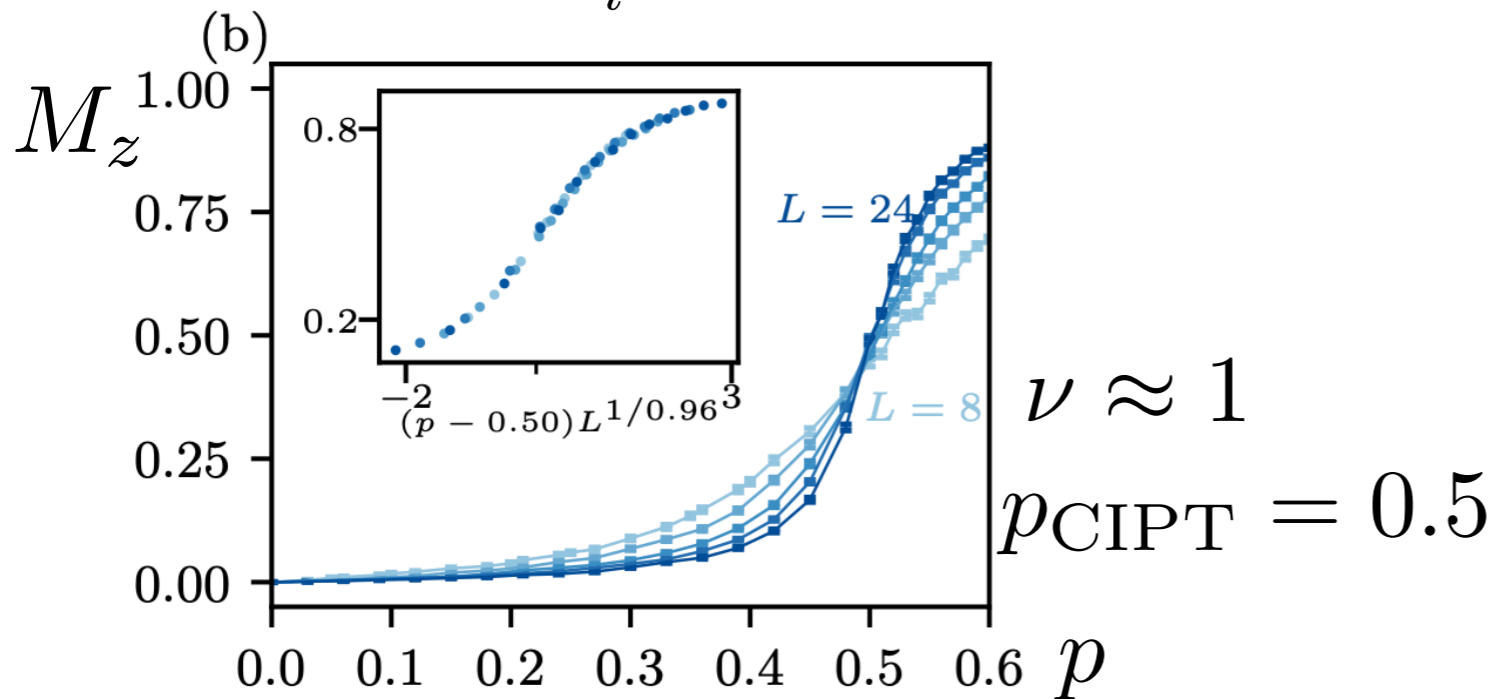
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Control map is now local

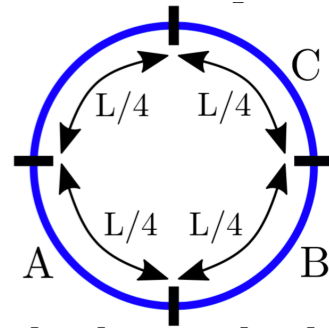
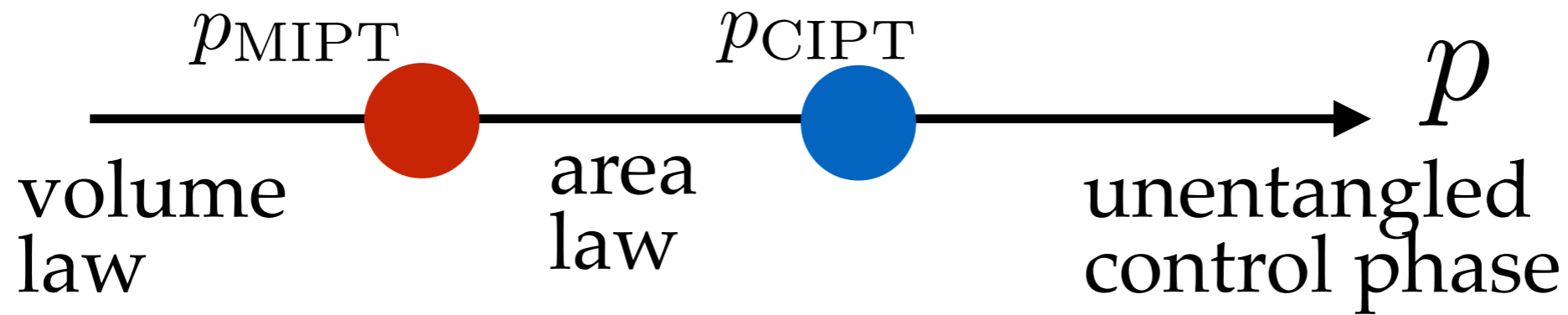
Recall Lecture 3:

$$\mathcal{I}_{3,n}(A, B, C) \equiv S_n(A) + S_n(B) + S_n(C) - S_n(A \cup B) - S_n(A \cup C) - S_n(B \cup C) + S_n(A \cup B \cup C). \quad (1)$$

$$M_z = \frac{1}{L} \sum_i \frac{1}{2} (1 - \sigma_i^z)$$



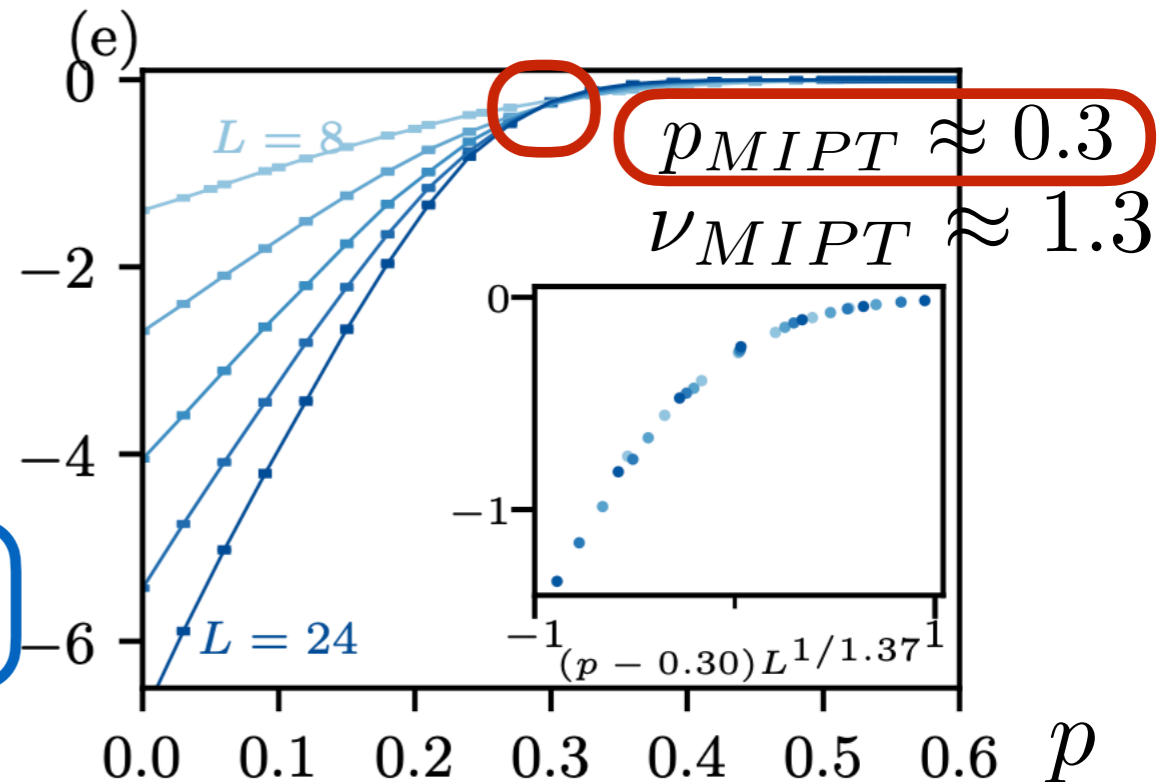
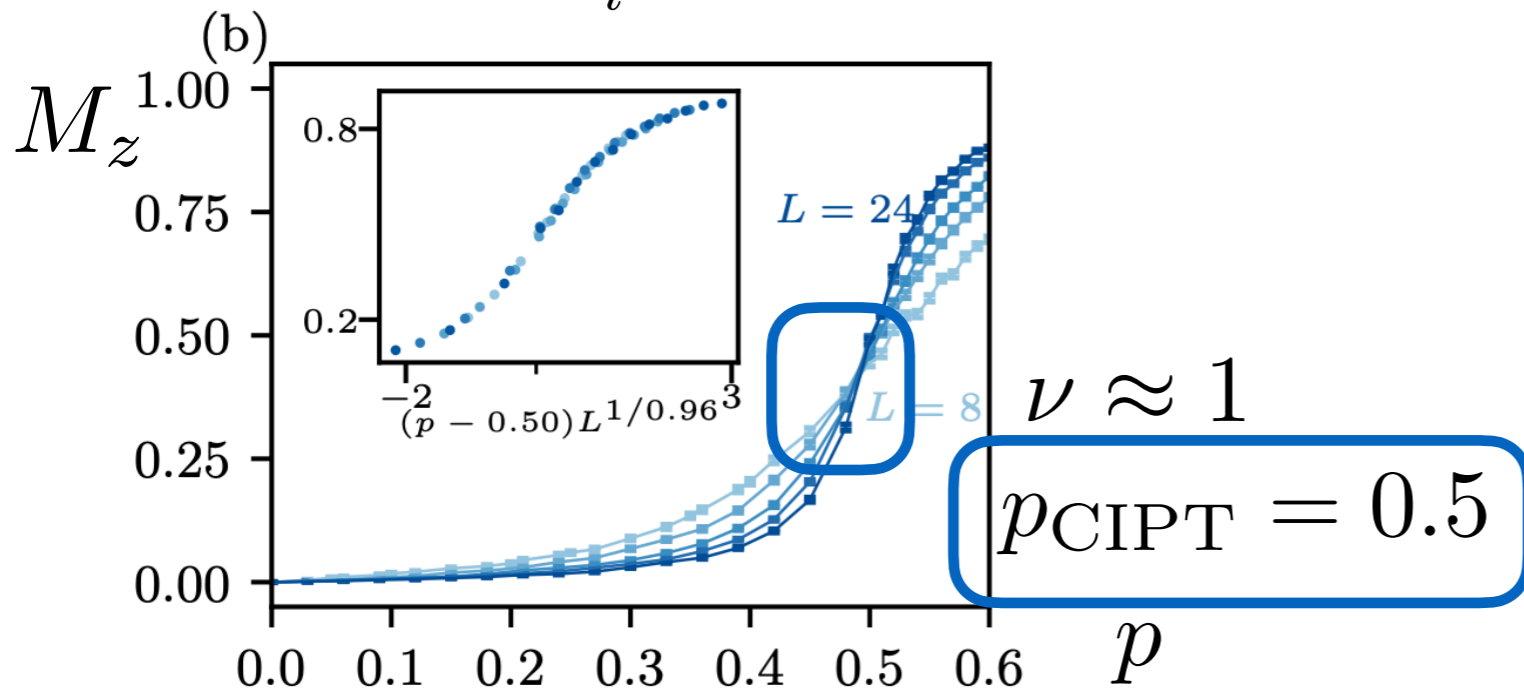
LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT



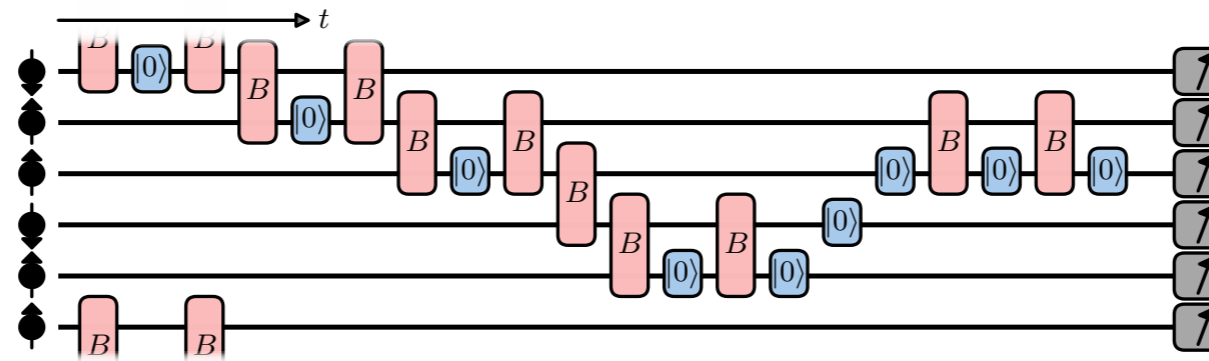
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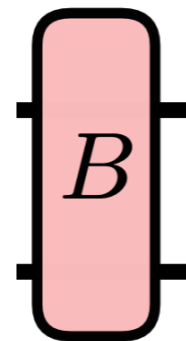


CONTROLLED LIMITS: LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT

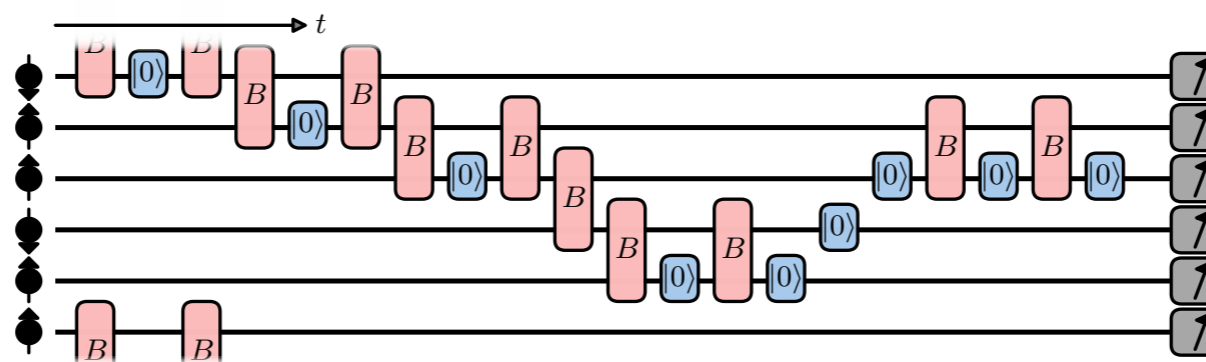


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Recall, lecture 3: Controlled limits, by moving away from Haar random gates



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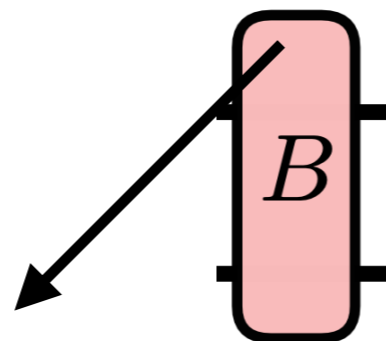


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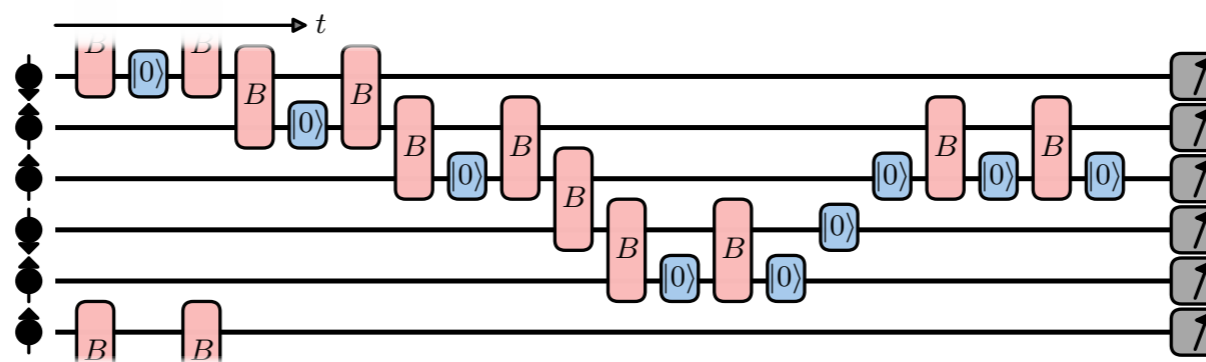
2-qubit Clifford gates

Stabilizer circuits CAN be simulated with classical efficiency!



Gottesman–Knill theorem

CONTROLLED LIMITS: LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT



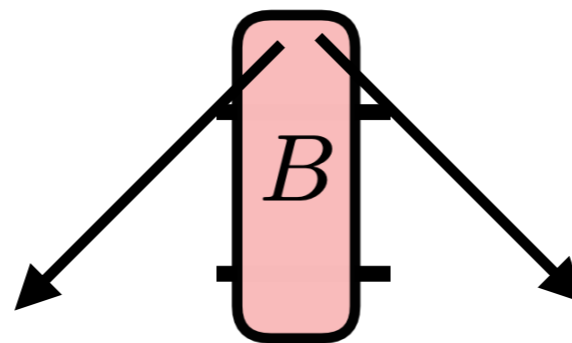
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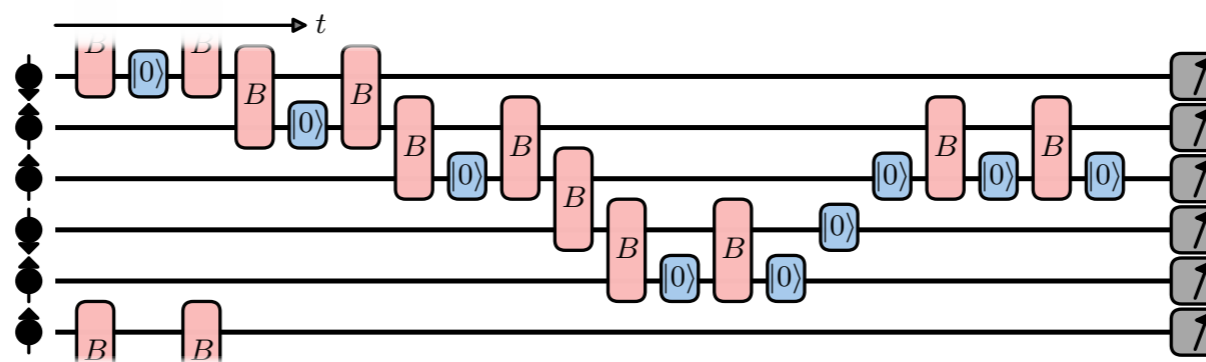


2-qudit gate

A “semiclassical” limit taking the local Hilbert space $q \rightarrow \infty$

Analytically tractable

CONTROLLED LIMITS: LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT



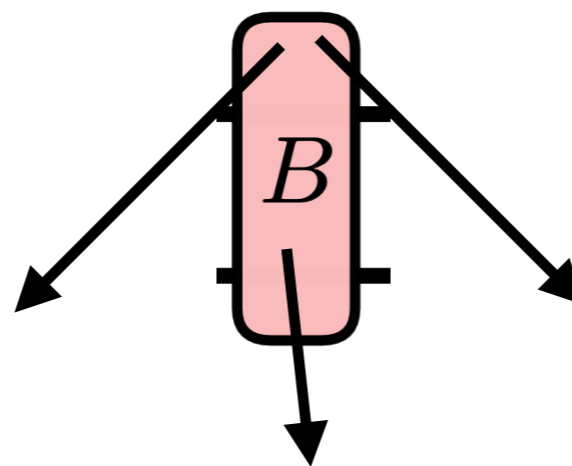
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$n=0$ Renyis

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CONTROLLED LIMITS: STABILIZER CONTROL TRANSITION

2-qubit Clifford gates

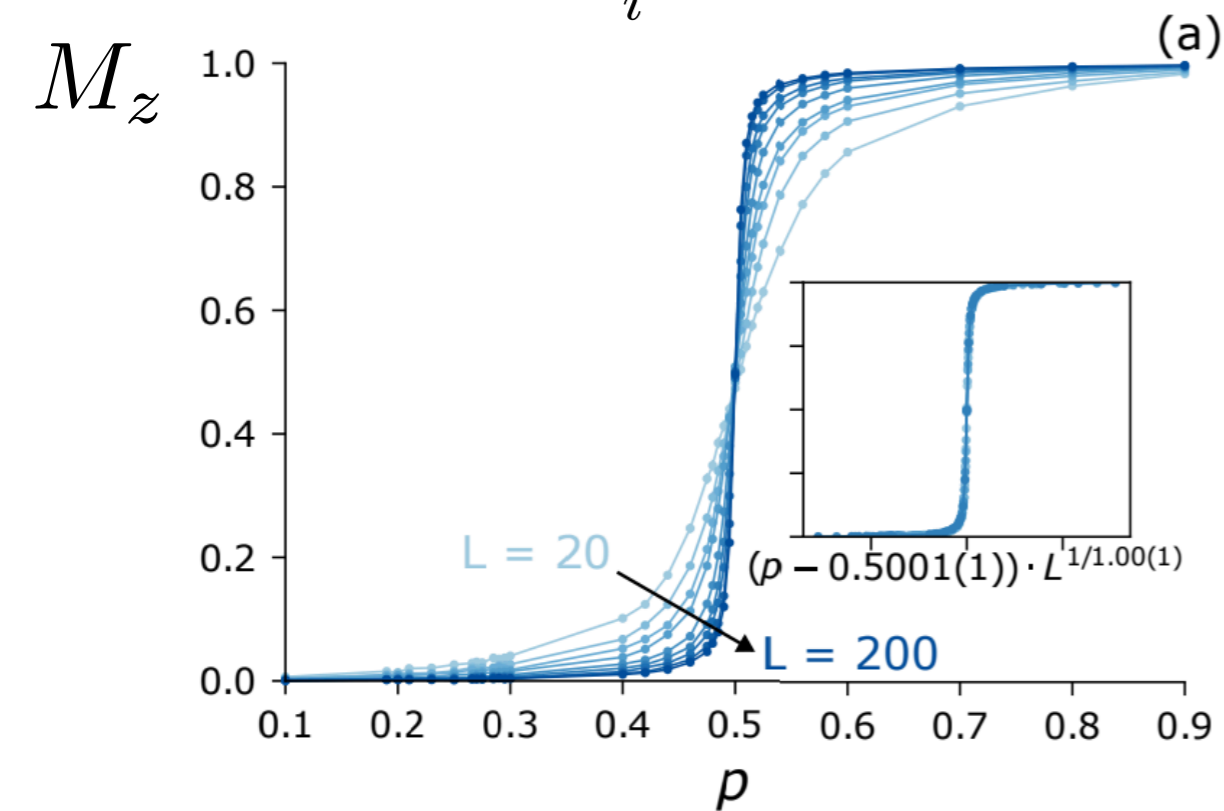
CONTROLLED LIMITS: STABILIZER CONTROL TRANSITION

2-qubit Clifford gates

Control transition remains at

$$p_{\text{CIPT}} = 0.5 \quad \nu = 1.006$$

$$M_z = \frac{1}{L} \sum_i \frac{1}{2} (1 - \sigma_i^z)$$



CONTROLLED LIMITS: STABILIZER CONTROL TRANSITION

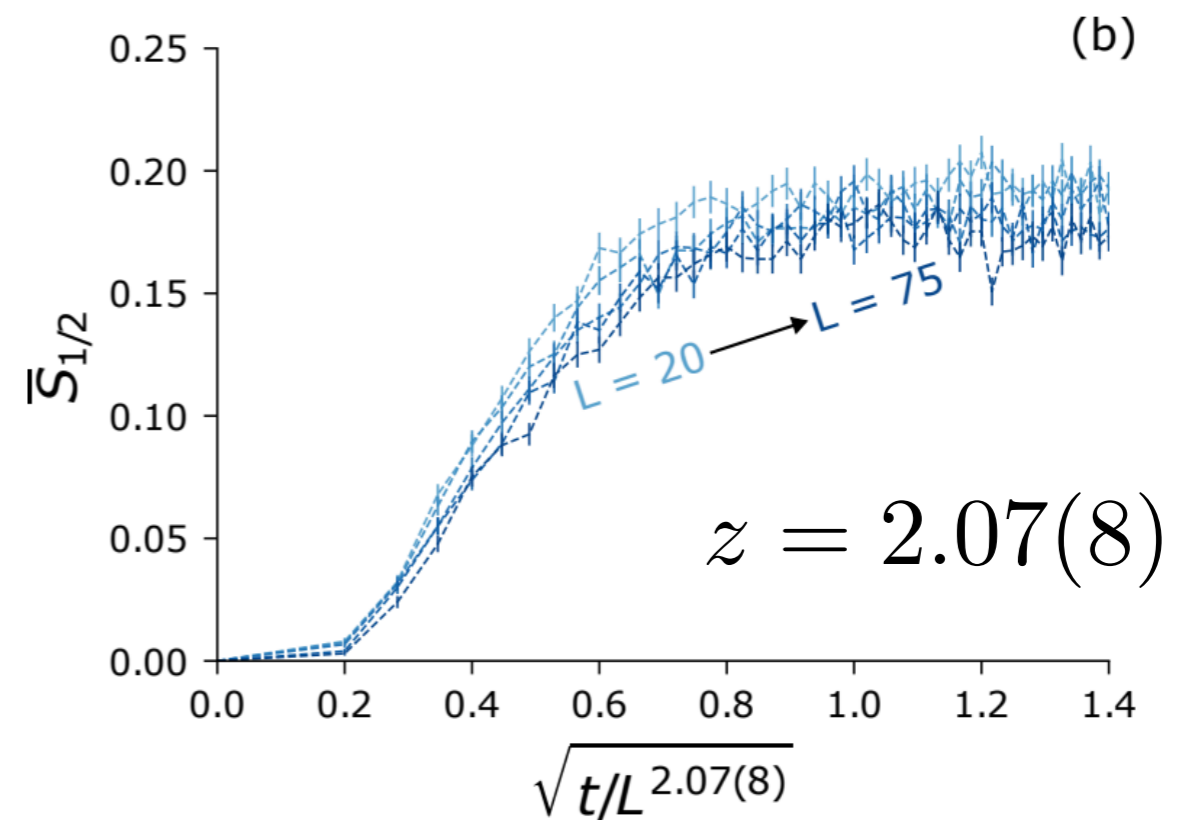
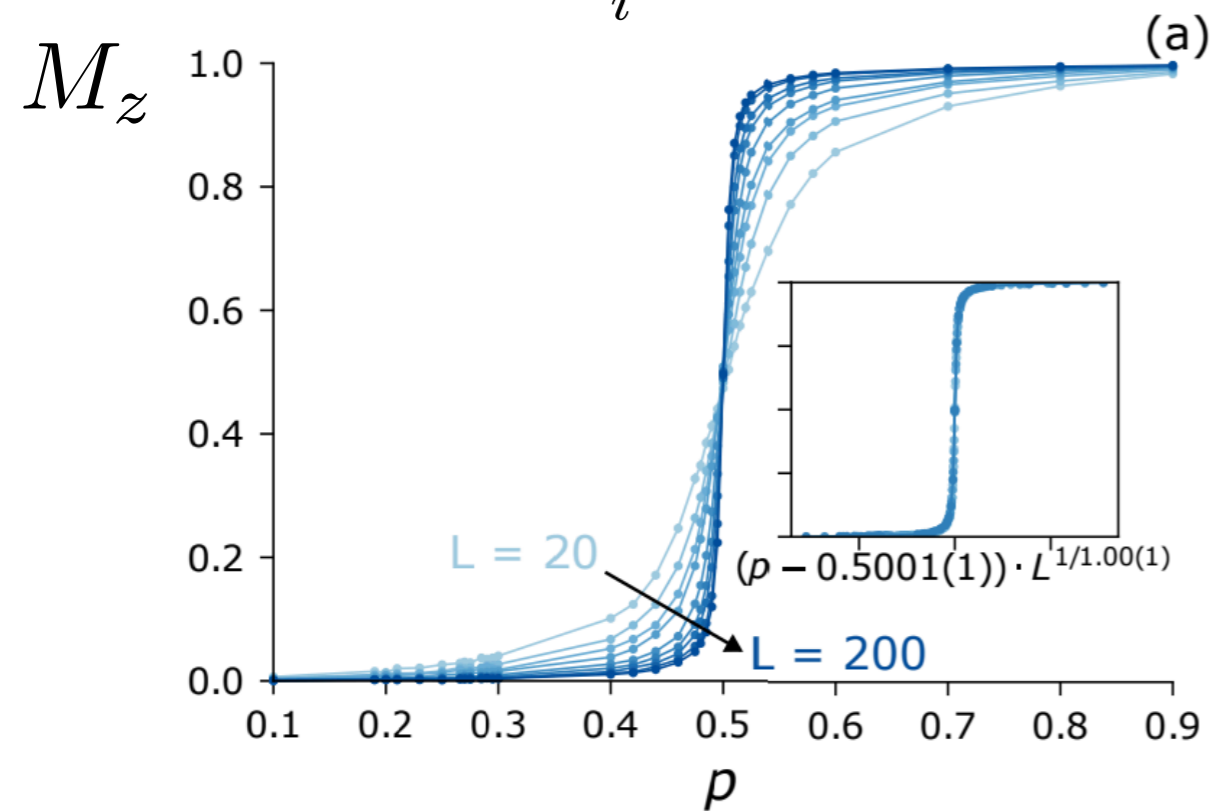
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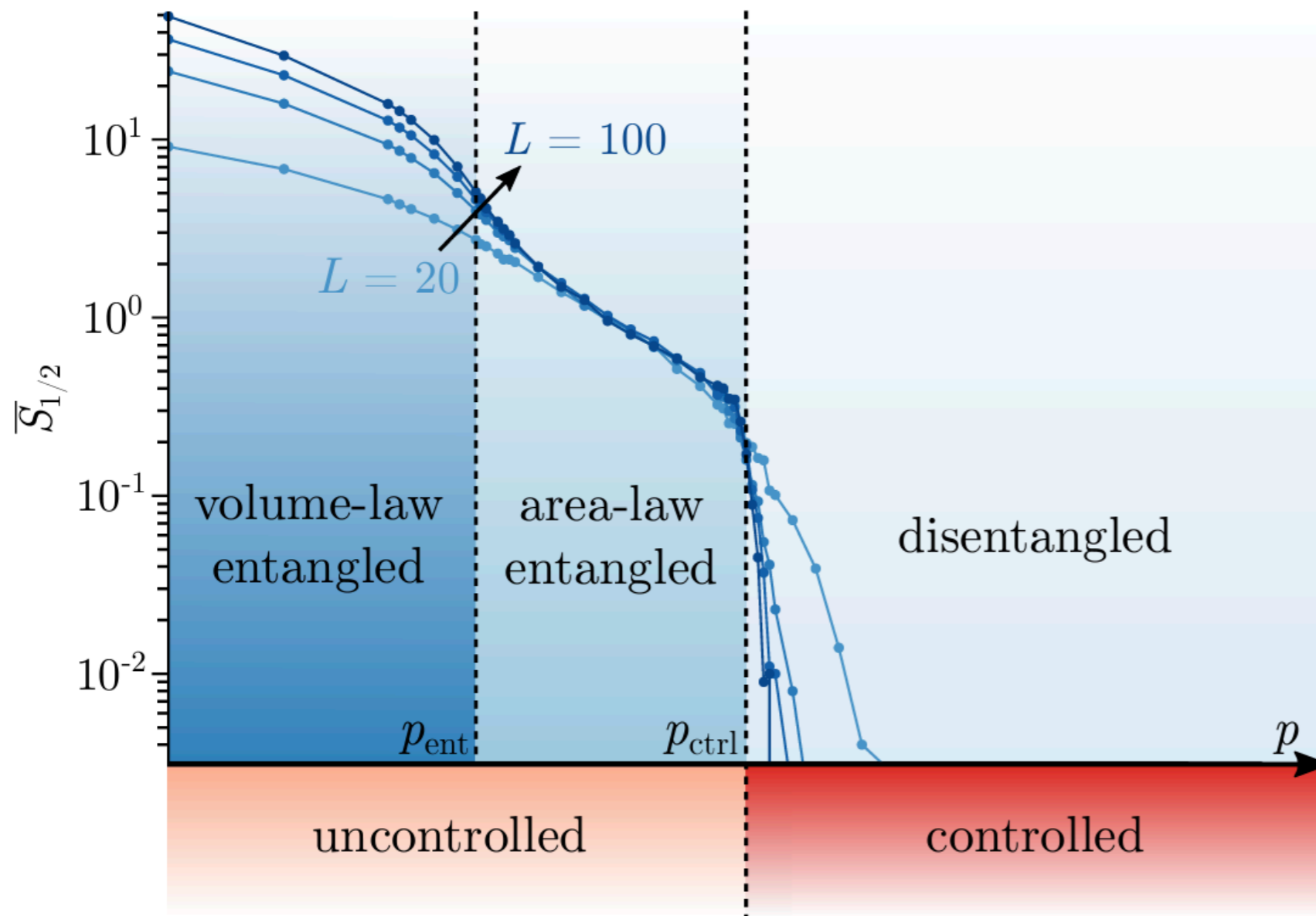
Entanglement dynamics is diffusive and agrees with small sizes



CONTROLLED LIMITS: STABILIZER CONTROL TRANSITION

2-qubit Clifford gates

Entanglement transition splits off



$$p_{\text{MIPT}} = 0.266$$

$$\nu_{\text{MIPT}} = 1.24$$

$$p_{\text{CIPT}} = 0.5$$

$$\nu_{\text{CIPT}} = 1.006$$

CONTROLLED LIMITS: LARGE LOCAL HILBERT SPACE

Make contact with large local Hilbert space limit of the monitored circuit problem as well.

