RUTGERS
THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS CENTER FOR Materials Theory

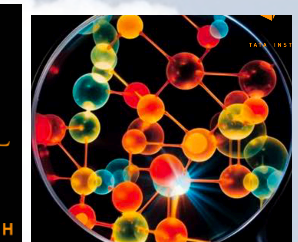
FLATIRON INSTITUTE
Center for Computational Quantum Physics

# Lecture 3: Monitored quantum dynamics in random quantum circuits

Jed Pixley

ICTS Lecture Series:

INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH



Quantum Dynamics in the pre-fault tolerant era

6/8/2026

Office of Naval Research Science & Technology

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# Lecture Series Overview

Quantum dynamics in the pre fault tolerant era

Lecture 1: From Classical and quantum chaos to thermalization in isolated quantum systems

Lecture 2: Analog and digital quantum simulators from unitary dynamics to midcircuit measurements.

**Lecture 3: Monitored quantum dynamics in random quantum circuits**

Lecture 4: Adaptive quantum circuits and control induced phase transitions

Lecture 5: Open quantum dynamics software tutorial

# Lecture Series, learning goals

I. Lecture 1: Classical and Quantum Chaos, from single particle to many-body

II. Lecture 2: Quantum platforms

**III. Lecture 3: Entanglement phases and phase transitions driven by interplay of unitary and projective evolution.**

IV. Lecture 4: Adaptive dynamics, controlling quantum dynamics

V. Lecture 5: Numerical approaches to adaptive quantum dynamics

# Lecture 3: Learning Goals

## Monitored Quantum Dynamics

Understand the role of random local measurements on the dynamics of quantum many body systems

This produces novel dynamical phases and phase transitions in the entanglement structure.

We will understand the universality classes discovered, their stability, and the role of symmetries. We will also discuss several extensions including projector only models, long range interactions, and free fermions.

The models commonly used will also be presented

# Measuring Quantum Many Body Systems

As discussed in Lecture 2.

To build a fully working quantum computer, error correction is essential, making mid circuit measurements a necessity.

Measurements produce a non-linear effect on the wavefunction, its projective nature collapses the wavefunction and makes the evolution non-unitary.

# Measuring Quantum Many Body Systems

A central question of the talk:

Is entangling unitary dynamics stable to a small but non-zero local measurement rate?

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Is entangling unitary dynamics stable to a small but non-zero local measurement rate?

YES!

Li, Chen, Fisher PRB (2018); PRB (2019)
Skinner, Ruhman, Nahum PRX (2019)

Vasseur, Potter, You, Ludwig PRB (2018)

A central question of the talk:

Is entangling unitary dynamics stable to a small but non-zero local measurement rate?

YES!

the competition between entangling dynamics and disentangling measurements drives a **phase transition in the dynamics of the entanglement**

Li, Chen, Fisher PRB (2018); PRB (2019)

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# Measuring Quantum Many Body Systems

Consider replacing local measurements with gate errors

Indicative of a "quantum computer" (i.e. unitary time evolution) with each gate having a **finite error rate** $\eta$

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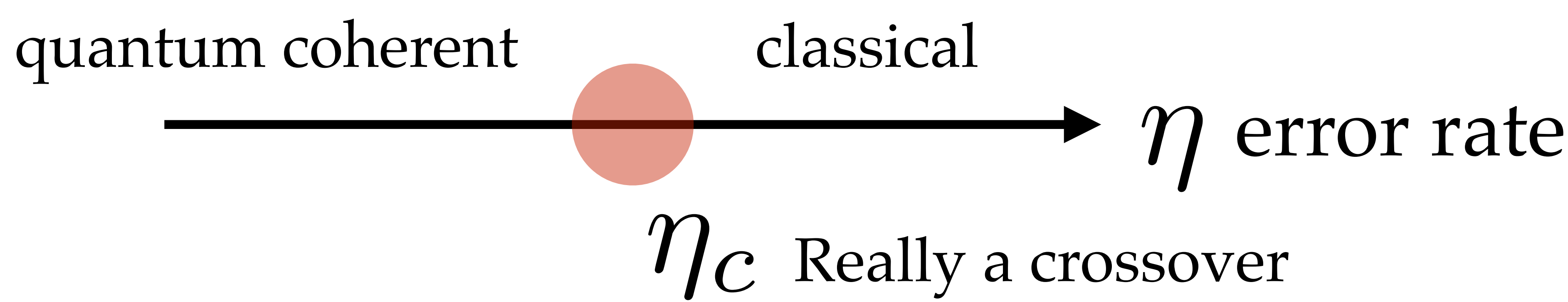
Indicative of a "quantum computer" (i.e. unitary time evolution) with each gate having a **finite error rate** $\eta$

PHYSICAL REVIEW A, VOLUME 62, 062311

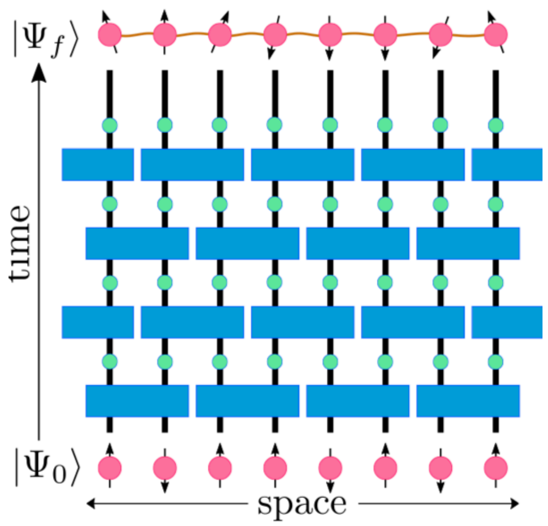
**Quantum to classical phase transition in noisy quantum computers**

Dorit Aharonov*
*Computer Science Division, University of California–Berkeley, Berkeley, California 94720-1776*
(Received 21 October 1999; published 14 November 2000)



# Outline

## I. Motivation



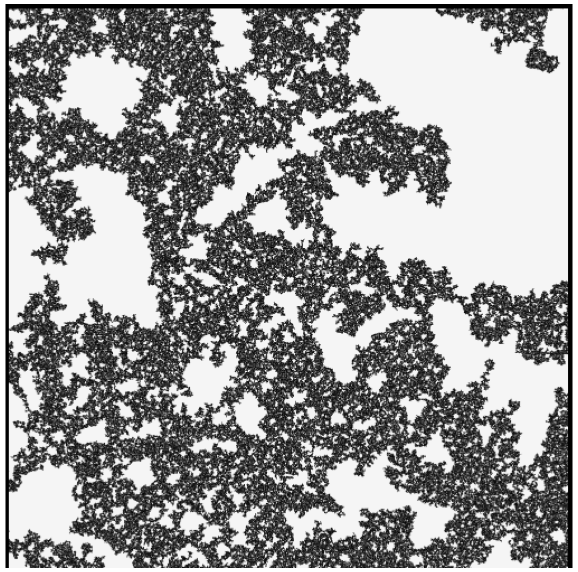

## II. Measurement Induced Transition (MIT)

- Models
- Controlled limits
- Numerical solution for qubits

$$|\Psi(t)\rangle \to \frac{P_i^m|\Psi(t)\rangle}{\langle\Psi(t)|P_i^m|\Psi(t)\rangle}$$

## III. Extensions

- Weak measurements
- Projector only models
- Free fermions



## IV. Stability of the MIPT

- Presence of symmetry
- Static and temporal perturbations

# Quantum Circuits

Want to construct the simplest model that captures the competition between **entangling gates** and **collapsing** the state with **measurements**

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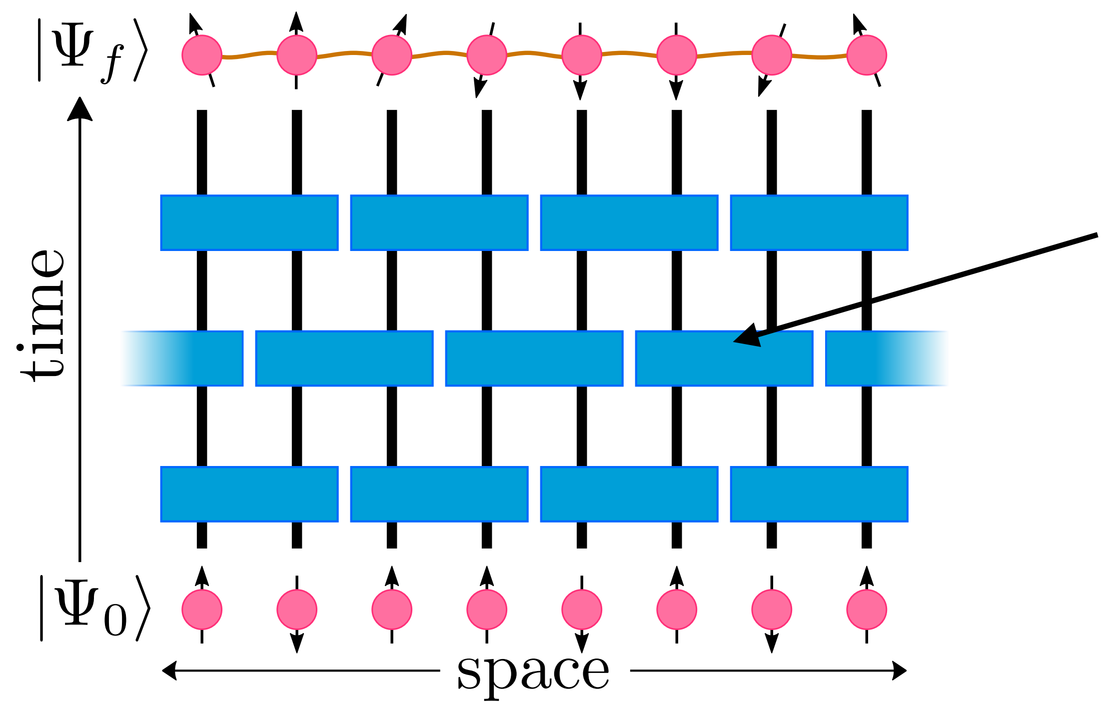



2-qubit gate $U_{i,i+1}$

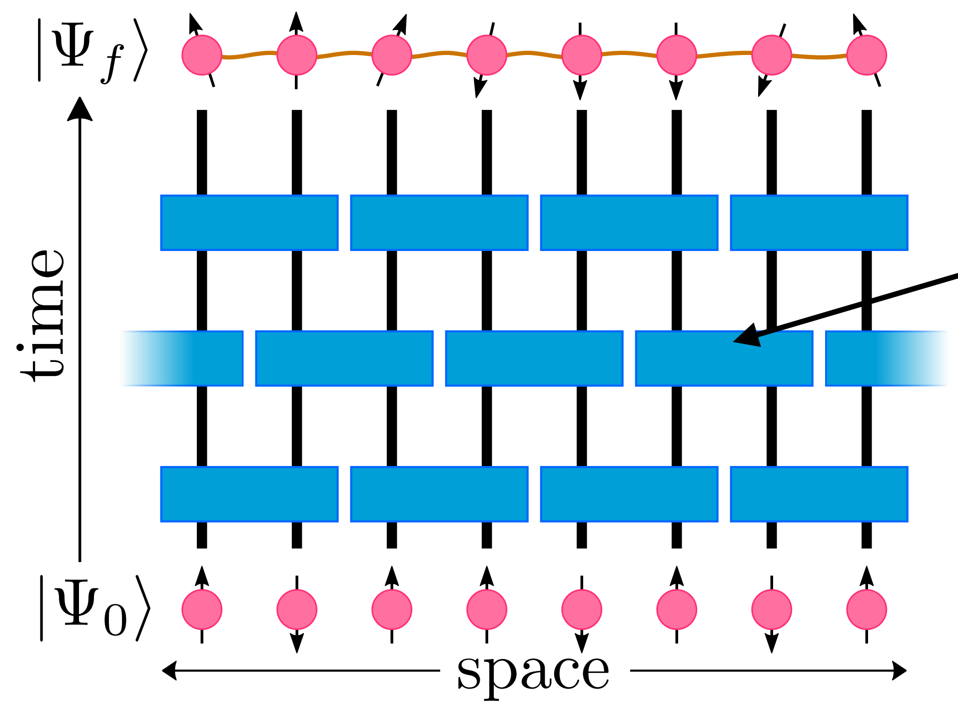
most "generic" quantum gate is a random U(4) matrix (i.e. a **Haar radom gate**)

Cannot be simulated with classical efficiency

$$|\Psi(t)\rangle = U(t,t-1)U(t-1,t-2)\cdots U(2,1)U(1,0)|\Psi_0\rangle$$

# Quantum Circuits

Want to construct the simplest model that captures the competition between **entangling gates** and **collapsing** the state with **measurements**




2-qudit gate $U_{i,i+1}$

A "semiclassical" limit taking the local Hilbert space $q \to \infty$

Analytically tractable

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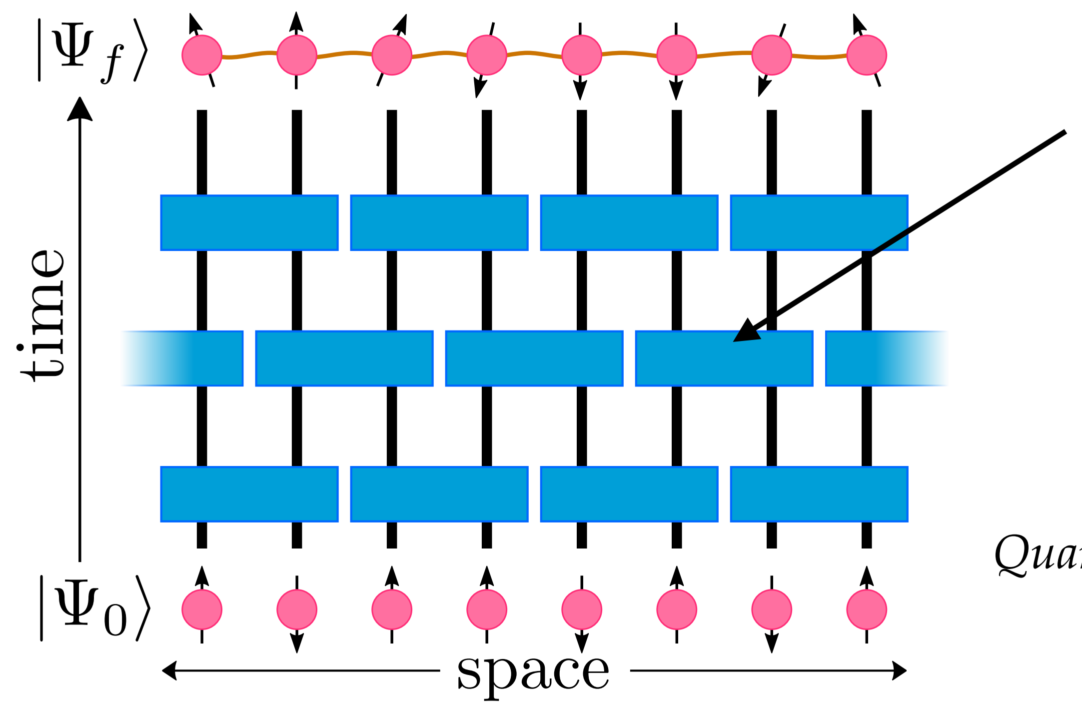



2-qubit Clifford gate $U_{i,i+1}$

Stabilizer circuits: take gates from the **Clifford group** that act on **stabilizer states**

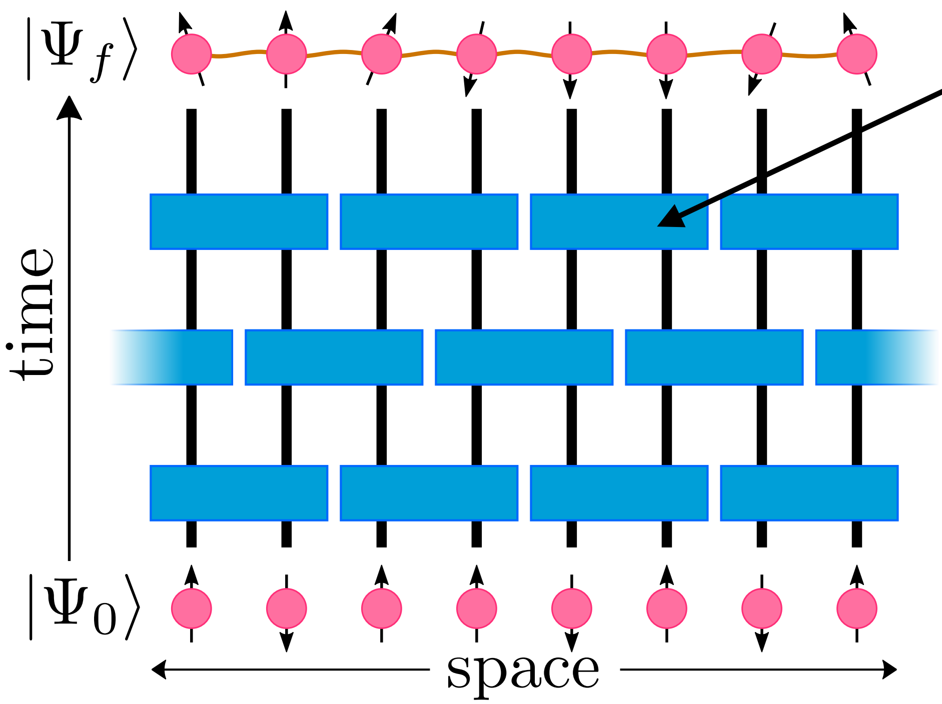
*Quantum Computation and Quantum Information*, Chuang and Nielsen

CAN be simulated with classical efficiency!
(akin to a distinct and non-trivial semiclassical limit)

Gottesman–Knill theorem

$$|\Psi(t)\rangle = U(t,t-1)U(t-1,t-2)\cdots U(2,1)U(1,0)|\Psi_0\rangle$$

# Quantum Circuits




2-qubit Clifford gate $U_{i,i+1}$

Stabilizer circuits: take **gates** from the **Clifford group**, generated by Hadamard $H$, Phase $P$, and CNOT gates

$$H = \frac{i}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad P = \begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, \quad U_{\text{CNOT}}|s_1, s_2\rangle = |s_1, s_1 \oplus s_2\rangle$$

modulo 2 addition

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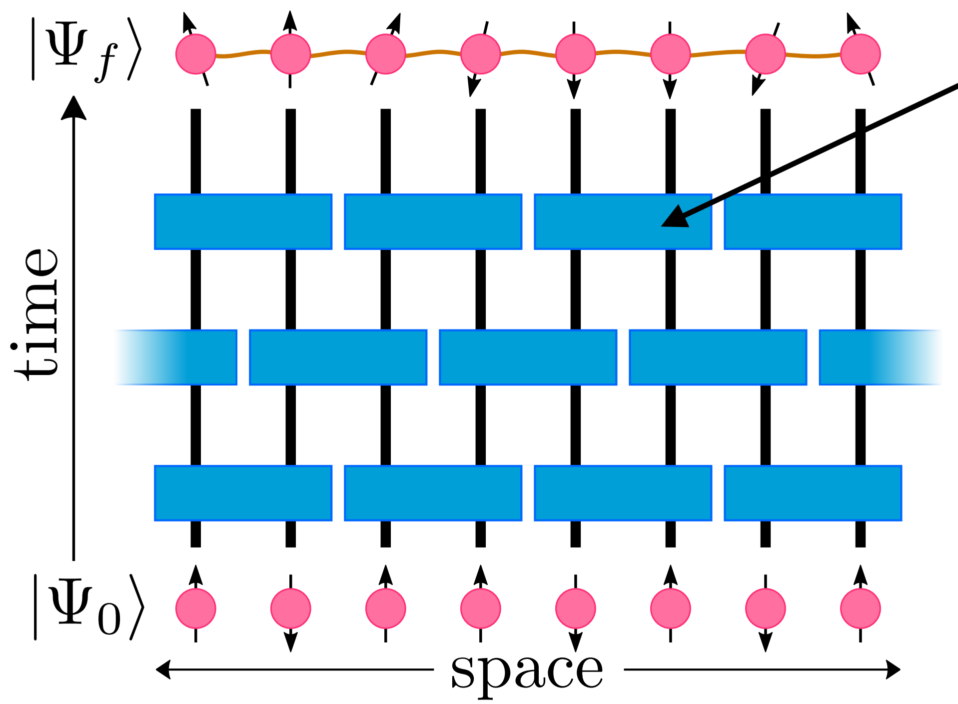
# Quantum Circuits

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That act on **stabilizer states**, which are generated by acting on $|0\rangle^{\otimes L}$ with gates from the Clifford group.

These states can be represented as Pauli strings

$$i^{\ell}\mathbb{I} \otimes Z \otimes X \otimes Y \otimes \cdots$$

Member of the
Pauli group $\mathscr{P}_L$ on $L$ qubits

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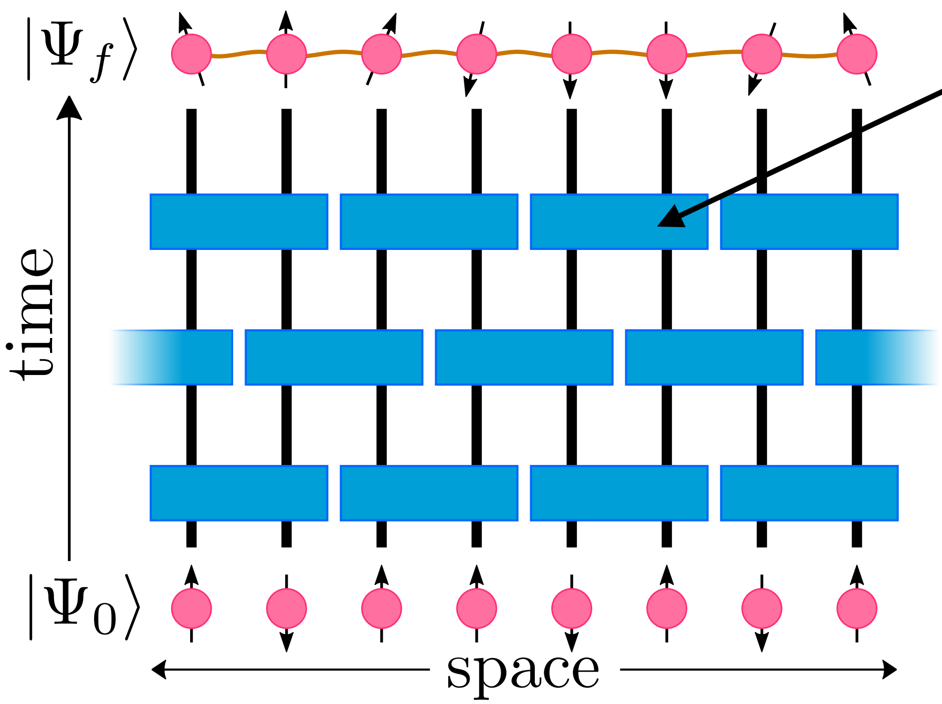


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**Gottesman–Knill theorem**: a polynomially efficient algorithm exists to simulate this dynamics.

These states can be represented as Pauli strings

$$i^{\ell}\mathbb{I} \otimes Z \otimes X \otimes Y \otimes \cdots \to ((0,0,1,0,0,1,1,1,\ldots)|\ell/2)$$

Member of the Pauli group $\mathscr{P}_L$ on $L$ qubits

$\mathbb{I} \to (0,0),\ Z \to (1,0),\ X \to (0,1),$ and $Y \to (1,1)$

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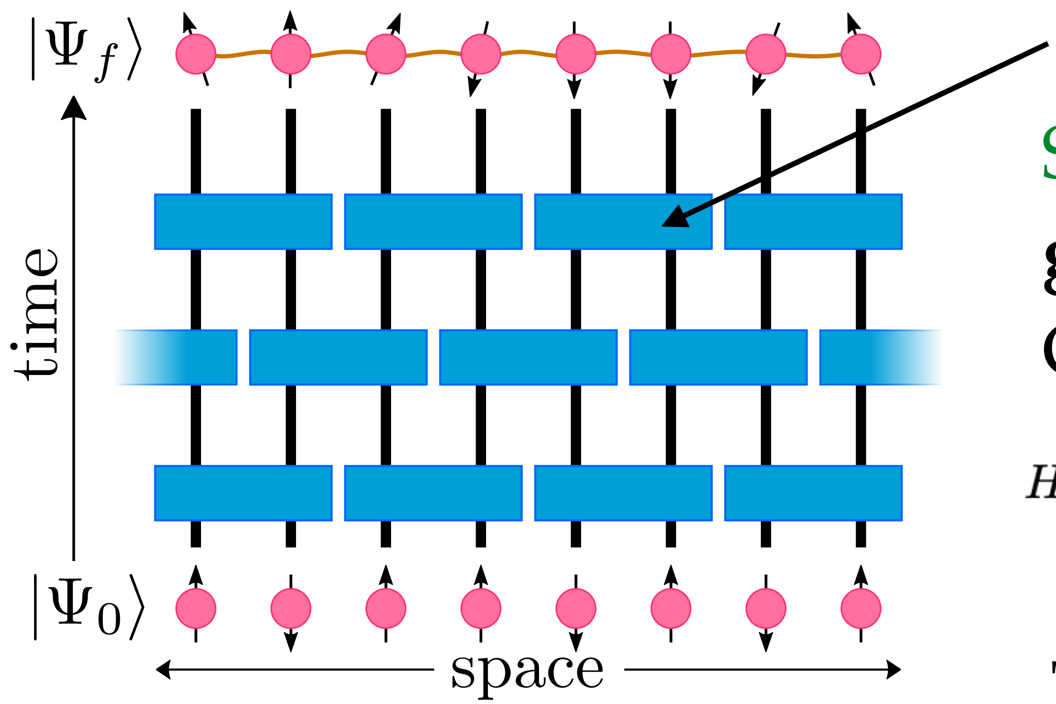


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$\mathbb{I} \to (0,0), \; Z \to (1,0), \; X \to (0,1),$ and $Y \to (1,1)$

For depth *t*, *L* qubits, and *n* measurements

\# of operations $\sim O(tL + nL^2)$

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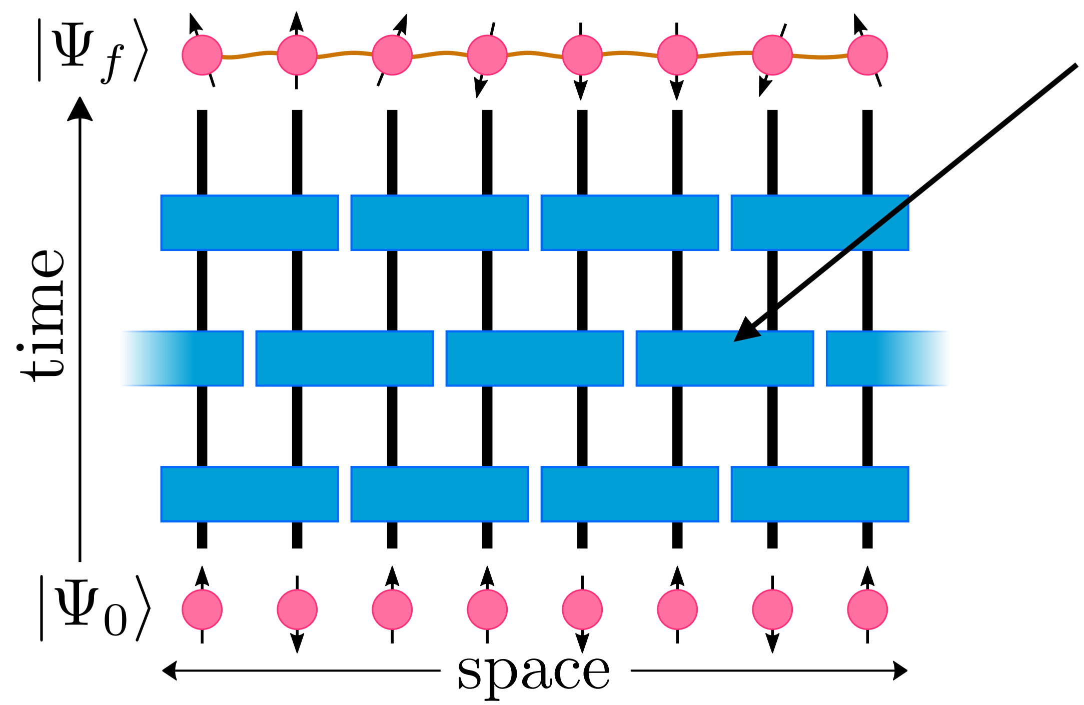
Want to construct the simplest model that captures the competition between **entangling gates**
and **collapsing** the state with **measurements**




2-qubit dual unitary gate $U_{i,i+1}$

Unitary in space and time

Has analytical tractability in some cases

Piroli, Bertini, Cirac, Prosen PRB (2020)

$$|\Psi(t)\rangle = U(t,t-1)U(t-1,t-2)\cdots U(2,1)U(1,0)|\Psi_0\rangle$$

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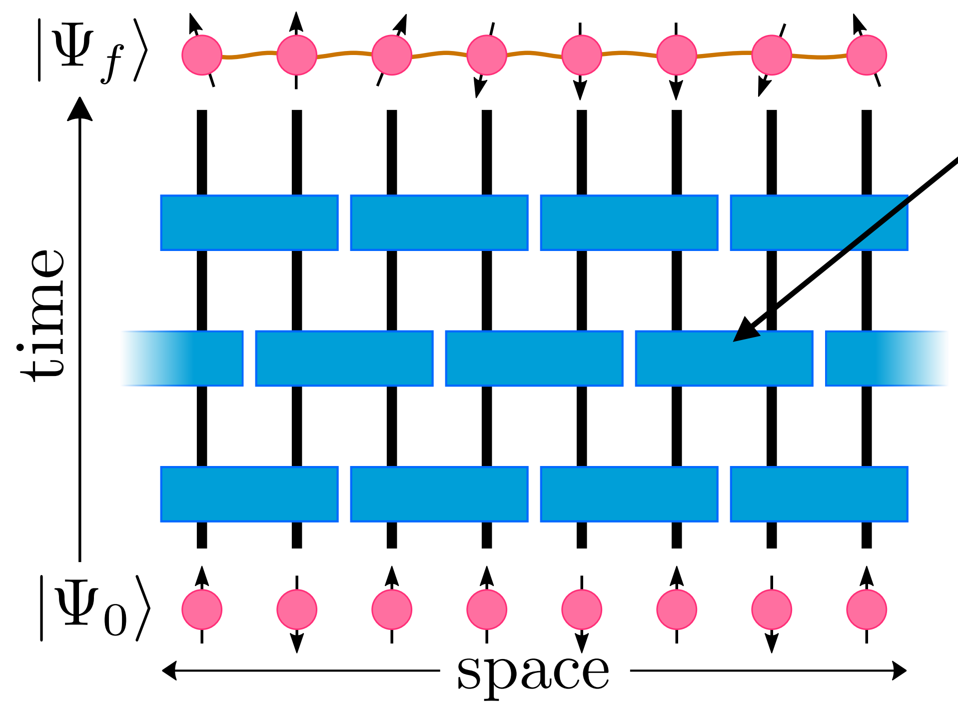


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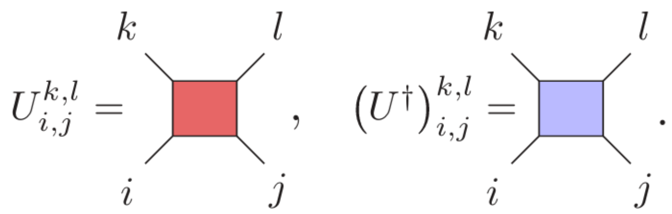
$$\langle k| \otimes \langle \ell|\tilde{U}|i\rangle \otimes |j\rangle = \langle j| \otimes \langle \ell|U|i\rangle \otimes |k\rangle .$$

Unitary along time AND space

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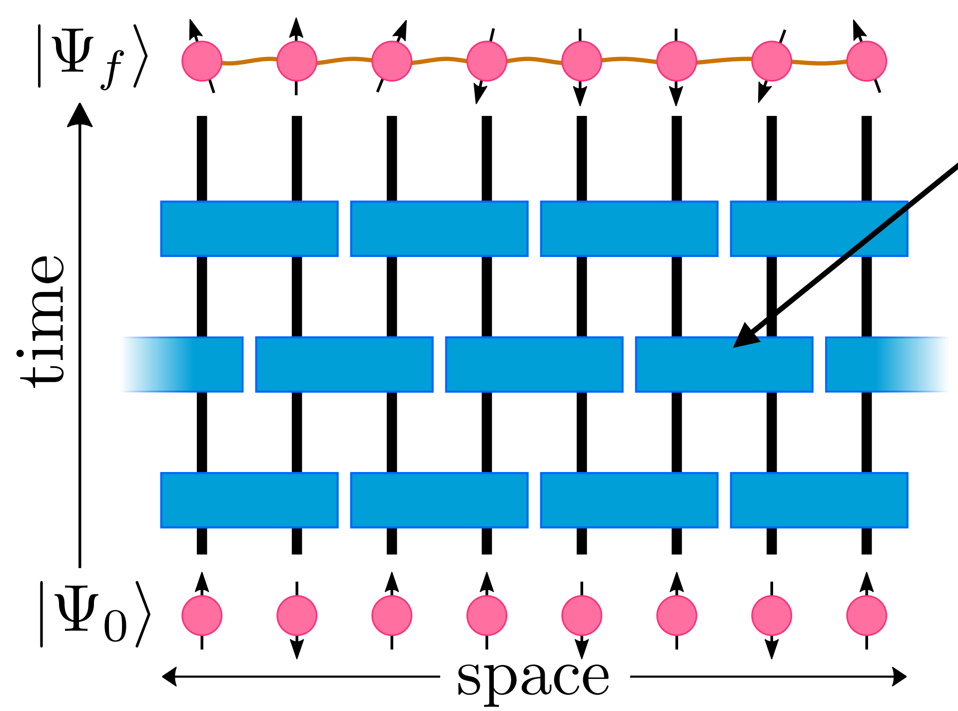


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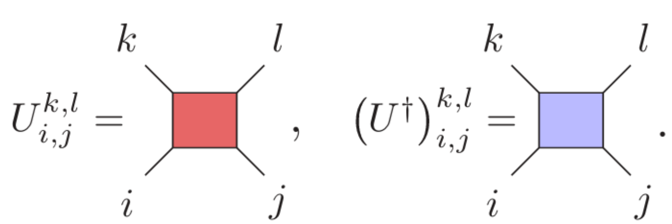
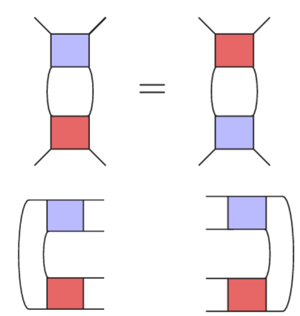



$$UU^\dagger = U^\dagger U = \mathbb{1},$$
$$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger\tilde{U} = \mathbb{1}.$$

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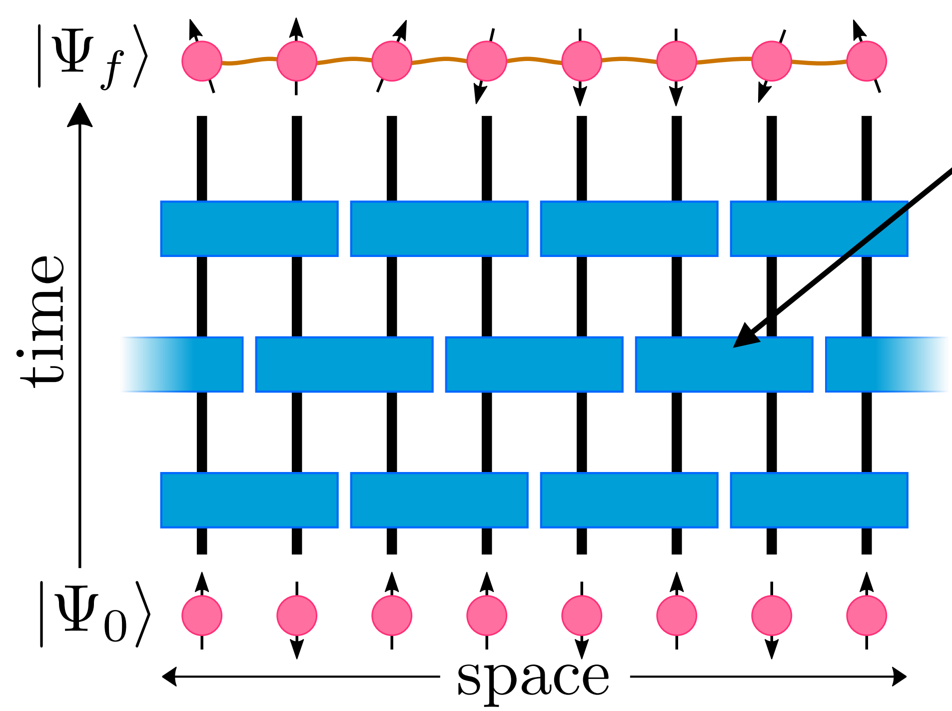


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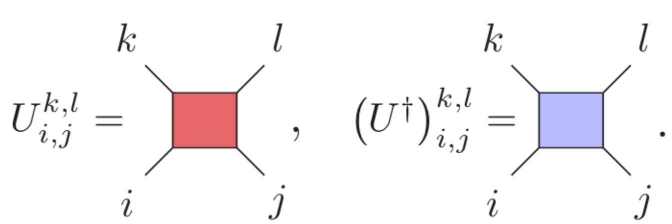


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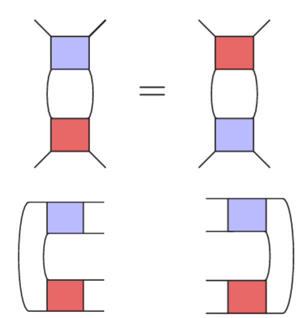



Unitary along time AND space

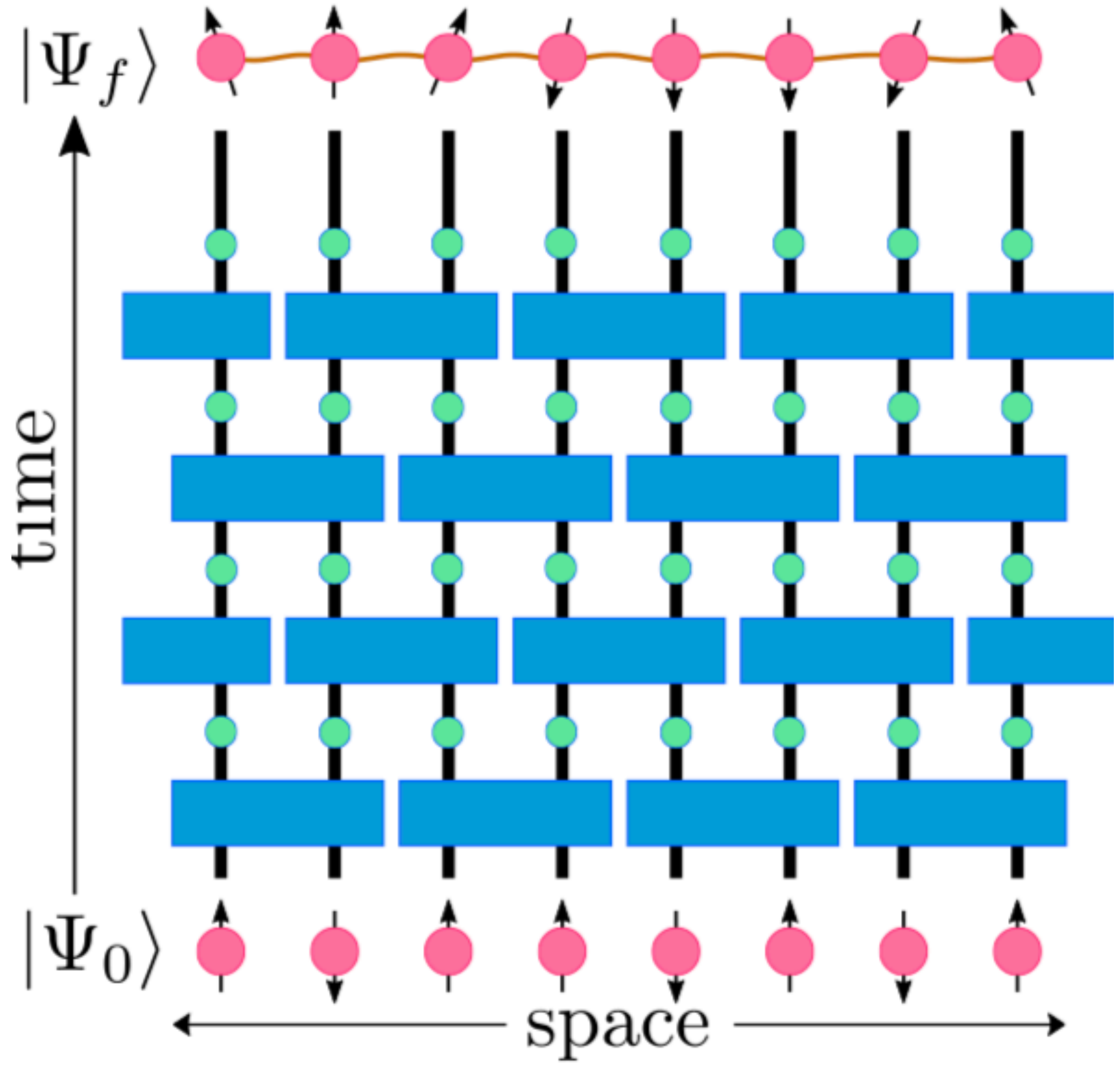
$$U = e^{i\phi}(u_+ \otimes u_-)V[J](v_- \otimes v_+)$$

$$V[J] = \exp\left[-i\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right)\right]$$

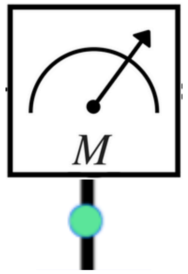
$$\phi, J \in \mathbb{R},\ u_\pm,\ v_\pm \in \mathrm{SU}(2)$$

$$|\Psi(t)\rangle = U(t, t-1)U(t-1, t-2)\cdots U(2,1)U(1,0)|\Psi_0\rangle$$

# Monitored Quantum Circuits




Intersperse
local measurements
in the circuit
with probability $p$




competition between **entangling** gates
and **collapsing** the state with measurements

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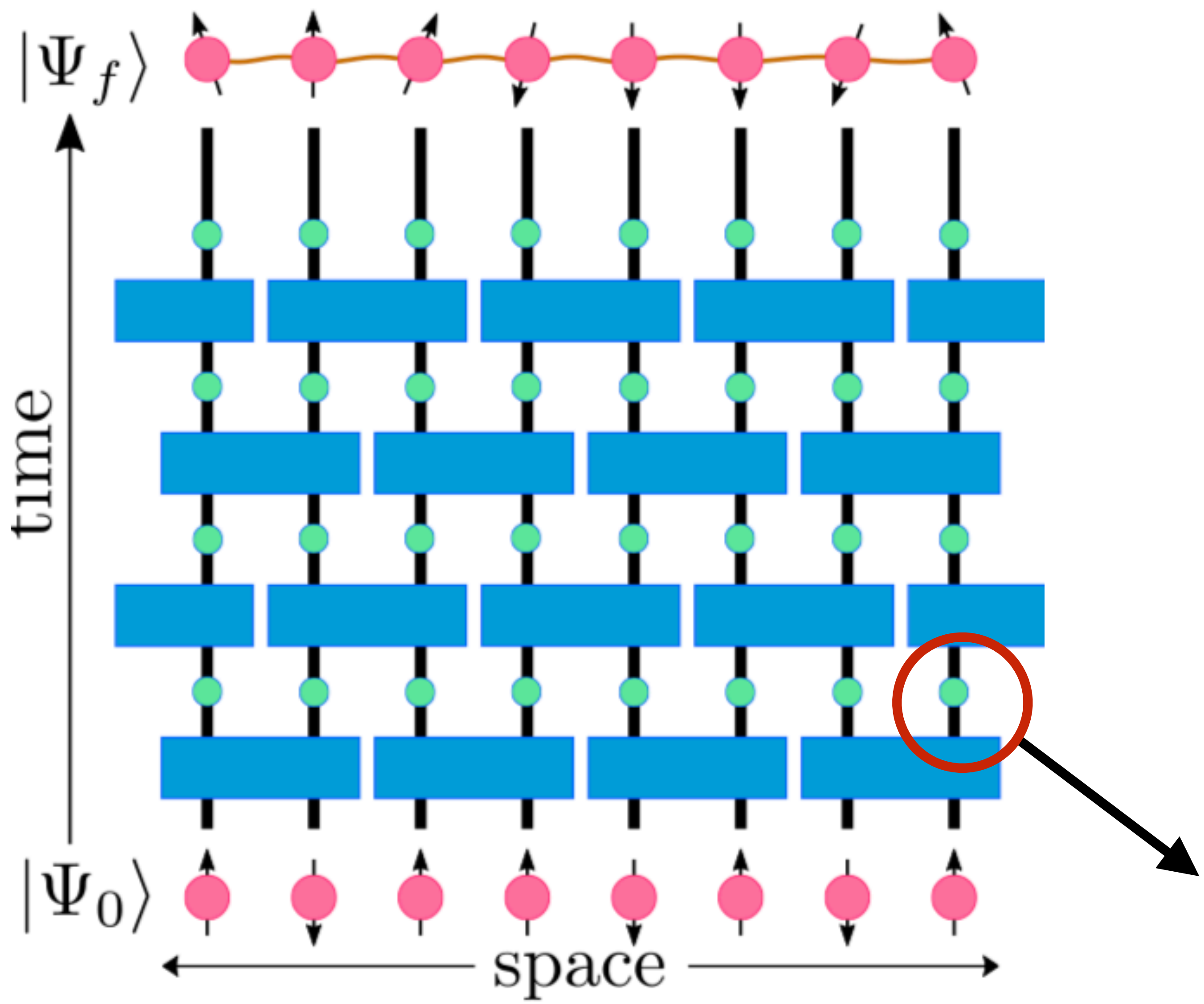


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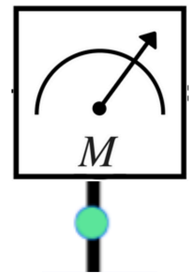


measure $S_i^z$ with probability $p$

competition between **entangling** gates and **collapsing** the state with measurements

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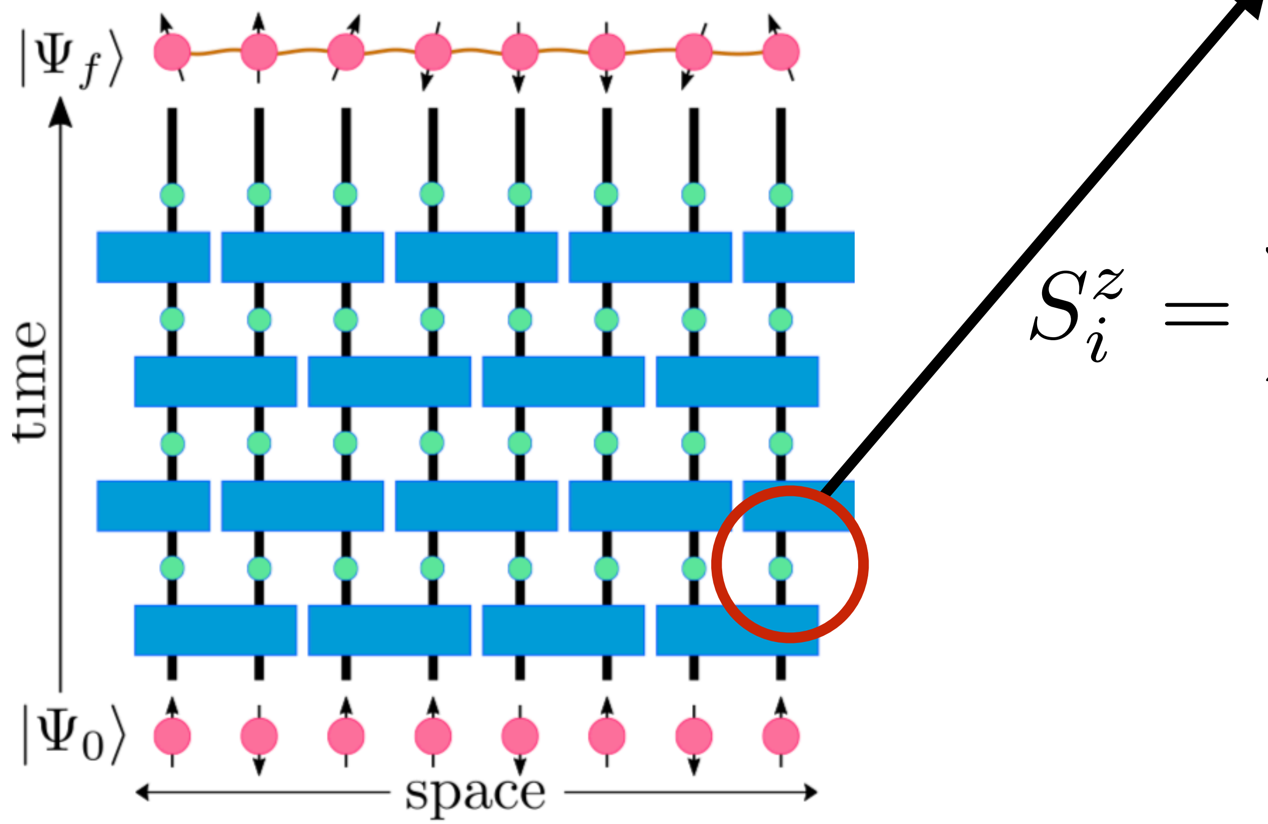


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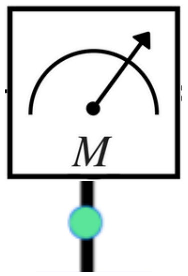


$$S_i^z = \sum_m e_m P_i^m = \frac{1}{2}(P_i^+ - P_i^-)$$

$$P_i^{\pm} = \frac{1}{2}(1 \pm \sigma_i^z)$$

*Quantum Computation and Quantum Information*, Nielsen and Chuang

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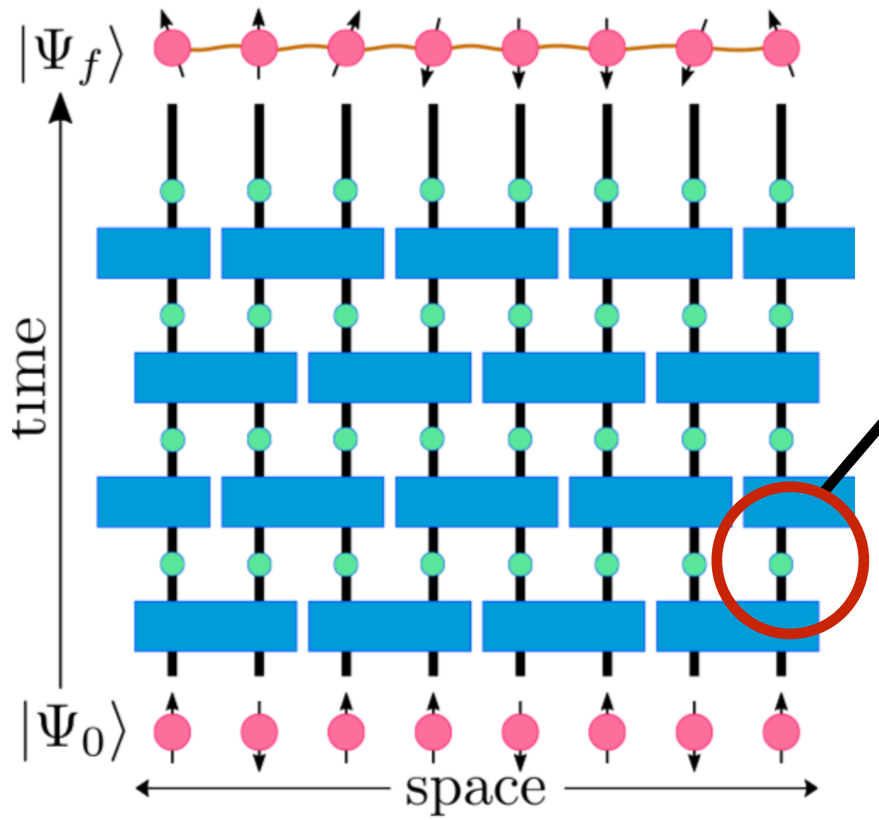


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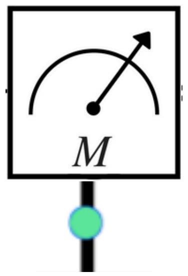


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For a projective (strong) measurement:

Project onto spin up w/ probability: $\langle \Psi(t)|P_i^+|\Psi(t)\rangle$

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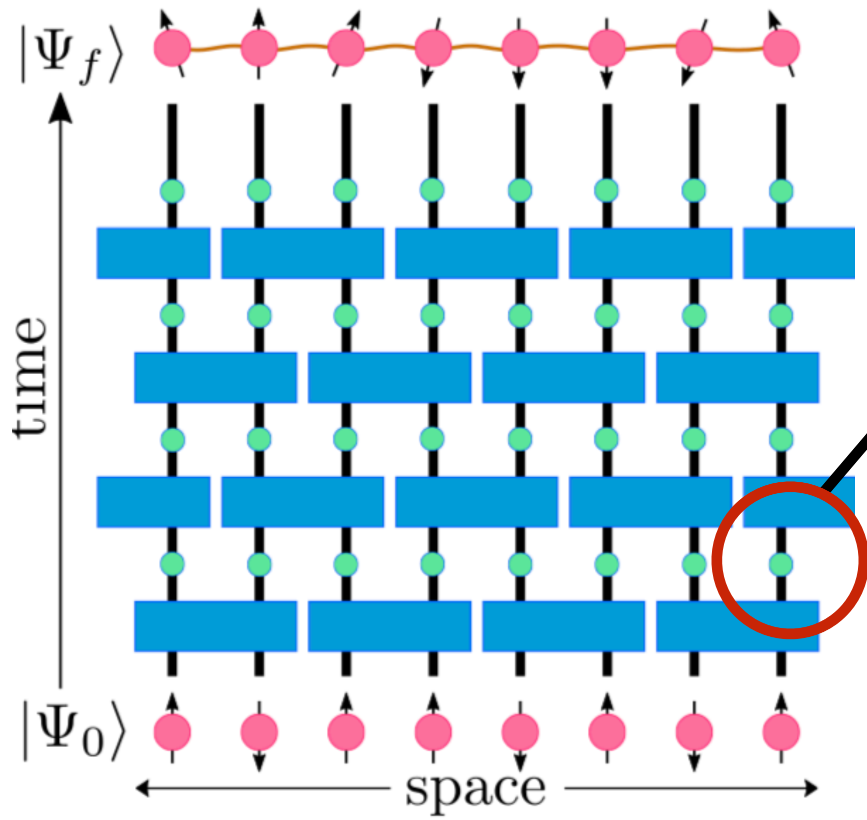


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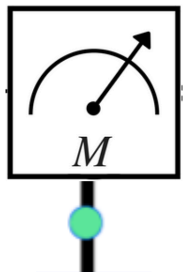


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For a projective (strong) measurement:

Project onto spin up w/ probability: $\langle\Psi(t)|P_i^+|\Psi(t)\rangle$

Update wave function post measurement $\quad |\Psi(t)\rangle \rightarrow \frac{P_i^m|\Psi(t)\rangle}{\langle\Psi(t)|P_i^m|\Psi(t)\rangle}$

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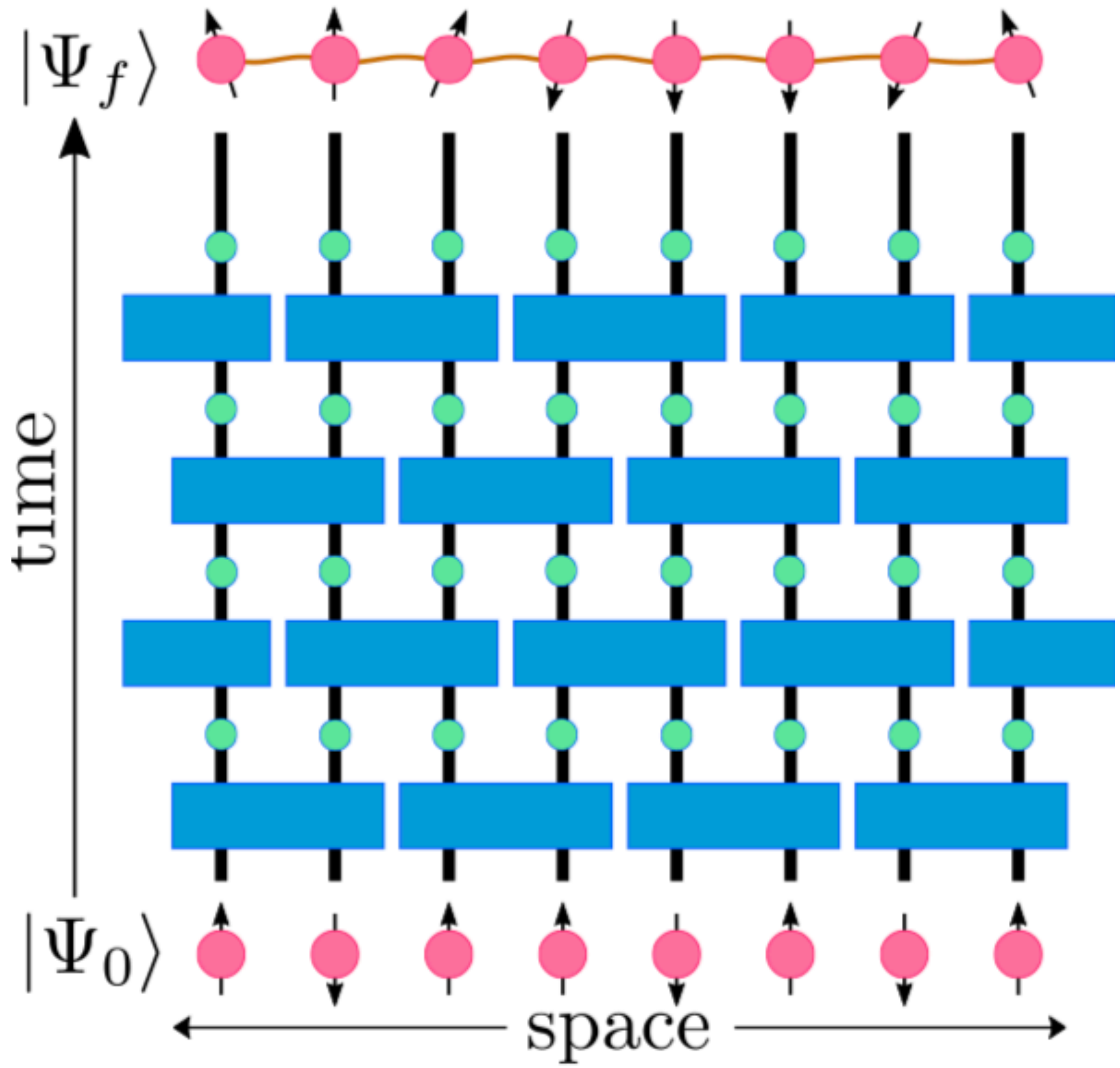
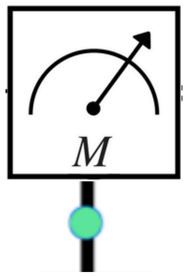
# Monitored Quantum Circuits

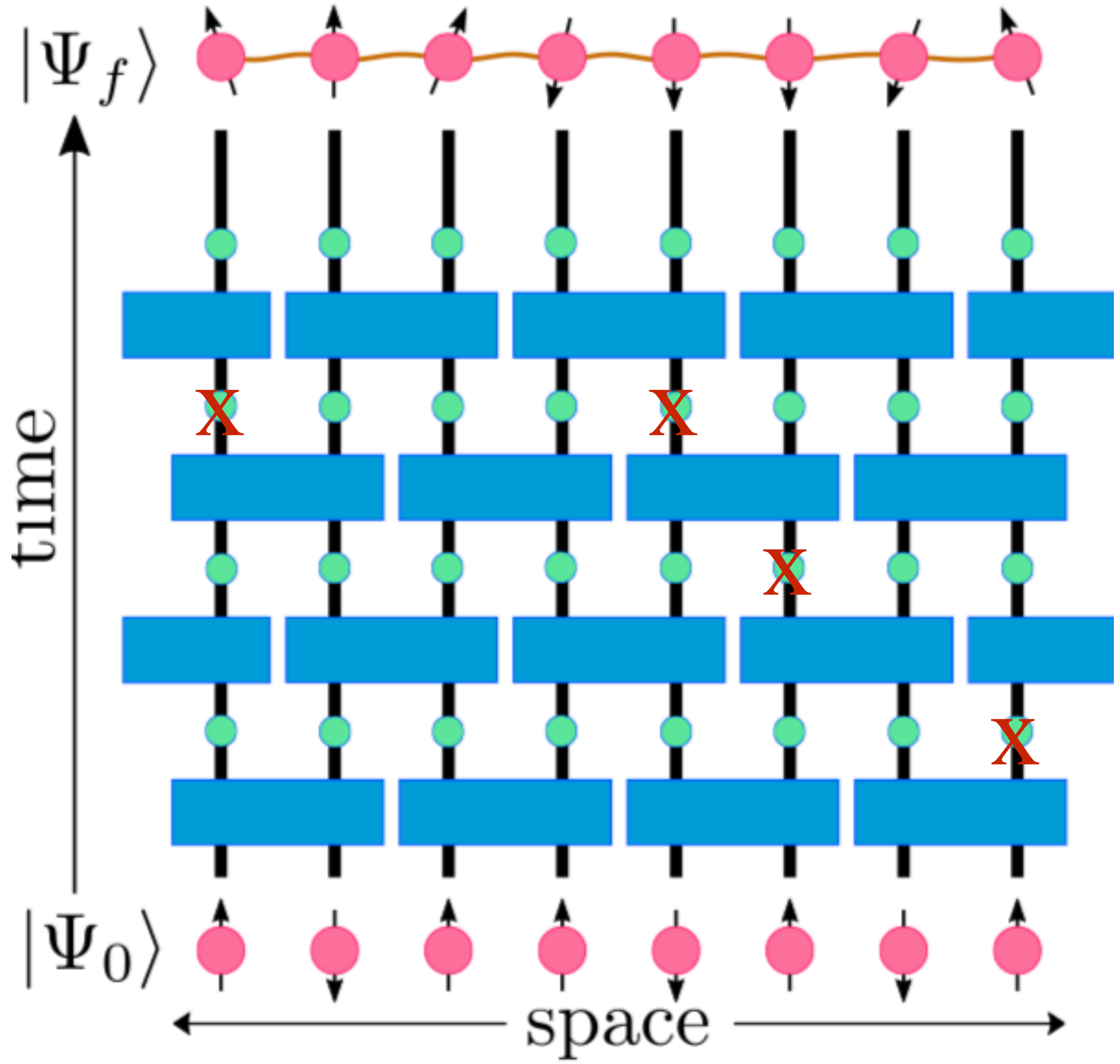



Intersperse
local measurements
of $S_i^z$ in the circuit
with probability $p$




$$S_i^z = \frac{1}{2}(P_i^{\uparrow} - P_i^{\downarrow})$$

# Monitored Quantum Circuits

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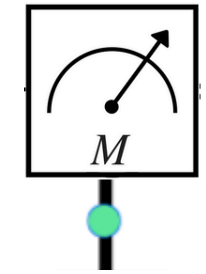


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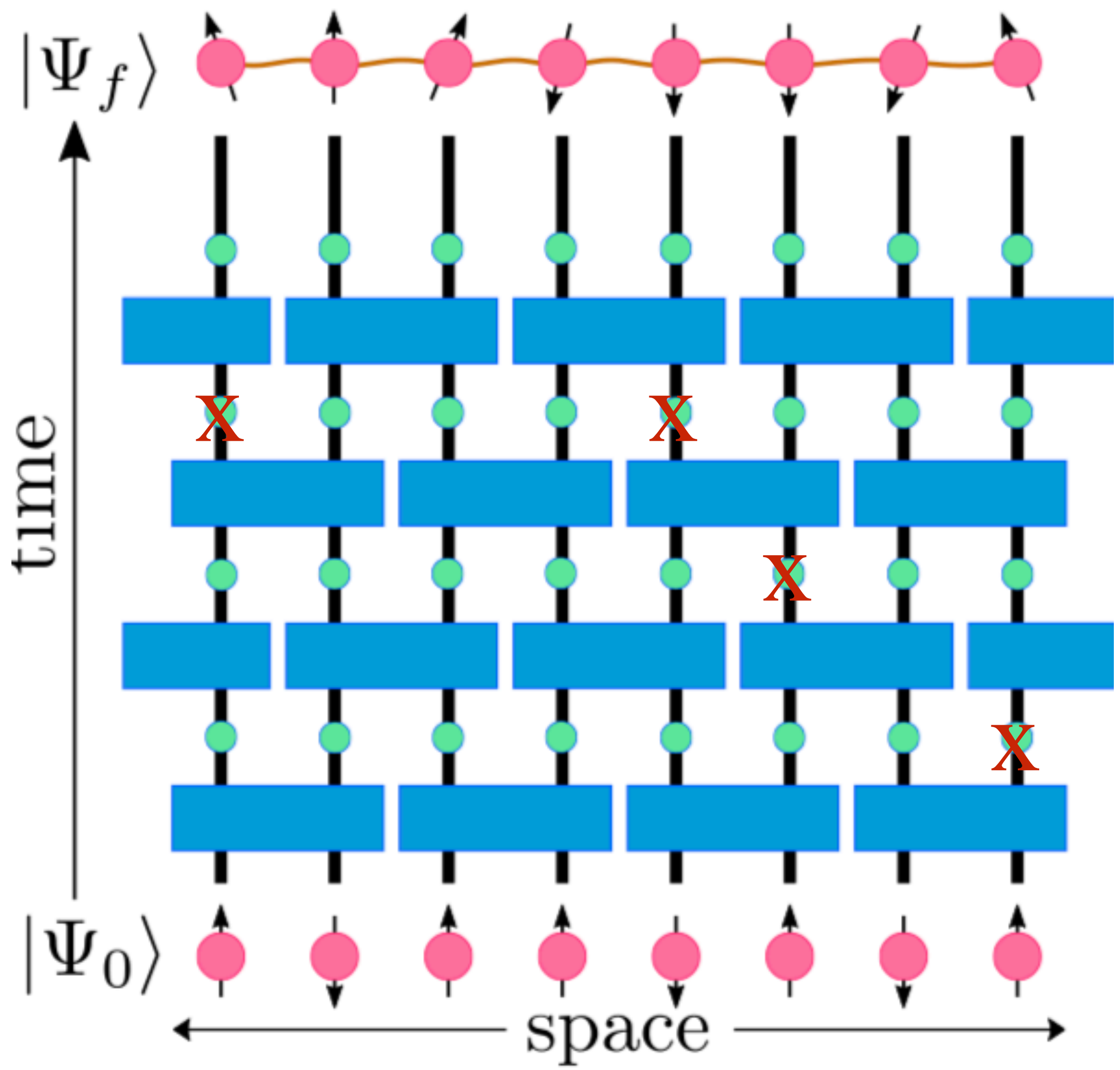
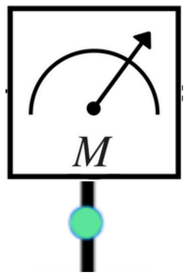



Intersperse local measurements of $S_i^z$ in the circuit with probability $p$




$$S_i^z = \frac{1}{2}(P_i^{\uparrow} - P_i^{\downarrow})$$

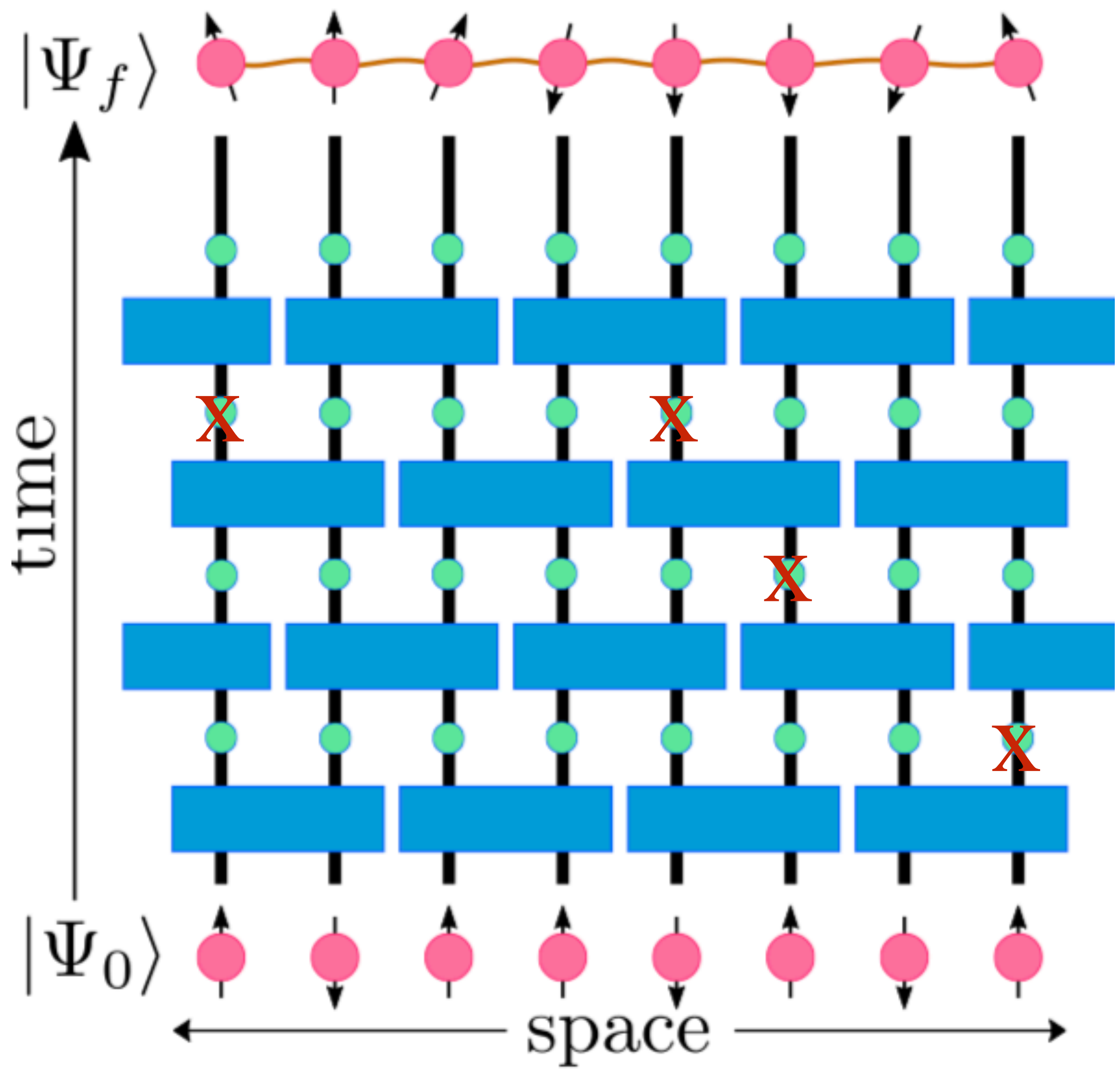
**Dynamics is non-unitary!**

$$|\Psi_{\mathbf{m}}(t)\rangle = K_{m_t} U(t, t-1) \ldots K_{m_2} U(2,1) K_{m_1} U(1,0) |\Psi_0\rangle / \sqrt{p_{\mathbf{m}}(\Psi_0)}$$

$K_{m_t}$ Kraus operator, product of the operators across sites $\{\mathbf{1}, P_i^{\uparrow}, P_i^{\downarrow}\}$

$\mathbf{m} = (\vec{m}_1, \vec{m}_2, \ldots, \vec{m}_t)$ measurement history

# Monitored Quantum Circuits

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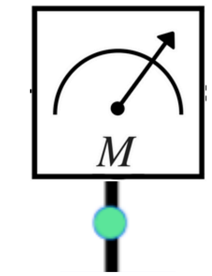


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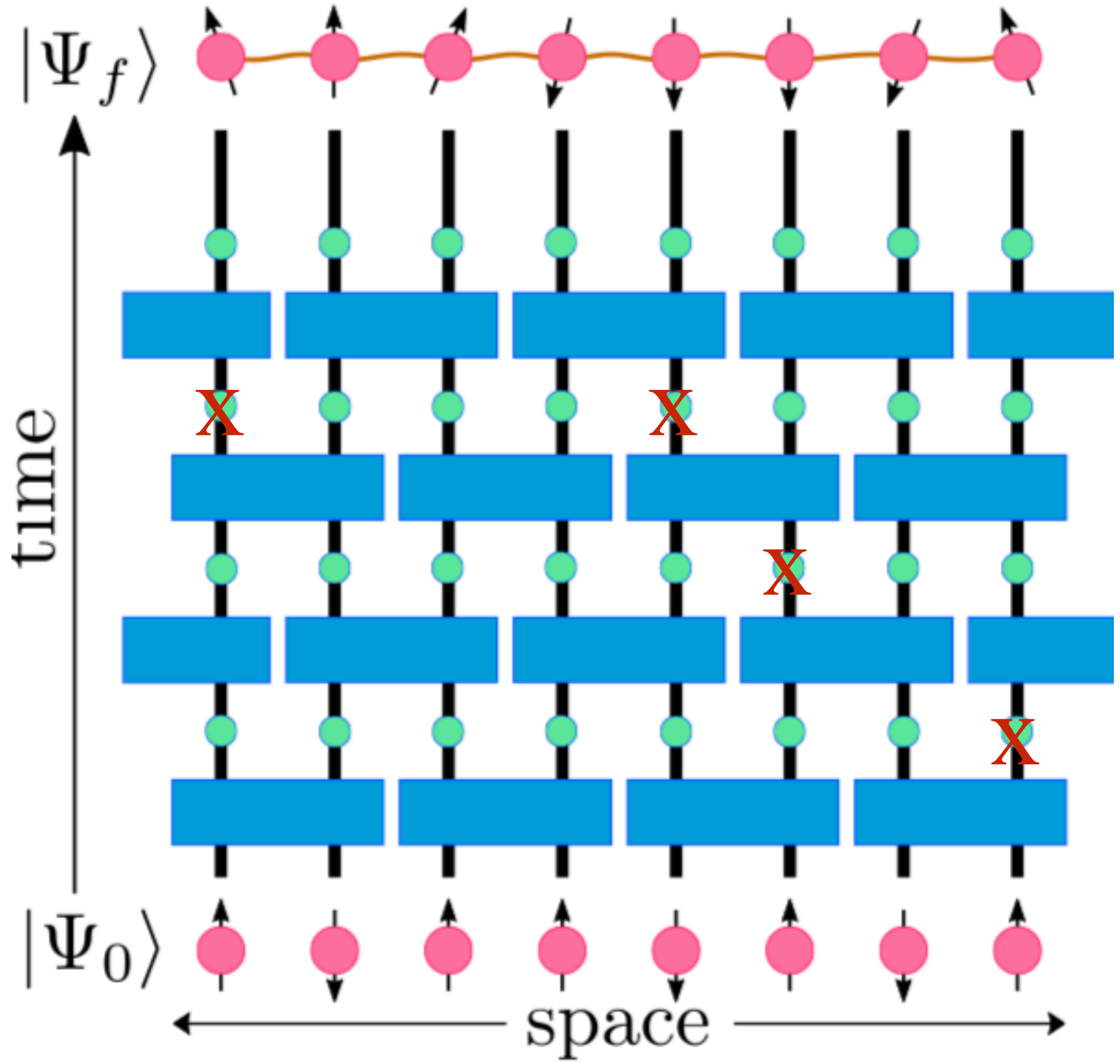
Observables depend on the measurement history $\langle \Psi(t) | \mathcal{O} | \Psi(t) \rangle \equiv \mathcal{O}_{\mathbf{m}}(t)$

# Monitored Quantum Circuits

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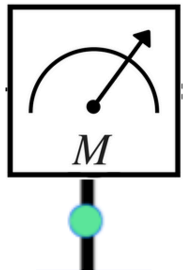


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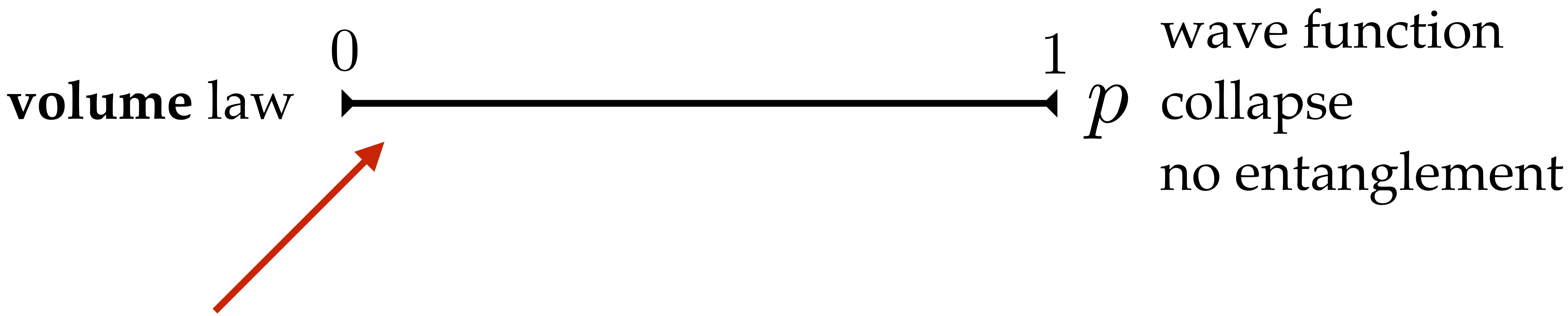
Averages over the Born probability $\overline{\langle \mathcal{O} \rangle} = \sum_{\mathbf{m}} p_{\mathbf{m}} \mathcal{O}_{\mathbf{m}}$ $\quad \langle \Psi(t)|\mathcal{O}|\Psi(t)\rangle \equiv \mathcal{O}_{\mathbf{m}}(t)$

$$p_{\mathbf{m}} = ||K_{m_t} U(t, t-1) \ldots K_{m_2} U(2,1) K_{m_1} U(1,0) |\Psi_0\rangle||^2$$

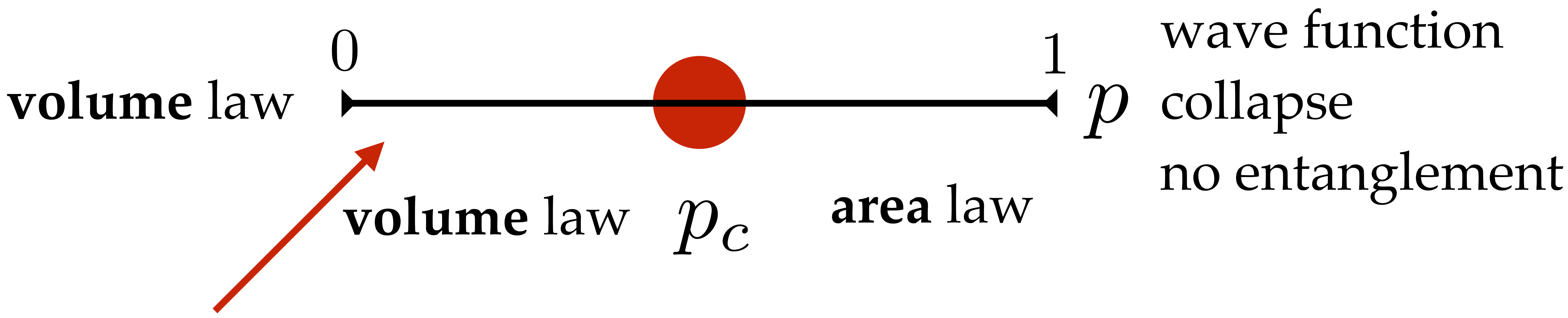
# Measurement Transition

# Measurement Transition




**Does the volume law phase survive a finite p?**

# Measurement Transition




**Does the volume law phase survive a finite p?**

**YES!**


Vasseur, Potter, You, Ludwig PRB (2018)
Choi, Bao, Qi, Altman, PRL (2020)
Li, Chen, Fisher PRB (2018); PRB (2019)
Skinner, Ruhman, Nahum PRX (2019)

# Controlled Limits: Large Local Hilbert Space

Controlled Limit: 

$$q \to \infty$$

qudit gates with
a **q**-size Hilbert space

Replicated field theory of the Renyi entropy $S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}(\rho_A^n)$

Vasseur, Potter, You, Ludwig PRB (2018)

Jian, You, Vasseur, Ludwig PRB (2020)

*Entanglement in Spin Chains. Quantum Science and Technology,*
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$$p_{\mathbf{m}} = \mathrm{tr}\rho_{\mathbf{m}}$$

$$\rho_{\mathbf{m}} = |\psi_{\mathbf{m}}\rangle\langle\psi_{\mathbf{m}}|$$

Avg. over unitaries & measurement outcomes

Vasseur, Potter, You, Ludwig PRB (2018)

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$$\log \left[ \frac{\mathrm{tr}\rho_{A,\mathbf{m}}^n}{(\mathrm{tr}\rho_{\mathbf{m}})^n} \right] = \log \left[ \mathrm{tr}\rho_{A,\mathbf{m}}^n \right] - \log \left[ (\mathrm{tr}\rho_{\mathbf{m}})^n \right]$$

Replica trick

$$\log x = \lim_{k\to 0} \frac{x^k - 1}{k}$$

$$S_A^{(n)} = \lim_{k\to 0} \mathbb{E}_U \sum_{\mathbf{m}} \frac{p_{\mathbf{m}}}{(1-n)k} \left( (\mathrm{tr}\rho_{A,\mathbf{m}}^n)^k - (\mathrm{tr}\rho_{\mathbf{m}}^{\otimes kn}) \right)$$

Analytic continuation in *k*,
subtle business

Note on notation

$$(\mathrm{tr}\rho_{\mathbf{m}}^{\otimes kn}) = (\mathrm{tr}\rho_{\mathbf{m}})^{kn}$$

Vasseur, Potter, You, Ludwig PRB (2018)

Jian, You, Vasseur, Ludwig PRB (2020)

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Using

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$$p_{\mathbf{m}} (\mathrm{tr}\rho_{\mathbf{m}})^{kn} = (\mathrm{tr}\rho_{\mathbf{m}})^{kn+1}$$

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Combined replica index $Q = kn + 1$

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Jian, You, Vasseur, Ludwig PRB (2020)

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Permutation operator $\mathcal{S}_{A,n}$ allows us to evaluate the partial trace

$$p_{\mathbf{m}} \mathrm{Tr}_A (\rho_{A,\mathbf{m}}^n)^k = \mathrm{Tr} \left( \mathcal{S}_{n,A} \rho_{\mathbf{m}}^{\otimes Q} \right)$$

$$\mathcal{S}_{A,n} |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle = |\psi_n\rangle \otimes |\psi_1\rangle \otimes \cdots |\psi_{n-1}\rangle$$


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The average over unitaries is mathematically involved (not get into here)

Roberts and Yoshida, JHEP (2017)

$$Z_A = \mathbb{E}_{U,\mathbf{m}} \mathrm{tr}\left[ \mathcal{S}_{A,n}^{\otimes k} \rho_{\mathbf{m}}^{\otimes Q} \right]$$
$$Z_0 = \mathbb{E}_{U,\mathbf{m}} \mathrm{tr}\left[ \rho_{\mathbf{m}}^{\otimes Q} \right],$$

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Vasseur, Potter, You, Ludwig PRB (2018)

Jian, You, Vasseur, Ludwig PRB (2020)

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Vasseur, Potter, You, Ludwig PRB (2018)

Jian, You, Vasseur, Ludwig PRB (2020)

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replica index

$$Q = kn + 1$$

=partition function of the $Q$ states Potts model


Vasseur, Potter, You, Ludwig PRB (2018)

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Replica limit $\xrightarrow[Q \to 1]{}$ **=Bond percolation on the square lattice!**

Cardy, J. Phys. A (1992)

Vasseur, Potter, You, Ludwig PRB (2018)

Jian, You, Vasseur, Ludwig PRB (2020)

*Entanglement in Spin Chains. Quantum Science and Technology*, Potter and Vasseur, (2021)

# Controlled Limits: *n=0*

The fully quantum limit has Haar random gates

In this case Renyi dependence is important

$$S_n = \frac{1}{1-n} \log_2 \mathrm{Tr} \rho_A^n$$

Taking *n=0* allows for a controlled calculation through a "minimal cut"

Skinner, Ruhman, Nahum PRX (2019)

# Controlled Limits: *n=0*

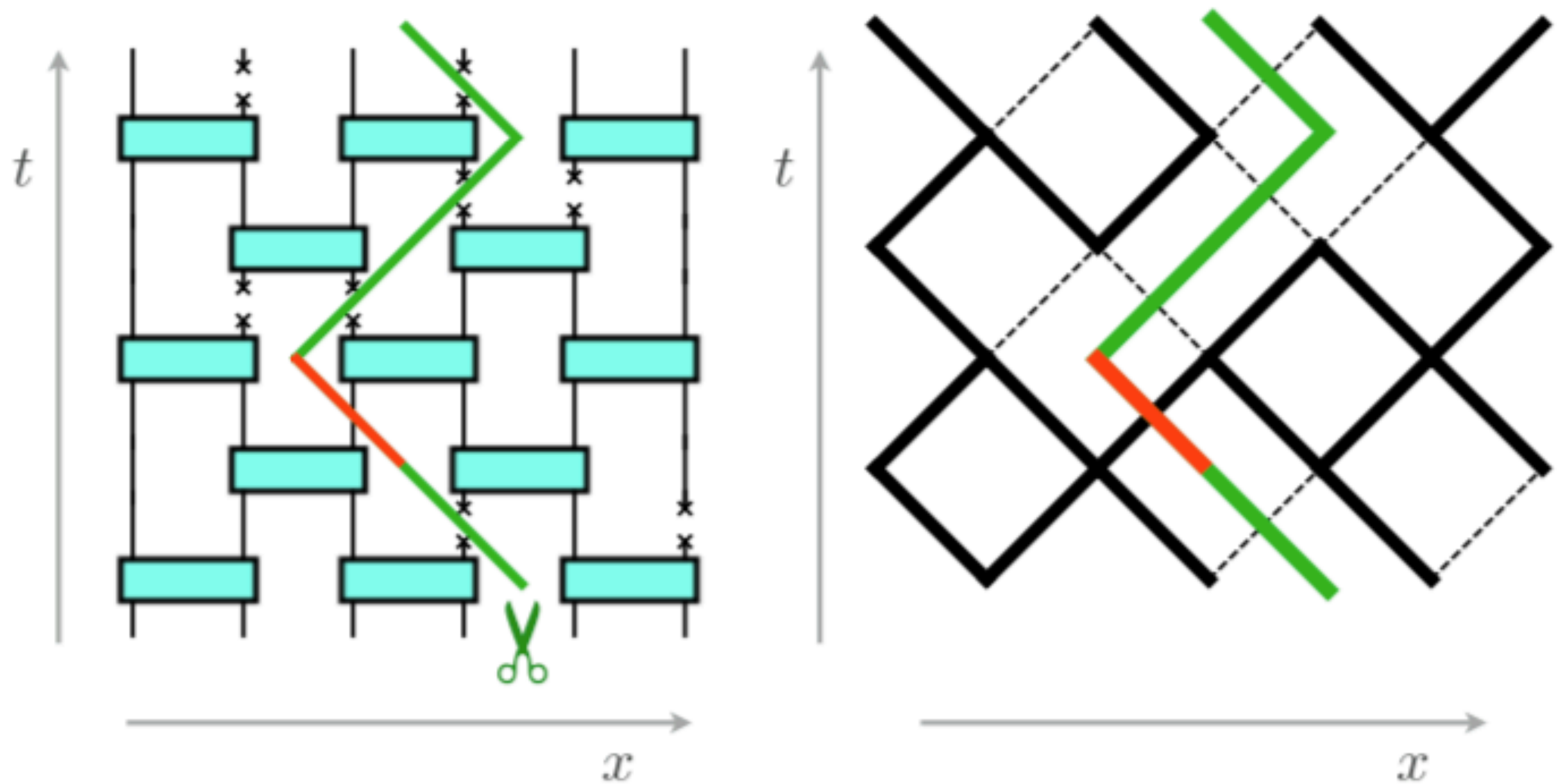
Taking *n=0* allows for a controlled calculation through a "minimal cut"

*= the number of bonds one must cut to separate the sub-region at the physical legs at the end*

Hartley entropy

$$S_0 = \log_2 N$$

*N*=Number of non-zero eigenvalues of the reduced density matrix




Skinner, Ruhman, Nahum PRX (2019)

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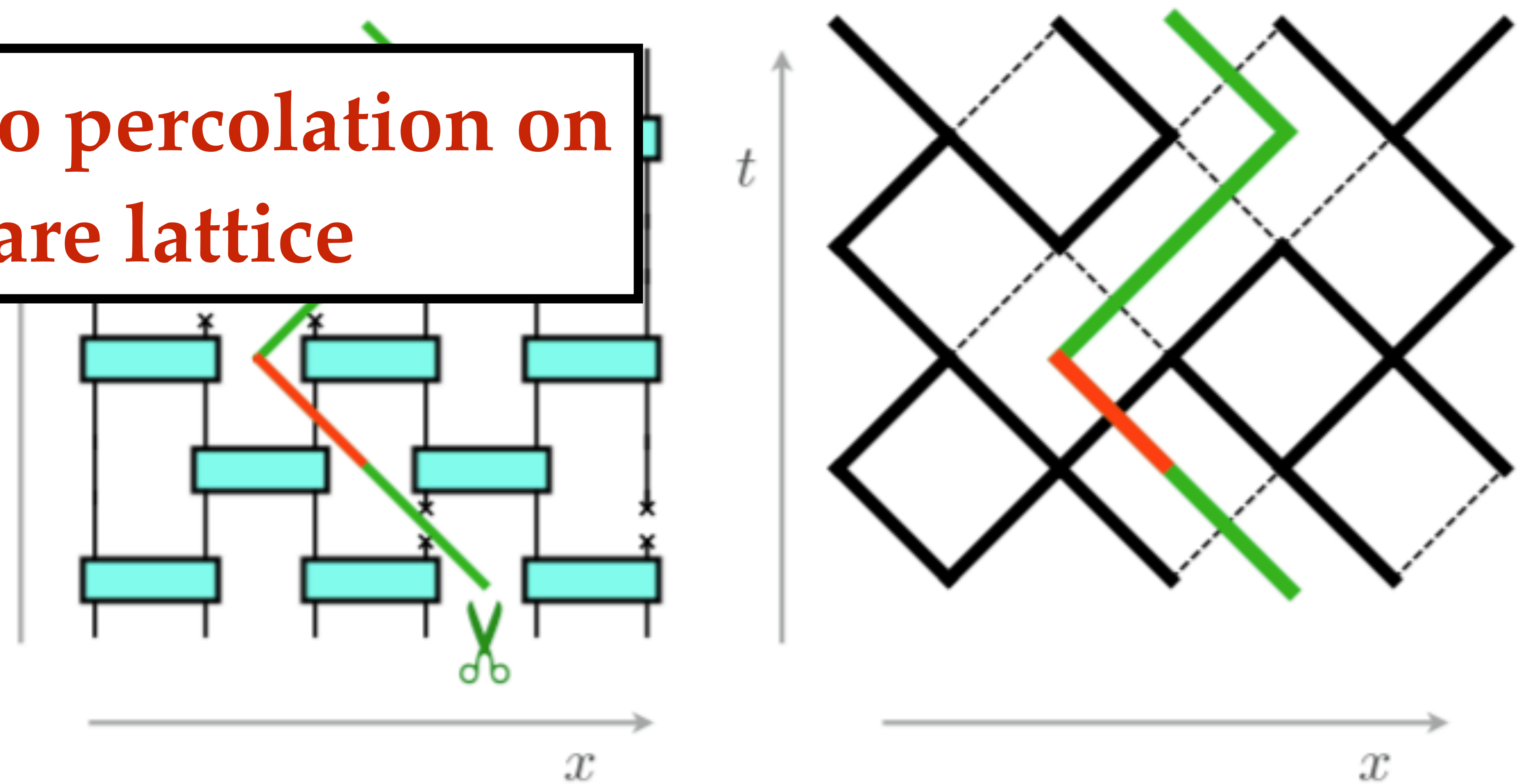
Hartley entropy

$S_0$

**Maps exactly to percolation on the square lattice**

$N$=Num

eigenvalues of the reduced density matrix



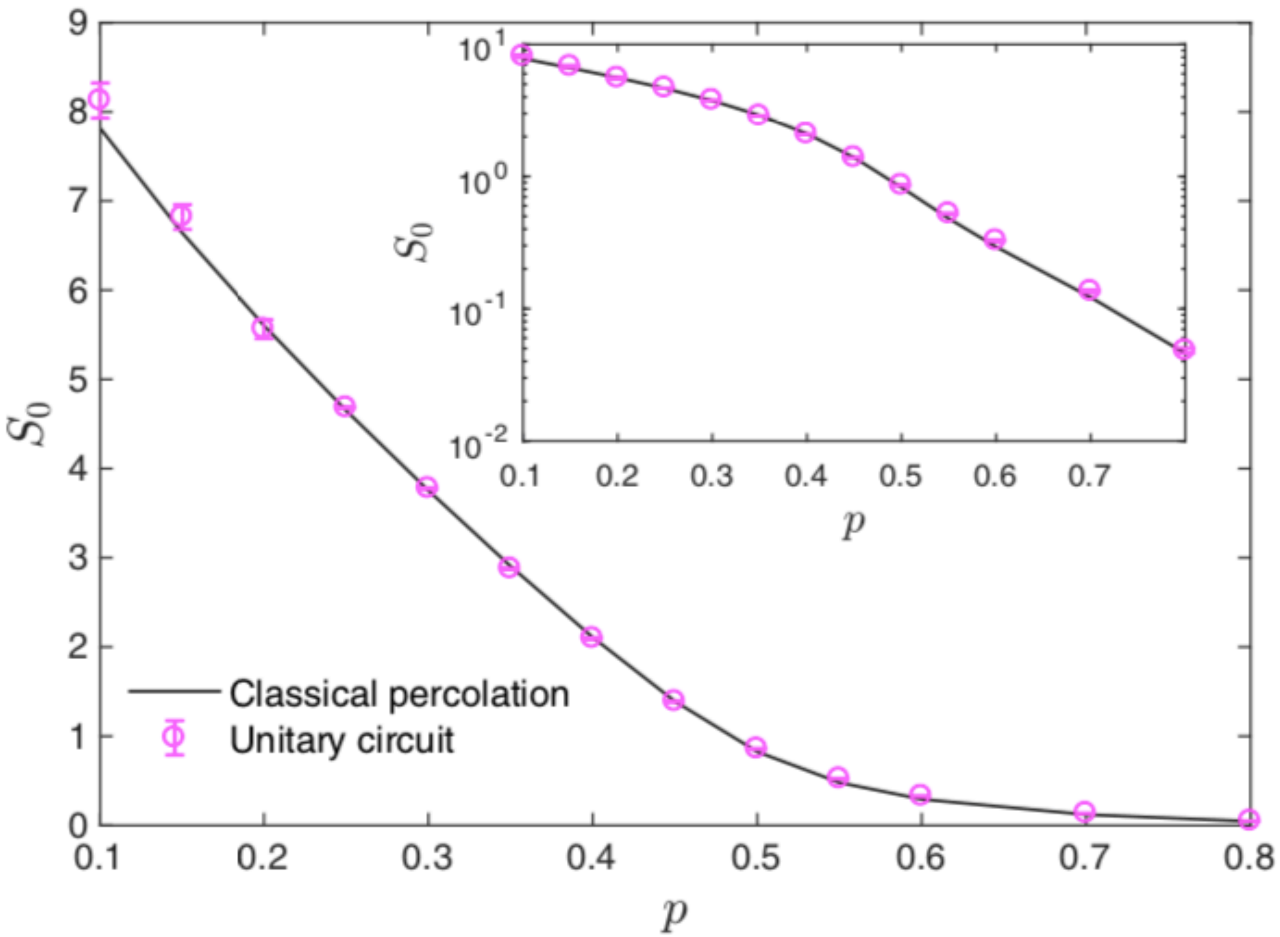

Skinner, Ruhman, Nahum PRX (2019)

# Controlled Limits: $n$=0

$$S_0 = \log_2 N$$

Maps exactly to **percolation**

$$p_c = 1/2$$
$$\nu = 4/3$$
$$z = 1$$




$$S_0 \sim A \log L$$
$$A = (1/2)\sqrt{3}/\pi$$

Cardy arXiv:math-ph/0103018 (2001)

Skinner, Ruhman, Nahum PRX (2019)

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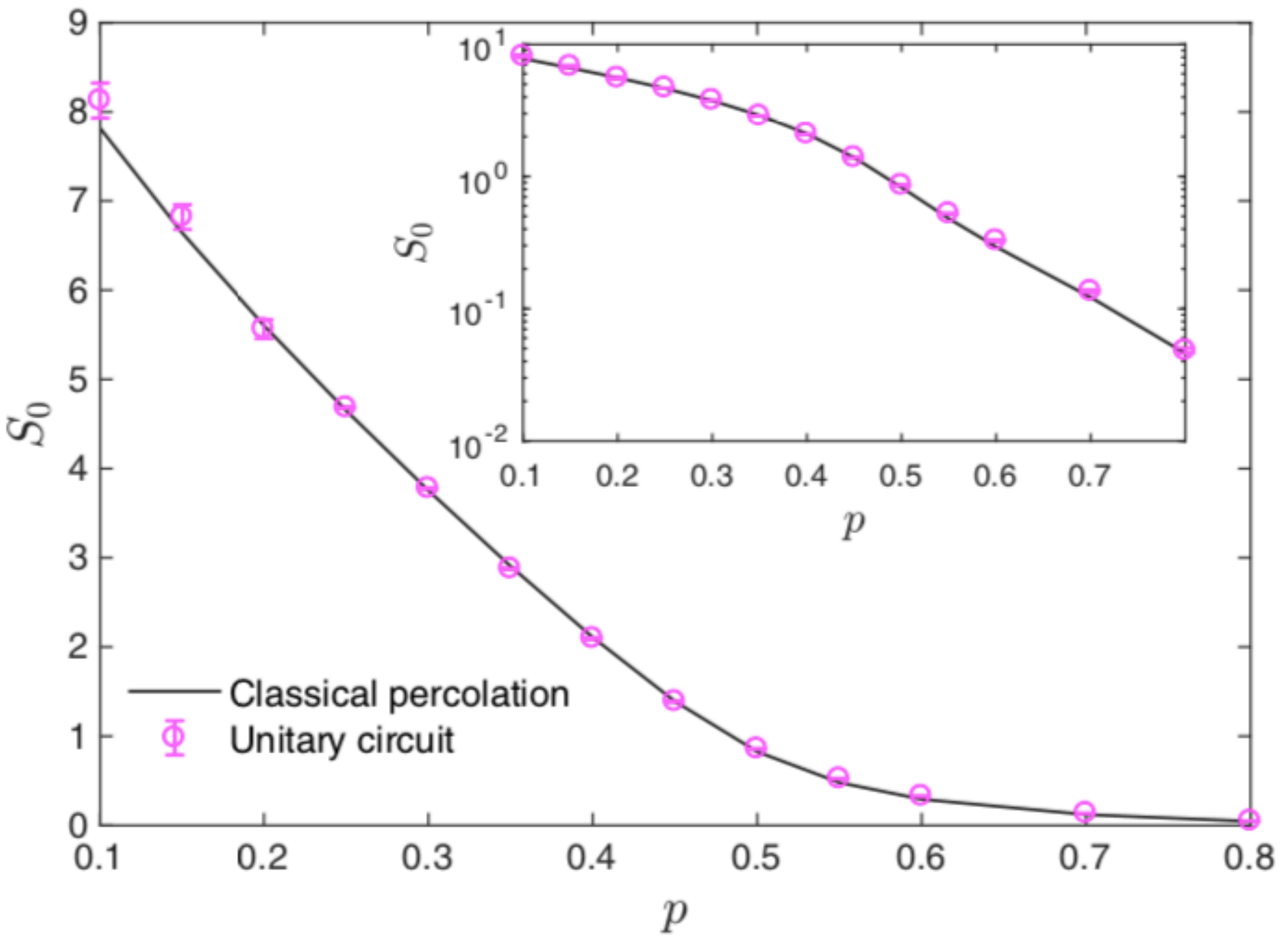
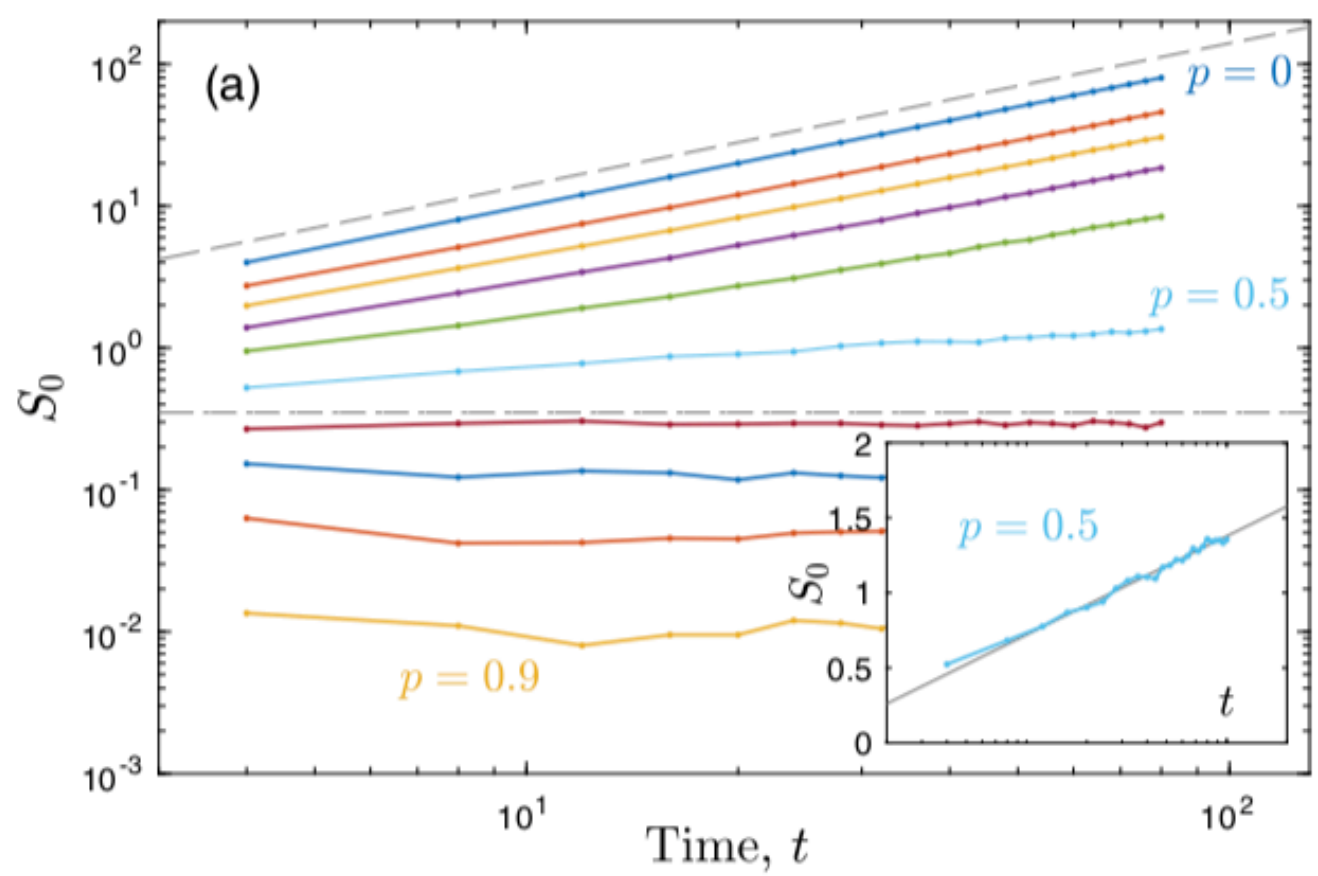



Skinner, Ruhman, Nahum PRX (2019)



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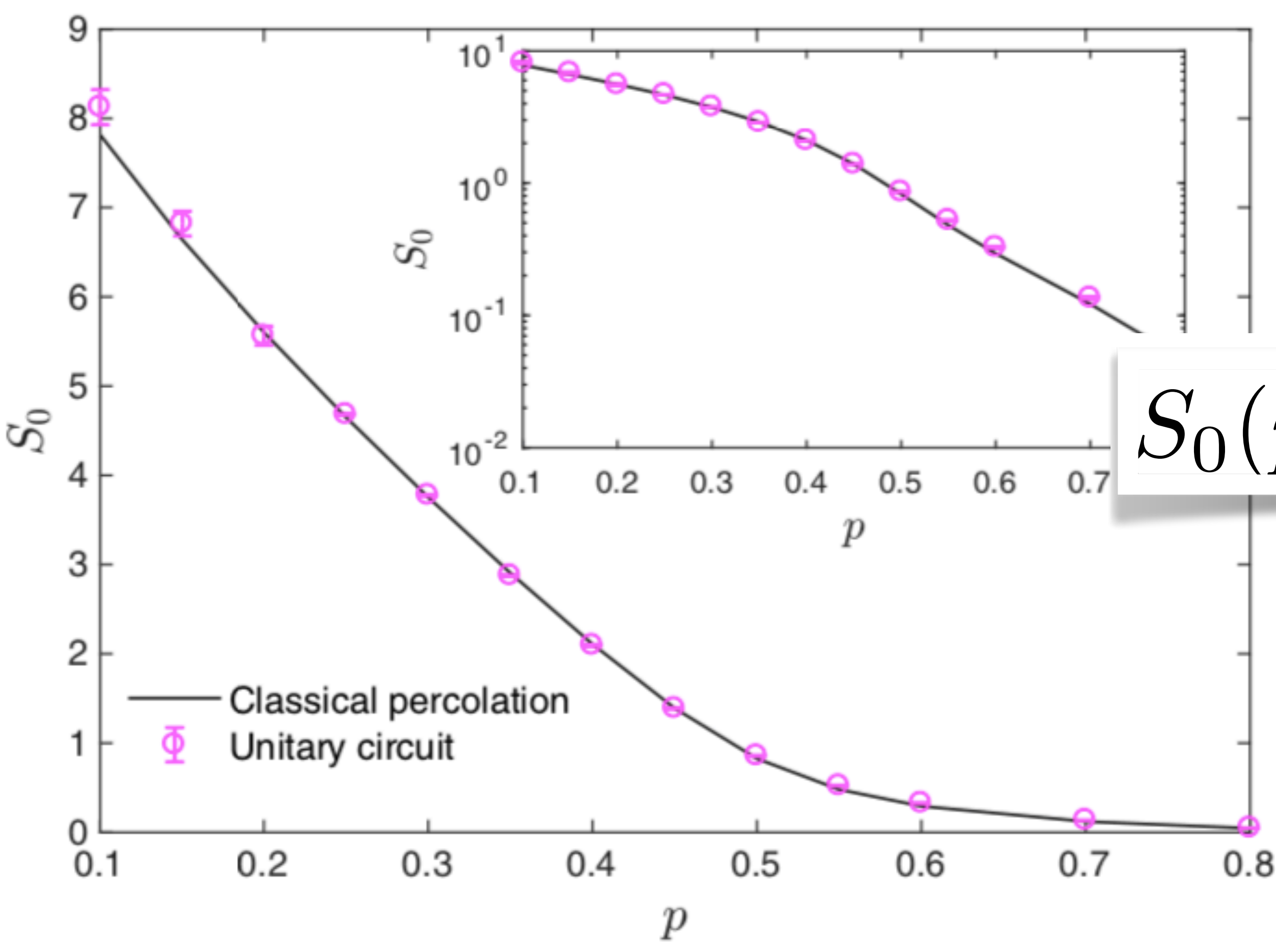
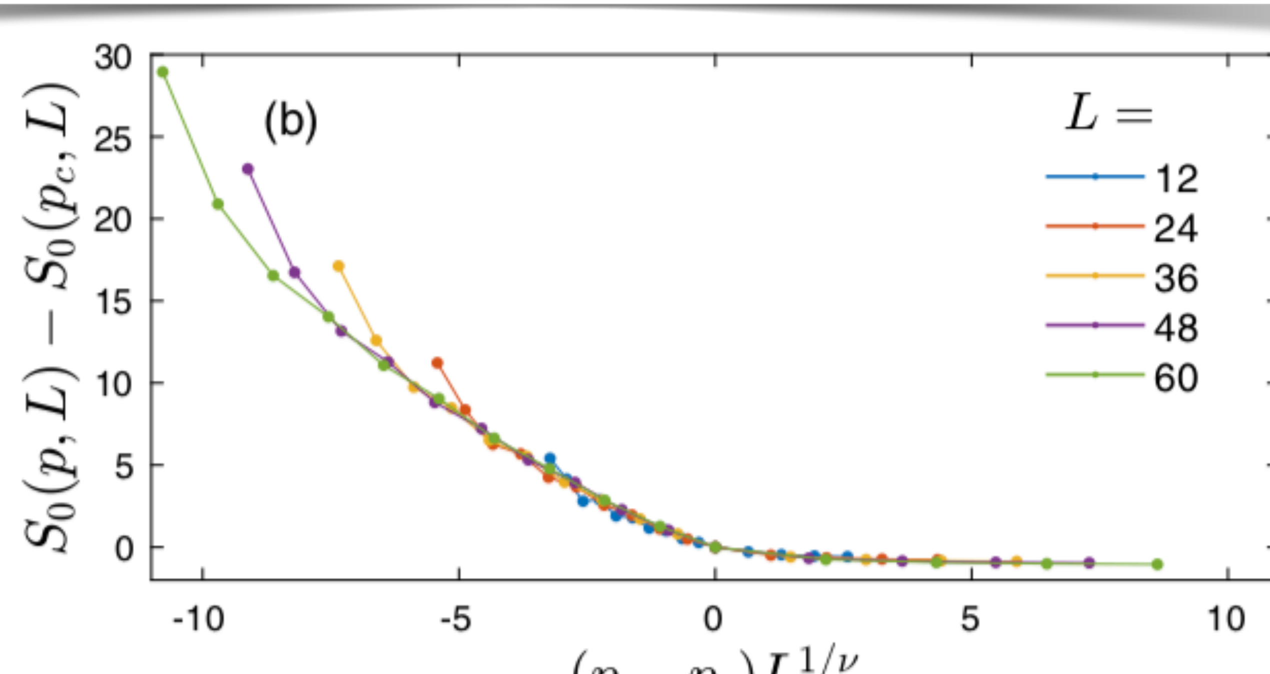
$$S_0(p_c, L) \sim \alpha \log L + f\left[(p - p_c)L^{1/\nu}\right]$$



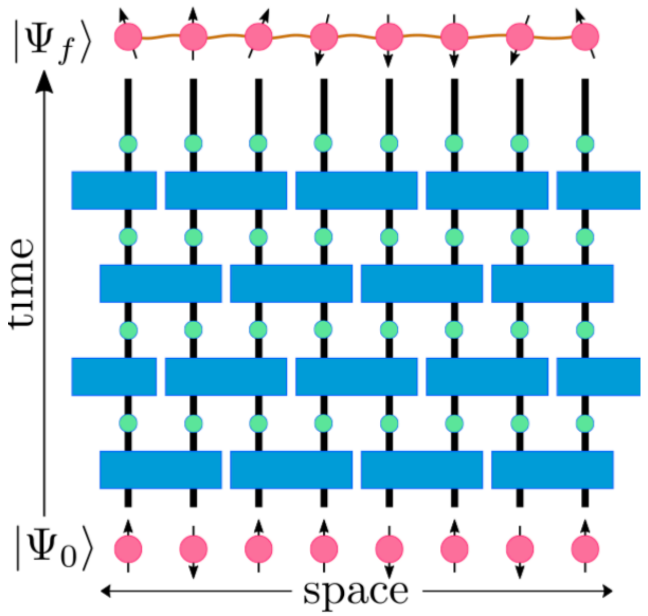

Skinner, Ruhman, Nahum PRX (2019)

# Controlled Limits: Large Local Hilbert Space

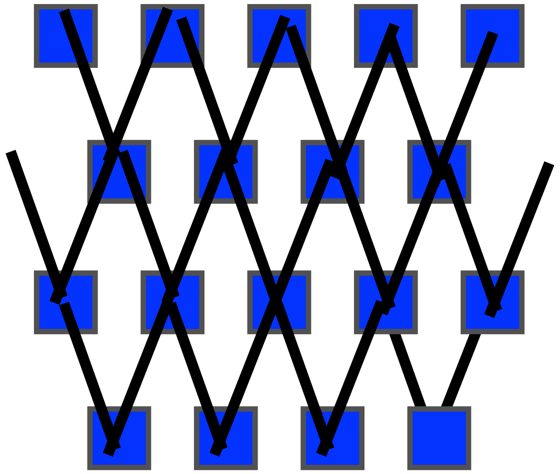
Controlled Limits: 

$q \to \infty$ or $n = 0$ are equivalent!

Maps to a **bond percolation problem** in *D=d+1*




unitaries build a connected network in space-time



Vasseur, Potter, You, Ludwig PRB (2018)

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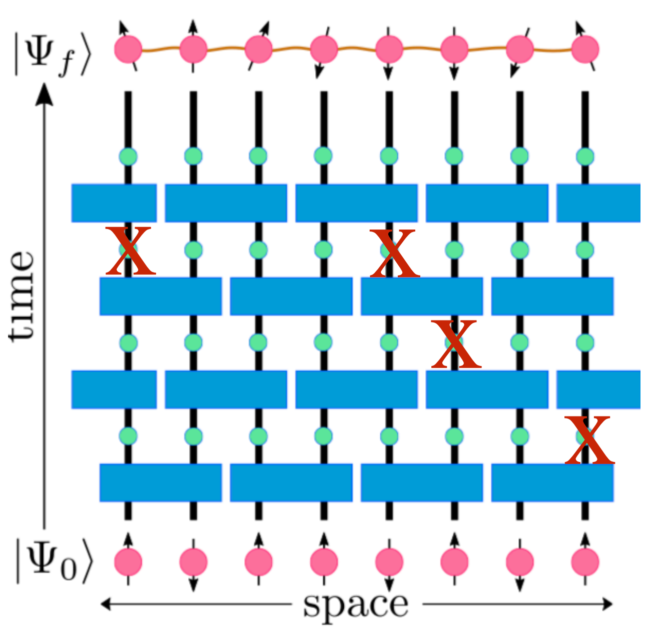
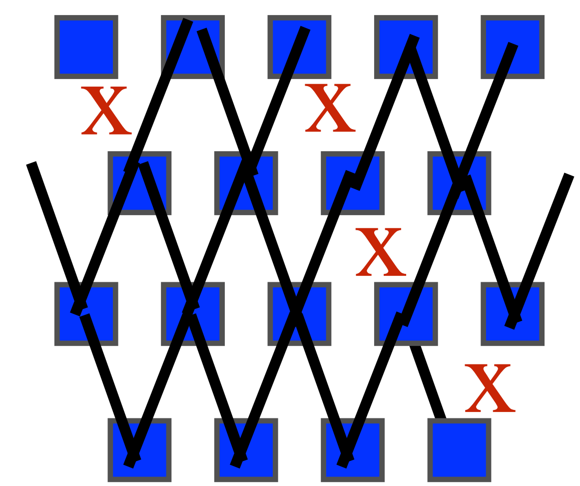



measurements "cut" it apart

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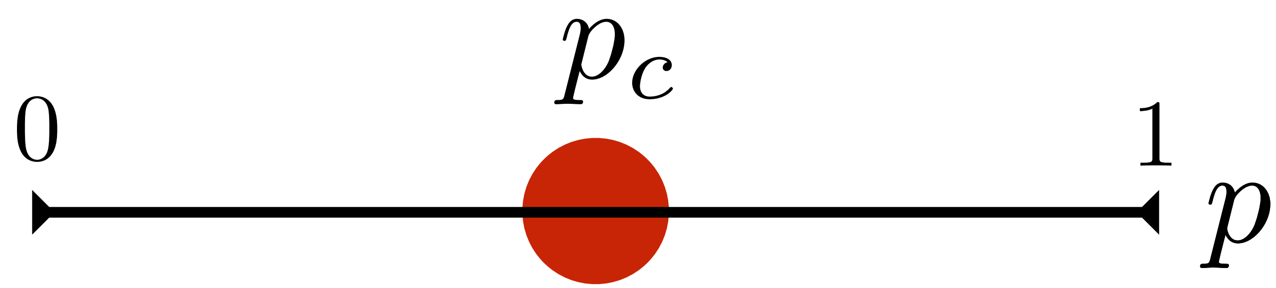


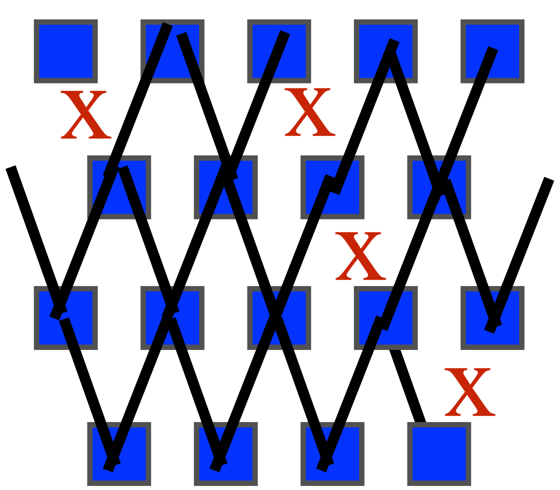

wave function collapse

no entanglement

**volume** law

**area** law

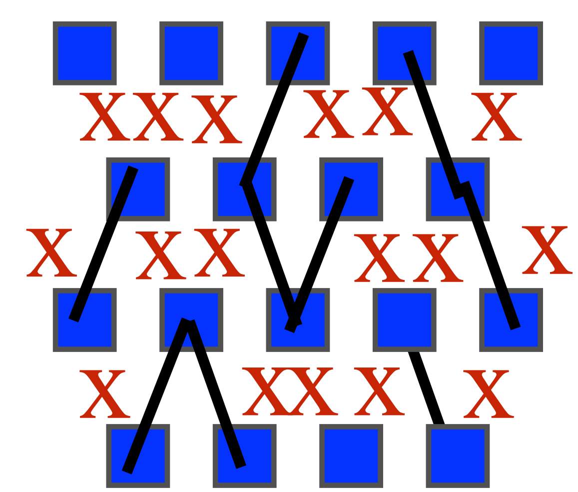
connected network



$S_n(L, t \to \infty) \sim L$

Jian, You, Vasseur, Ludwig PRB (2020)

disconnected network



$S_n(L, t \to \infty) \sim \text{const}$

Skinner, Ruhman, Nahum PRX (2019)

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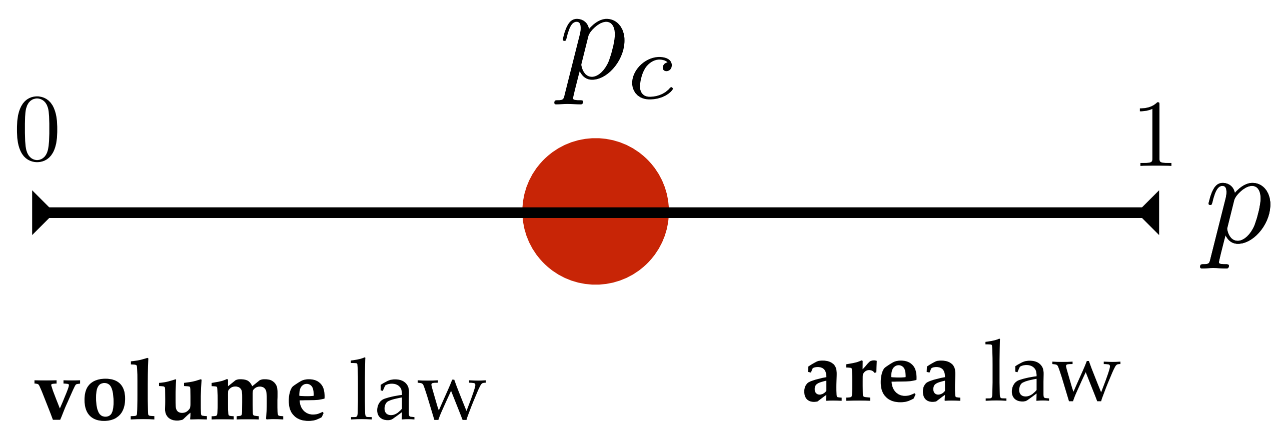
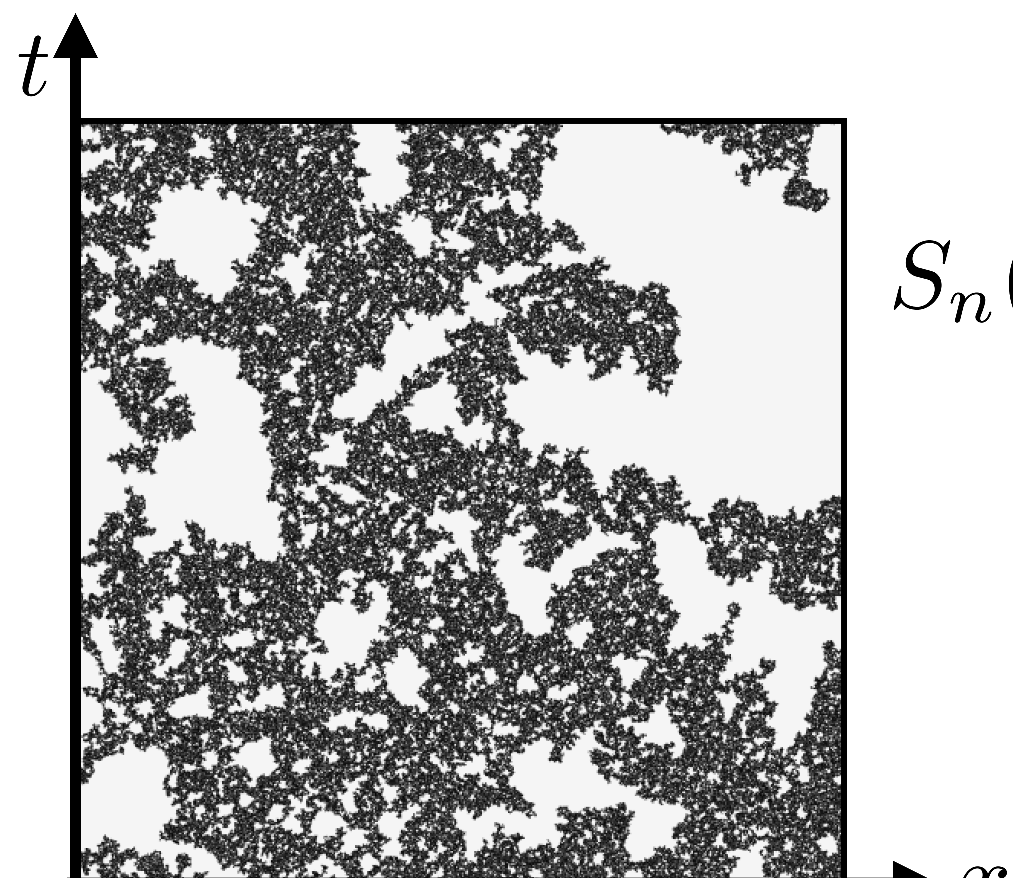



wave function collapse no entanglement

$$p = p_c$$

This predicts the transition is in the **bond-percolation universality class**

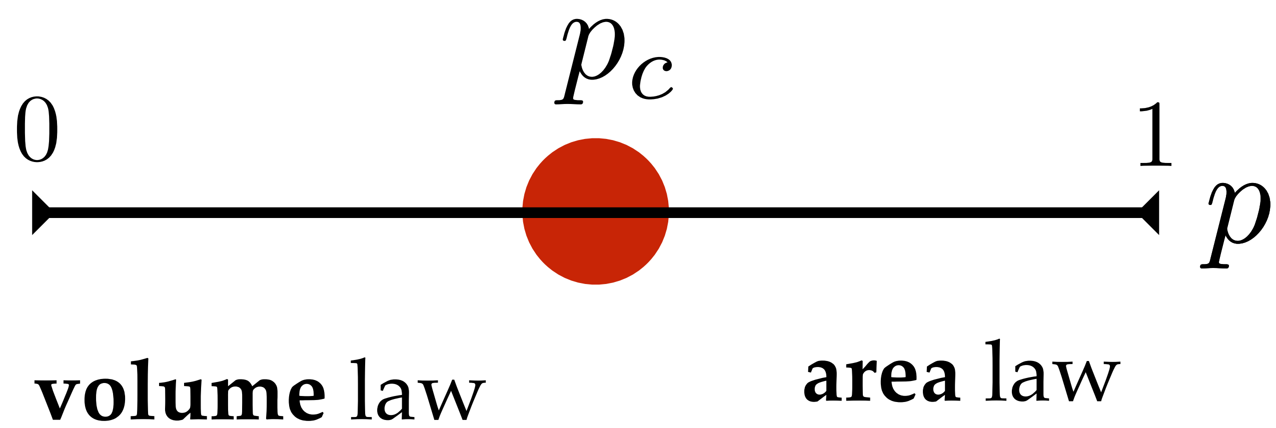



$$S_n(L, t \to \infty) \sim \log L$$

*Introduction to Percolation Theory*
Stauffer and Aharony

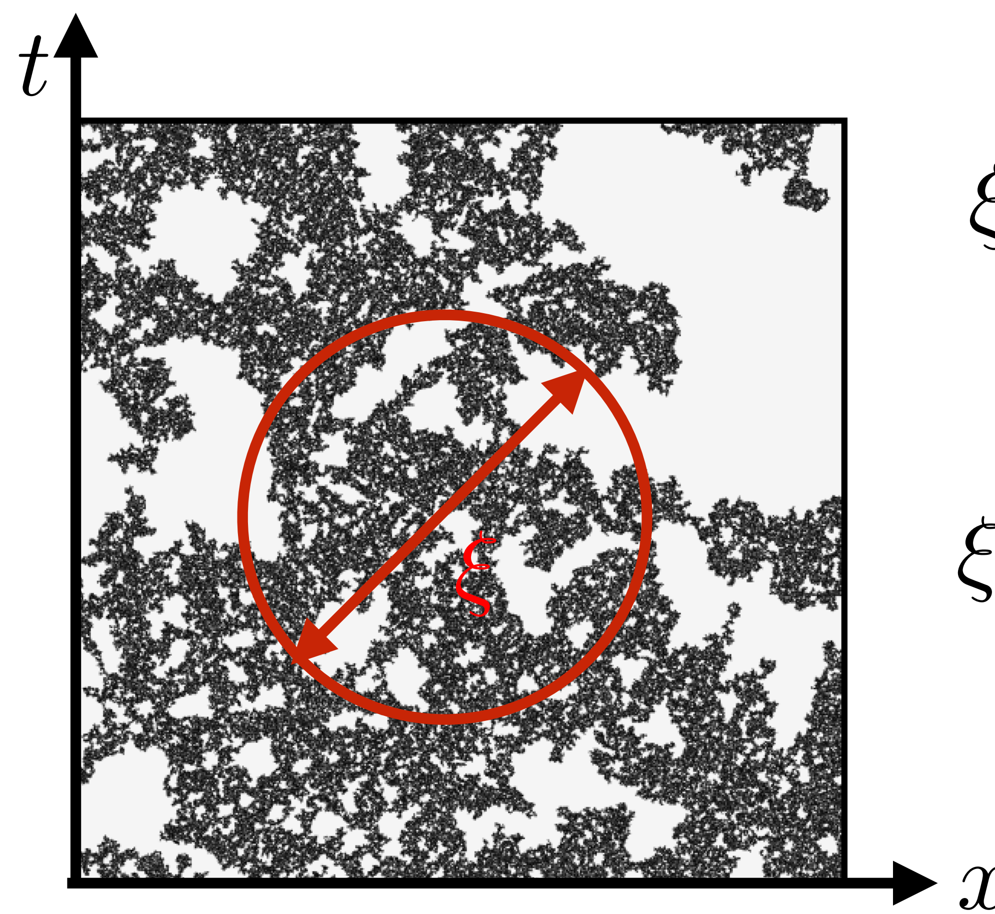
# Controlled Limits: Large Local Hilbert Space

Controlled Limits:  $q \to \infty$ or $n = 0$




wave function collapse
no entanglement

$p = p_c$




$\xi \sim |p - p_c|^{-\nu}$

$\nu = 4/3$

$\xi_t \sim \xi \qquad z = 1$

emergent Lorentz invariance!

This predicts the transition is in the **bond-percolation universality class**

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# Controlled Limits: Large Local Hilbert Space

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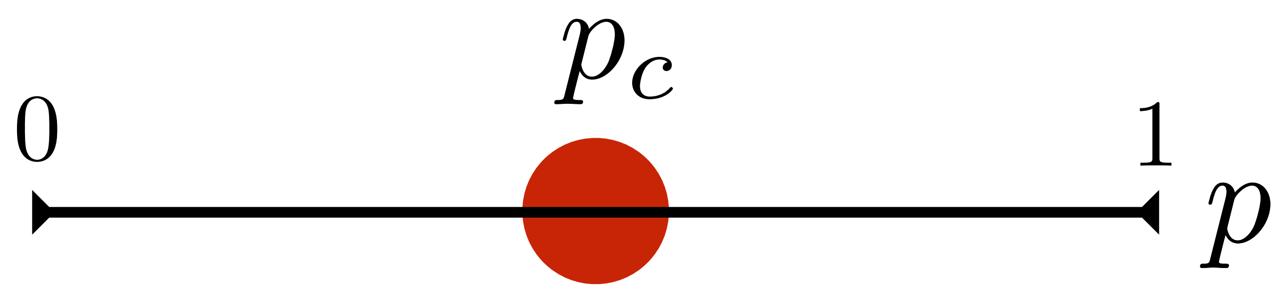
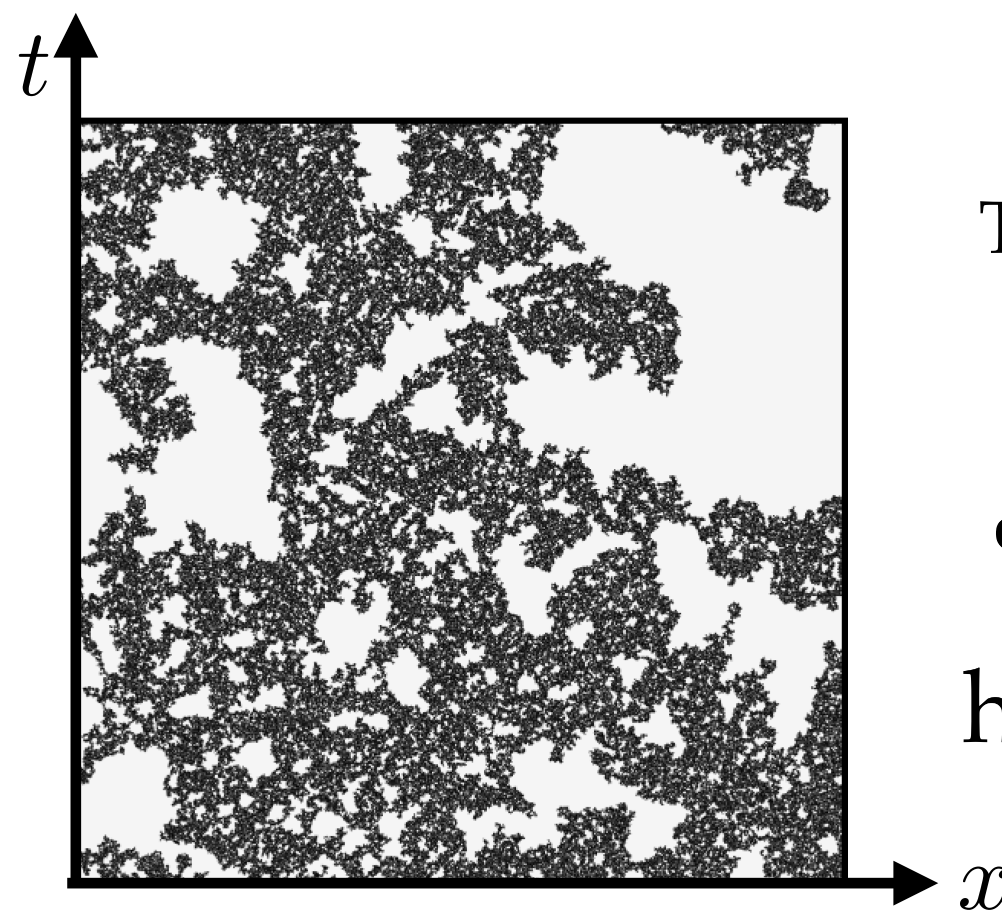



wave function collapse no entanglement

**volume** law

**area** law

$p = p_c$




This predicts the transition is in the **bond-percolation universality class**

*Introduction to Percolation Theory*
Stauffer and Aharony

This predicts the **critical point** described by a **logarithmic conformal field theory**

has a central charge

$c = 0$

# Controlled Limits: Stabilizers

Numerics on **stabilizer circuits** can simulate the circuit in polynomial time (but is it really quantum?)

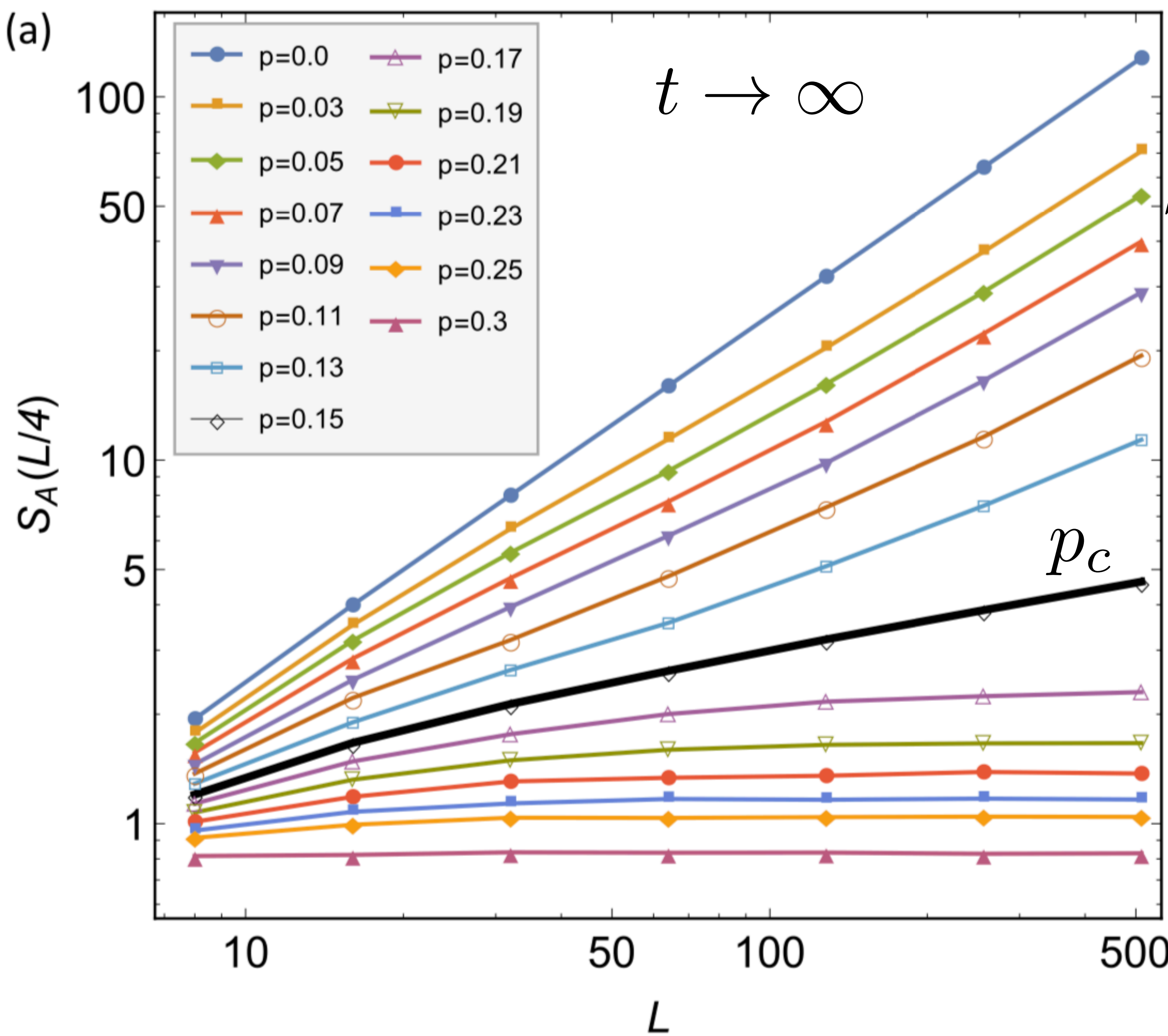
choose from the Clifford group

converts a Pauli string to a Pauli string — NO superpositions of Pauli strings are generated

Li, Chen, Fisher PRB (2018)

# Controlled Limits: Stabilizers

Numerics on **stabilizer circuits** can simulate the circuit in polynomial time (but is it really quantum?)



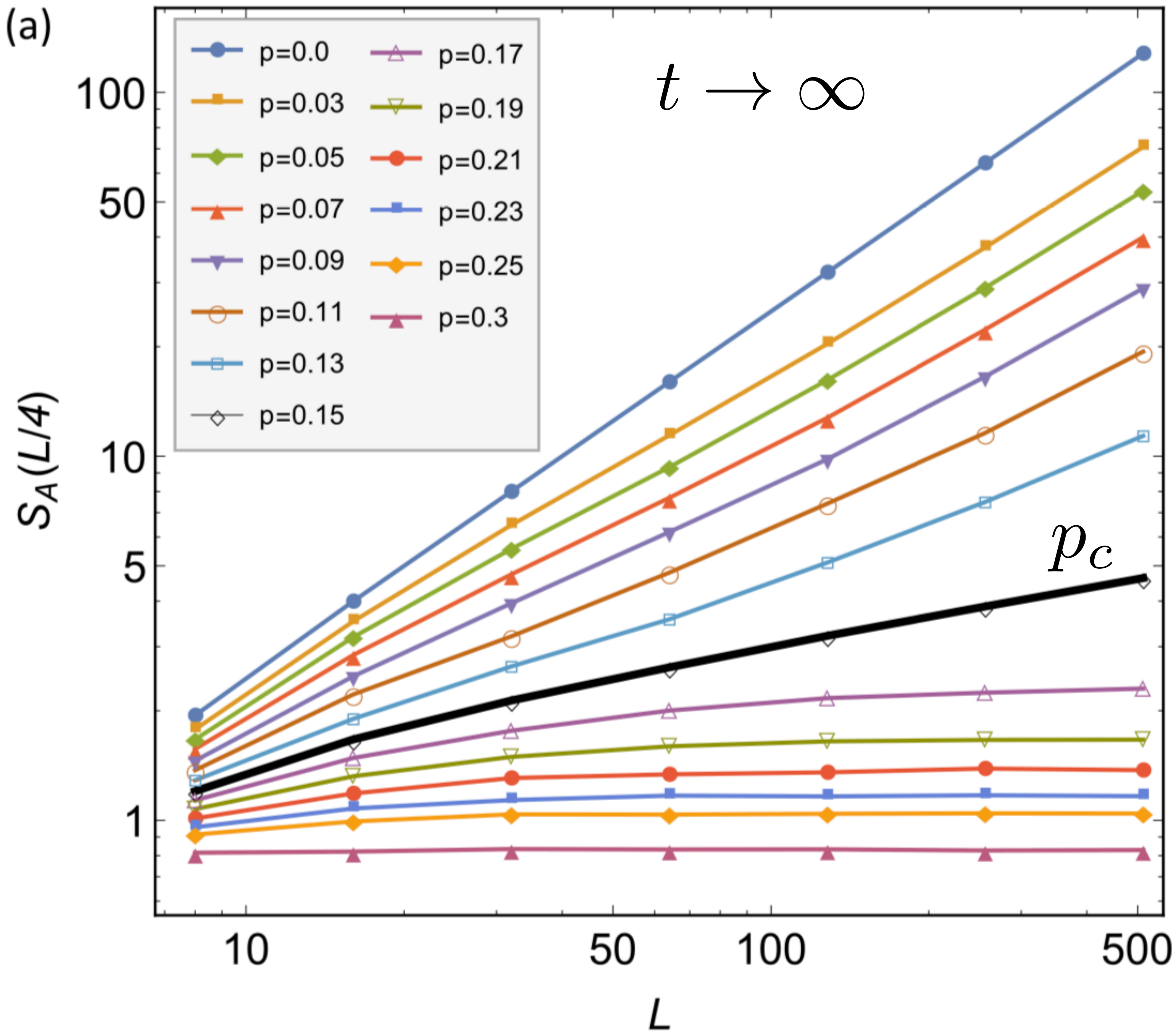

$S \sim L$ volume law

$S \sim \text{const}$ area law

Li, Chen, Fisher PRB (2018)

# Controlled Limits: Stabilizers

Numerics on **stabilizer circuits** can simulate the circuit in polynomial time (but is it really quantum?)




$p_c \approx 0.15$

At $p_c$ $\quad S_n \sim A \log L$

$A \approx 1.6 > A_{\text{perc}} \approx 0.55$

Li, Chen, Fisher PRB (2018)

# Fully Quantum Measurement Transition

Consider ***q*=2**

Haar Gates

**Have to use numerics**

i.e. random *U(4)* matrices

Gates are represented as sparse matrices in many body Hilbert space, use sparse matrix-vector operations to perform time evolution. Measurements are efficient, project and normalize.

Does the fully quantum problem belong to a different universality class?