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Apply the **measurement and reset** with probability p 

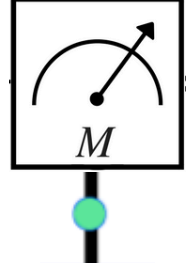
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(With $p = 0$, similar to **lecture 3** but is an adaptive circuit) 

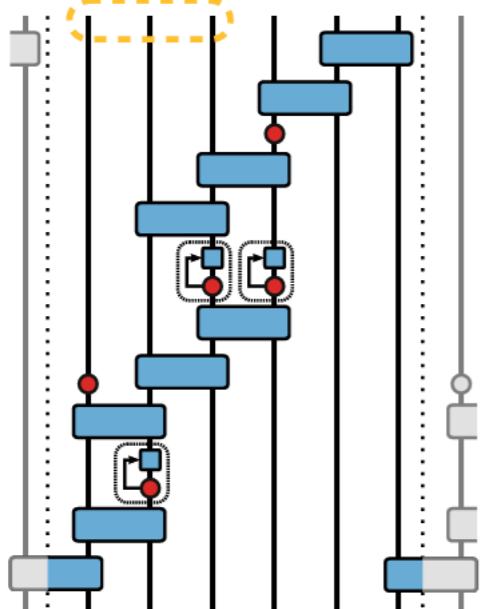
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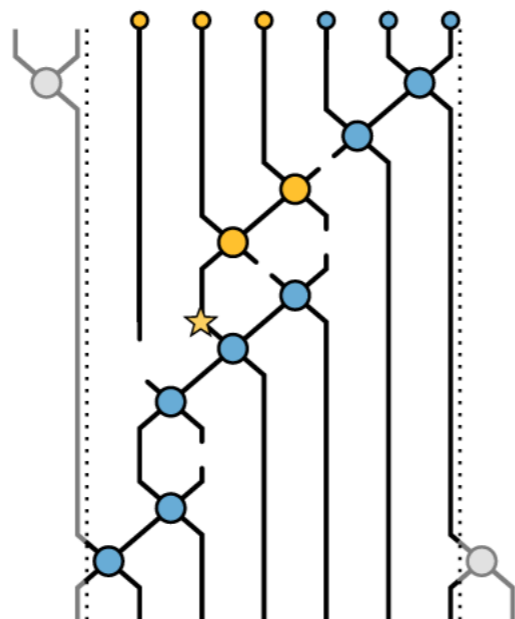
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(d) Stochastic d -adic circuit



(e) Potts model (irregular lattice)



lecture 3

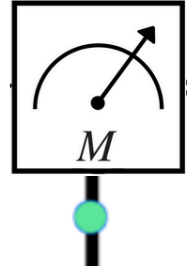
$$\overline{S_A^{(n)}} = \frac{n}{1-n} \lim_{Q \rightarrow 1} \frac{Z_A - Z_0}{Q - 1}$$

$$\sum_{\text{clusters}} p^{\#\text{empty links}} (1-p)^{\#\text{occupied links}} (Q!)^{\#\text{clusters}}$$

Now this Q -state Potts is defined on a stochastically defined graph.
 Solve stat mech model numerically!

CONTROLLED LIMITS: LARGE LOCAL HILBERT SPACE

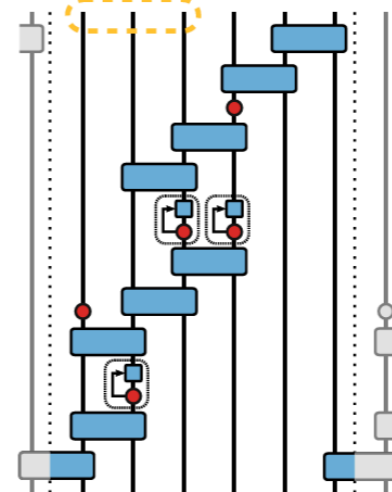
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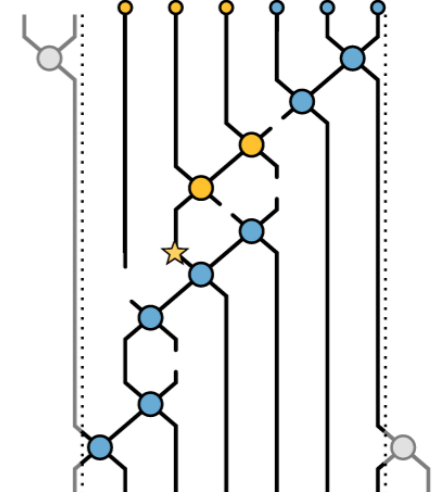
lecture 3 $\overline{S_A^{(n)}} = \frac{n}{1-n} \lim_{Q \rightarrow 1} \frac{Z_A - Z_0}{Q - 1} \longrightarrow \sum_{\text{clusters}} p^{\#\text{empty links}} (1-p)^{\#\text{occupied links}} (Q!)^{\#\text{clusters}}$

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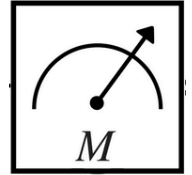



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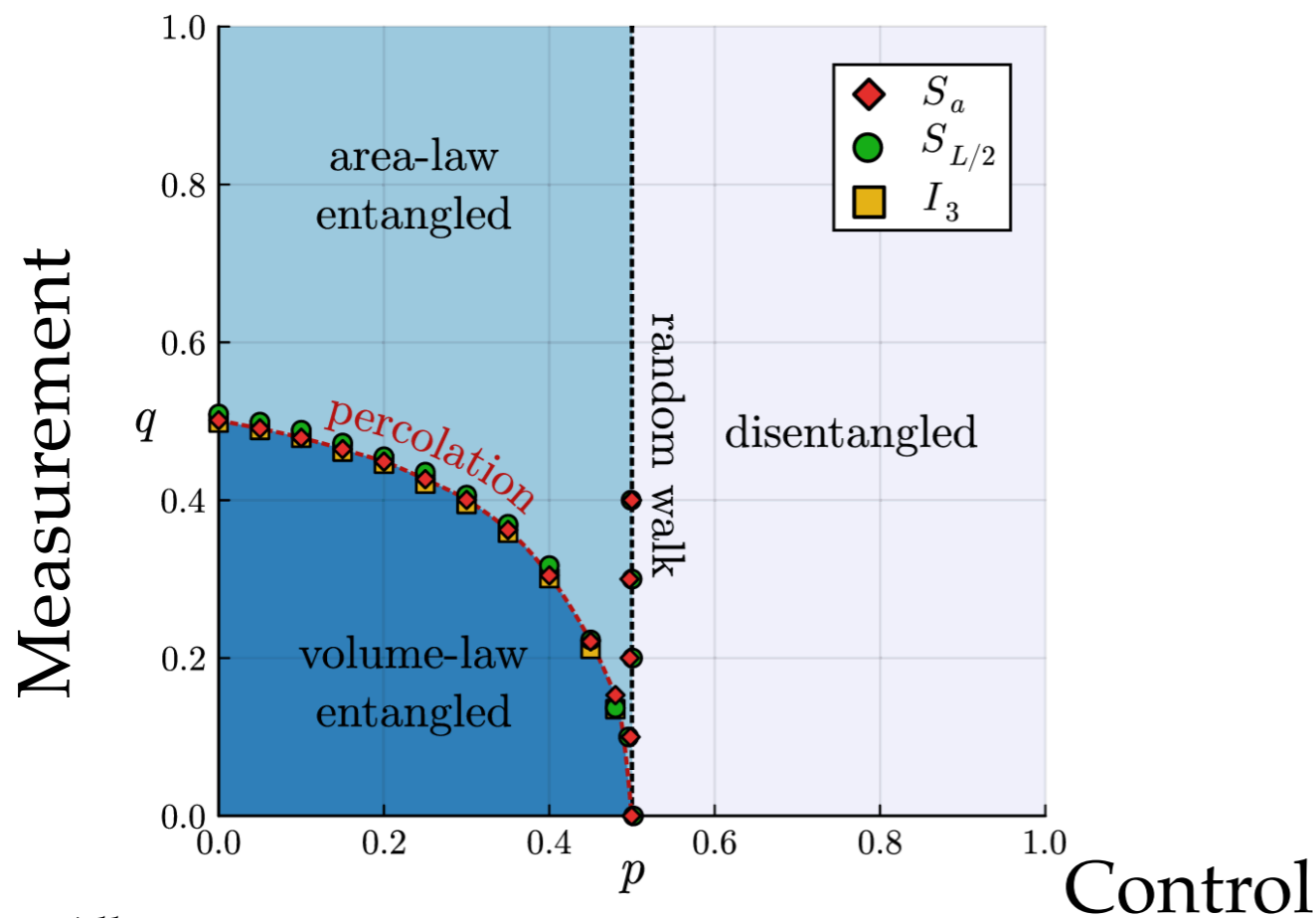
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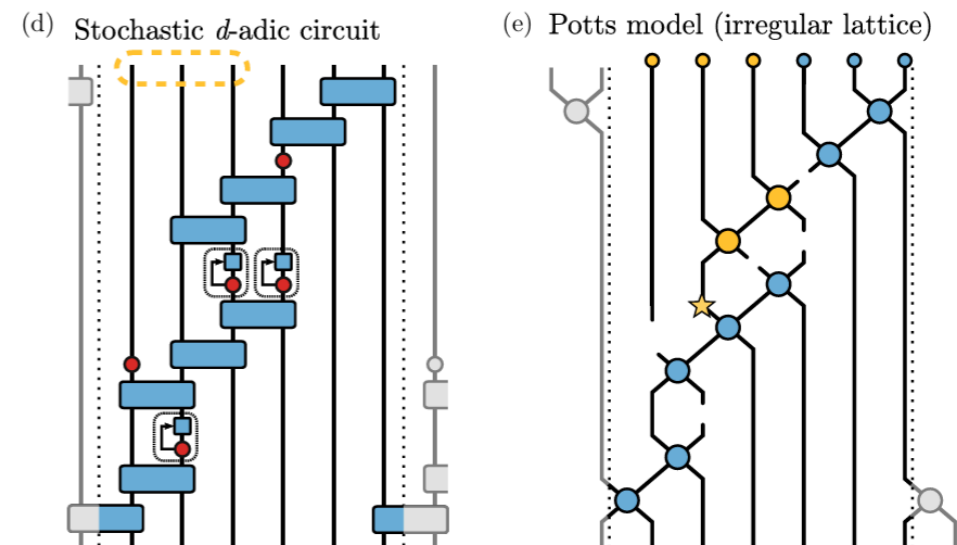
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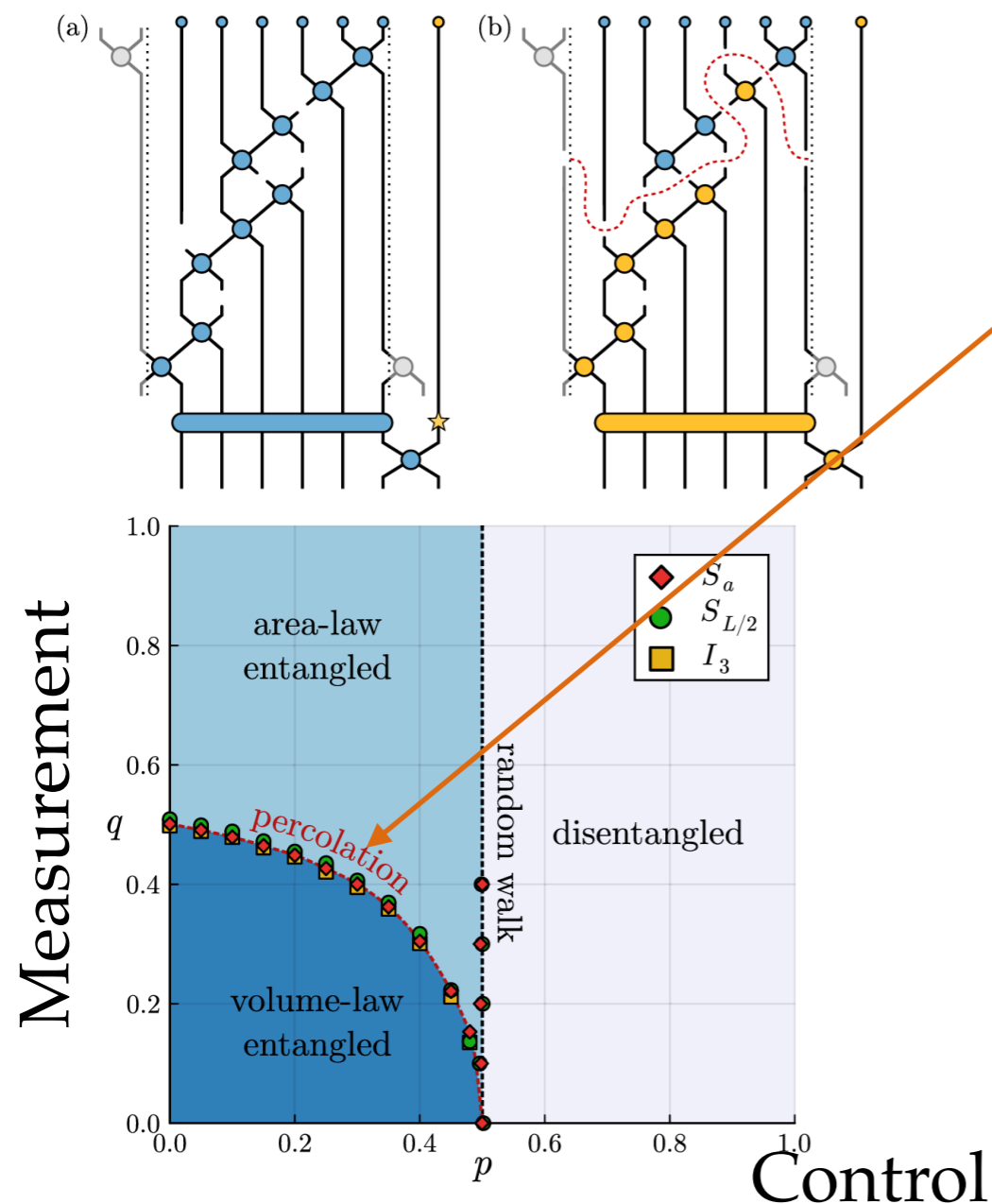


Alloca, LeMaire, Iadecola, Wilson PRE (2024)

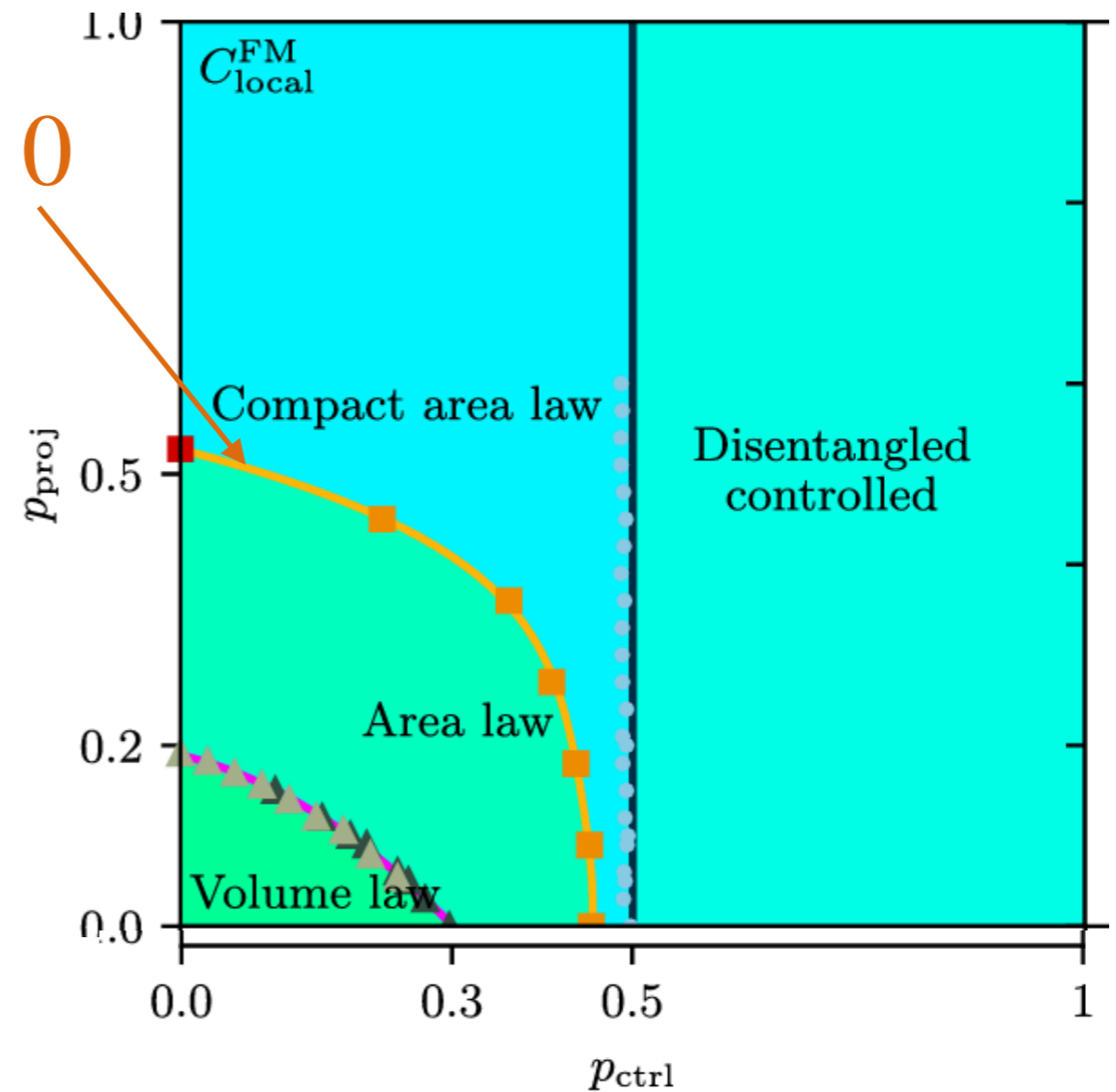


CONTROLLED LIMITS: $n = 0$ RENYI ENTROPY

In the $n = 0$ limit we can use min-cut and simulate the zeroth Renyi entropy



Full simulation



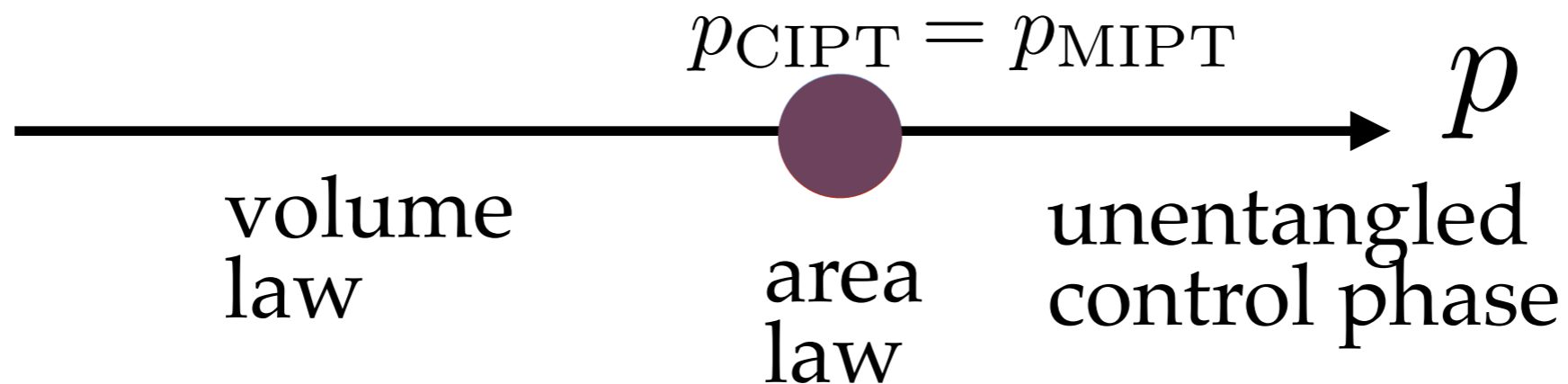
SUMMARY CONTROL TRANSITIONS

Summary: Whether the transitions coincide or not depends on if the control operation is **local** or **global**

Pan, et al to appear (2024)

Sierant and Turkeshi ACTA PHYSICA POLONICA A (2023)

Qubits with global control operations, or $q \rightarrow \infty$, or $n = 0$



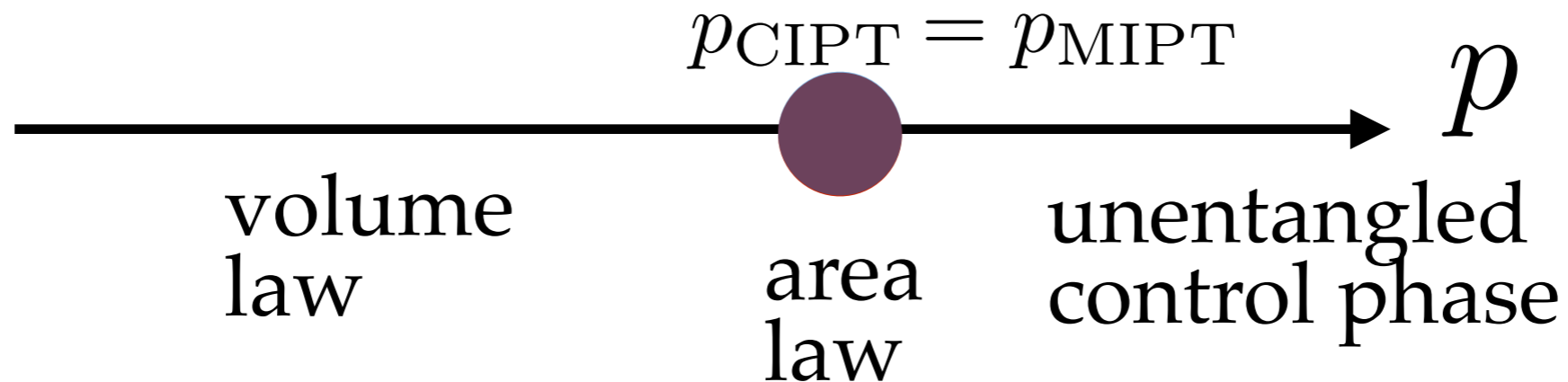
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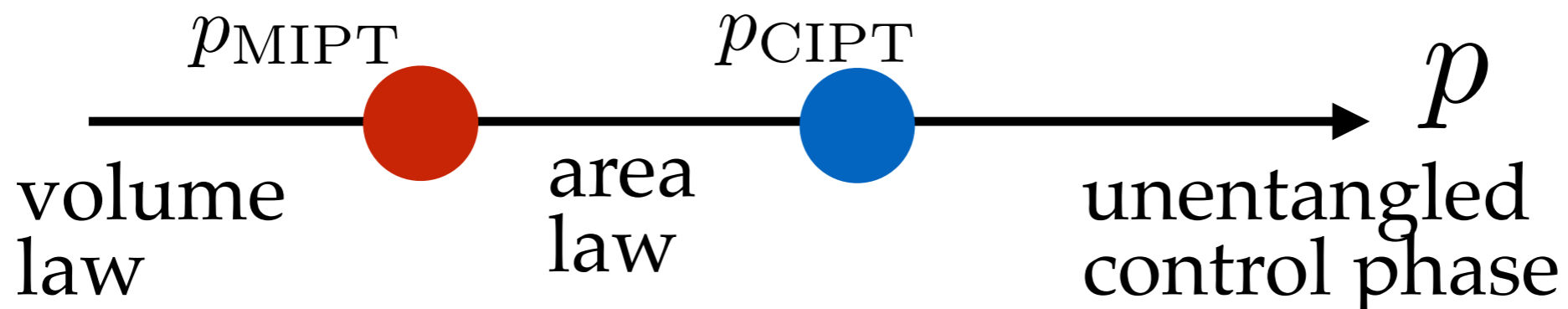
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Local control operations, qubits and stabilizers



OUTLINE

I. Motivation

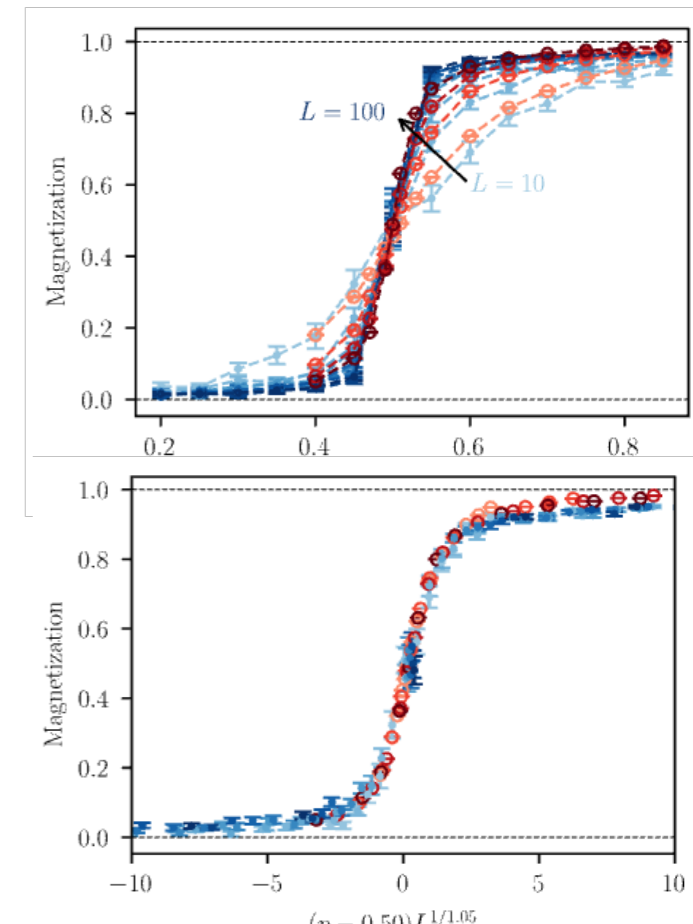
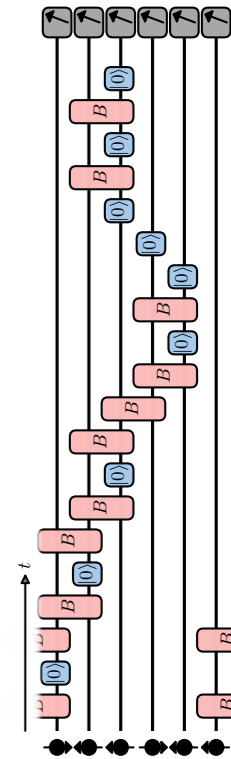
- Controlling classical chaos
- Examples of classical control transitions

II. Quantum Control Induced Transition

- Control with an Exact Dark State
- Controlling the quantum top
- Controlling the Bernoulli map

III. Realization across circuits

- IBM's quantum computer
- Accessing Quantum fluctuations



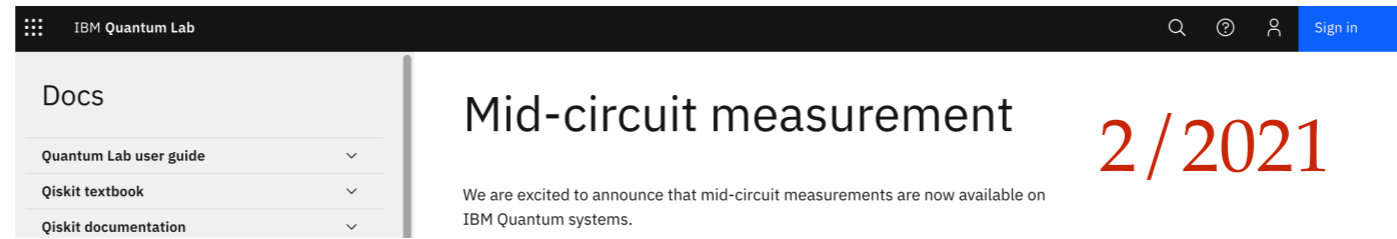
DATA ON IBM'S QUANTUM COMPUTER



Midcircuit measurements and resets pose a significant challenge for timing and classical data processing. Out of reach when we first conceived of this idea in 2022.

Iadecola, Ganeshan, JHP, Wilson PRL (2023)

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Rutgers P&A Colloquium,
November 2022

Barbara Jones, IBM

Department of Physics and Astronomy

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Events

Calendar

Observatory Nights

Colloquium

Seminars

Upcoming Conferences

Calendar of Events

Events Calendar

By Year By Month By Week Today Jump to month

Simulation of some Open Quantum Systems on Near-term Quantum Computers

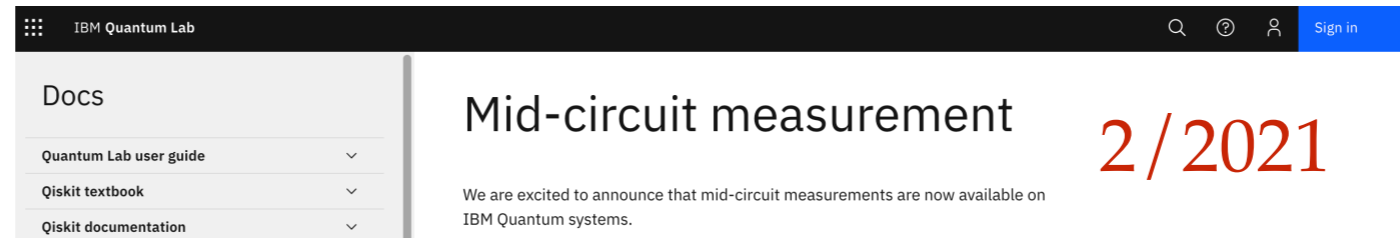
Date and Time: Wednesday, November 16, 2022, 03:30pm - 04:30pm

Location: Room 330W and via Zoom

Speaker: Barbara Jones, IBM Quantum



DATA ON IBM'S QUANTUM COMPUTER



Superconducting qubits

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Feedback is NOT yet accessible, till now!



Barbara Jones



Bibek Pokharel



Abhinav Deshpande



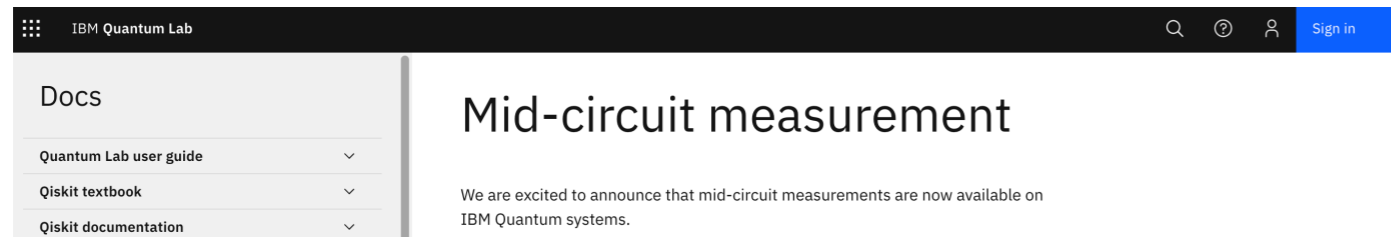
Luke Govia



Maika Takita

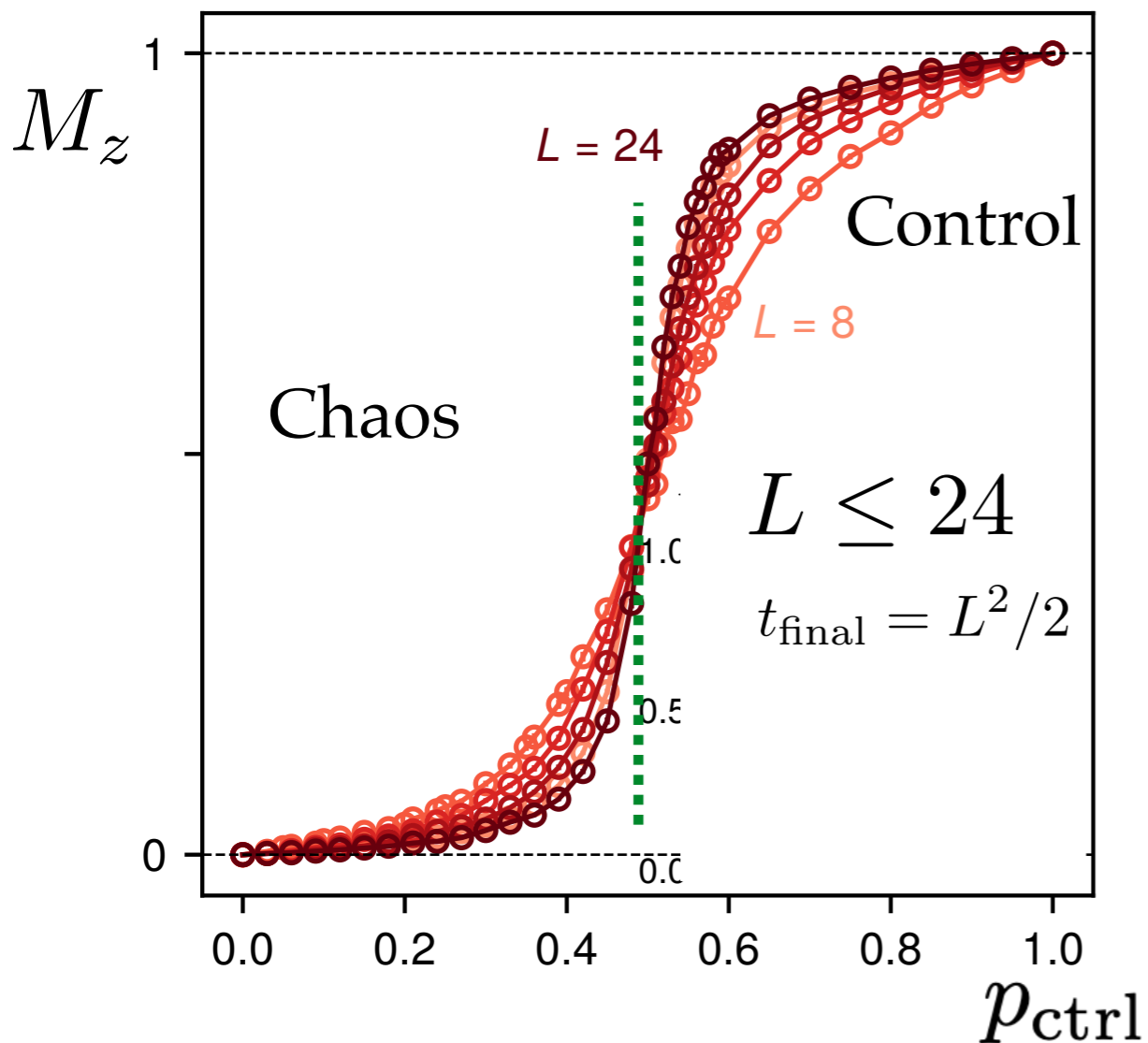
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IBM Q

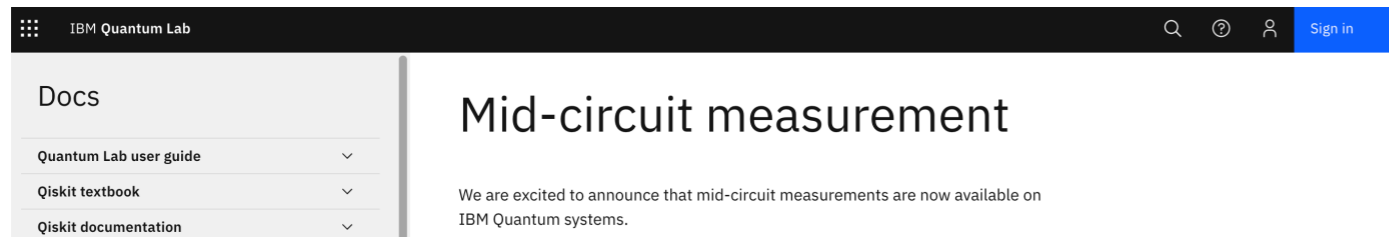


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Theory (in the steady state)