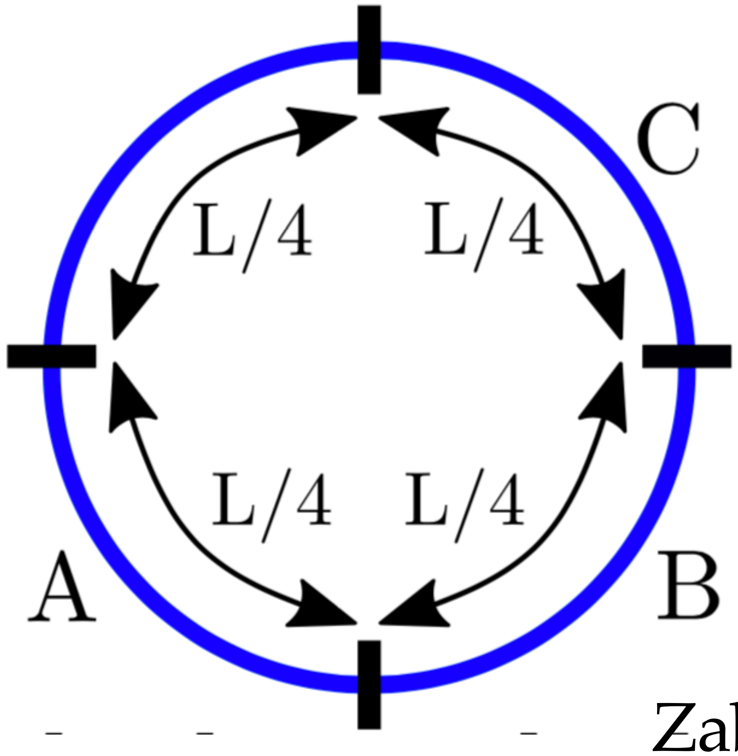
# New approach to the Critical Properties

Scaling of the entanglement entropy spoiled by the log$L$ for small $L$

Consider the tripartite mutual information

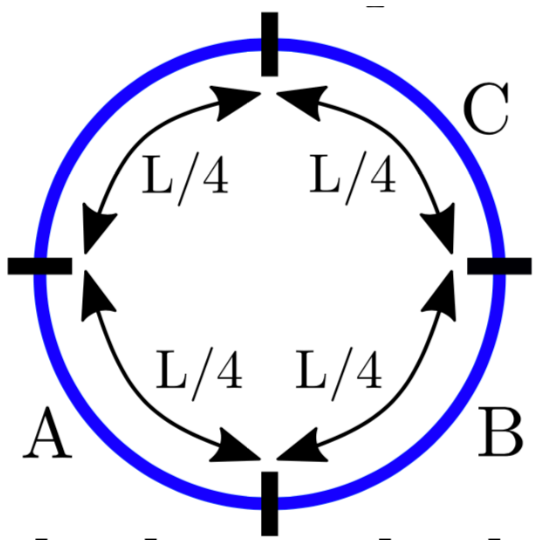
$$\mathcal{I}_{3,n}(A,B,C) \equiv S_n(A) + S_n(B) + S_n(C) - S_n(A\cup B) - S_n(A\cup C) - S_n(B\cup C) + S_n(A\cup B\cup C). \quad (1)$$

Kitaev and Preskill PRL (2006)   Levin and Wen PRL (2006)

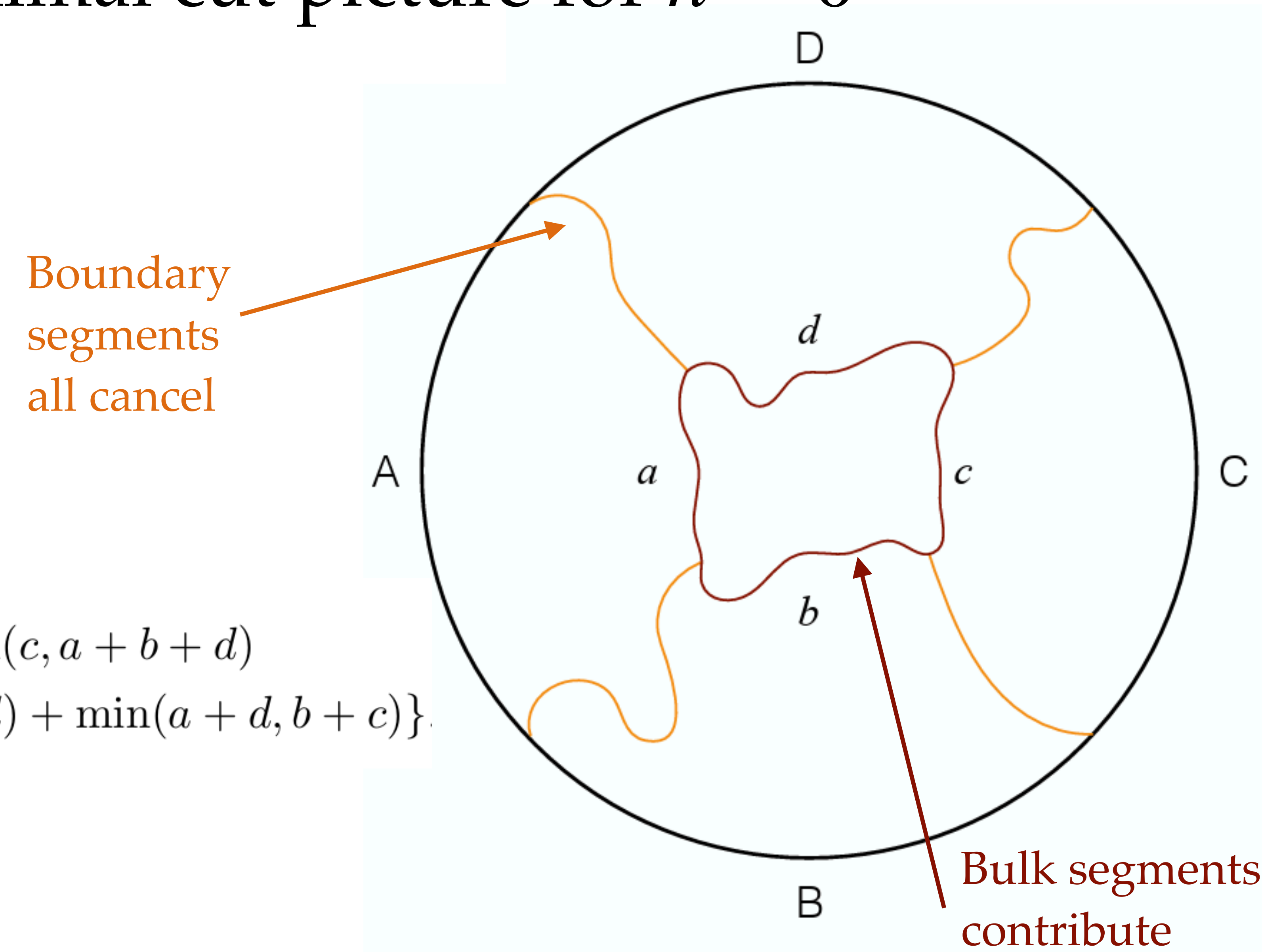



Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

# Tripartite Mutual Information




## Minimal cut picture for $n = 0$




$\mathcal{I}_{3,n=0}$

$$= \min(a, b+c+d) + \min(b, a+c+d) + \min(c, a+b+d) - \{\min(a+b, c+d) + \min(a+c, b+d) + \min(a+d, b+c)\}.$$

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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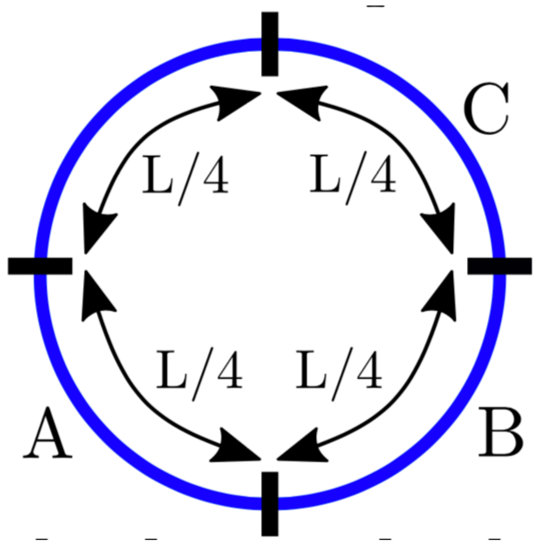


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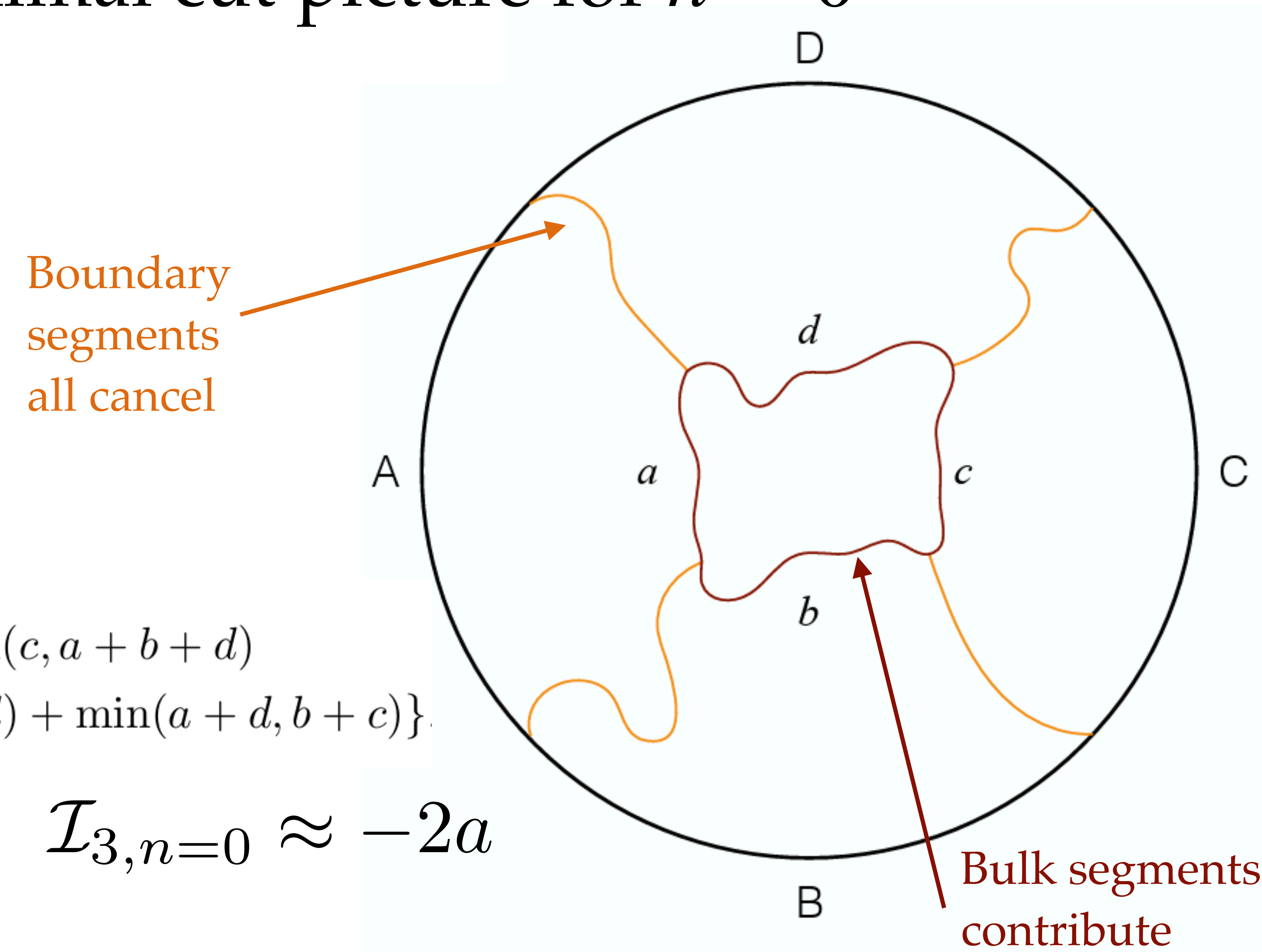
In the volume law phase, $a \approx b \approx c \approx d$ $\quad \mathcal{I}_{3,n=0} \approx -2a$




Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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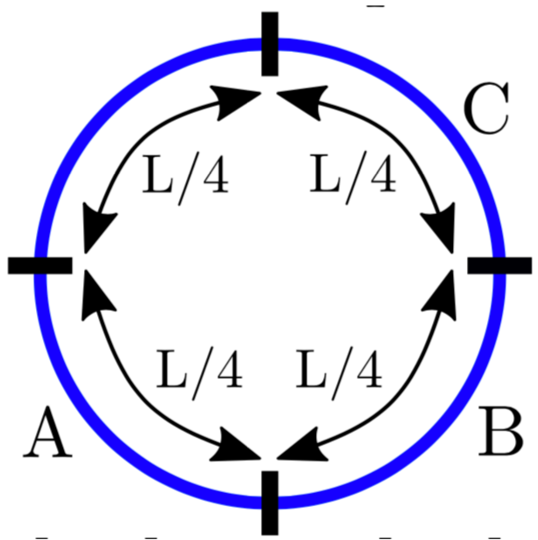


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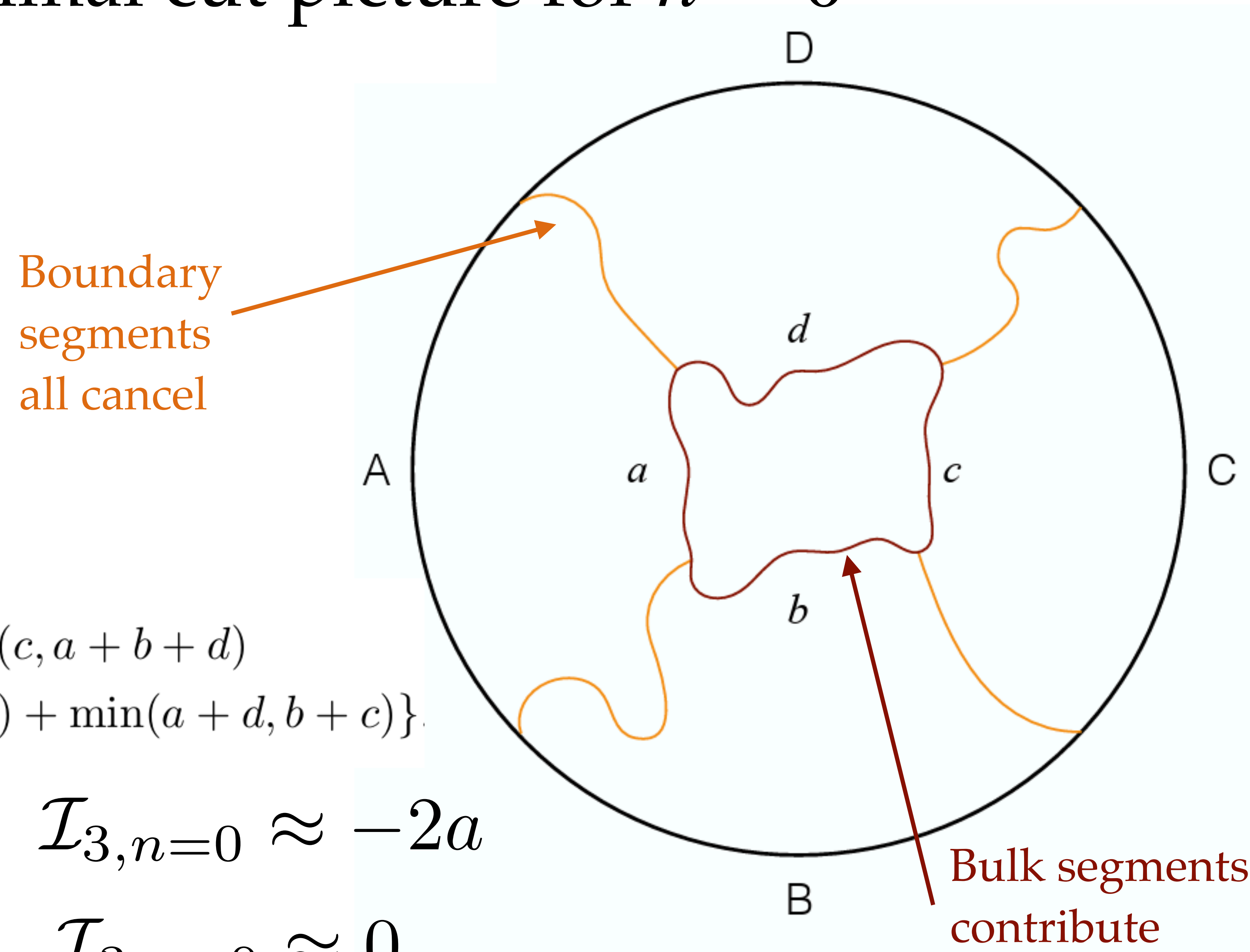


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Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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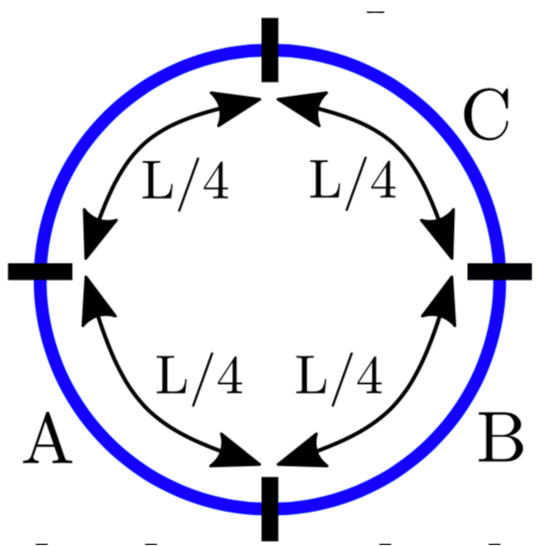


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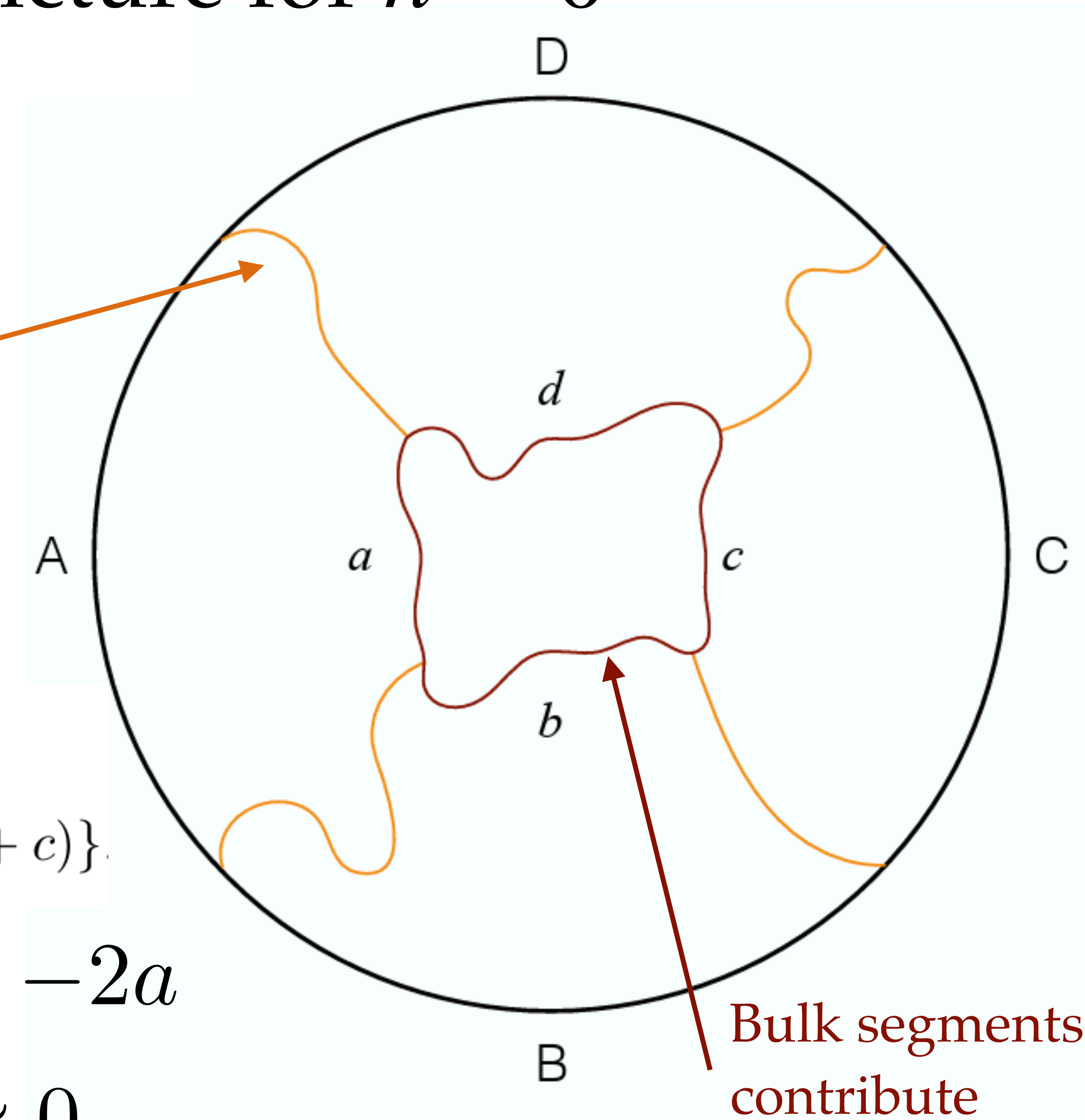


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At the critical point , bulk contribution is $O(1)$ $\quad \mathcal{I}_{3,n=0} \approx \text{const}$

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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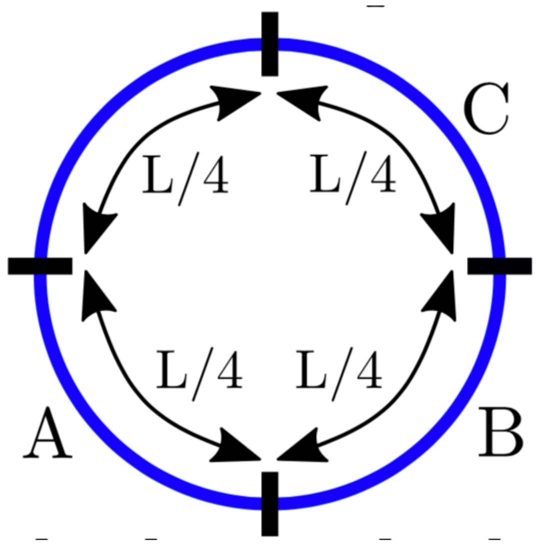


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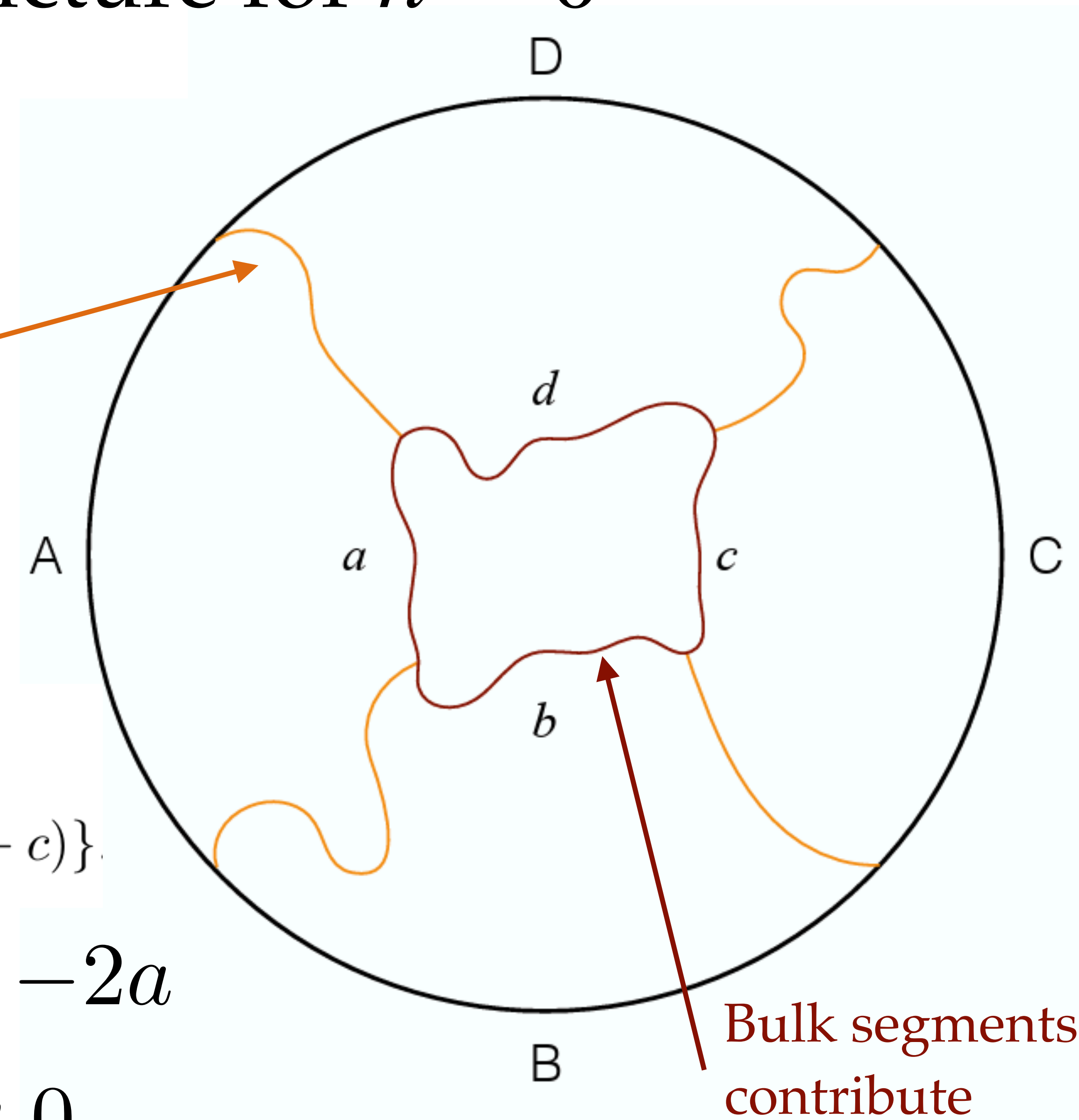


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$$\mathcal{I}_{3,n=0} \leq 0$$

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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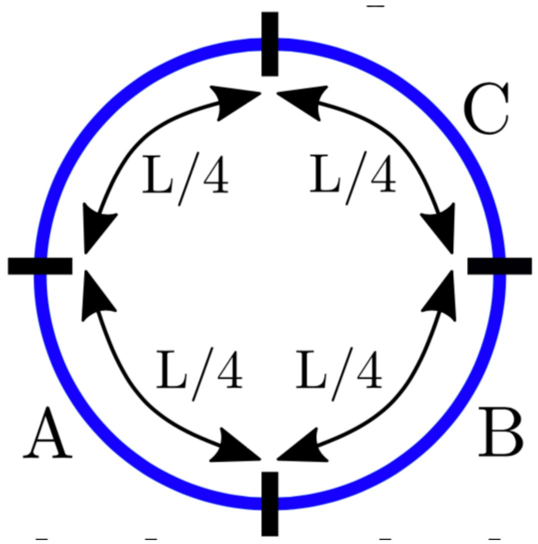


Cancels the log$L$ at criticality!

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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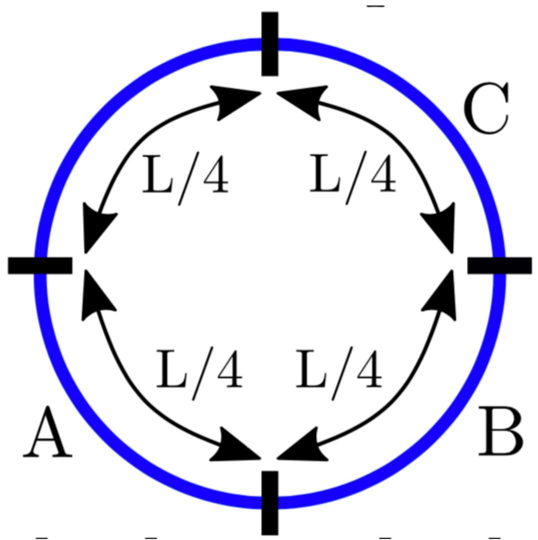


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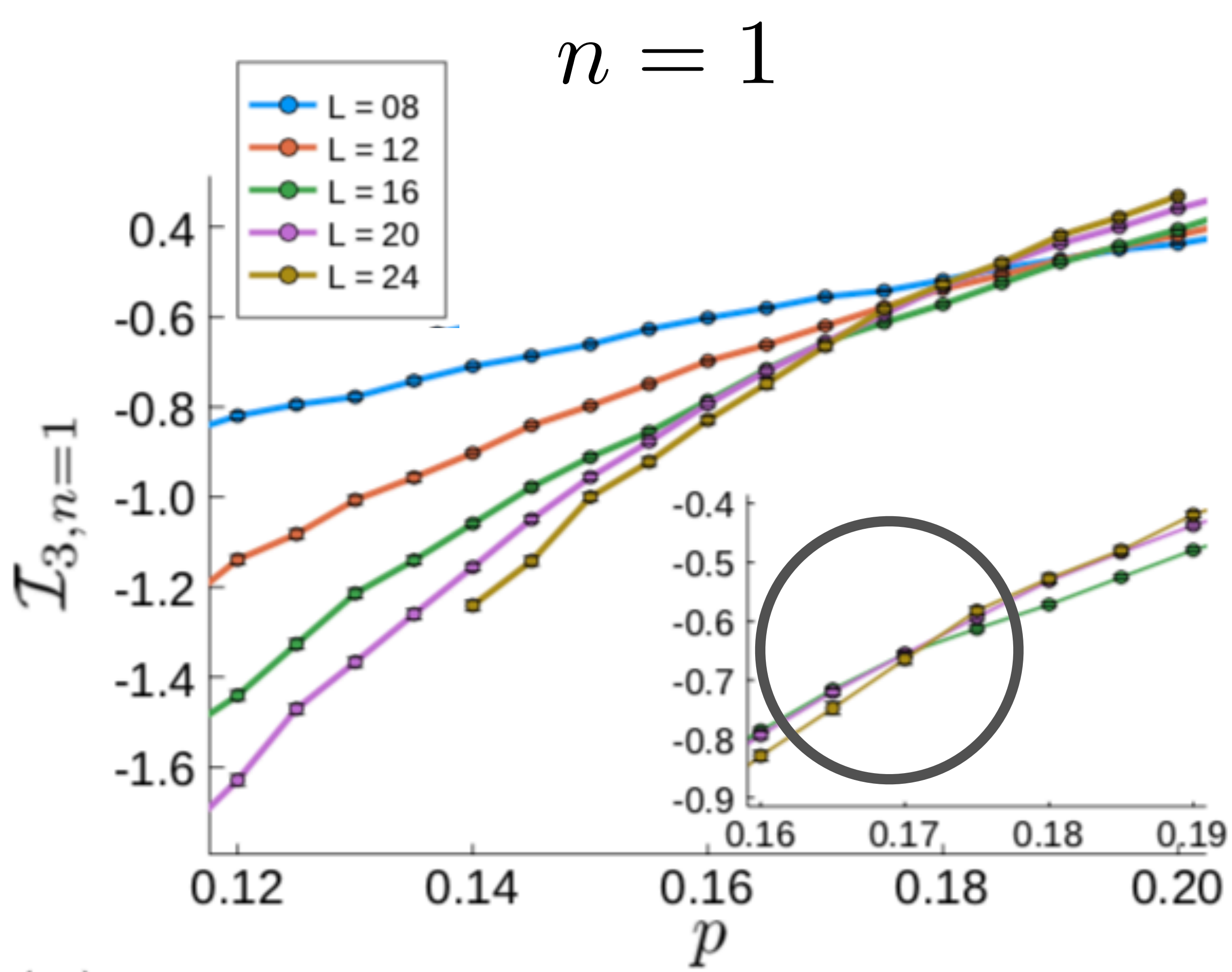


Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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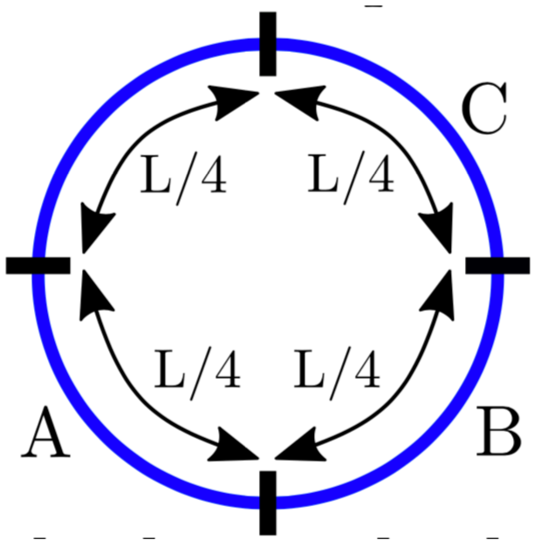
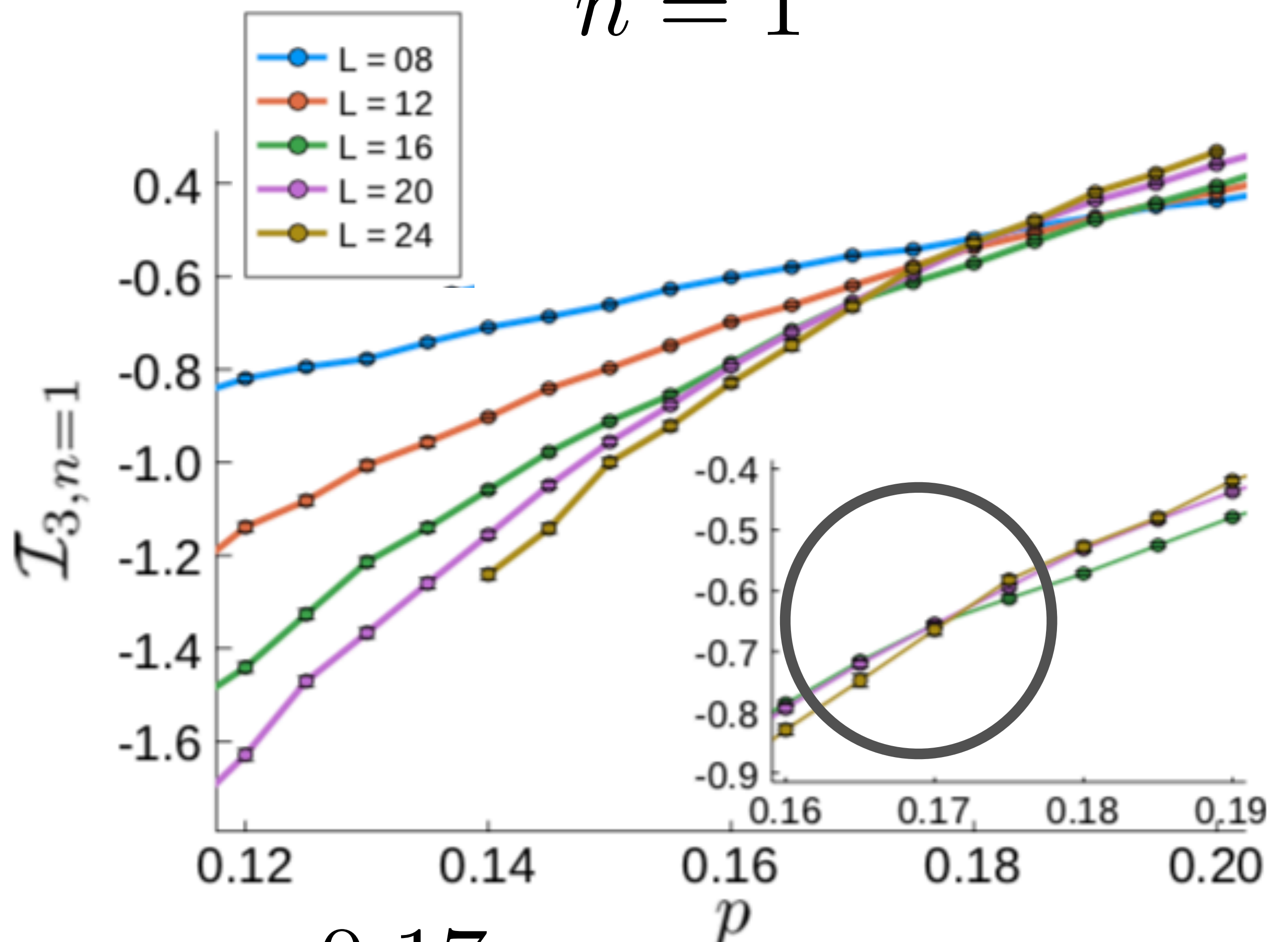
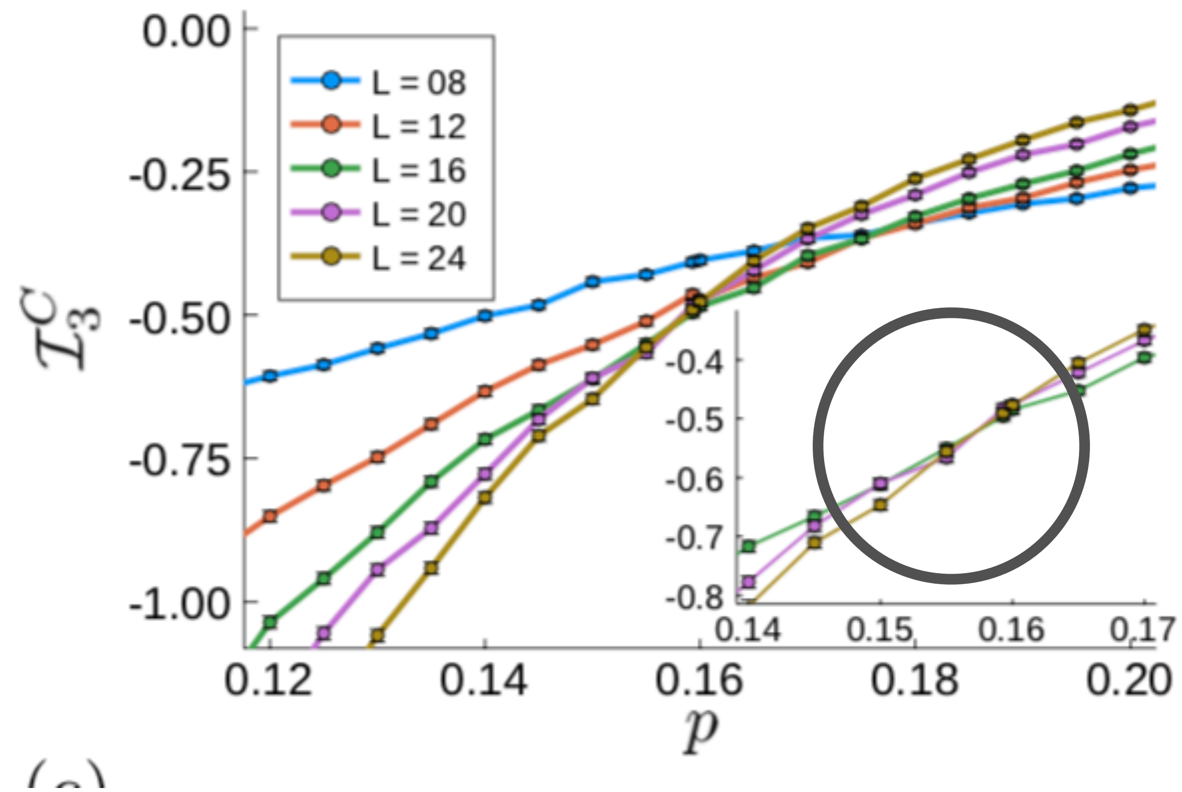



Cancels the log$L$ at criticality!

Compare with **Clifford Circuits**

$n = 1$

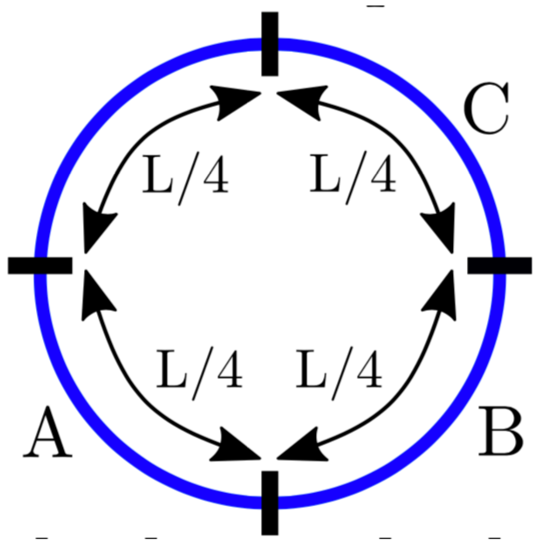


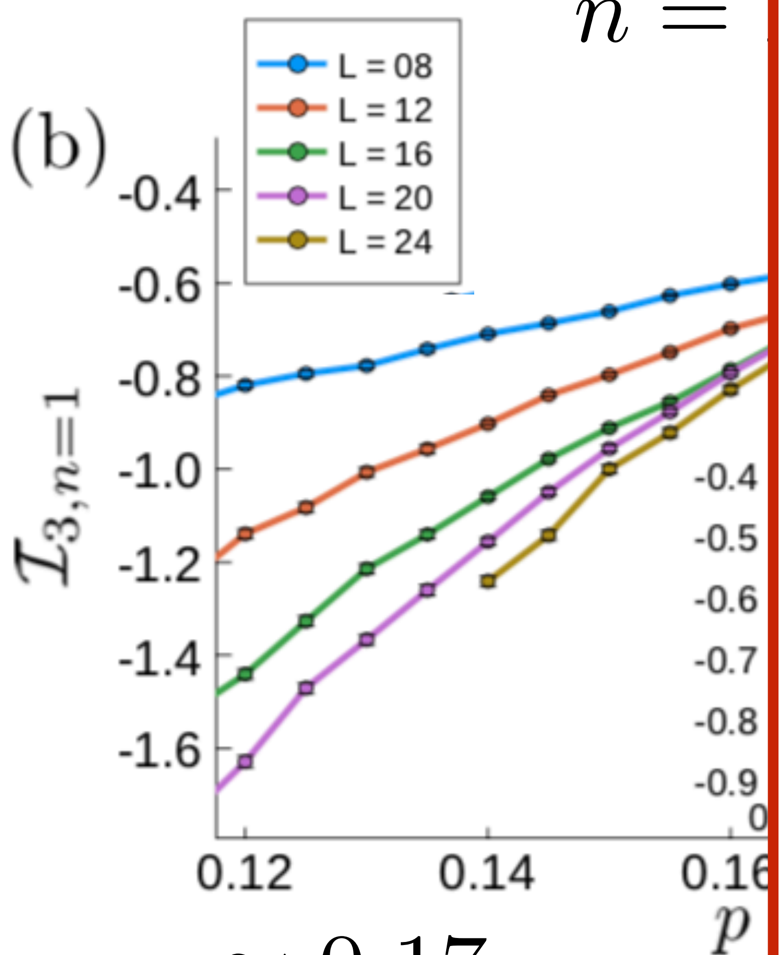

$p_c \approx 0.17$

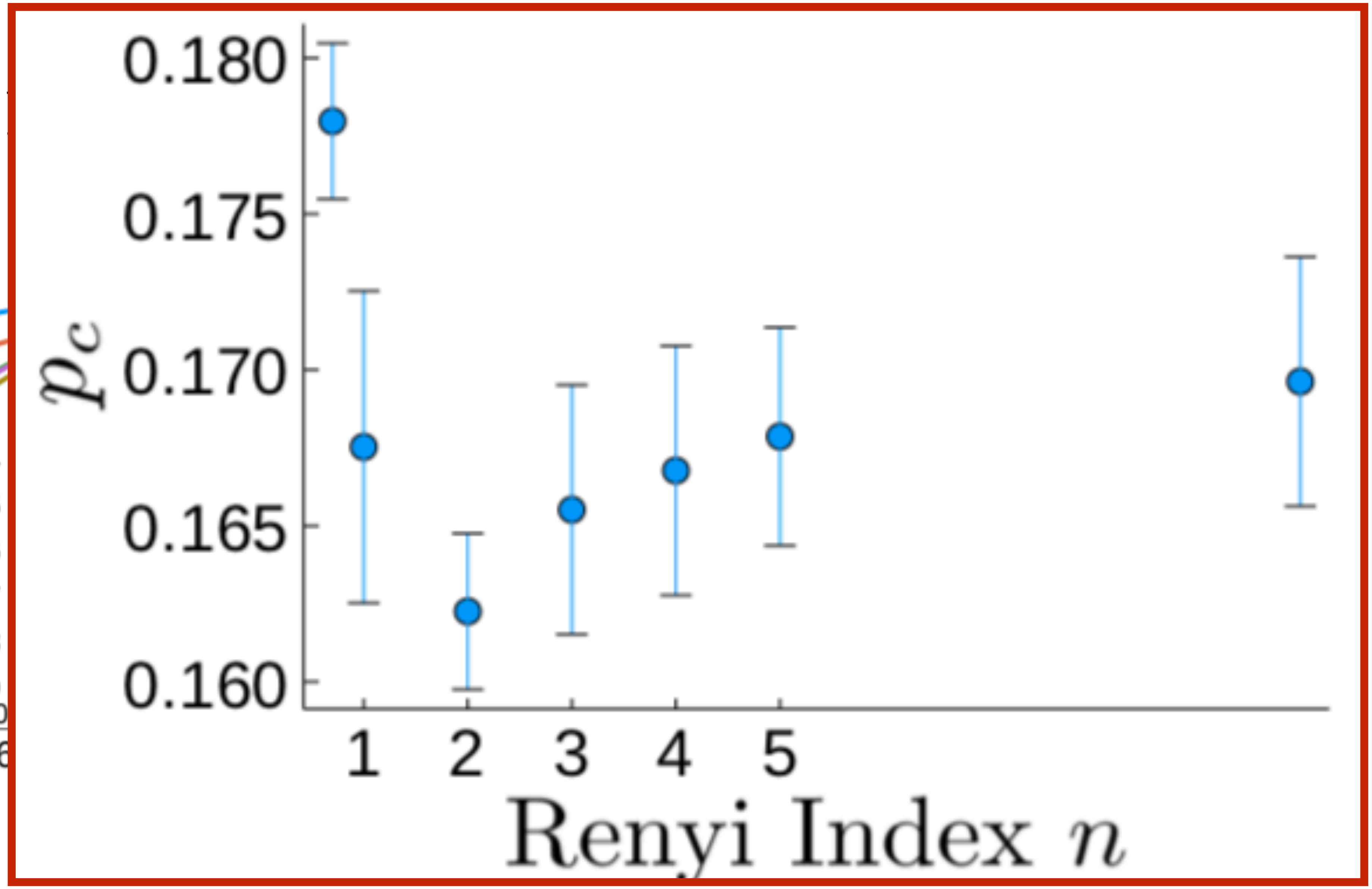
$p_c \approx 0.154$

# Tripartite Mutual Information




$n =$

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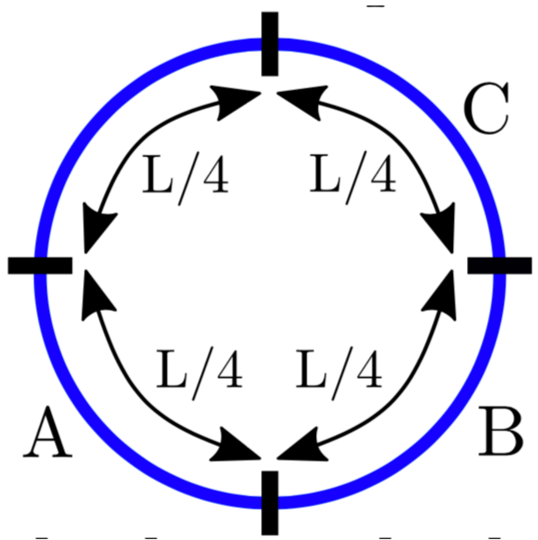
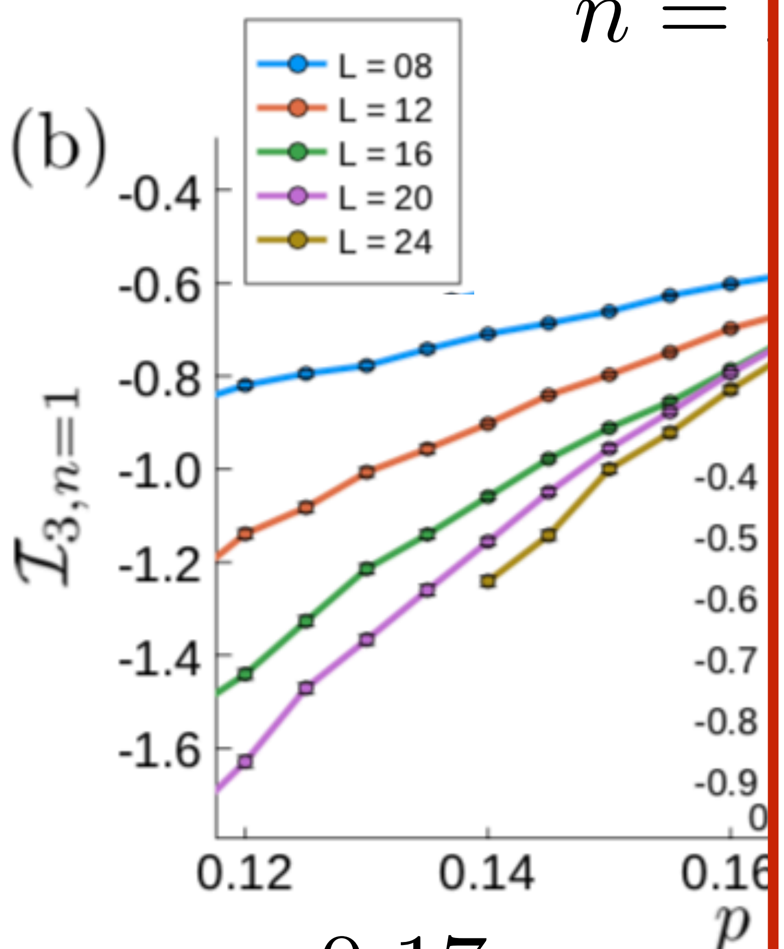



$n > 1$

$$S_\infty \leq S_n \leq \frac{n}{n-1} S_\infty$$

$\longrightarrow$ $p_c(n) = p_c$

$n =$

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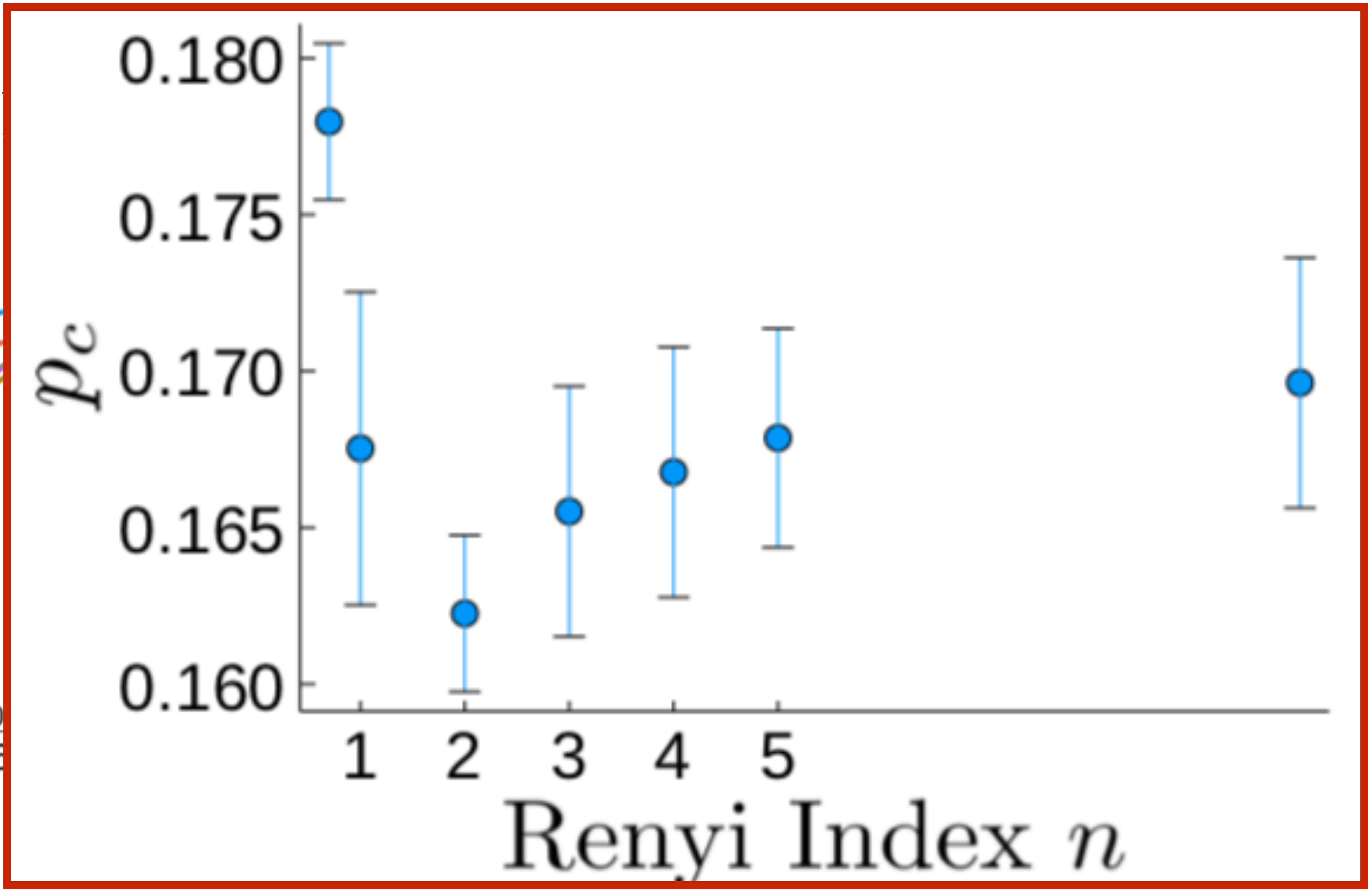
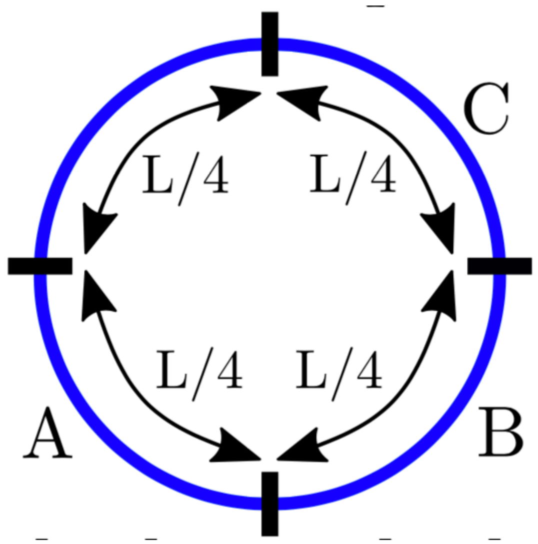


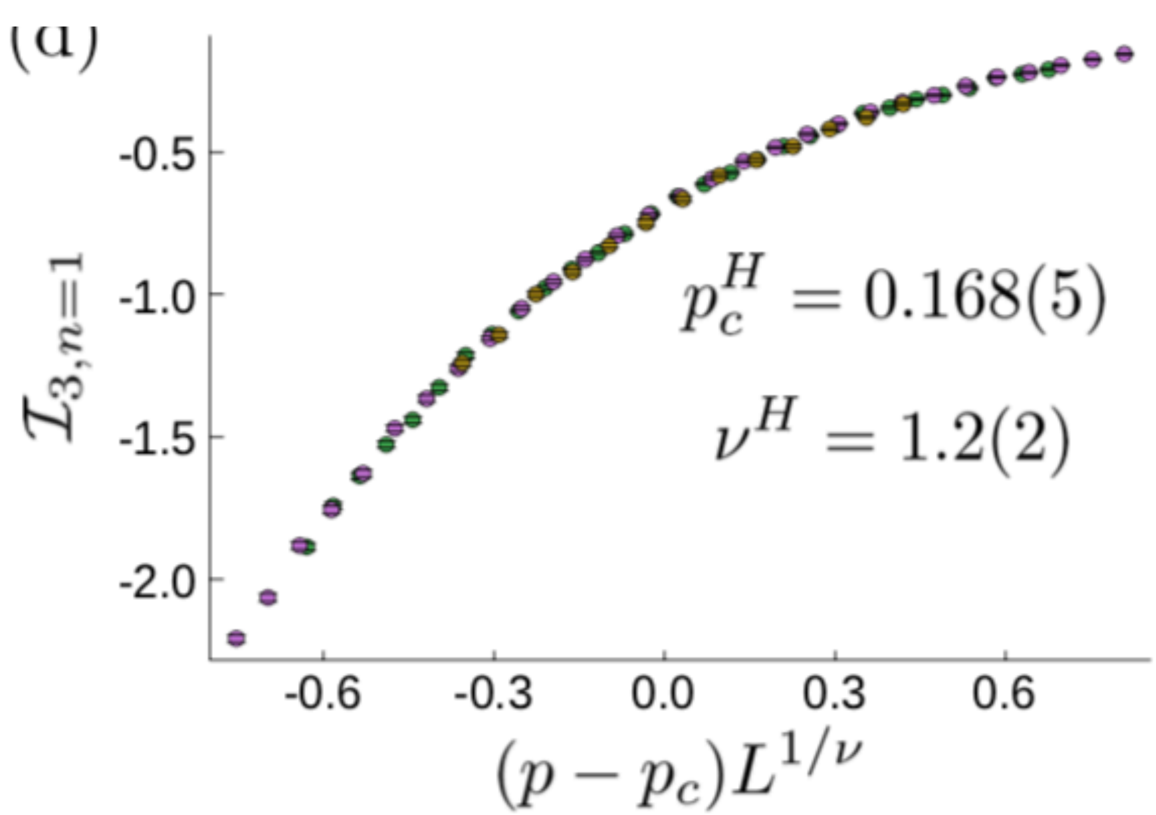
# Scaling Collapse

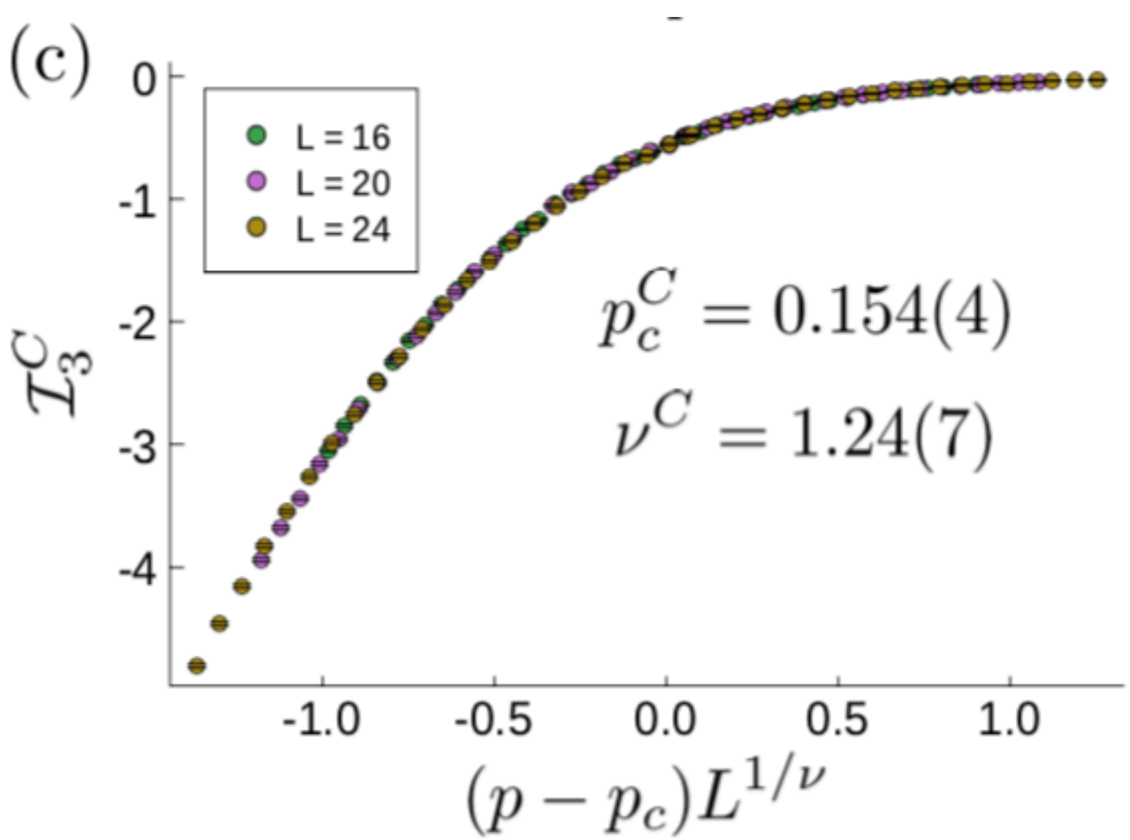



$$\mathcal{I}_{3,n=1}(p, L) \sim f(L^{1/\nu}(p - p_c))$$

$n = 1$

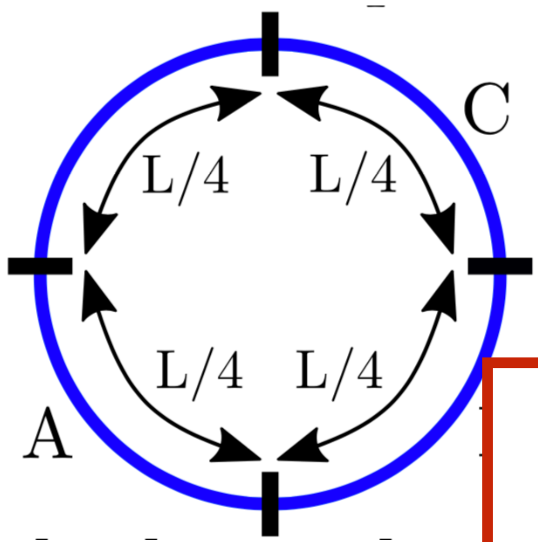
Compare with **Clifford Circuits**




Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

# Scaling Collapse for different $n$

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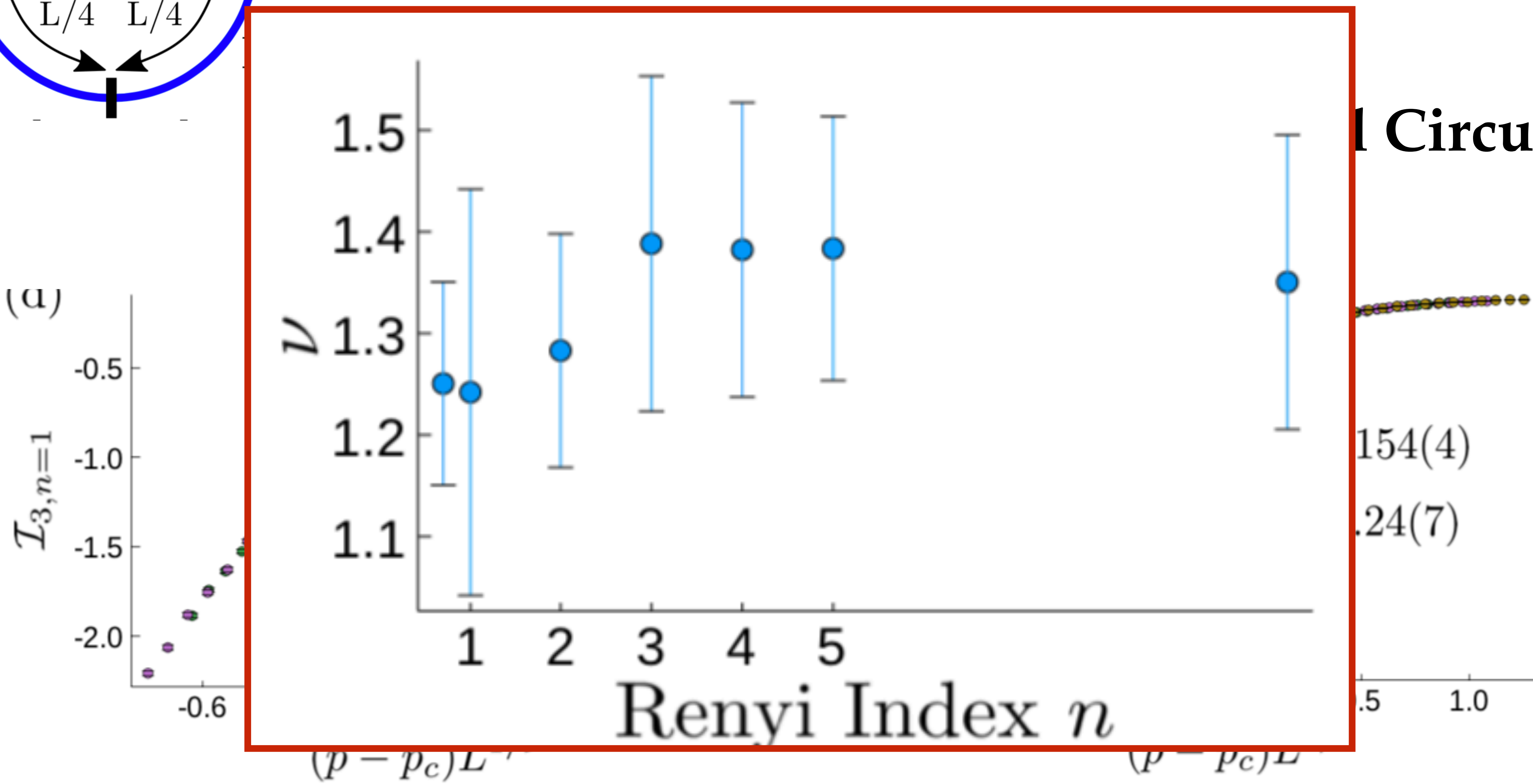
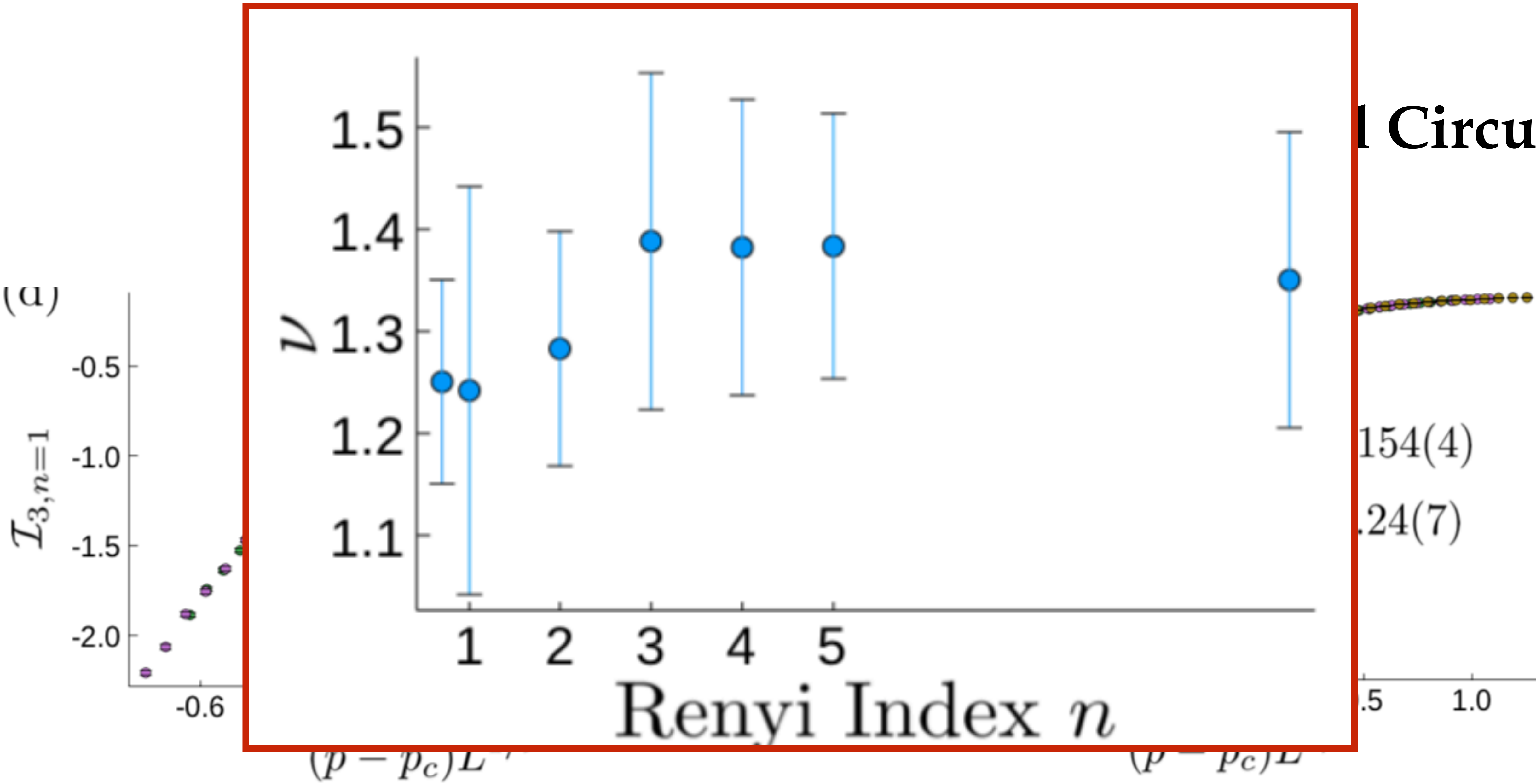


# Scaling Collapse for different $n$

$$S_\infty \le S_n \le \frac{n}{n-1} S_\infty \longrightarrow \xi_\infty/\xi_n \to \text{const} \longrightarrow \nu_n = \nu$$

$$\xi_n \sim |p - p_c|^{-\nu_n}$$

$n > 1$




Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP arXiv (2019)

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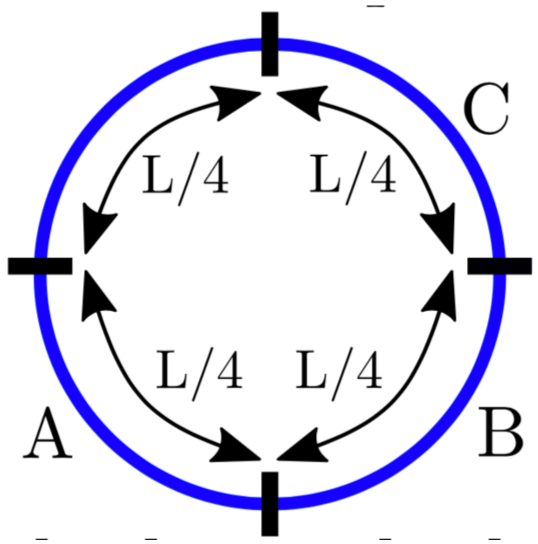


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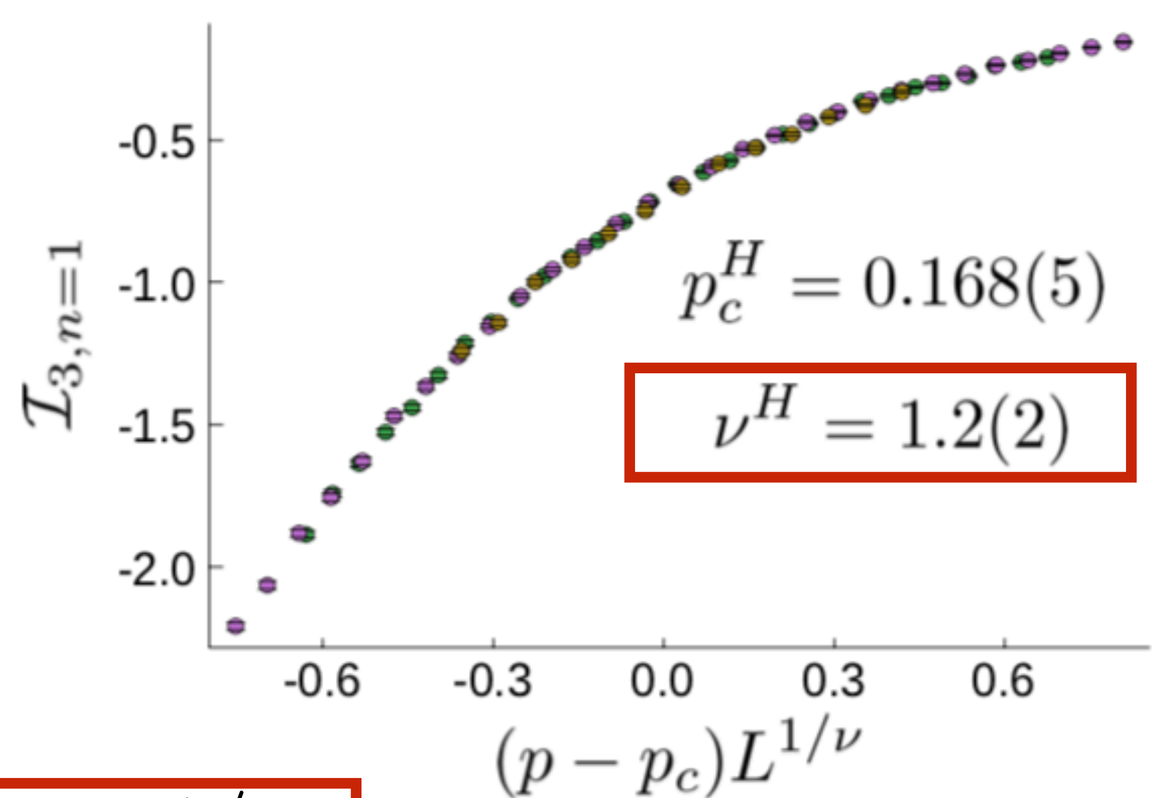
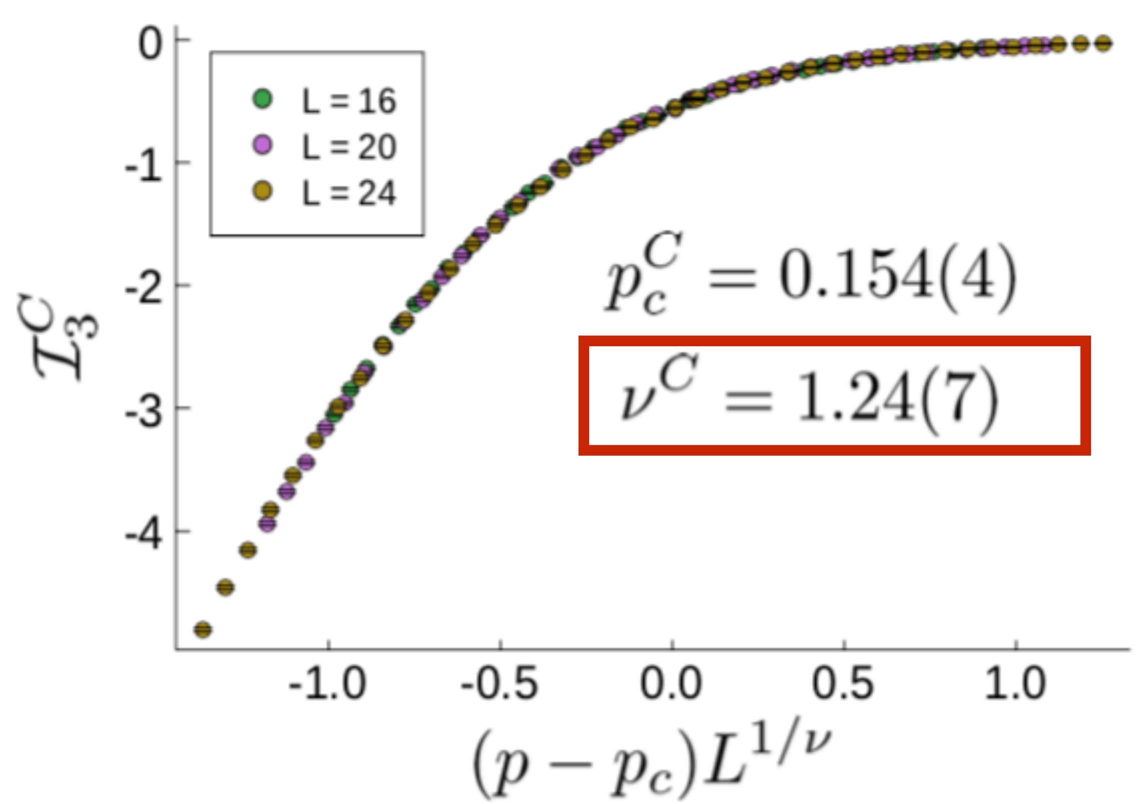


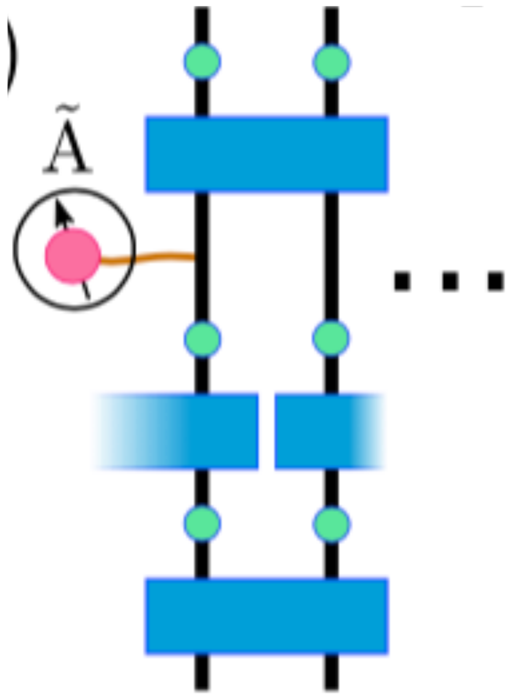




$\nu_P = 4/3$

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

# Order Parameter

$\tilde{A} = (r, t_0)$




Entangle one ancilla qubit in a Bell state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\Uparrow\downarrow\rangle - |\Downarrow\uparrow\rangle)$$

and track its **entanglement entropy**

Gullans and Huse PRL (2020)

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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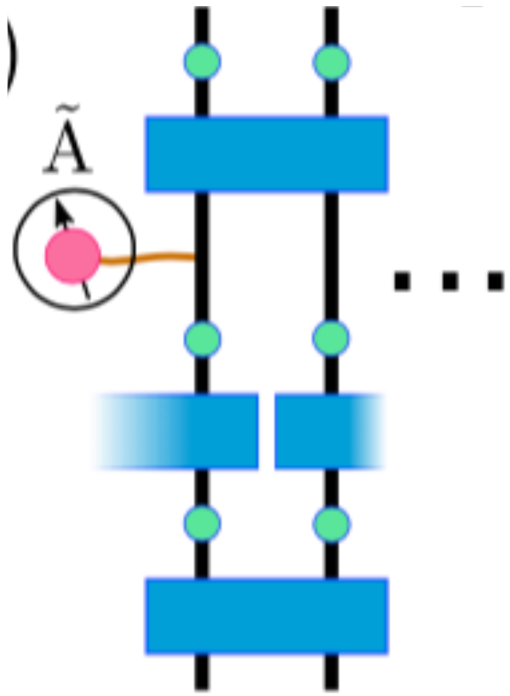


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"order parameter" = $S_1(\rho_R)$

reduced density matrix of ancilla

Gullans and Huse PRL (2020)

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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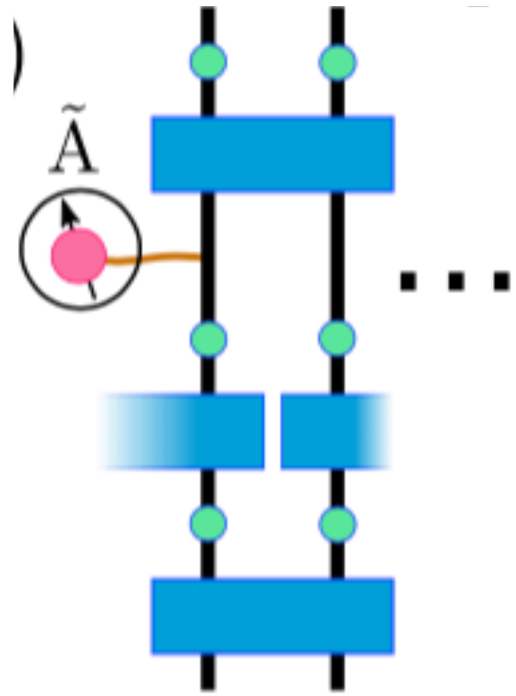


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remains **entangled** in the volume law phase

**vanishes** in the area law phase

Gullans and Huse PRL (2020)

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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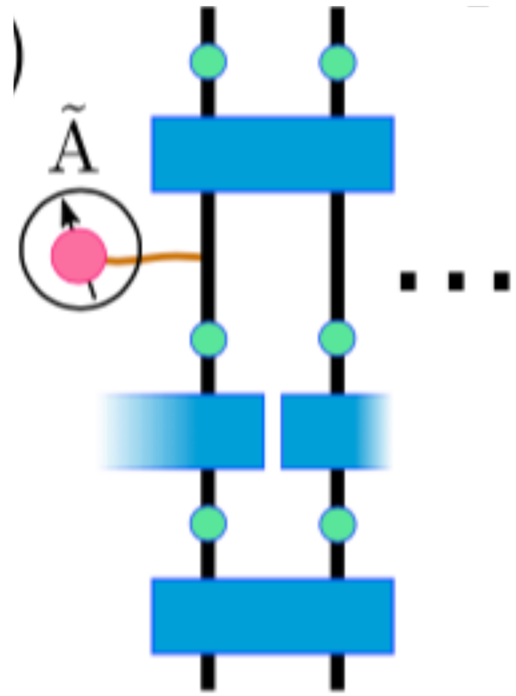


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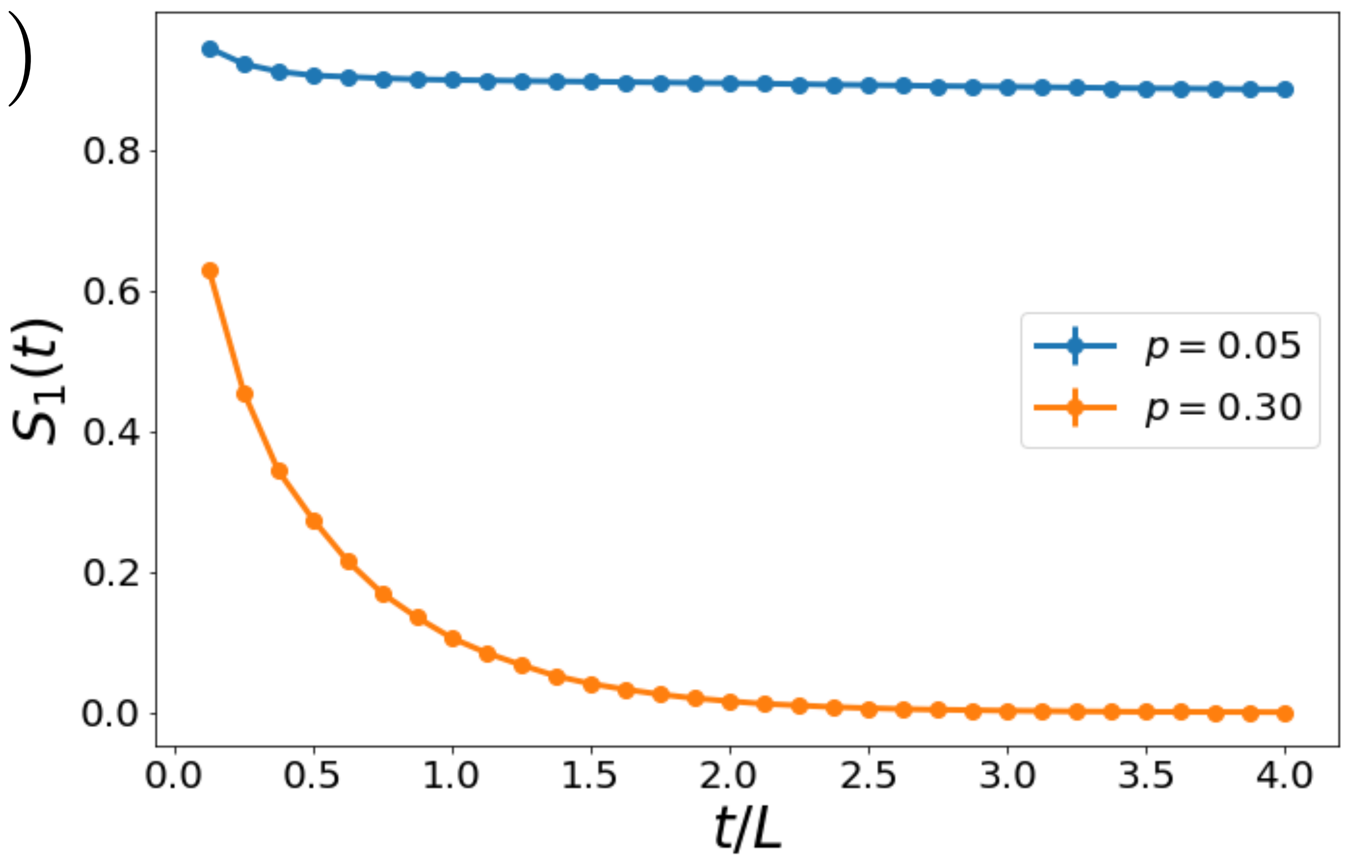
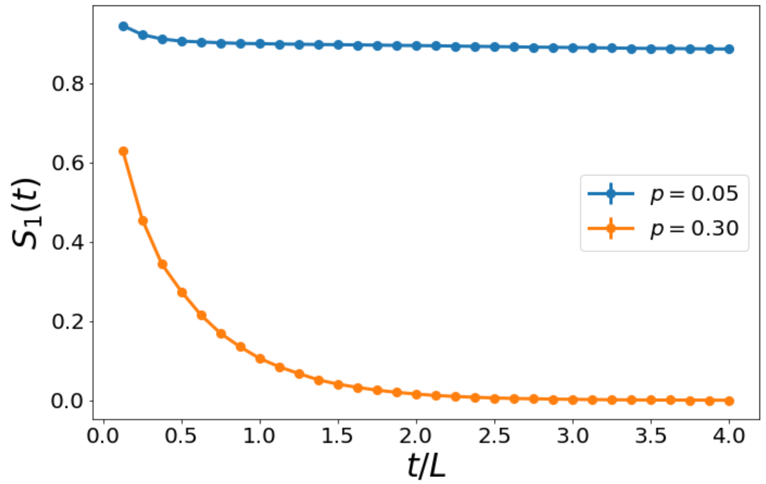
$p < p_c$ remains **entangled**