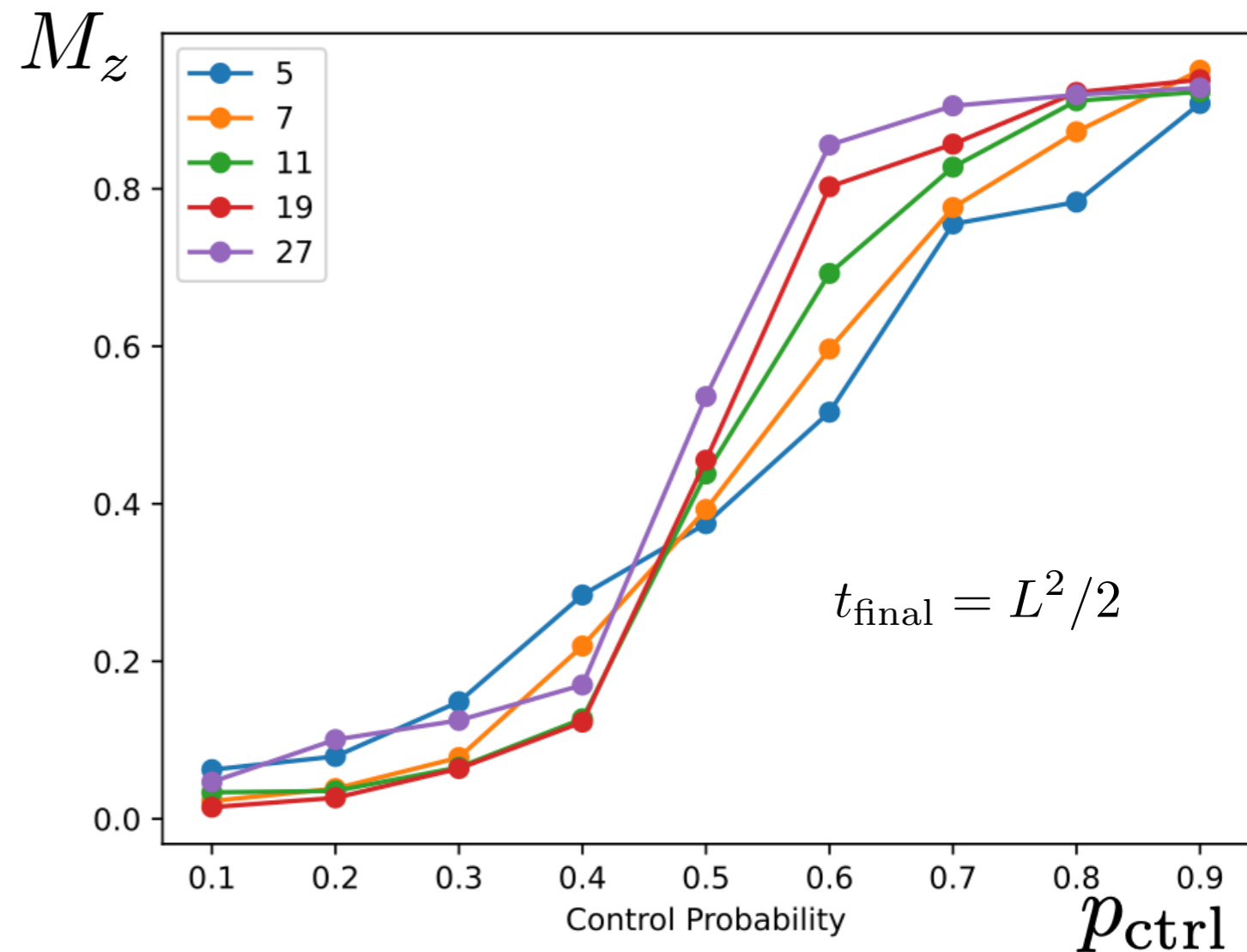
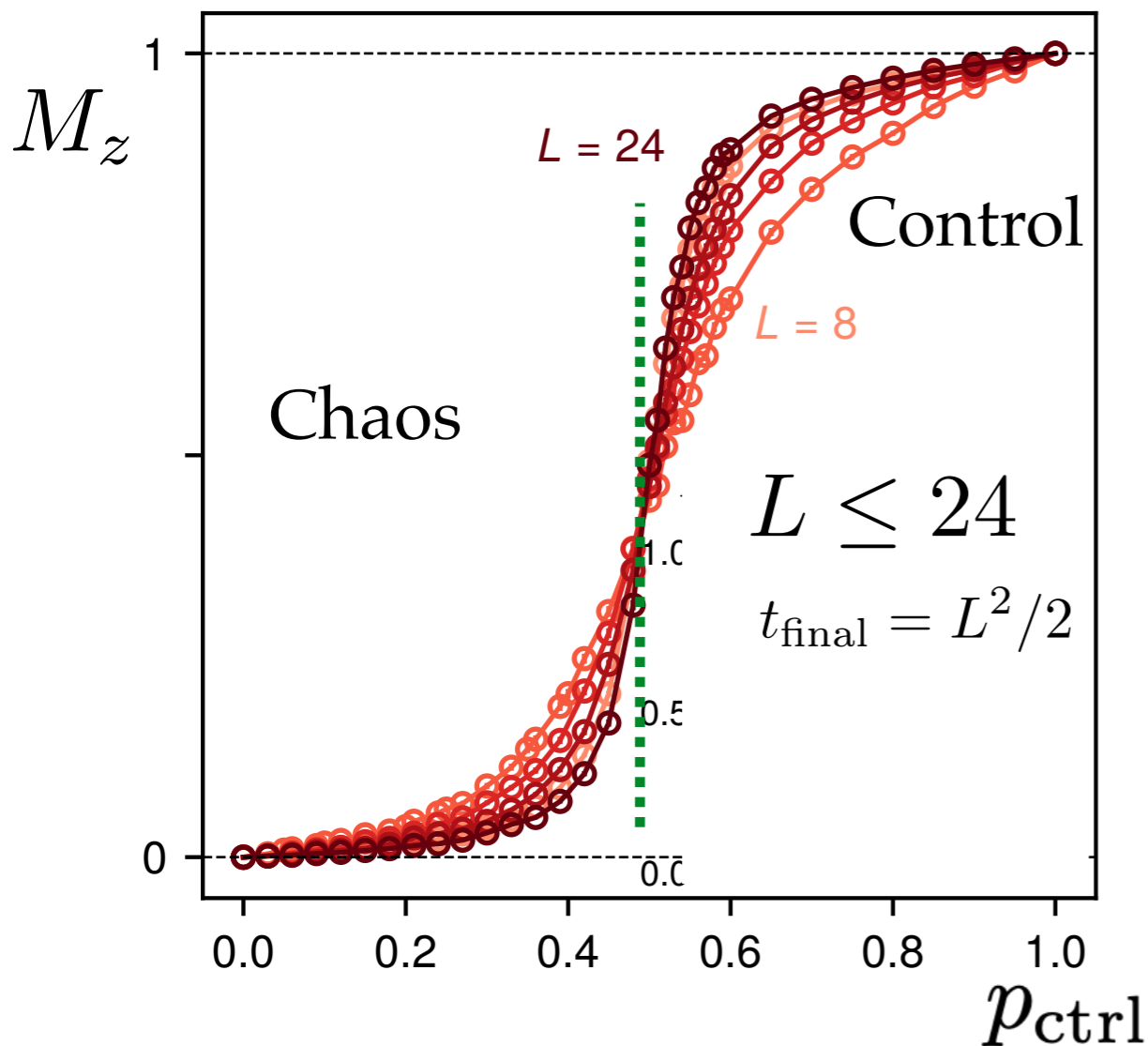
DATA ON IBM'S QUANTUM COMPUTER



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Experiment (first real go)

Theory (in the steady state)



Results from quantum backend Sapporo

DATA ON IBM'S QUANTUM COMPUTER

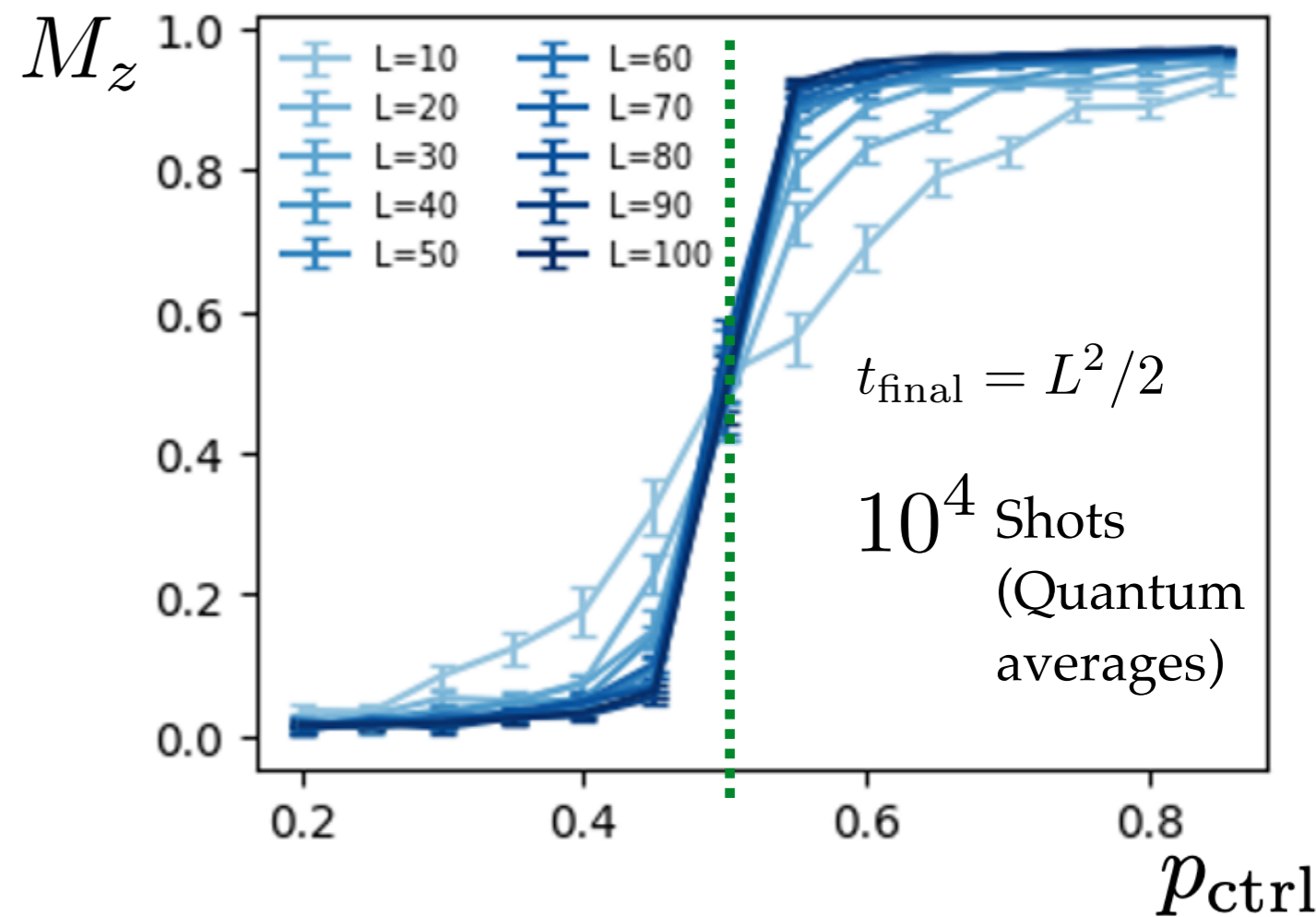
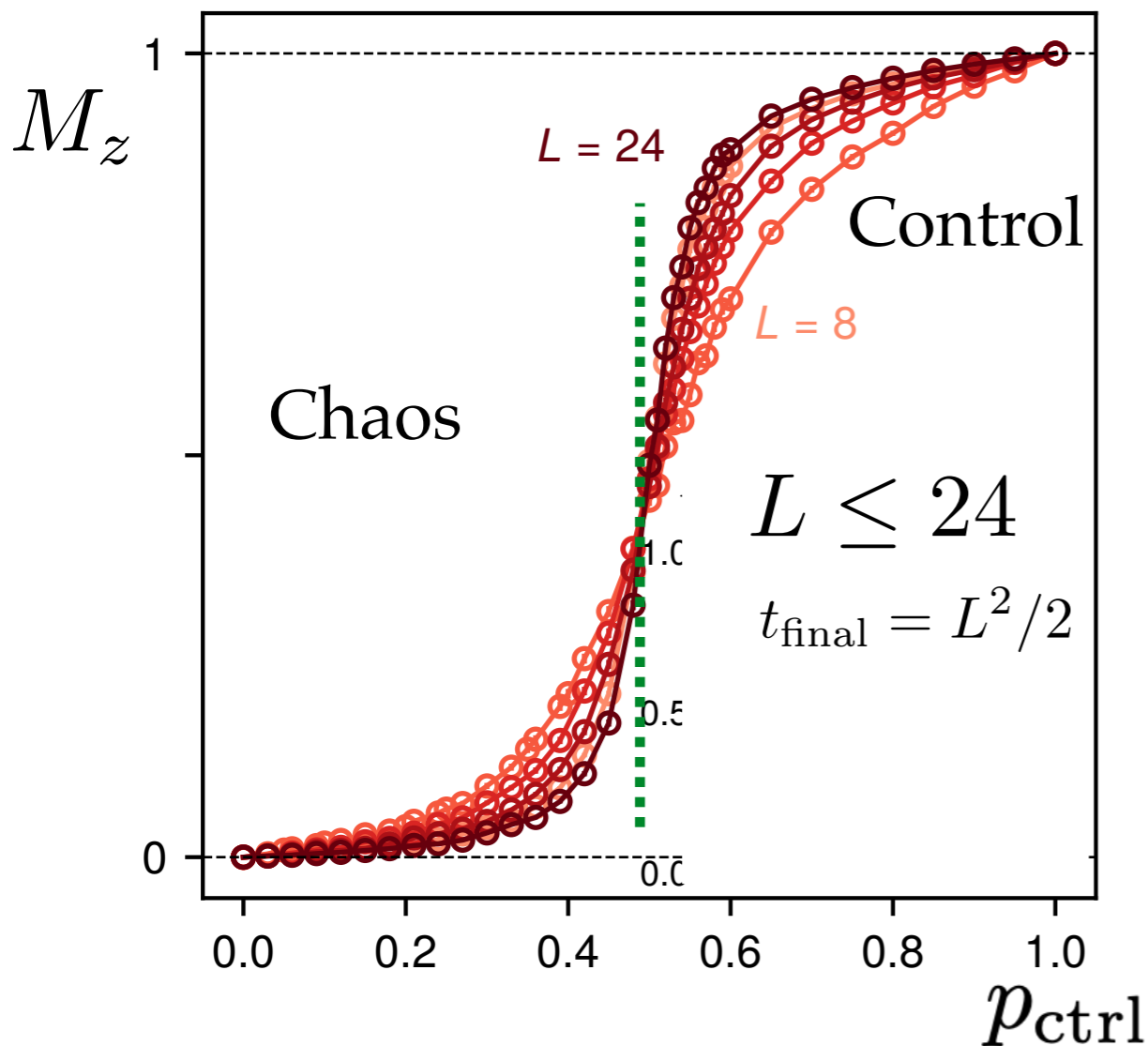
IBM Q



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Experiment (improved)
With no error mitigation

Theory (in the steady state)

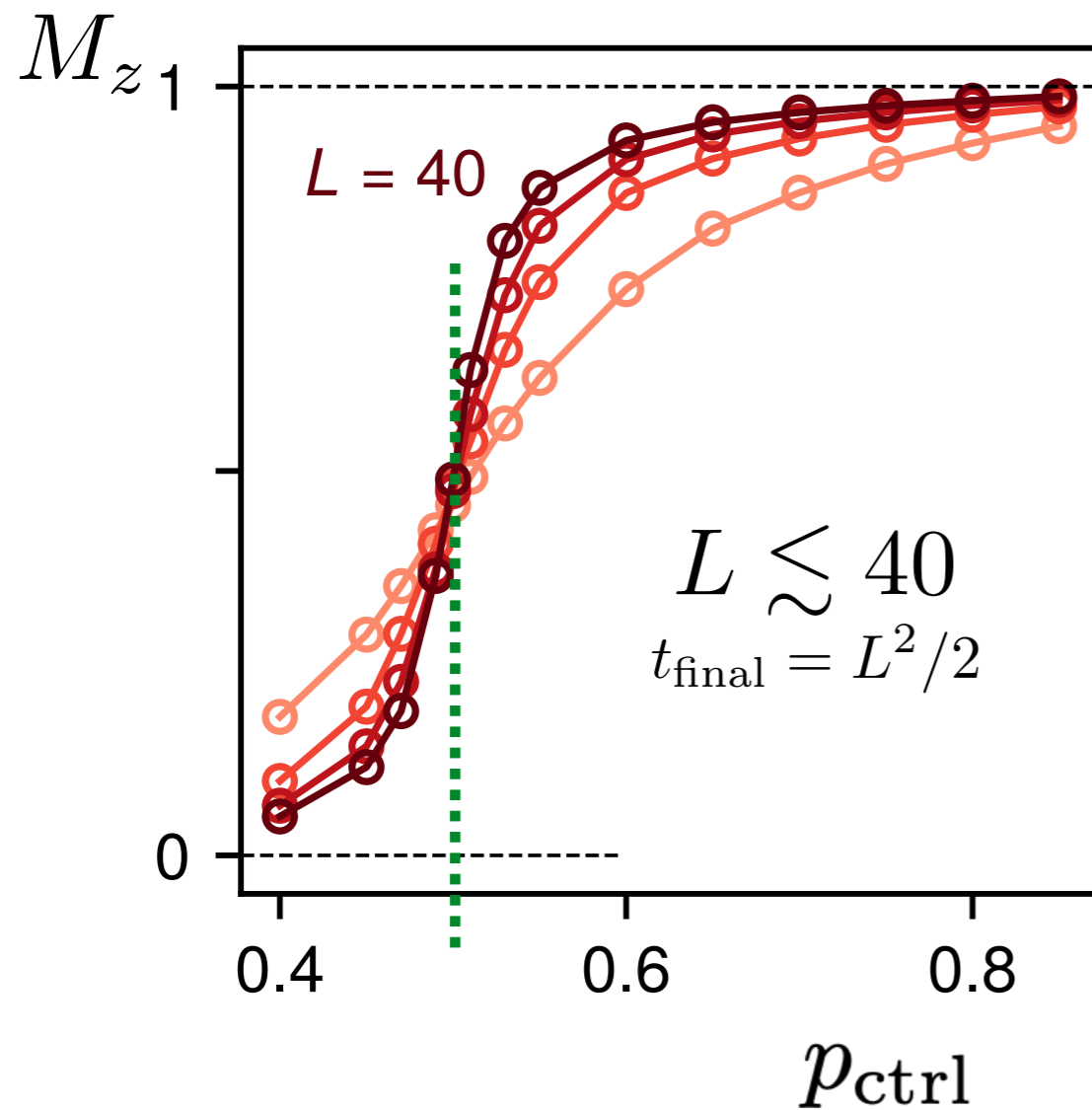


Quantum computer: Fez w/ 156 qubits

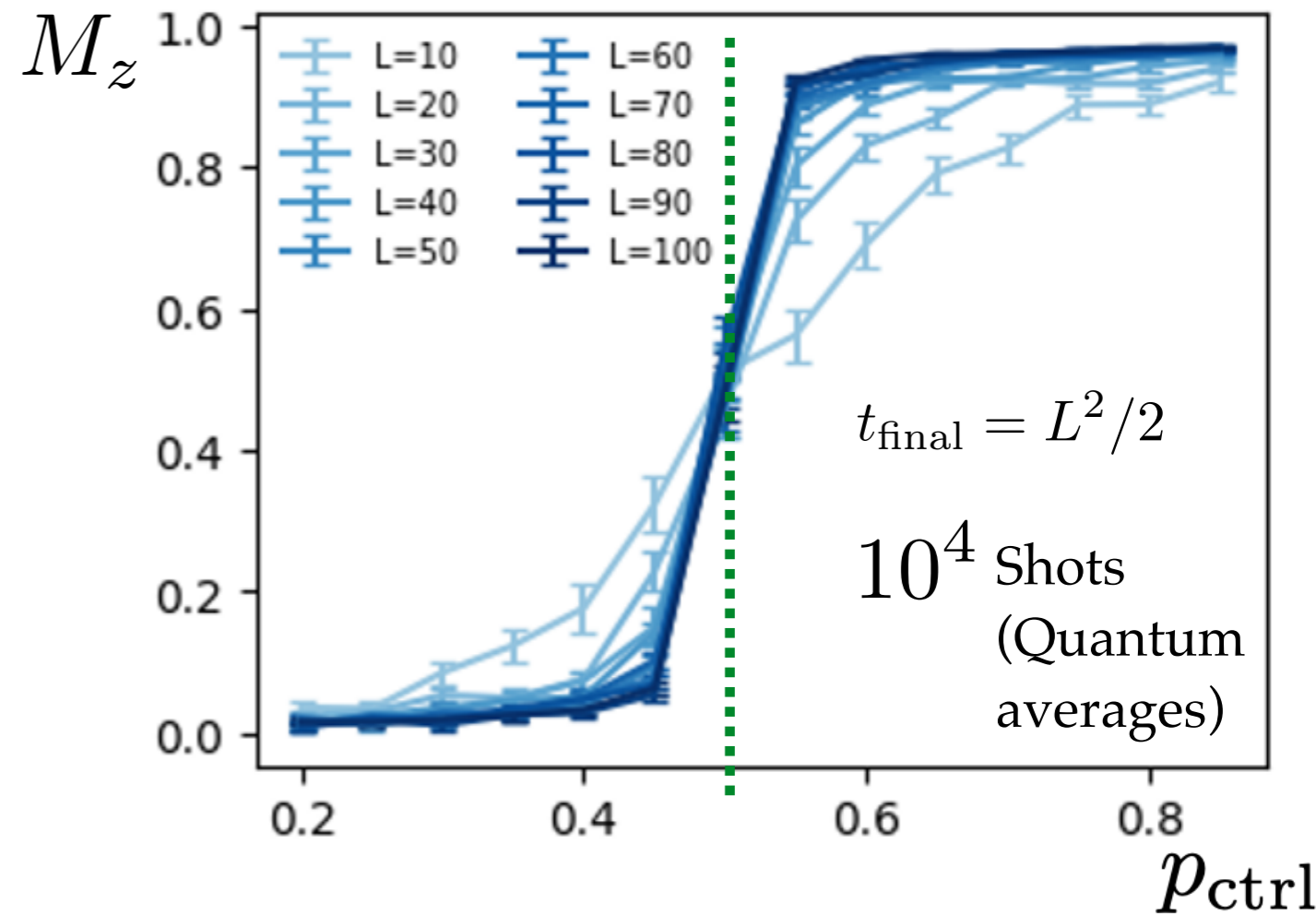
DATA ON IBM'S QUANTUM COMPUTER

IBM Q

Theory To compare with experiment use **matrix product states**



Experiment (improved)
With no error mitigation

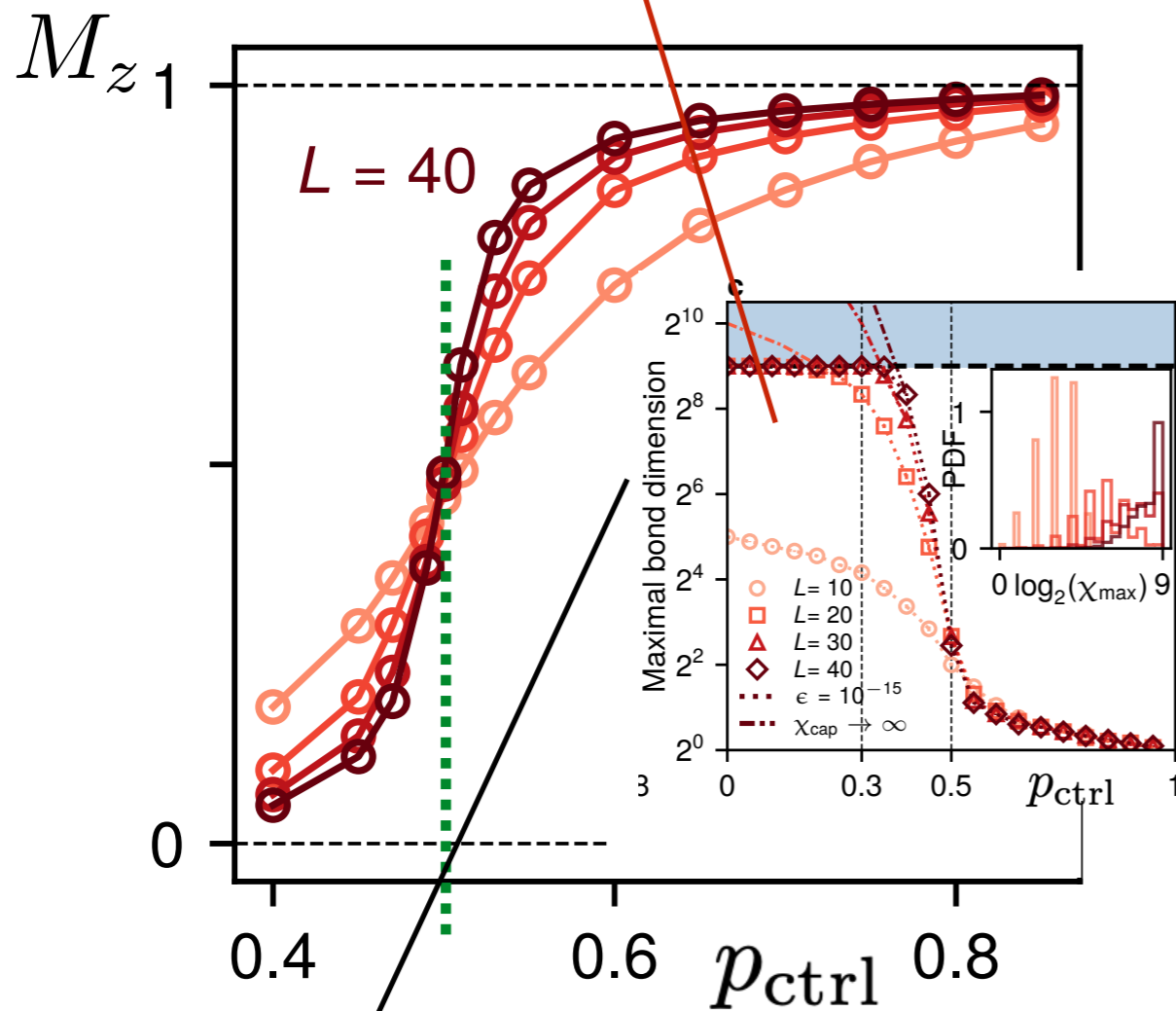


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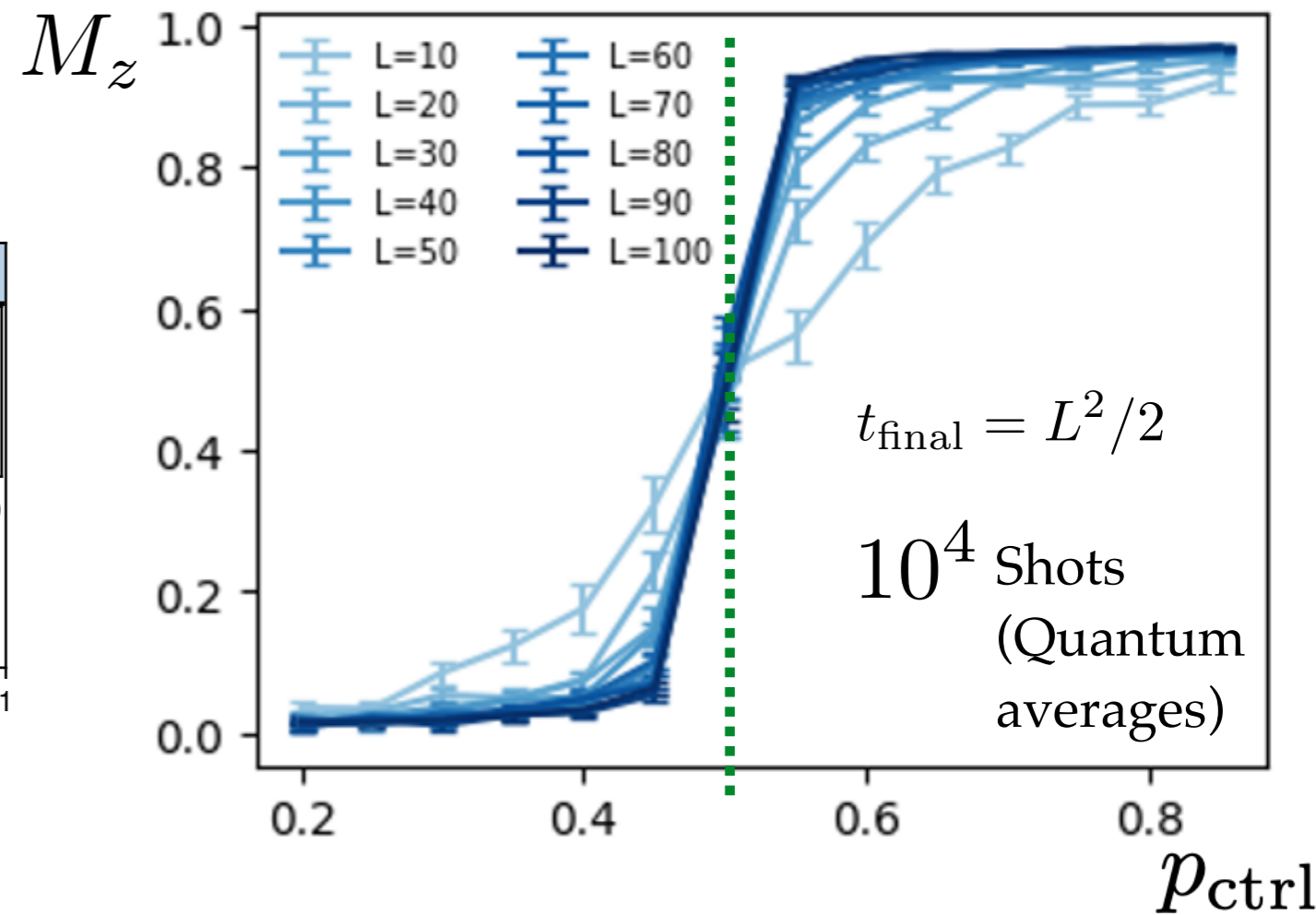
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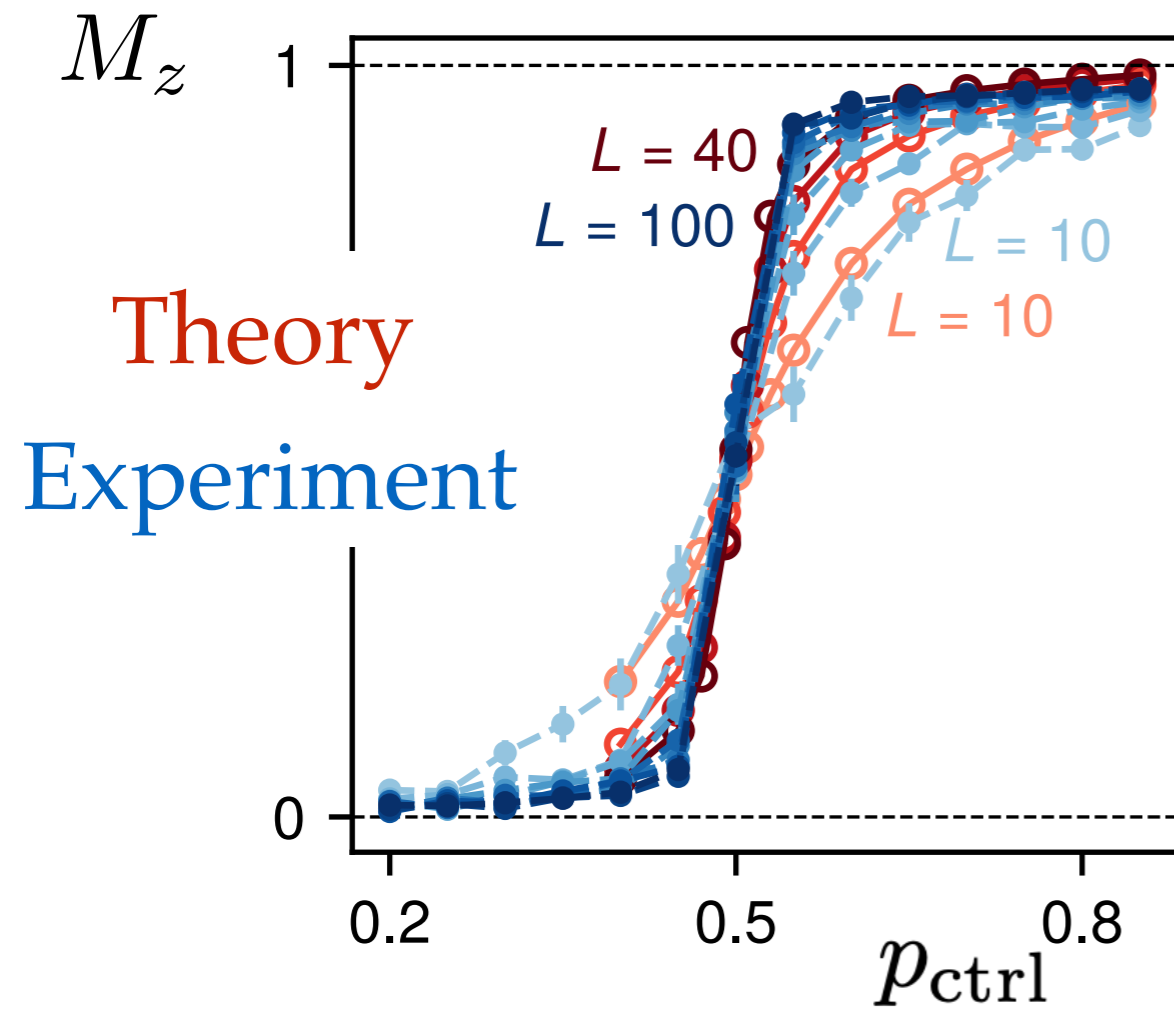
Quantum computer: Fez w / 156 qubits