$p > p_c$ **vanishes**



# Order Parameter

Use the order parameter to determine $p_c$



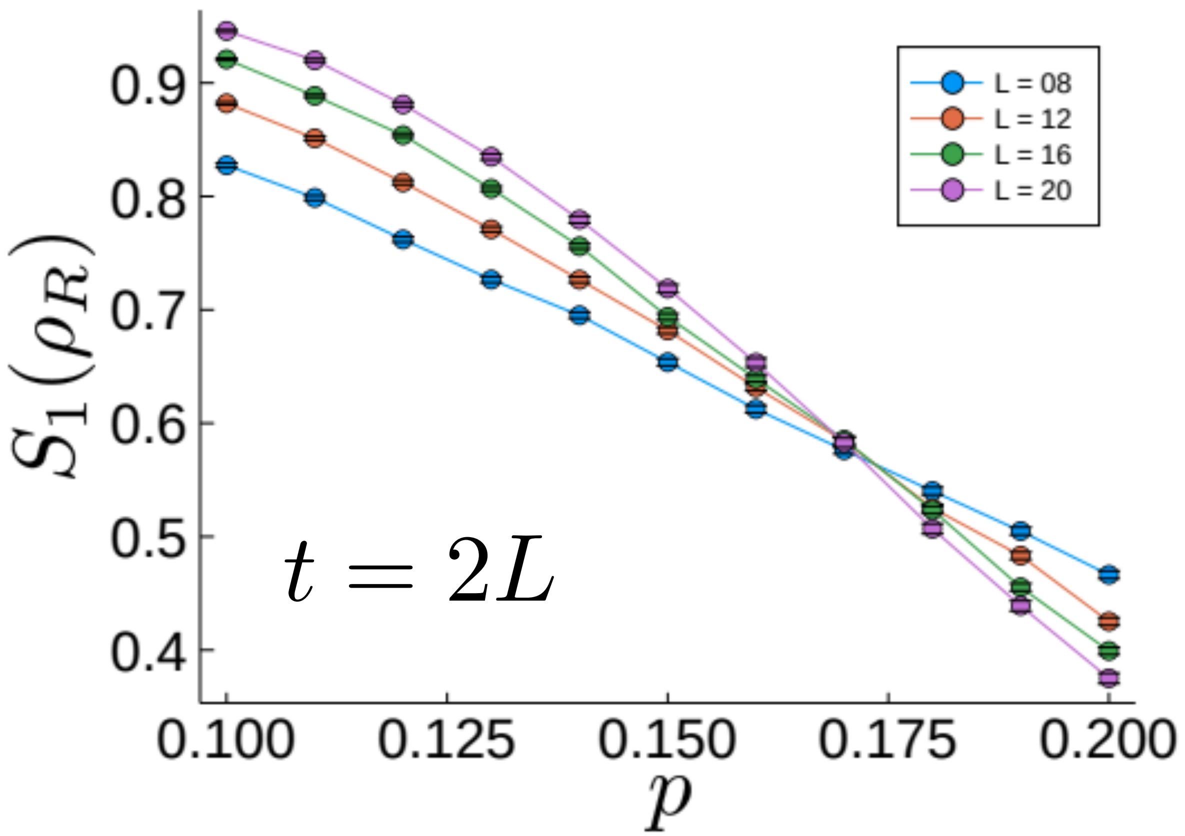

"order parameter" = $S_1(\rho_R)$

$p < p_c$ remains **entangled**

$p > p_c$ **vanishes**

**near $p_c$**

$$S_1(\rho_R) \sim F((p - p_c)L^{1/\nu}, t/L^z)$$



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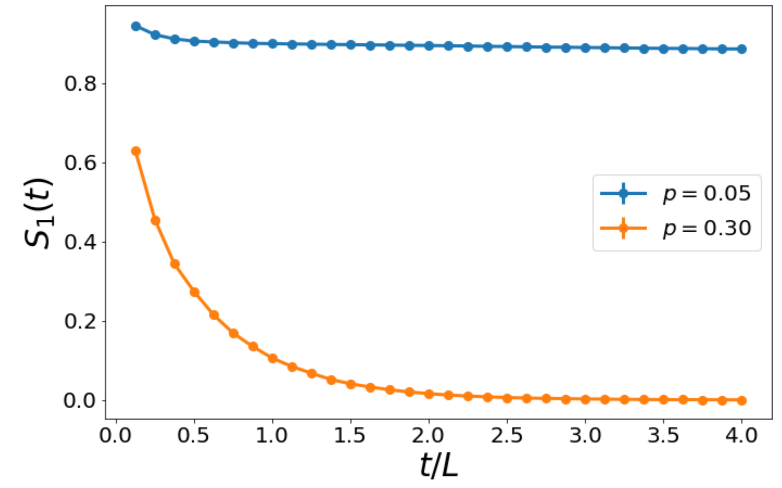


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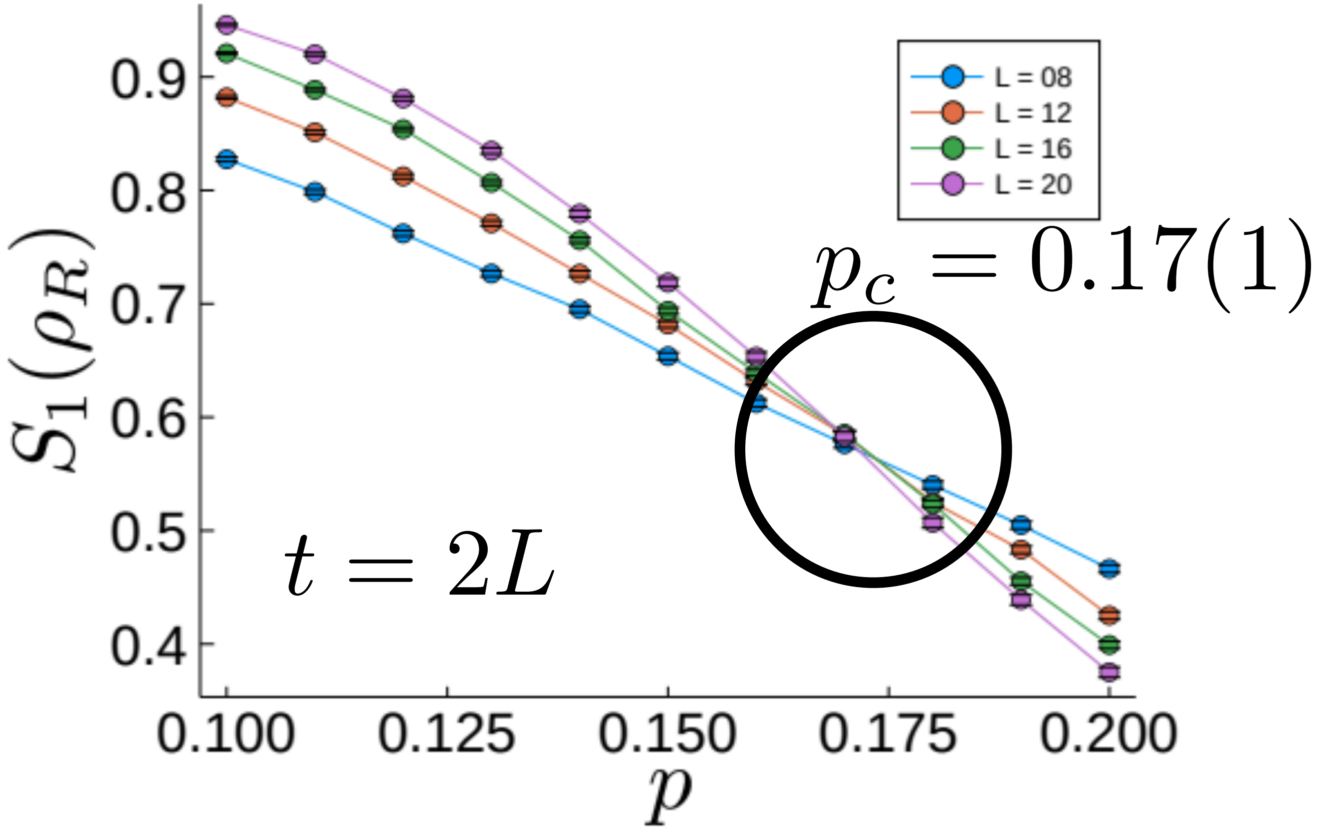
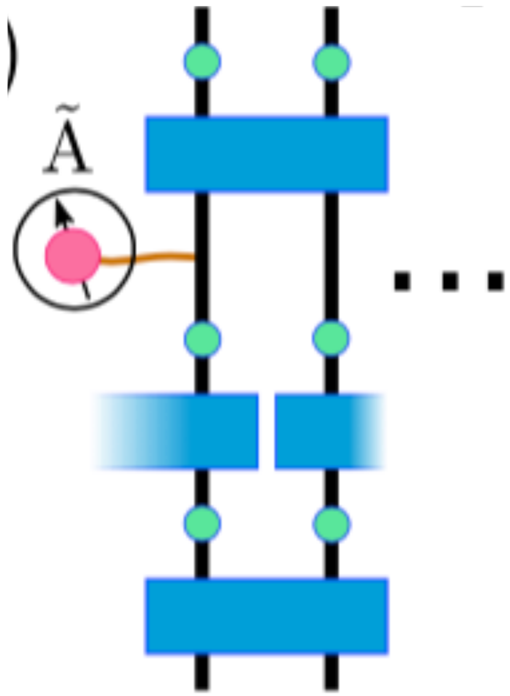
**near $p_c$**

$$S_1(\rho_R) \sim F((p - p_c)L^{1/\nu}, t/L^z)$$



# Order Parameter

$\tilde{A} = (r, t_0)$



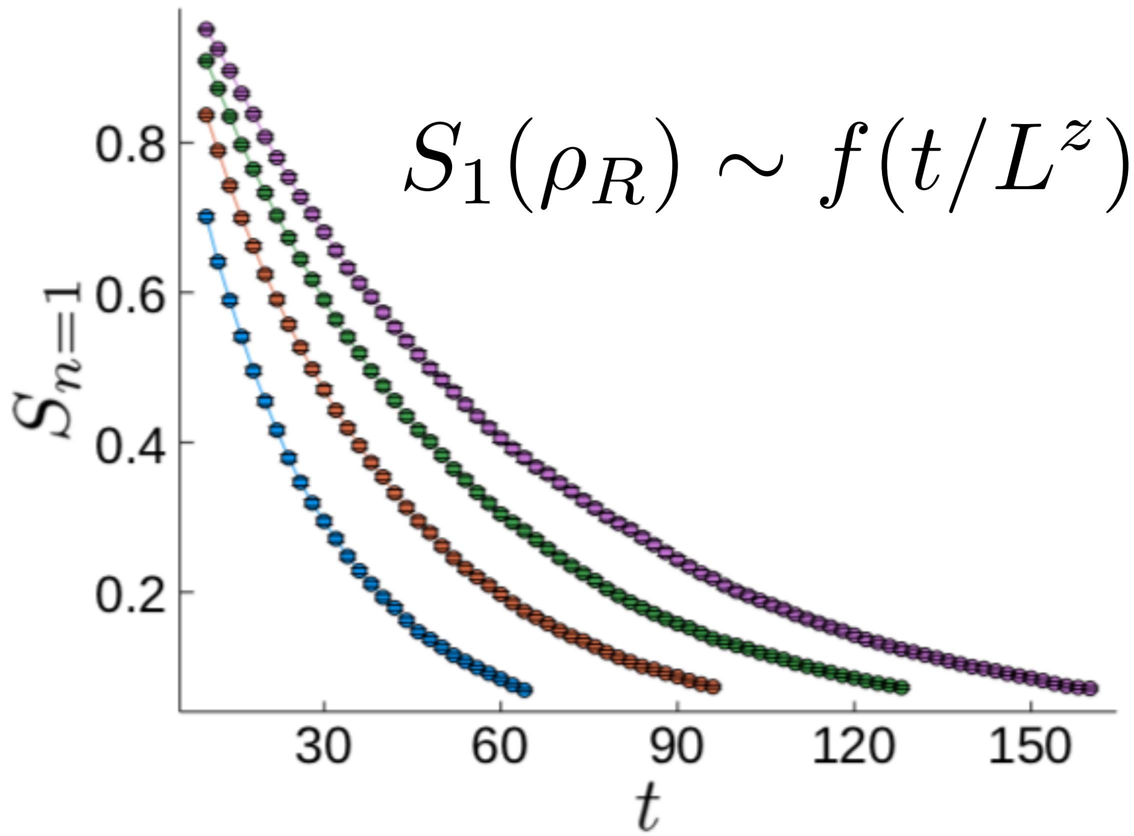

Entangle one ancilla qubit in a Bell state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\Uparrow\downarrow\rangle - |\Downarrow\uparrow\rangle)$$

and track its **entanglement entropy**

At the transition

$p = p_c$




$S_1(\rho_R) \sim f(t/L^z)$

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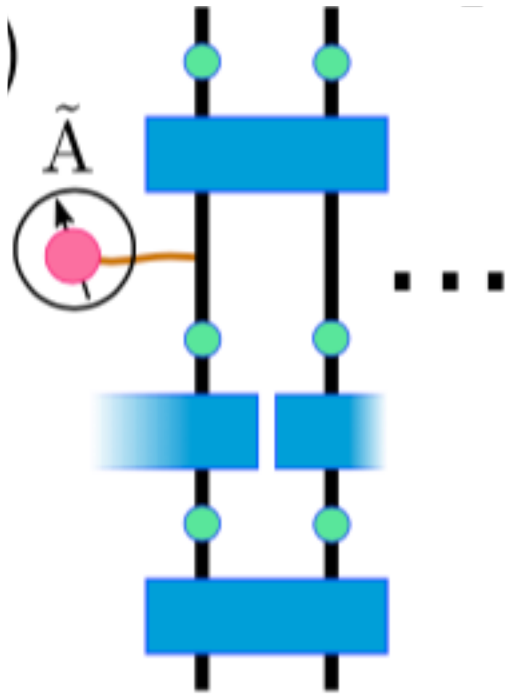


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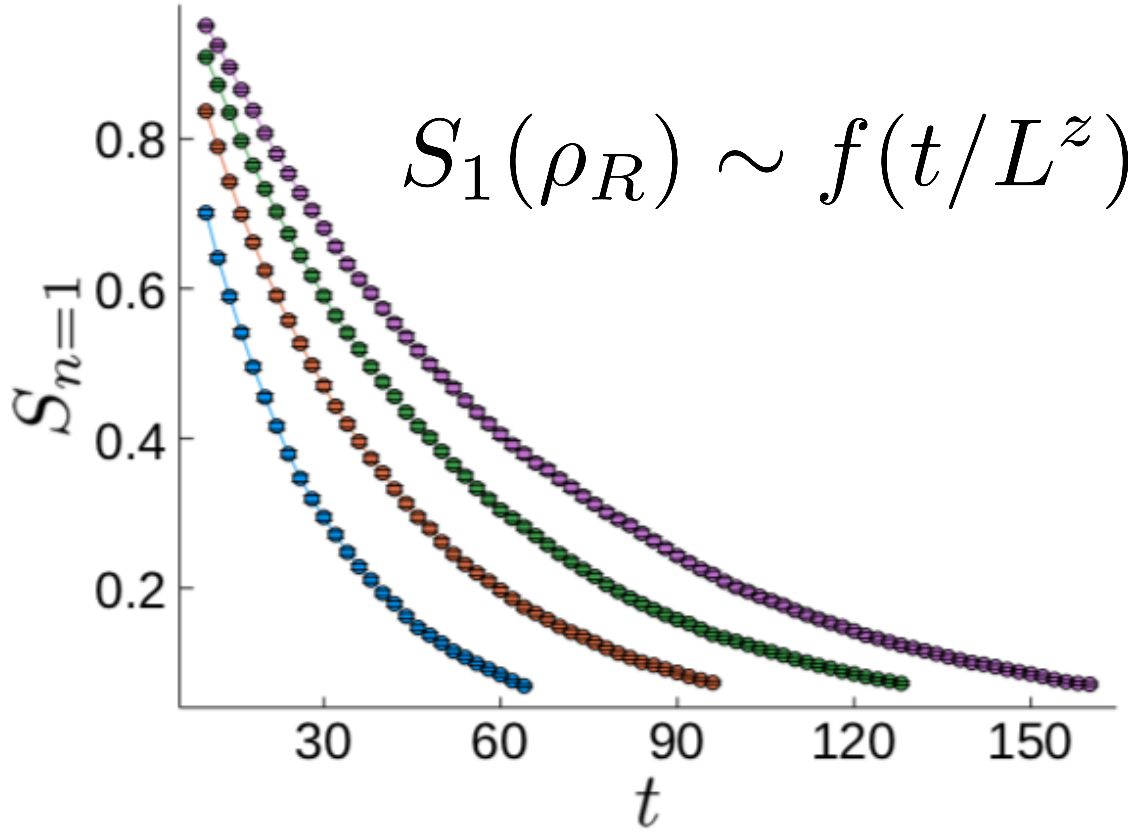


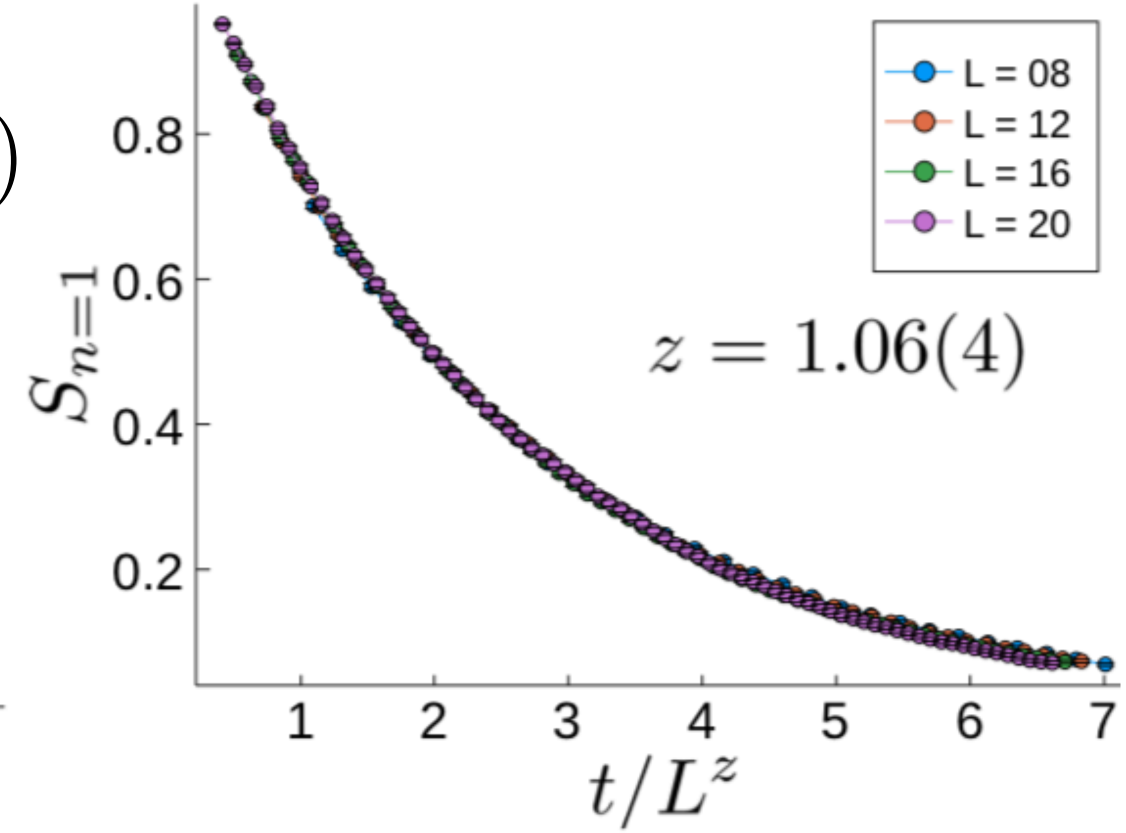

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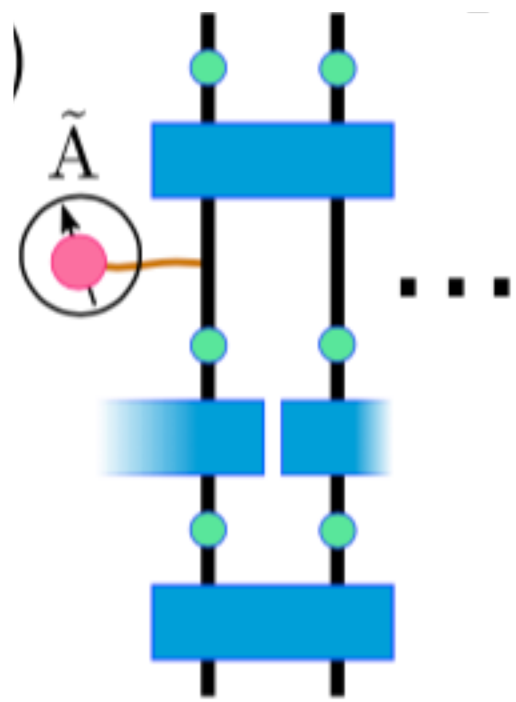


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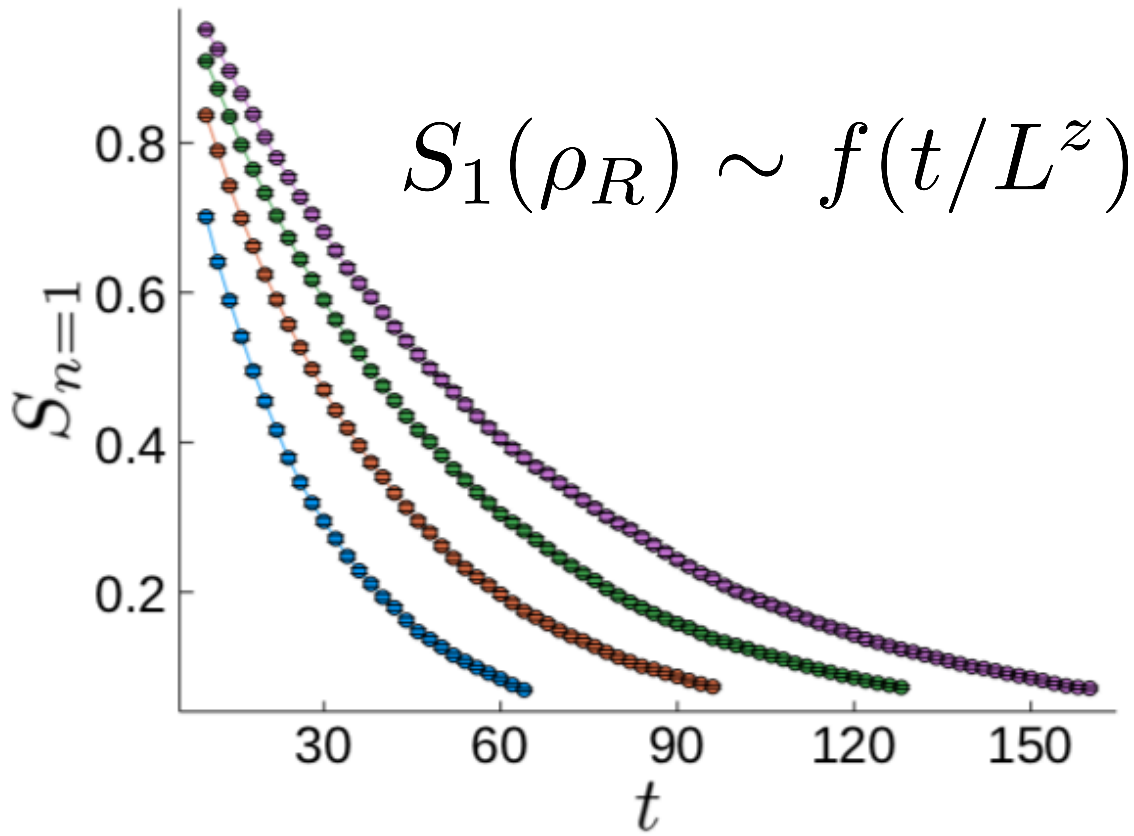


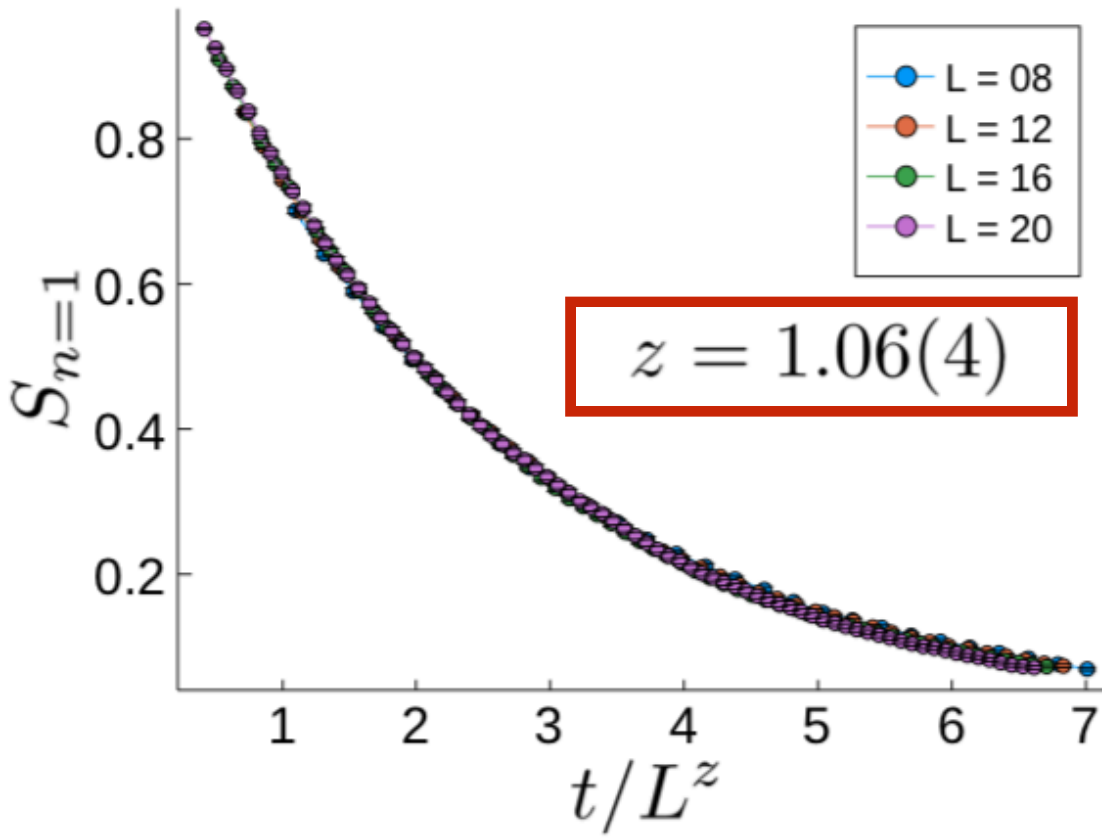

# Order Parameter: "Correlation Function"

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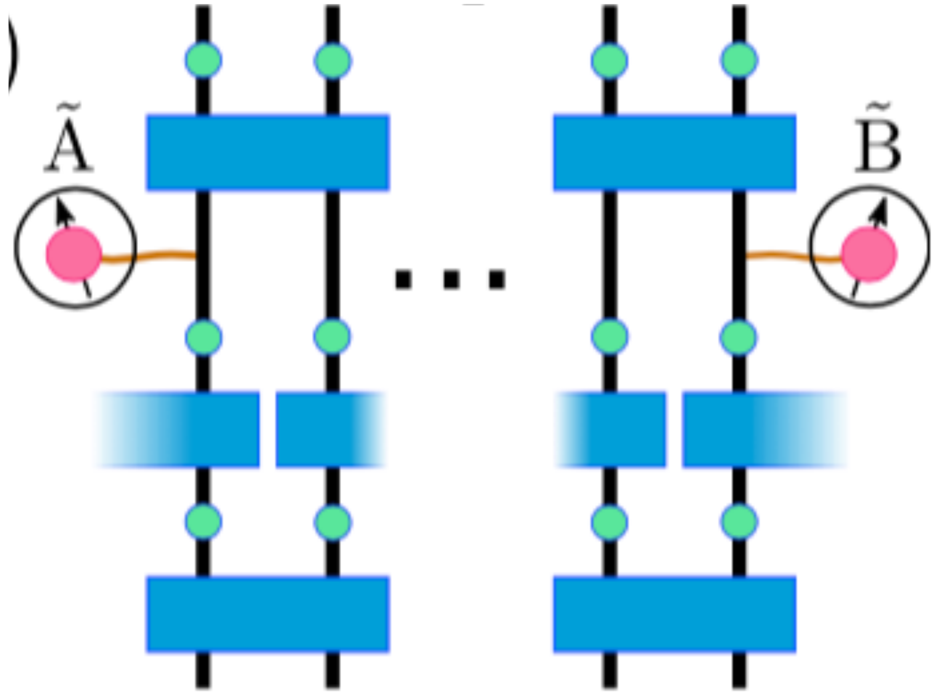


$\tilde{B} = (r', t_0)$

Entangle two ancilla qubits and track their mutual information

$$C(r - r'; t - t_0) = I_1(\tilde{A}, \tilde{B}) = S_1(\tilde{A}) + S_1(\tilde{B}) - S_1(\tilde{A} \cup \tilde{B})$$

Gullans and Huse PRL (2020)

Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, JHP PRB(R) (2020)

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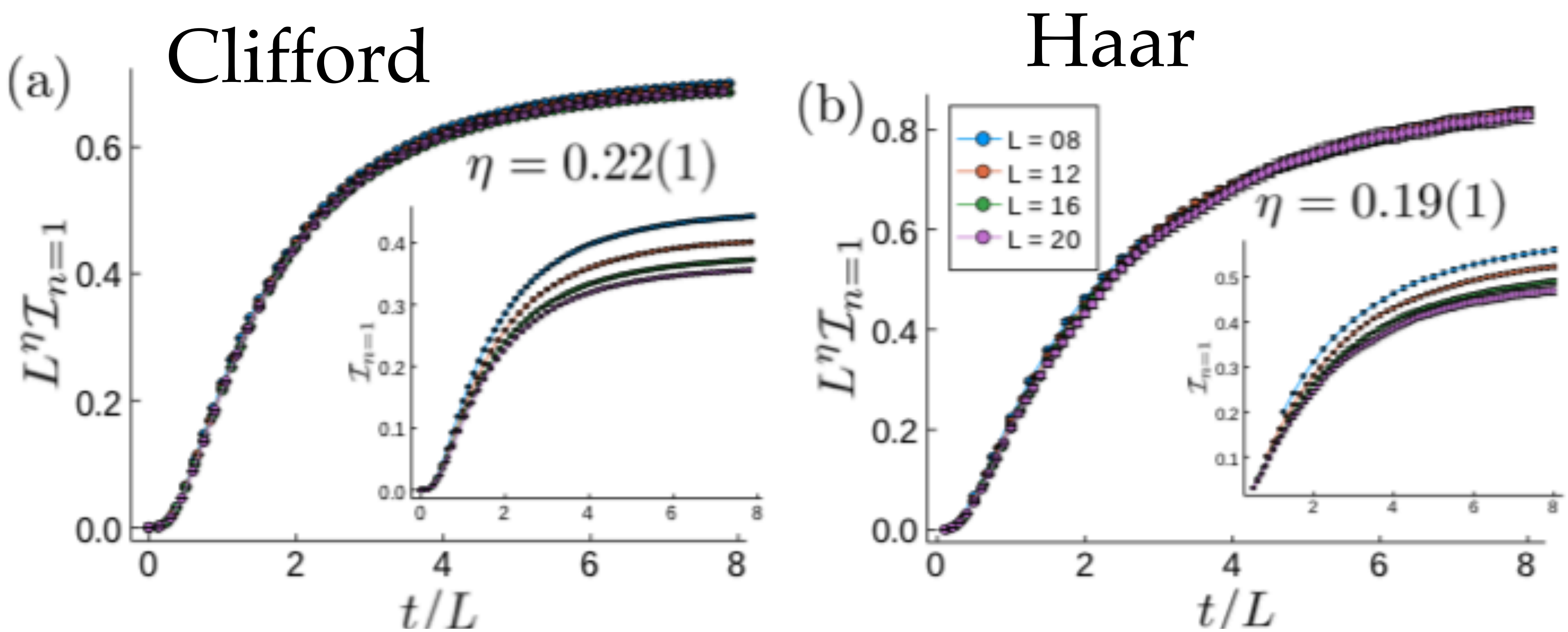



$\eta_P = 5/24 \approx 0.21$

$|r-r'| = L/2$

# Critical Properties Summary

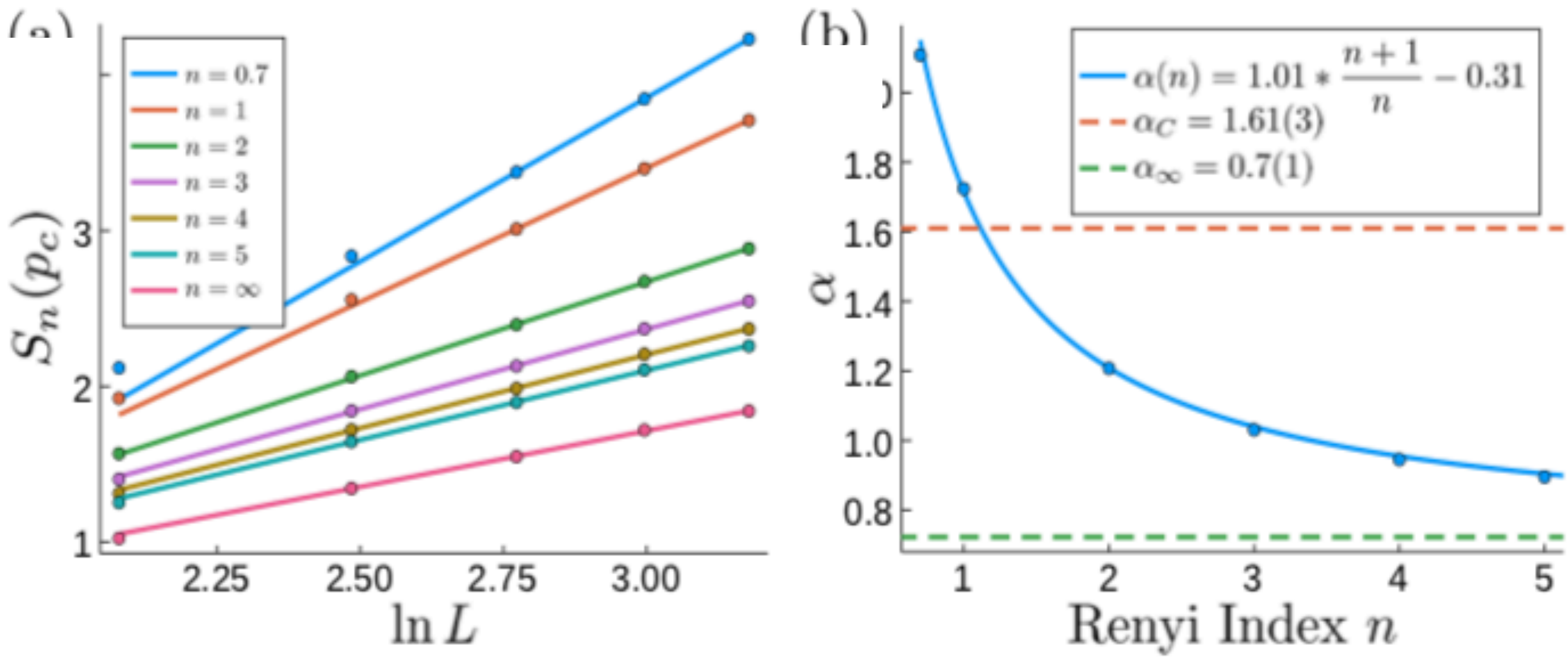
Critical exponents for both Clifford and Haar gates are very close to percolation (Lorentz invariant)

| $n$ | 1 | 2 | 5 | $\infty$ | C | P |
|---|---|---|---|---|---|---|
| $p_c$ | 0.168(5) | 0.162(3) | 0.168(4) | 0.170(4) | 0.154(4) | 0.5 |
| $\nu$ | 1.2(2) | 1.3(1) | 1.4(1) | 1.4(1) | 1.24(7) | 1.33 |
| $\eta$ | 0.19(1) | 0.25(1) | 0.26(1) | 0.26(1) | 0.22(1) | 0.21 |

Renyi dependence of the entanglement entropy?

# Renyi Entropies at Criticality

Renyi dependence of the entanglement entropy




$$S_n \sim \alpha(n) \log L \qquad \alpha(n) = 0.7(1) + 1.0(1)/n$$

conventional (i.e unitary) CFT $\alpha \propto (1 + 1/n)$

$$A_{\text{Clifford}} \approx 1.6 > A_{\text{perc}} \approx 0.55$$

# Critical Properties Summary

Critical exponents for both Clifford and Haar gates are very close to percolation

| | Haar | Clifford | Percolation |
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$S \sim A \log L$ $\quad \alpha(n) \quad A_{\text{Clifford}} \approx 1.6 \quad A_{\text{perc}} \approx 0.55$

$\alpha(n) = 0.7(1) + 1.0(1)/n$

Accuracy of critical exponents doesn't allow a conclusion on the nature of the universality class.

**Is the transition in Haar random circuits simply percolation?**

# Characterizing the Confromal Field Theory

The field theory of the percolation transition is known as a logarithmic conformal field theory (log-CFT for short).

Cardy, J. Phys. A (1992)

Jeng and Ludwig, NPB (2001); Cardy, and Jacobsen, NPB (1998); Cardy, J. Phys. A (2013)

# Characterizing the Confromal Field Theory

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This replicated field theory is rather unusual:

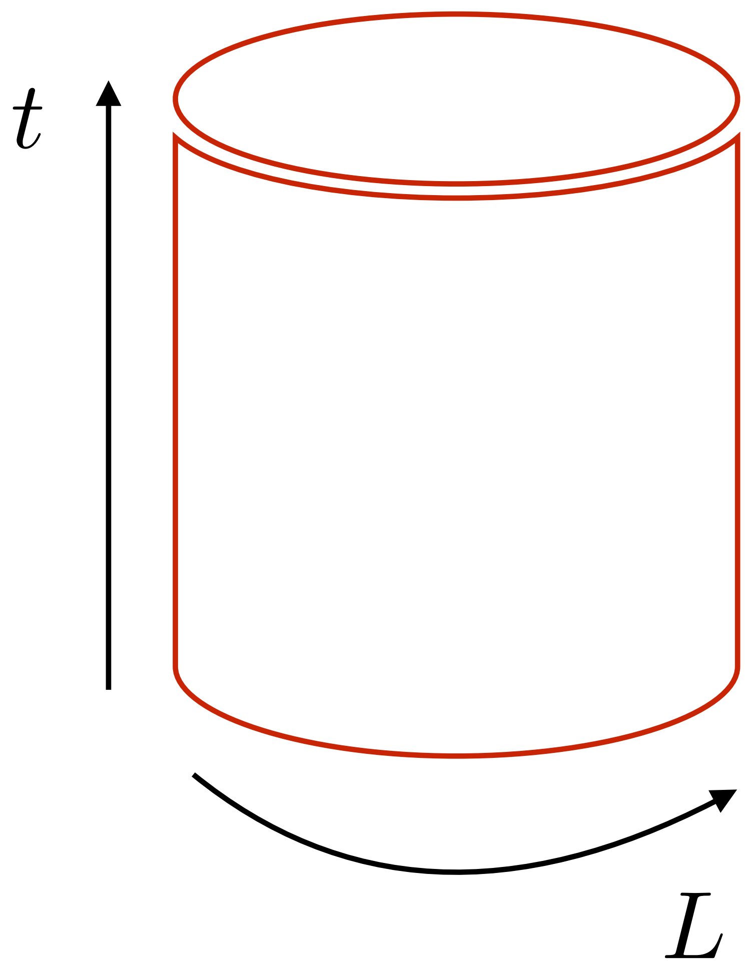
It has a central charge $c=0$.

Instead the log-CFT is characterized by a so-called **effective central charge.**

Jeng and Ludwig, NPB (2001); Cardy, and Jacobsen, NPB (1998); Cardy, J. Phys. A (2013)

# Characterizing the Confromal Field Theory

With an integer number of replicas *r*, the Free energy density of a 1+1D CFT on a cylinder is given by




$$f = F/A = f_r - \frac{\pi c(r)}{6L^2} + \frac{d_r}{L^4} + \ldots$$

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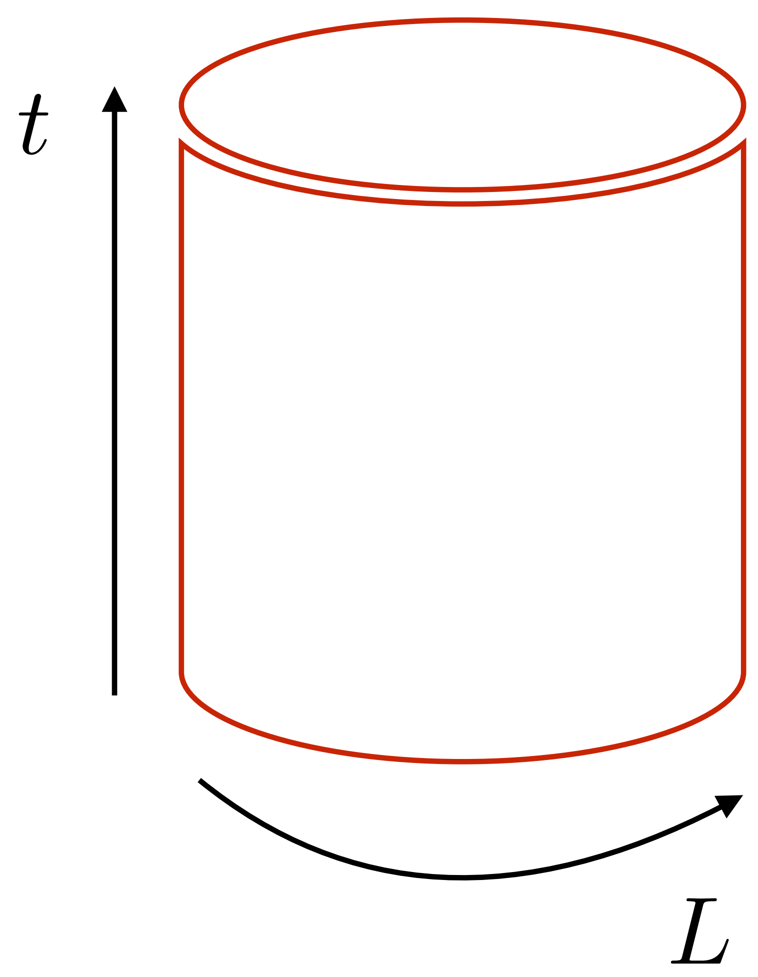


$$f = F/A = f_r - \frac{\pi c(r)}{6L^2} + \frac{d_r}{L^4} + \dots$$

For a theory with $c(0)=0$, we look at an analogous quantity $c_{eff}$

$$\frac{df}{dr}\Big|_{r=0} = \frac{1}{A}\frac{dF}{dr}\Big|_{r=0} = f_0' - \frac{\pi c_{\text{eff}}}{6L^2} + \frac{d_0'}{L^4}$$

$$\boxed{c_{\text{eff}} = \frac{dc(r)}{dr}\Big|_{r=0}}$$

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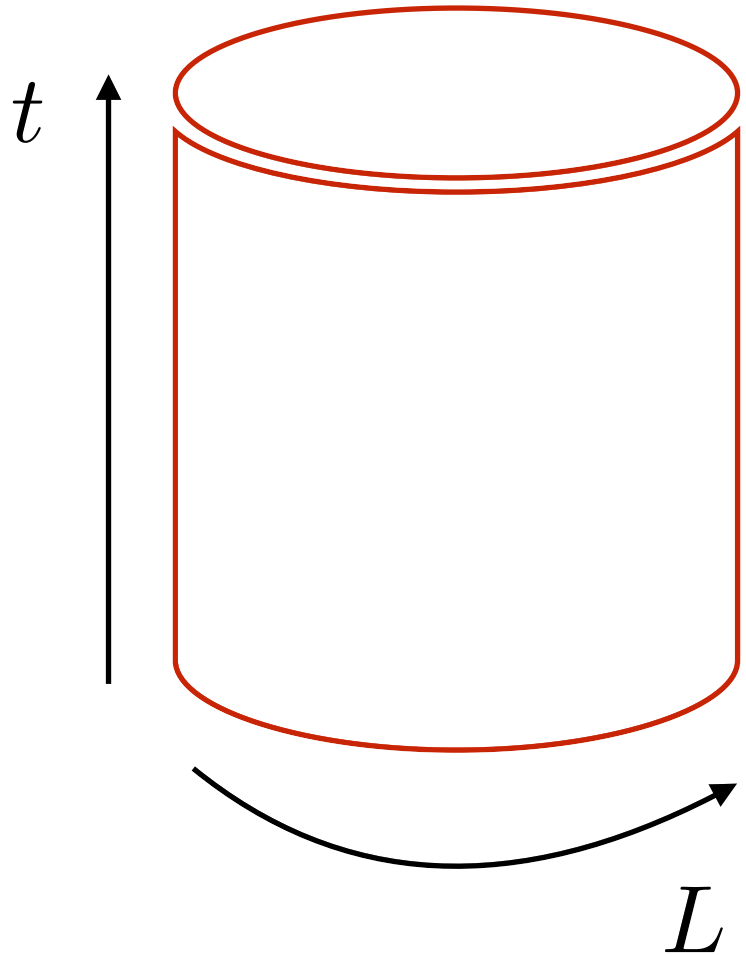


Goal: Compute $c_{eff}$
at the measurement transition

$$\frac{df}{dr}\Big|_{r=0} = \frac{1}{A}\frac{dF}{dr}\Big|_{r=0} = f_0' - \frac{\pi c_{\text{eff}}}{6L^2} + \frac{d_0'}{L^4}$$

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# From the Circuit to the CFT Free Energy

Consider the reduced density matrix of the measurement environment (trace out the spin-degrees of freedom)

$$\rho_E(\psi_0) = \sum_{\mathbf{m}} p_{\mathbf{m}}(\psi_0)|\mathbf{m}\rangle\langle\mathbf{m}|$$

$$p_{\mathbf{m}}(\psi_0) = ||K_{m_t}U(t,t-1)\ldots K_{m_2}U(2,1)K_{m_1}U(1,0)|\psi_0\rangle||^2$$

At late times, $p_{\boldsymbol{m}}$ is independent of $|\psi_0\rangle$

$$\rho_E(\psi_0) \to \sum_{\mathbf{m}} p_{\mathbf{m}}|\mathbf{m}\rangle\langle\mathbf{m}|$$

# From the Circuit to the CFT Free Energy

Connect with replicated field theory: Consider *r* replicas

$$(\rho_E)^{r+1} = (\sum_{\mathbf{m}} p_{\mathbf{m}} |\mathbf{m}\rangle\langle\mathbf{m}|)^{r+1} = \sum_{\mathbf{m}} p_{\mathbf{m}}^{r+1} |\mathbf{m}\rangle\langle\mathbf{m}|$$

The partition function of this replicated density matrix is then:

$$\boxed{Z = \sum_{\mathbf{m}} p_{\mathbf{m}}^{r+1}}$$

$$r \to 0 \qquad Z \to 1$$

immediately implies, $c=0$

# From the Circuit to the CFT Free Energy

Connect with replicated field theory: Consider *r* replicas

$$\boxed{Z = \sum_{\mathbf{m}} p_{\mathbf{m}}^{r+1}}$$

$$F = -\log Z = -\log\left(\sum_{\mathbf{m}} p_{\mathbf{m}} e^{r \log p_{\mathbf{m}}}\right)$$ in the physical limit $r \to 0$

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$$\approx -\log\left(\sum_{\mathbf{m}} p_{\mathbf{m}}(1 + r\log p_{\mathbf{m}})\right) \approx -r\sum_{\mathbf{m}} p_{\mathbf{m}} \log p_{\mathbf{m}}$$

$$\frac{dF}{dr}\Big|_{r=0} = -\sum_{\mathbf{m}} p_{\mathbf{m}} \log p_{\mathbf{m}} \equiv S_0(\rho_E)$$

entropy of the measurement record

# From the Circuit to the CFT Free Energy

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entropy of the measurement record

For a log-CFT

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$$\boxed{\to S_0(\rho_E)/A = f_0' - \frac{\pi c_{\text{eff}}}{6L^2} + \ldots}$$

# Entropy of the Measurement Record

$$S_0(\rho_E)/A = f_0' - \frac{\pi c_{\text{eff}}}{6L^2} + \ldots$$

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$$S(\rho_E) = -\sum_{\mathbf{m}} \langle \log p_{\mathbf{m}} \rangle = -\sum_{i=1}^{N_{\rm meas}} \langle \log p(m_i | m_{i-1}, \ldots, m_1) \rangle$$

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**Estimated from the Born probabilities we compute as we run the circuit!**

# Area of the Cylinder

$$\boxed{S_0(\rho_E)/A = f_0' - \frac{\pi c_{\mathrm{eff}}}{6L^2} + \ldots}$$

area of the cylinder of the log-CFT $\quad A = \alpha t L$

unknown anisotropy factor between space and time, model dependent

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area of the cylinder of the log-CFT $\quad A = \alpha t L$

unknown anisotropy factor between space and time, model dependent

Handle this 2 ways:

(1) Compute it directly (using conformal mappings, not covering).

(2) Choice a convenient set of gates with $\alpha = 1$

Use **dual unitary gates**:

$$V[J] = \exp\left[-i\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right)\right]$$

Bertini, Kos, and Prosen, Phys. Rev. Lett. (2019)

$$\alpha = 1$$

# The effective central charge

$$S_0(\rho_E)/A = f_0' - \frac{\pi c_{\text{eff}}}{6L^2} + \dots$$

computing $S_0$ and $A$

Percolation ($q \to \infty$)

$p_c = 0.5$

$$c_{\text{eff}}^{d\to\infty} = \frac{5\sqrt{3}(1-\gamma)}{4\pi} \approx 0.291$$

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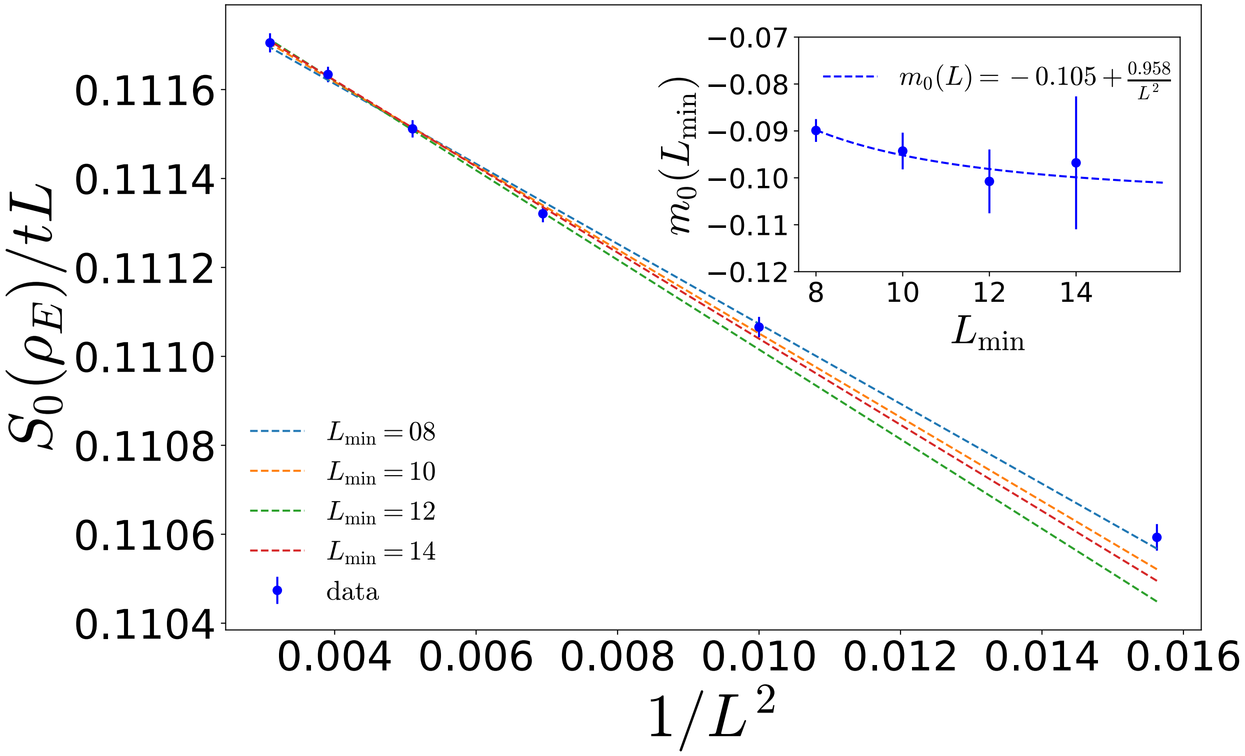
**Haar random gates**

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$$c_{\text{eff}}^H = 0.25 \pm 0.03$$

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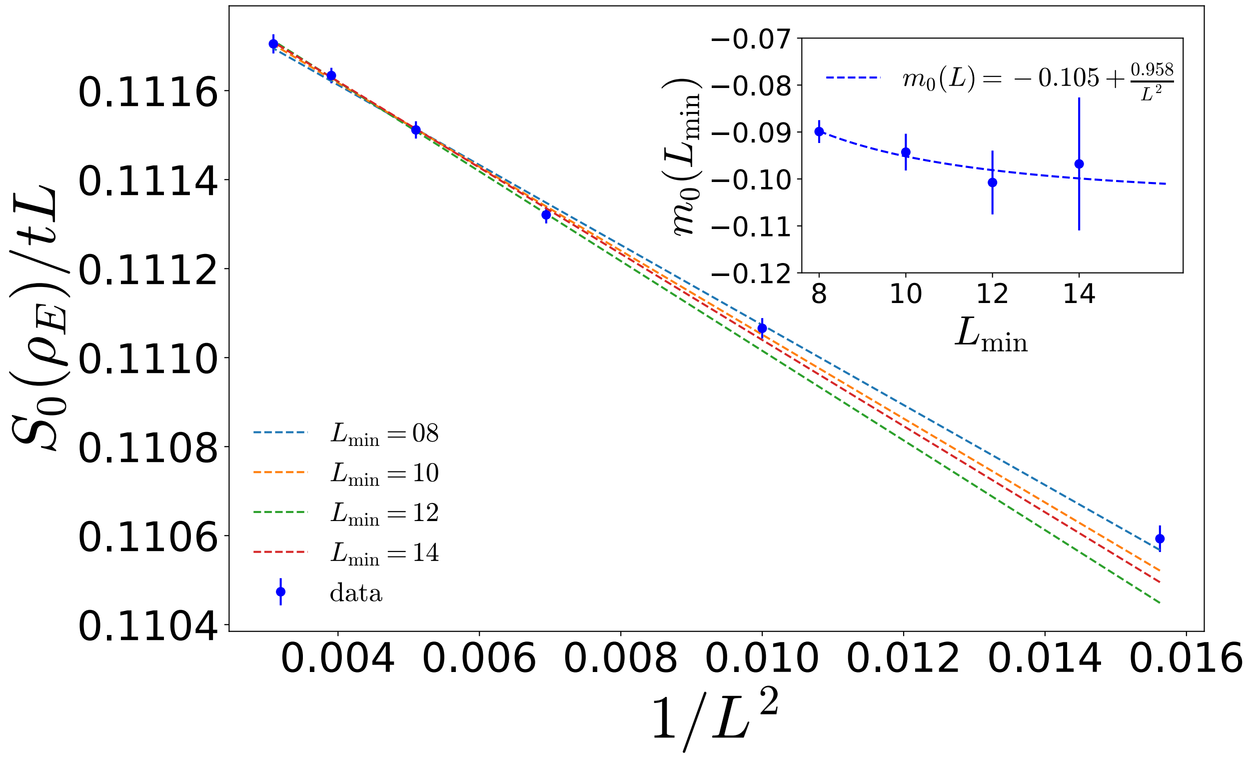
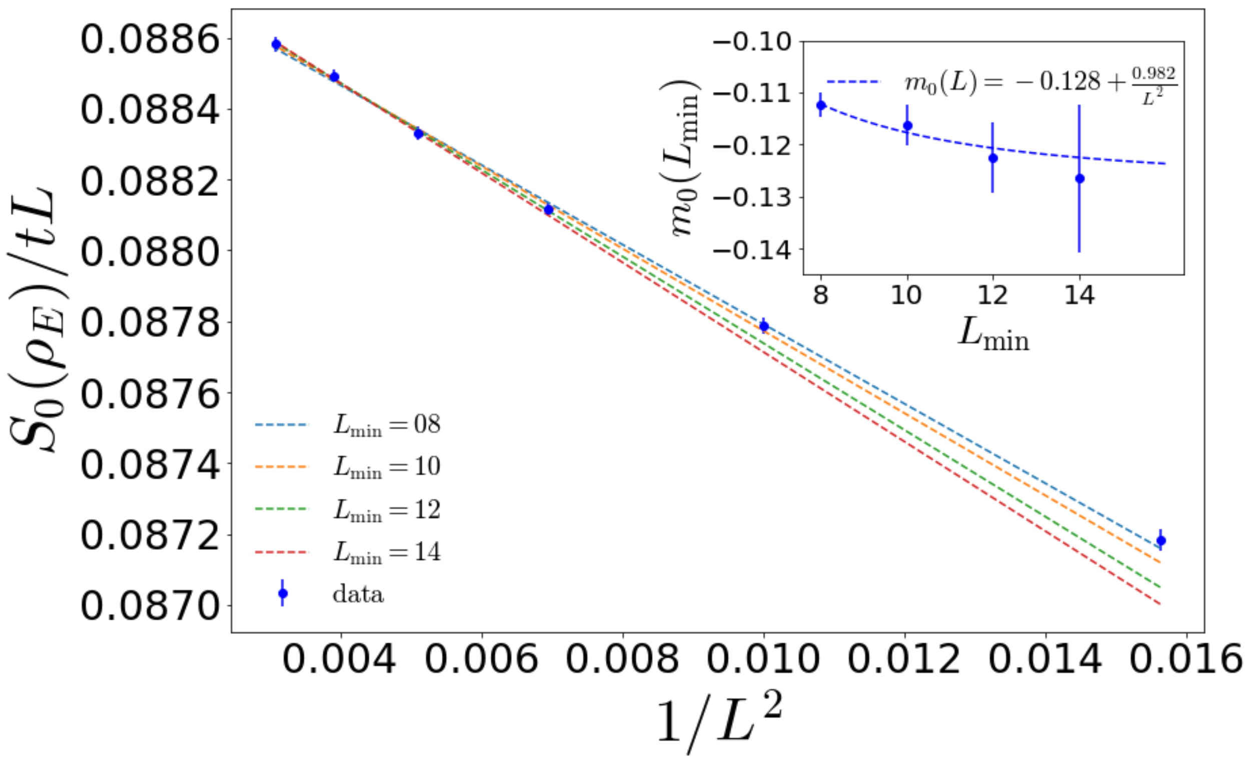



**random dual unitary gates**

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$c_{\text{eff}}^{DU} = 0.24 \pm 0.02$

# THE EFFECTIVE CENTRAL CHARGE

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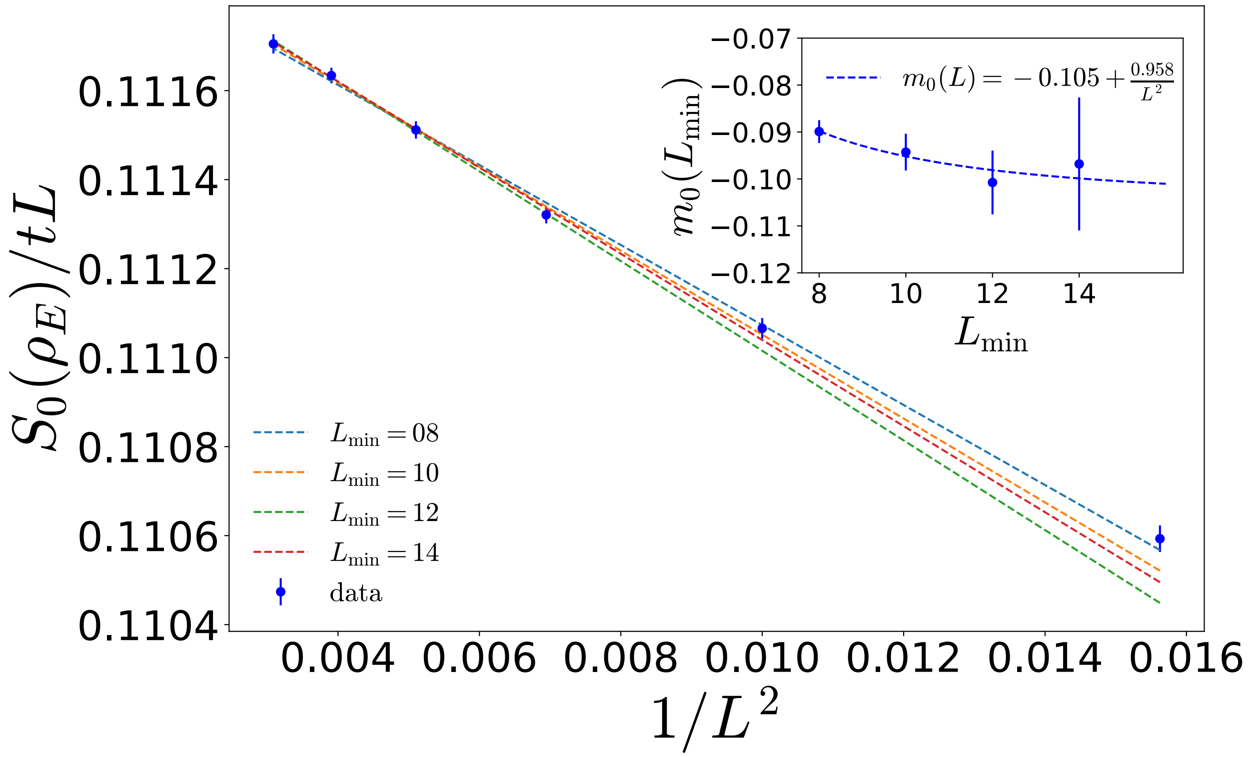
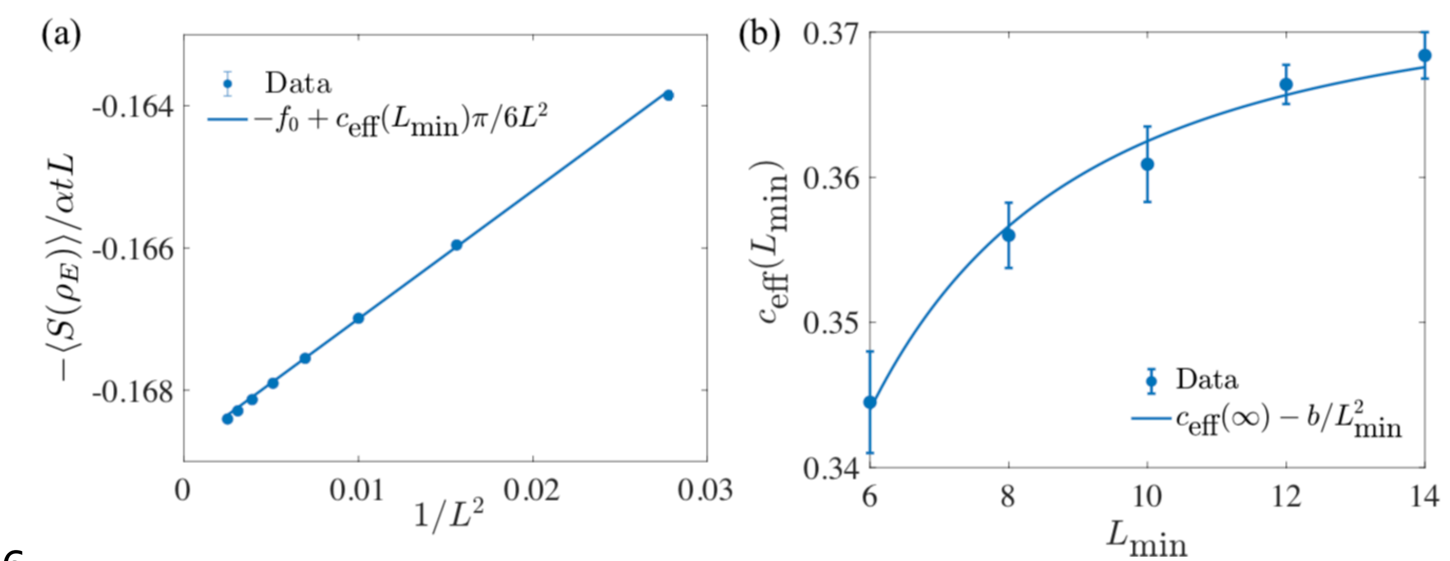



**Stabilizer circuits**

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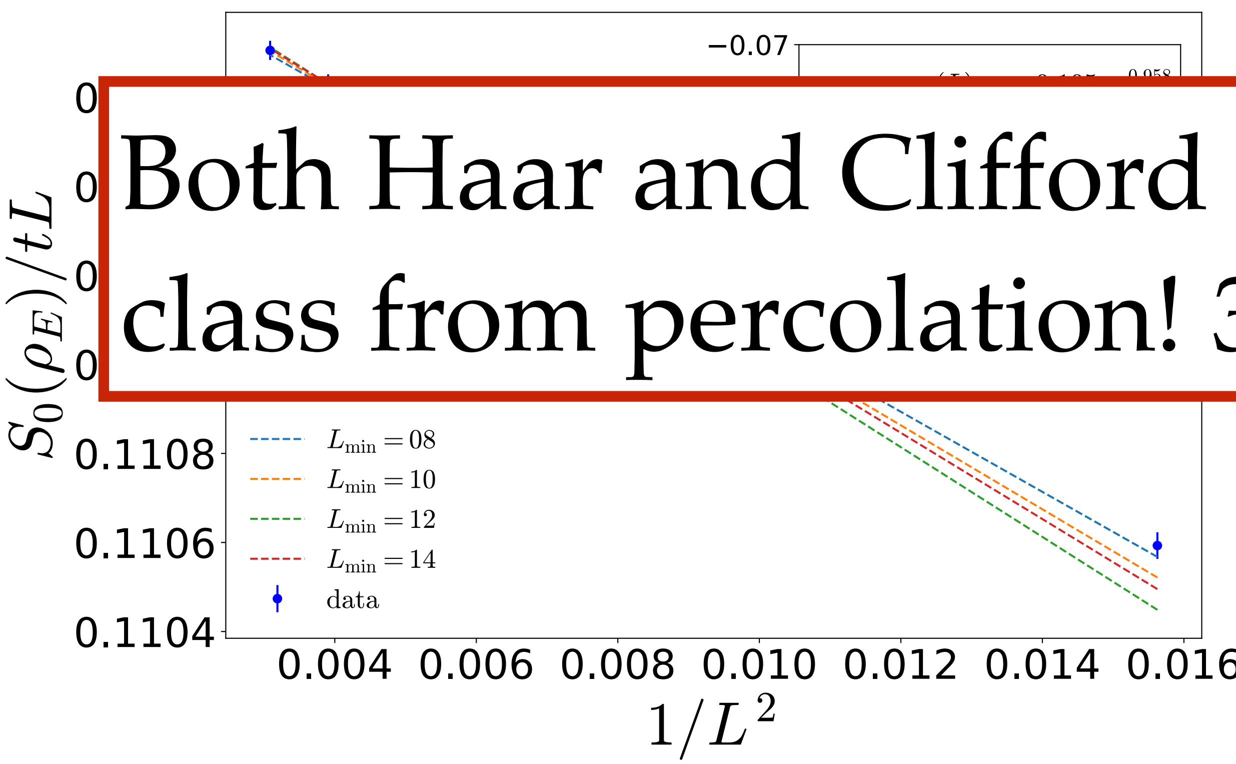


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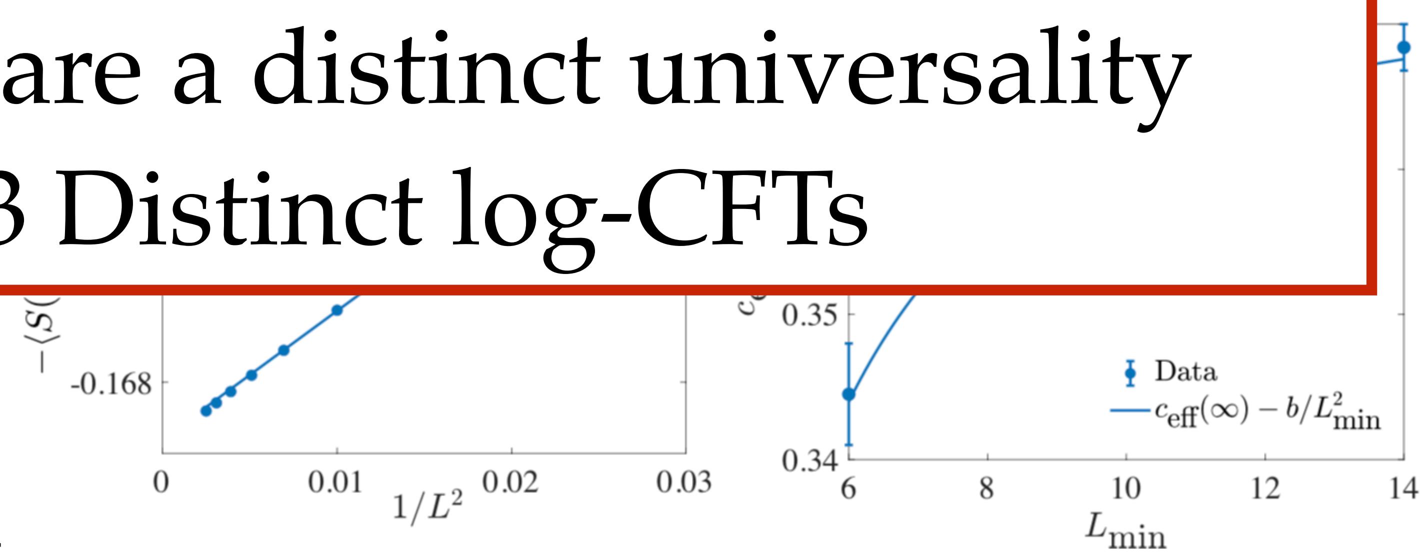


Both Haar and Clifford are a distinct universality class from percolation! 3 Distinct log-CFTs

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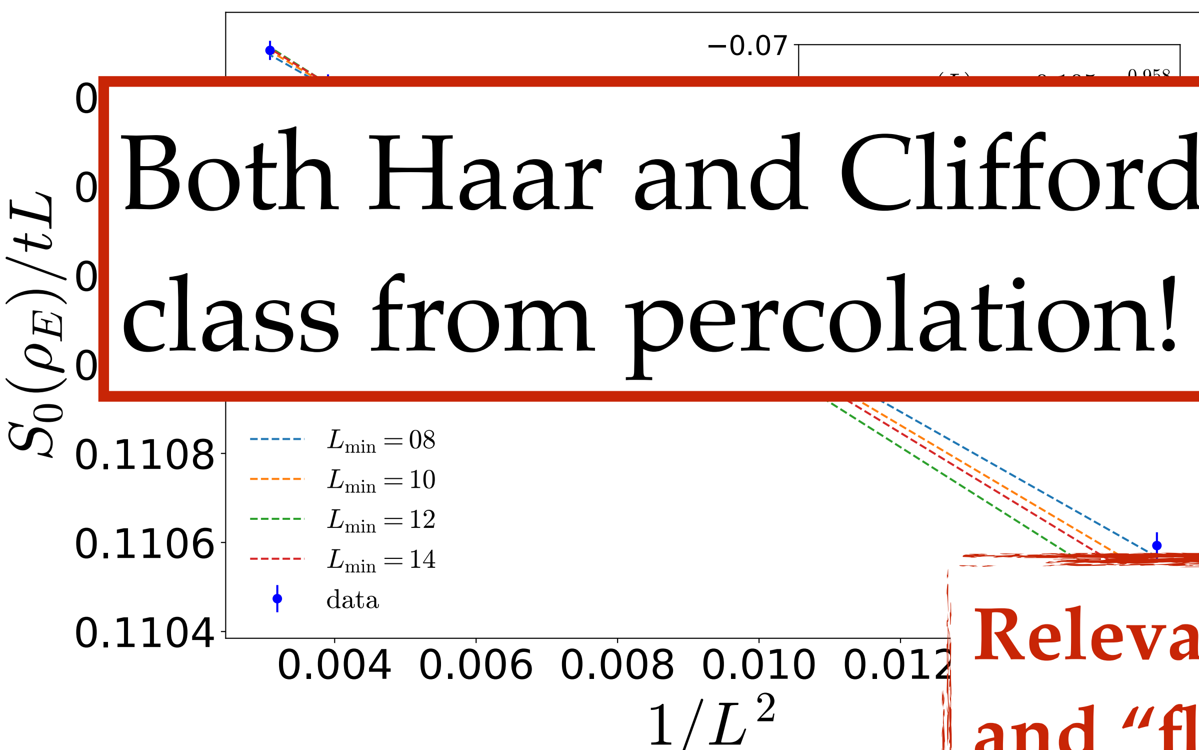


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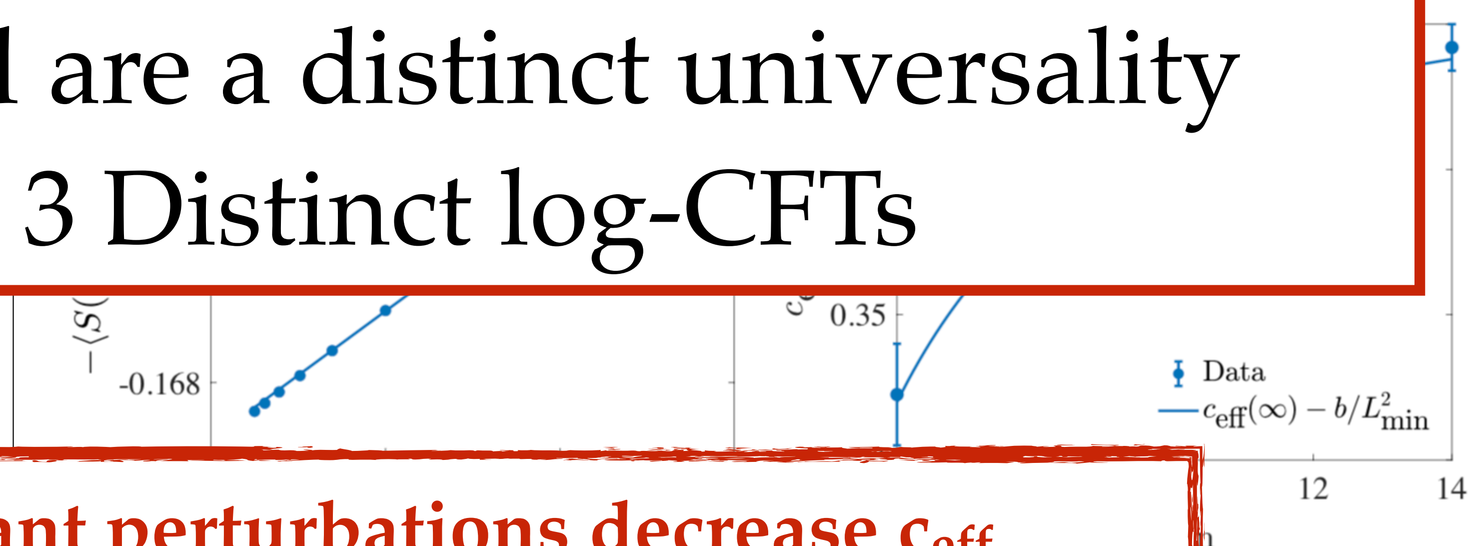


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**Relevant perturbations decrease $c_{\text{eff}}$ and "flow" to the general quantum limit**

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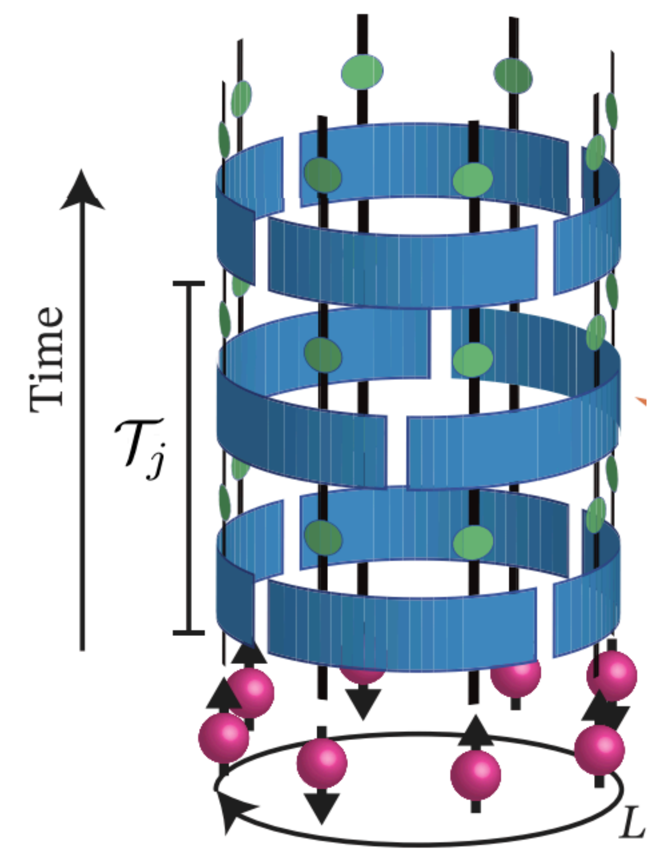
# Characterizing the Confromal Field Theory

The log-CFT of the measurement driven transition can be characterized by its transfer matrix!

Zabalo, Gullans, Wilson, Vasseur, Ludwig, Gopalakrishnan, Huse, JHP PRL (2022)

$$|\Psi_{\mathbf{m}}(t)\rangle = K_{m_t}U(t,t-1)\ldots \underbrace{K_{m_2}U(2,1)K_{m_1}U(1,0)}_{\mathcal{T}_1}|\Psi_0\rangle/\sqrt{p_{\mathbf{m}}(\Psi_0)}$$

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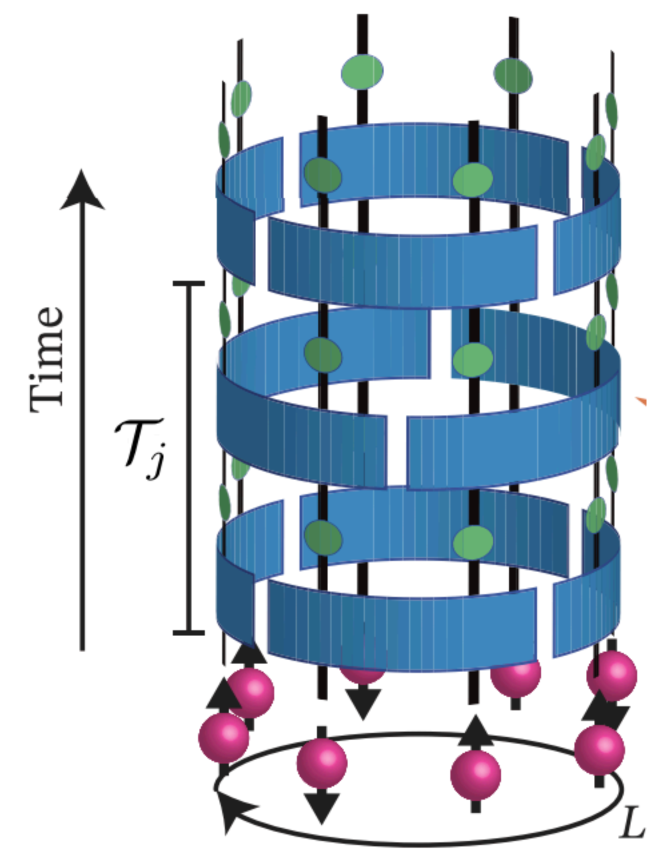


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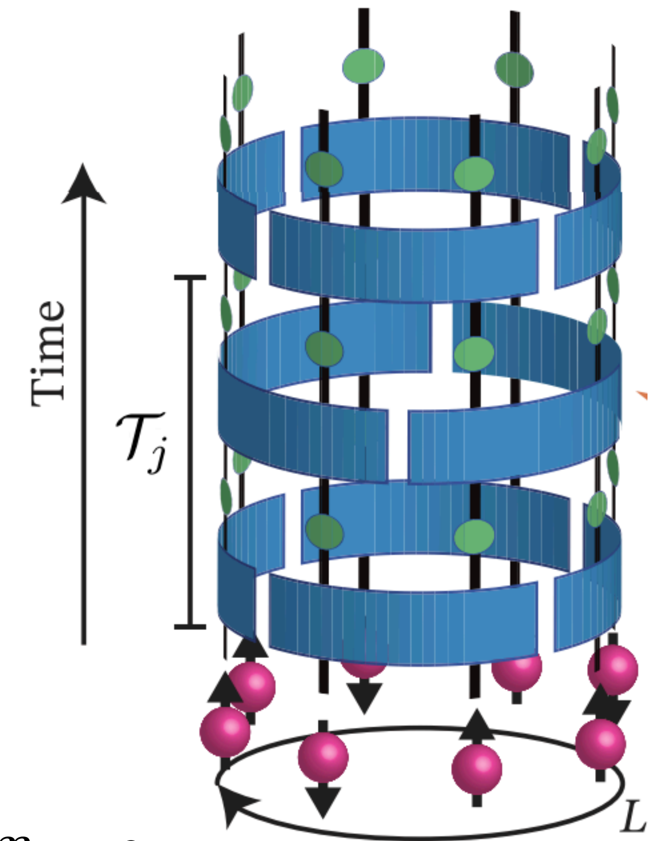
Singular values of $\prod_j \mathcal{T}_j$ decay $[\sigma_i^{\mathbf{m}}(t)]^2 = e^{\lambda_i^{\mathbf{m}} t}$




$\lambda_i^{\mathbf{m}} < 0$

$t \gg L$

# Characterizing the Confromal Field Theory

The log-CFT of the measurement driven transition can be characterized by its transfer matrix!

Zabalo, Gullans, Wilson, Vasseur, Ludwig, Gopalakrishnan, Huse, JHP PRL (2022)

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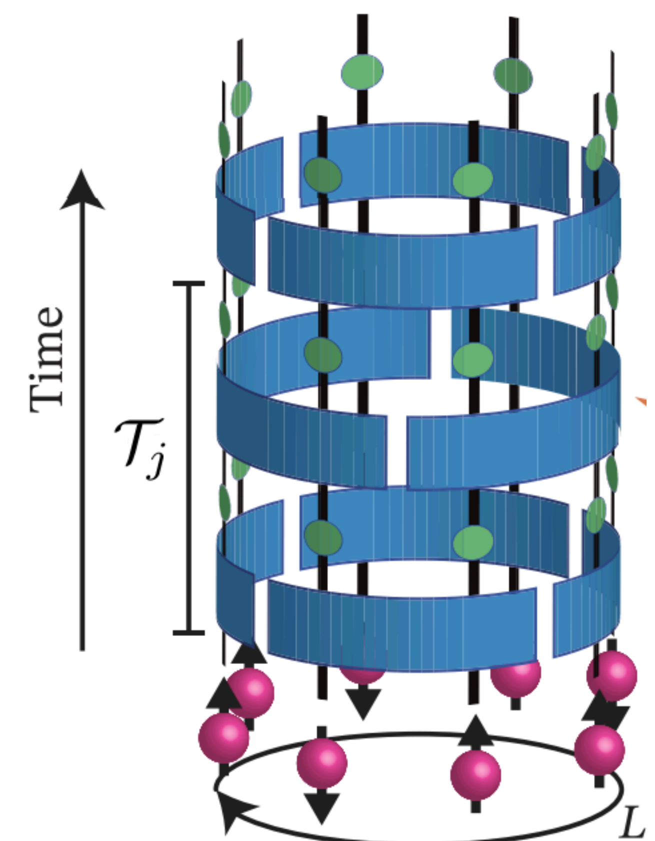


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allowing us to obtain the Lyapunov exponents $\lambda_i = \sum_{\mathbf{m}} p_{\mathbf{m}}\lambda_i^{\mathbf{m}}$

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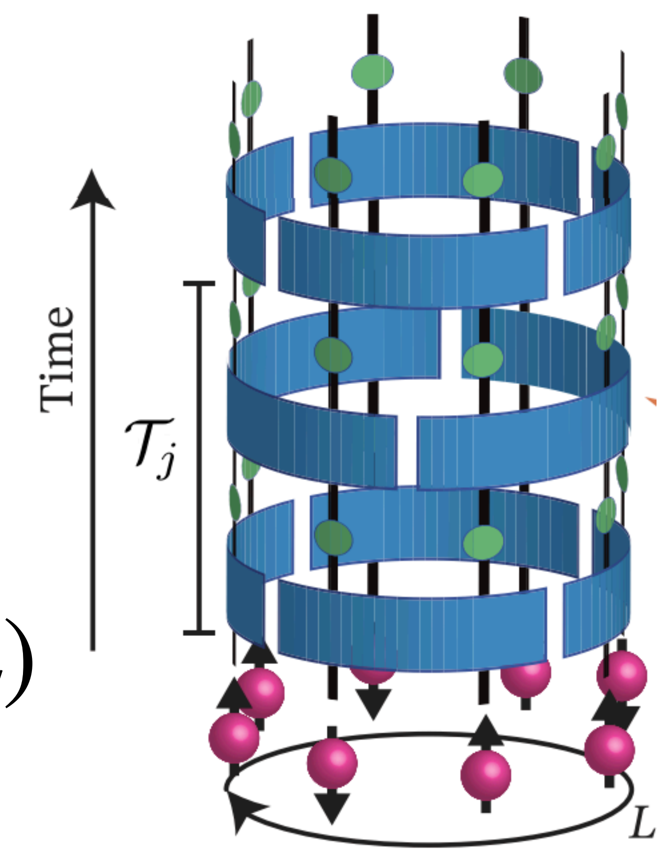
$$\lambda_i = \sum_{\mathbf{m}} p_{\mathbf{m}} \lambda_i^{\mathbf{m}}$$

$$f_i(L) = -\lambda_i/(\alpha L)$$




$t \gg L$

The scaling dimensions $x_i$ of operators can be extracted from the generalized free energies $f_i(L)$

$$f_i(L) - f(L) = 2\pi x_i^{\text{typ}}/L^2$$

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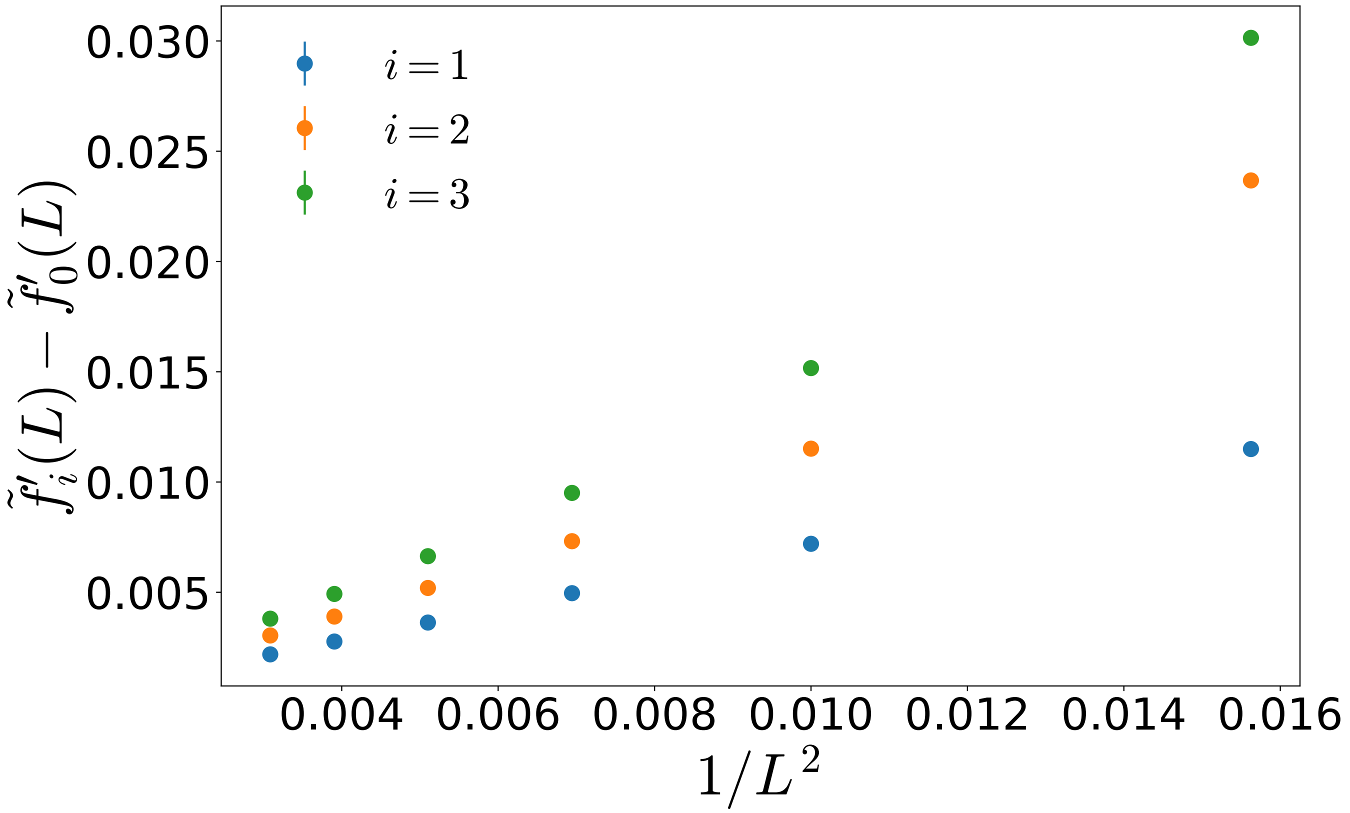
| Model | $p_c$ | $\alpha$ | $c_{\text{eff}}$ | $x_1^{\text{typ}}$ | $x_2^{\text{typ}}$ | $x_3^{\text{typ}}$ |
|---|---|---|---|---|---|---|
| Haar | 0.170 | 0.808(90) | 0.248(34) | 0.138(16) | 0.175(20) | 0.228(26) |
| Dual Unitary | 0.140 | 1.0 | 0.244(16) | 0.132(2) | 0.184(4) | 0.234(4) |



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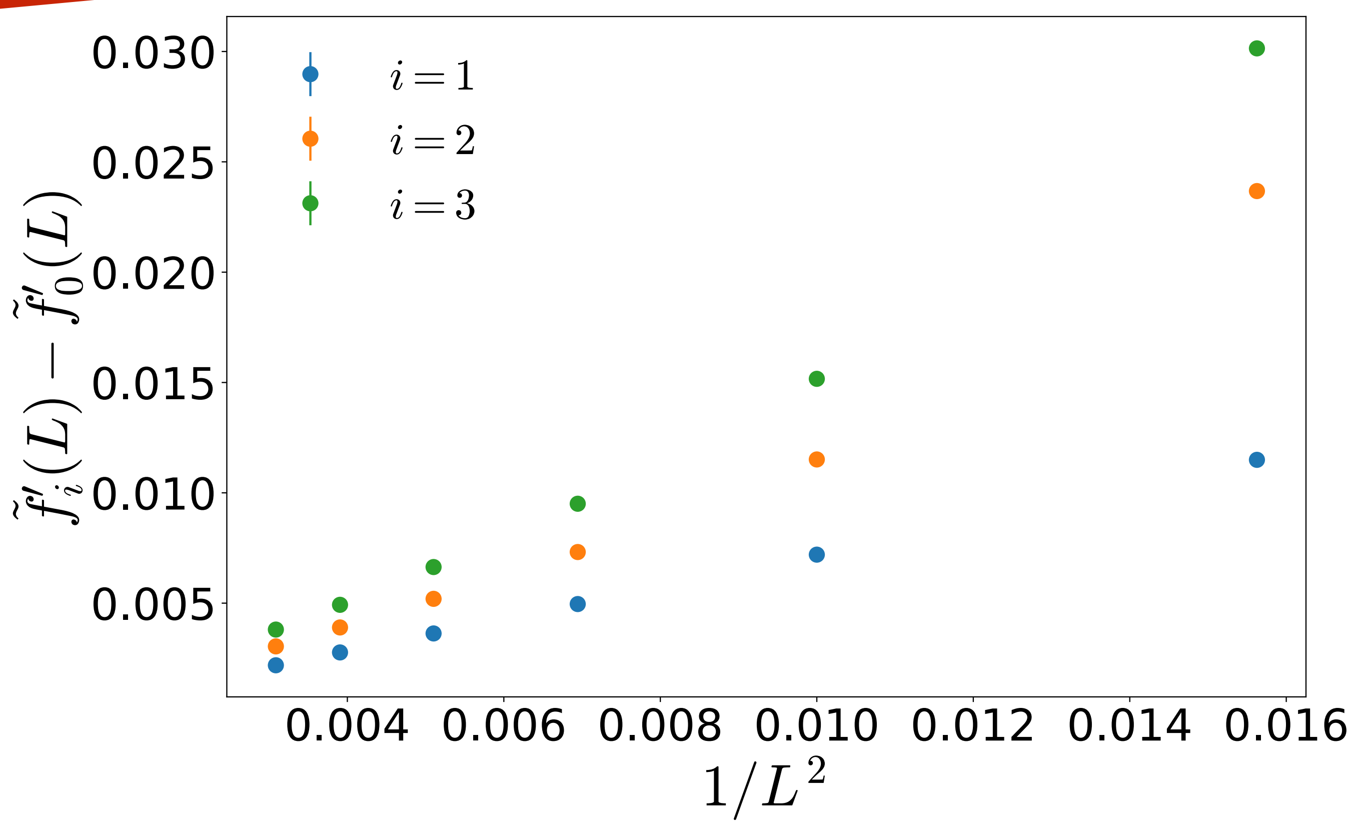
$$x_1 = \eta/2$$

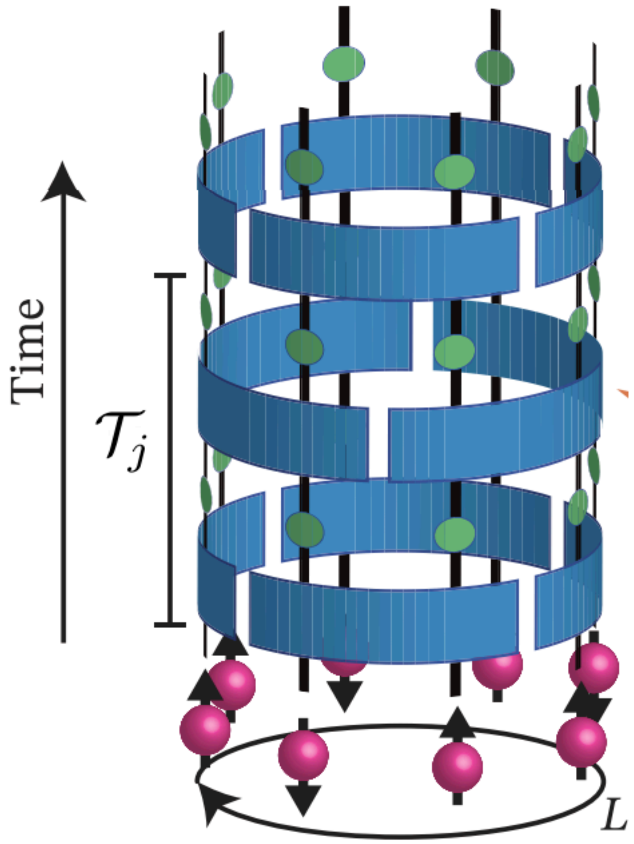
Our previous estimate

$$\eta/2 \approx 0.125$$



# Distribution of Correlation functions

$$\ln G_j^{\mathbf{m}}(t) = t(\lambda_j^{\mathbf{m}} - \lambda_0^{\mathbf{m}})$$




$$t \gg L$$

In the percolation and Clifford limits

Correlation functions have fractal scaling dimensions

$$\overline{G_j^{\mathbf{m}}(t)} \sim \frac{1}{t^{x_j}}$$

NOT multifractal!

$$\overline{[G_j^{\mathbf{m}}(t)]^n} \sim \frac{1}{t^{n*x_j}}$$

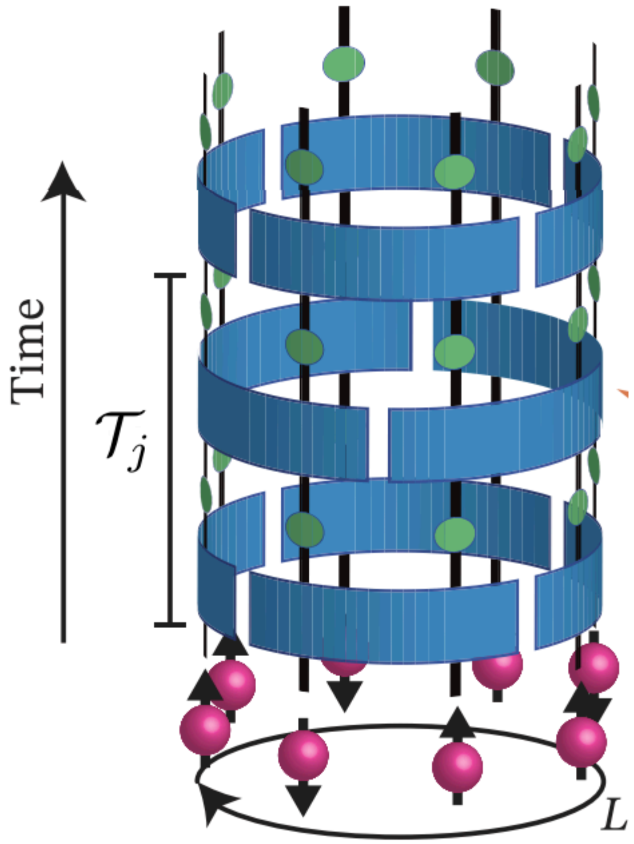
This is an artifact of these semiclassical limits!

Zabalo, Gullans, Wilson, Vasseur, Ludwig, Gopalakrishnan, Huse, JHP arXiv:2107.03393

# Distribution of Correlation functions

$$\ln G_1^{\mathbf{m}}(t) = t(\lambda_1^{\mathbf{m}} - \lambda_0^{\mathbf{m}})$$

Keeping the full quantum nature of the gates




$t \gg L$

Is the distribution of correlation functions multi-fractal?

$$\overline{[G_1^{\mathbf{m}}(t)]^n} \sim \frac{1}{t^{x_1(n)}}$$

i.e. is $x_1(n)$ non-linear in *n* ???

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$$x_1(n) = n x_1^{(1)} + \frac{n^2}{2!} x_1^{(2)} + \ldots$$

can compute via cumulants $k_i$ of $\log G_1^{\mathbf{m}}$

$$k_i / tL \sim -2\pi x_1^{(i)} / L^2$$

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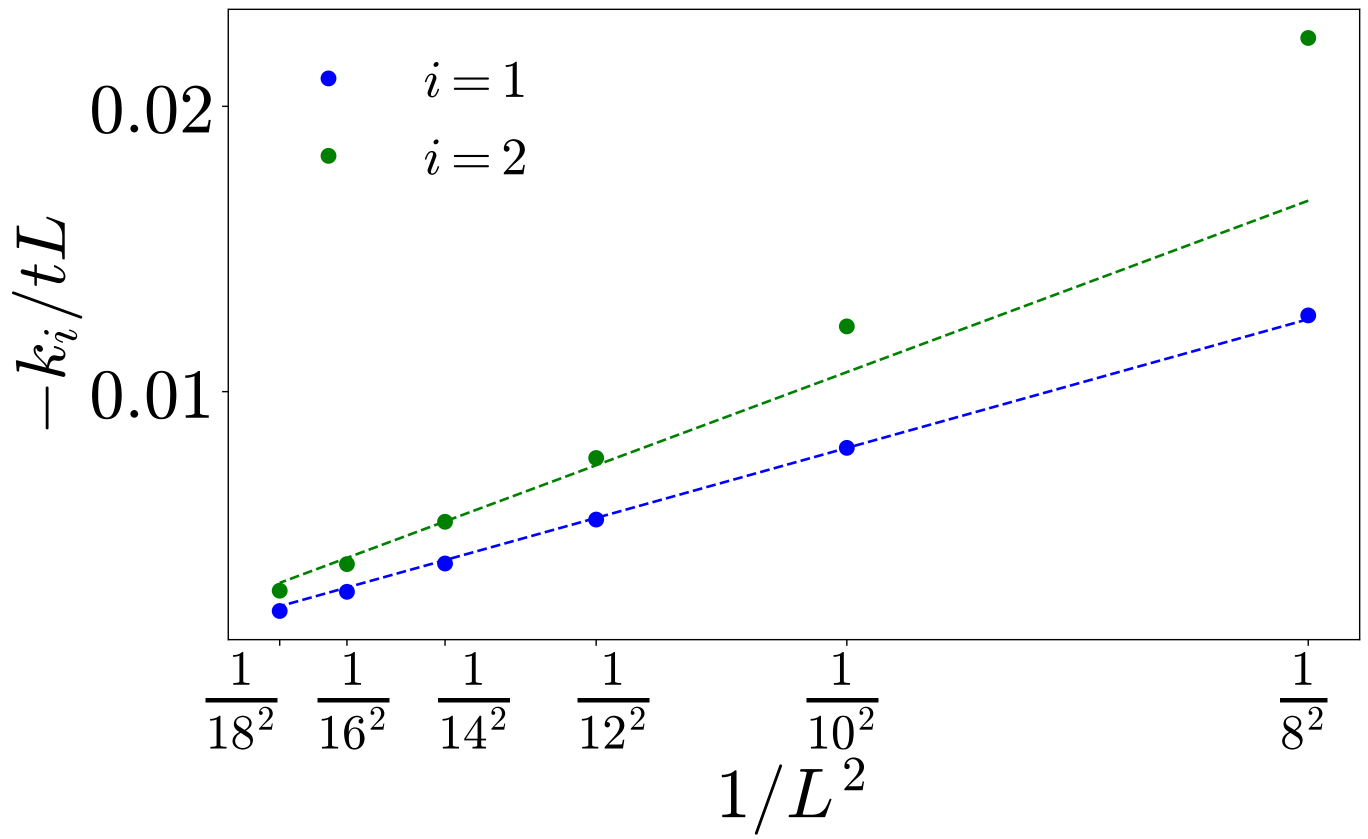


$$x_1(n) = 0.122n + 0.145\frac{n^2}{2!} + \dots$$

**multifractal!**

Zabalo, Gullans, Wilson, Vasseur, Ludwig, Gopalakrishnan, Huse, JHP arXiv:2107.03393

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Multifractal nature should appear as universal scaling of the distribution of

$$Y(t) \equiv -\ln G_1^{\mathbf{m}}(t)$$

$$P\left[Y(t)\right] \sim \left(\frac{2\pi\alpha t}{L}\right)^{-1/2} \exp\left[-\frac{2\pi\alpha t}{L} H\left(\frac{Y(t)}{2\pi\alpha t/L}\right)\right]$$

universal scaling function

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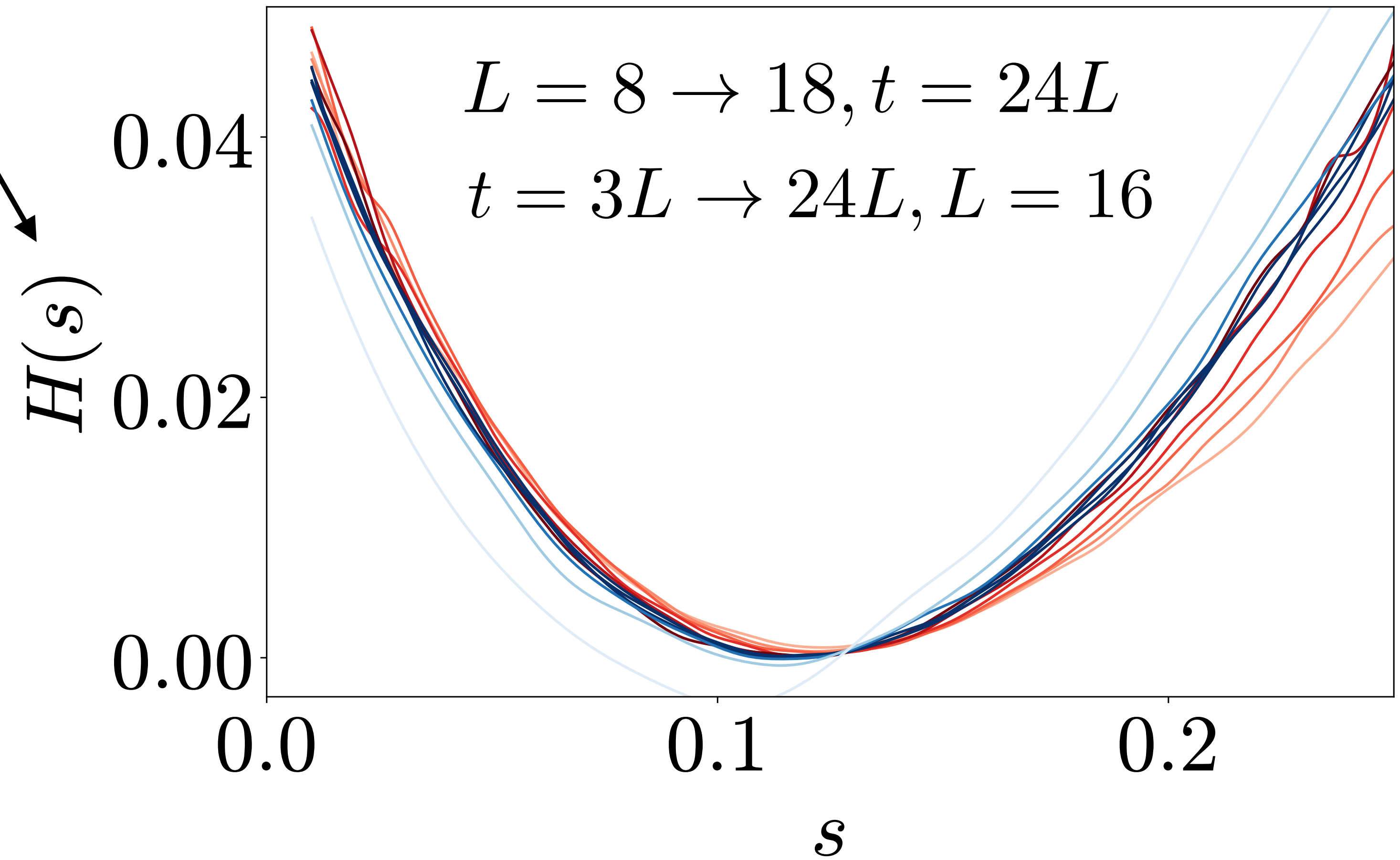

# Summary so far

Have identified 3 log-CFTs:
Percolation
Clifford transition
Haar transition

| | Haar | Dual unitary | Clifford | Dual Clifford | $d=\infty$ Haar/Clifford |
|---|---|---|---|---|---|
| $c_{\mathrm{eff}}$ | 0.25(3) | 0.24(2) | 0.37(1) | | 0.2914/0.3652 |
| $x_1$ | $0.14(2)^{\dagger}$ | $0.122(1)^{\dagger}$ | 0.120(5) | 0.111(1) | 0.1042 |
| MF | ✓ | ✓ | ✗ | ✗ | ✗ |

Quantitative Distinction: Critical exponents all look rather similar, effective central charge can tell them apart.

Qualitative distinction: Only the Haar transition is multifractal

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Quantitative Distinction: Critical exponents all look rather similar, effective central charge can tell them apart.

Qualitative distinction: Only the Haar transition is multifractal

But entanglement takes exponential resources,
Can this be seen in experiment?

# Now you see it, now you don't (Post Selection Problem)

Unusual phase transition, takes place in the **information**, spatial correlation functions averaged over measurement outcomes do not see it!

Skinner, Ruhman, Nahum PRX (2019)

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$$C_{\mathbf{m}}(x,y) = \langle \Psi_{\mathbf{m}}(t)|S^z(x)S^z(y)|\Psi_{\mathbf{m}}(t)\rangle \quad C(x,y) = \mathbb{E}_{\mathbf{m}} C_{\mathbf{m}}(x,y)$$

$$\mathbf{m} = (\vec{m}_1, \vec{m}_2, \ldots, \vec{m}_t)$$ measurement record

avg. over measurement outcomes

$$C(x,y) = \mathrm{Tr}\left(\rho_{\mathrm{av}} S_z(x) S_z(y)\right)$$

at late times will approach the infinite temperature Gibbs ensemble **for any *p***

$$\rho_{\mathrm{av}} = \frac{1}{2^L}\sum_{\mathbf{m}} |\Psi_{\mathbf{m}}(t)\rangle\langle\Psi_{\mathbf{m}}(t)|$$

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avg. over measurement outcomes

For correlations to "witness" the transition must construct non-linear averages over the measurement outcomes e.g.

$$C^2(x,y) = \mathbb{E}_{\mathbf{m}}\left[C_{\mathbf{m}}(x,y)^2\right] - \left[\mathbb{E}_{\mathbf{m}} C_{\mathbf{m}}(x,y)\right]^2$$

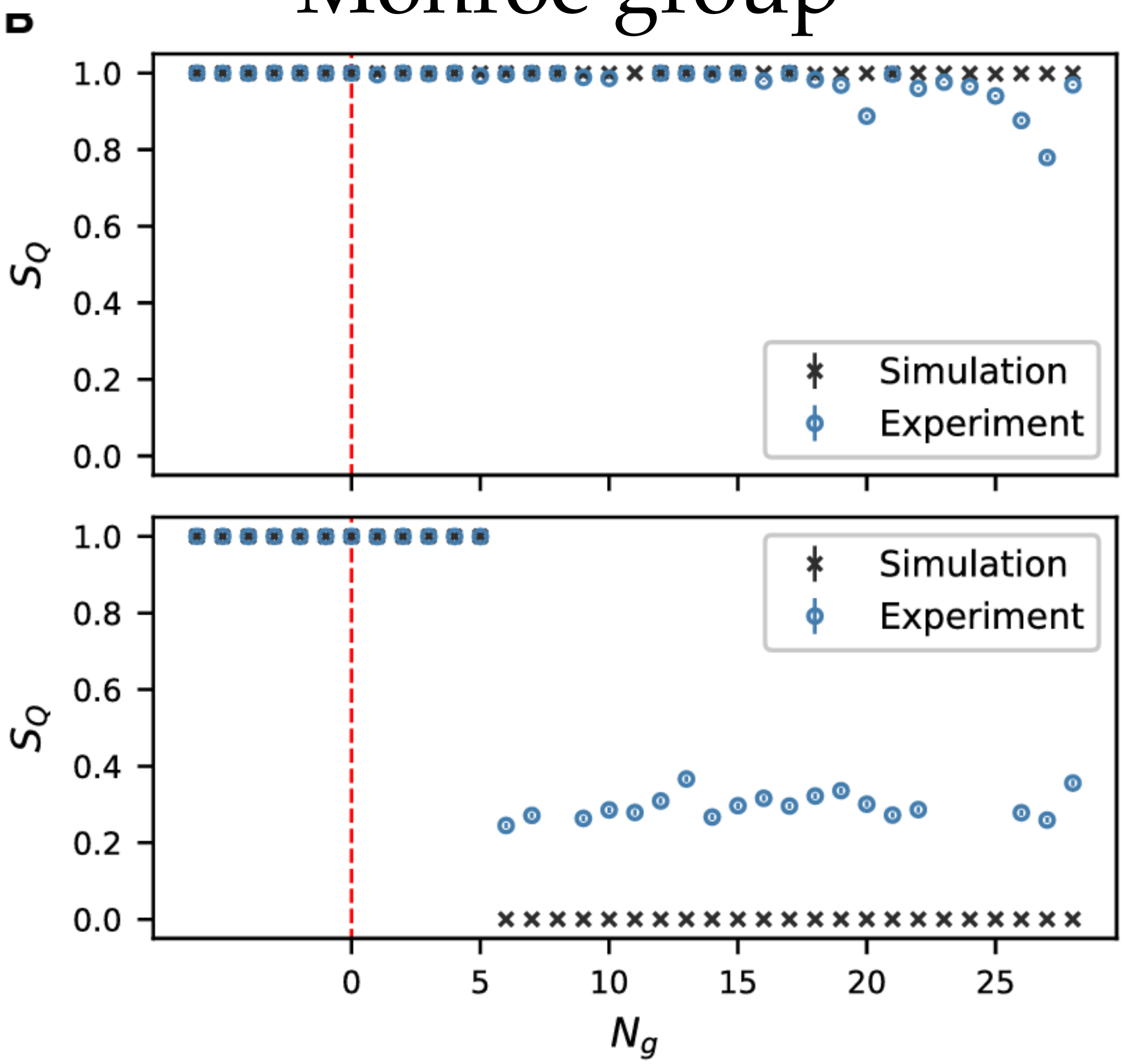
$S_E = -\mathbb{E}_{\mathbf{m}} \mathrm{Tr}_A(\rho_A^{\mathbf{m}} \log \rho_A^{\mathbf{m}})$ $\quad \rho_A^{\mathbf{m}} = \mathrm{Tr}\left(|\psi_{\mathbf{m}}(t)\rangle\langle\psi_{\mathbf{m}}(t)|\right)$

Skinner, Ruhman, Nahum PRX (2019)

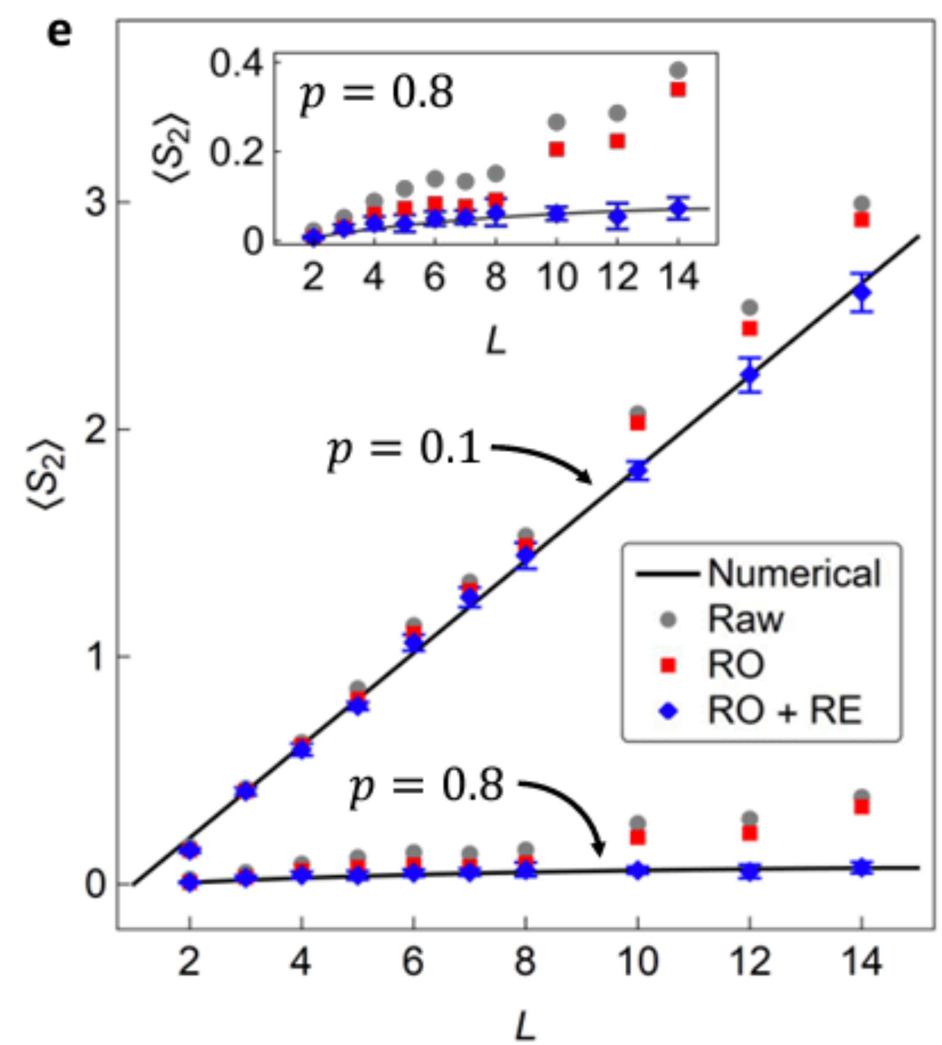
# Experimental Data

Experiments on trapped ions and superconducting qubits have seen signatures of the two phases using a great deal of post-selection

Monroe group




IBM



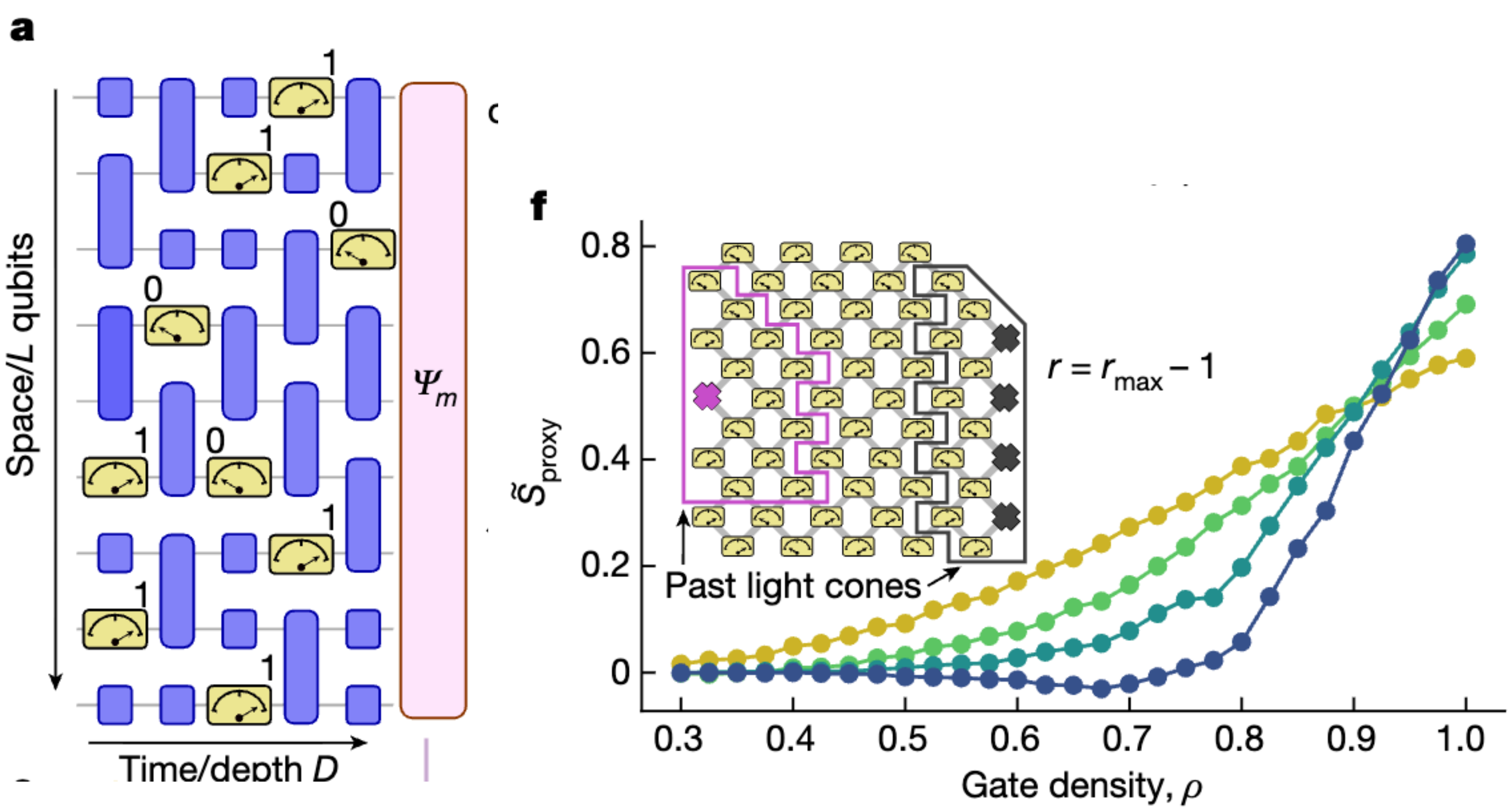

Noel et al Nat. Phys. (2022)

Koh, Sun, Motta, Minnich, Nat. Phys (2023)

# Experimental Data

Google used a short depth circuit but lots of qubits, to probe a version of this transition at finite time. Bao, Block, Altman PRL (2024)

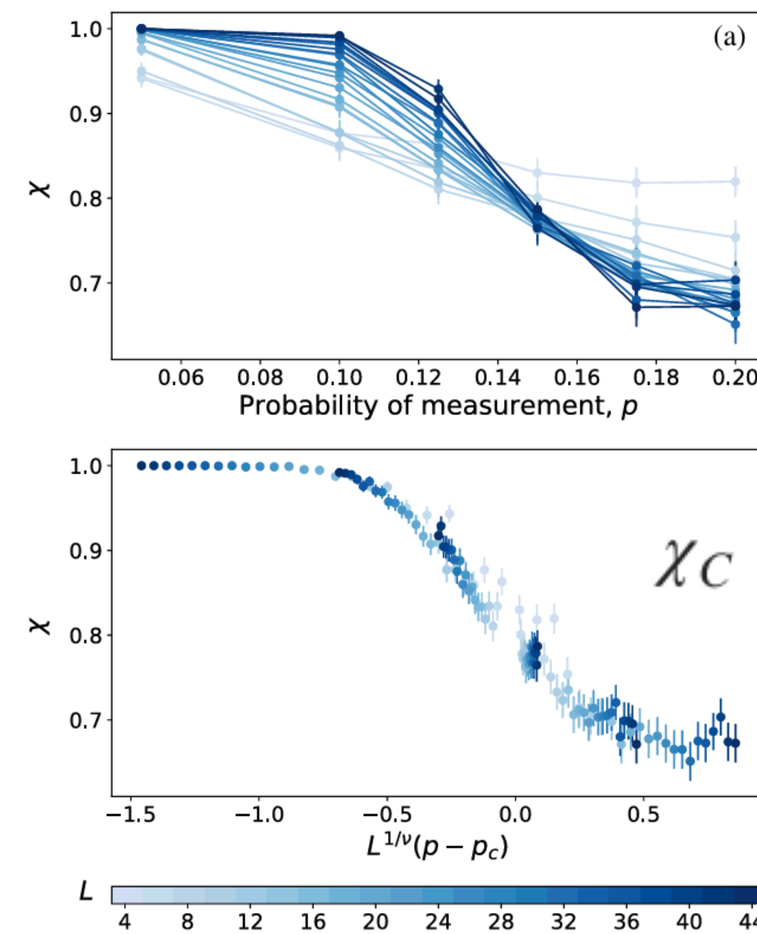
## Google




Google Quantum AI and Collaborators, Nature (2023)

## IBM

### Cross entropy benchmark

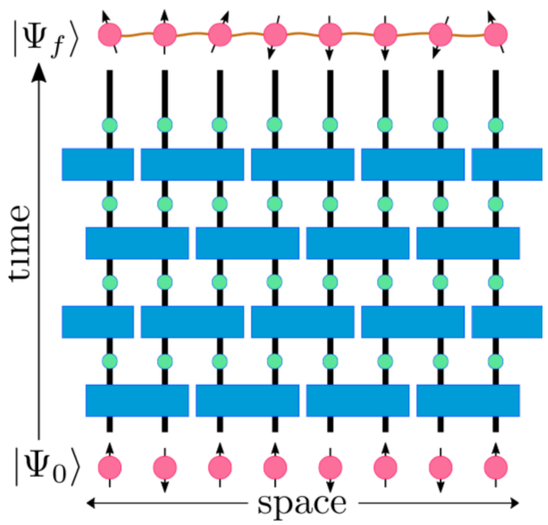



$$\chi_C = \frac{\sum_{\mathbf{m}} p^{\rho}_{\mathbf{m}} p^{\sigma}_{\mathbf{m}}}{\sum_{\mathbf{m}} (p^{\sigma}_{\mathbf{m}})^2}$$

Kamakari et al PRL (2025)

# Outline

I. Motivation




II. Measurement Induced Transition (MIT)

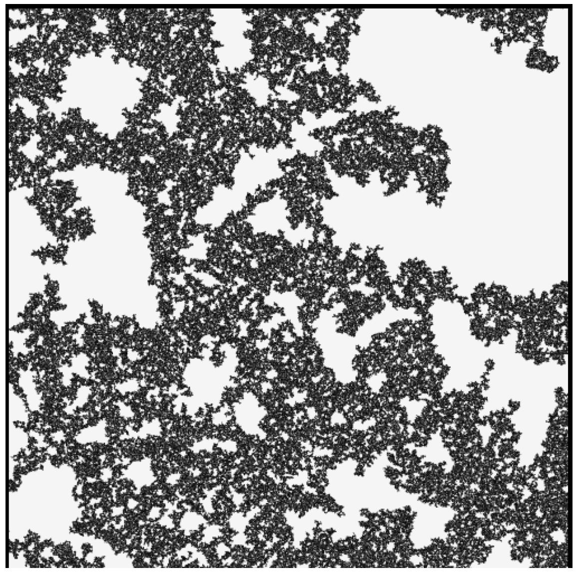
- Models
- Controlled limits
- Numerical solution for qubits

Time for a break?

III. Extensions

$$|\Psi(t)\rangle \to \frac{P_i^m|\Psi(t)\rangle}{\langle\Psi(t)|P_i^m|\Psi(t)\rangle}$$

- Weak measurements
- Projector only models
- Free fermions

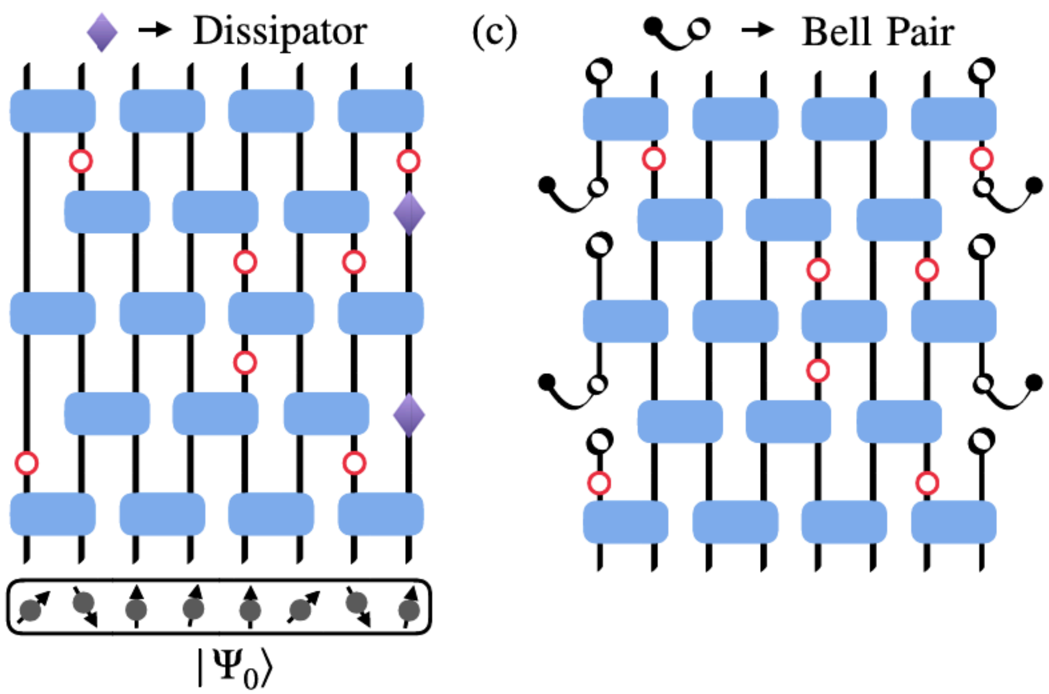


IV. Stability of the MIPT

- Presence of symmetry
- Static and temporal perturbations

# Extensions

Transfer matrix description can be generalized to only probe boundary operators




PHYSICAL REVIEW B **109**, 014303 (2024)

**Boundary transfer matrix spectrum of measurement-induced transitions**

Abhishek Kumar,[1] Kemal Aziz,[2] Ahana Chakraborty,[2] Andreas W. W. Ludwig,[3]
Sarang Gopalakrishnan,[4] J. H. Pixley,[2,5] and Romain Vasseur[1,*]

$$f_{(\alpha|\beta)} = f(L=\infty) + \frac{f_s^{(\alpha|\beta)}}{L} + \frac{\pi h_{\alpha|\beta}}{L^2} - \frac{\pi c_{\text{eff}}}{24L^2}$$

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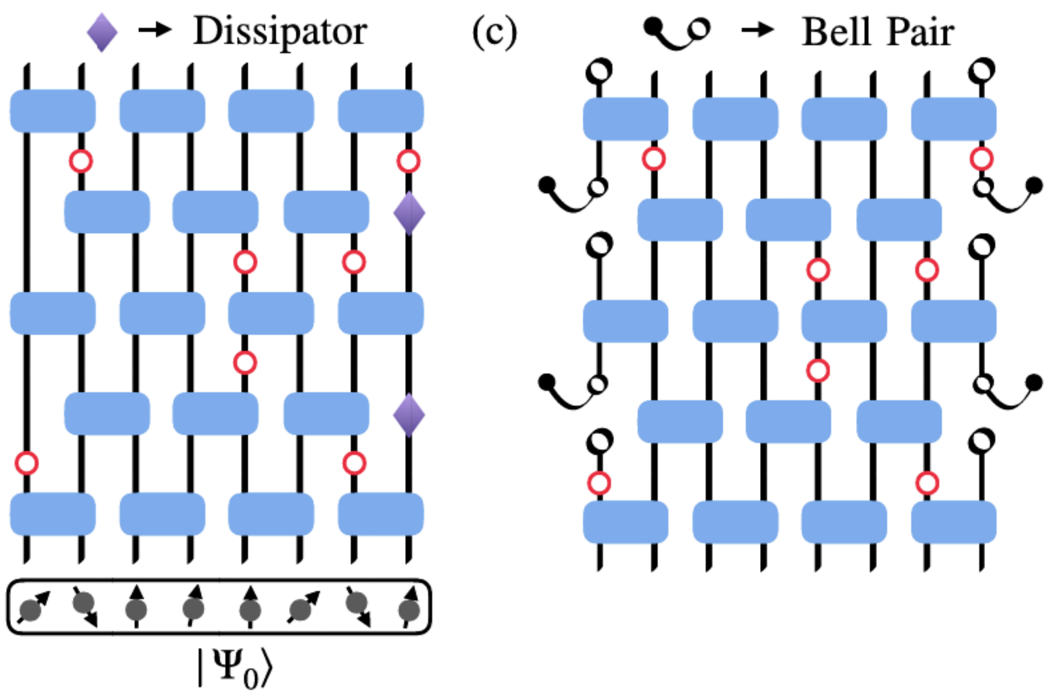


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$$S_n \sim A \log L \qquad\qquad S_A \sim 2h_{a|b} \ln L_A$$

Coefficient of the log is the scaling dimension of a boundary operator.

Very demanding simulations, over 1 million samples/data point

# Extensions

Circuits (are of course) not essential: Unitary time evolution from chaotic many-body Hamiltonians. $U(\delta t) = e^{-i\delta t H}$

Tang and Zhu, PRR (2020)

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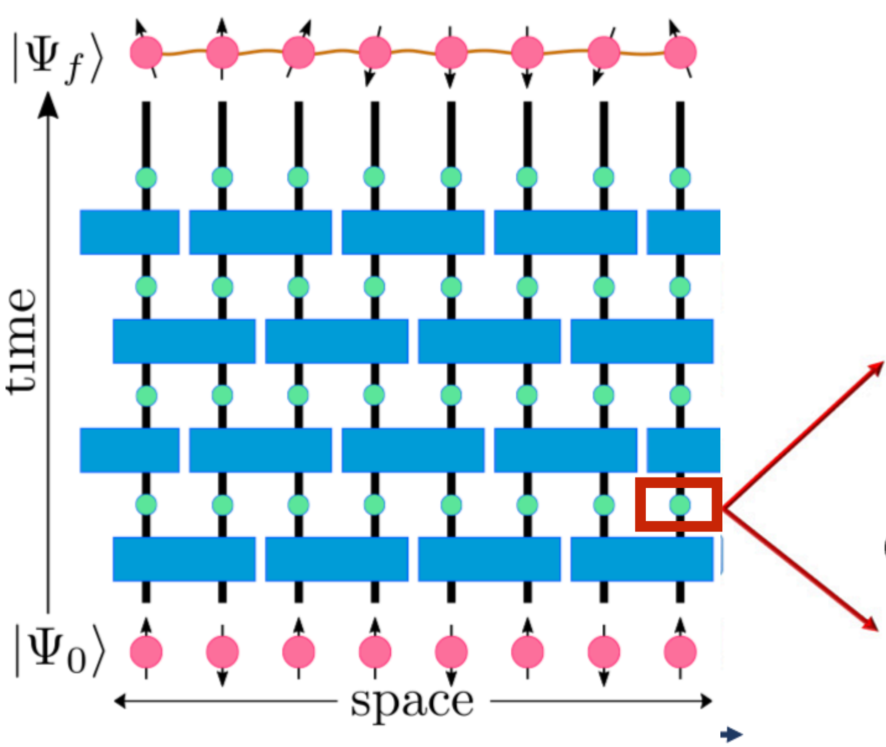
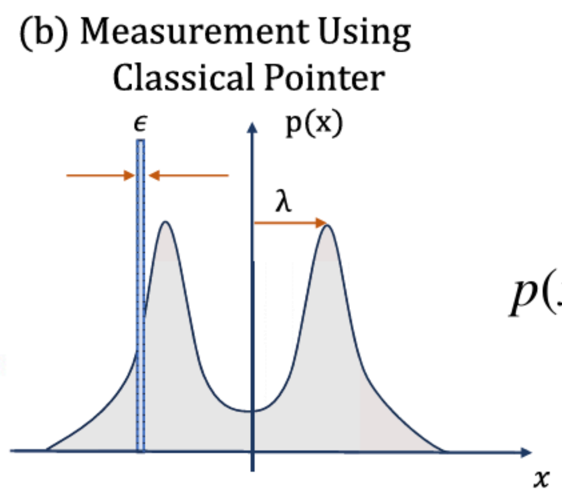
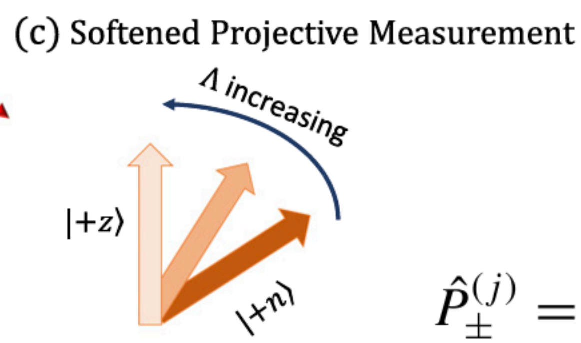



$$\hat{H}_{\text{int}} = \lambda\Theta(t)\Theta(\delta t - t)\sigma_z^{(j)} \otimes \hat{q}.$$

$$p(x_o;\lambda/\Delta) = \langle\psi|\Pi_+^{(j)}|\psi\rangle G_\Delta^2(x_o - \lambda) + \langle\psi|\Pi_-^{(j)}|\psi\rangle G_\Delta^2(x_o + \lambda)$$

$$\Pi_\pm^{(j)} = [(\mathbb{I} \pm \sigma_z^{(j)})/2] \otimes_{i\neq j} \mathbb{I}^{(i)} \qquad G_\Delta(x) = \frac{e^{\frac{-x^2}{2\Delta^2}}}{\pi^{1/4}\Delta^{1/4}}$$




$$\hat{P}_\pm^{(j)} = \frac{1 \pm \Lambda\sigma_z^{(j)}}{\sqrt{2(1+\Lambda^2)}}$$

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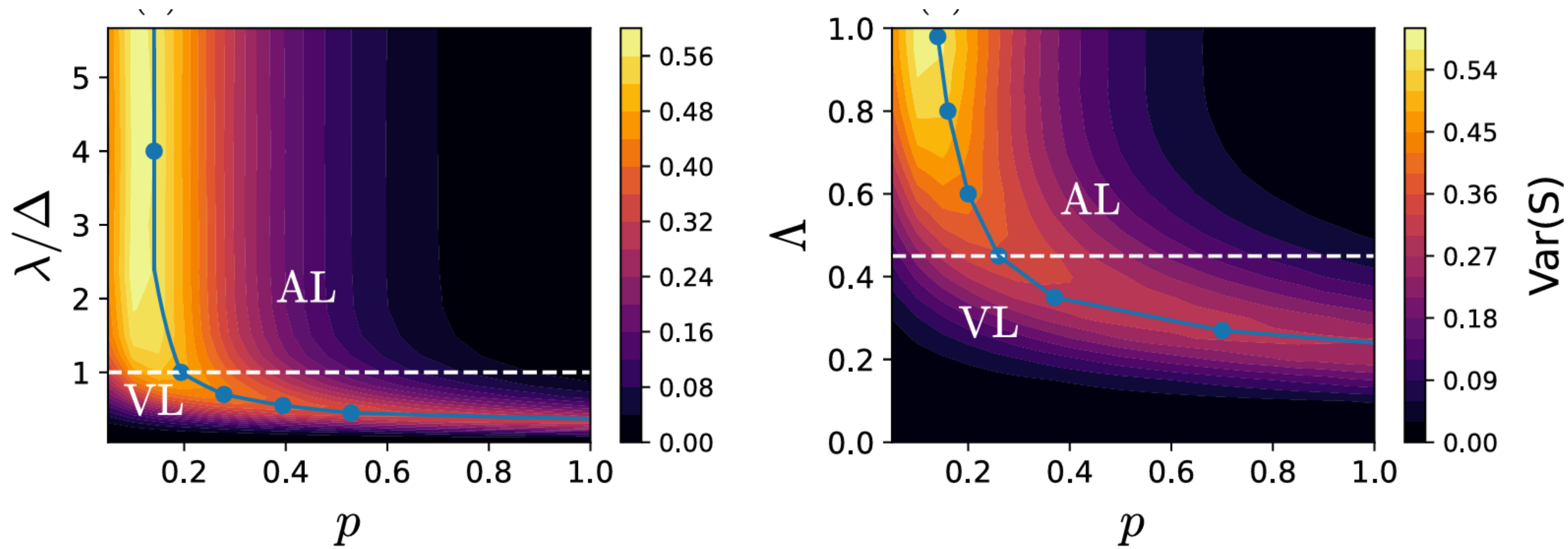

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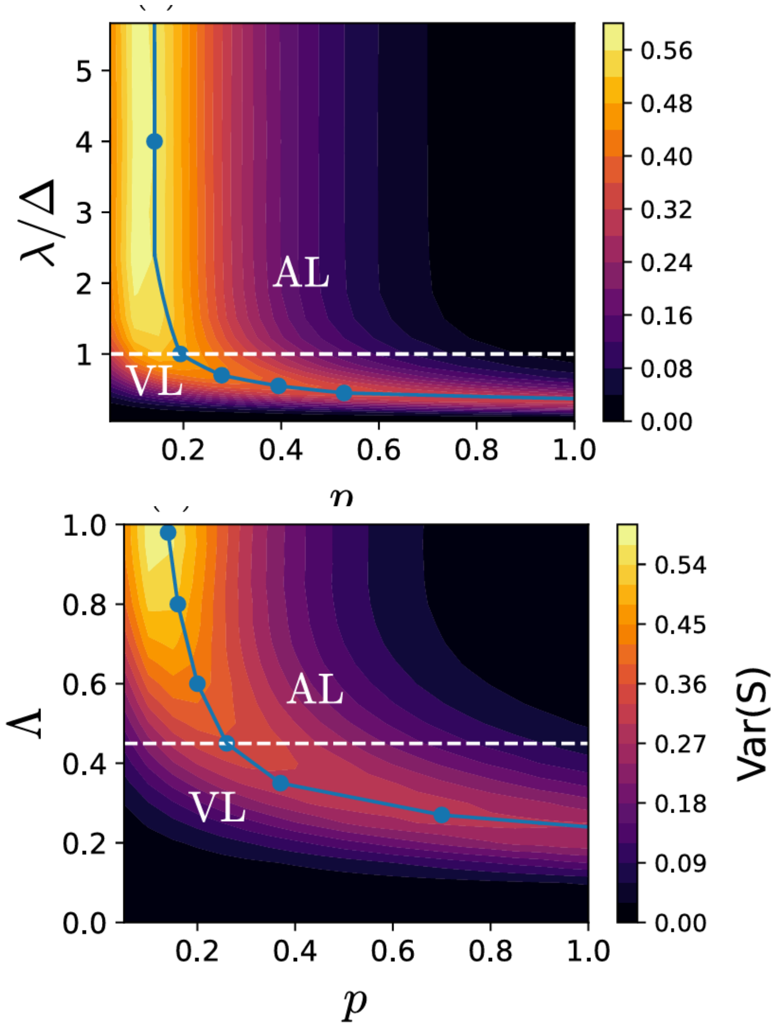


| | $\frac{\lambda}{\Delta} = \infty$ | $\frac{\lambda}{\Delta} = 1$ | $\Lambda = 0.45$ |
|---|---|---|---|
| $p_c$ | 0.14(1) | 0.19(1) | 0.28(2) |
| $\nu$ | 1.3(3) | 1.3(3) | 1.6(3) |
| $z$ | 0.98(8) | 0.94(6) | 0.95(5) |
| $\eta$ | 0.23(2) | 0.21(2) | 0.19(3) |
| $c_{\text{eff}}$ | 0.24(2) | 0.25(3) | 0.26(2) |
| $x_1^{\text{typ}}$ | 0.12(2) | 0.14(2) | 0.12(2) |
| $x_1^{(2)}$ | 0.14(2) | 0.19(2) | 0.14(2) |

$$x_1^{\text{typ}} = \eta/2$$

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Models with *fully non-commuting projective* dynamics.

Lang and Büchler, PRB (2020)

Ippoliti, Gullans, Gopalakrishnan, Huse, Khemani, PRX (2021)

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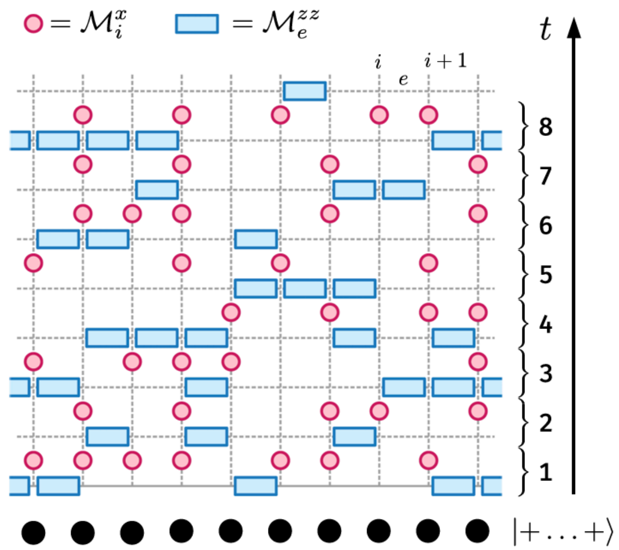


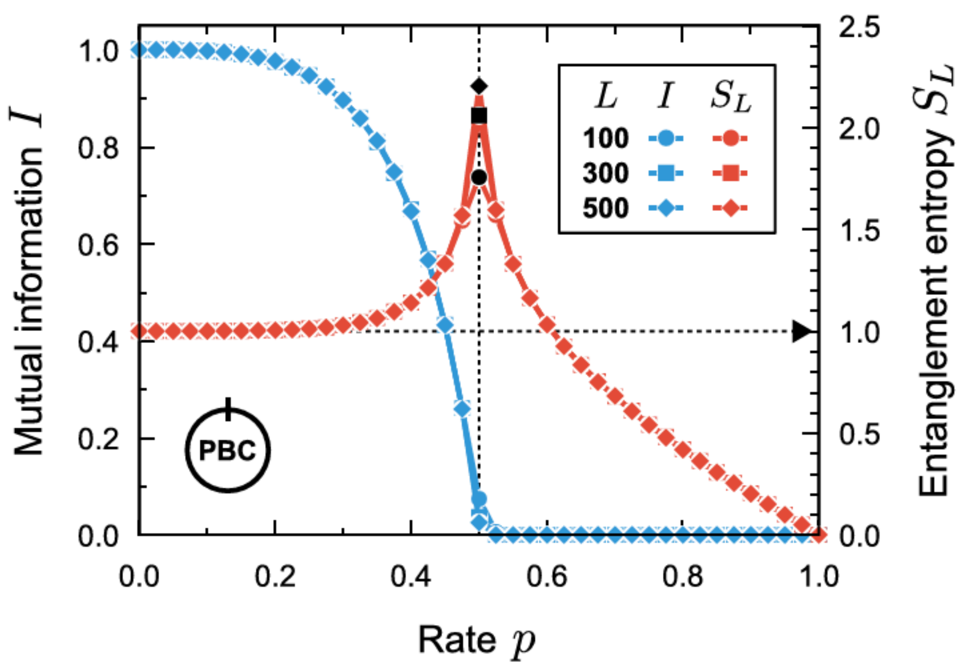

$$\mathcal{M}[O](|\Psi\rangle) = \frac{P_\lambda|\Psi\rangle}{\sqrt{\langle\Psi|P_\lambda|\Psi\rangle}}$$

$$\mathcal{M}_i^x \equiv \mathcal{M}[\sigma_i^x] \quad \text{with} \quad P_\lambda = \tfrac{1}{2}(\mathbb{1} + \lambda\,\sigma_i^x),$$

$$\mathcal{M}_e^{zz} \equiv \mathcal{M}[\sigma_i^z\sigma_{i+1}^z] \quad \text{with} \quad P_\lambda = \tfrac{1}{2}(\mathbb{1} + \lambda\,\sigma_i^z\sigma_{i+1}^z)$$

Lang and Büchler, PRB (2020)

Ippoliti, Gullans, Gopalakrishnan, Huse, Khemani, PRX (2021)



# Extensions

Circuits (are of course) not essential: Unitary time evolution from chaotic many-body Hamiltonians. $U(\delta t) = e^{-i\delta t H}$

Tang and Zhu, PRR (2020)

Relaxing *projective* measurements to *weak* measurements.

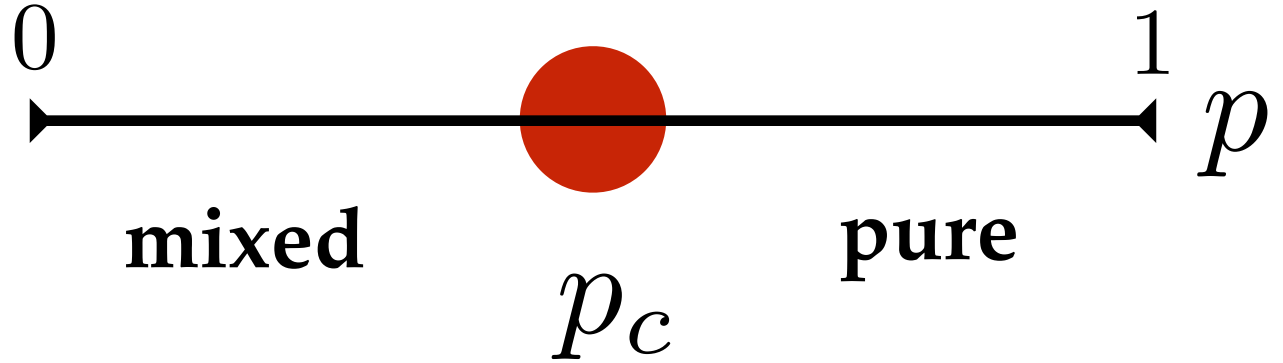
Szyniszewski, Romito, Schomerus, PRB (2019)

Models with *fully non-commuting projective* dynamics.

Lang and Büchler, PRB (2020)

Ippoliti, Gullans, Gopalakrishnan, Huse, Khemani, PRX (2021)

*Long-range interactions* remove the separation from volume law to area law: the transition then appears as a purification transition.




Gullans and Huse, PRX (2020)

# Extensions

## Higher dimensions (Stabilizer simulations confirm percolation like transition in any d+1)

| | Stabilizer circuits | | | Classical percolation | | |
|---|---|---|---|---|---|---|
| Exponent | (1 + 1)D | (2 + 1)D | (3 + 1)D | 2D | 3D | 4D |
| $\nu$ | 1.265(15) | 0.87(2) | 0.68(2) | 1.333 | 0.8774(13) | 0.686(2) |
| $\eta$ | 0.212(4) | −0.02(3) | −0.04(5) | 0.208 | − 0.03(1) | −0.084(4) |
| $\eta_{\parallel}$ | 0.70(2) | 0.99(4) | 1.5(2) | 0.667 | 1.08(10) | 1.37(13) |
| $\eta_{\perp}$ | 0.461(8) | 0.43(5) | 0.42(10) | 0.438 | 0.5(1) | 0.65(7) |
| $\beta$ | 0.129(8) | 0.44(1) | 0.60(3) | 0.139 | 0.429(4) | 0.658(1) |
| $\beta_s$ | 0.46(2) | 0.86(2) | 1.14(9) | 0.444 | 0.854(2) | 1.09(8) |
| $z$ | 1.00(1) | 1.01(2) | 1.02(4) | 1 | 1 | 1 |

Sierant, Schiro, Lewenstein, Turkeshi, PRB (2022)

Turkeshi, Fazio, Pal, PRB (2021)

O. Lunt, M. Szyniszewski, and A. Pal, PRB (2021)

# Extensions

## Higher dimensions (Stabilizer simulations confirm percolation like transition in any d+1)

| Exponent | Stabilizer circuits (1 + 1)D | Stabilizer circuits (2 + 1)D | Stabilizer circuits (3 + 1)D | Classical percolation 2D | Classical percolation 3D | Classical percolation 4D |
|---|---|---|---|---|---|---|
| $\nu$ | 1.265(15) | 0.87(2) | 0.68(2) | 1.333 | 0.8774(13) | 0.686(2) |
| $\eta$ | 0.212(4) | −0.02(3) | −0.04(5) | 0.208 | − 0.03(1) | −0.084(4) |
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Sierant, Schiro, Lewenstein, Turkeshi, PRB (2022)

Turkeshi, Fazio, Pal, PRB (2021)

O. Lunt, M. Szyniszewski, and A. Pal, PRB (2021)

## Free fermions: Turned out to be a subtle and difficult problem

PHYSICAL REVIEW B **106**, 134206 (2022)

Editors' Suggestion

**Criticality and entanglement in nonunitary quantum circuits and tensor networks of noninteracting fermions**

Chao-Ming Jian,[1] Bela Bauer,[2] Anna Keselman,[2,3] and Andreas W. W. Ludwig[4]

The ten fold way can now be used!

Entanglement *d*-dimensional monitored free fermions →

Localization in *(d+1)*-dimensional unitary disordered fermions

# Extensions

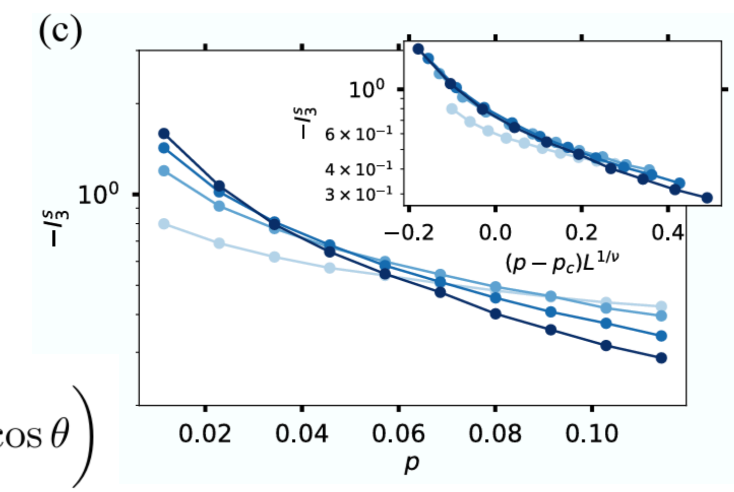
Measurement induced localization (Zeno effect) is incompatible with disorder induced localization.