Classical computational complexity depends on p_{ctrl}

DATA ON IBM'S QUANTUM COMPUTER

IBM Q

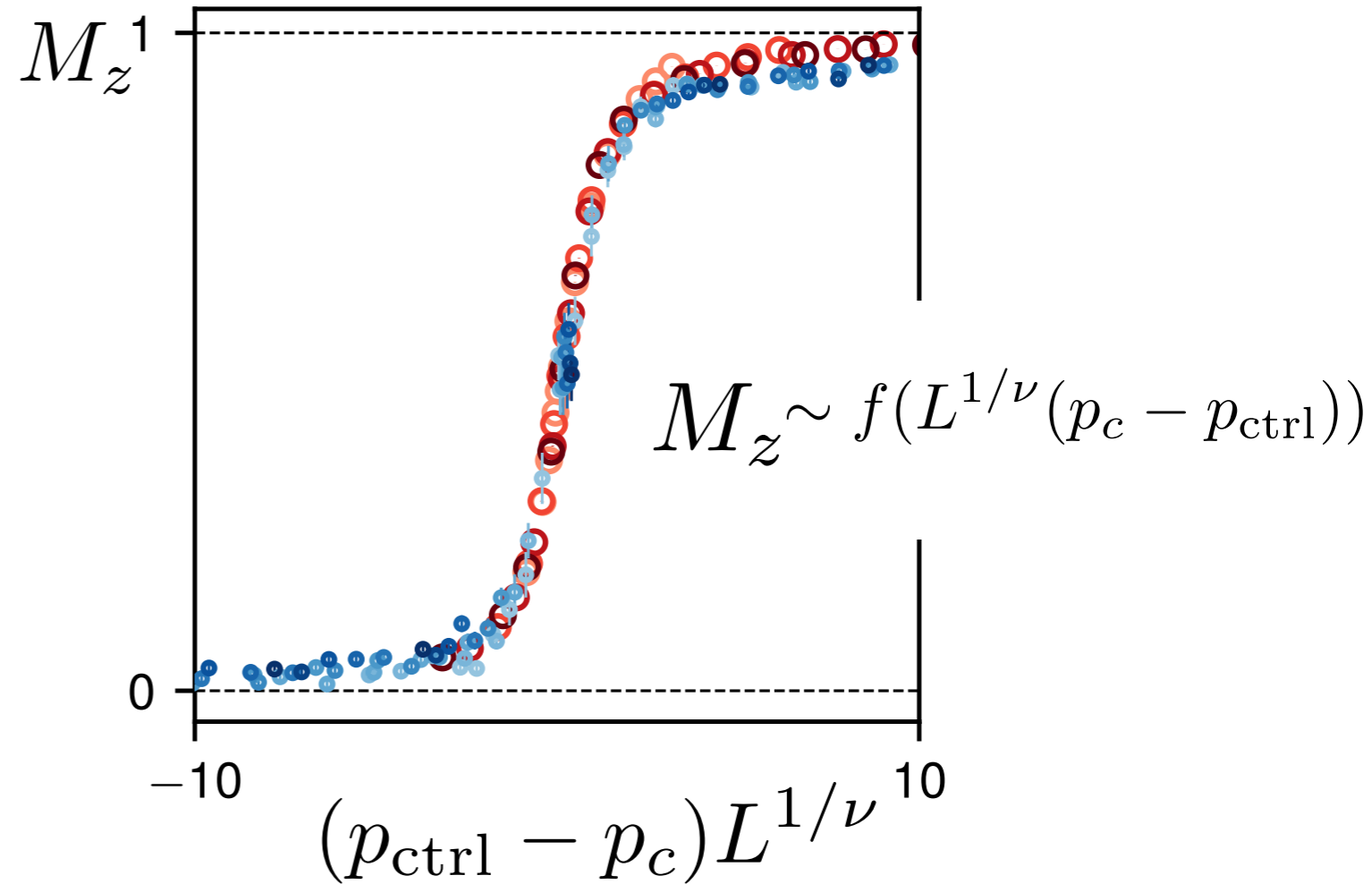
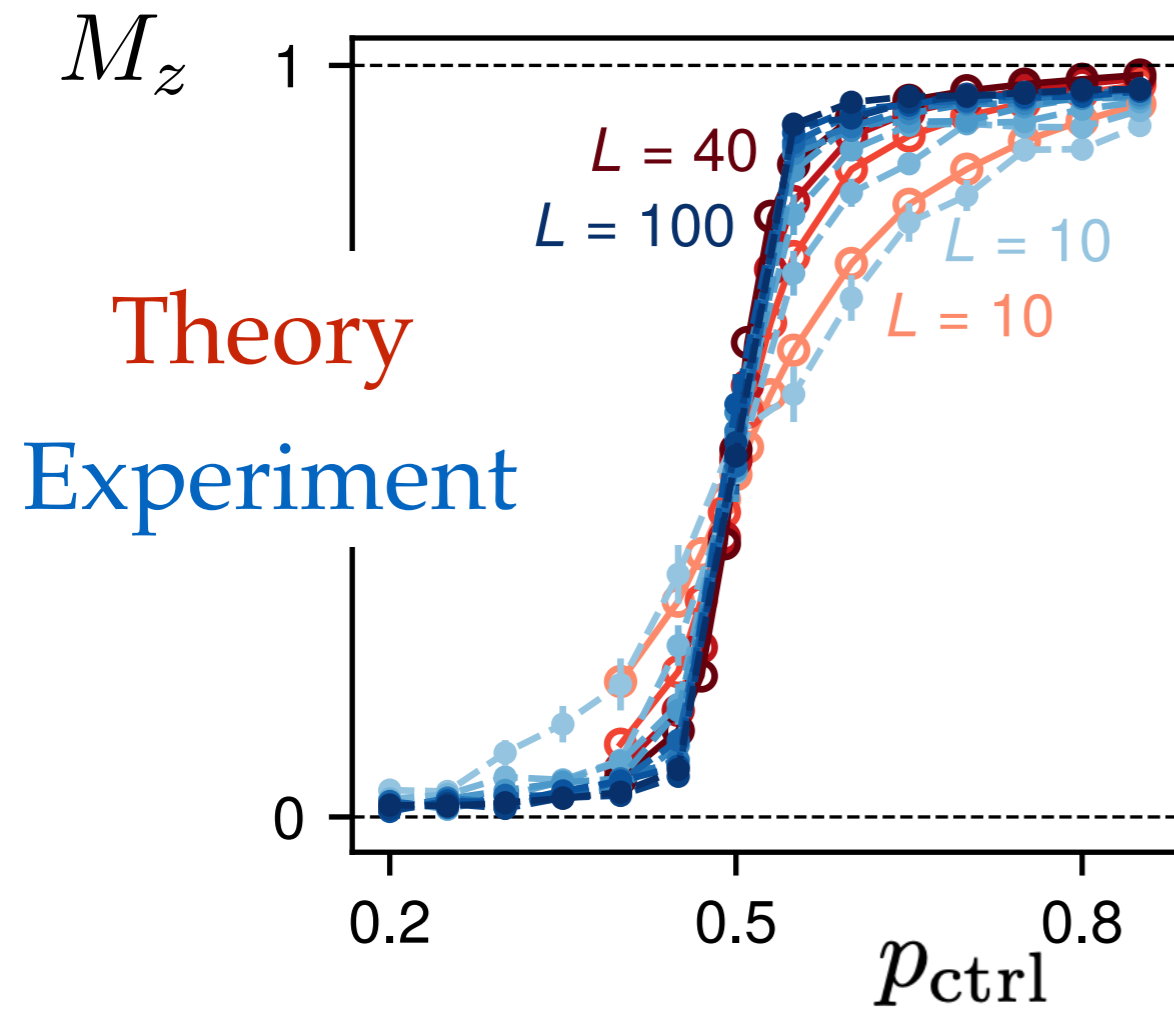
Put it together! $t_{\text{final}} = L^2/2$



DATA ON IBM'S QUANTUM COMPUTER

IBM Q

Critical properties in the steady state $t_{\text{final}} = L^2/2$



Matrix product state simulations
(on a classical computer)

$$p_c = 0.4947(5) \quad \nu = 1.05(2)$$

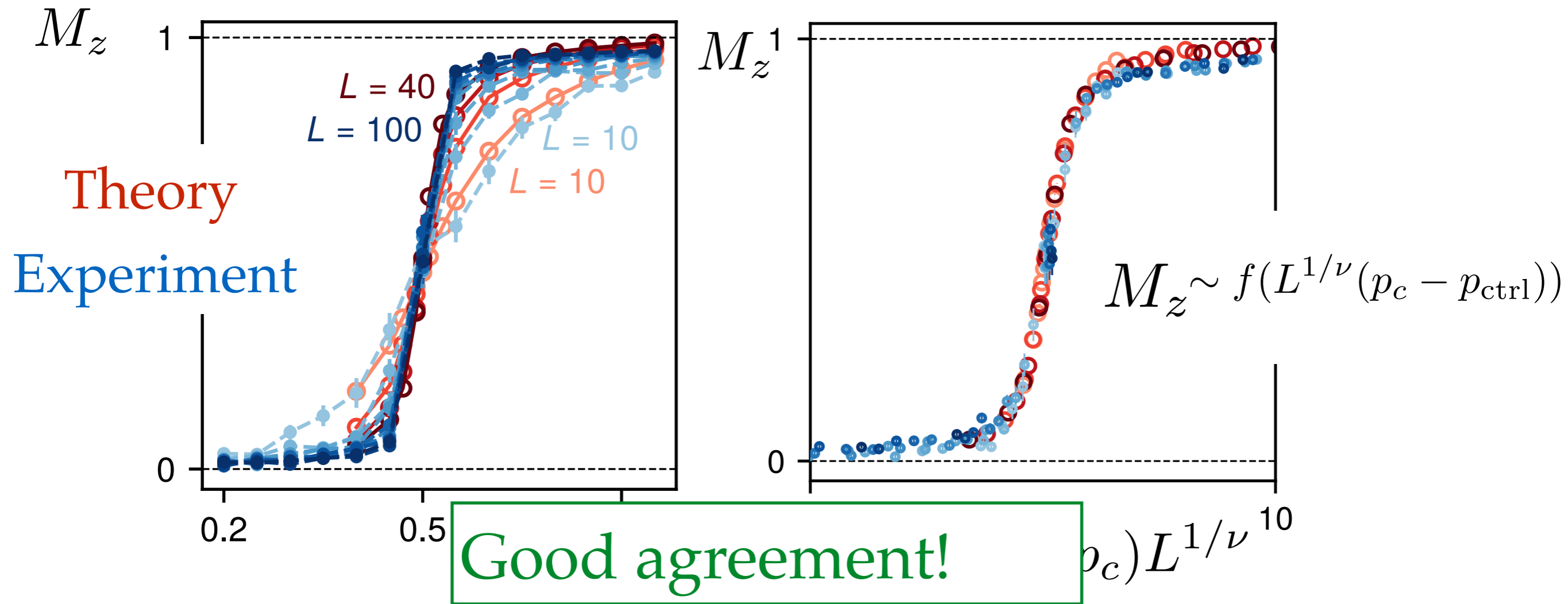
Experimental data from IBM's
quantum computer Fez

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IBM Q

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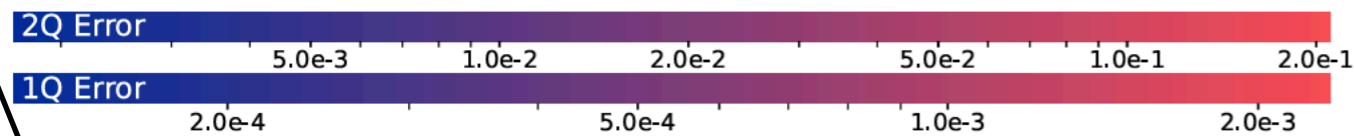
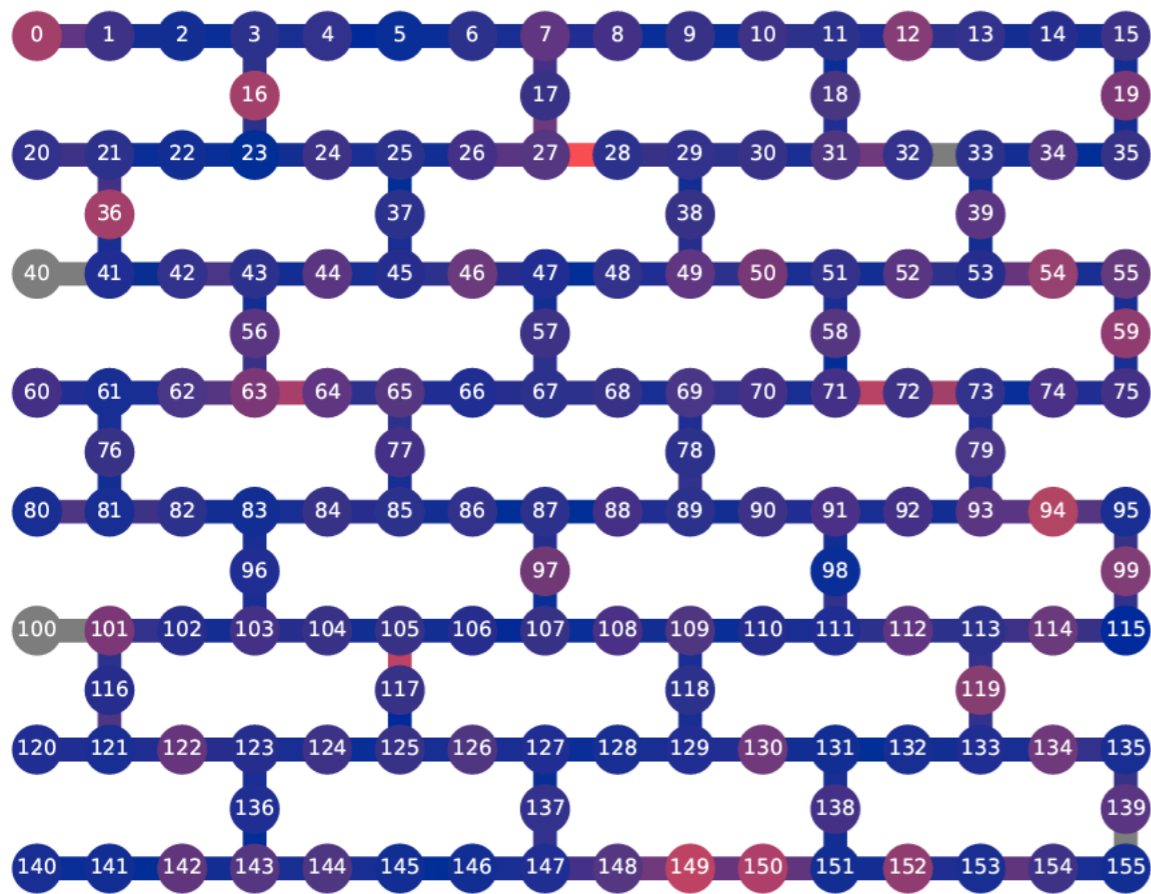
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OPERATIONS ON THE QC



IBM's Fez

Superconducting qubits



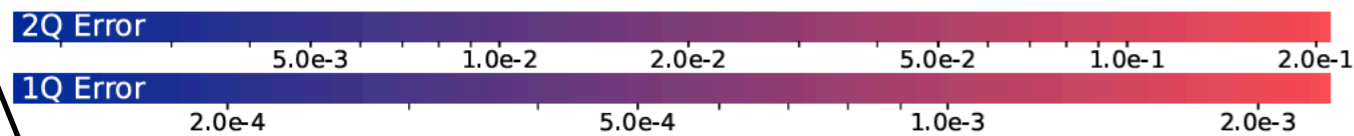
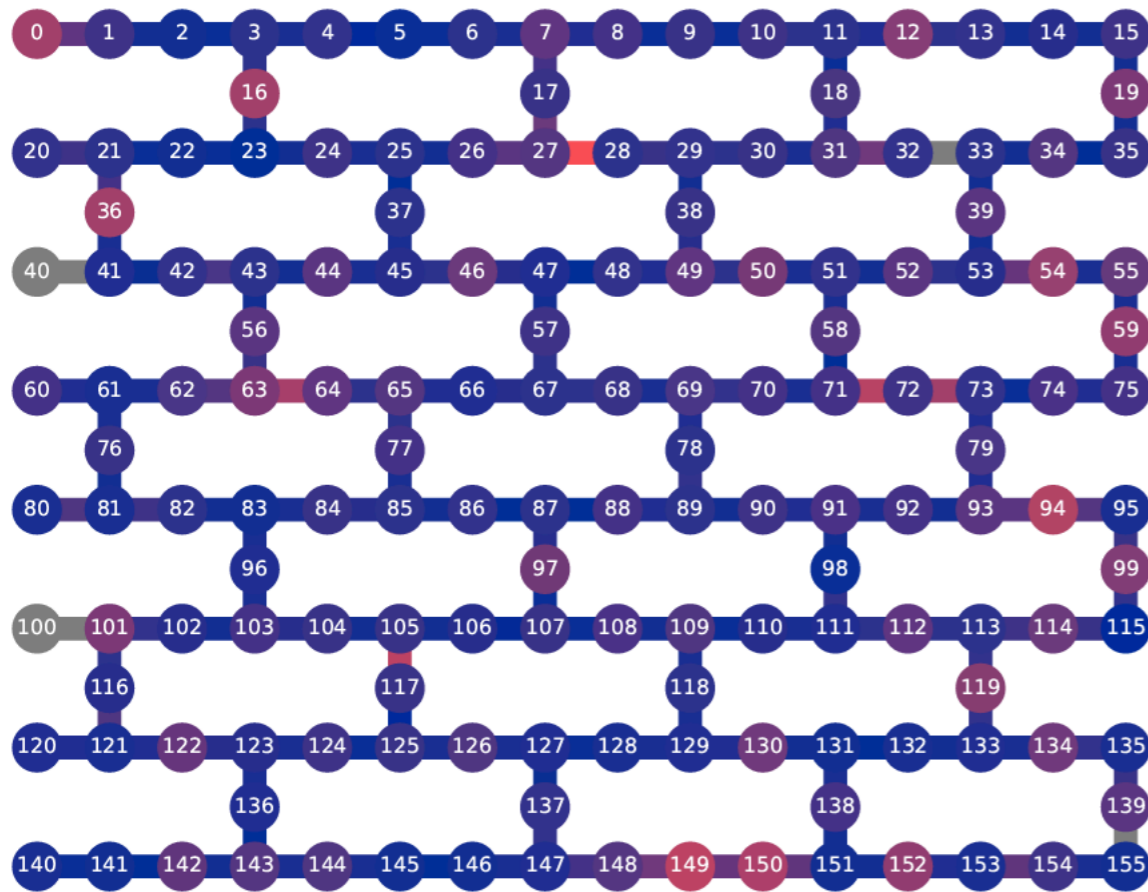
2qubit, 1qubit Error rates

OPERATIONS ON THE QC

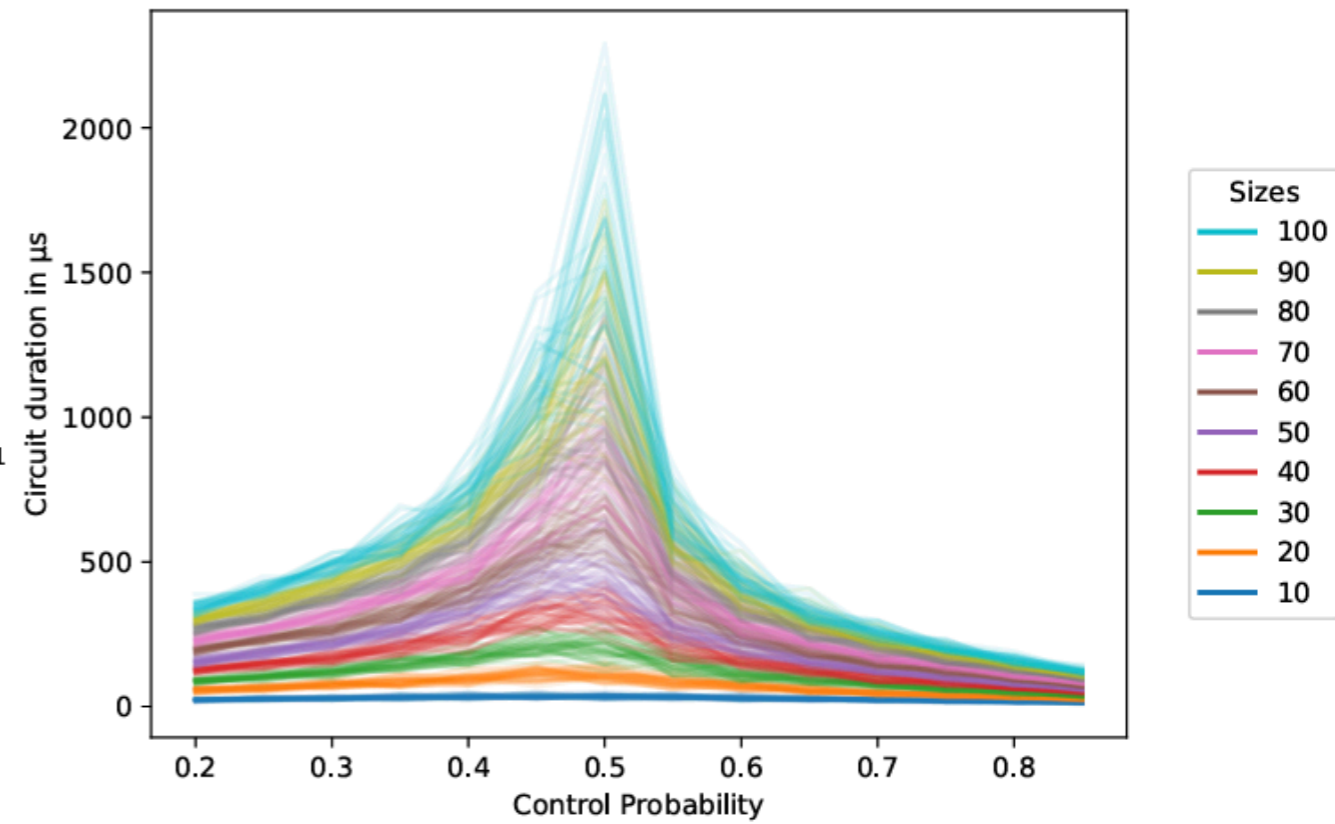
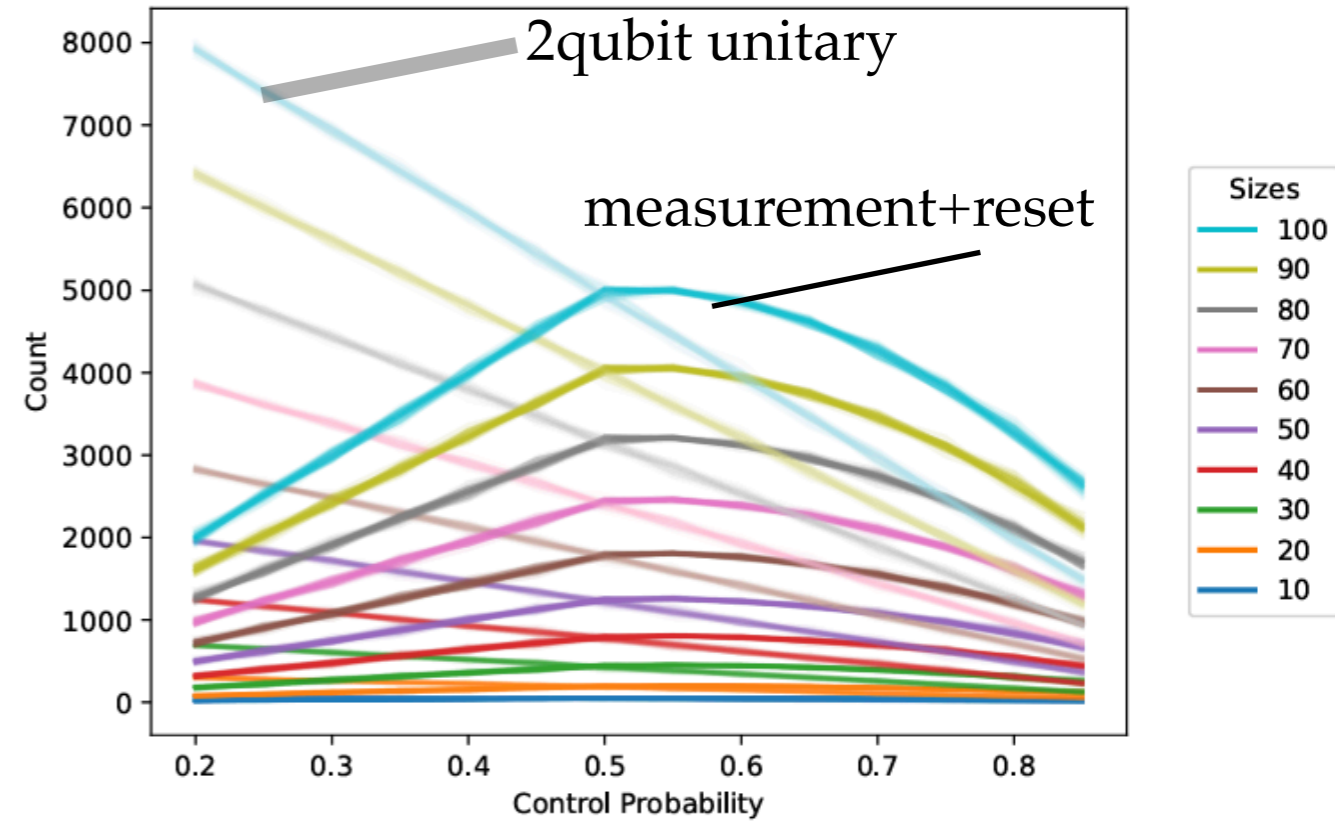


IBM's Fez

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CONTROL TRANSITIONS ROBUSTNESS TO NOISE

IBM Q

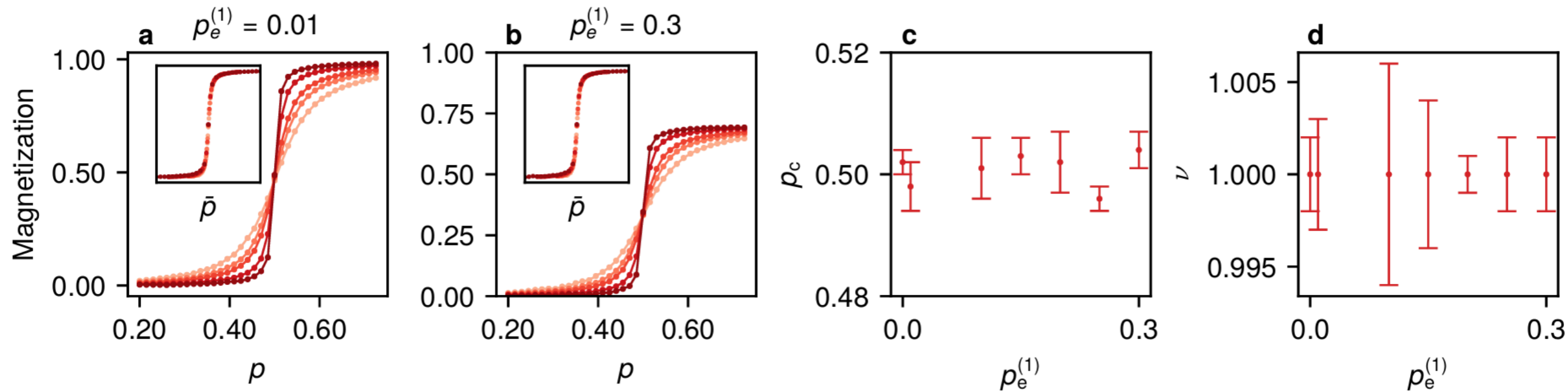


Fig. 7 | Control Transition in the Presence of Noise. Results are shown for mechanics model calculations (red circles; $L = 16 - 128$). **a** The magnetization density averaged over initial states, measurement outcomes and circuits (denoted $\langle M_z \rangle$) as a function of the control probability p for various system sizes L , and single qubit depolarizing error $p_e^{(1)} = 0.01$ and two qubit depolarizing error $p_e^{(2)} = 0.001$. (Inset) Data collapse as a function of $\tilde{p} = L^{1/\nu}(p - p_c)$ with $p_c = 0.498(1)$ and $\nu = 1.000(2)$ **b** The magnetization density for single qubit depolarizing error $p_e^{(1)} = 0.3$ and two qubit depolarizing error $p_e^{(2)} = 0.03$. (Inset) Data collapse as a function of $\tilde{p} = L^{1/\nu}(p - p_c)$ with $p_c = 0.504(3)$ and $\nu = 1.000(2)$ **c** The fitted p_c as a function of the single qubit depolarizing error rate $p_e^{(1)}$, where $p_e^{(2)} = p_e^{(1)}/10$ is fixed. **d** The fitted ν as a function of the single qubit depolarizing error rate $p_e^{(1)}$.

DATA ON IBM'S QUANTUM COMPUTER

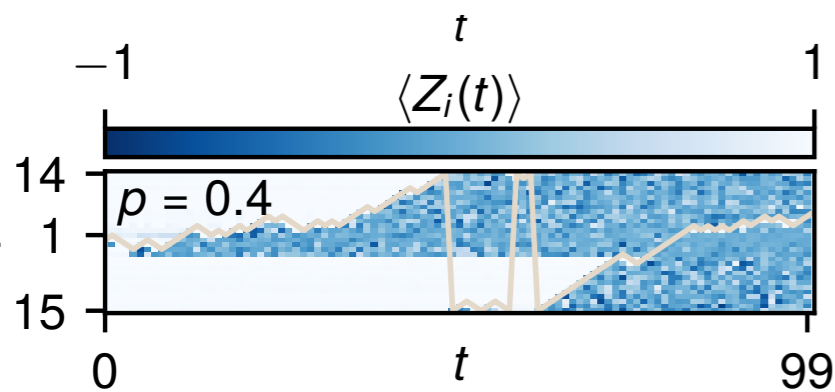
IBM Q

Dynamics in each regime on the quantum computer, $L=30$

(Below are one realization of the circuit averaged over 10^4 shots)

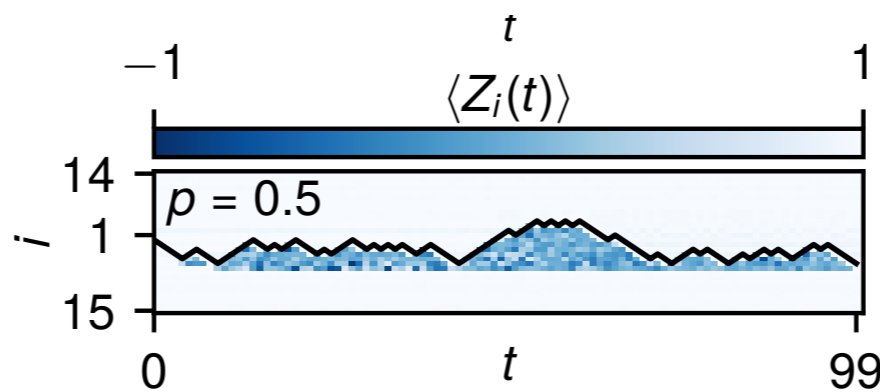
$$p = 0.4$$

Chaotic



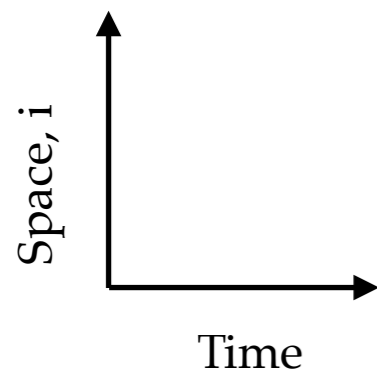
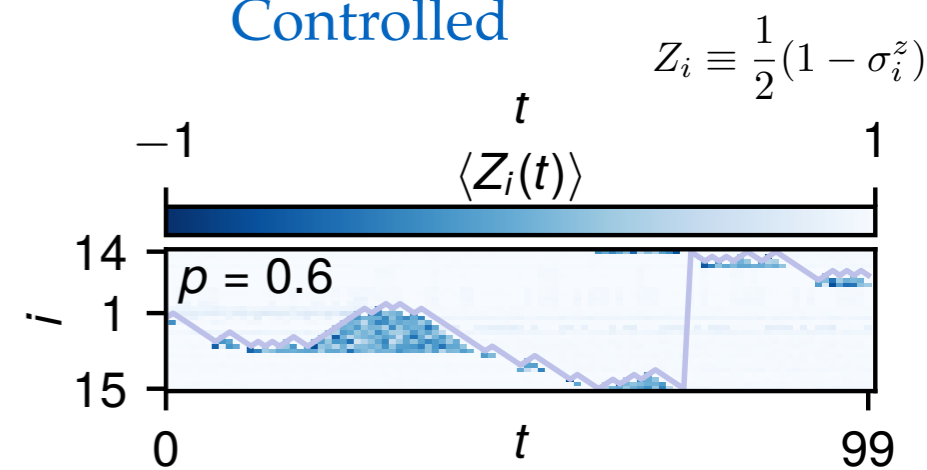
$$p_{\text{ctrl}} = 0.5$$

Critical



$$p_{\text{ctrl}} = 0.6$$

Controlled



DATA ON IBM'S QUANTUM COMPUTER

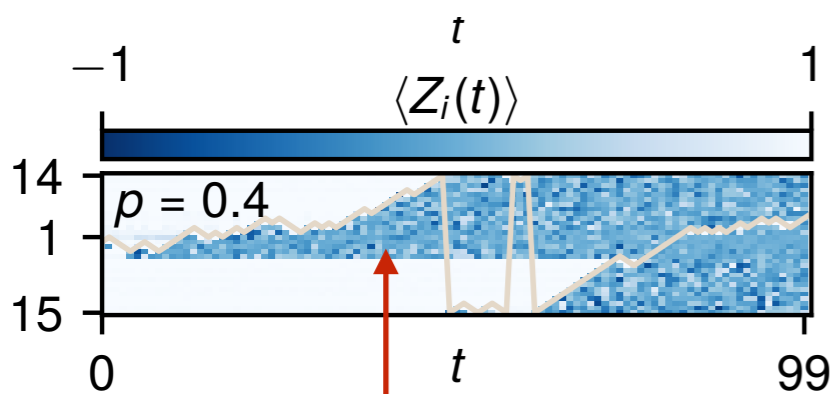
IBM Q

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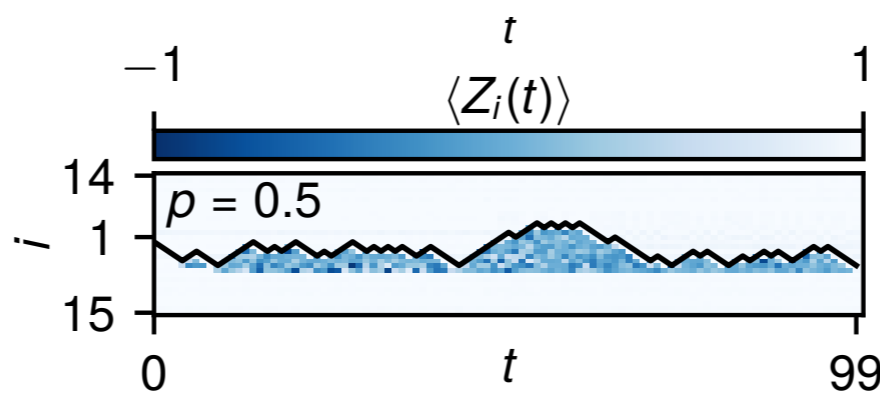
$$p = 0.4$$