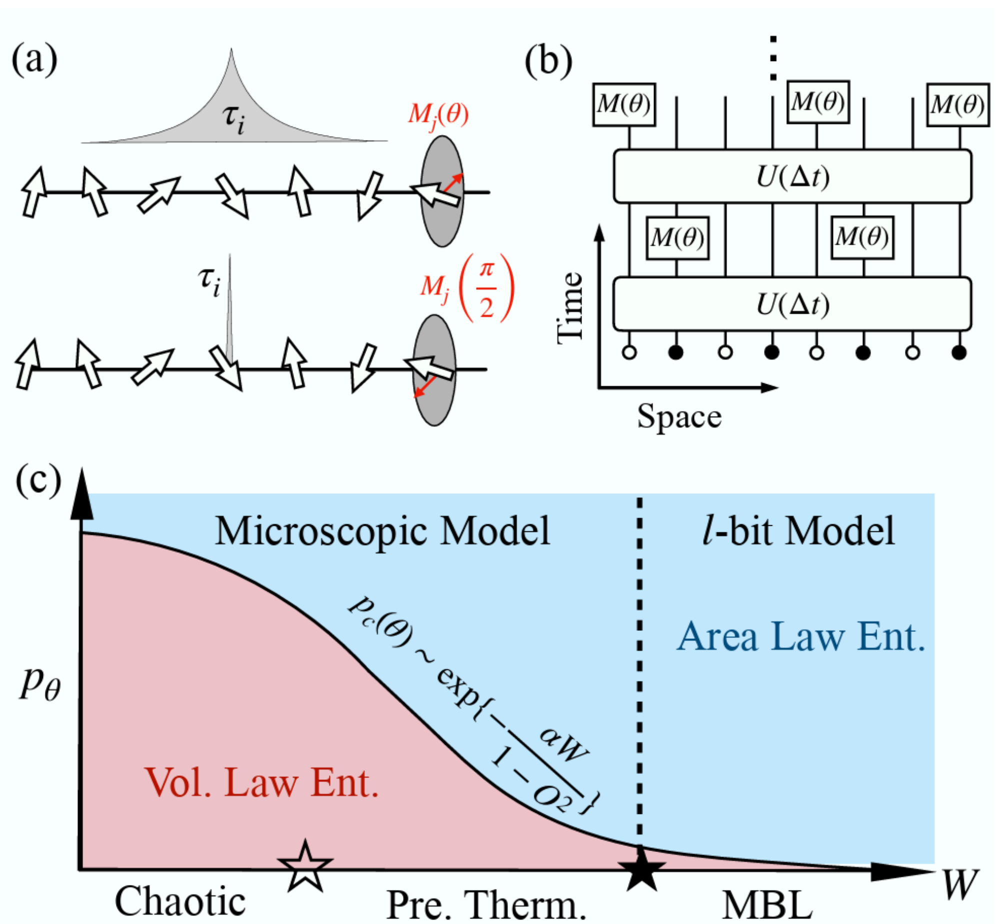
## Instability of MBL

Ippoliti, Gullans, Gopalakrishnan, Huse, Khemani, PRX (2021)

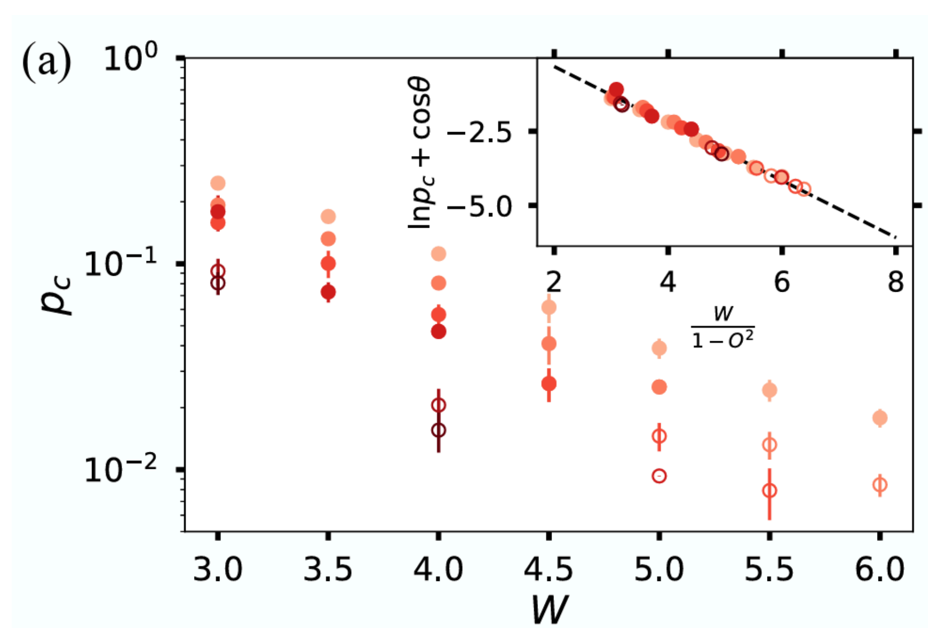
Sun, Trigueros, Tang, Heyl, PRB (2025)

Tang, Kattel, Pal, Yuzbashyan, JHP arXiv (2026)




$$p_c(W,\theta) = p_0 \exp\left(-\frac{\alpha W}{1-O(\theta,W)^2} - \cos\theta\right)$$



# Extensions

Measurement induced localization (Zeno effect) is incompatible with disorder induced localization.

Ippoliti, Gullans, Gopalakrishnan, Huse, Khemani, PRX (2021)

## Instability of MBL

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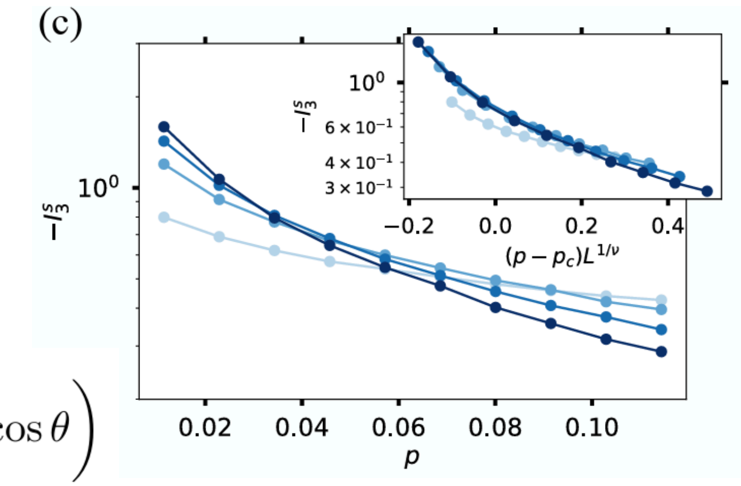


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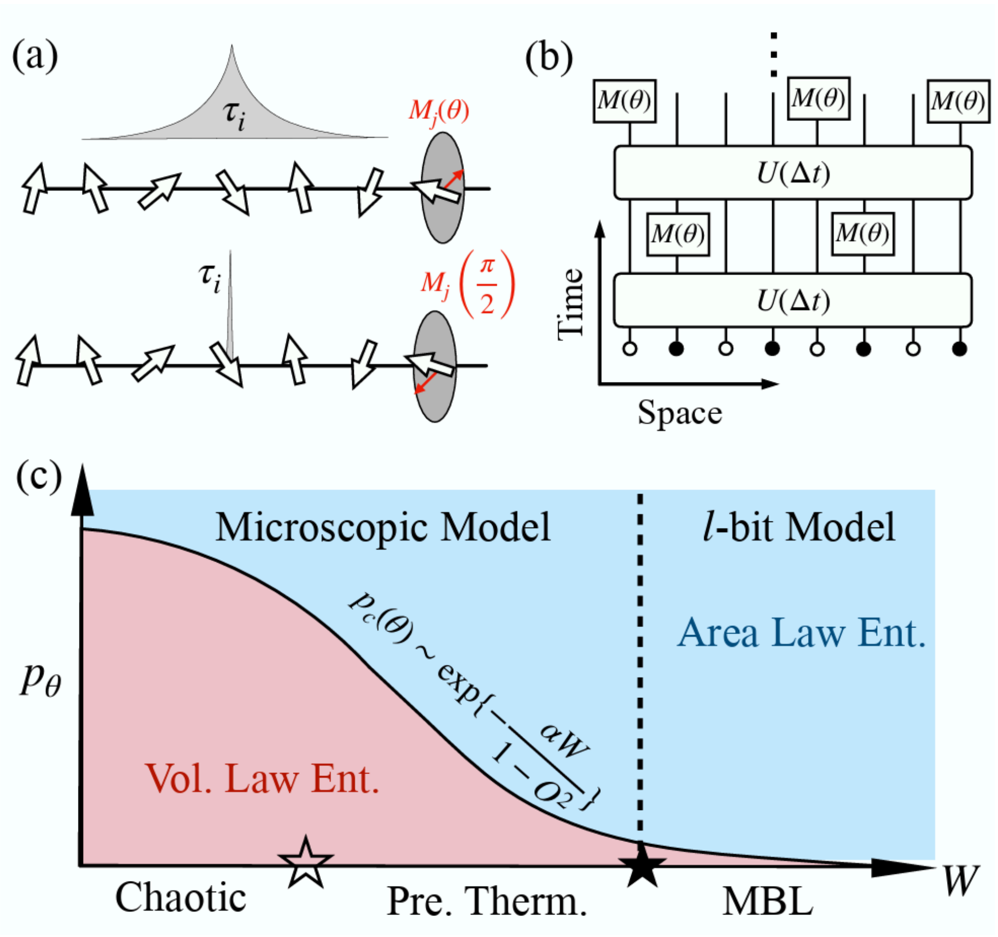
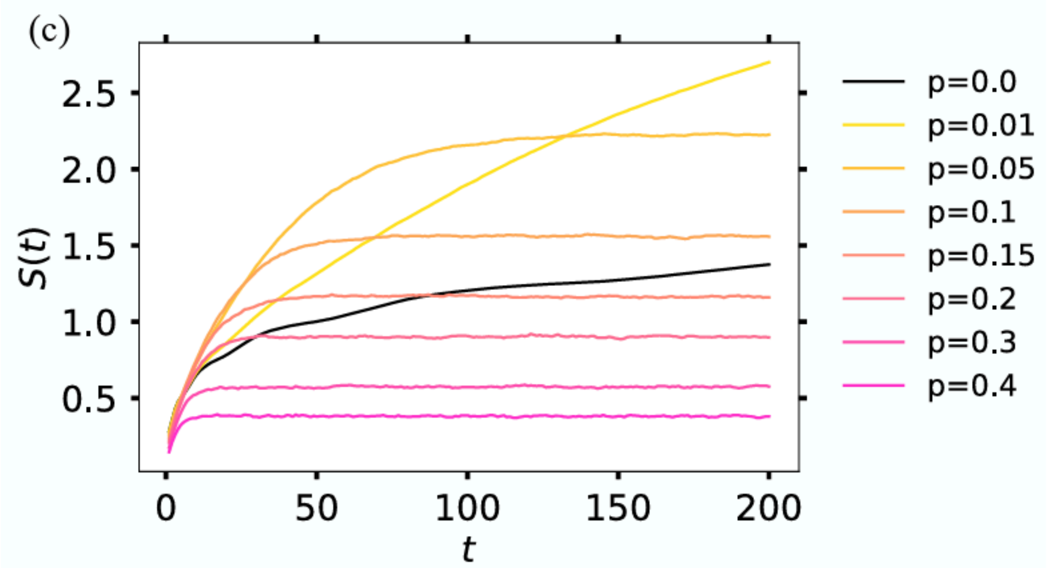
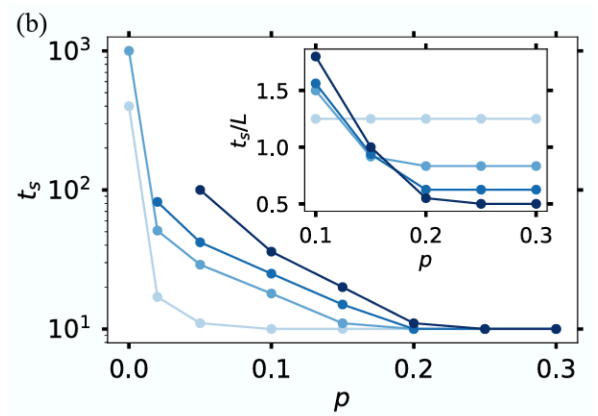
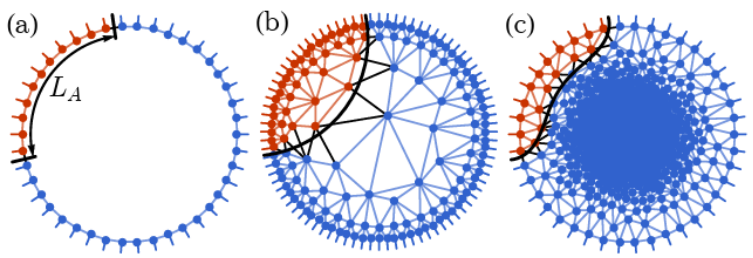


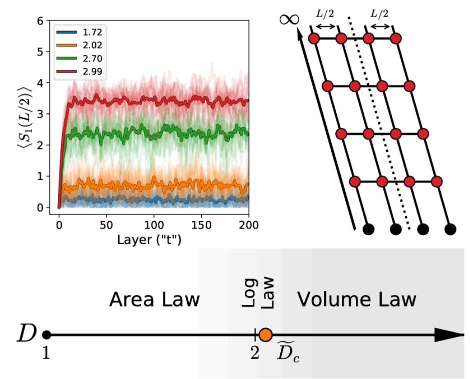



# Extensions (NOT Covered)

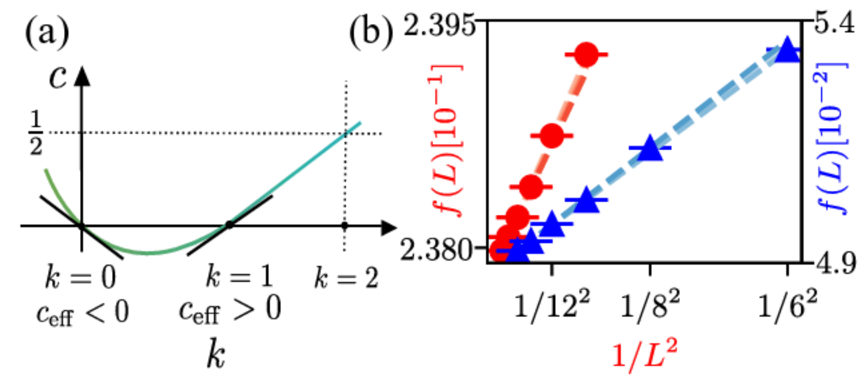
## Tensor network transitions and the post selection transition




Vasseur, Potter, You, Ludwig PRB (2019)

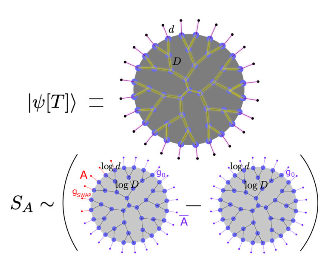



Levi and Clark, PRB (2022)

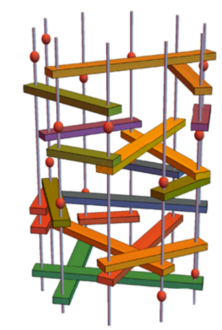



Nambi, et al, arXiv (2026)

## Mean field limits on non-trivial graphs/all to all connectivity

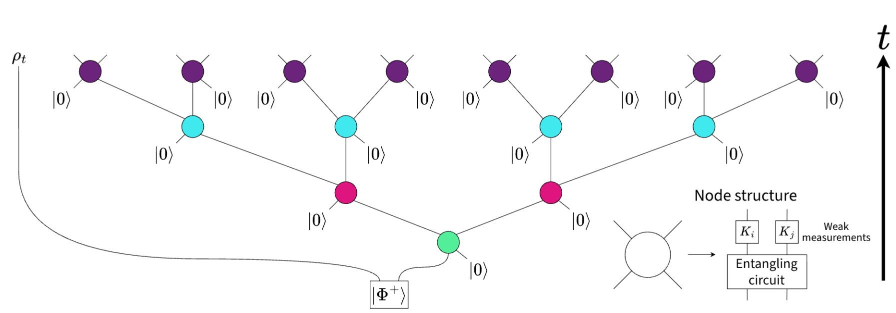


Lopez-Piqueres, Ware, Vasseur PRB (2020)

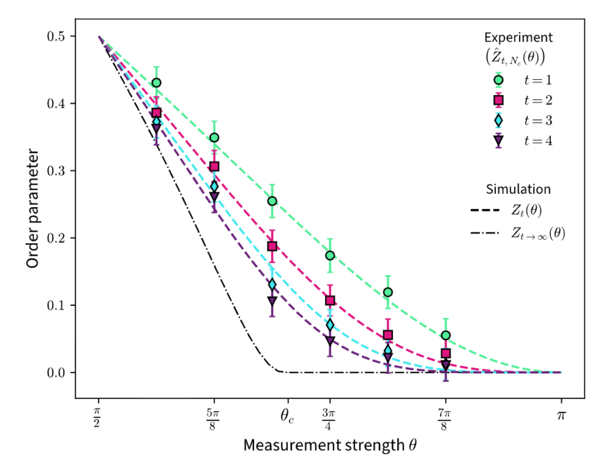


Nahum, Roy, Skinner, Ruhman PRX Quantum (2021)

## Measurement induced transition observable on trees without postselection.



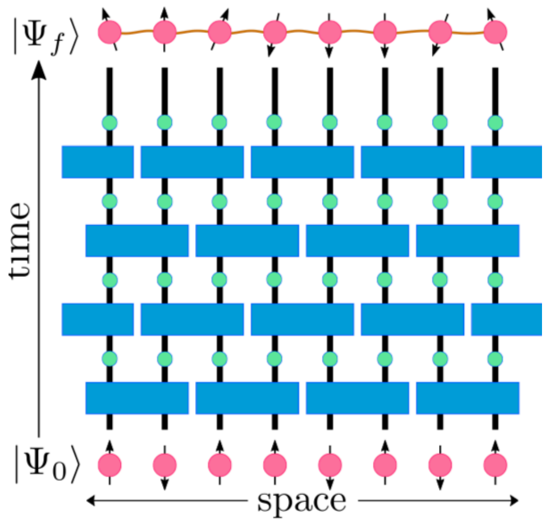

Feng, Cote, Kourtis, Skinner, Comm. Phys. (2026)




Data from trapped ions

# Outline

## I. Motivation




## II. Measurement Induced Transition (MIT)

- Models
- Controlled limits
- Numerical solution for qubits

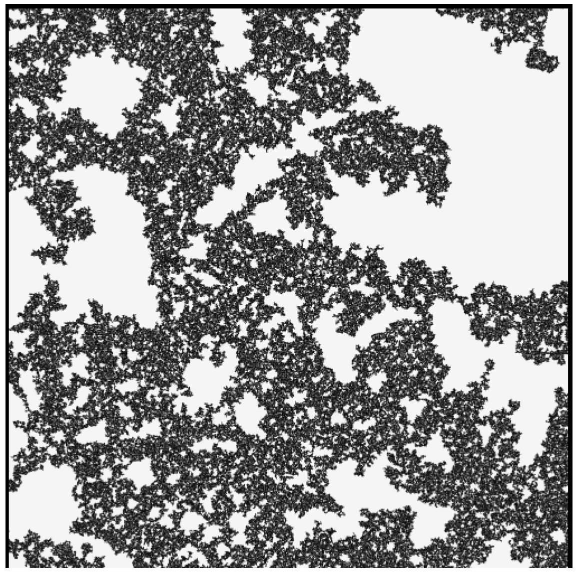
$$|\Psi(t)\rangle \to \frac{P_i^m|\Psi(t)\rangle}{\langle\Psi(t)|P_i^m|\Psi(t)\rangle}$$

## III. Extensions

- Weak measurements
- Projector only models
- Free fermions

## IV. Stability of the MIPT

- Presence of symmetry
- Static and temporal perturbations



# Relevant Perturbations

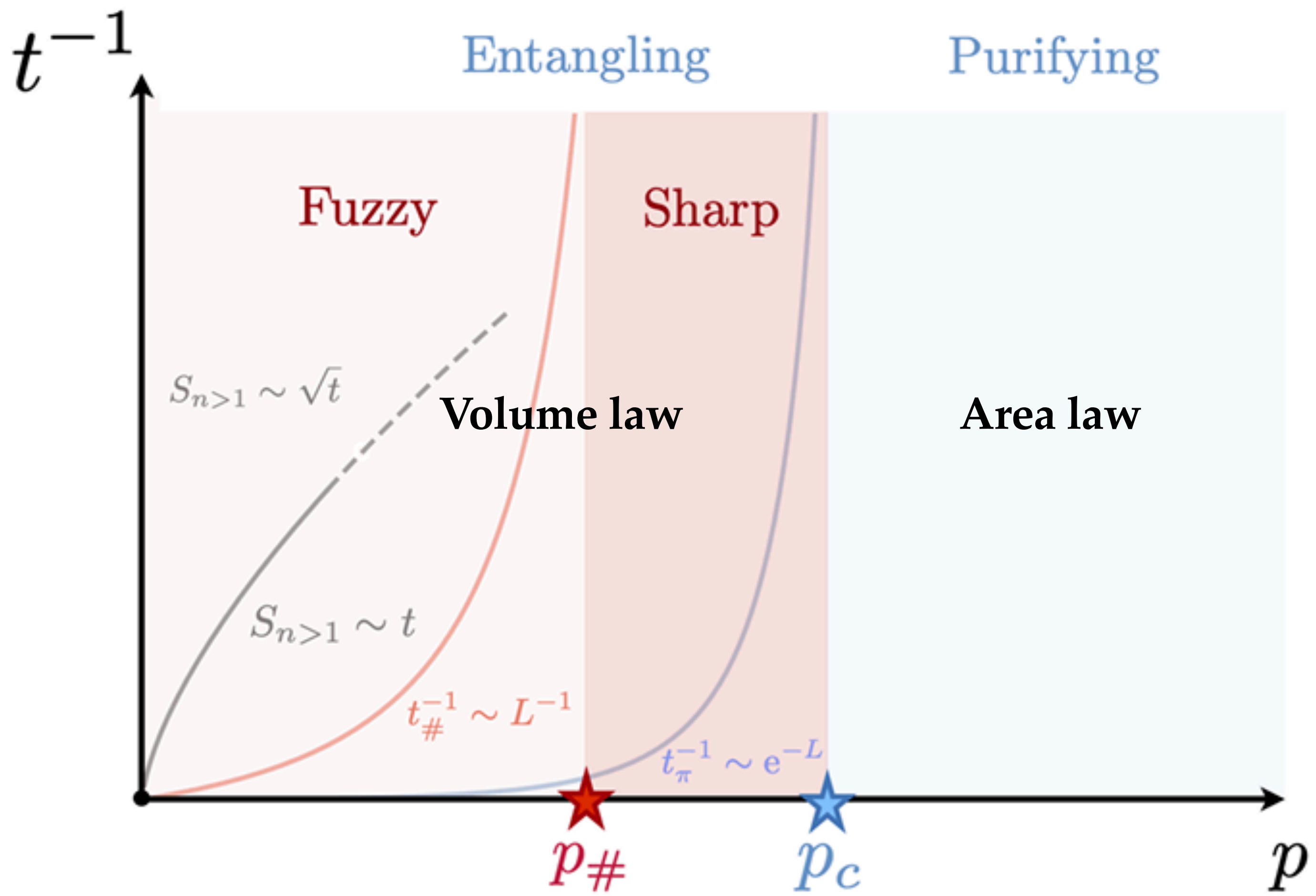
We now consider modifications to the circuit dynamics that fundamentally alter the problem.

Enforcing continuous symmetries in the hybrid dynamics

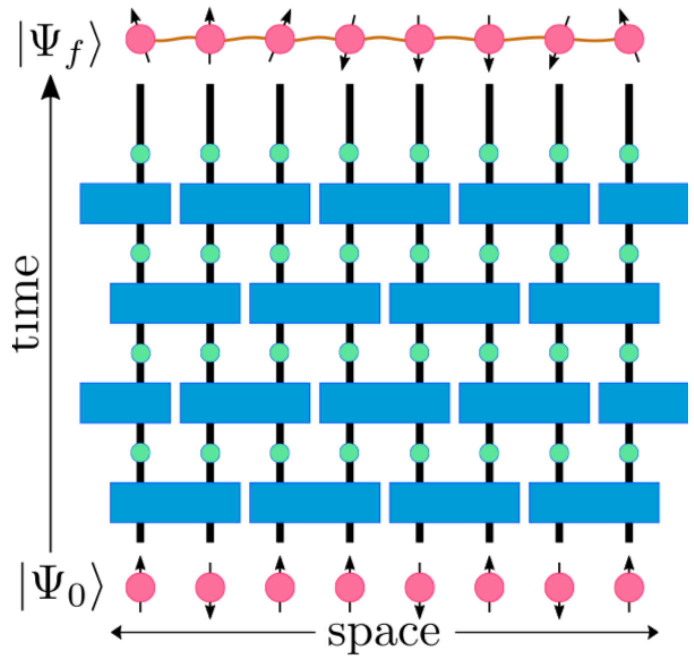
Including static and temporal perturbations

# Charge sharpening transition

U(1) Conserving monitored dynamics




Sharpening transition

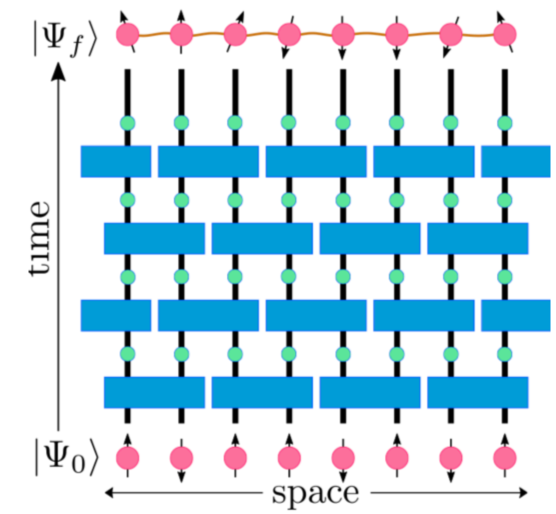


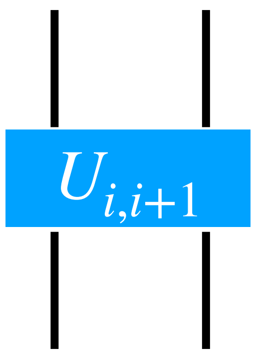

Gates and measurements preserve the symmetry

Agrawal, Zabalo, Chen, Wilson, Potter, **JHP**, Gopalakrishnan, Vasseur PRX (2022)

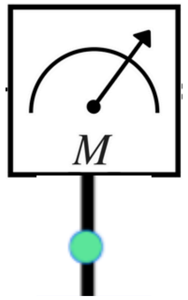
# Charge sharpening transition: Models

U(1) Conserving monitored dynamics: **Qubits**




$$U_{i,i+1} = \begin{bmatrix} e^{i\phi_0} & 0 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{$U_{2\times 2}$}} & 0 \\ 0 & & & 0 \\ 0 & 0 & 0 & e^{i\phi_1} \end{bmatrix}$$




$$S_i^z = \frac{1}{2}(P_i^{\uparrow} - P_i^{\downarrow})$$

Conserved quantity

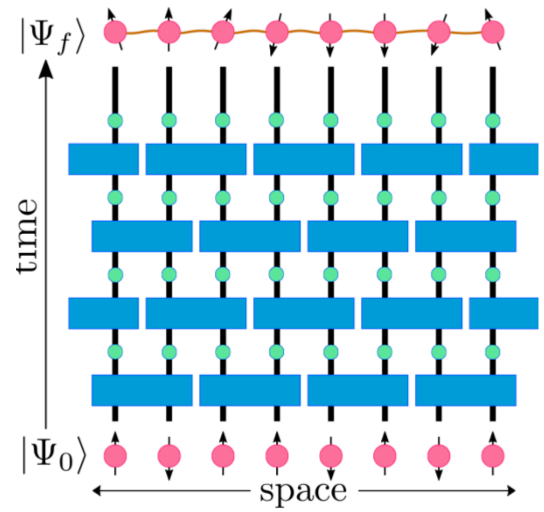
$$S^z_{\text{tot}} = \sum_i S_i^z$$

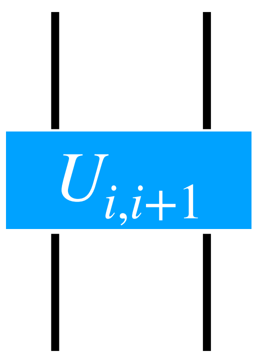
"Charge"

$$\mathcal{Q} = S^z_{\text{tot}} + L/2$$

# Charge sharpening transition: Models

## U(1) Conserving monitored dynamics: Qubits

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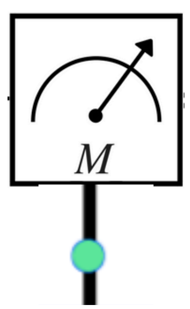



$$S_i^z = \frac{1}{2}(P_i^{\uparrow} - P_i^{\downarrow})$$

Conserved quantity

$$S^z_{\text{tot}} = \sum_i S_i^z$$

"Charge"

$$Q = S^z_{\text{tot}} + L/2$$

## U(1) Conserving monitored dynamics: **Qudits**

$$\mathcal{Q} \equiv \sum_i \mathfrak{q}_i \otimes \mathbb{I}_i,$$

$$\mathfrak{q}_i = (\sigma_i^z + 1)/2$$

Local Hilbert space

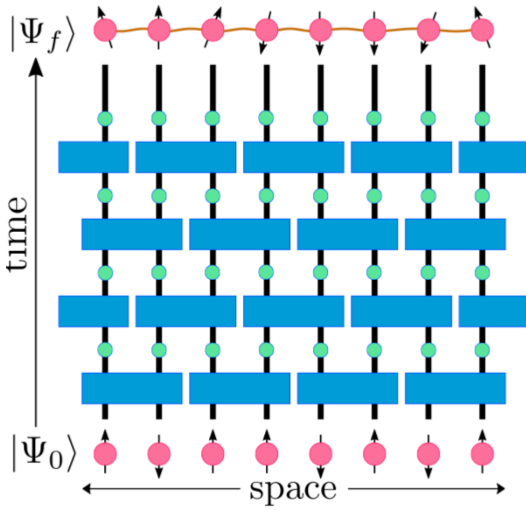
$\mathbb{C}^2 \otimes \mathbb{C}^d$ for $d > 1$

$$d \to \infty$$

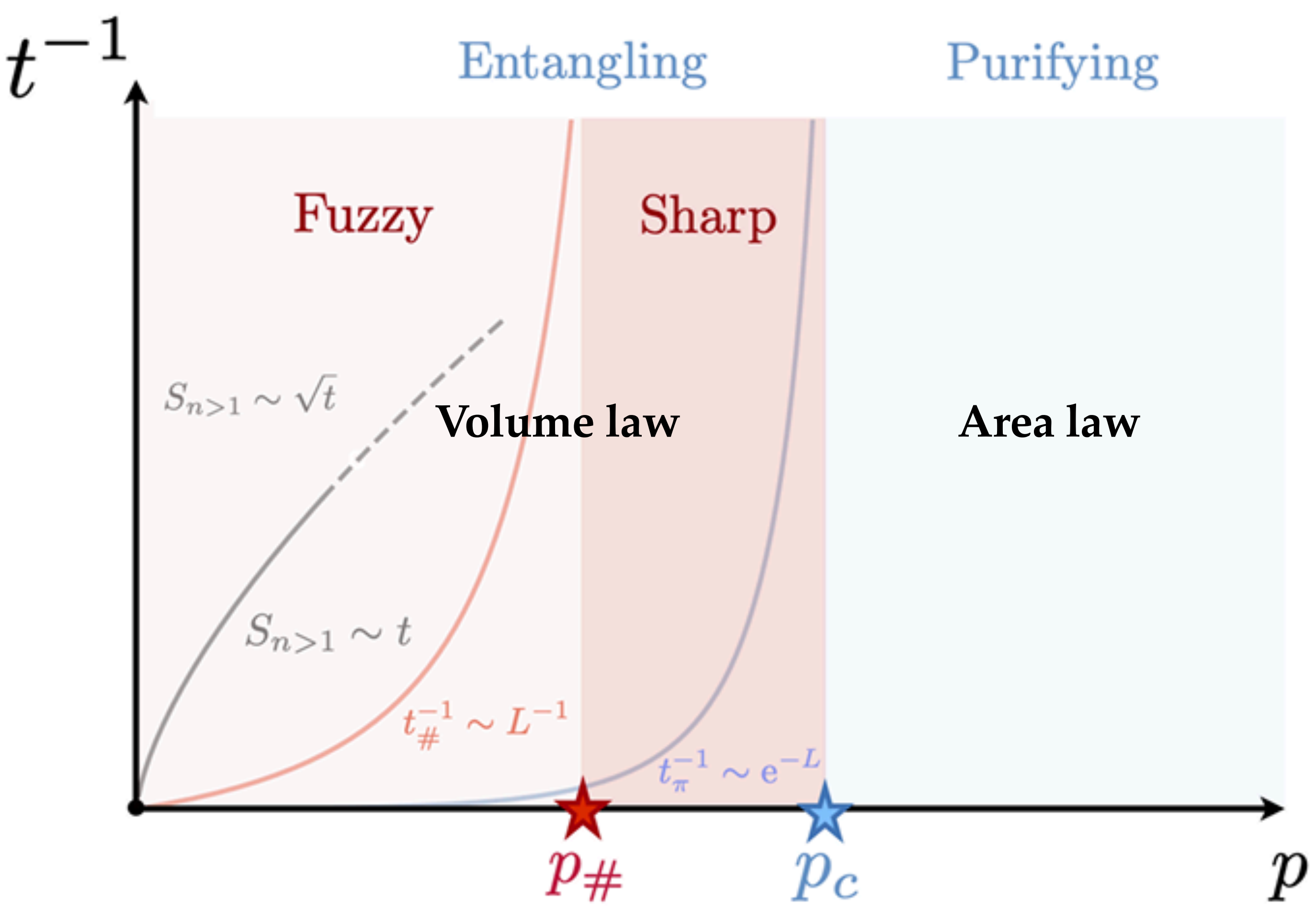
$$U_{i,i+1} = \begin{pmatrix} U^0_{d^2\times d^2} & 0 & 0 \\ 0 & U^1_{2d^2\times 2d^2} & 0 \\ 0 & 0 & U^2_{d^2\times d^2} \end{pmatrix}$$

# Charge sharpening transition

U(1) Conserving monitored dynamics




$$S^z_{\text{tot}} = \sum_i S^z_i \qquad \mathcal{Q} = S^z_{\text{tot}} + L/2$$




Sharpening transition

Consider the variance of the charge

$$[\delta\mathcal{Q}^2] = [\langle\mathcal{Q}^2\rangle_{\mathbf{m}} - \langle\mathcal{Q}\rangle^2_{\mathbf{m}}]$$

$$|\psi_{\text{initial}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}} |S^z_{\text{tot}}, m\rangle$$

Agrawal, Zabalo, Chen, Wilson, Potter, **JHP**, Gopalakrishnan, Vasseur PRX (2022)

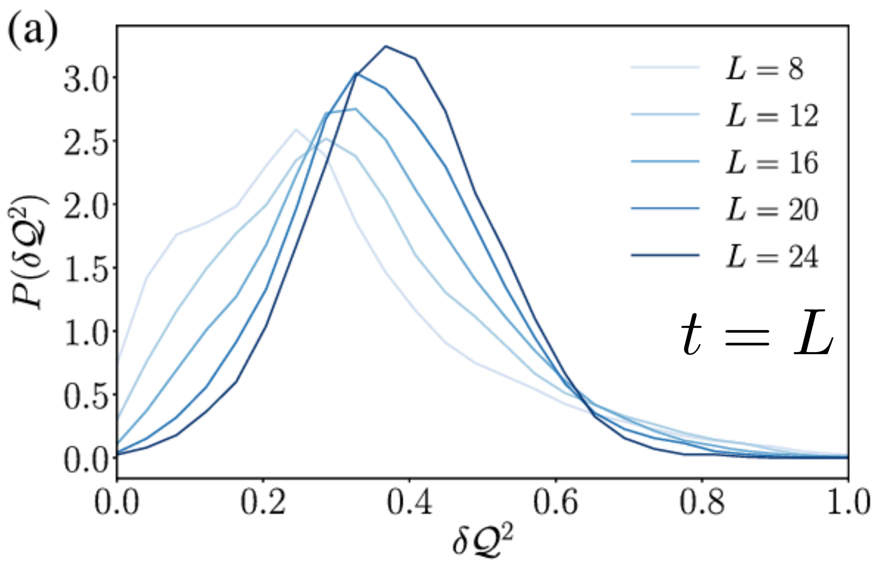
# Charge sharpening transition

## U(1) Conserving monitored dynamics

$$|\psi_{\text{initial}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}} |S^z_{\text{tot}}, m\rangle$$

$$S^z_{\text{tot}} = \sum_i S^z_i \qquad \mathcal{Q} = S^z_{\text{tot}} + L/2$$

**charge fuzzy** $p < p_\#$




$$|\psi_{\text{final}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}}(t) |S^z_{\text{tot}}, m\rangle$$

Still spread across sectors!

Consider the variance of the charge

$$[\delta\mathcal{Q}^2] = [\langle\mathcal{Q}^2\rangle_{\mathbf{m}} - \langle\mathcal{Q}\rangle^2_{\mathbf{m}}]$$

Agrawal, Zabalo, Chen, Wilson, Potter, **JHP**, Gopalakrishnan, Vasseur PRX (2022)

# Charge sharpening transition

## U(1) Conserving monitored dynamics

$$|\psi_{\text{initial}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}} |S^z_{\text{tot}}, m\rangle$$

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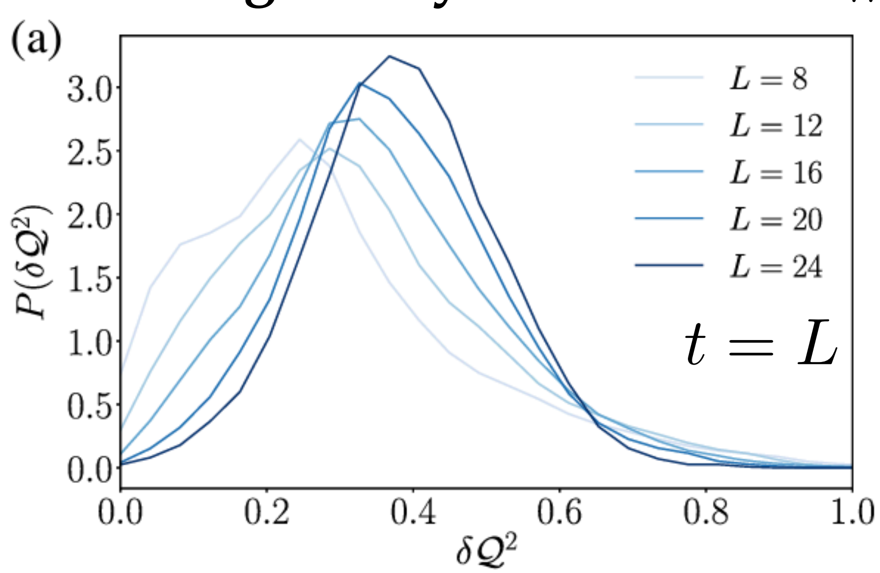
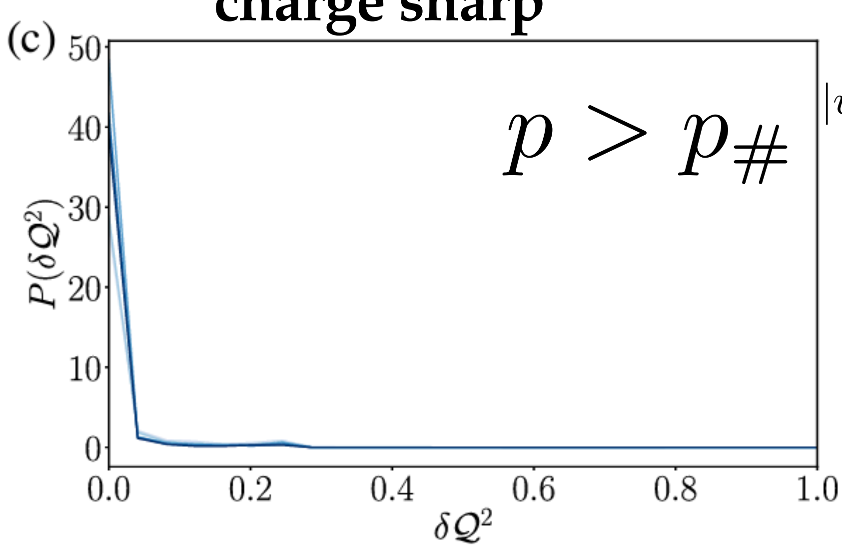



$$|\psi_{\text{final}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}}(t) |S^z_{\text{tot}}, m\rangle$$

Still spread across sectors!

**charge sharp**

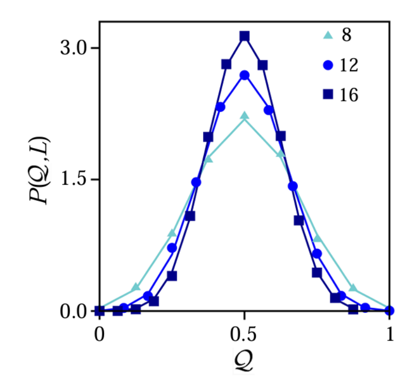



$$|\psi_{\text{final}}\rangle \approx \sum_m a_m^{S^z_{\text{tot}}=0}(t) |S^z_{\text{tot}} = 0, m\rangle$$

Dynamics projects into one conserving sector!

Most likely

$$P(\mathcal{Q}, L) = \binom{L}{\mathcal{Q}} / 2^{L-1}$$




Chakraborty, Chen, Zabalo, Wilson, JHP, PRB (2024)

Agrawal, Zabalo, Chen, Wilson, Potter, **JHP**, Gopalakrishnan, Vasseur PRX (2022)

# Charge sharpening transition

## U(1) Conserving monitored dynamics

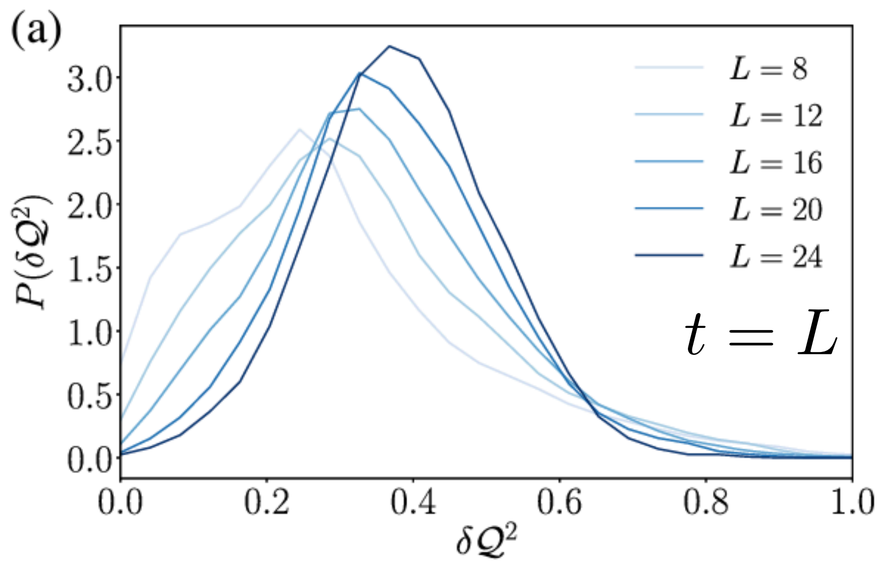
$$|\psi_{\text{initial}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}} |S^z_{\text{tot}}, m\rangle$$

$$S^z_{\text{tot}} = \sum_i S^z_i \qquad \mathcal{Q} = S^z_{\text{tot}} + L/2$$

Consider the variance of the charge

$$[\delta\mathcal{Q}^2] = [\langle \mathcal{Q}^2\rangle_{\mathbf{m}} - \langle \mathcal{Q}\rangle^2_{\mathbf{m}}]$$

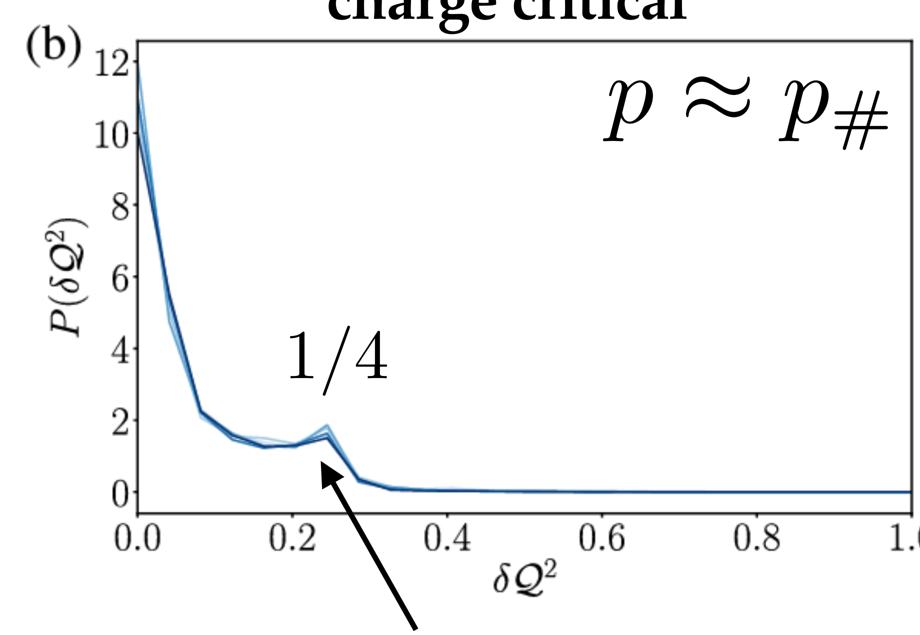
**charge fuzzy** $p < p_\#$



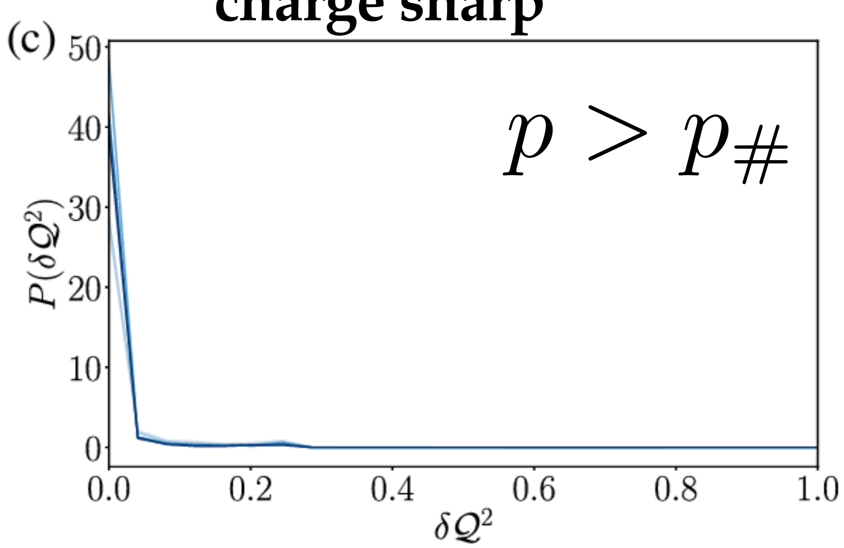

$$|\psi_{\text{final}}\rangle = \sum_{m, S^z_{\text{tot}}} a_m^{S^z_{\text{tot}}}(t) |S^z_{\text{tot}}, m\rangle$$

Still spread across sectors!

**charge critical**




**charge sharp**




$$|\psi_{\text{final}}\rangle \approx \sum_m a_m^{S^z_{\text{tot}}=0}(t) |S^z_{\text{tot}} = 0, m\rangle$$

Dynamics projects into one conserving sector!

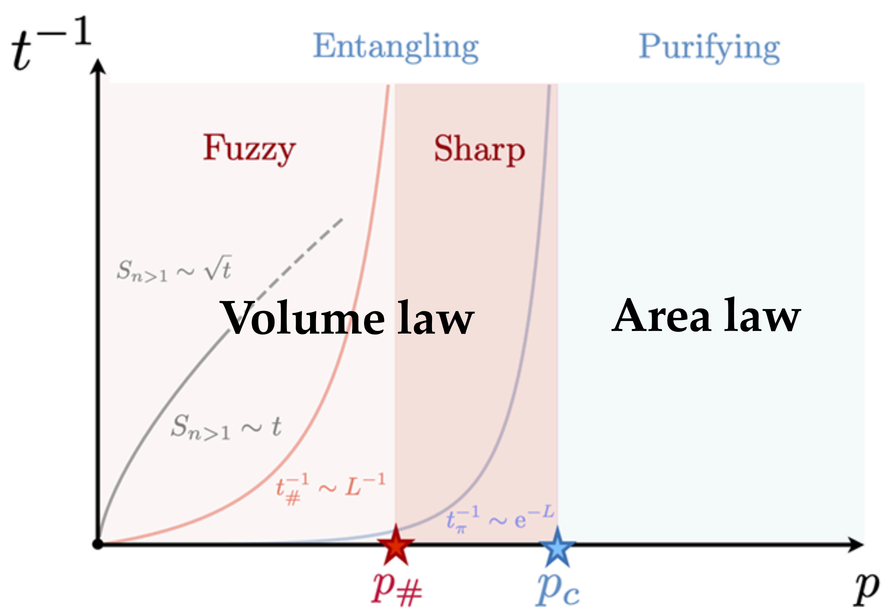
$$|\psi_{\text{final}}\rangle \approx \sum_m a_m^{S^z_{\text{tot}}=0}(t) |S^z_{\text{tot}} = 0, m\rangle + \sum_m a_m^{S^z_{\text{tot}}=\pm 1}(t) |S^z_{\text{tot}} = \pm 1, m\rangle$$

OR $|\psi_{\text{final}}\rangle \approx \sum_m a_m^{S^z_{\text{tot}}=0}(t) |S^z_{\text{tot}} = 0, m\rangle$

# Including a U(1) Conservation law

The problem is greatly enriched by adding a continuous symmetry that is maintained by the measurements.

Novel sharpening transition within the volume law phase




Conserved quantity

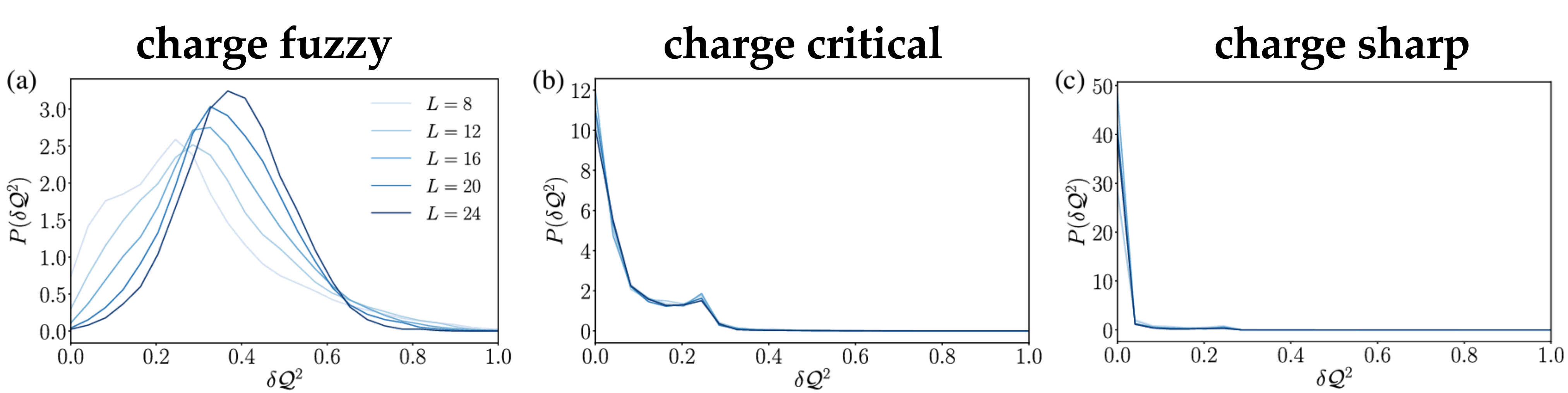
$$S^z_{\text{tot}} = \sum_i S^z_i$$

"Charge"

$$\mathcal{Q} = S^z_{\text{tot}} + L/2$$

Consider the variance of the charge

$$[\delta \mathcal{Q}^2] = [\langle \mathcal{Q}^2 \rangle_{\mathbf{m}} - \langle \mathcal{Q} \rangle^2_{\mathbf{m}}]$$




Agrawal, Zabalo, Chen, Wilson, Potter, **JHP**, Gopalakrishnan, Vasseur PRX (2022)

# CHARGE SHARPENING TRANSITION: UNIVERSALITY

**Qudits**

Charge Sharpening in 1D quantum circuits is akin to a Berzinski-Kosterlitz-Thouless (BKT) Transition

$p \approx p_{\#}$ Local Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^d$ $d \to \infty$

$$\mathcal{L}_{\text{dual}} = \frac{1}{8\pi^2\bar{\rho}}\left[\left(\partial_t\bar{\vartheta}\right)^2 + D^2\left(\partial_x^2\bar{\vartheta}\right)^2\right] + \\ + \frac{1}{8\pi^2\rho_s}(\Pi\partial_\mu\vartheta)^2 - \lambda\sum_{a\neq b}\cos(\vartheta_a - \vartheta_b),$$

String order parameter

$$W_{[0,r]} = \prod_{0<i<r} \sigma_i^z,$$

$$C_W(r) = \mathcal{E}\left[\langle W_{[0,r]}\rangle^2\right]$$

$$C_W(r) \sim 1/|r|^{2\pi\rho_s}$$

Barratt, Agrawal, Gopalakrishnan, Huse, Vasseur, Potter PRL (2022)

# Charge sharpening transition: Universality

## Qudits

Charge Sharpening in 1D quantum circuits is akin to a Berzinski-Kosterlitz-Thouless (BKT) Transition

$p \approx p_{\#}$ Local Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^d$ $d \to \infty$

$$\mathcal{L}_{\text{dual}} = \frac{1}{8\pi^2\bar{\rho}}\left[\left(\partial_t\bar{\vartheta}\right)^2 + D^2\left(\partial_x^2\bar{\vartheta}\right)^2\right] + \\ + \frac{1}{8\pi^2\rho_s}(\Pi\partial_\mu\vartheta)^2 - \lambda\sum_{a\neq b}\cos(\vartheta_a - \vartheta_b),$$

String order parameter

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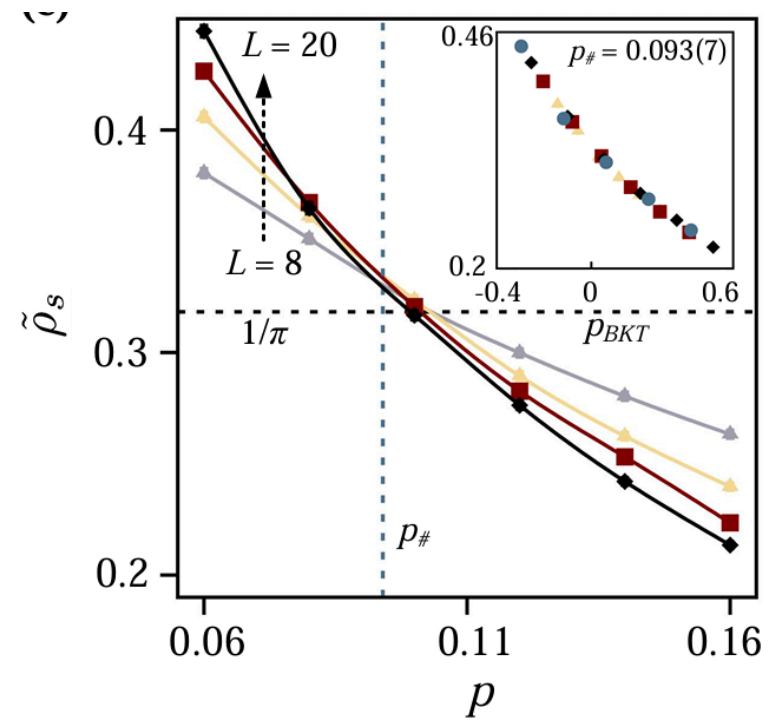
Barratt, Agrawal, Gopalakrishnan, Huse, Vasseur, Potter PRL (2022)

## Qubits

Jump in the putative stiffness at the sharpening transition




Chakraborty, Chen, Zabalo, Wilson, **JHP**, PRB (2024)

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Barratt, Agrawal, Gopalakrishnan, Huse, Vasseur, Potter PRL (2022)

Interesting questions related to breaking particle hole symmetry

Ha, Pandey, Gopalakrishnan, Huse, PRB (2024)

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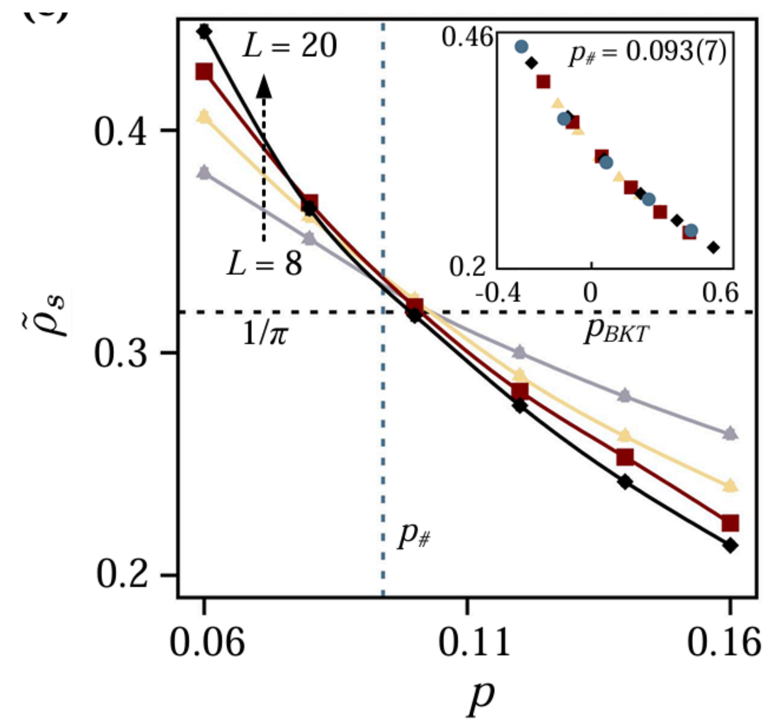
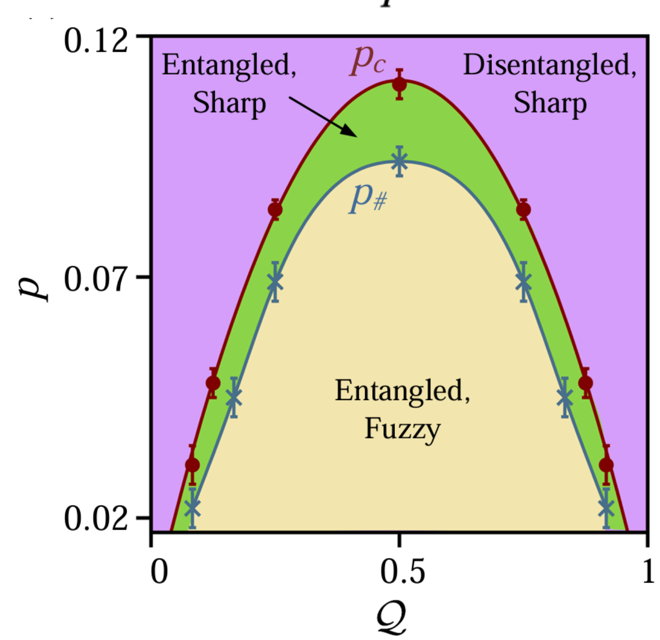
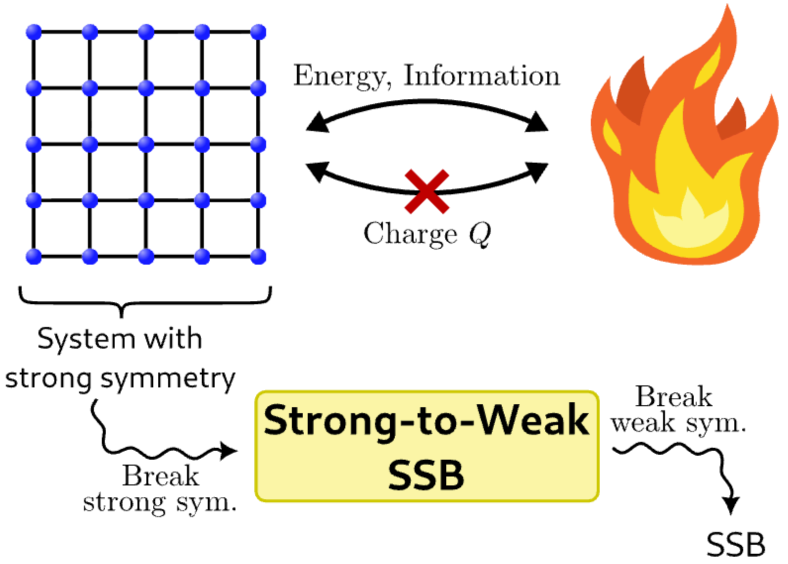



Chakraborty, Chen, Zabalo, Wilson, **JHP**, PRB (2024)

# Charge sharpening is a Weak to Strong Symmetry Breaking transition (SWSSB)

Generalization to non-equilibrium phases of matter with a local order parameter description.




Density matrix evolution of a mixed state $\rho$

Symmetry generator $U$

Strong Symmetry $U\rho = e^{i\theta}\rho$

Weak Symmetry $U\rho \neq e^{i\theta}\rho, \quad U\rho U^{\dagger} = \rho,$

Ogunnaike, Feldmeier, and Lee PRL (2023)

Lee, Jiang, Xu PRX Quantum (2023)

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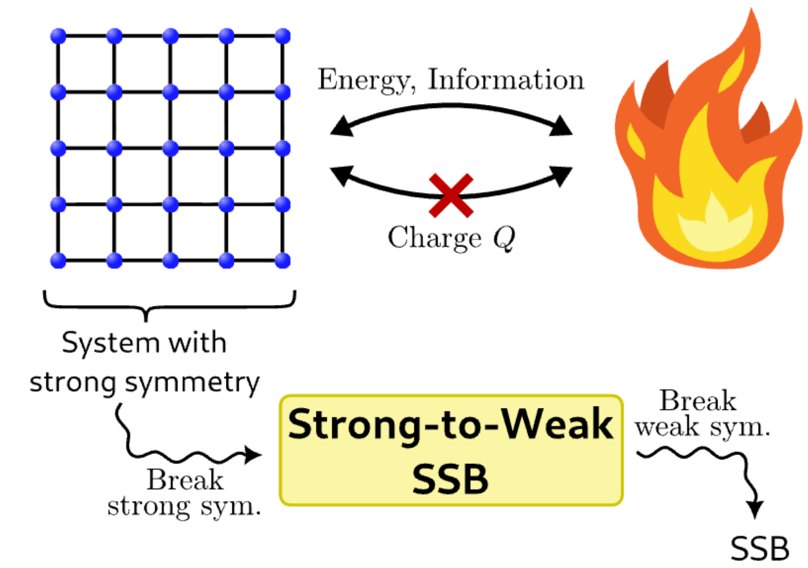


Symmetry generator $U$

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Weak Symmetry $U\rho \neq e^{i\theta}\rho, \quad U\rho U^{\dagger} = \rho,$

Detected through

Fidelity Correlator

$$\langle O \rangle_F = \mathrm{Tr}\sqrt{\sqrt{\rho} O \rho O^{\dagger} \sqrt{\rho}}$$

Renyi-2 correlator

$$R^{(2)}(x,y) := \frac{\mathrm{Tr}\left(O(x)O^{\dagger}(y)\rho O(y) O^{\dagger}(x)\rho\right)}{\mathrm{Tr}\rho^2}$$

Ogunnaike, Feldmeier, and Lee PRL (2023)

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# Spontaneous Symmetry Breaking in mixed states and Charge Sharpening

How about weak and strong symmetries for pure states?

$$\rho = |\psi\rangle\langle\psi|$$

Consider a state with a U(1) symmetry at all times

e.g. particle number conservation, with conserved charge $\hat{Q}$

Gopalakrishnan, McCulloch, Vasseur, PRX (2026)

Zerba, Gopalakrishnan, Knap, arXiv (2025)

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$$U|N, m\rangle = e^{i\theta_N}|N, m\rangle$$

$$\langle\psi|\hat{Q}^2|\psi\rangle - \langle\psi|\hat{Q}|\psi\rangle^2 = 0$$

Charge is sharp

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$$\langle\psi|\hat{Q}^2|\psi\rangle - \langle\psi|\hat{Q}|\psi\rangle^2 \neq 0$$

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Weak symmetry

Gopalakrishnan, McCulloch, Vasseur, PRX (2026)

Zerba, Gopalakrishnan, Knap, arXiv (2025)

Lee arXiv (2026)

Tang, Kattel, Pixley arXiv (2026)

# Relevant Perturbations

We now consider modifications to the circuit dynamics that fundamentally alter the problem.

Enforcing continuous symmetries in the hybrid dynamics

Work on continuous non-Abelian symmetries not discussed, Majidy et al PRB (2023)

**Including static and temporal perturbations**

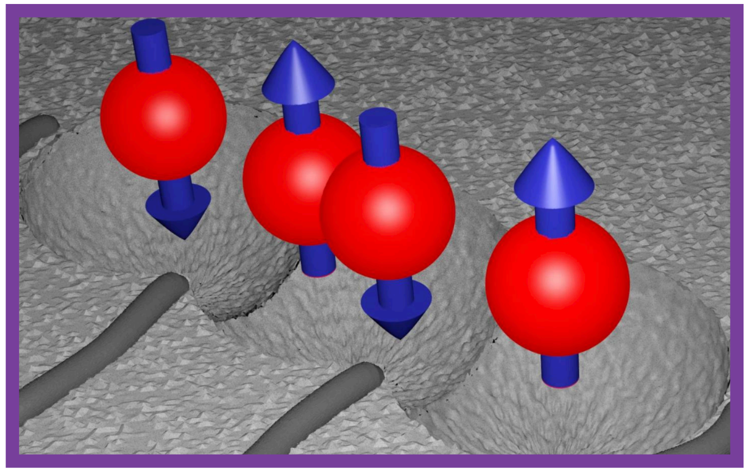
# Is the Transition Stable to static Disorder?

How do we understand the stability of the measurement induced transition in the presence of static disorder?

Apply the Harris criteria: For the fixed point to remain stable it MUST satisfy

$$\nu \geq 2/d$$

All solid state qubit realizations have some form of static disorder.



Maximilian Russ/Guido Burkard

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

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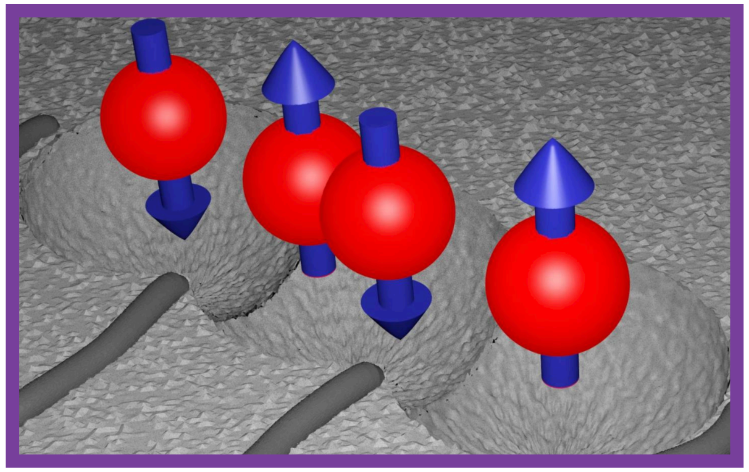
Apply the Harris criteria: For the fixed point to remain stable it MUST satisfy $\nu \geq 2/d$ $\nu \approx 1.3$

Thus, the transition should be unstable!

Where does it flow to?

All solid state qubit realizations have some form of static disorder.



Maximilian Russ/Guido Burkard

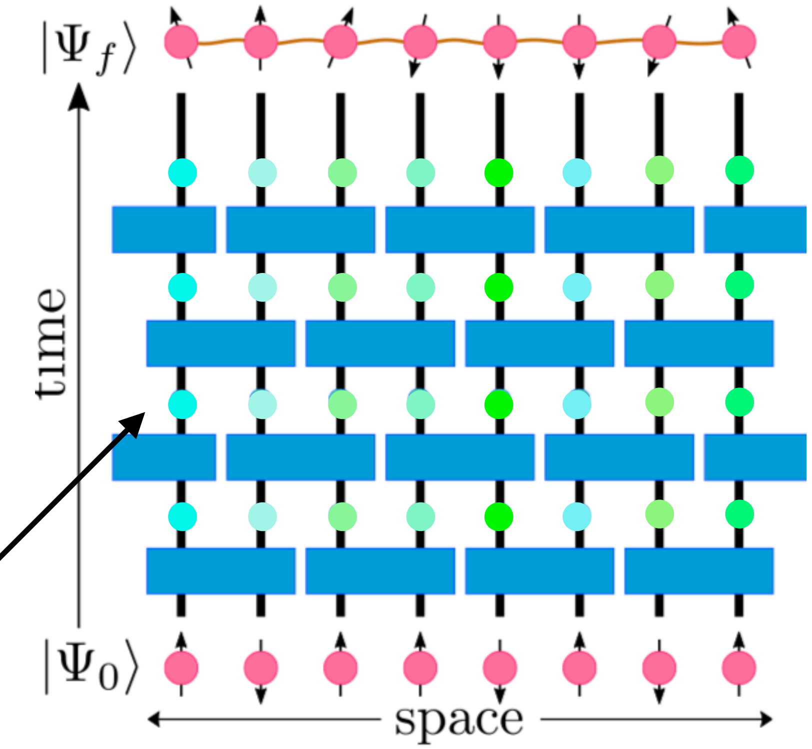
Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Introducing Static Disorder

Have a choice how to introduce the static perturbation, could have modified the gates

Nahum, Ruhman, Huse PRB (2018)

Fix the measurement rate values to be static




value of p is **random in space, fixed in time**

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Introducing Static Disorder

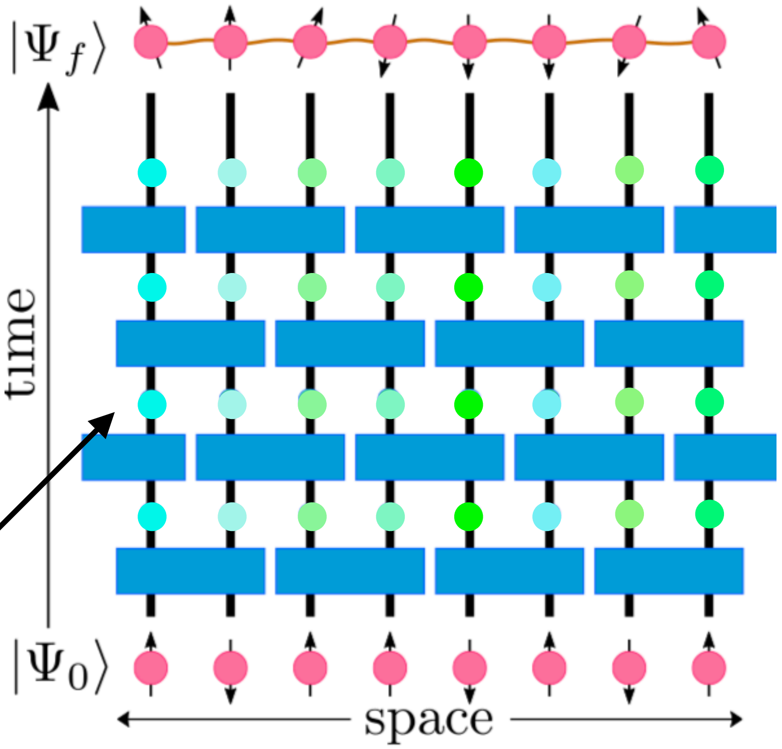
Fix the measurement rate values to be static

$$p(x) = (r_x)^n \qquad \bar{p} = 1/(n+1)$$

amplify rare events
with tails to small/large $p$

$r_x$ Random real number

$$0 < r_x < 1$$

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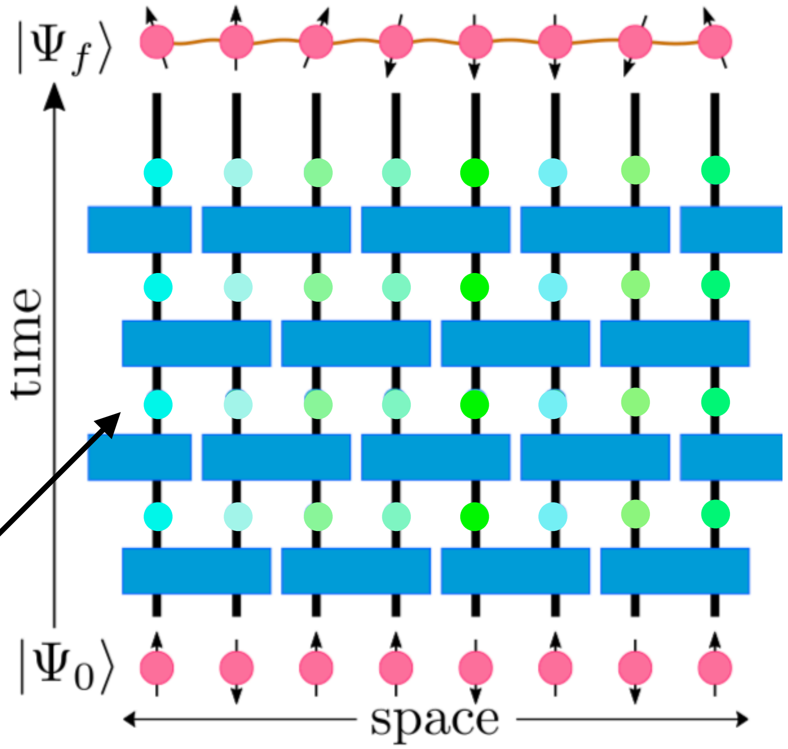


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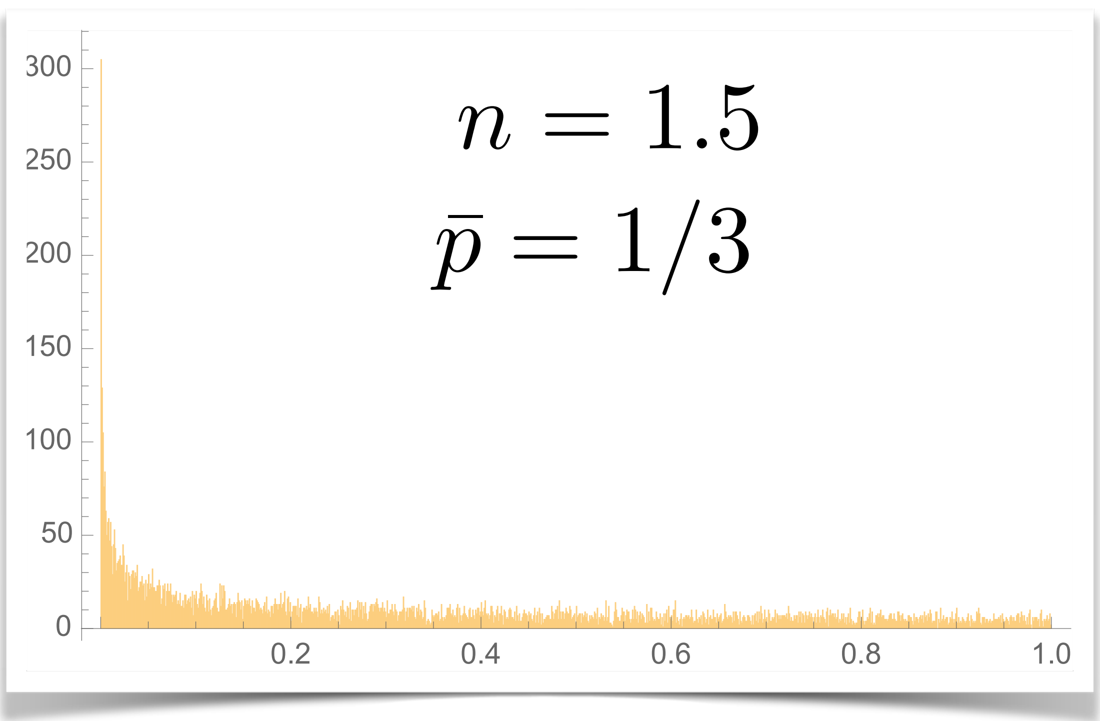
$P[p(x)]$




$p$

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Finding the Infinite Randomness Fixed Point

Fix the measurement rate values to be static $p(x) = (r_x)^n$

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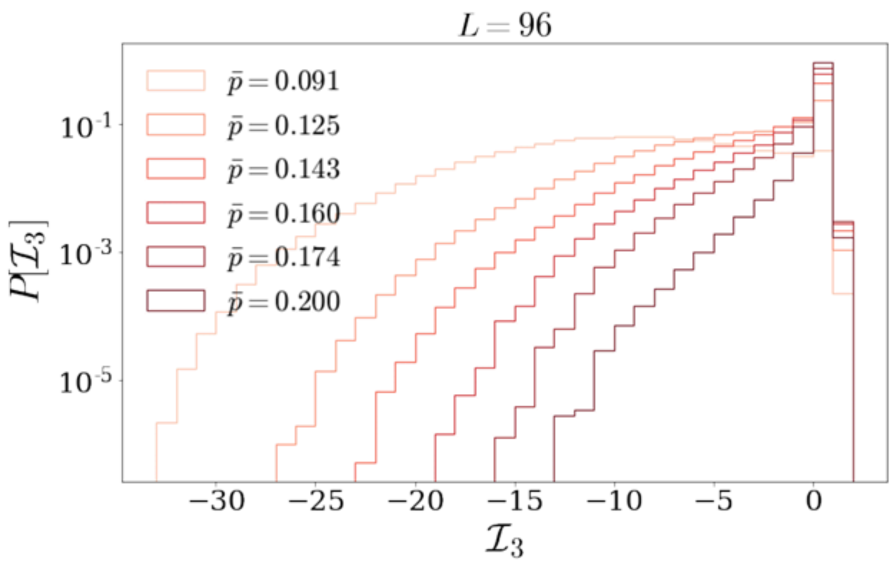
Broad distribution near the transition cannot use the mean!

$\mathcal{I}_3 \to P[\mathcal{I}_3]$

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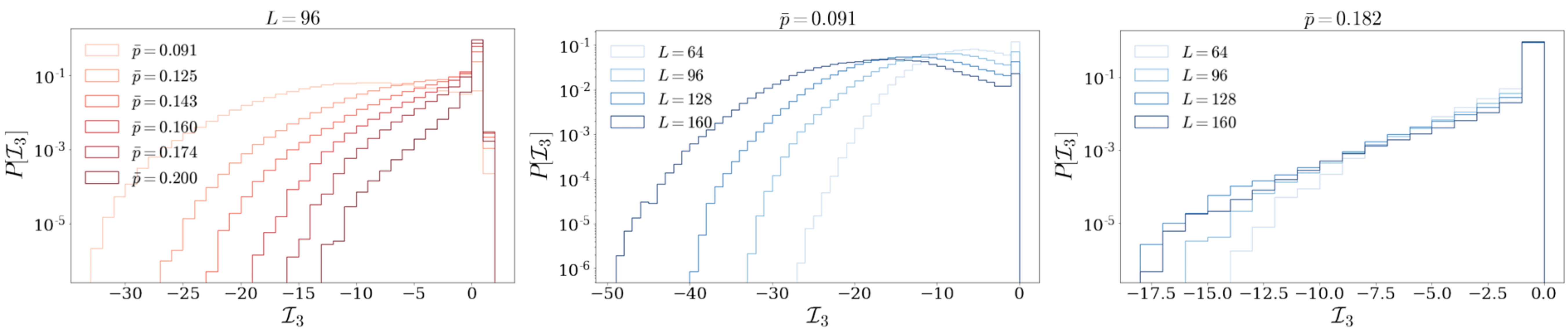
volume law

area law

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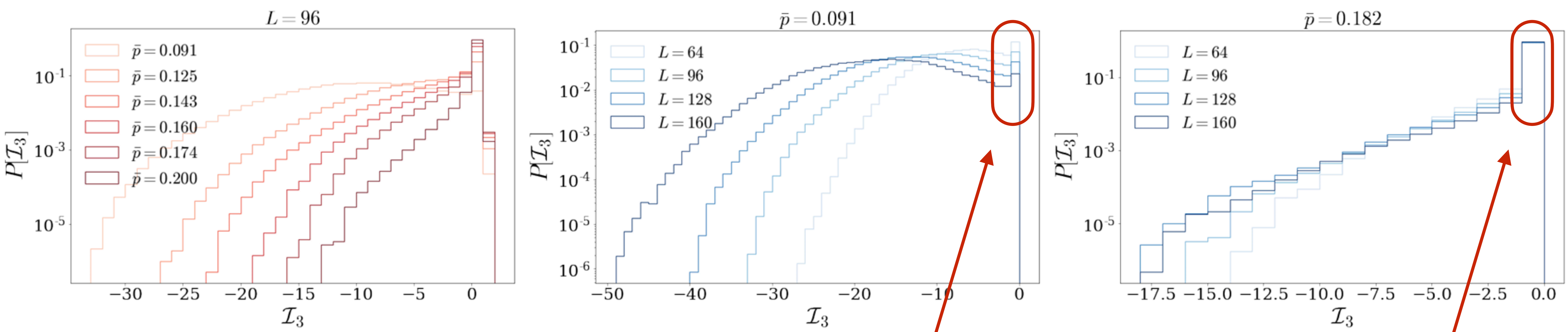


*L*-dependence at $I_3$=0 distinct in the two phases

focus on → $P[\mathcal{I}_3 = 0]$

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

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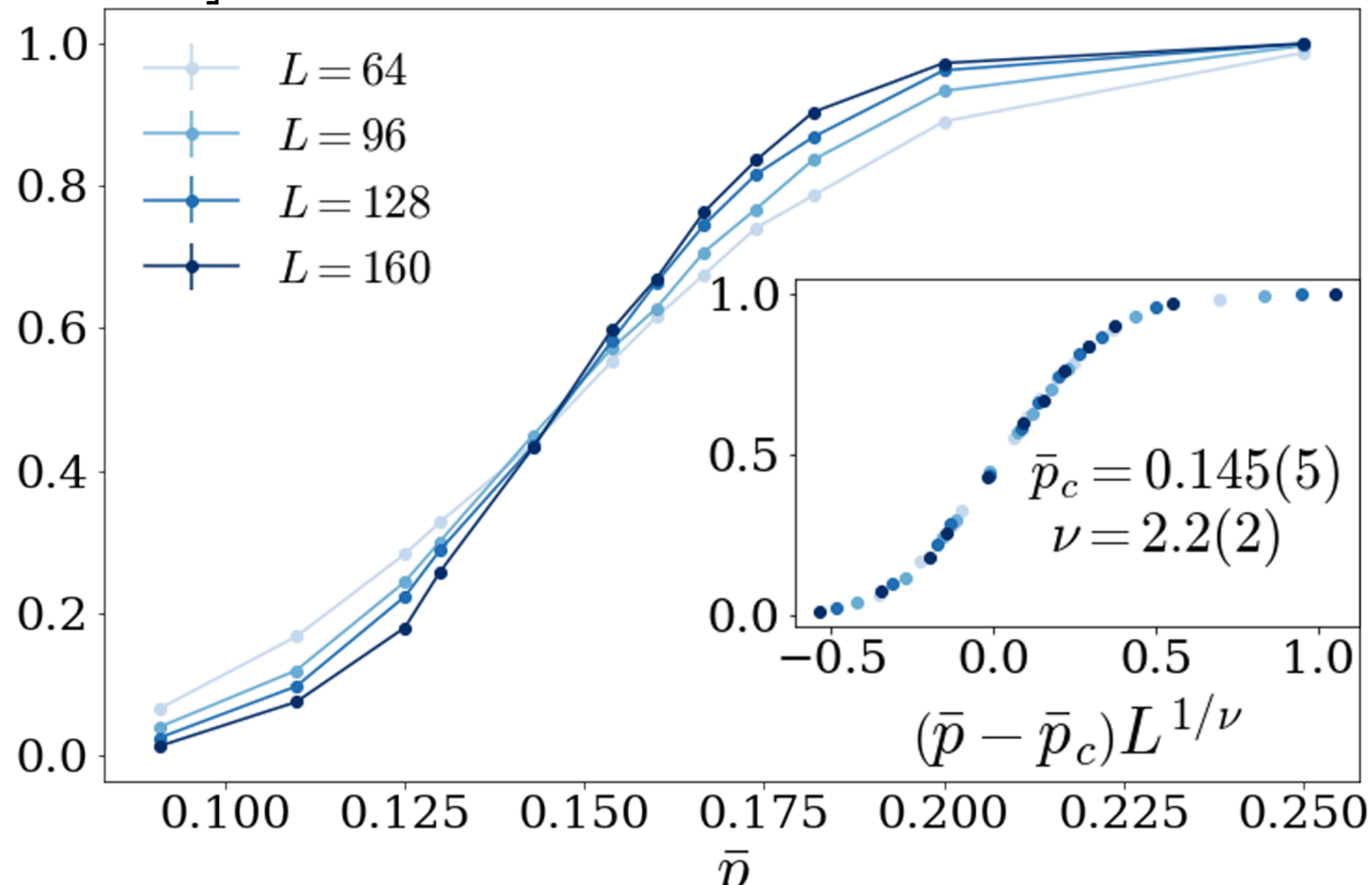


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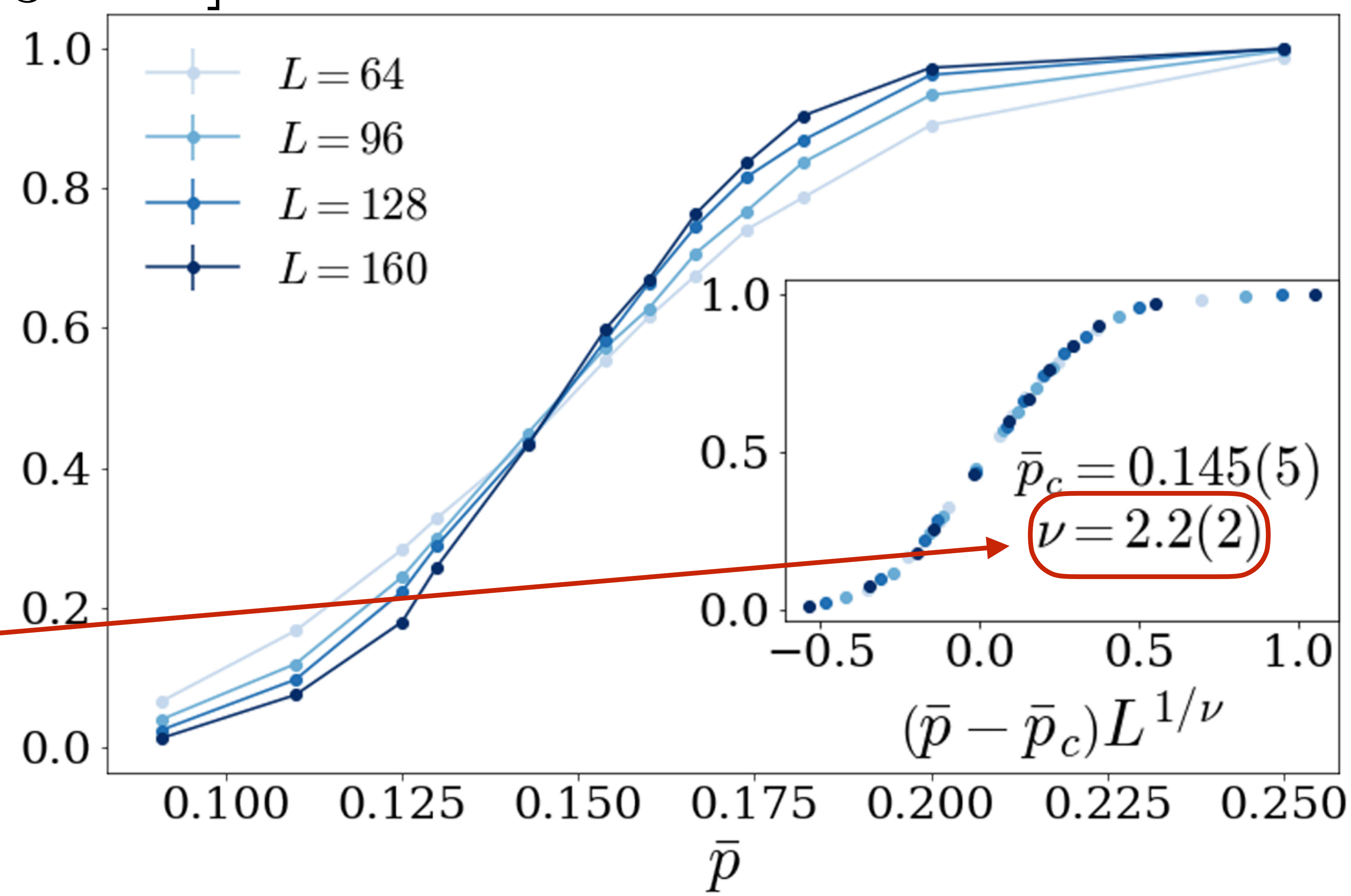



appears to saturate the bound

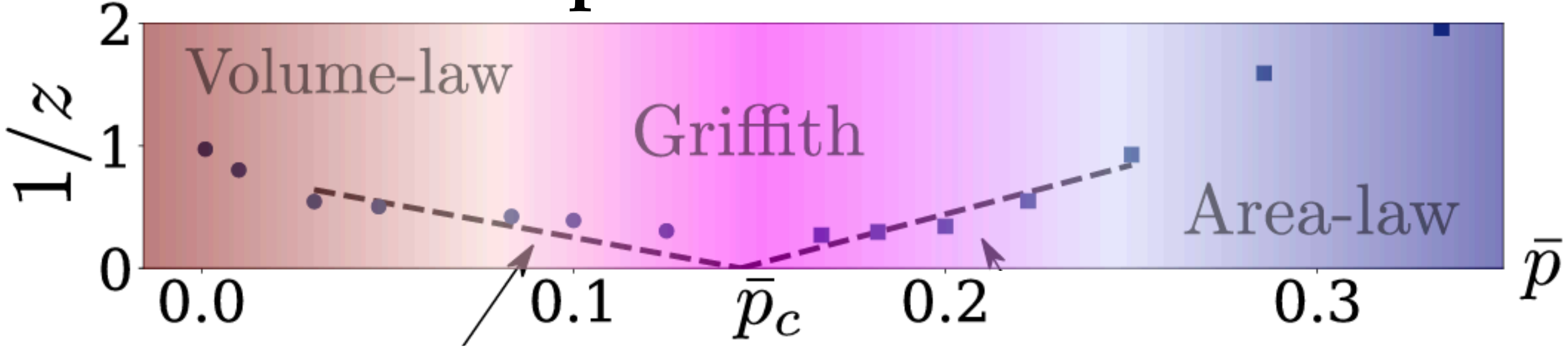
$\nu \geq 2/d$

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Phase Diagram and Griffith Effects

Rare regions dominate near the transition

**Volume law phase**




$$z \sim |p - p_c|^{-\nu\psi}$$

Rare regions of the measurement rate create Griffith effects, i.e. local regions that are in the "wrong" phase

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

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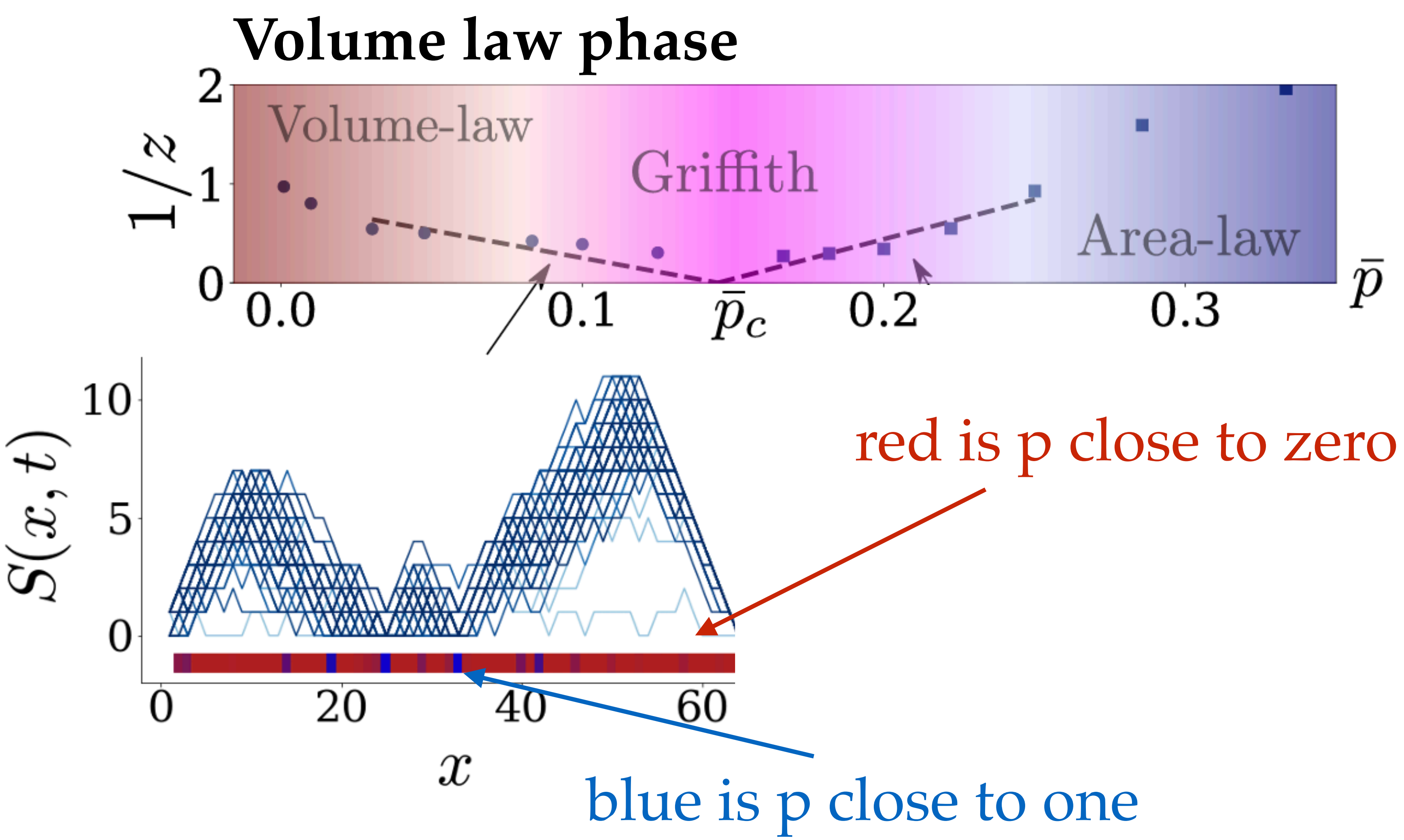


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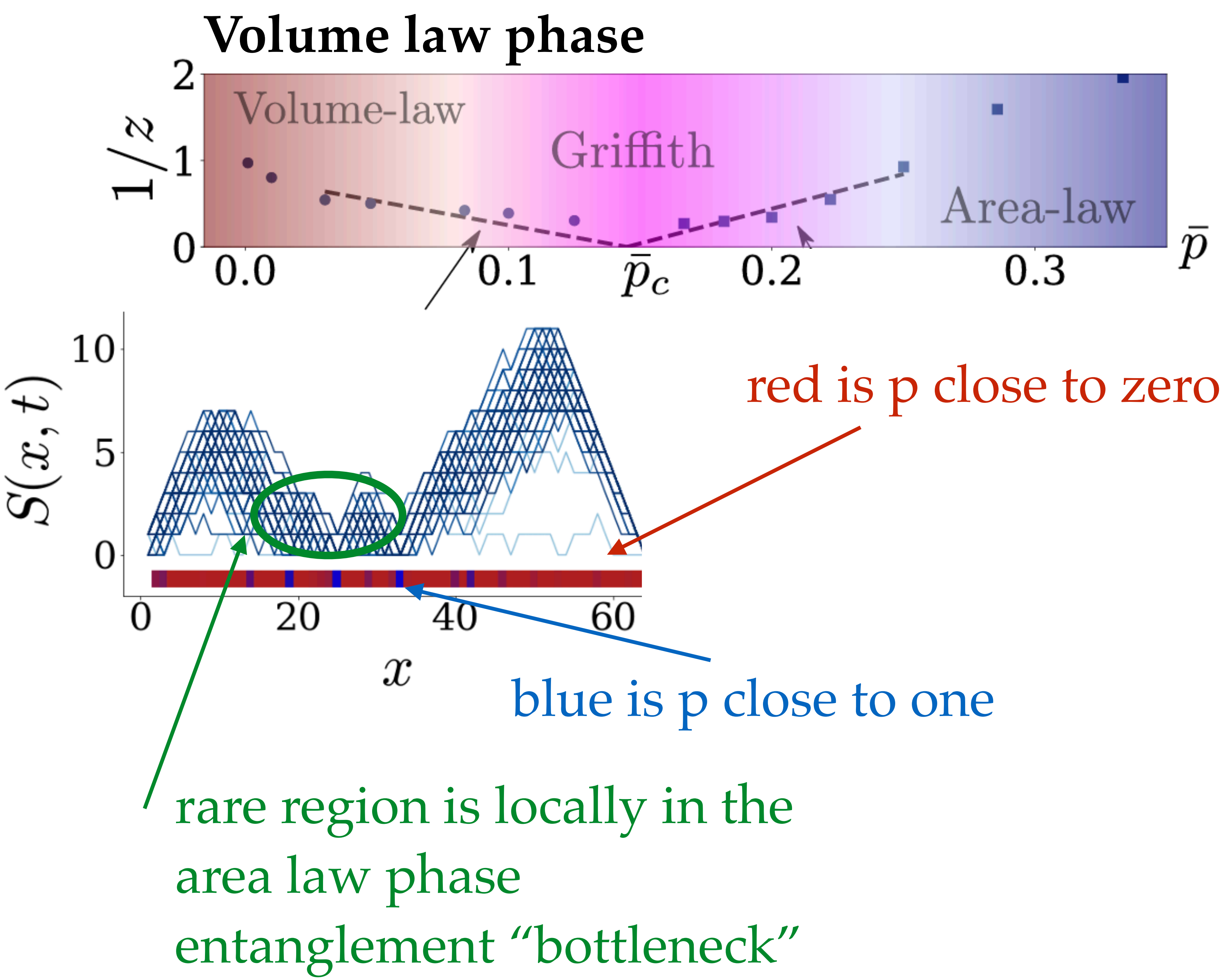


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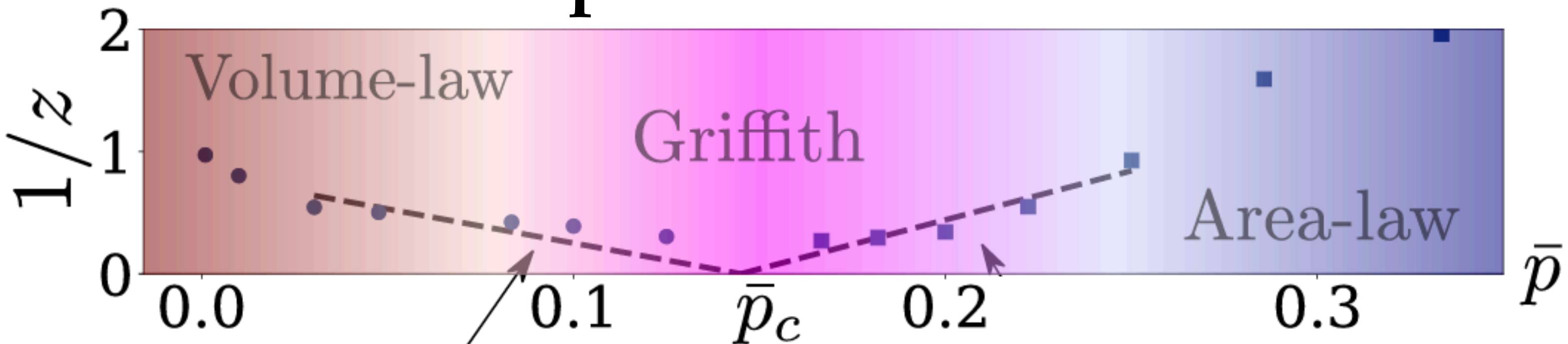


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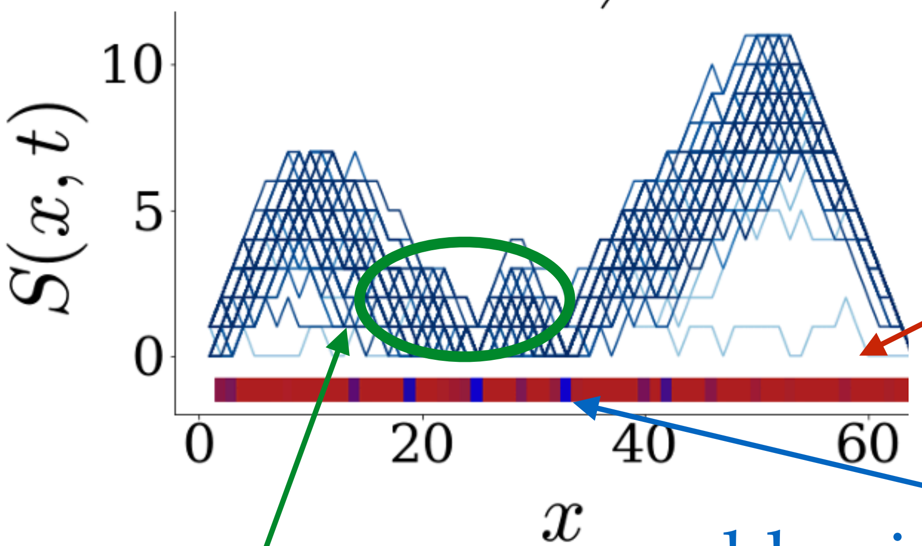


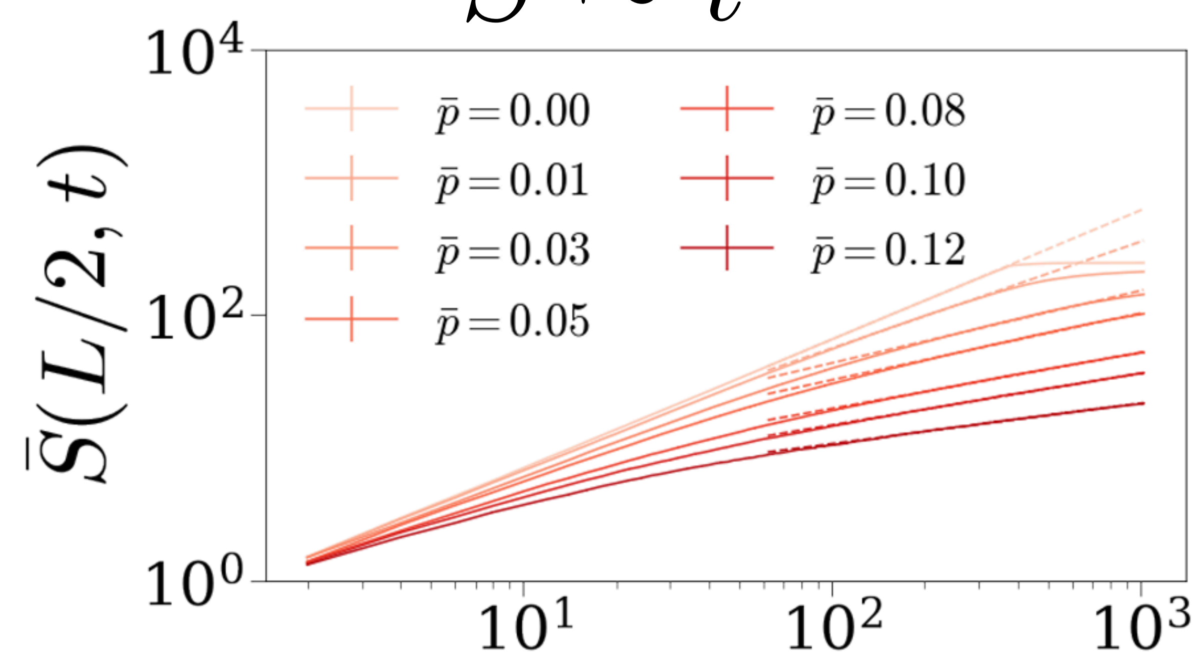

red is p close to zero

blue is p close to one

rare region is locally in the area law phase
entanglement "bottleneck"

non-ballistic entanglement propagation

$$\bar{S} \sim t^{1/z}$$

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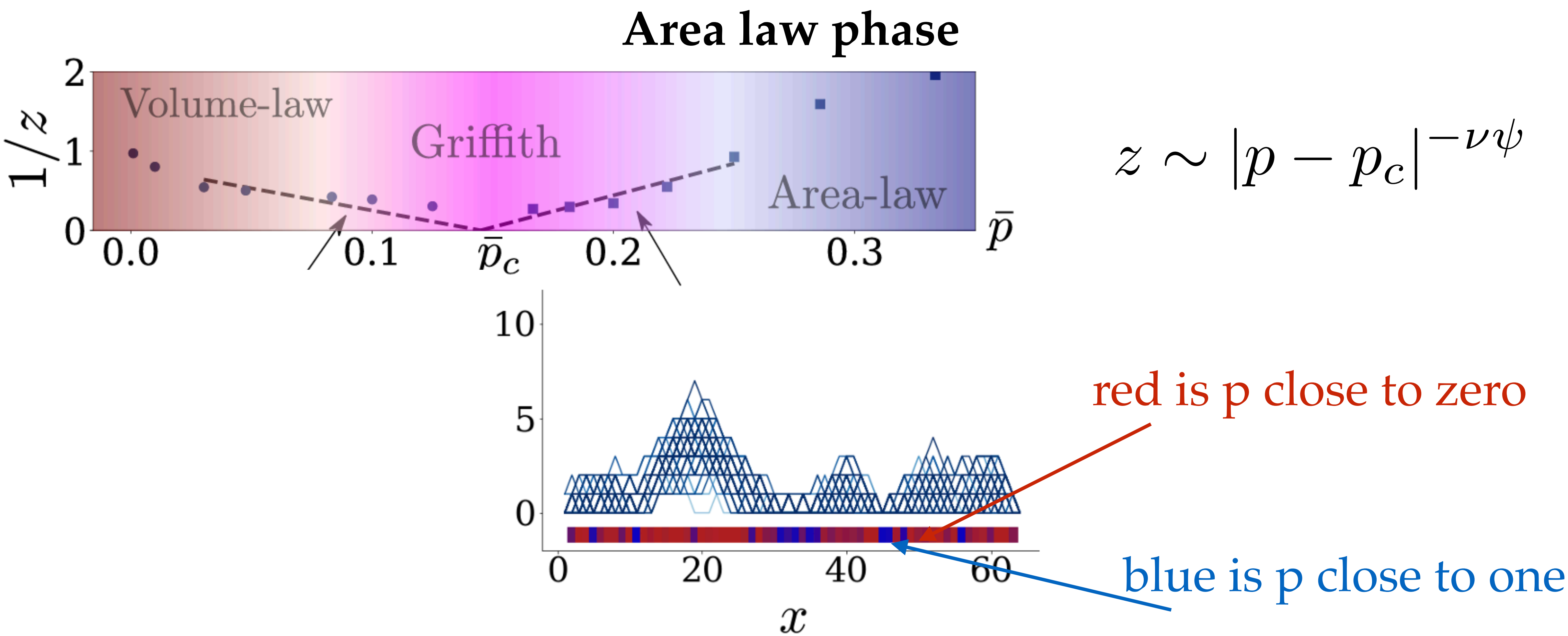


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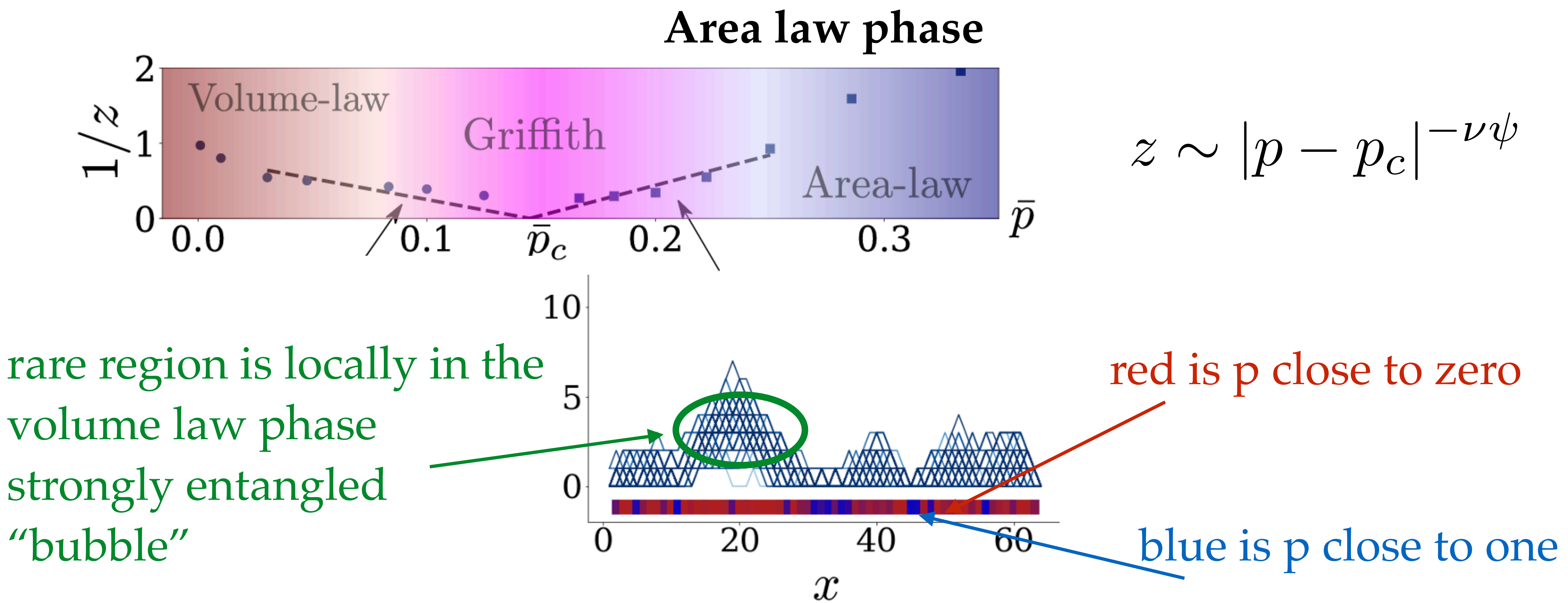


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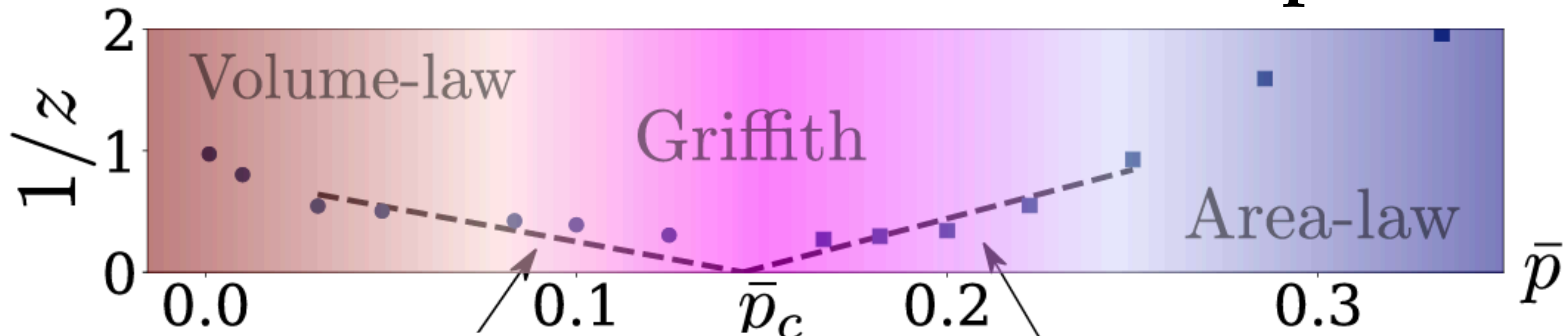


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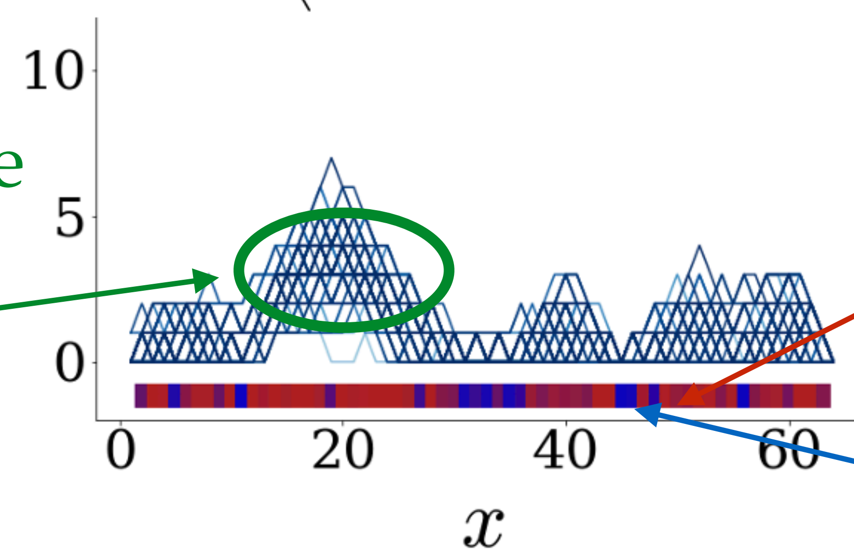
rare region is locally in the volume law phase strongly entangled "bubble"

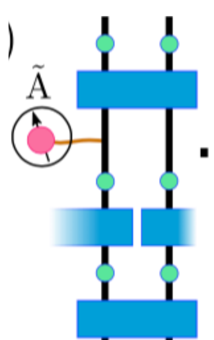



red is p close to zero

blue is p close to one

probe via long purification time

coupling to ancilla qubit



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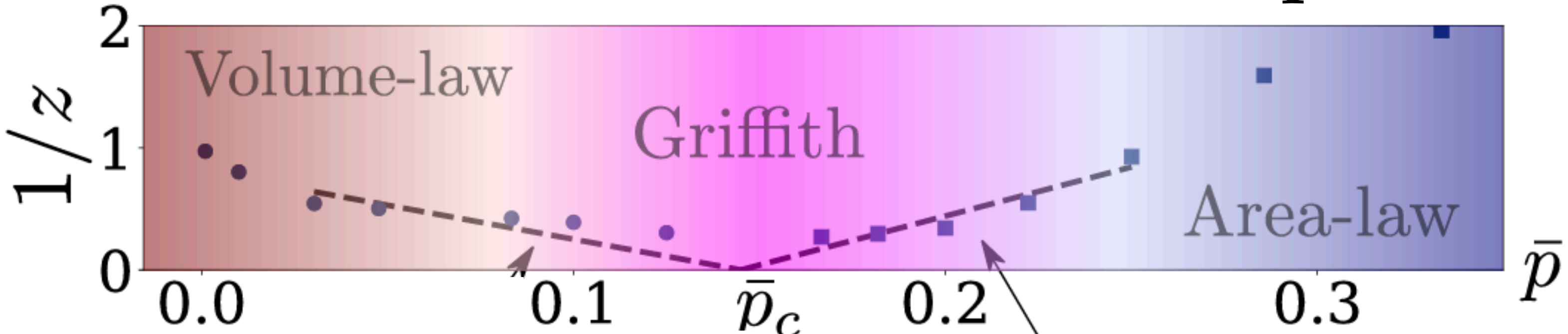


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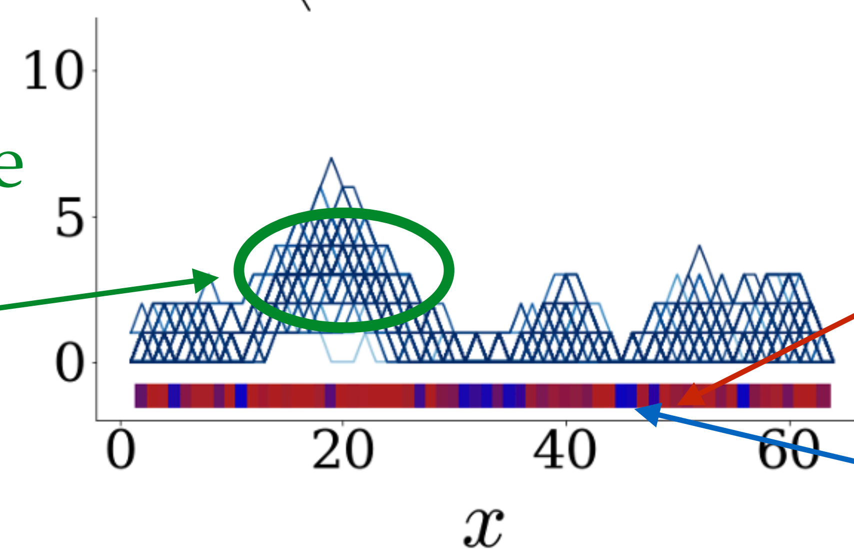


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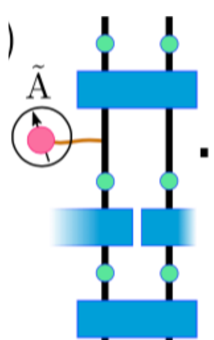
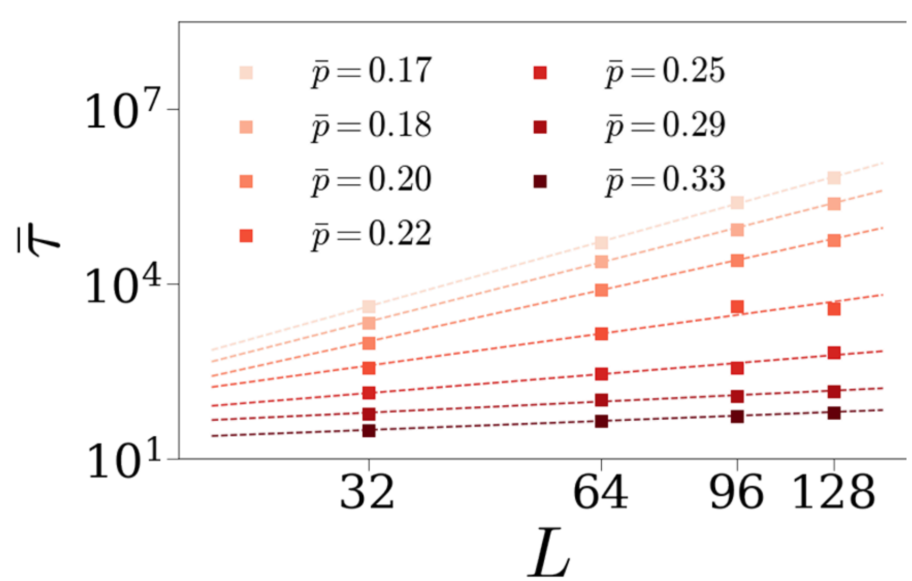
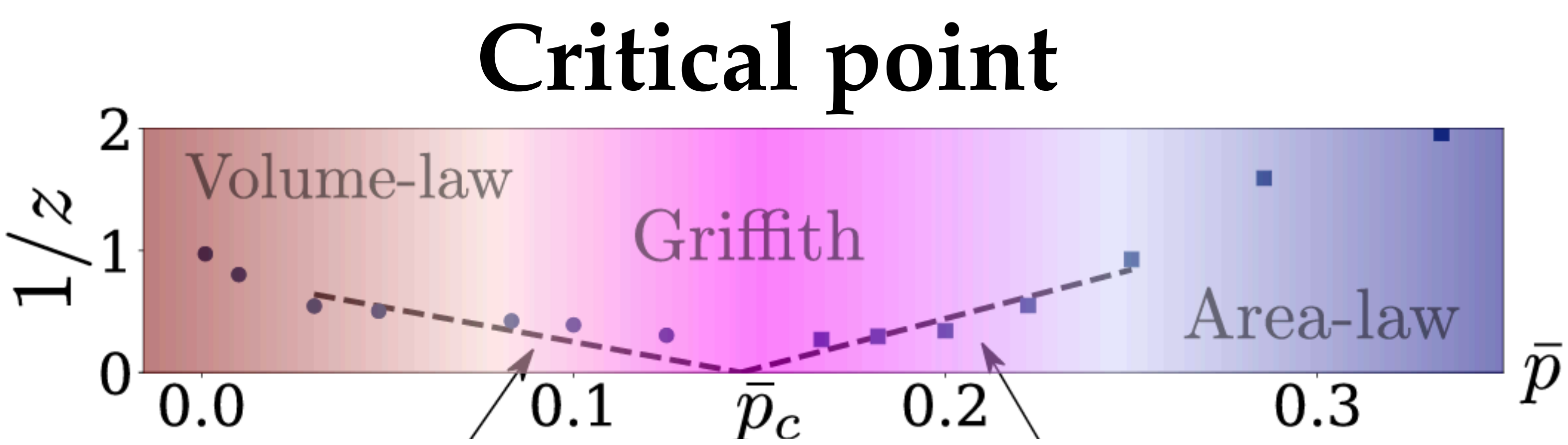


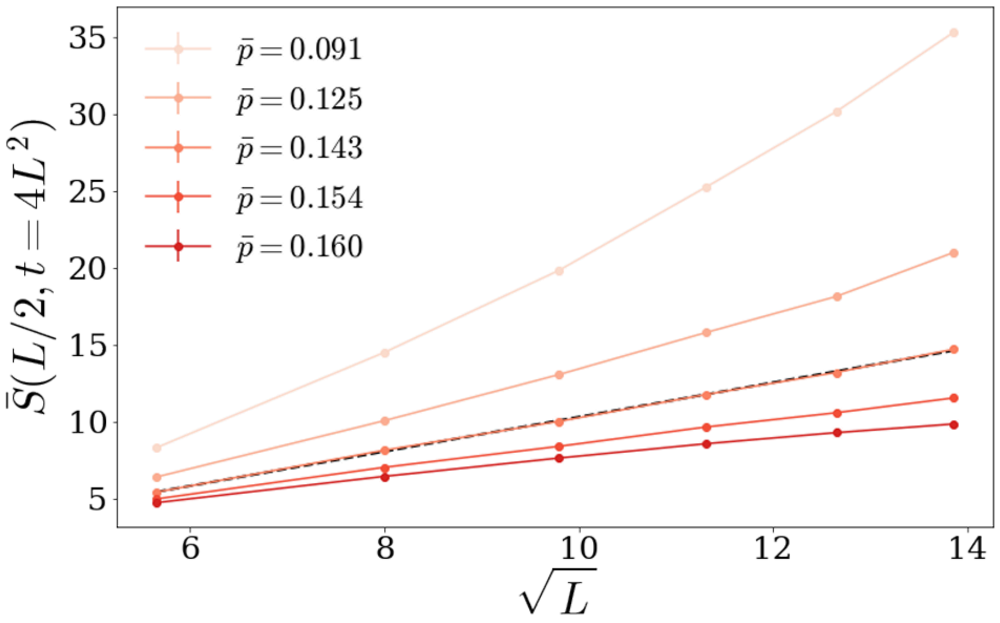




time to purify an ancilla qubit

$$\bar{\tau} \sim L^z$$

# Infinite Randomness Fixed Point



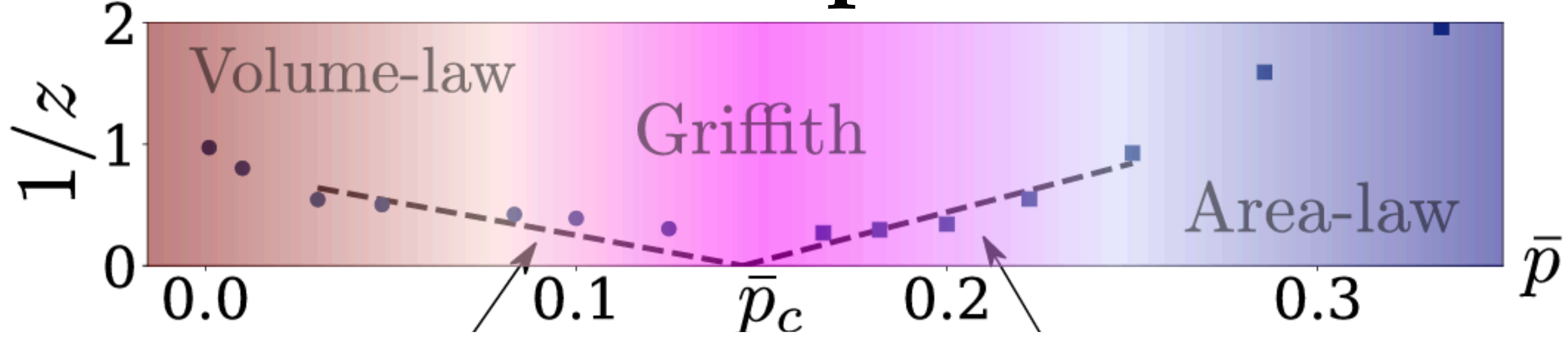



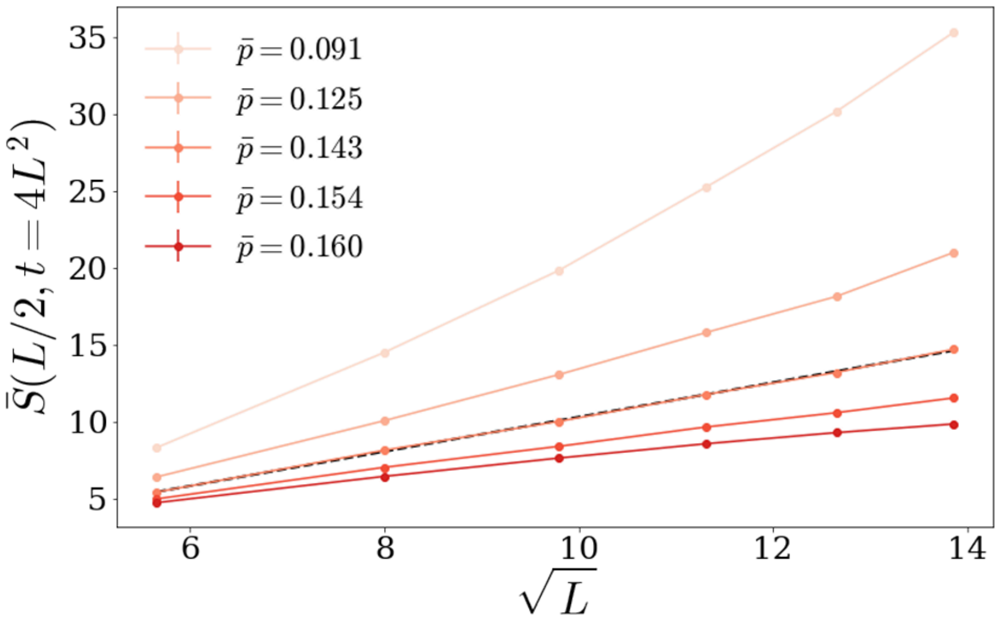

$$\bar{S}(L, t = \infty) \sim \sqrt{L}$$

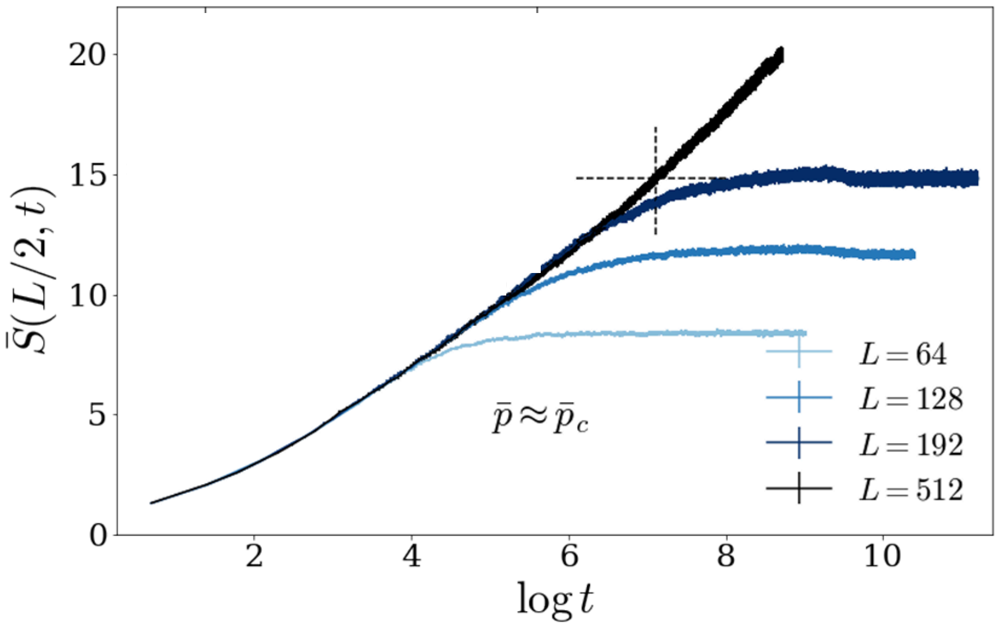
Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Infinite Randomness Fixed Point

**Critical point**



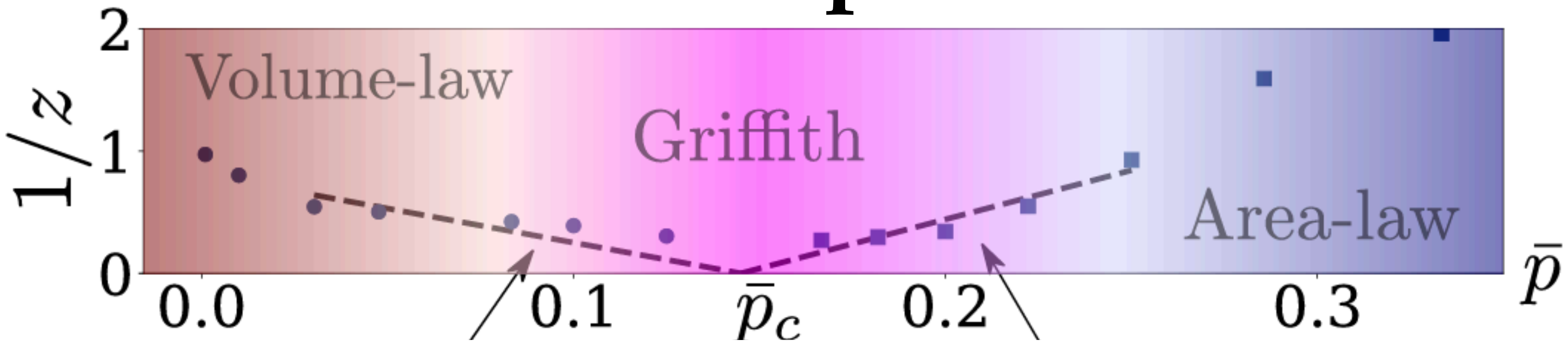

$$\bar{S}(L, t=\infty) \sim \sqrt{L}$$

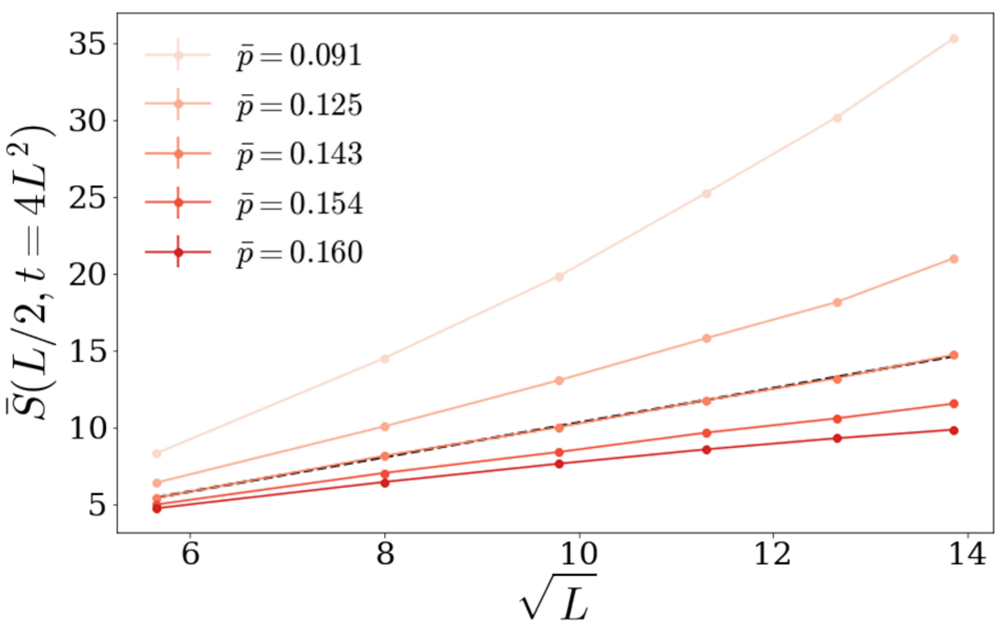
$$\bar{S}(L=\infty, t) \sim \log(t)$$

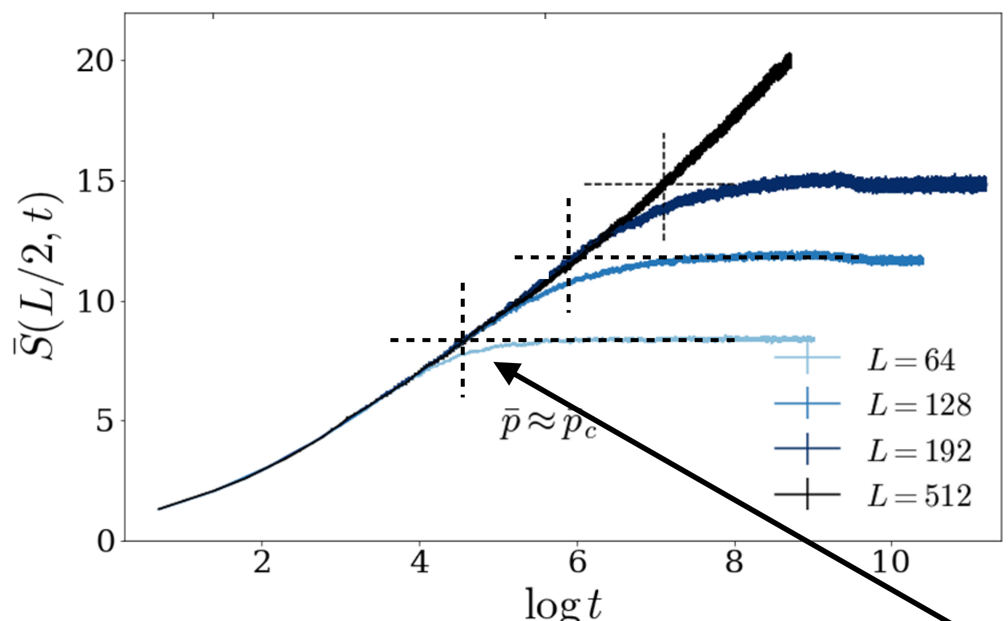
Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Infinite Randomness Fixed Point

**Critical point**




relaxation time

$$t^* \sim e^{a\sqrt{L}}$$

$$z \to \infty$$

$$\bar{S}(L, t = \infty) \sim \sqrt{L}$$

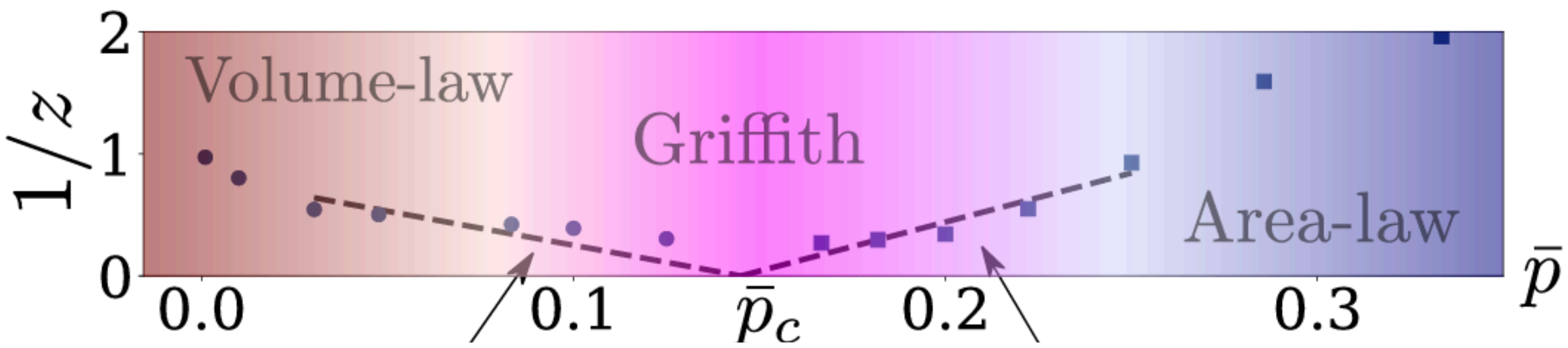
$$\bar{S}(L = \infty, t) \sim \log(t)$$

Extract relaxation times $t^*$

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Infinite Randomness Fixed Point

**Critical point**



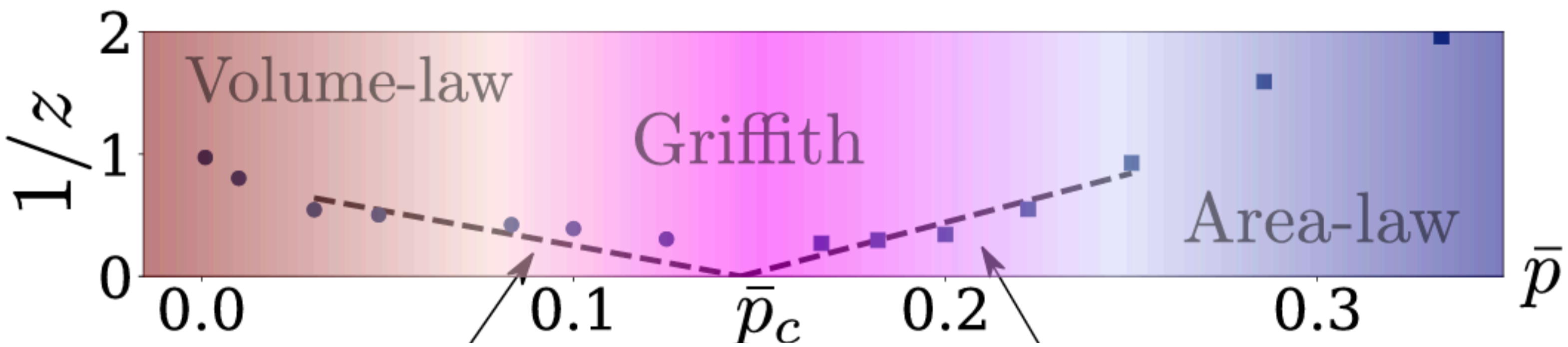

Probe the diverging dynamic exponent

$$z \sim \xi^{\psi} \sim |p - p_c|^{-\nu\psi}$$

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Infinite Randomness Fixed Point

## Critical point

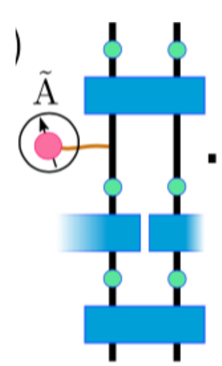


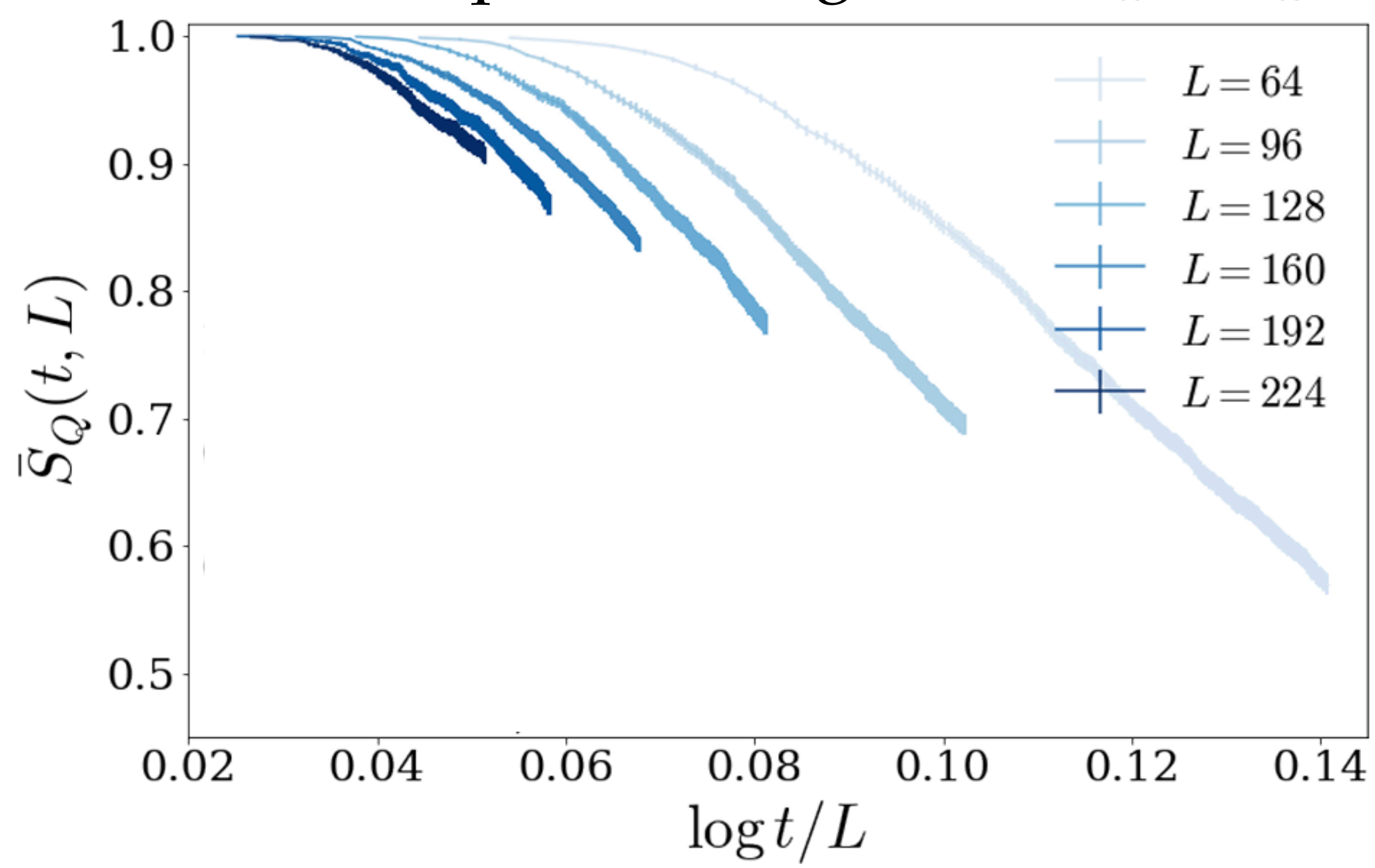

Probe the diverging dynamic exponent

$$z \sim \xi^{\psi} \sim |p - p_c|^{-\nu\psi}$$

Ancilla qubit entanglement $\bar{p} = \bar{p}_c$

coupling to ancilla qubit




Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

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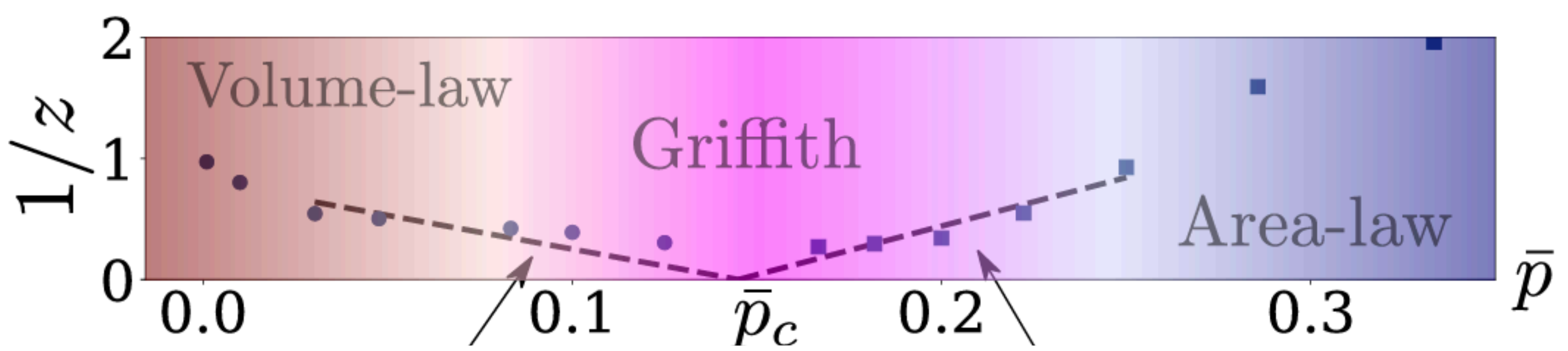
## Critical point




Probe the diverging dynamic exponent

$$z \sim \xi^{\psi} \sim |p - p_c|^{-\nu\psi}$$

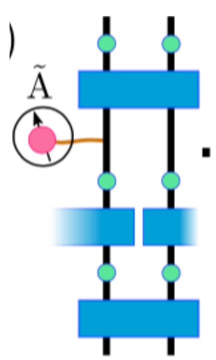
Activated dynamical scaling

$$S_Q \sim f(\log t / L^{\psi})$$

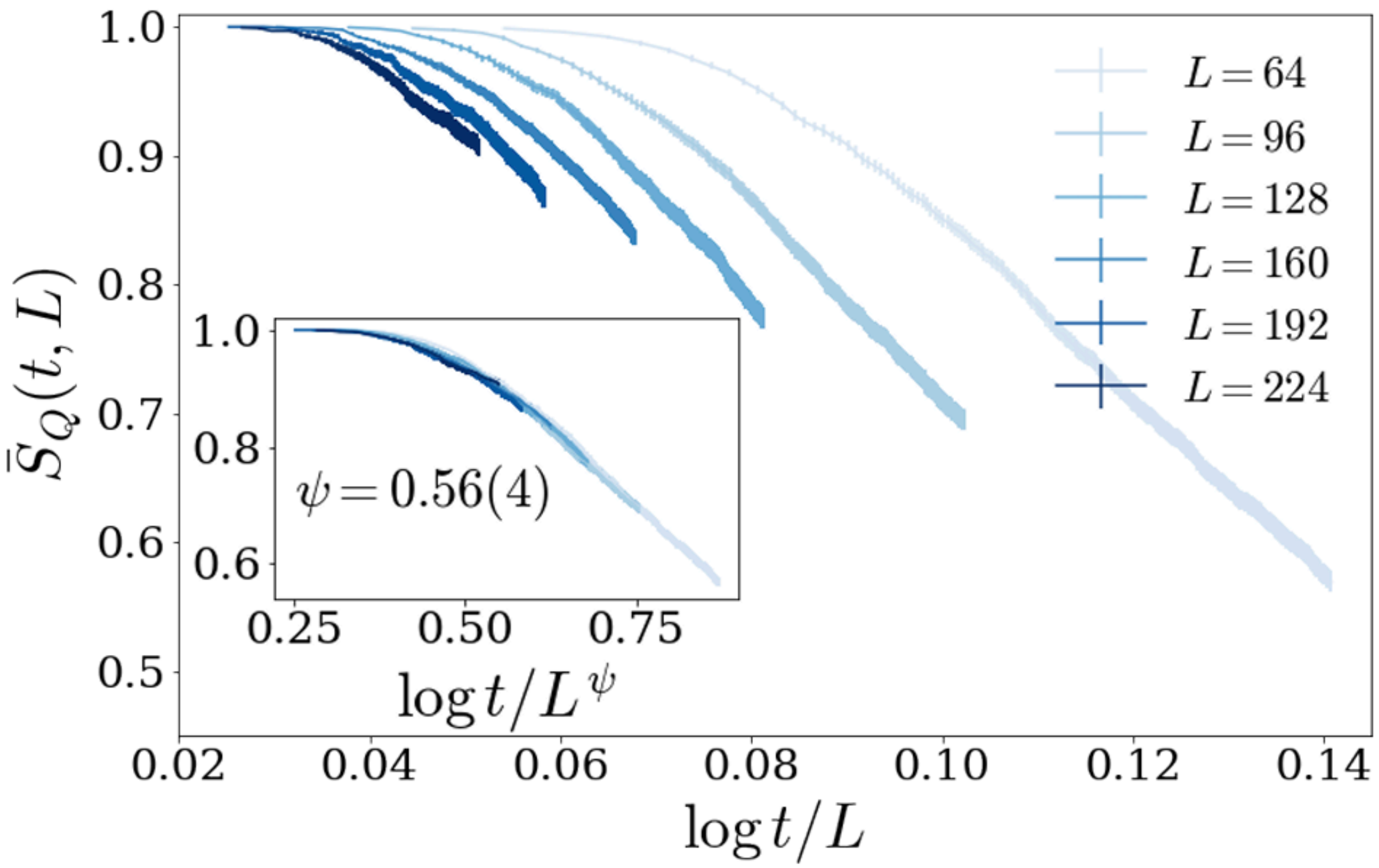
$$z \sim |p - p_c|^{-\nu\psi}$$

$$\nu \approx 2 \quad \psi \approx 0.5$$

coupling to ancilla qubit



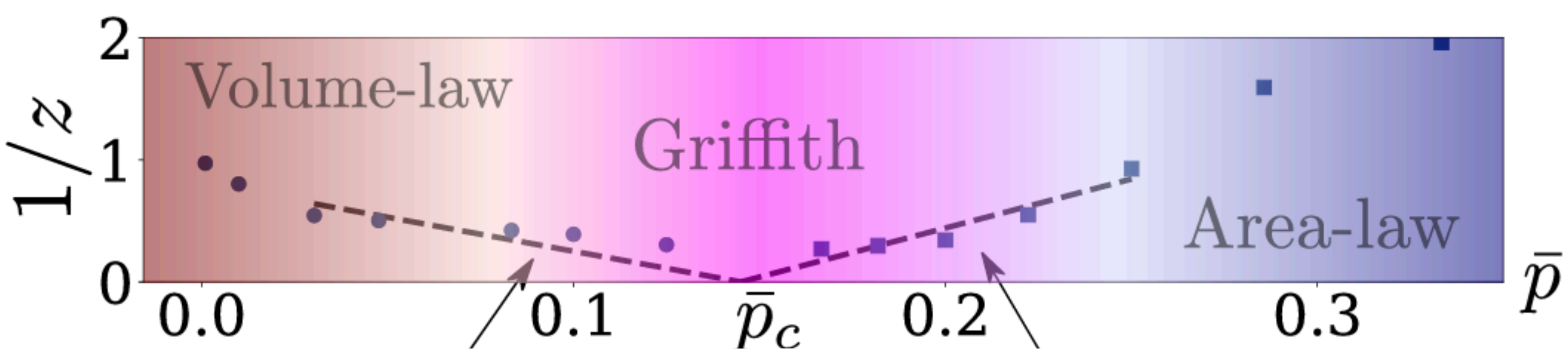
Ancilla qubit entanglement $\bar{p} = \bar{p}_c$




Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Infinite Randomness Fixed Point

## Critical point




$$z \sim \xi^{\psi} \sim |p - p_c|^{-\nu\psi}$$

Critical point well described by real space renormalization group (RSRG)

In the controlled limit of qudit gates with a **q**-size Hilbert space $q \to \infty$

The problem maps to a Q-state Potts model with columnar disorder that admits an RSRG description in the replica limit Q —> 1.

| RSRG | | Numerics | |
|---|---|---|---|
| $\nu = 2$ | $\psi = 1/2$ | $\nu \approx 2$ | $\psi \approx 0.5$ |

Zabalo, Wilson, Gullans, Vasseur, Gopalakrishnan, Huse, JHP PRB (2023)

# Infinite randomness in time?

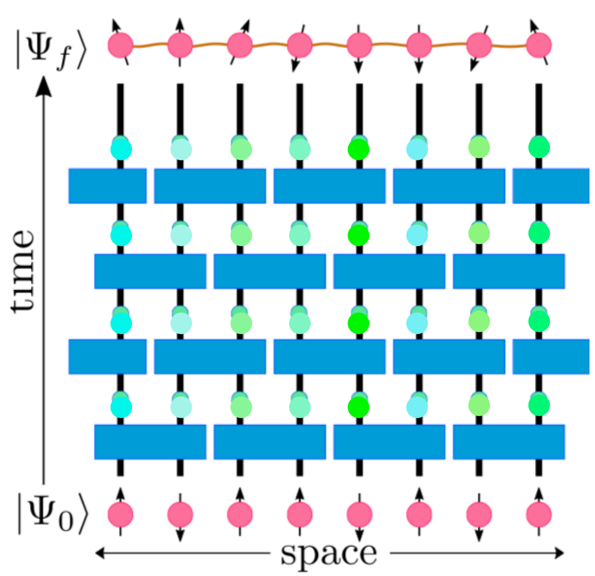

Spacetime duality between localization transitions and measurement-induced transitions