Chaotic



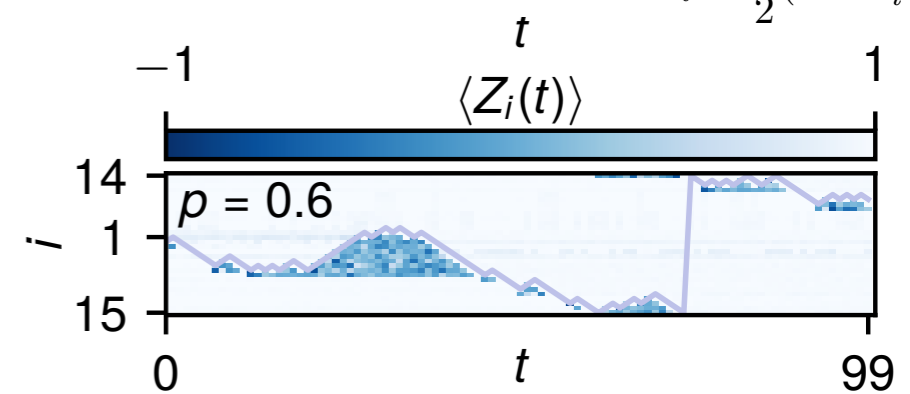
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Critical

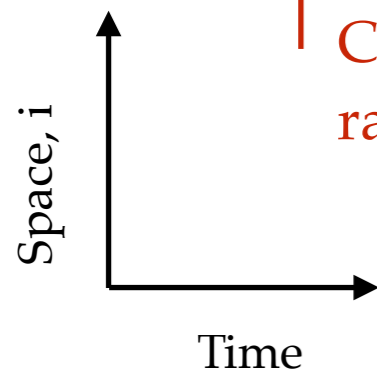


$$p_{\text{ctrl}} = 0.6$$

Controlled



Chaotic dynamics
randomizing all sites



DATA ON IBM'S QUANTUM COMPUTER

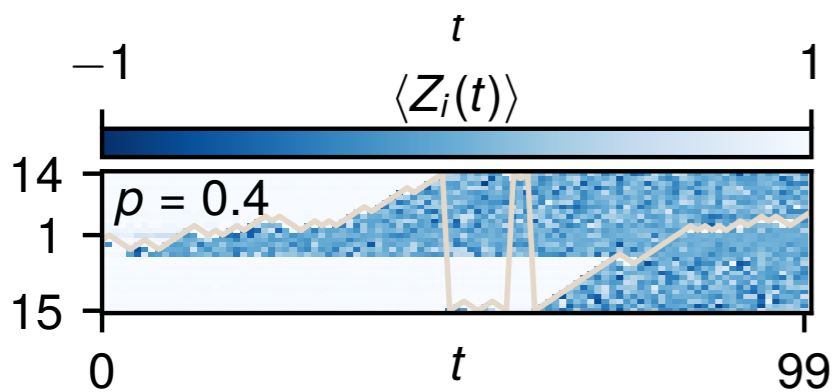
IBM Q

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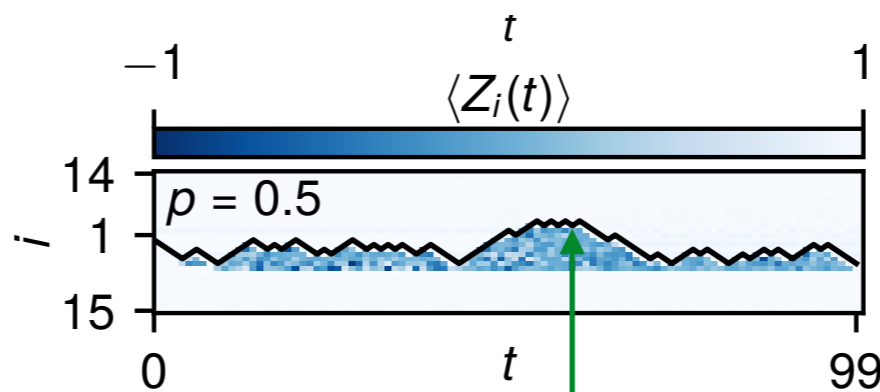
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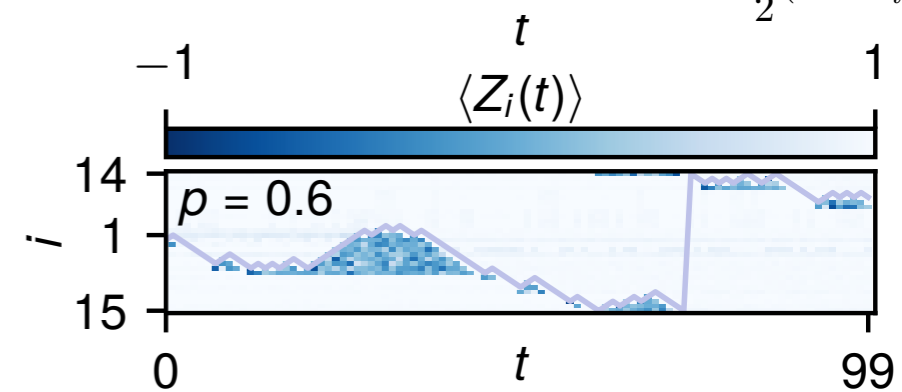
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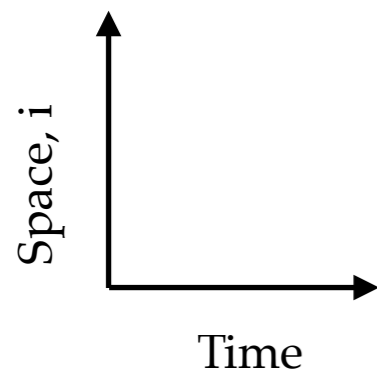


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Controlled



$$Z_i \equiv \frac{1}{2}(1 - \sigma_i^z)$$



DATA ON IBM'S QUANTUM COMPUTER

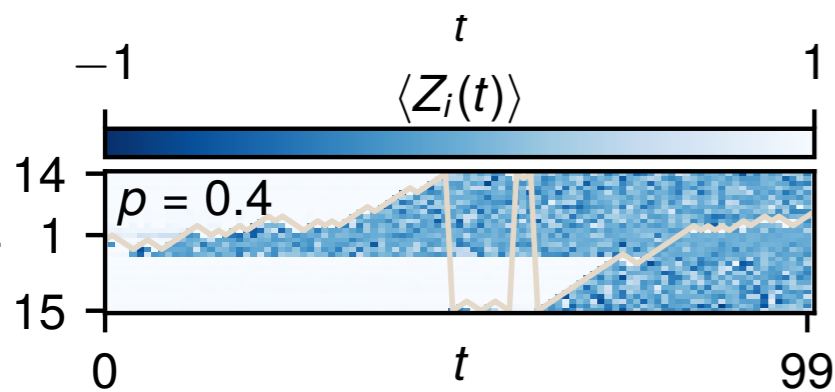
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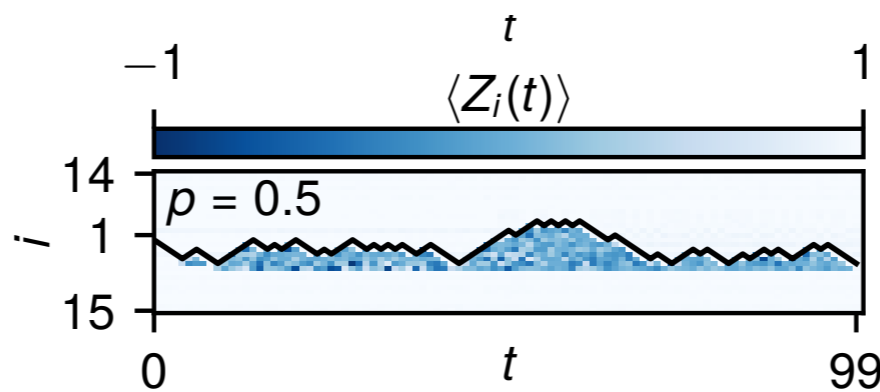
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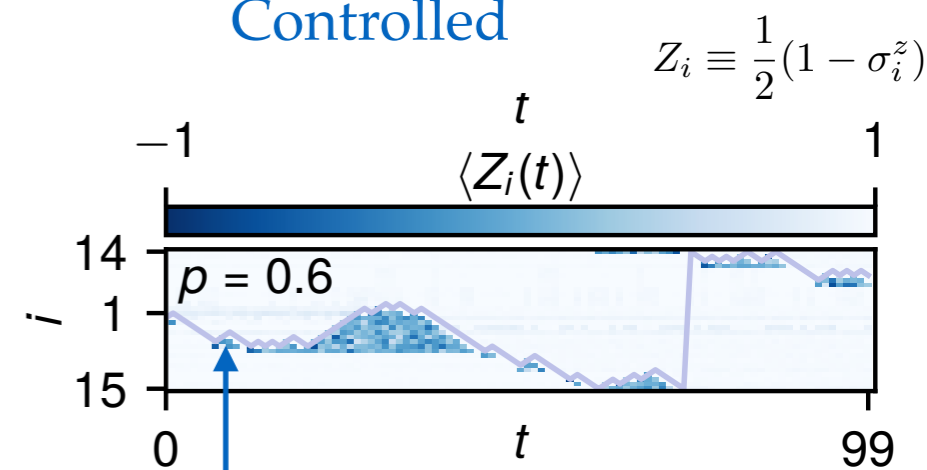
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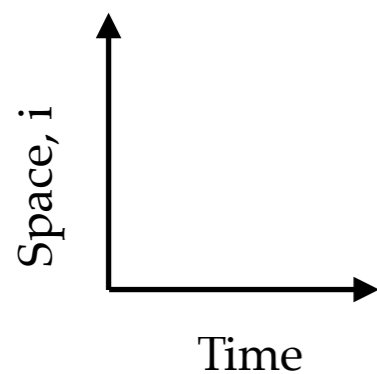


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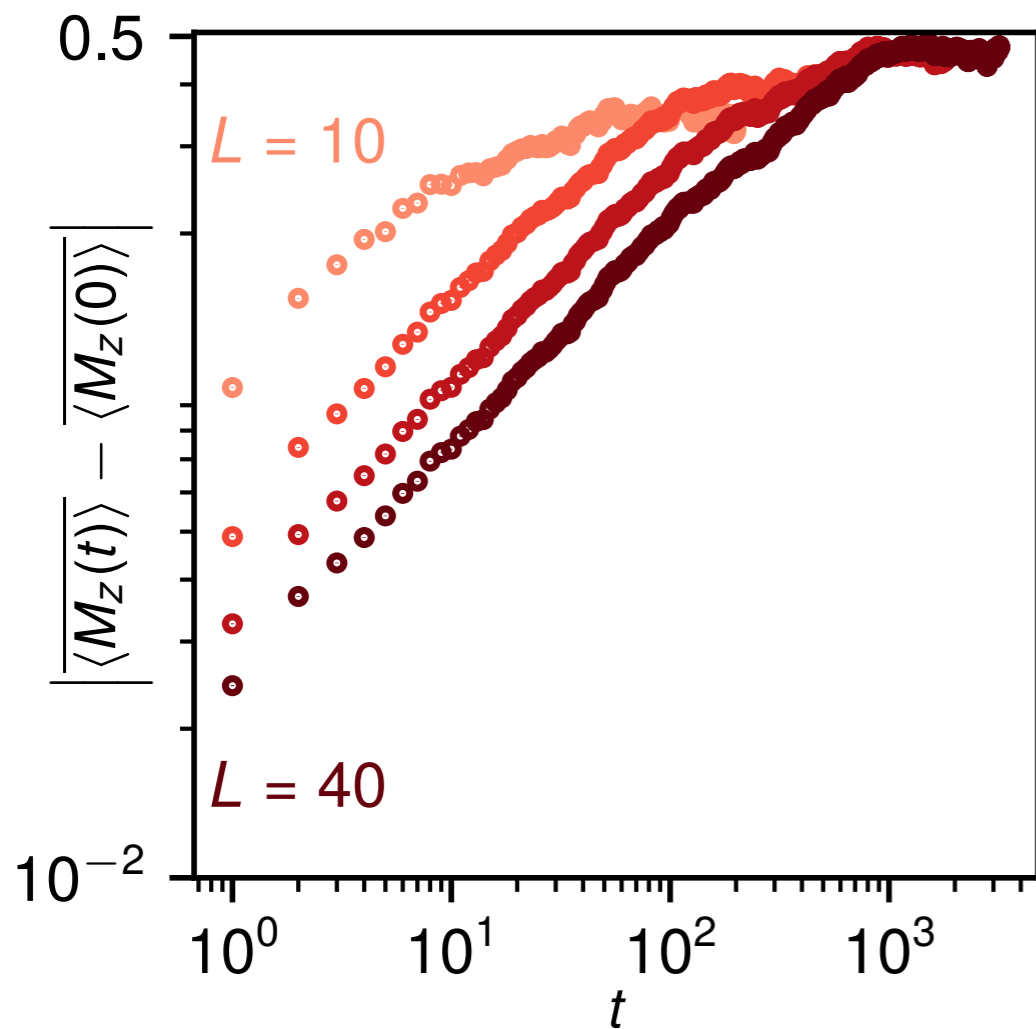
Resets are zeroing out and controlling the configuration

DATA ON IBM'S QUANTUM COMPUTER

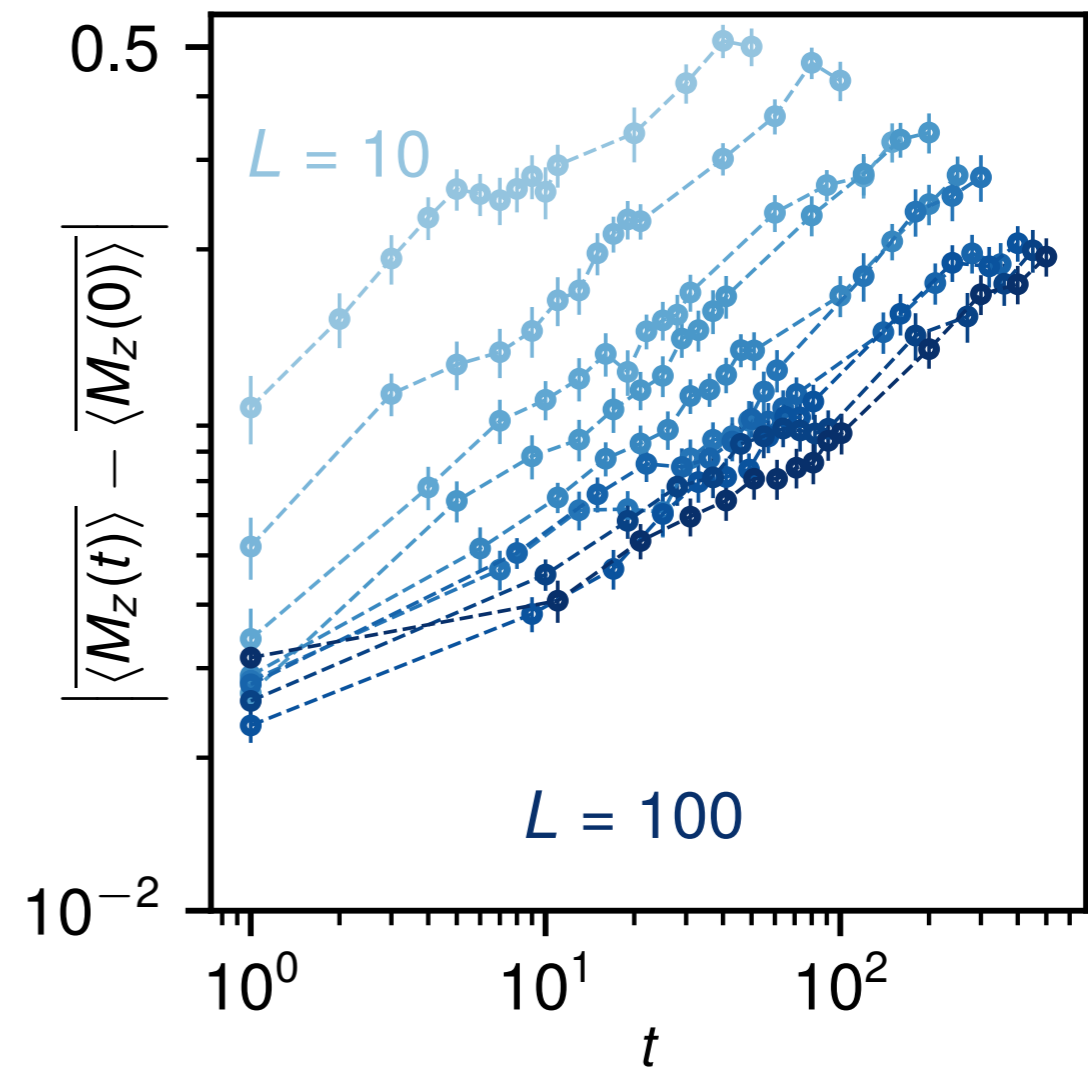
IBM Q

Critical properties of the dynamics $p_{\text{ctrl}} = p_c$

Theory



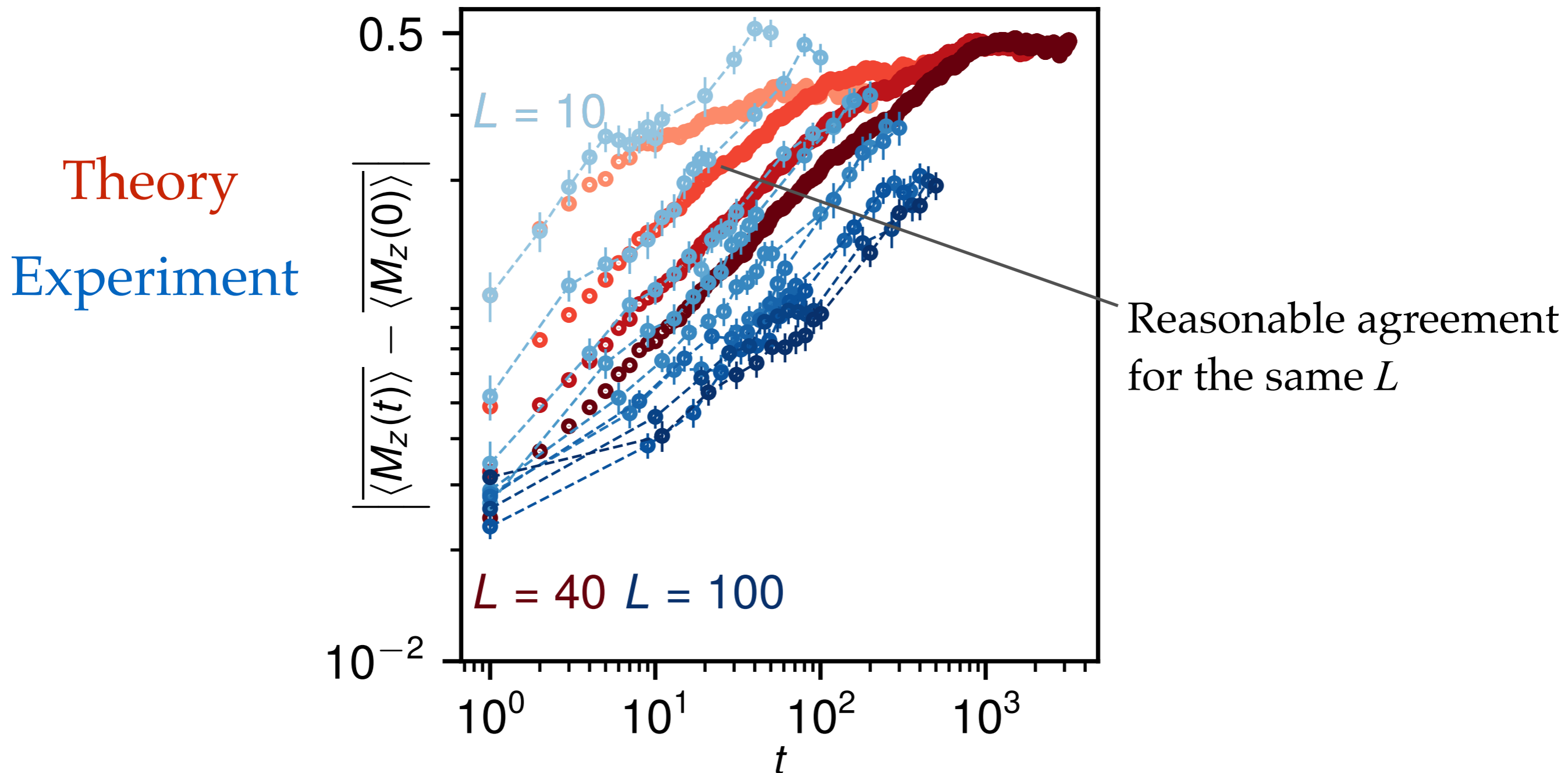
Experiment



DATA ON IBM'S QUANTUM COMPUTER

IBM Q

Critical properties of the dynamics $p_{\text{ctrl}} = p_c$

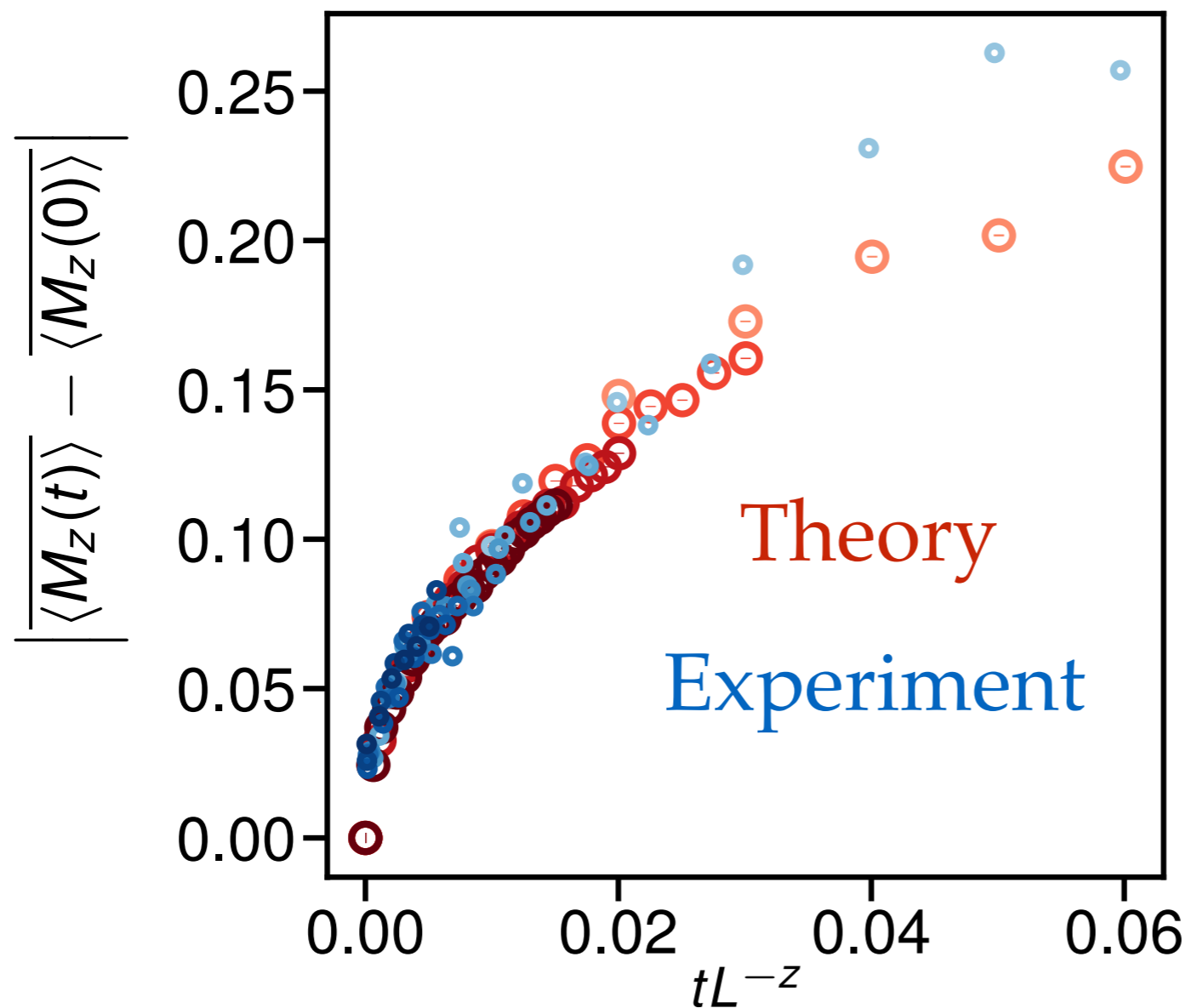


DATA ON IBM'S QUANTUM COMPUTER

IBM Q

Critical properties of the dynamics $p_{\text{ctrl}} = p_c$

Apply finite size scaling to the critical point



$$M_z \sim g(t/L^z)$$

Matrix product state simulations (on a classical computer)

$$z = 2.00(2)$$

Experimental data from IBM's quantum computer Fez

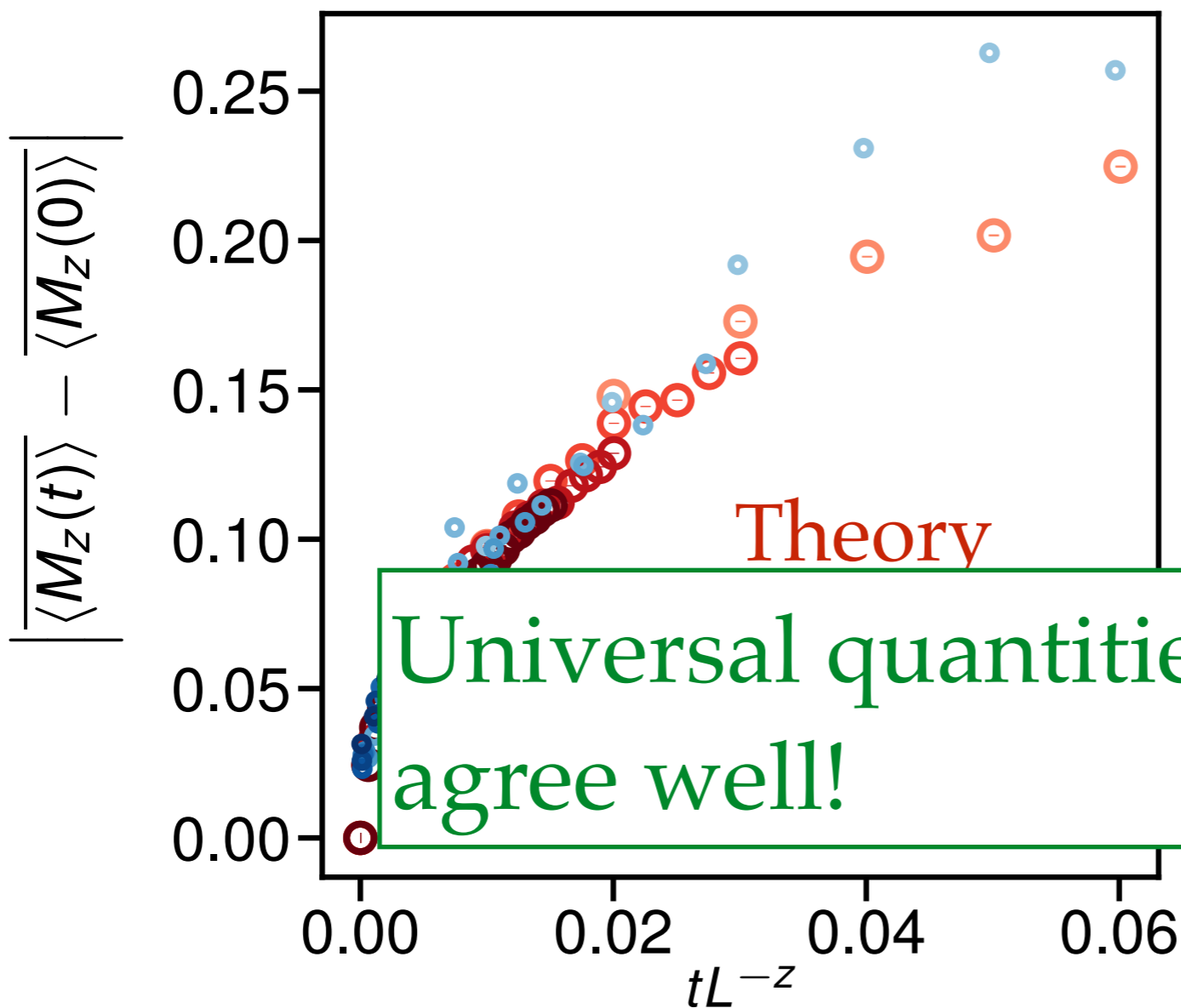
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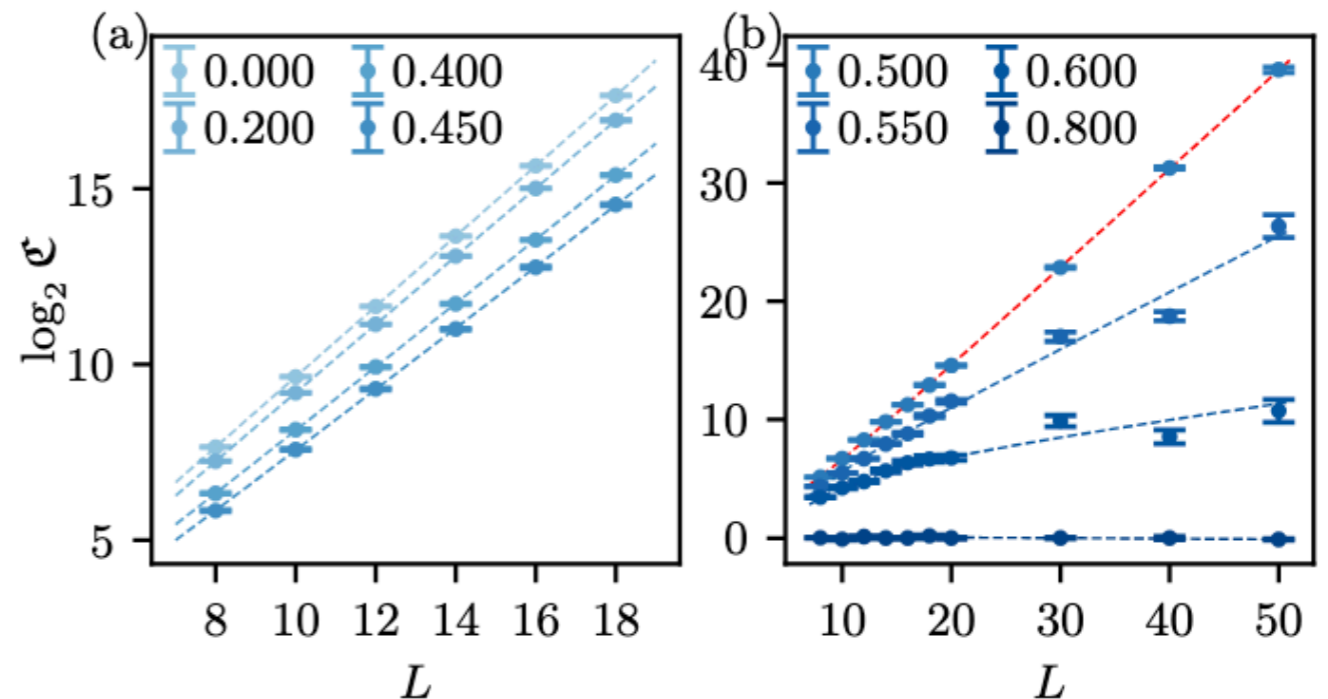
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NUMERICAL EVIDENCE OF STRONG QUANTUM FLUCTUATIONS

Consider the quantum coherence of the wavefunction

$$\mathfrak{C} = \mathbb{E}_M \left[\sum_{x \neq x'} |(\rho_M)_{x,x'}| \right]$$

$$(\rho_M)_{x,x'} = |x\rangle\langle x'| = \bigotimes_{i=1}^L |b_i\rangle\langle b'_i|$$



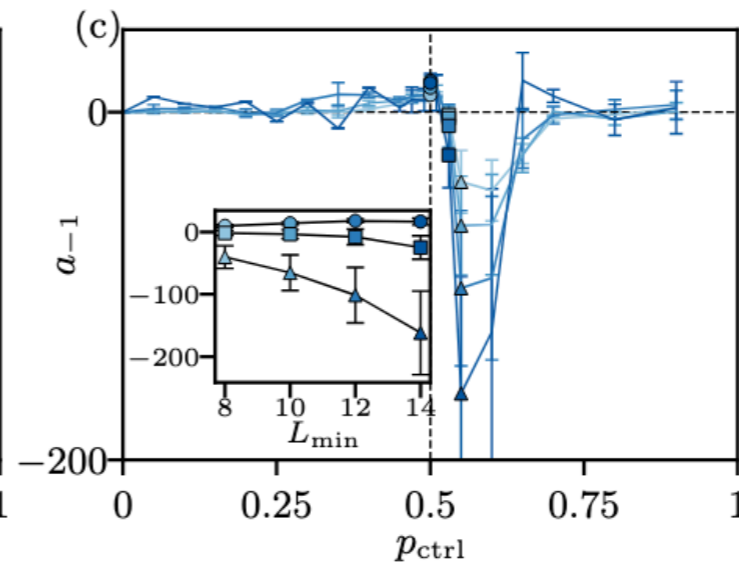
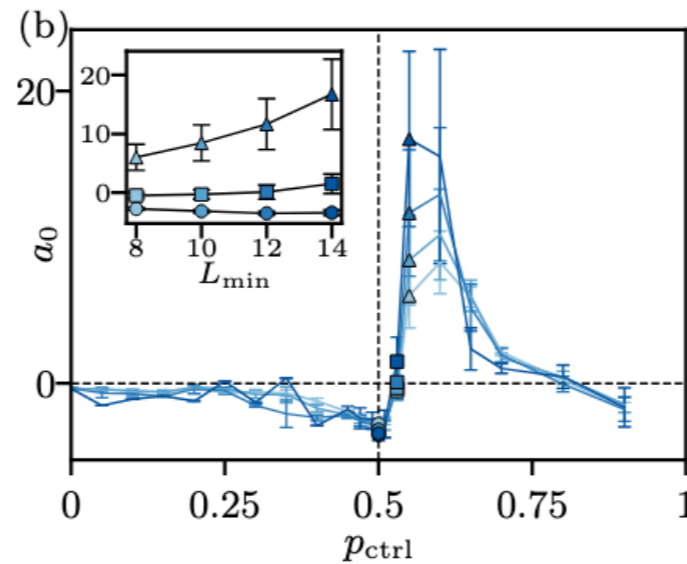
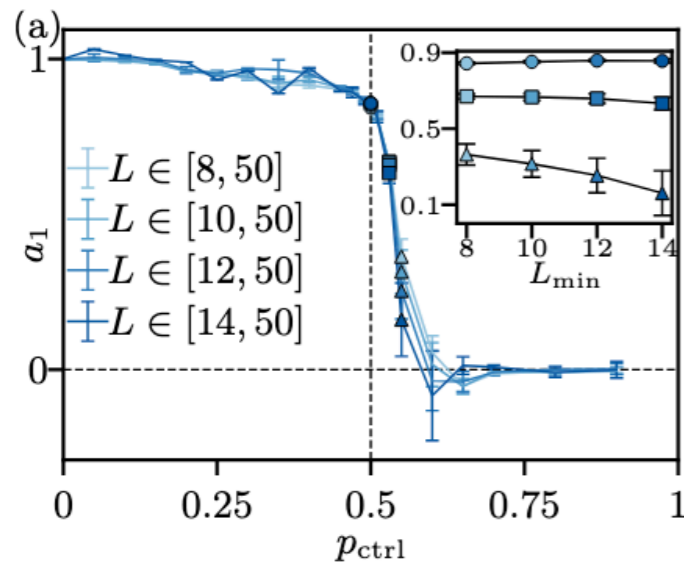
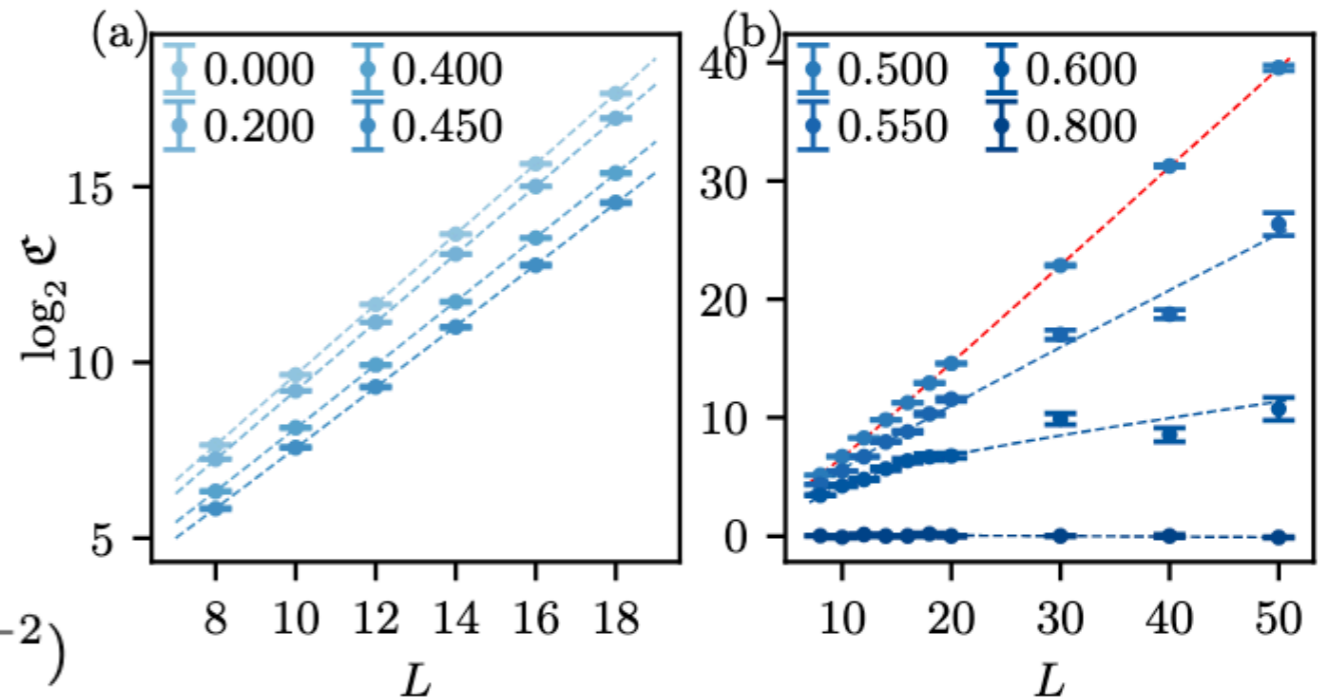
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$$\log_2 \mathfrak{C} \sim a_1(p)L + a_0(p) + a_{-1}(p)L^{-1} + O(L^{-2})$$



QUANTUM FLUCTUATIONS AT THE CONTROL TRANSITION

Can do more than averages:

Variance over measurement outcomes and circuits

Averages over samples: $\mathbb{E}_M[\dots] = \sum_M p_M[\dots] = \sum_C p_C \sum_{m_C} p_{m_C|C}[\dots]$ $M = \{C, m_C\}$

$|\psi\rangle = |\psi_{C, m_C}\rangle$

$= \mathbb{E}_C [\mathbb{E}_{m_C}[\dots]]$

Average over Circuits realizations (green arrow pointing to \mathbb{E}_C)

Average over measurement outcomes (blue arrow pointing to \mathbb{E}_{m_C})

Circuits realizations (green arrow pointing to C)

Measurement outcomes for this circuit (blue arrow pointing to m_C)

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Circuits realizations (green arrow pointing to C)

Measurement outcomes for this circuit (blue arrow pointing to m_C)

Consider the **variance** of an observable O

$$\sigma_M^2[O] = \mathbb{E}_M [O_M^2] - (\mathbb{E}_M [O_M])^2 \quad O_M = \langle \psi_{C, m_C} | O | \psi_{C, m_C} \rangle$$

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Can break up the variance into two parts

$$\begin{aligned} \sigma_M^2[O] &= \mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}^2] - (\mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}])^2 \\ &= \underbrace{\mathbb{E}_C \left[(\mathbb{E}_{m_C} [O_{m_C}])^2 \right] - (\mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}])^2}_{\text{circuit fluctuations}} + \underbrace{\mathbb{E}_C \left[\mathbb{E}_{m_C} [O_{m_C}^2] - (\mathbb{E}_{m_C} [O_{m_C}])^2 \right]}_{\text{quantum fluctuations}} \\ &= \sigma_C^2[O] + \mathbb{E}_C [\sigma_{Q,C}^2[O]] \quad , \end{aligned}$$

QUANTUM FLUCTUATIONS AT THE CONTROL TRANSITION

Can do more than averages:

Variance over measurement outcomes and circuits

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Circuits realizations (green arrow pointing to C)

Measurement outcomes for this circuit (blue arrow pointing to m_C)

Consider the **variance** of an observable O

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$|\psi_0\rangle = |0\dots 01\rangle$

Can break up the variance into two parts

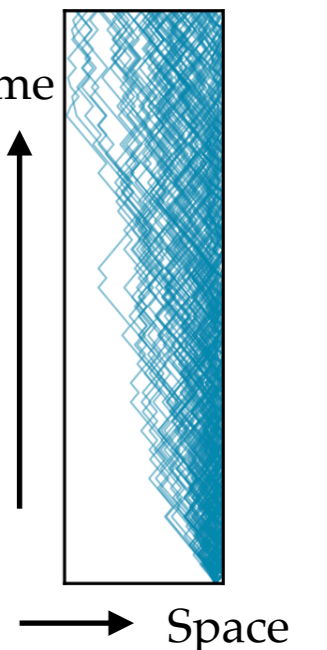
$$\sigma_M^2[O] = \mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}^2] - (\mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}])^2$$

$$= \underbrace{\mathbb{E}_C \left[(\mathbb{E}_{m_C} [O_{m_C}])^2 \right] - (\mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}])^2}_{\text{circuit fluctuations}} + \underbrace{\mathbb{E}_C \left[\mathbb{E}_{m_C} [O_{m_C}^2] - (\mathbb{E}_{m_C} [O_{m_C}])^2 \right]}_{\text{quantum fluctuations}}$$

$$= \sigma_C^2[O] + \mathbb{E}_C [\sigma_{Q,C}^2[O]] ,$$

$$O = \hat{r}^{\text{time}}$$

$$= \sum_x r_x |x\rangle \langle x|$$



QUANTUM FLUCTUATIONS AT THE CONTROL TRANSITION

Can do more than averages:

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Average over measurement outcomes (blue arrow pointing to \mathbb{E}_{m_C})

Circuits realizations (green arrow pointing to C)

Measurement outcomes for this circuit (blue arrow pointing to m_C)

Using **post selection** can break up the variance into **three parts**

$$\sigma_M^2[O] = \mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}^2] - (\mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}])^2$$

$$= \underbrace{\mathbb{E}_C [(\mathbb{E}_{m_C} [O_{m_C}])^2] - (\mathbb{E}_C \mathbb{E}_{m_C} [O_{m_C}])^2}_{\text{circuit fluctuations}} + \underbrace{\mathbb{E}_C [\mathbb{E}_{m_C} [O_{m_C}^2] - (\mathbb{E}_{m_C} [O_{m_C}])^2]}_{\text{quantum fluctuations}}$$

$$= \sigma_C^2[O] + \mathbb{E}_C [\sigma_{Q,c}^2[O]]$$

$$\sigma_{Q,c}^2[O] = \underbrace{\mathbb{E}_{m_C} [\langle O \rangle_{m_C}^2] - (\mathbb{E}_{m_C} [\langle O \rangle_{m_C}])^2}_{\text{trajectory fluctuations}} + \underbrace{\mathbb{E}_{m_C} [\langle O^2 \rangle_{m_C} - \langle O \rangle_{m_C}^2]}_{\text{state fluctuations}}$$

$$= \sigma_t^2[O] + \mathbb{E}_{m_C} [\sigma_{s,m_C}^2[O]]$$

QUANTUM FLUCTUATIONS AT THE CONTROL TRANSITION

Using **post selection** can break up the variance into **three parts**

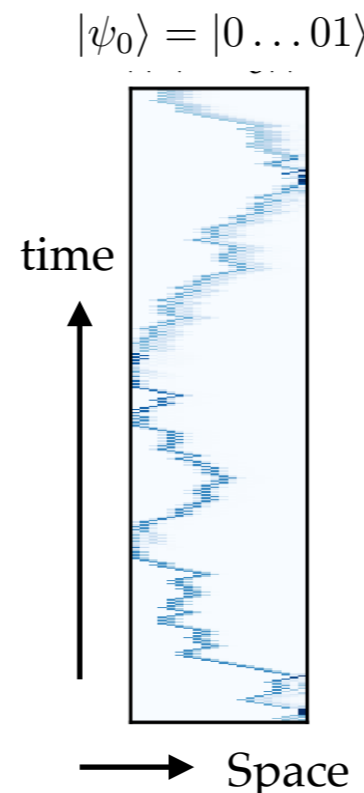
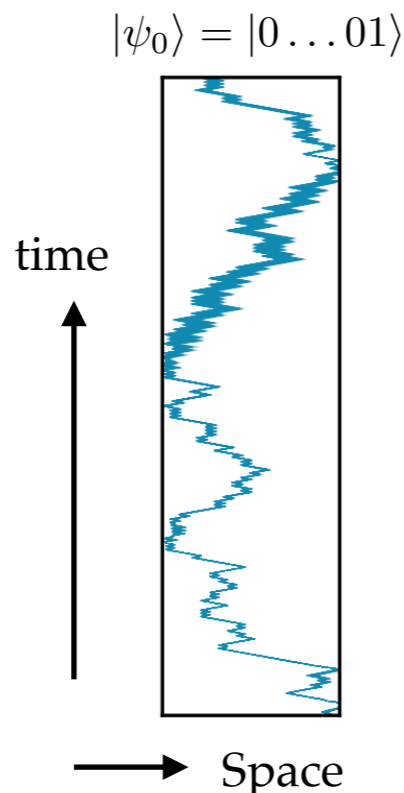
$$O_M = \langle \psi_{c,m_c} | O | \psi_{c,m_c} \rangle$$

$$\begin{aligned} \sigma_M^2[O] &= \mathbb{E}_c \mathbb{E}_{m_c} [O_{m_c}^2] - (\mathbb{E}_c \mathbb{E}_{m_c} [O_{m_c}])^2 \\ &= \underbrace{\mathbb{E}_c [(\mathbb{E}_{m_c} [O_{m_c}])^2] - (\mathbb{E}_c \mathbb{E}_{m_c} [O_{m_c}])^2}_{\text{circuit fluctuations}} + \underbrace{\mathbb{E}_c [\mathbb{E}_{m_c} [O_{m_c}^2] - (\mathbb{E}_{m_c} [O_{m_c}])^2]}_{\text{quantum fluctuations}} \\ &= \sigma_c^2[O] + \mathbb{E}_c [\sigma_{Q,c}^2[O]] \end{aligned}$$

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$$O = \hat{r} = \sum r_x |x\rangle \langle x|$$

Fluctuations from **measurement outcomes**



Fluctuations from **Wave function superpositions**

DATA ON IBM'S QUANTUM COMPUTER

IBM Q

Can do more than averages:

Variance over measurement outcomes and circuits

$$\underbrace{\mathbb{E}_C \left[\mathbb{E}_{m_c} [O_{m_c}^2] - (\mathbb{E}_{m_c} [O_{m_c}])^2 \right]}_{\text{quantum fluctuations}}$$

Implement on experiment

$$\text{Var}_{m_c} [M_z] = \mathbb{E}_{m_c} \left[[M_z]_{m_c}^2 \right] - (\mathbb{E}_{m_c} [[M_z]_{m_c}])^2. \quad O = \hat{M}_z = \frac{1}{L} \sum_x \sigma_x^z$$

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$$\text{Var}_Q [M_z] = \text{Var}_C [\text{Var}_{m_c} [M_z]]$$

$$\text{var}_Q [M_z] (t) \sim L^{-2\beta/\nu} f'_Q [t/L^z]$$

$t \rightarrow \infty$

$\beta_{\text{classical}} = 0$

DATA ON IBM'S QUANTUM COMPUTER



Can do more than averages:

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$$\underbrace{\mathbb{E}_C \left[\mathbb{E}_{m_c} [O_{m_c}^2] - (\mathbb{E}_{m_c} [O_{m_c}])^2 \right]}_{\text{quantum fluctuations}}$$

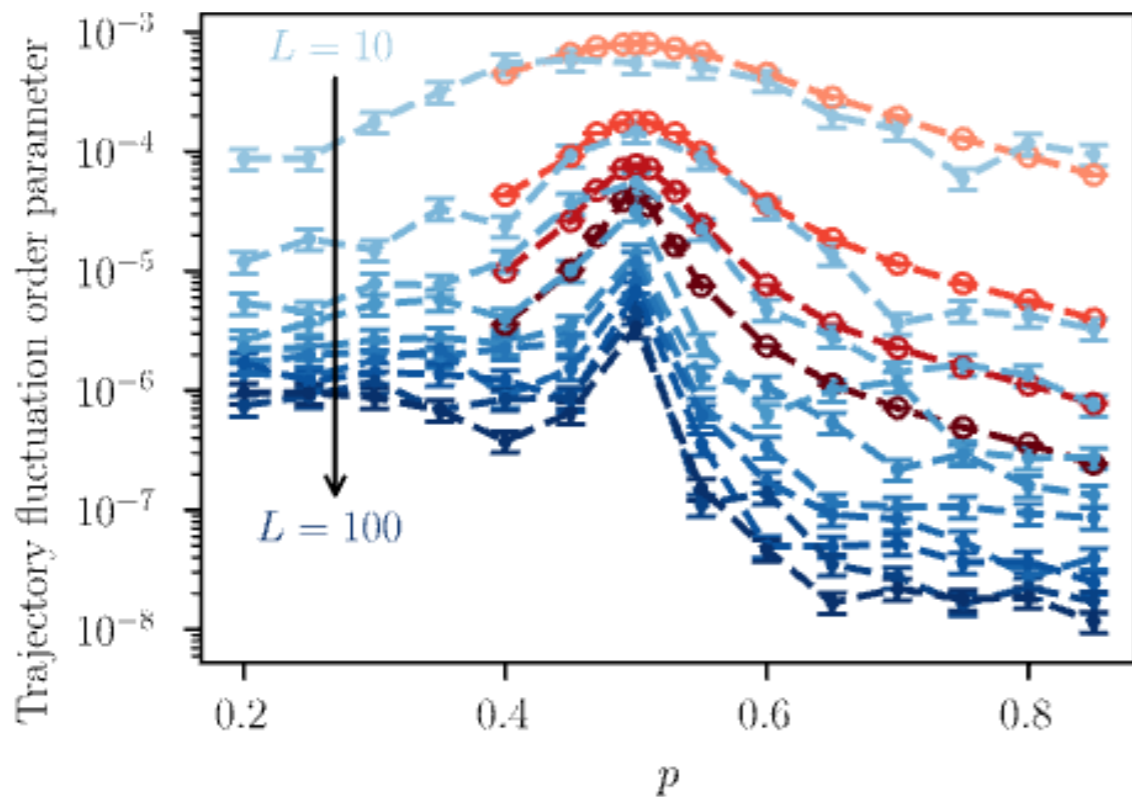
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Theory
Experiment

$$\beta_{\text{classical}} = 0$$

DATA ON IBM'S QUANTUM COMPUTER



Can do more than averages:

Variance over measurement outcomes and circuits

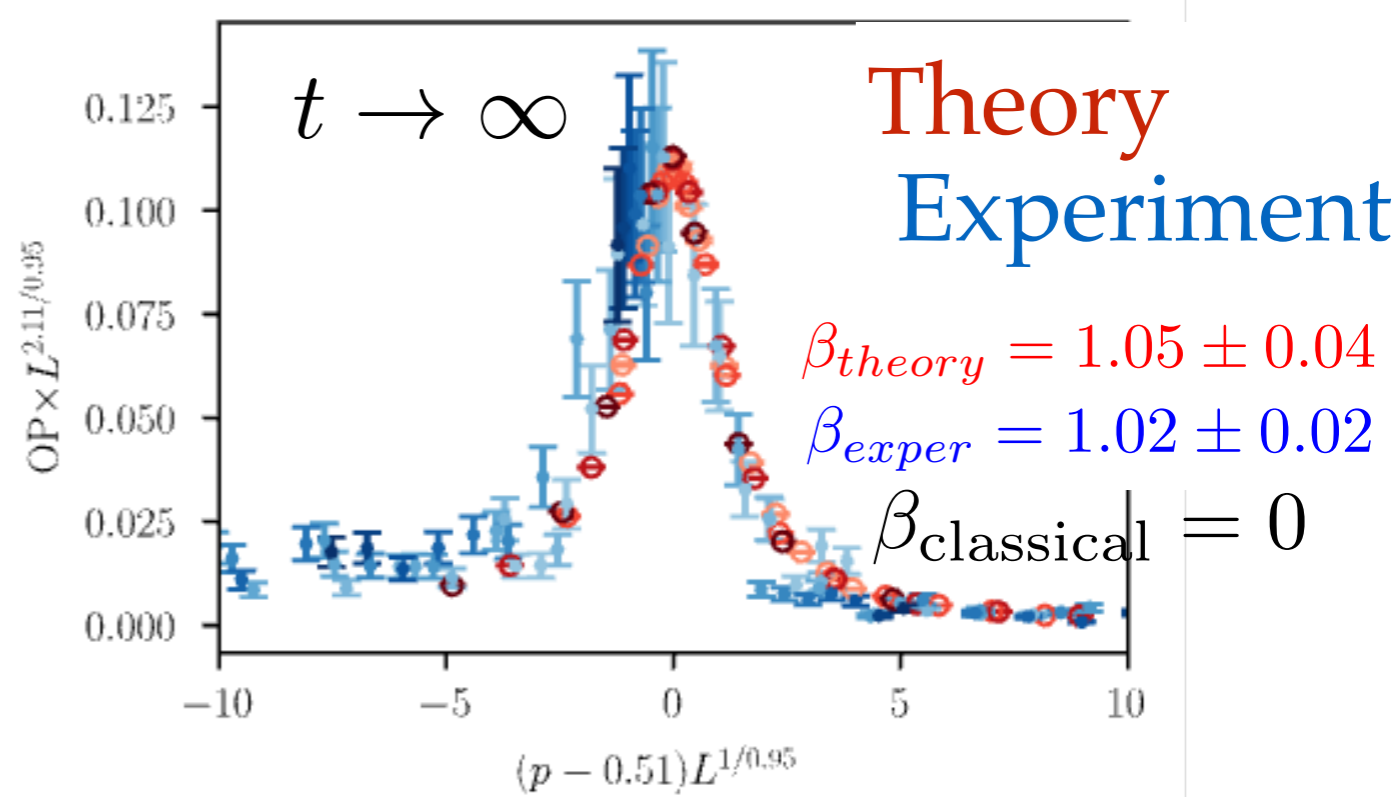
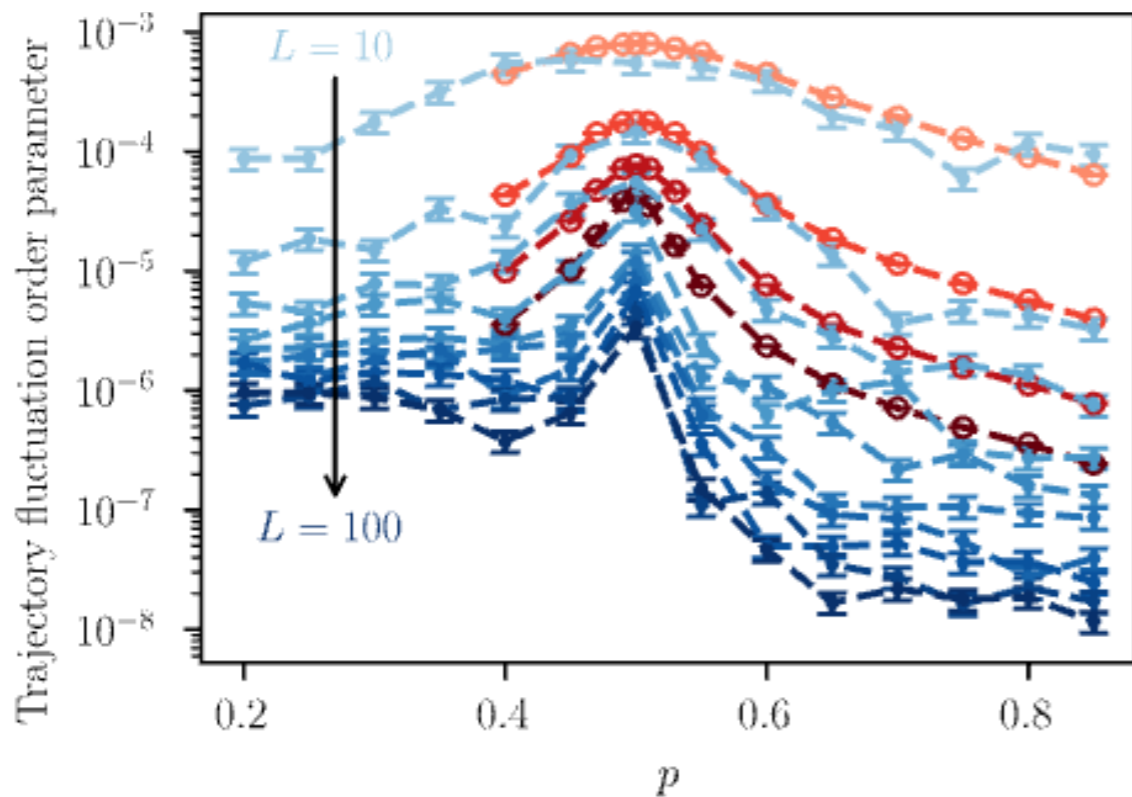
$$\underbrace{\mathbb{E}_C \left[\mathbb{E}_{m_c} [O_{m_c}^2] - (\mathbb{E}_{m_c} [O_{m_c}])^2 \right]}_{\text{quantum fluctuations}}$$

Implement on experiment

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DATA ON IBM'S QUANTUM COMPUTER

IBM Q

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Implement on experiment

$$\text{Var}_{m_C} [M_z] = \mathbb{E}_{m_C} \left[[M_z]_{m_C}^2 \right] - (\mathbb{E}_{m_C} [[M_z]_{m_C}])^2. \quad O = \hat{M}_z = \frac{1}{L} \sum_x \sigma_x^z$$

$$\text{Var}_Q [M_z] = \text{Var}_C [\text{Var}_{m_C} [M_z]]$$

Dynamics

DATA ON IBM'S QUANTUM COMPUTER



Can do more than averages:

Variance over measurement outcomes and circuits

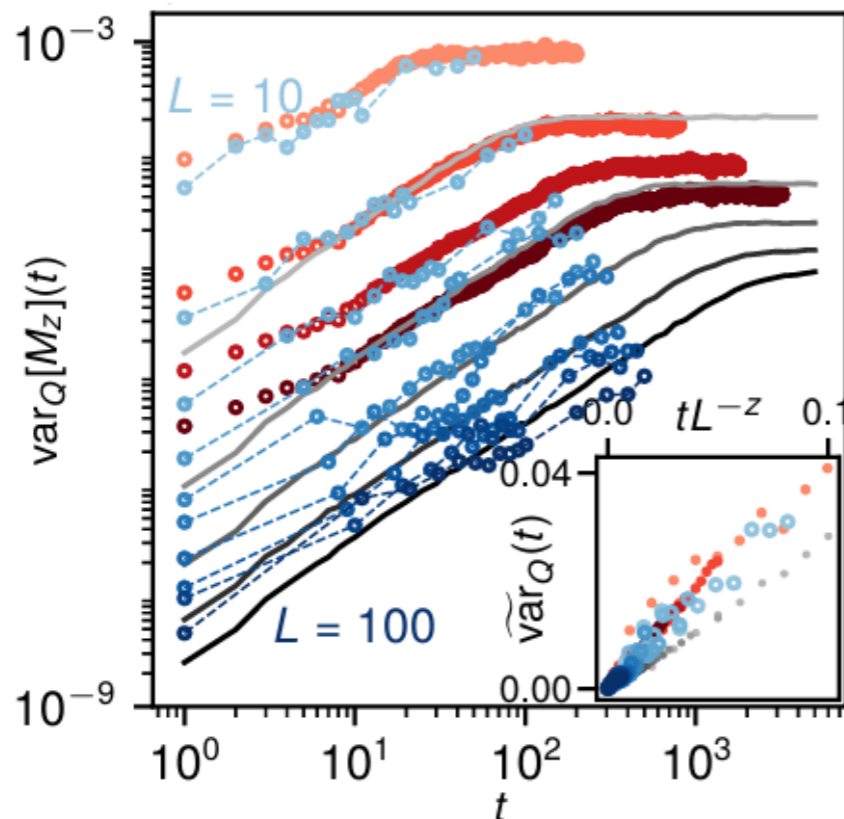
$$\underbrace{\mathbb{E}_C \left[\mathbb{E}_{m_c} [O_{m_c}^2] - (\mathbb{E}_{m_c} [O_{m_c}])^2 \right]}_{\text{quantum fluctuations}}$$

Implement on experiment

$$\text{Var}_{m_c} [M_z] = \mathbb{E}_{m_c} \left[[M_z]_{m_c}^2 \right] - (\mathbb{E}_{m_c} [[M_z]_{m_c}])^2. \quad O = \hat{M}_z = \frac{1}{L} \sum_x \sigma_x^z$$

$$\text{Var}_Q [M_z] = \text{Var}_C [\text{Var}_{m_c} [M_z]]$$

Dynamics

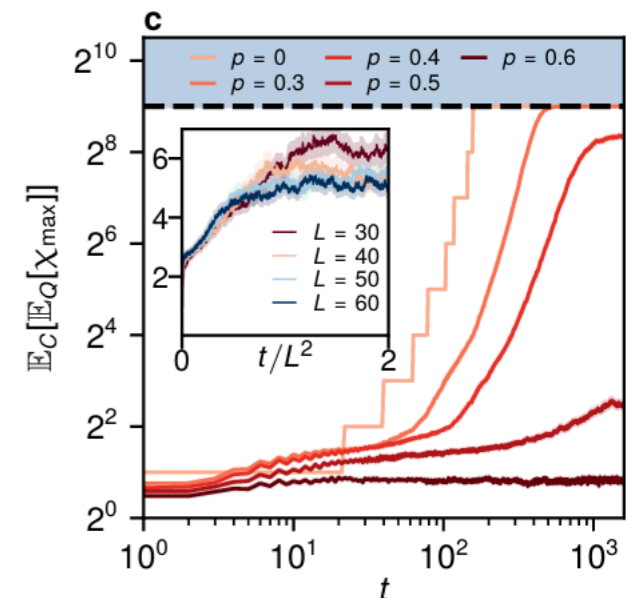


Theory
Experiment

$$z_{theory} = 2.001 \pm 0.003$$

$$z_{exper} = 2.08 \pm 0.02$$

Computational complexity



CONCLUSIONS AND OUTLOOK

- We identified a way to quantum systems following classical control of chaos.
- Control transitions may or may not coincide with the entanglement transition, depends on the structure of the feedback operation.
- Observed the Bernoulli control transition and critical properties on IBM's quantum computer with 100 qubits finding good agreement with the theoretical estimate on 24 qubits.
- Quantum fluctuation estimates are most likely not “spoofed” by (dephasing) noise.

Thank you for your attention!

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Iadecola, Ganeshan, JHP, Wilson PRL (2023)

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Prasad, Chakraborty, Iadecola, Kulkarni, JHP, Ganeshan, Wilson arXiv (2026)

NOISE COMPLICATES IDENTIFYING QUANTUM FLUCTUATIONS

Try to ensure our approach cannot be “fooled” with noise

$$\sigma_{Q,c}^2[O] = \underbrace{\mathbb{E}_{m_c} [\langle O \rangle_{m_c}^2] - (\mathbb{E}_{m_c} [\langle O \rangle_{m_c}])^2}_{\text{trajectory fluctuations}} + \underbrace{\mathbb{E}_{m_c} [\langle O^2 \rangle_{m_c} - \langle O \rangle_{m_c}^2]}_{\text{state fluctuations}}$$

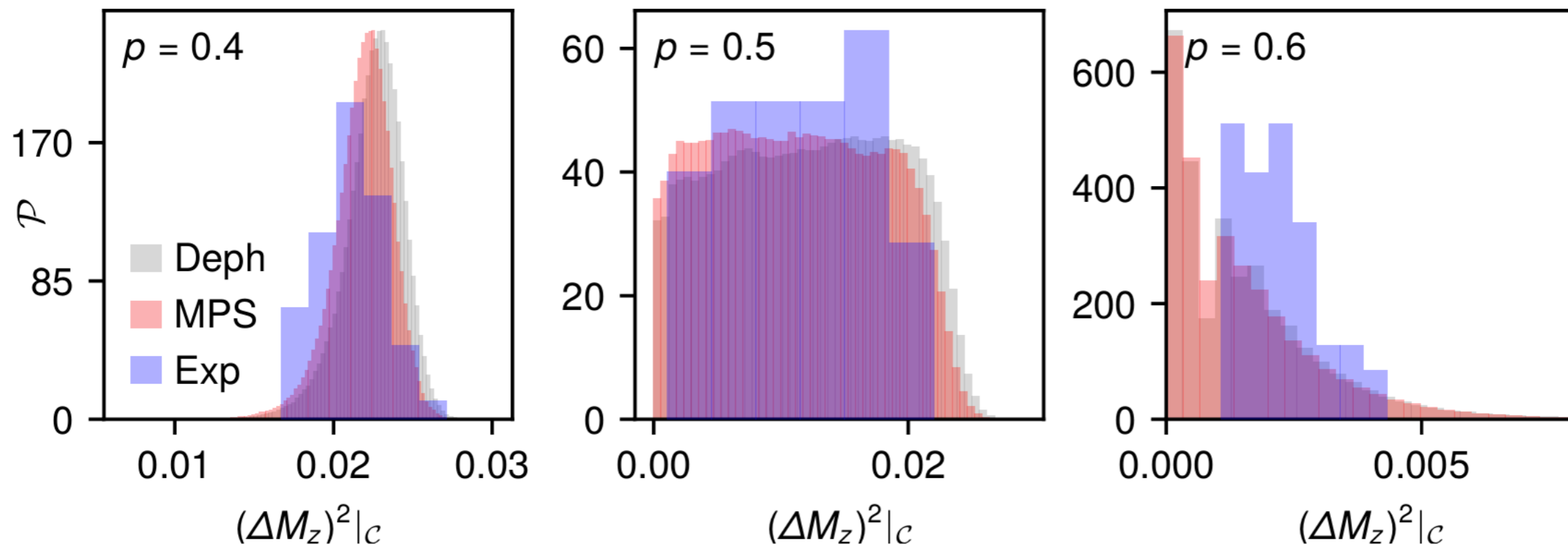
Consider, **dephasing noise channel**

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Consider, **dephasing noise channel** (grey classical, **red theory**, **blue experiment**)



$$p = 0.4 < p_{CIPT}$$

$$p = 0.5 = p_{CIPT}$$

$$p = 0.6 > p_{CIPT}$$

NOISE COMPLICATES IDENTIFYING QUANTUM FLUCTUATIONS

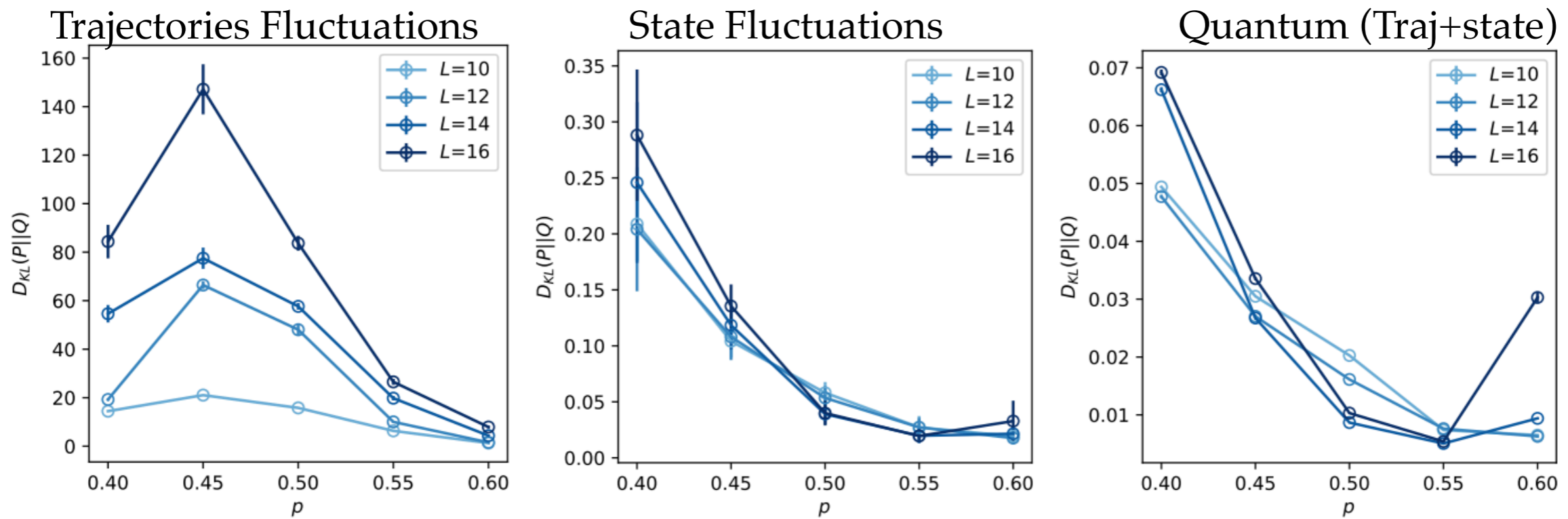
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Quantify difference in the distributions $p(x)$, $q(x)$

via the Kullback-Leibler (KL) Divergence $D_{\text{KL}}(P||Q) = \int_{-\infty}^{\infty} p(x) \log(p(x)/q(x)) dx$,

Compare with dephasing noise channel (Theory)



NOISE COMPLICATES IDENTIFYING QUANTUM FLUCTUATIONS

Try to ensure our approach cannot be “fooled” with noise

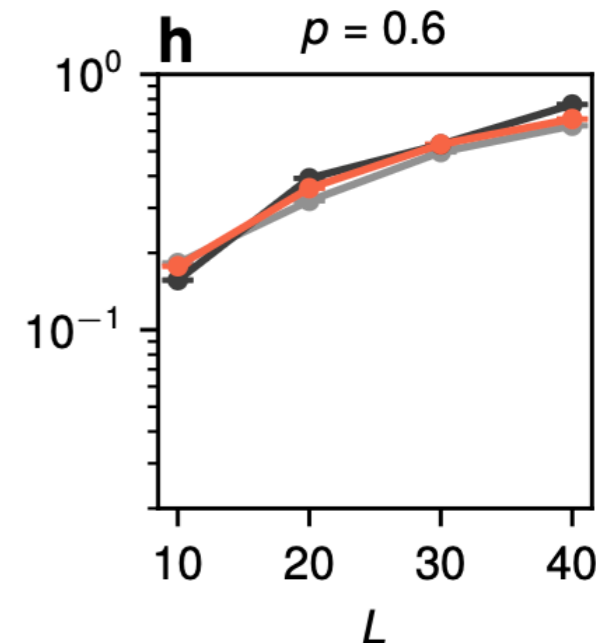
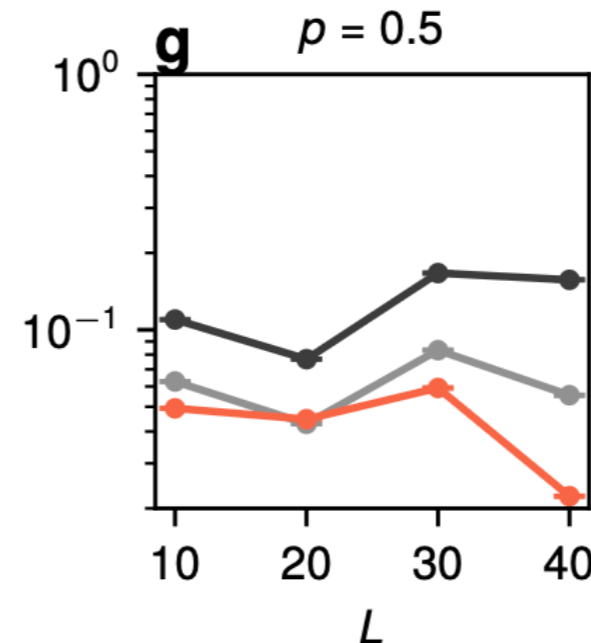
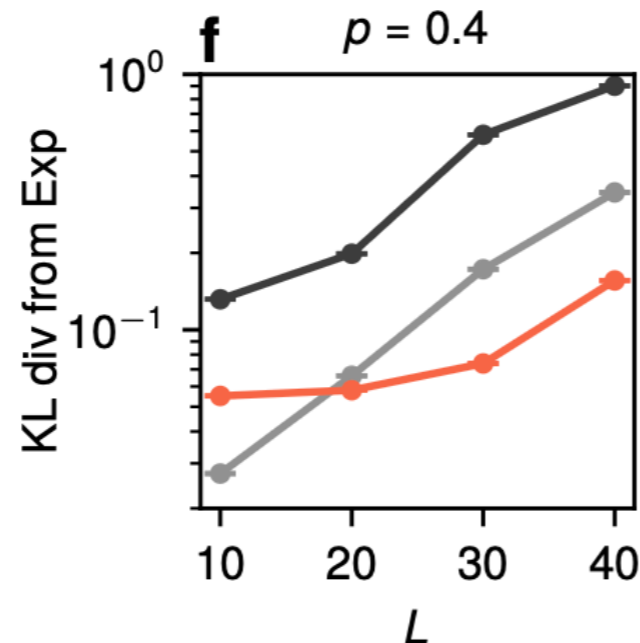
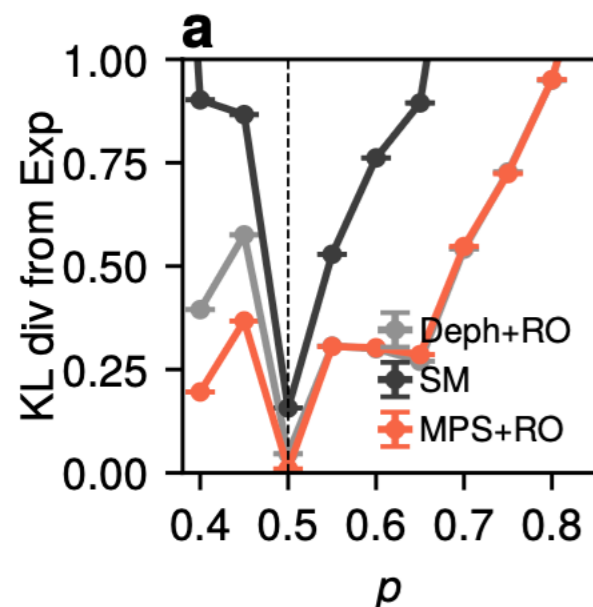
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$$D_{\text{KL}}(P||Q) = \int_{-\infty}^{\infty} p(x) \log(p(x)/q(x)) dx,$$

Compare with dephasing noise channel, theory, and experimental data



LOCAL CONTROL IN A STABILIZER MODEL

Actually straightforward to construct a stabilizer model from the quantum circuit with a local (identity) adder.

$$U_{\text{chaotic}}^\alpha = S_\alpha T$$

“scrambling operation”

REPLACE 2-qubit Haar random unitary with a **2-qubit Clifford gate**

$$C = A_{\text{ctrl}} T^{-1} R_L$$

Replaced the adder **with identity.**

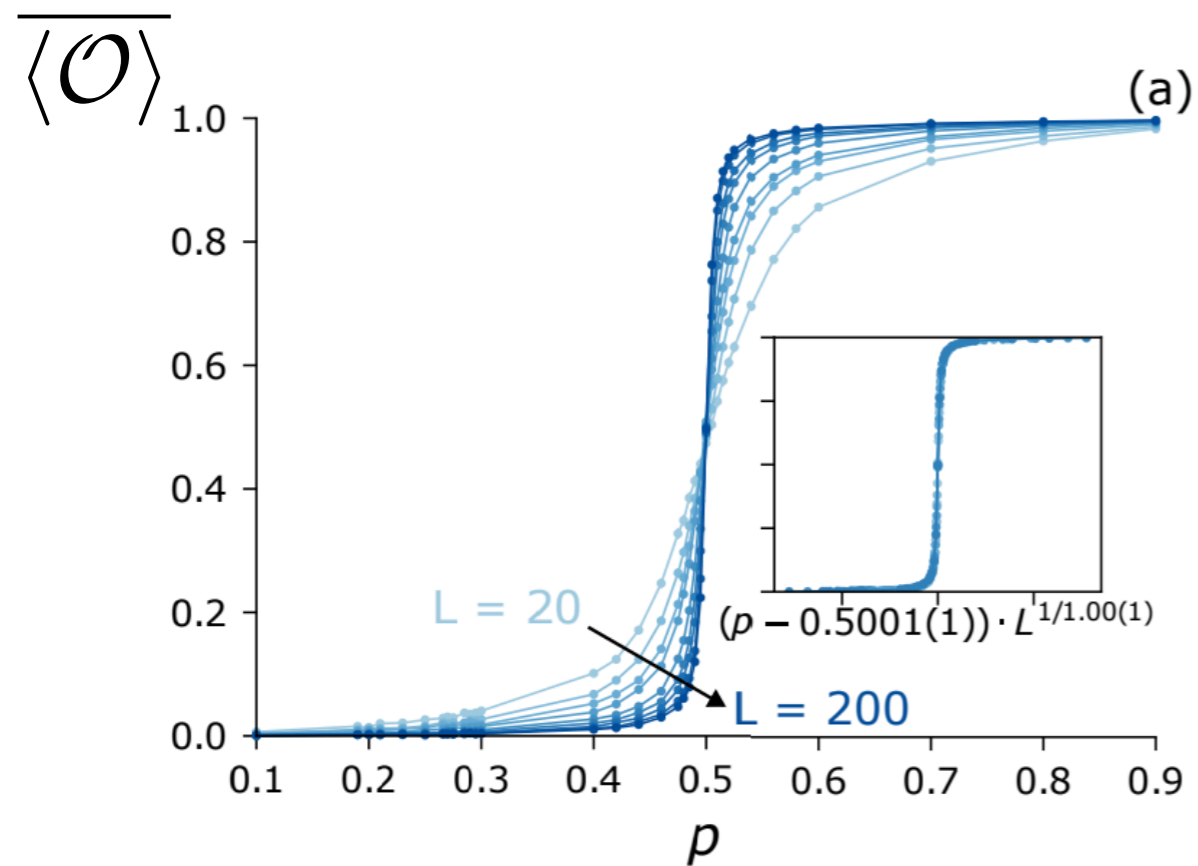
All of this is a stabilizer operation

STABILIZER LOCAL CONTROL TRANSITION

Control transition remains at

$$p_{\text{CIPT}} = 0.5 \quad \nu = 1.006$$

$$\mathcal{O} = \frac{1}{L} \sum_i S_i^z$$



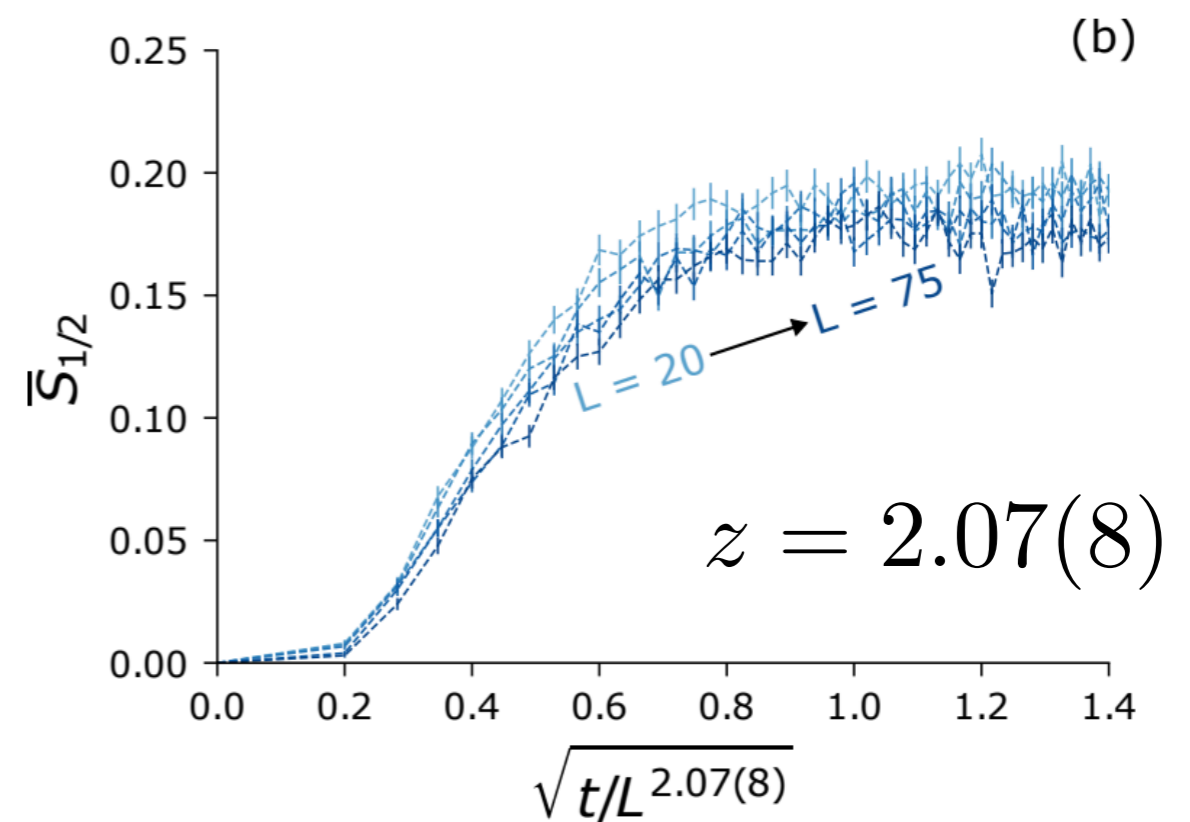
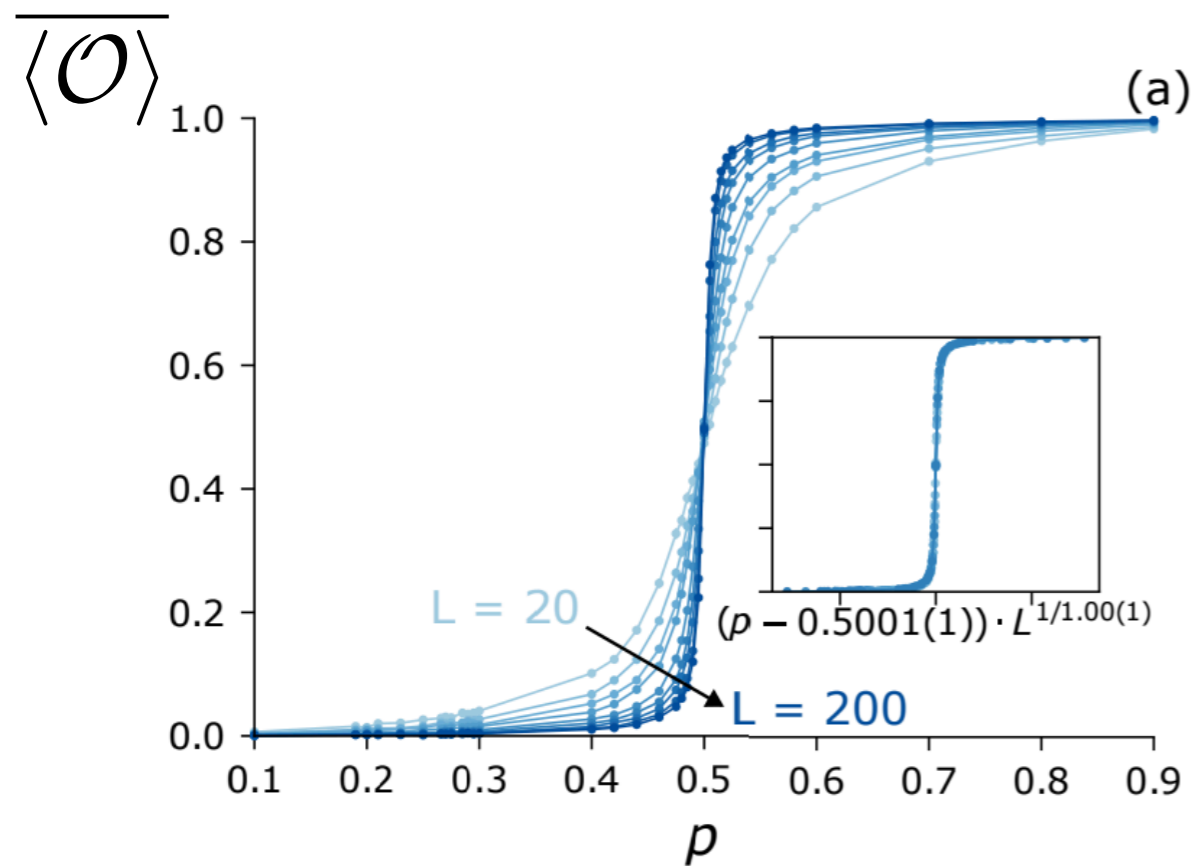
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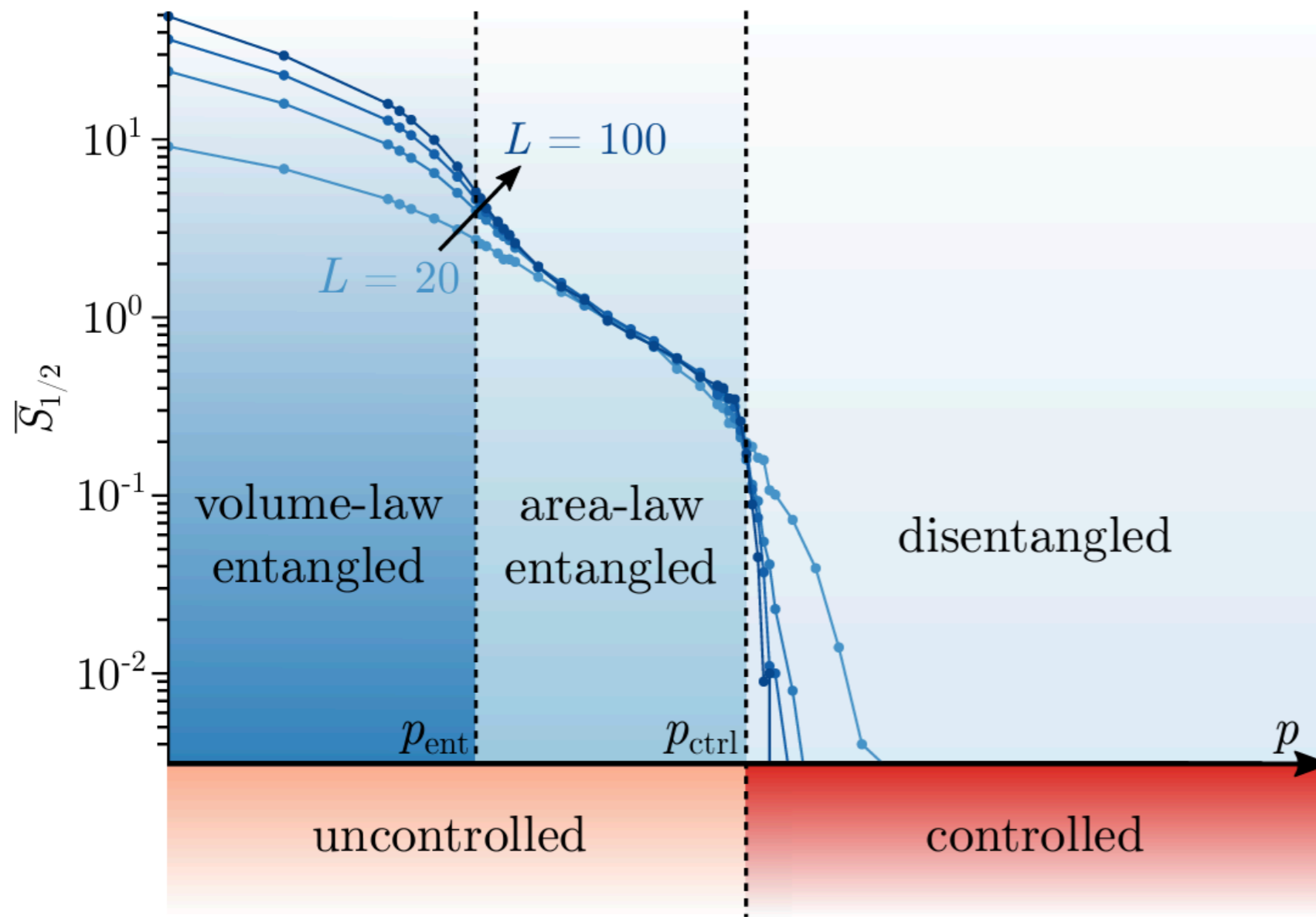
$$\mathcal{O} = \frac{1}{L} \sum_i S_i^z$$

Entanglement dynamics is diffusive and agrees with small sizes



STABILIZER LOCAL CONTROL TRANSITION

Entanglement transition splits off



$$p_{MIPT} = 0.266$$

$$\nu_{MIPT} = 1.24$$

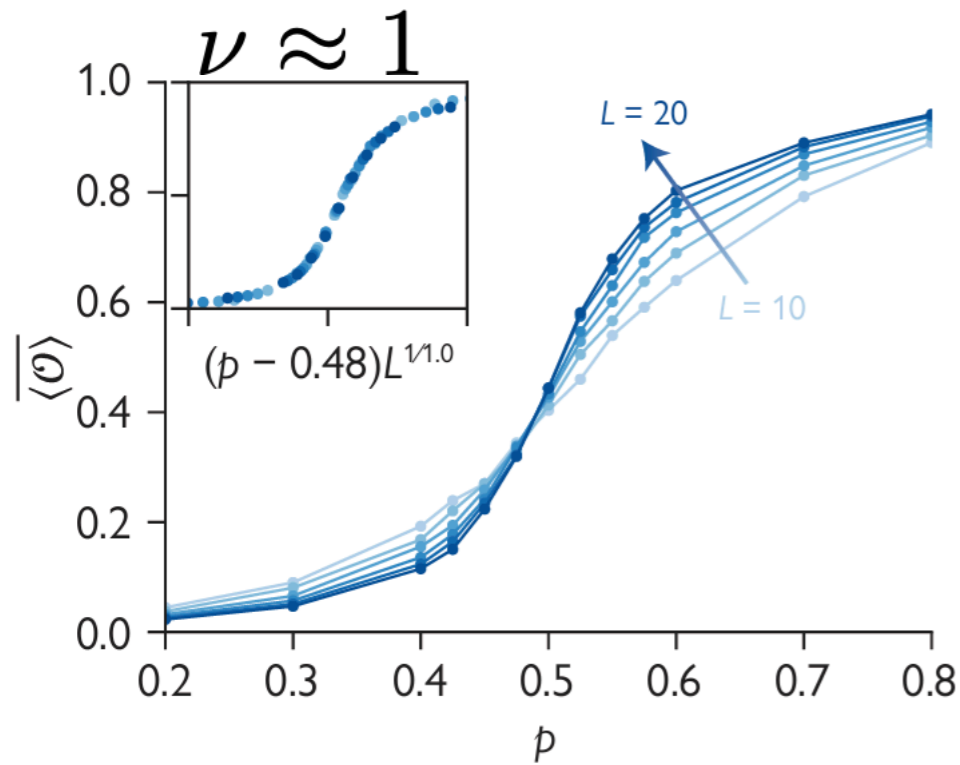
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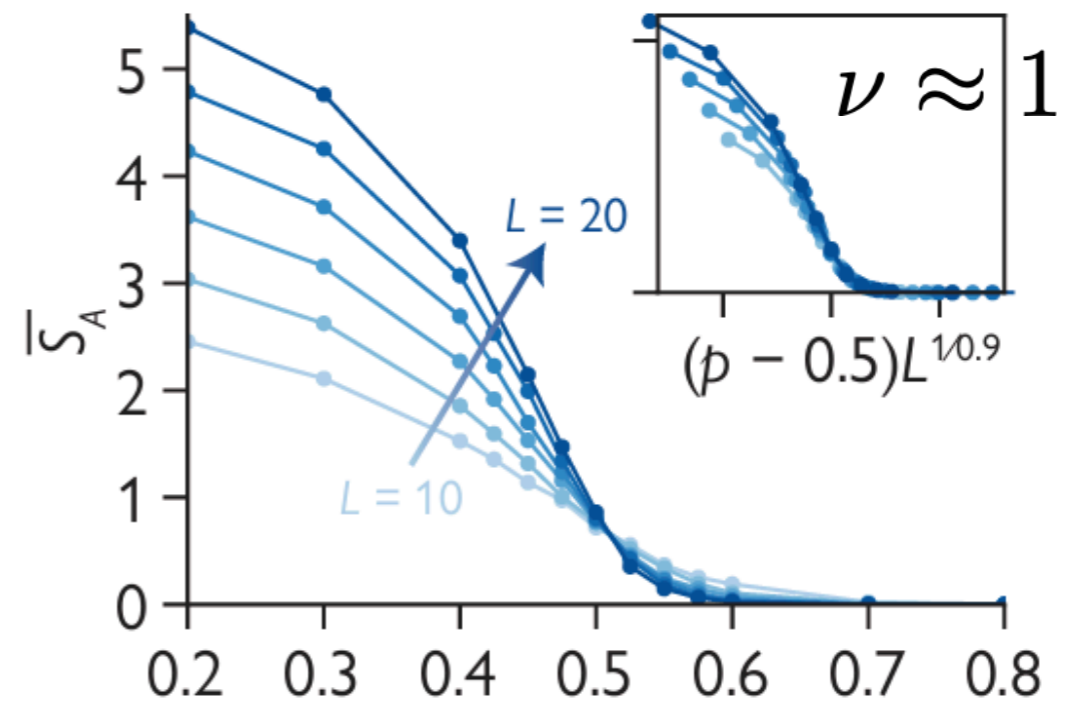
QUANTUM CONTROL TRANSITION

Local order parameter

$$\mathcal{O} = -\frac{1}{L} \sum_{i=1}^L Z_i Z_{i+1}$$



Entanglement entropy



$$\bar{S}(t \gg \overset{p}{L}) \sim L^0$$

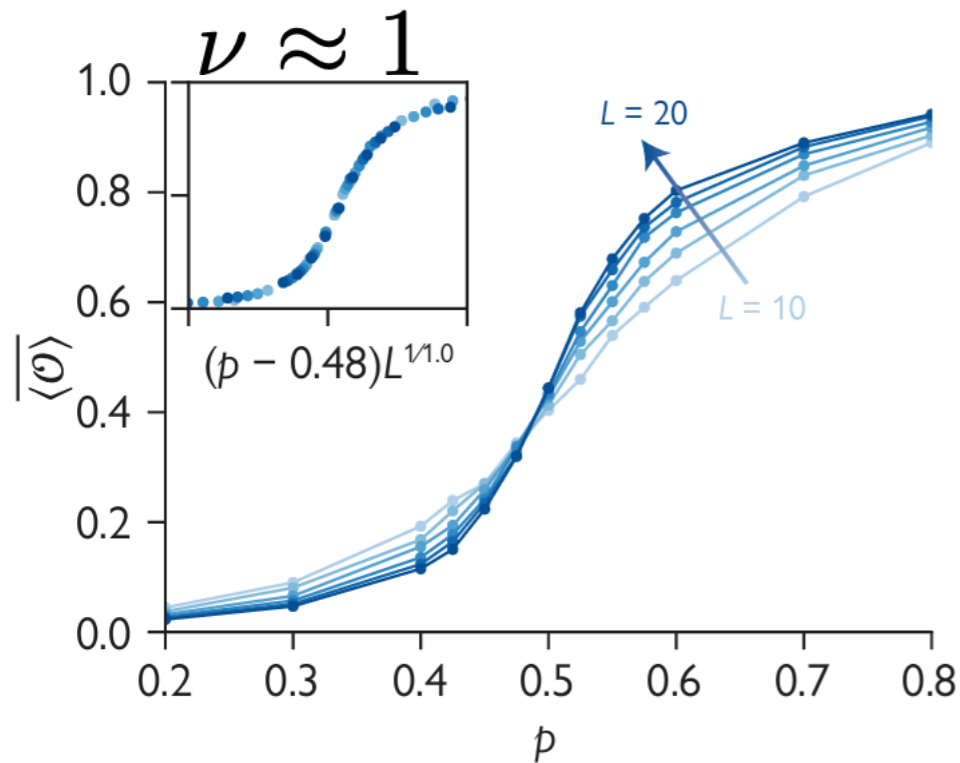
Area law at the transition!

Clear signature of the transition!

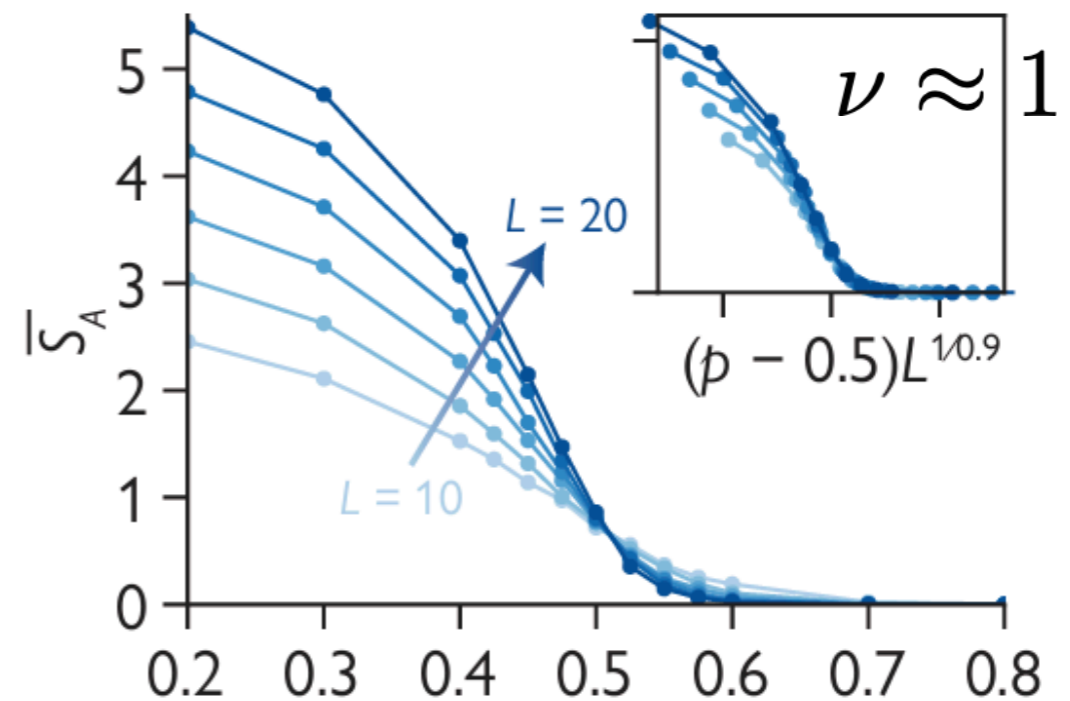
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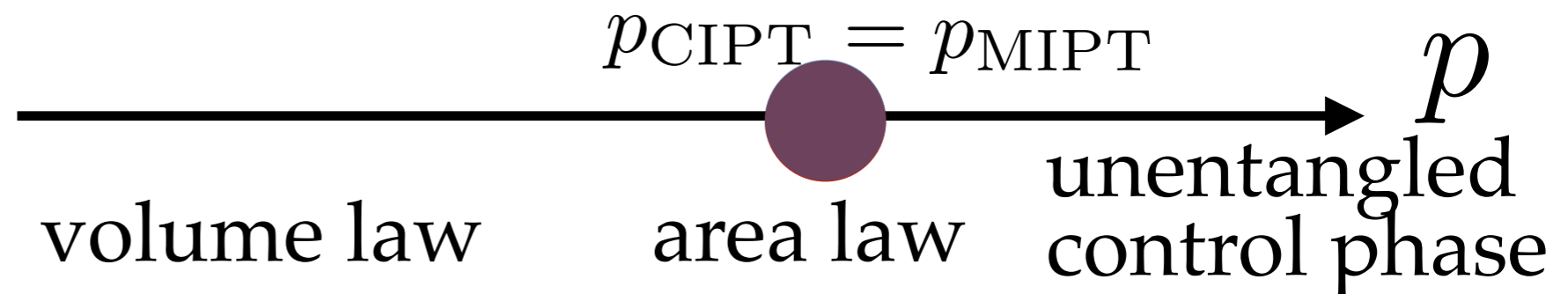
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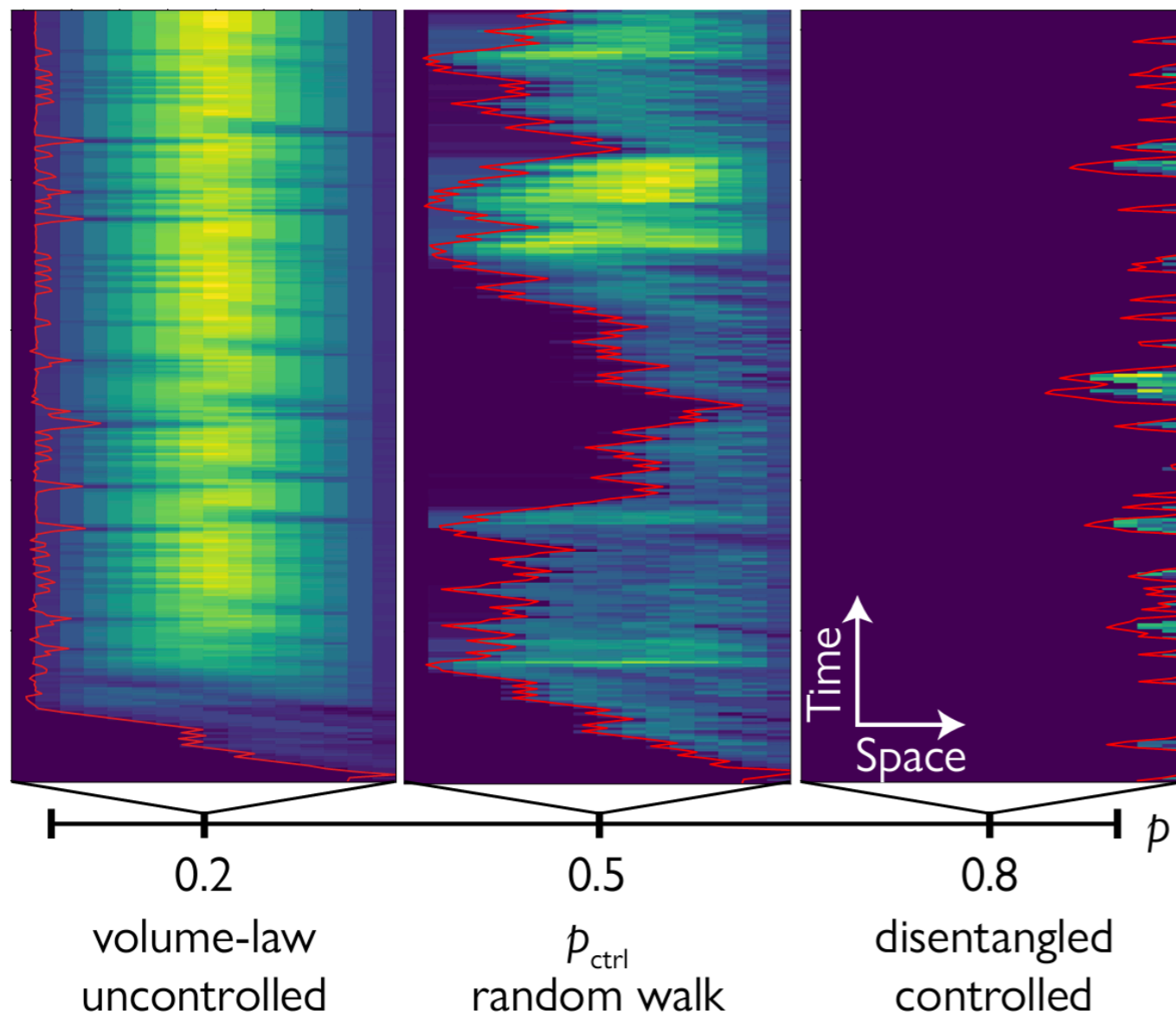


QUANTUM CONTROL TRANSITION, DYNAMICS

Brighter color = larger entanglement entropy

Red line = avg. location of 1st domain wall

Single trajectories



First domain wall

$|010101011011100\dots\rangle$

Controlled

Uncontrolled

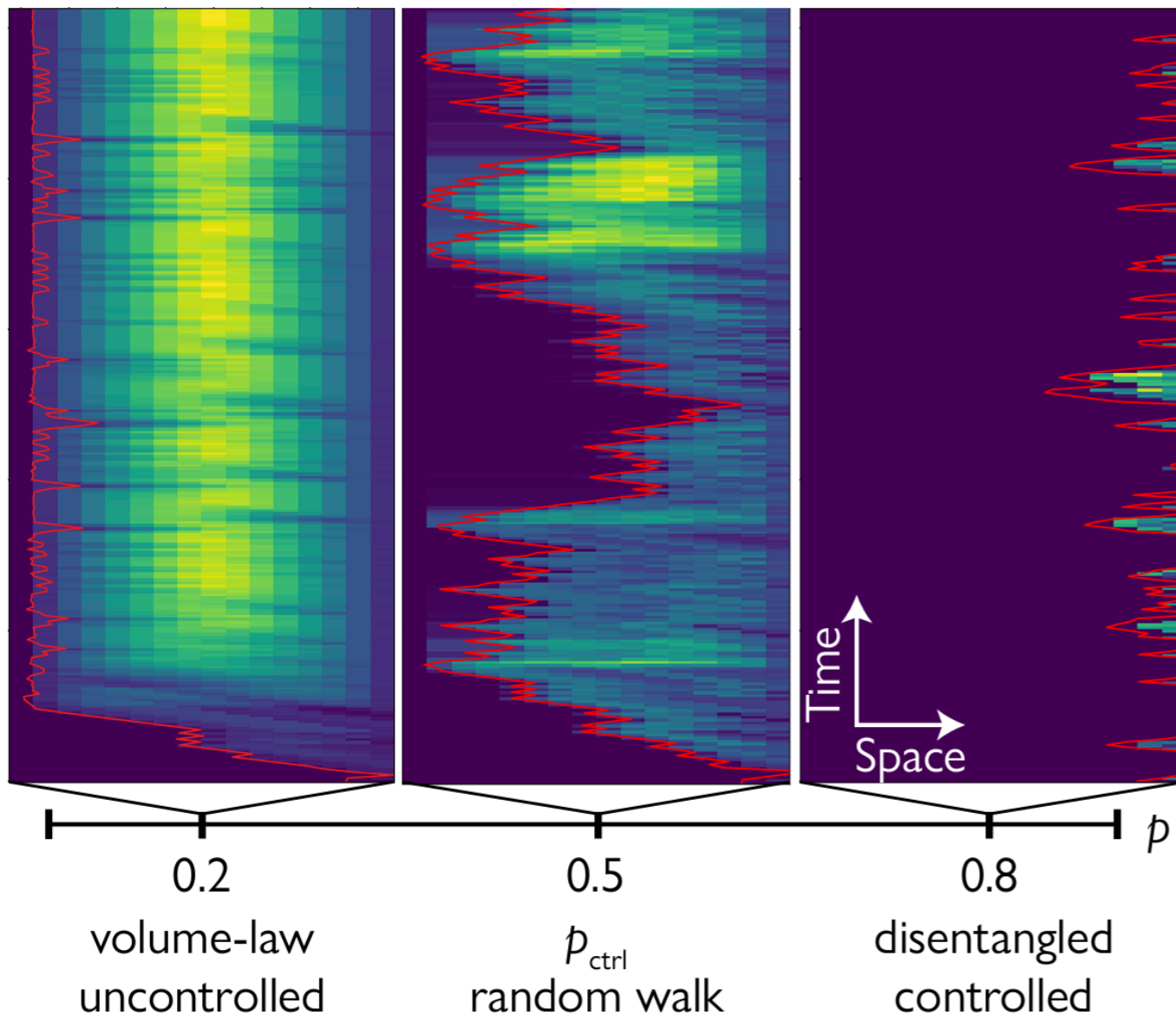
Becomes a wave packet
in the quantum limit

$$\langle r(t) \rangle = \sum_x |\langle x | \psi(t) \rangle|^2 r_x$$

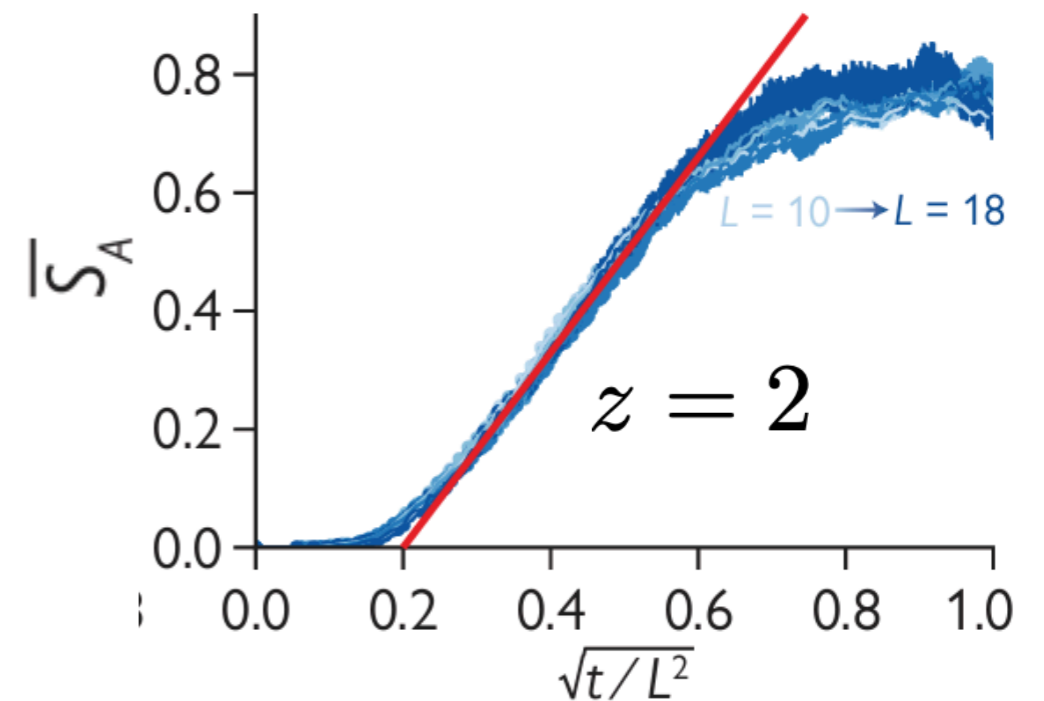
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Entanglement entropy dynamics at the transition



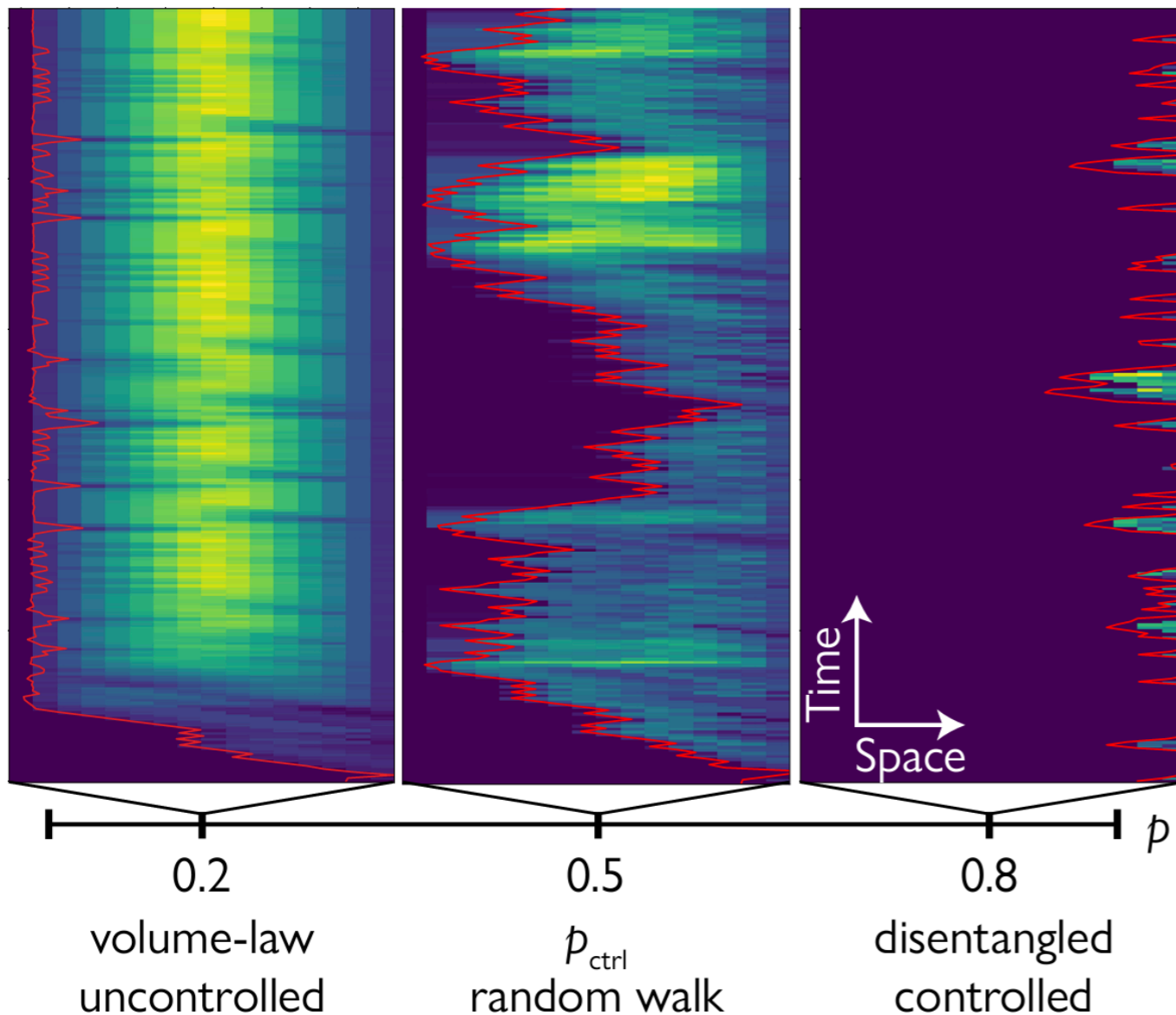
Diffusive growth

$$\bar{S}(t \ll L) \sim \sqrt{t/L^2}$$

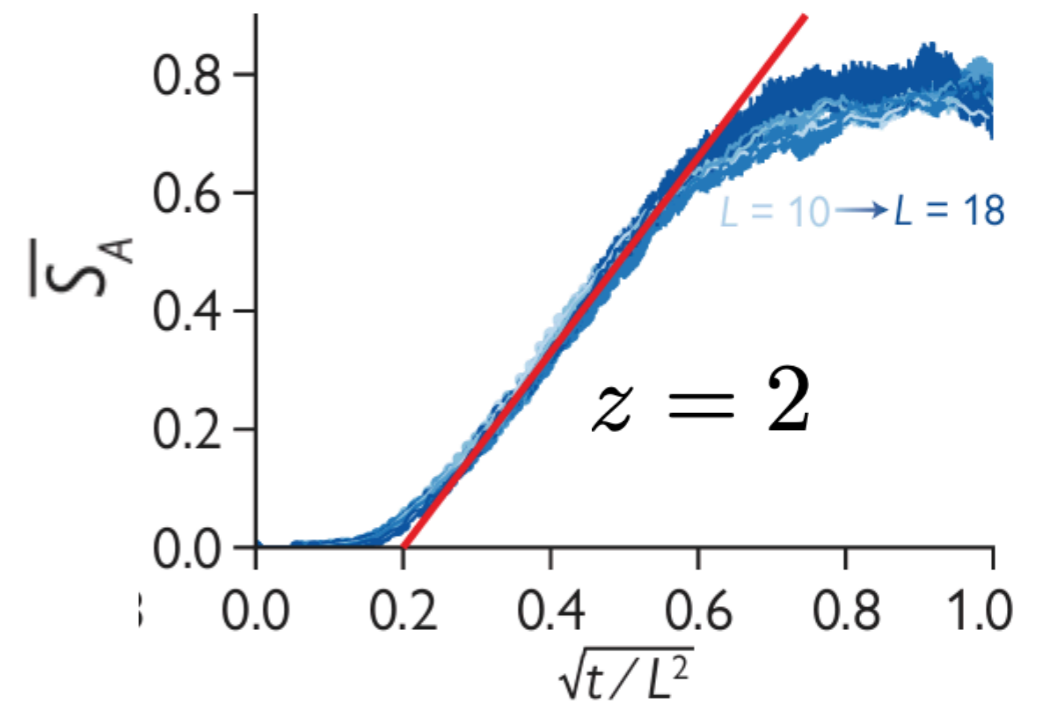
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Entanglement entropy
dynamics at the transition



Diffusive growth

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The transition is diffusive, very similar to the classical model

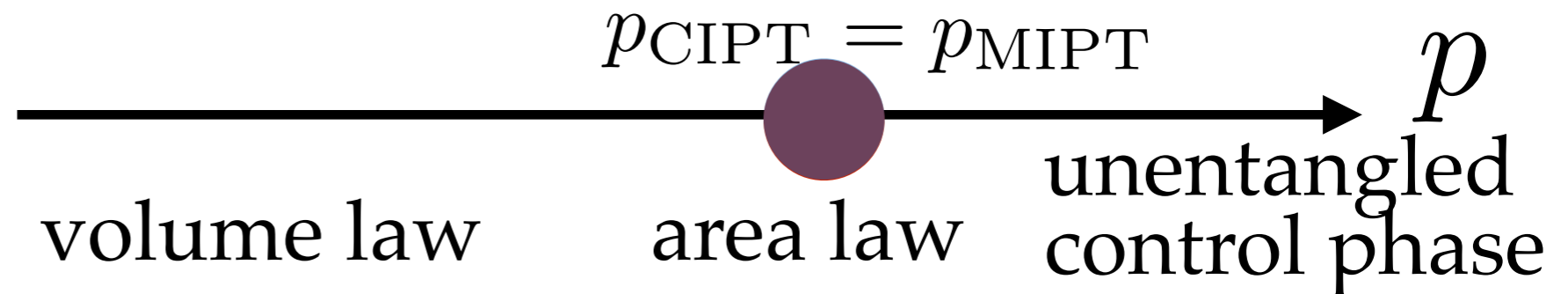
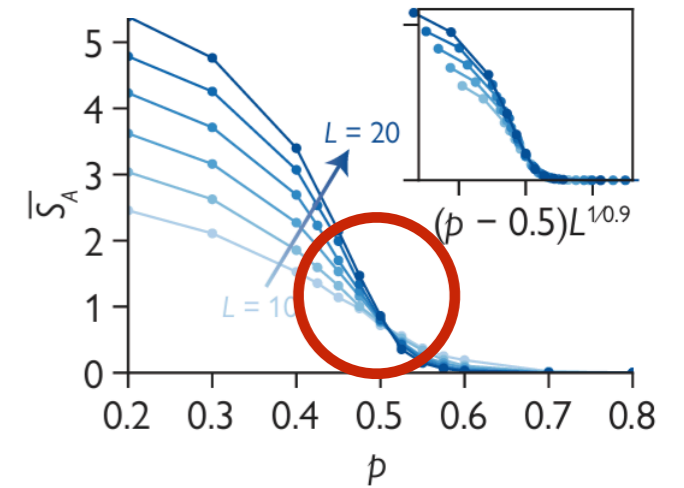
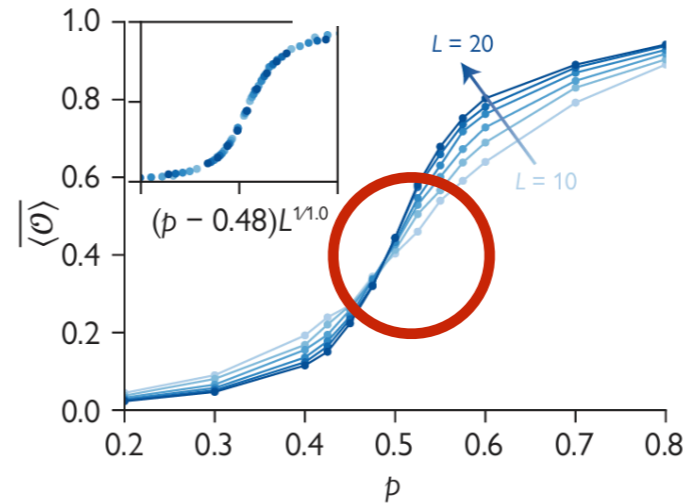
$$\nu \approx 1 \quad z = 2$$

SUMMARY: QUANTUM CONTROL TRANSITION

Here, the control and entanglement transitions coincide.

Is this fundamental?

$$\mathcal{O} = -\frac{1}{L} \sum_{i=1}^L Z_i Z_{i+1}$$

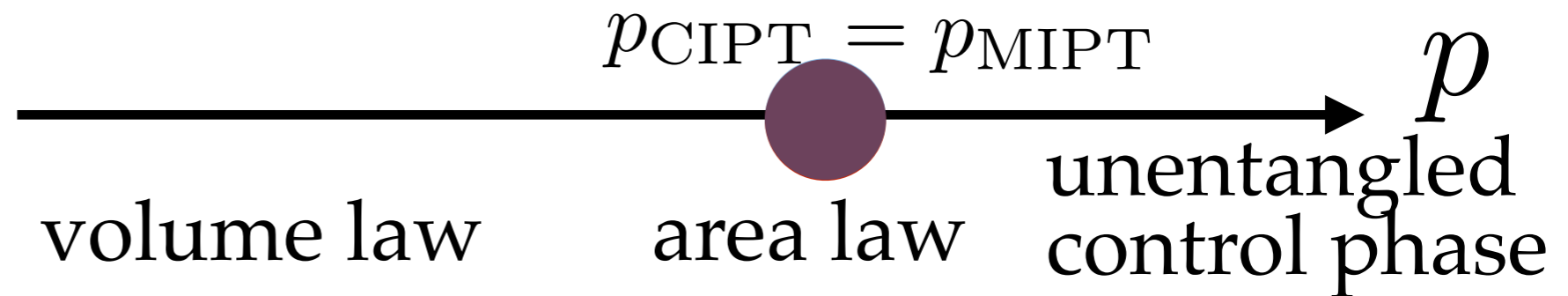
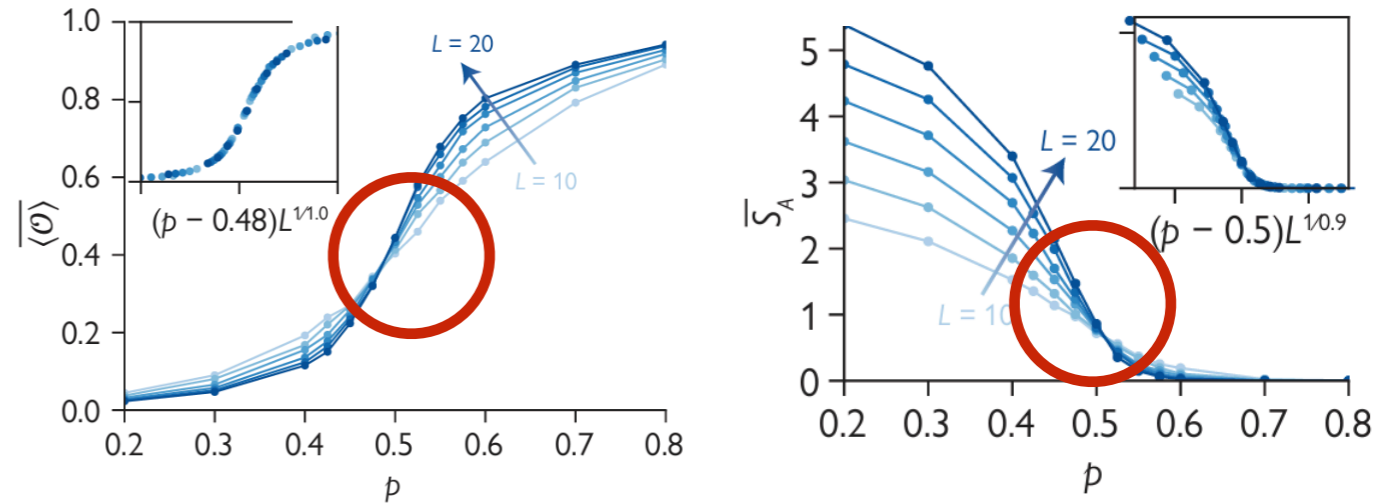


SUMMARY: QUANTUM CONTROL TRANSITION

Here, the control and entanglement transitions coincide.

Is this fundamental?

$$\mathcal{O} = -\frac{1}{L} \sum_{i=1}^L Z_i Z_{i+1}$$



Can we “pull” them apart?

Can we construct stabilizer models to reach larger system sizes?

LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT

Start with a simpler unstable fixed point, the single orbit ferromagnet

$$C = A_{\text{ctrl}} T^{-1} R_L$$


Replace the adder
with identity.

Now we are controlling
onto the ferromagnetic state

$|000 \dots 0\rangle$ **Control map is now local**

LOCAL ADDER AND THE FERROMAGNETIC FIXED POINT

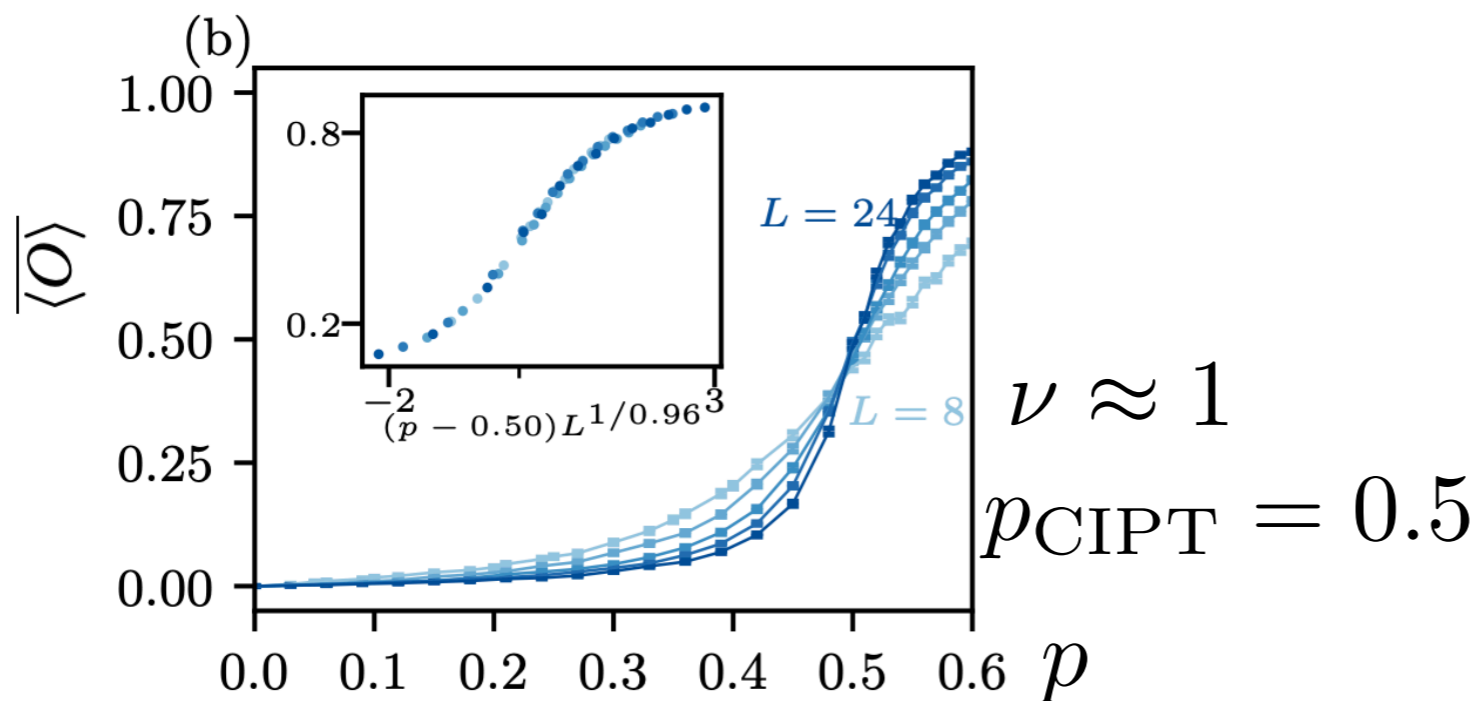
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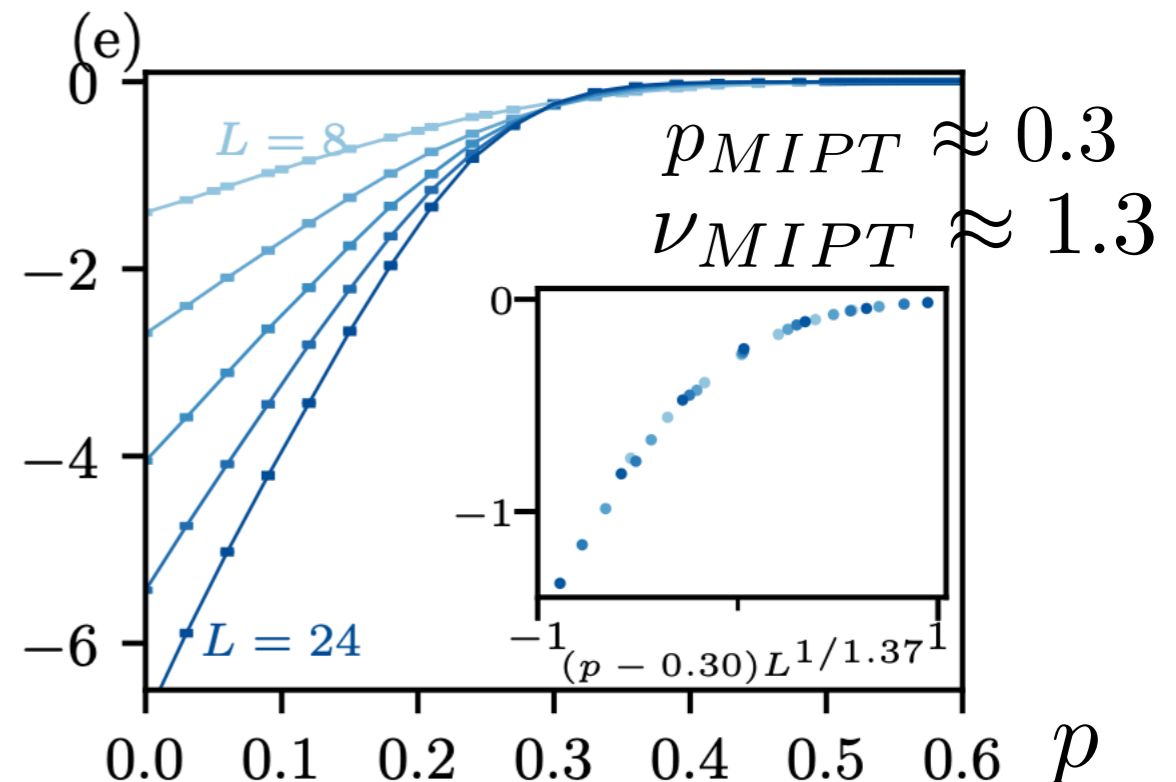
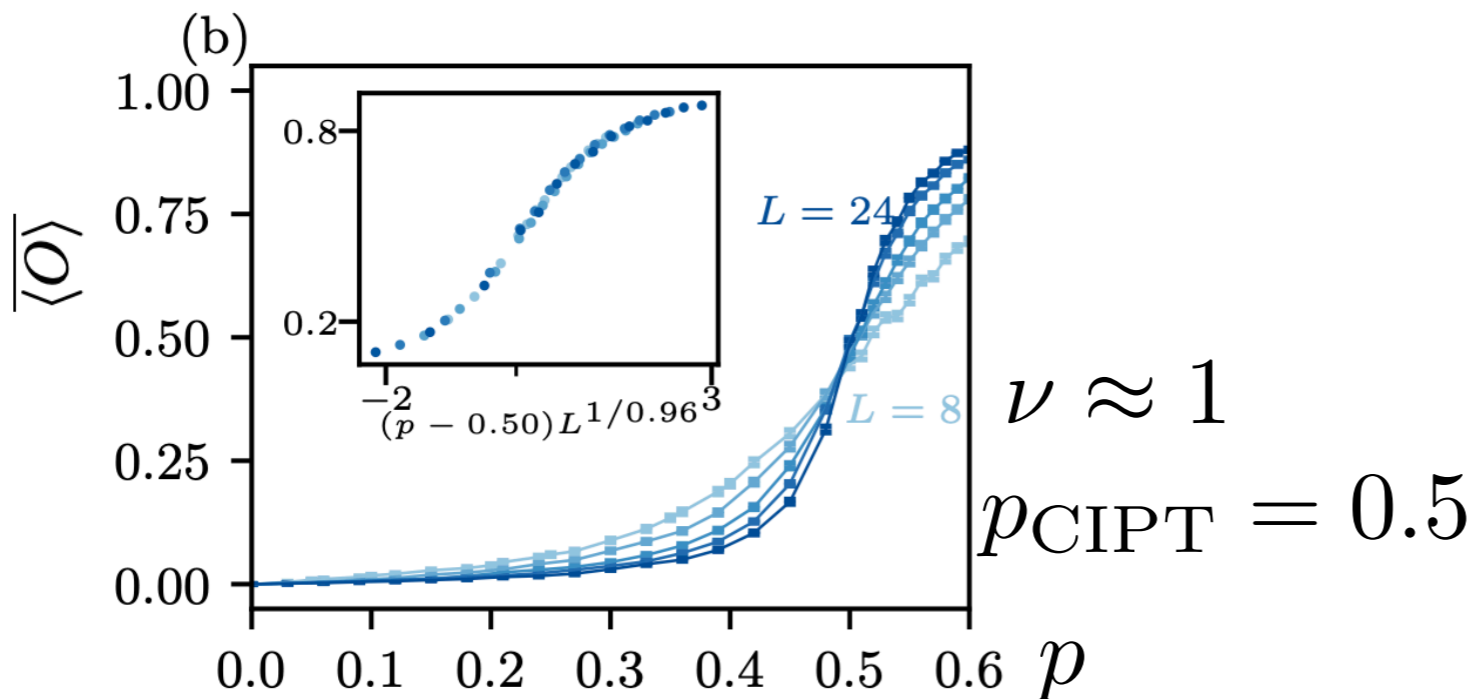
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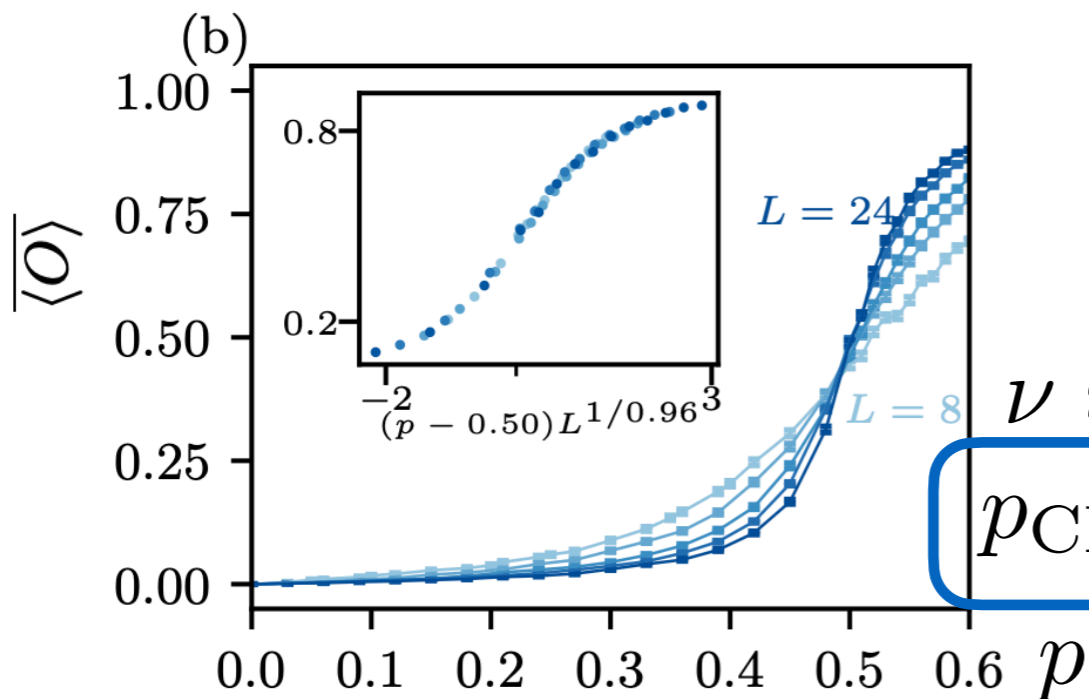
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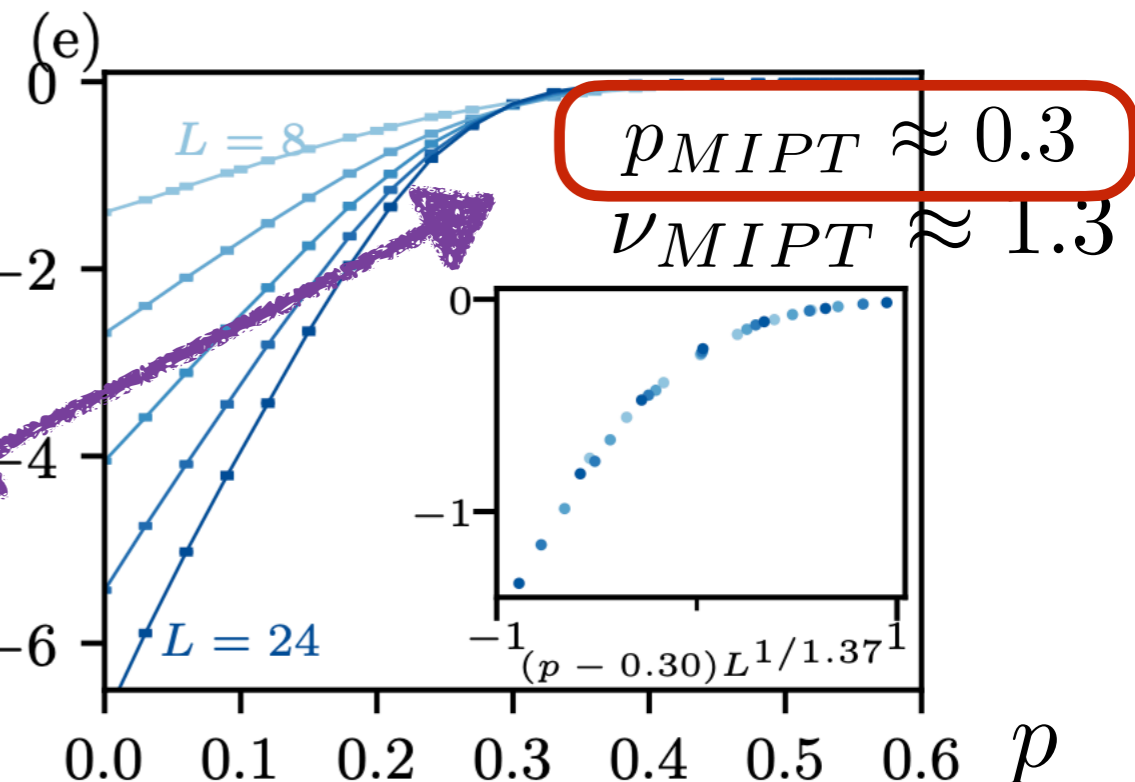
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Transitions now split!

$$\nu \approx 1$$



STABILIZER MODEL, WITH LOCAL ADDER

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REPLACE 2-qubit Haar random unitary with a **2-qubit Clifford gate**

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