Tsung-Cheng Lu and Tarun Grover
PRX Quantum **2**, 040319 – Published 28 October 2021

Fractal, Logarithmic, and Volume-Law Entangled Nonthermal Steady States via Spacetime Duality

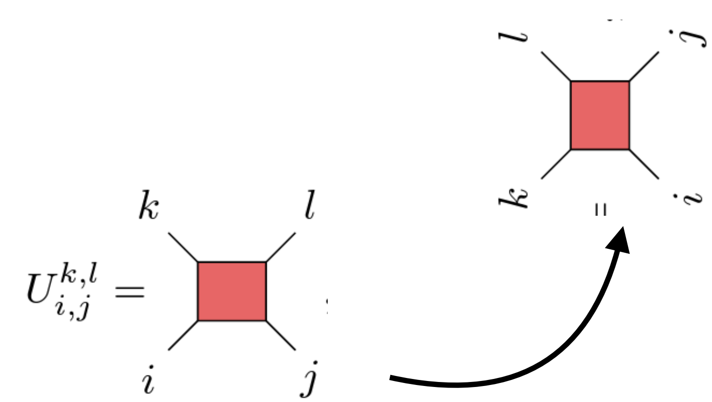
Matteo Ippoliti, Tibor Rakovszky, and Vedika Khemani
Phys. Rev. X **12**, 011045 – Published 9 March 2022


## Rotate space and time in the quantum circuit

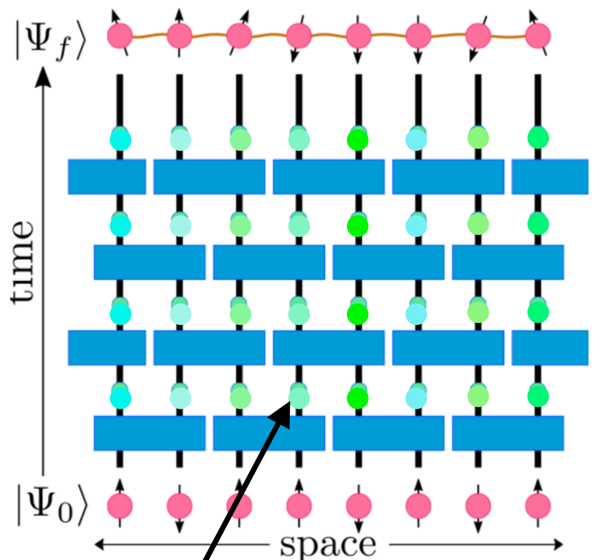



$$x \leftrightarrow t$$

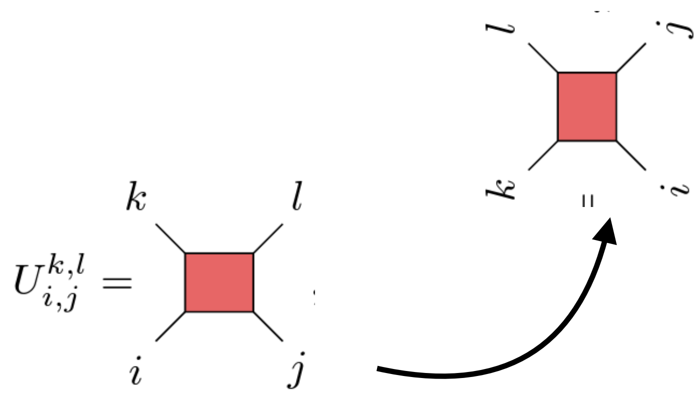
Think of the gates etc as a tensor network



# Apply a space-time rotation: $z \to 1/z$



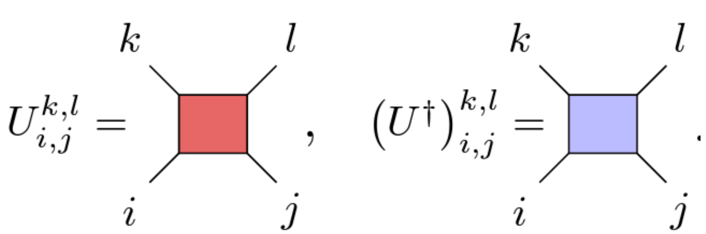

$x \to t$
$t \to -x$

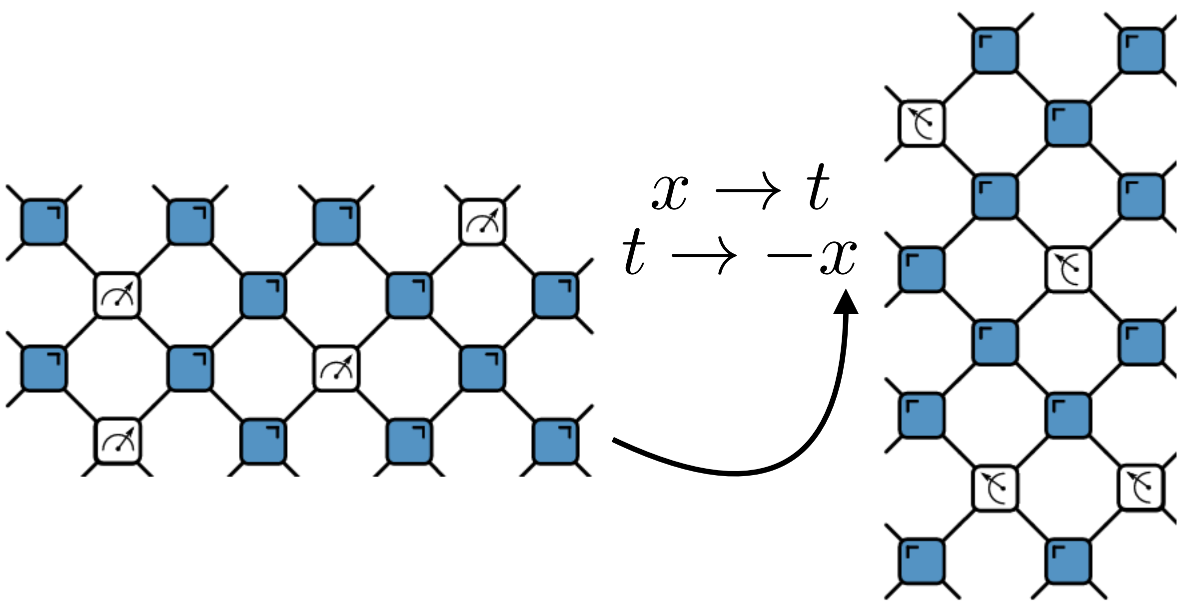



value of p is **random in space, fixed in time**

$$p(x) = (r_x)^n$$

Think of the circuit as a tensor network



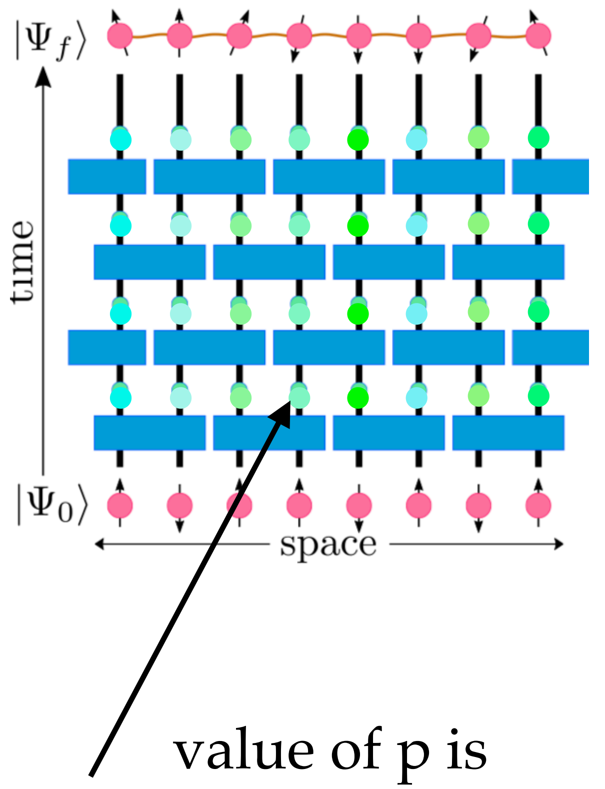



$x \to t$
$t \to -x$

Piroli, Bertini, Cirac, Prosen PRB (2020)

Claeys, Henry, Vicary, Lamacraft, PRR (2022)

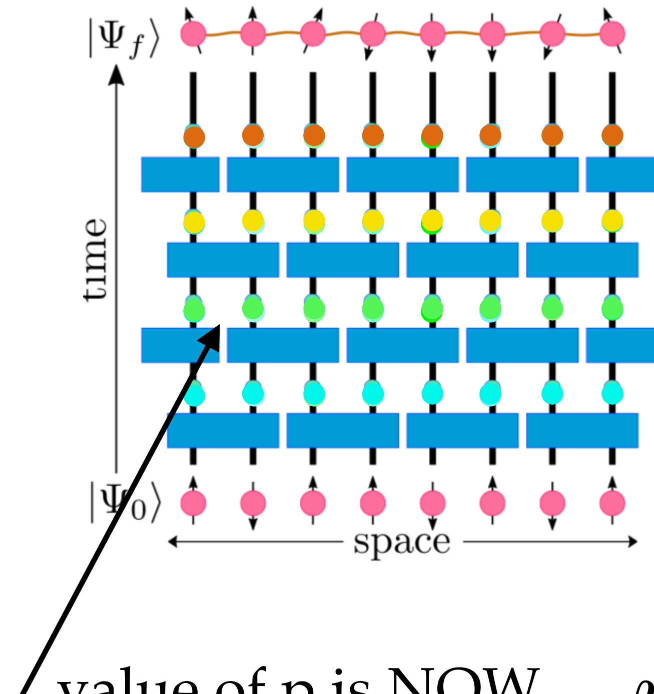
# APPLY A SPACE-TIME ROTATION: $z \to 1/z$




$x \to t$
$t \to -x$

value of p is **random in space, fixed in time**

$$p(x) = (r_x)^n$$

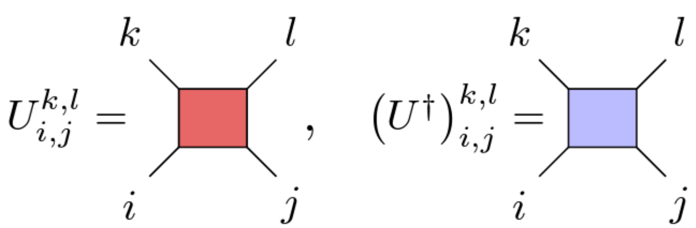


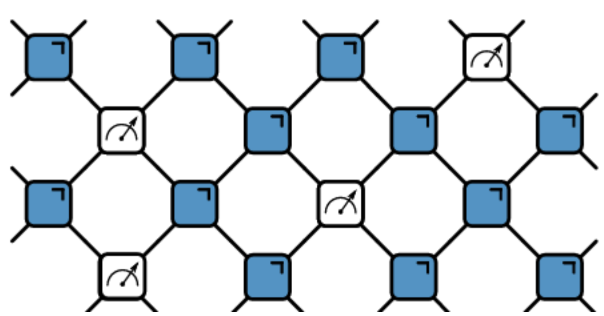

value of p is NOW **random in time, fixed in space**

$$p(t) = (r_t)^n$$

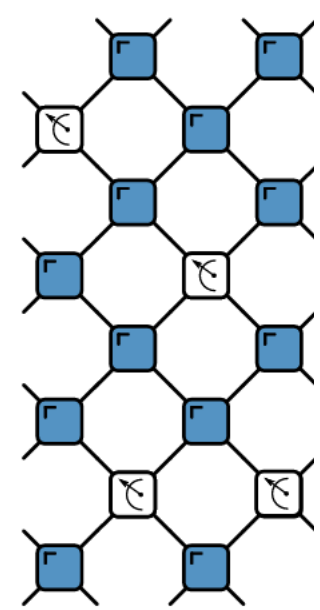
$$\bar{p} = 1/(n+1)$$

Think of the circuit as a tensor network



$x \to t$
$t \to -x$



Piroli, Bertini, Cirac, Prosen PRB (2020)

Claeys, Henry, Vicary, Lamacraft, PRR (2022)

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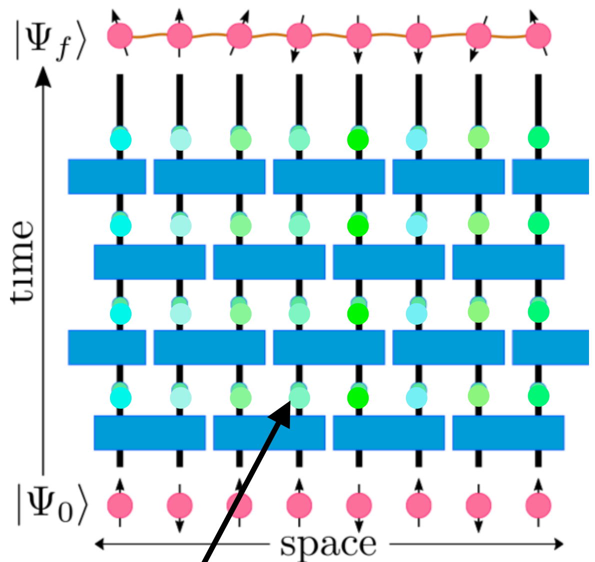
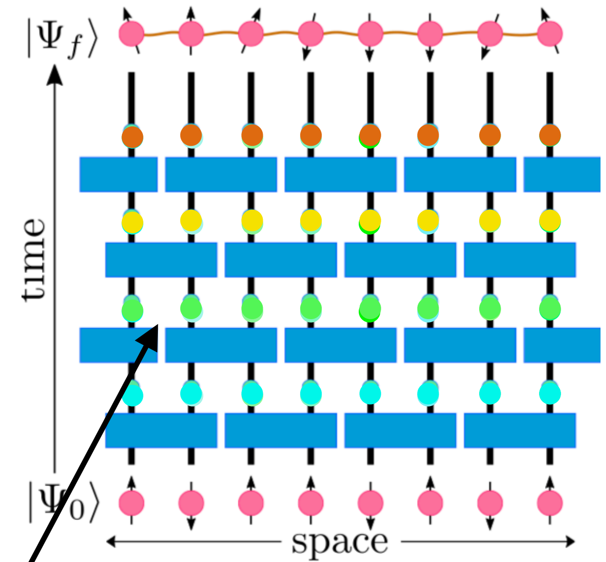



$x \to t$
$t \to -x$

value of p is
**random in space,**
**fixed in time**

$$p(x) = (r_x)^n$$



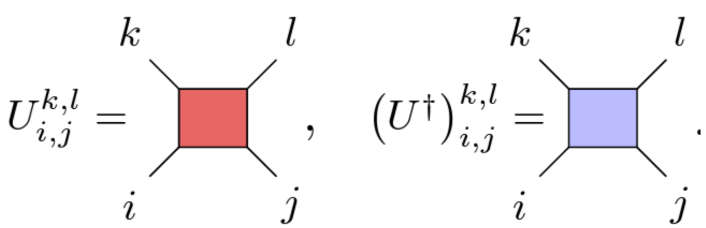

value of p is NOW
**random in time,**
**fixed in space**

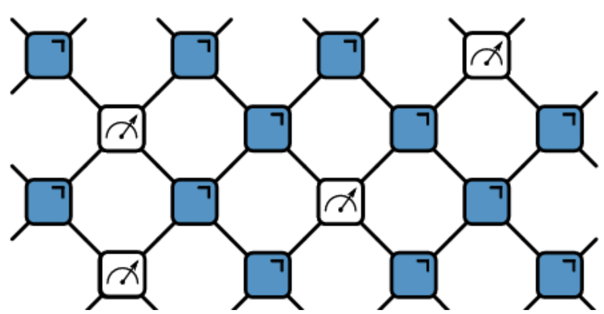
$$p(t) = (r_t)^n$$

$$\bar{p} = 1/(n+1)$$

(Ignoring rotation on gates, dual unitary circuit limit leaves them unchanged)

Think of the circuit as a tensor network



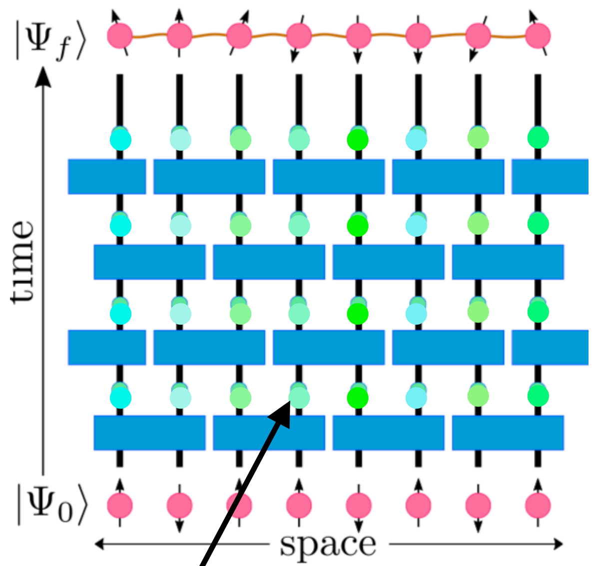



Dual unitary limit

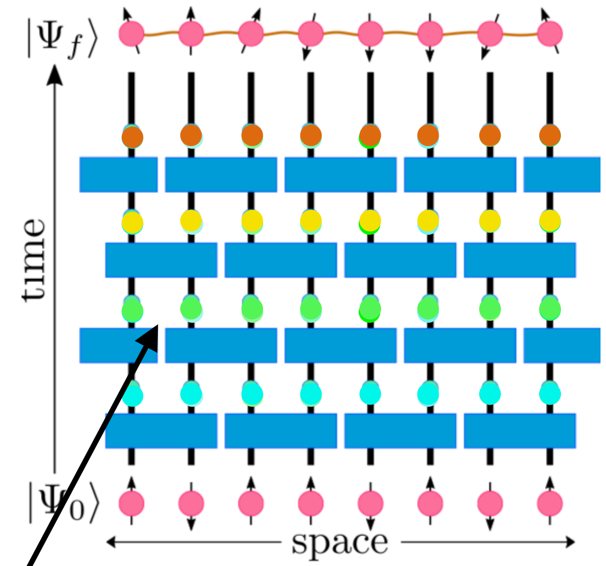
$$\langle k| \otimes \langle \ell|\tilde{U}|i\rangle \otimes |j\rangle = \langle j| \otimes \langle \ell|U|i\rangle \otimes |k\rangle \,.$$

Piroli, Bertini, Cirac, Prosen PRB (2020)

# Apply a space-time rotation: $z \to 1/z$




$x \to t$
$t \to -x$




value of p is **random in space, fixed in time**

$p(x) = (r_x)^n$

value of p is NOW **random in time, fixed in space**

$p(t) = (r_t)^n$

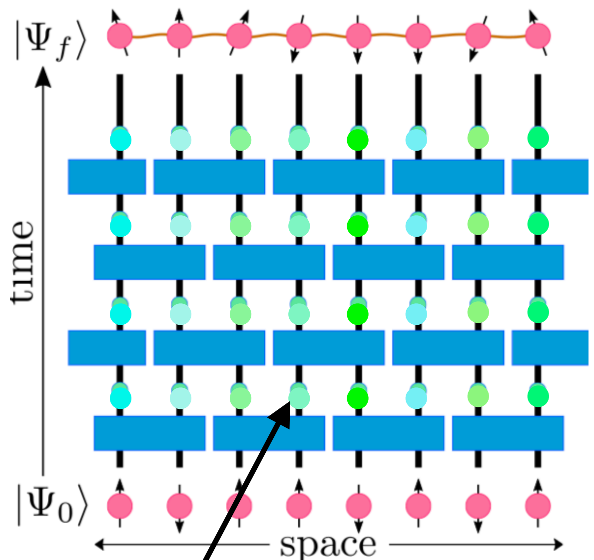
$\bar{p} = 1/(n+1)$

Apply the idea of a space time rotation to the infinite randomness fixed point

$z \sim |p - p_c|^{-\nu\psi}$ $\qquad x \leftrightarrow t \qquad$ $z \sim |p - p_c|^{+\nu\psi}$

$z \leftrightarrow 1/z$

# Apply a space-time rotation: $z \to 1/z$

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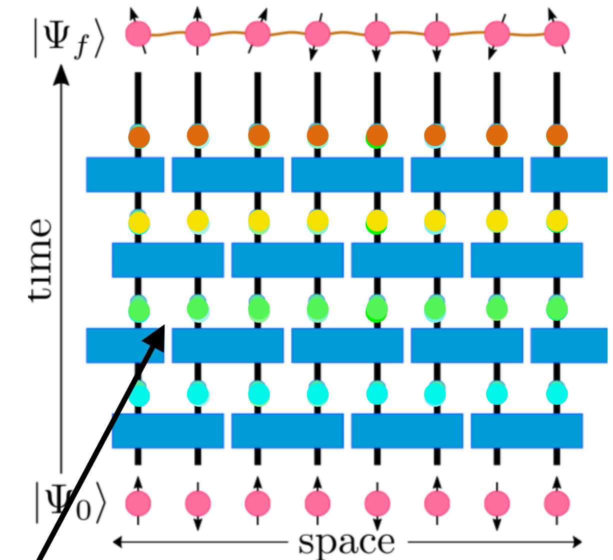


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Entanglement relaxation time scales $t \sim e^{a\sqrt{\xi}}$ $\quad z \leftrightarrow 1/z \quad$ $t^{\psi_\tau} \sim \log \xi$ Suggests an ultra-fast dynamics!

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Entanglement relaxation time scales $t \sim e^{a\sqrt{\xi}}$ ⟷ $z \leftrightarrow 1/z$ ⟷ $t^{\psi_\tau} \sim \log \xi$ Suggests an ultra-fast dynamics!

Generalized notion of Harris, spatio-temporal perturbation

$$\nu \geq 2/z$$

Vojta and Dickman, PRE (2016)

# Apply a space-time rotation: $z \to 1/z$

Apply the idea of a space time rotation to the infinite randomness fixed point

$$z \sim |p - p_c|^{-\nu\psi} \qquad x \leftrightarrow t \qquad z \sim |p - p_c|^{+\nu\psi}$$

Entanglement relaxation time scales

$$t \sim e^{a\sqrt{\xi}} \quad \longleftrightarrow_{z \leftrightarrow 1/z} \quad t^{\psi_\tau} \sim \log \xi$$

Suggests an ultra-fast dynamics!

Generalized notion of Harris, spatio-temporal perturbation

$$\nu \geq 2/z$$

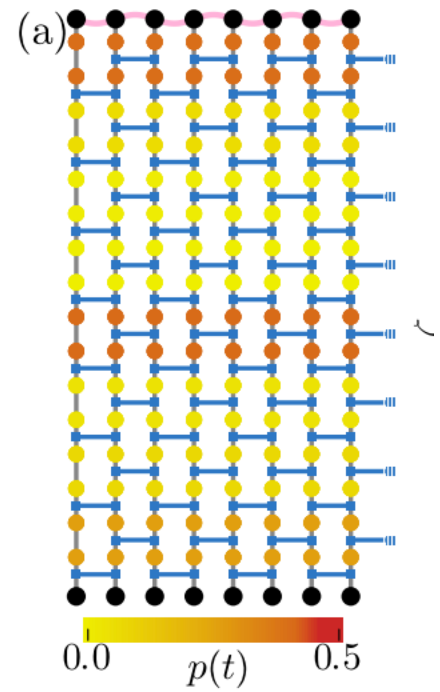
Where does it flow to?

Vojta and Dickman, PRE (2016)

# Phases of the temporal randomness model

What are the phases of the model now?

Is there a volume law phase at all? No!



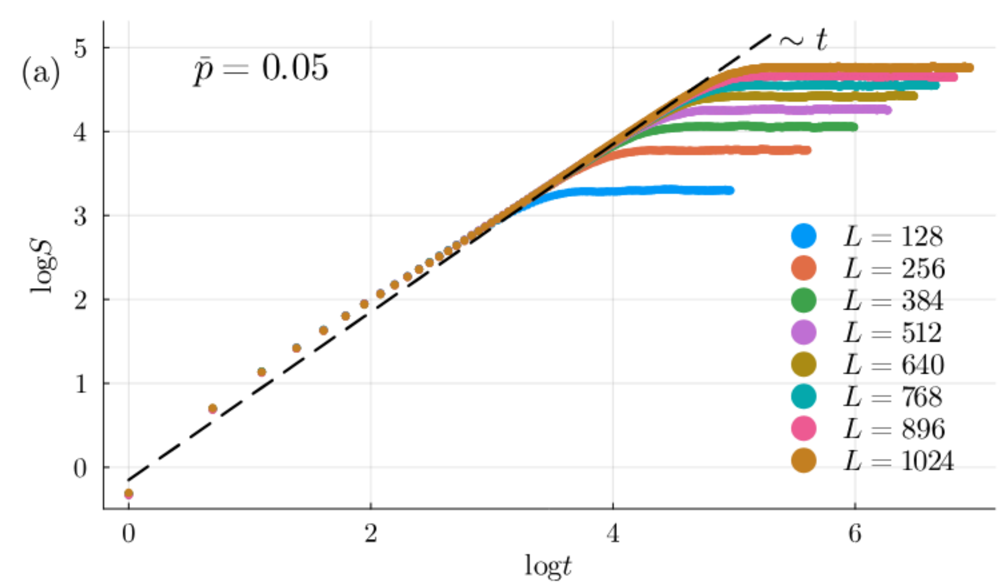

Volume law phase with static disorder

$$\bar{S} \sim L$$
$$\bar{S} \sim t^{1/z}$$

$$x \leftrightarrow t$$

Suggests that phase with temporal randomness

$$\bar{S} \sim L^z$$
$$\bar{S} \sim t$$



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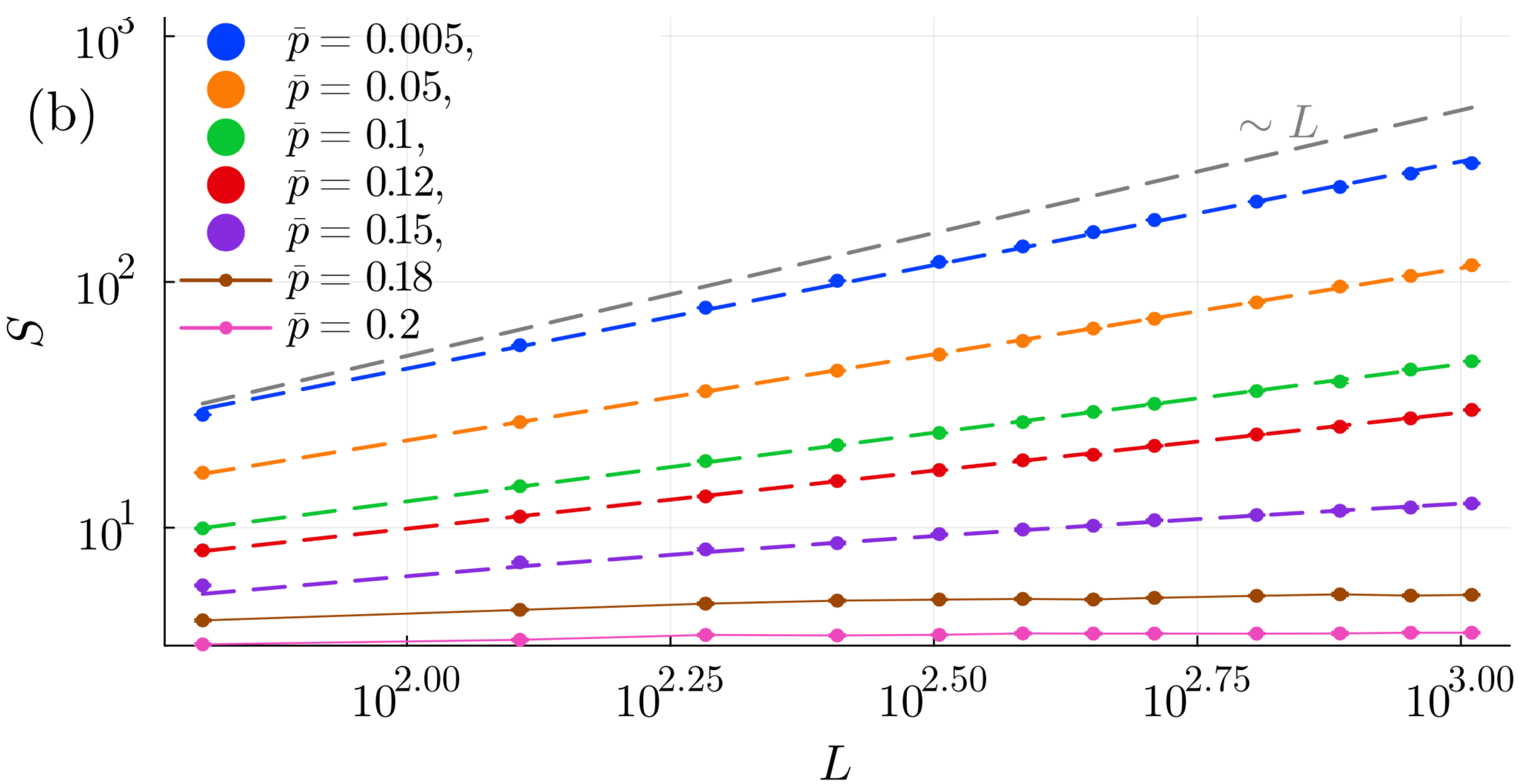
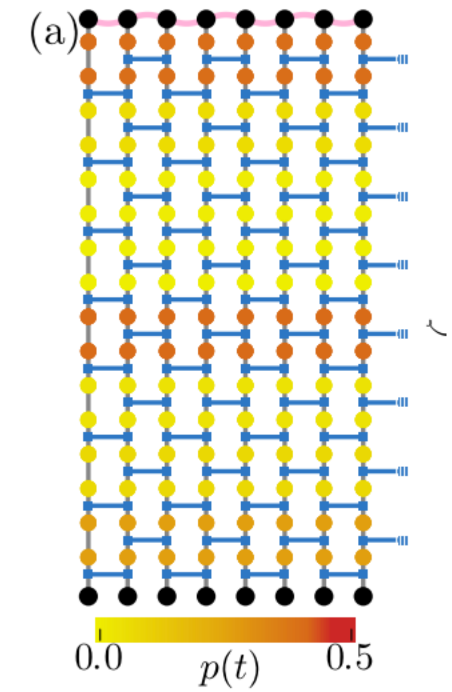
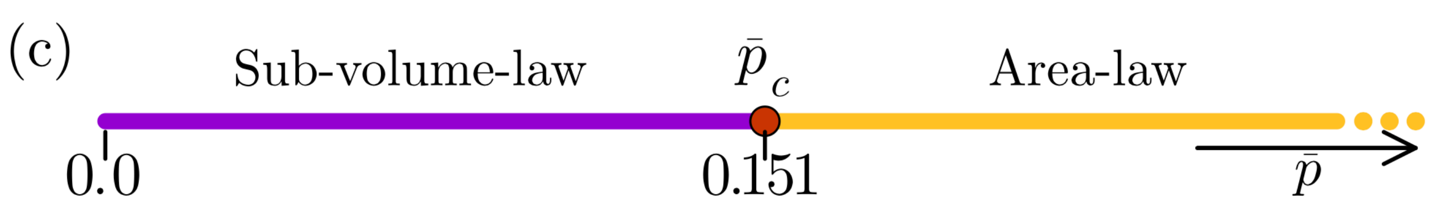
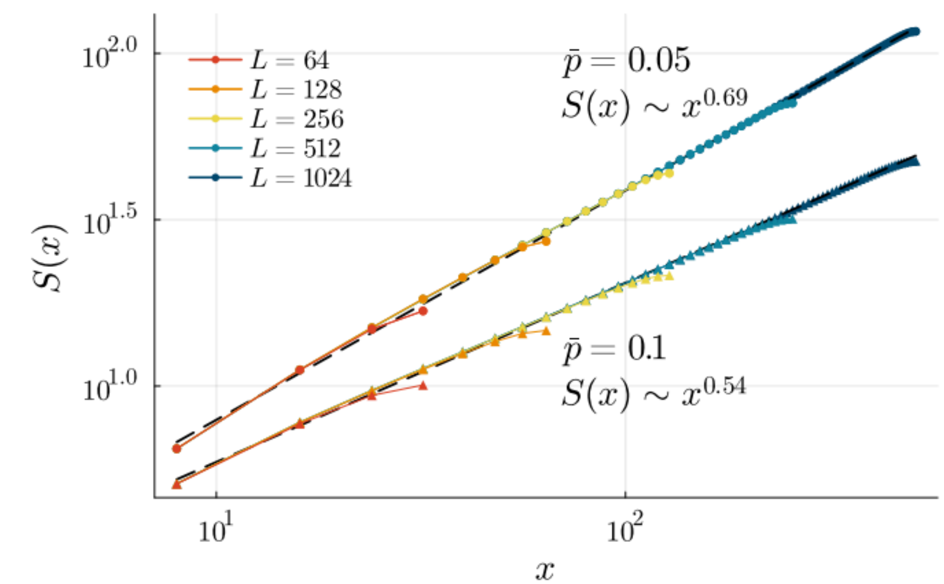


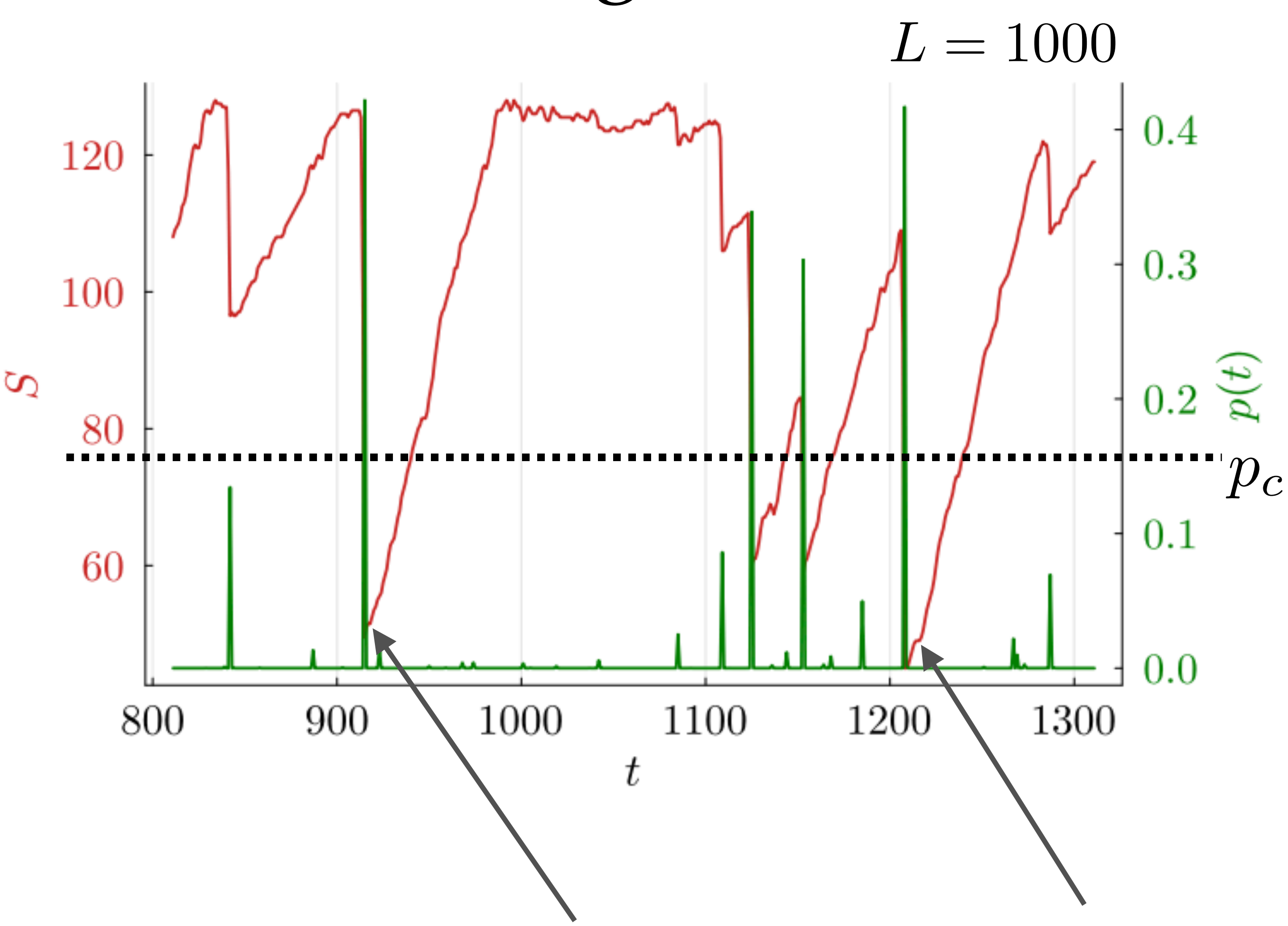

$$S(L, t \to \infty) \sim L^z$$



# Temporal Griffith effects, sub-volume phase

Consider a single realization

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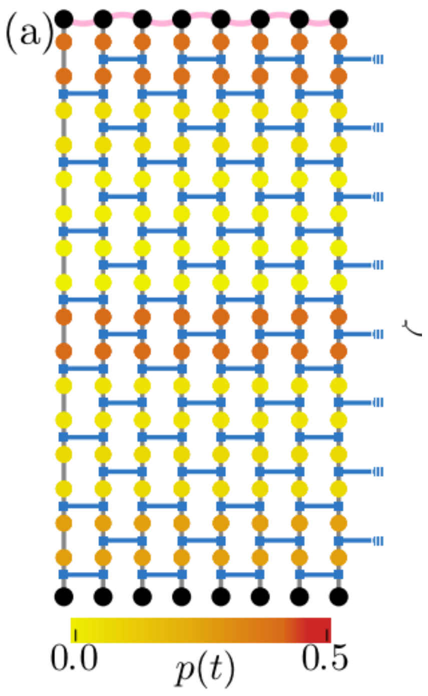


Saw tooth behavior: Can be understood as most p have $p<p_c$, but on occasion $p>p_c$ and S drops significantly

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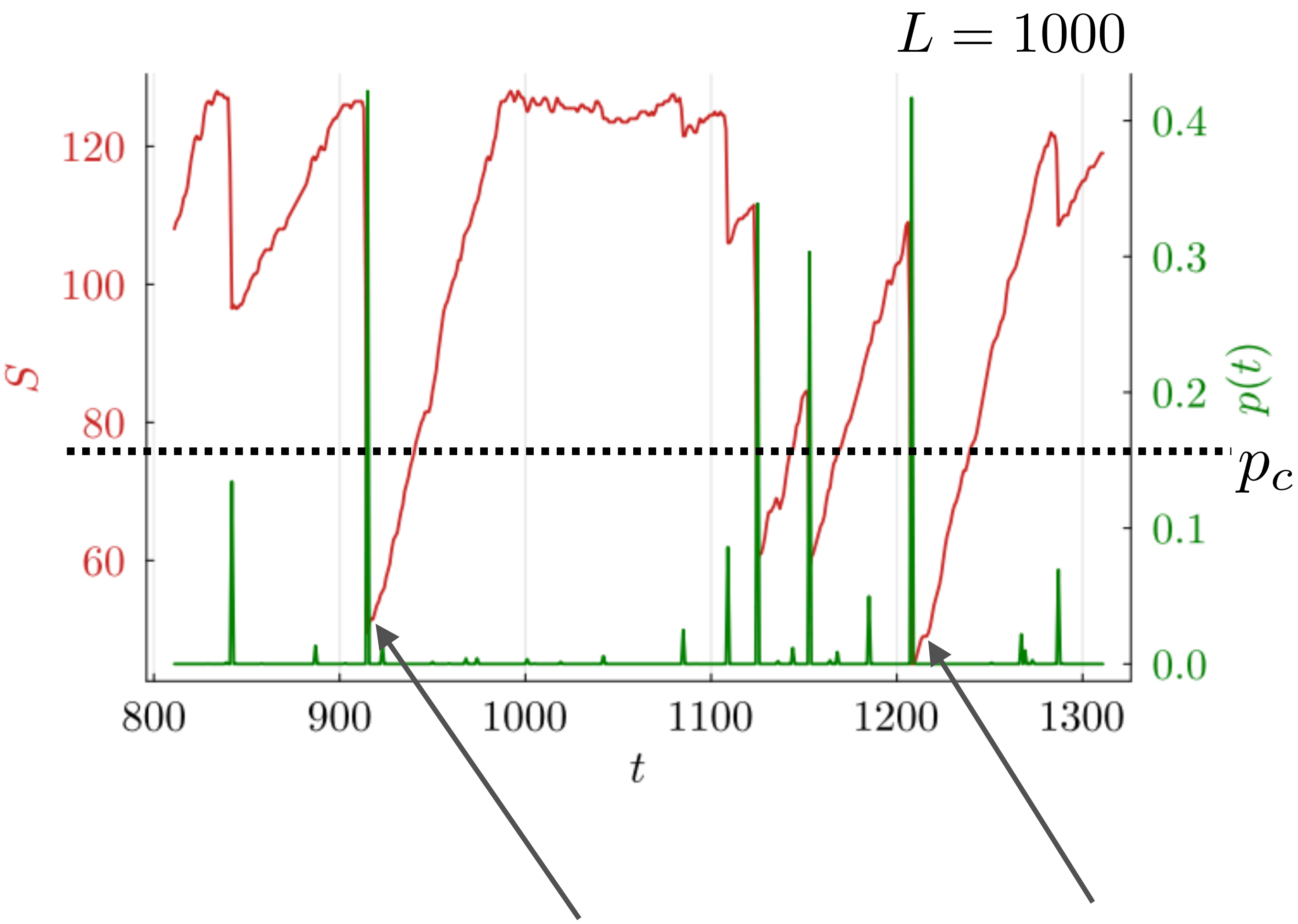


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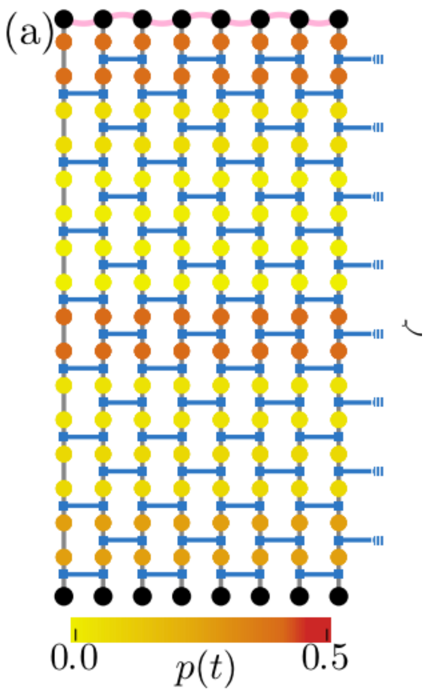


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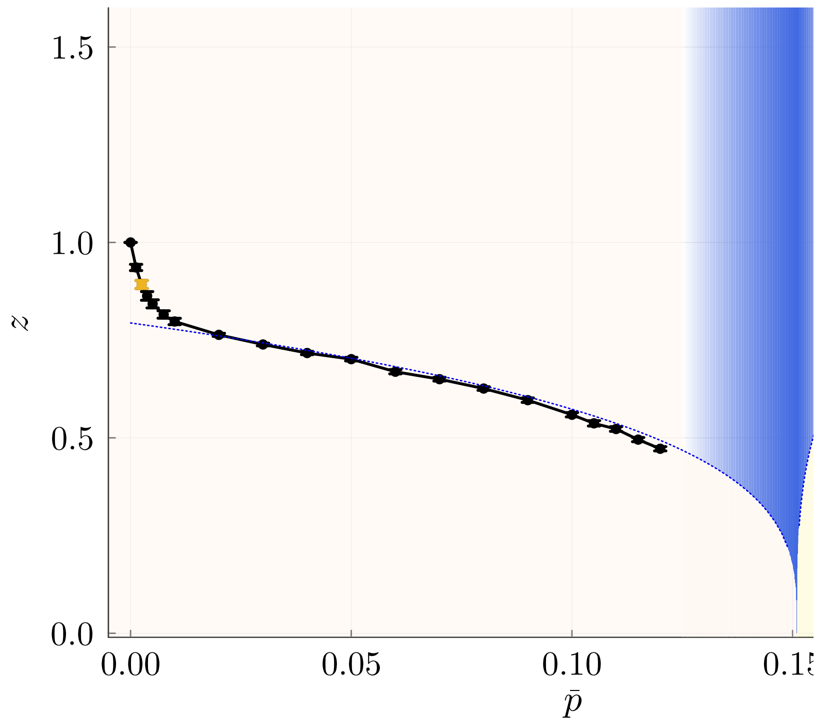
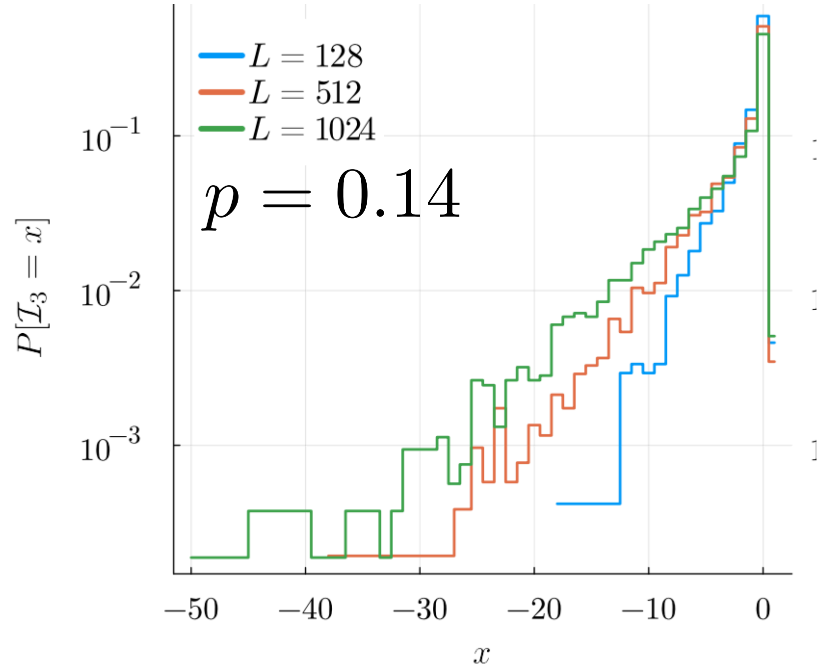


# Locating the transition

Tripartite mutual information has a broad distribution $\mathcal{I}_3 \to P[\mathcal{I}_3]$

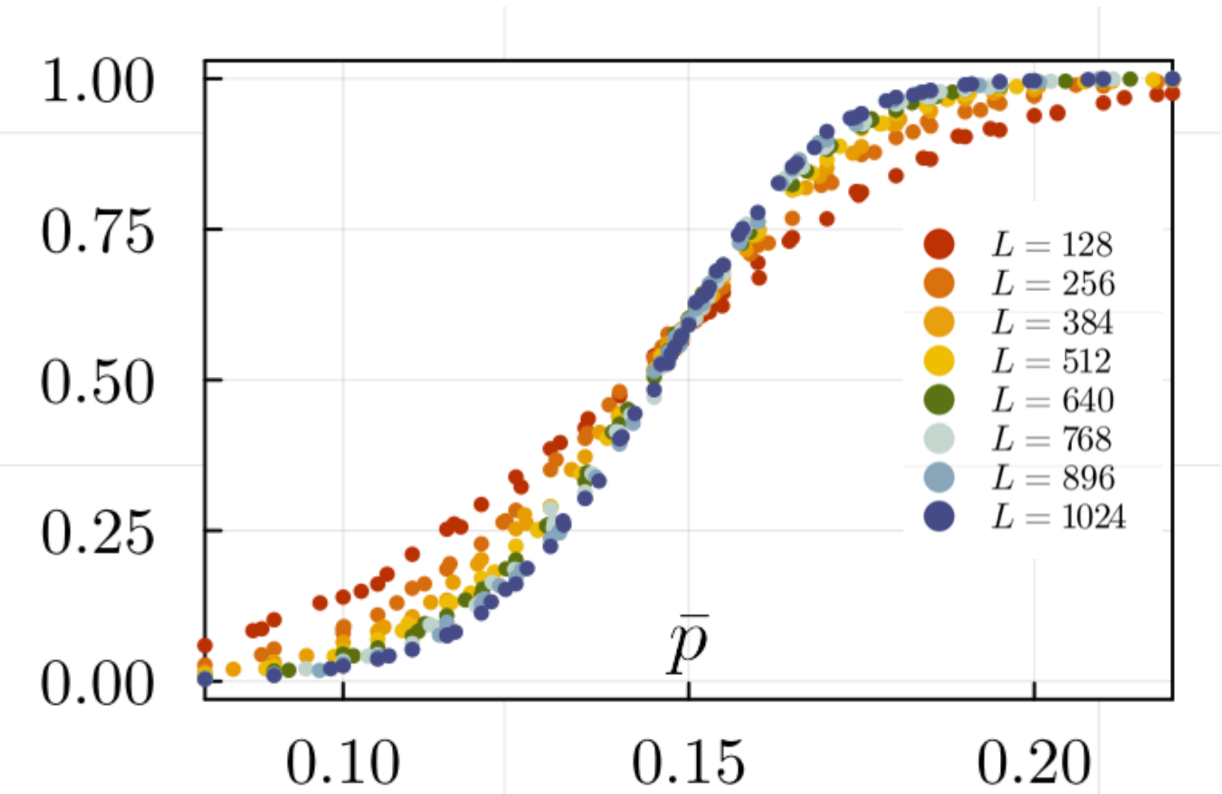
Broad distribution near the transition cannot use the mean!

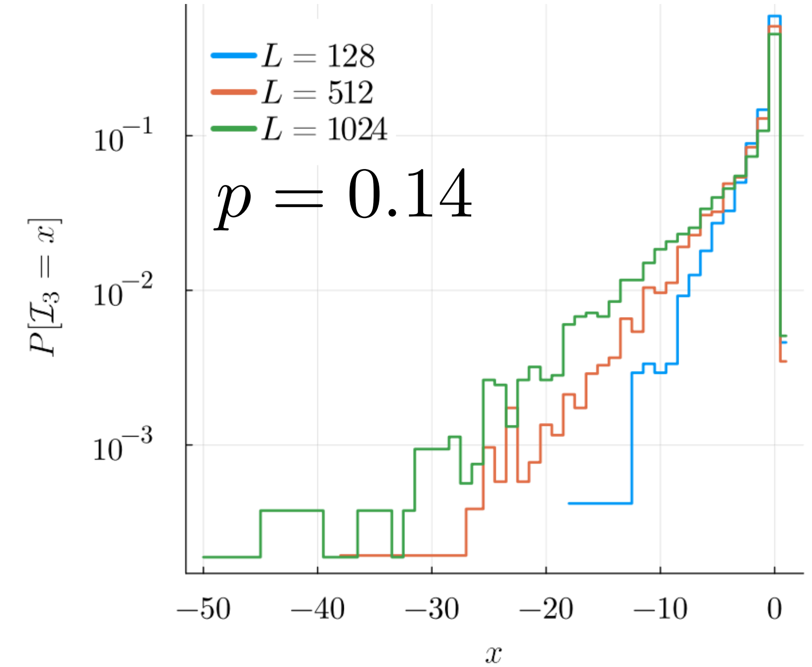


# Locating the transition

Tripartite mutual information has a broad distribution $\mathcal{I}_3 \to P[\mathcal{I}_3]$

$P[\mathcal{I}_3 = 0]$

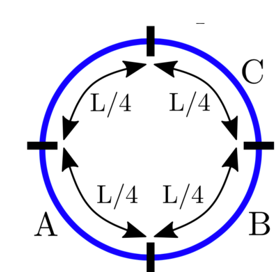



Define critical exponents for this dynamical transition $\xi_t \sim (p - p_c)^{-\nu_\tau}$

$$t^{\psi_\tau} \sim \log L$$

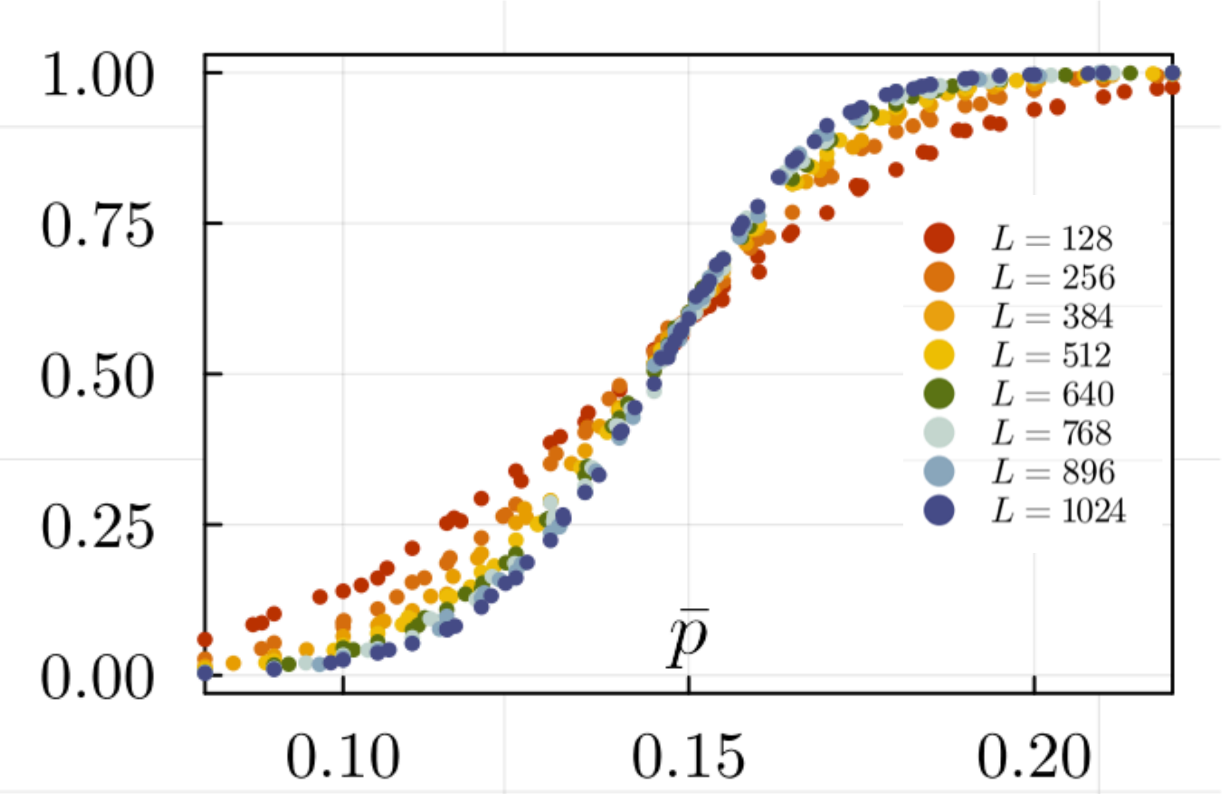
$$\mathcal{I}_{3,n}(A, B, C) \equiv S_n(A) + S_n(B) + S_n(C) - S_n(A \cup B) - S_n(A \cup C) - S_n(B \cup C) + S_n(A \cup B \cup C).$$



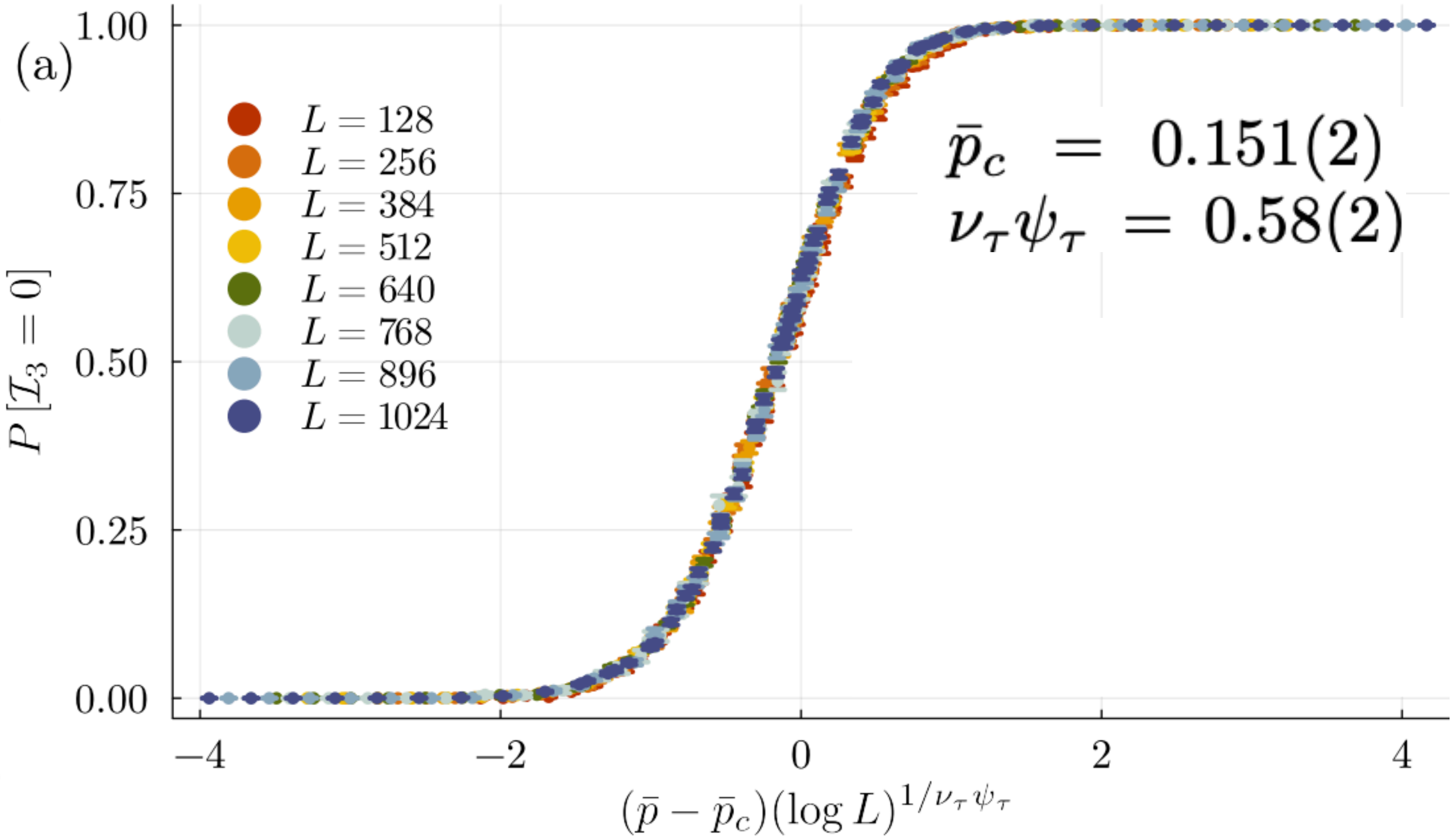
# Locating the transition

Tripartite mutual information has a broad distribution $\mathcal{I}_3 \rightarrow P[\mathcal{I}_3]$

$P[\mathcal{I}_3 = 0]$



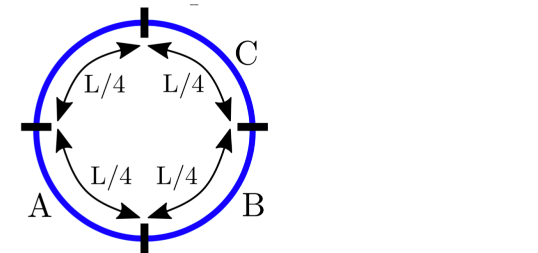

$P[\mathcal{I}_3 = 0]$




Define critical exponents for this dynamical transition $\xi_t \sim (p - p_c)^{-\nu_\tau}$

$t^{\psi_\tau} \sim \log L$

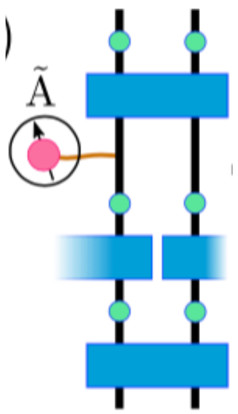
$$\mathcal{I}_{3,n}(A,B,C) \equiv S_n(A) + S_n(B) + S_n(C) - S_n(A \cup B) - S_n(A \cup C) - S_n(B \cup C) + S_n(A \cup B \cup C).$$

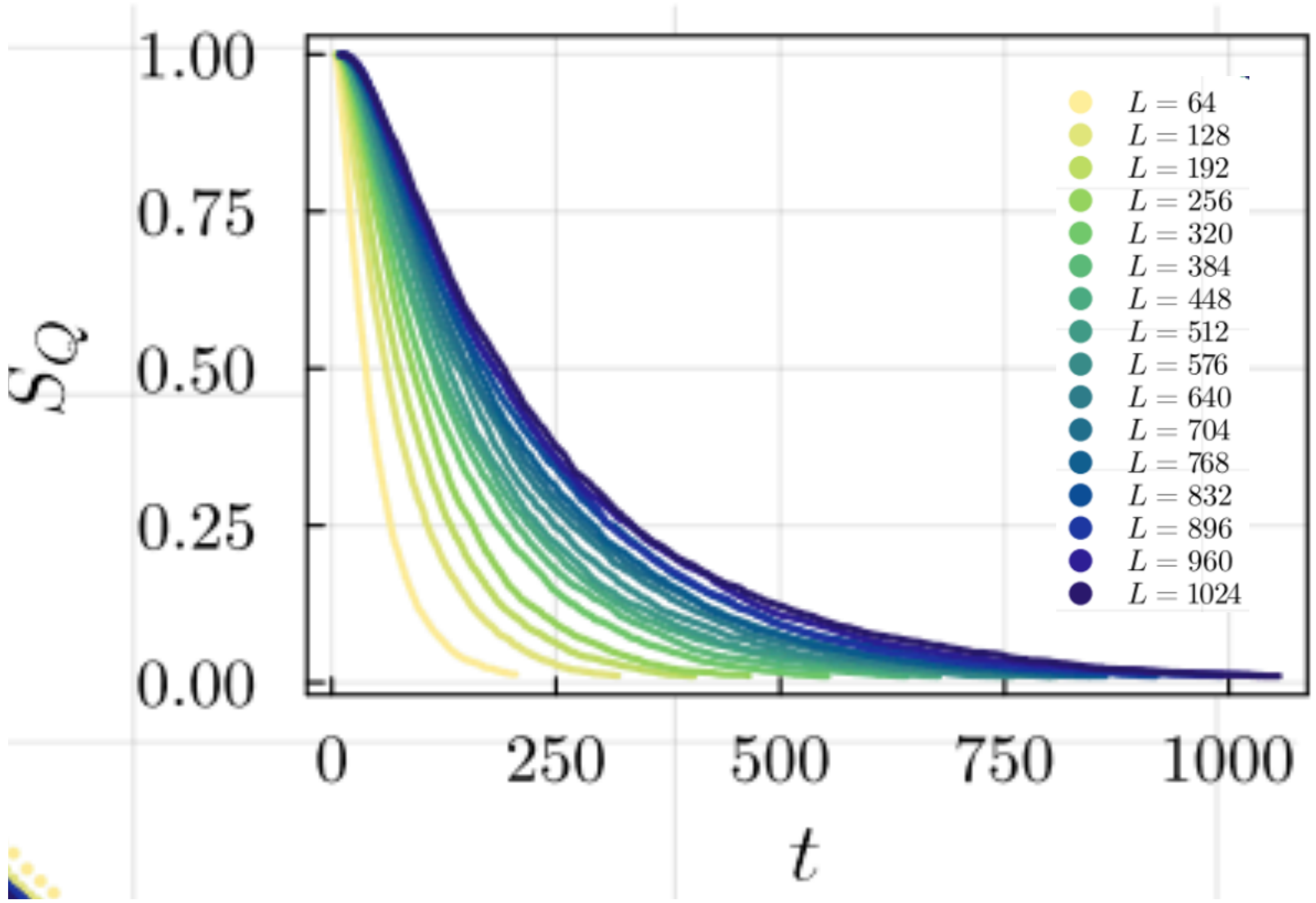


# Relaxation in the ancilla qubit at the transition

Directly test ultra-fast scaling ansatz $t^{\psi_\tau} \sim \log L$

coupling to
ancilla qubit




Defn of exponents

$$\xi_t \sim (p - p_c)^{-\nu_\tau}$$

$$t^{\psi_\tau} \sim \log L$$

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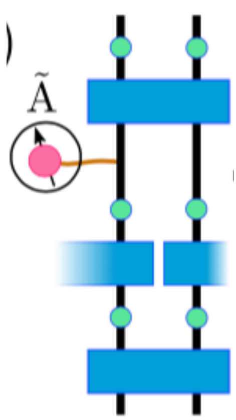
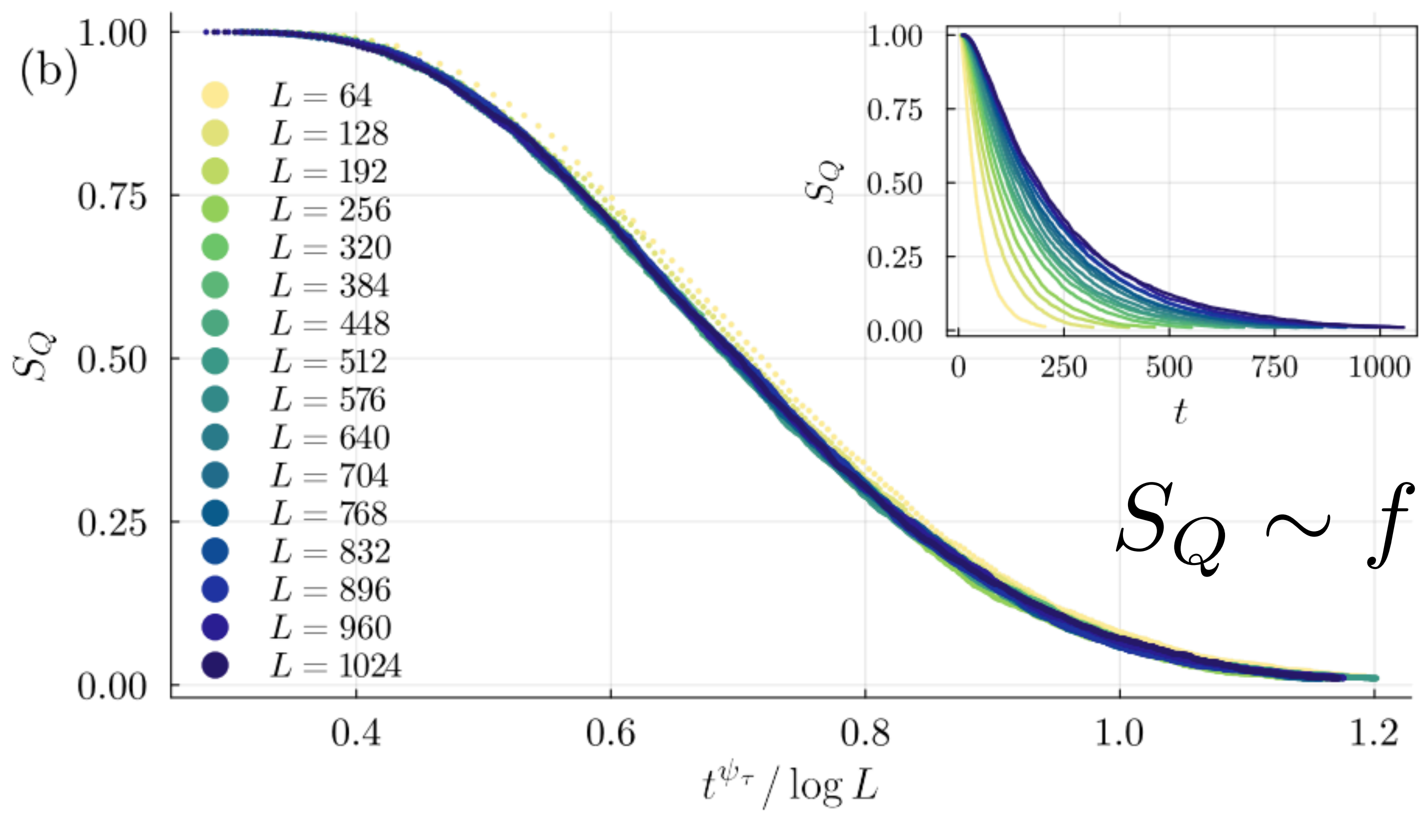



$S_Q \sim f(t^{\psi_\tau} / \log L)$

$\psi_\tau = 0.3$ Does not match infinite randomness

$\nu_\tau \approx 1.9(1)$

Matches infinite randomness and saturates a dynamical "Harris" bound

Defn of exponents

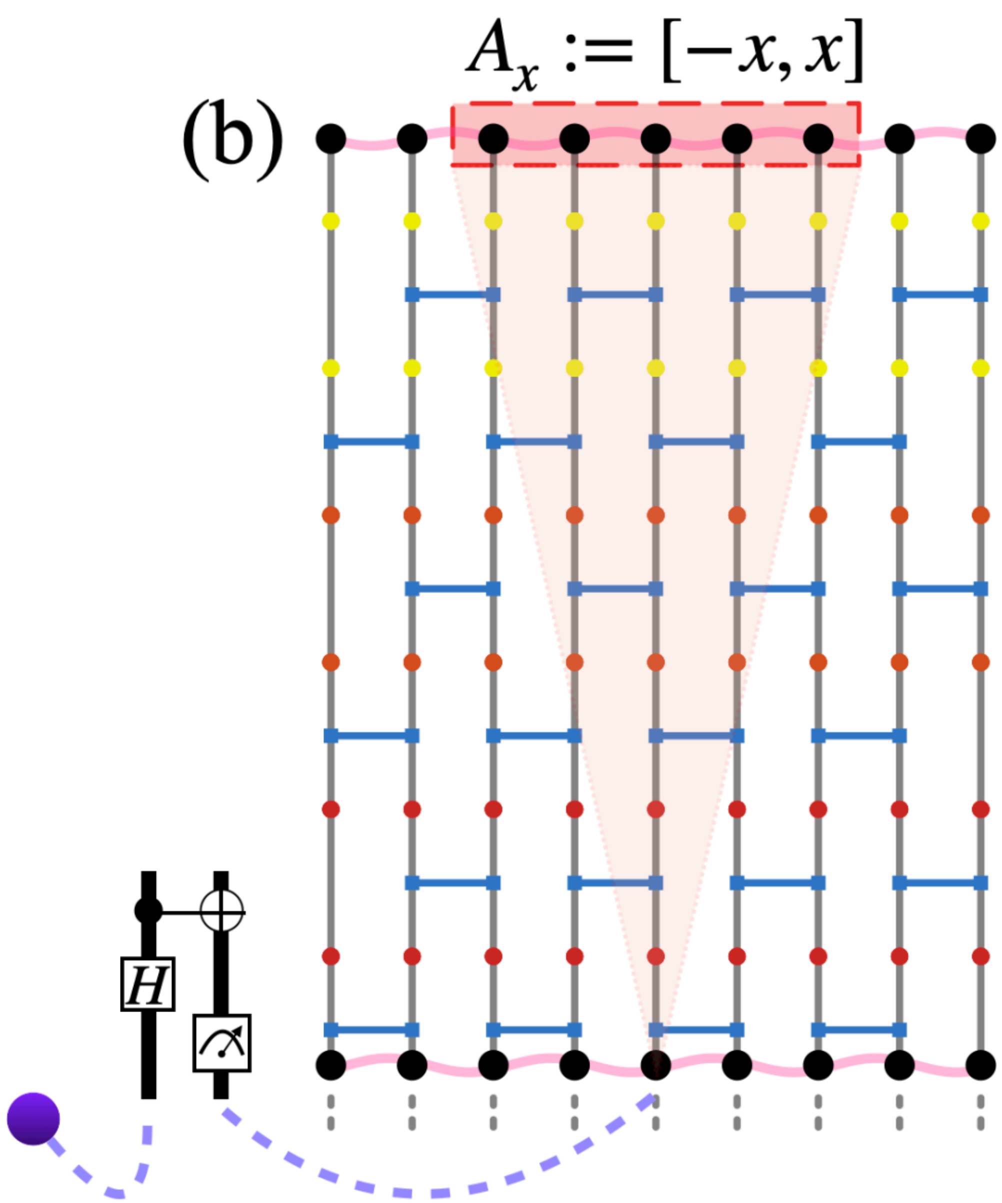
$\xi_t \sim (p - p_c)^{-\nu_\tau}$

$t^{\psi_\tau} \sim \log L$

# Does this Imply we can send information faster?

Yes!

Couple a qubit into a critical monitored circuit, "inject a bit of information"




Compute the mutual information

$$\mathcal{I}_2(A_x, B)$$

Find the $x$ where $\mathcal{I}_2(A_x, B) \to 0$

Call this $x_I$

Sang, Li, Hsieh, Yoshida, PRX Quantum (2023)

(Inject d qubits)

# Does this Imply we can send information faster?

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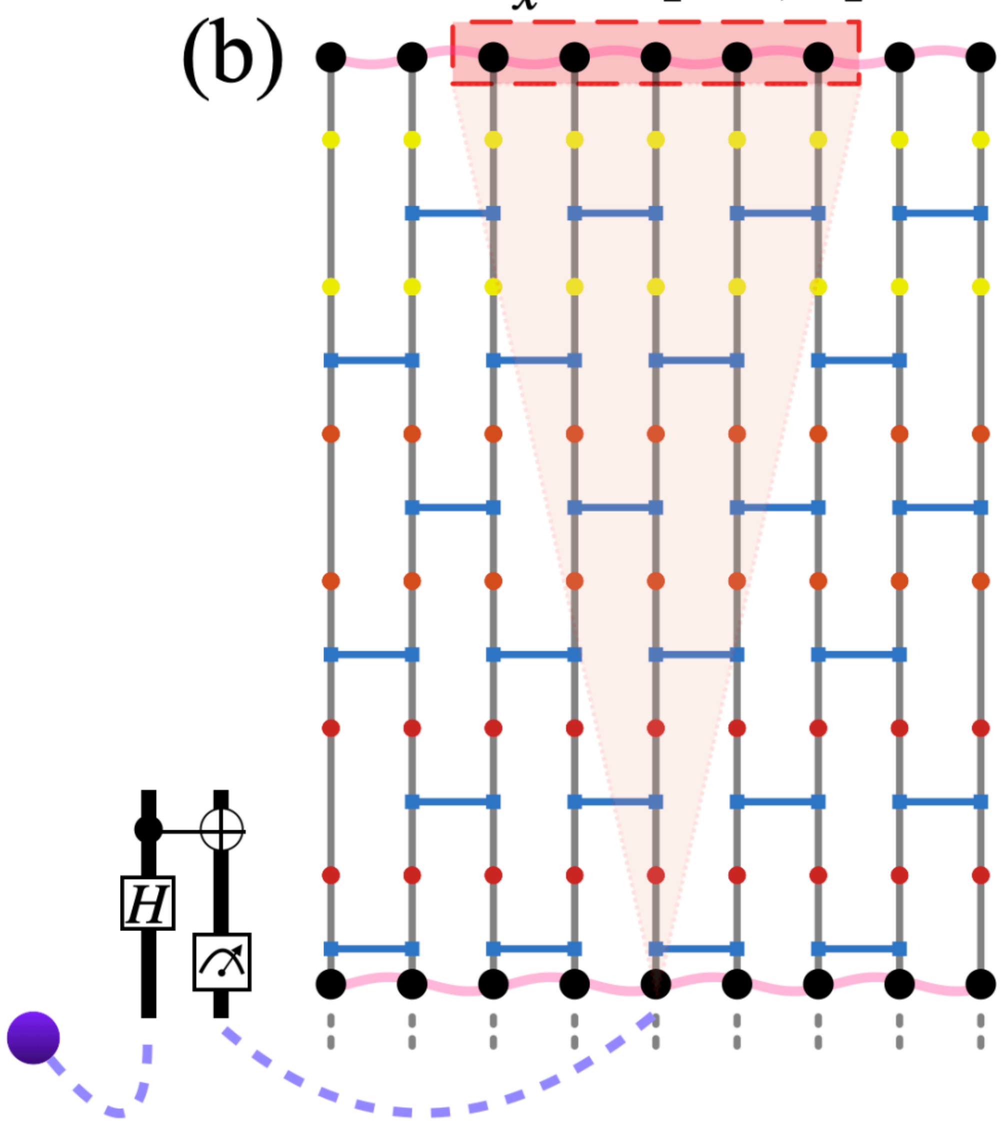
Couple a qubit into a critical monitored circuit, "inject a bit of information"

$A_x := [-x, x]$

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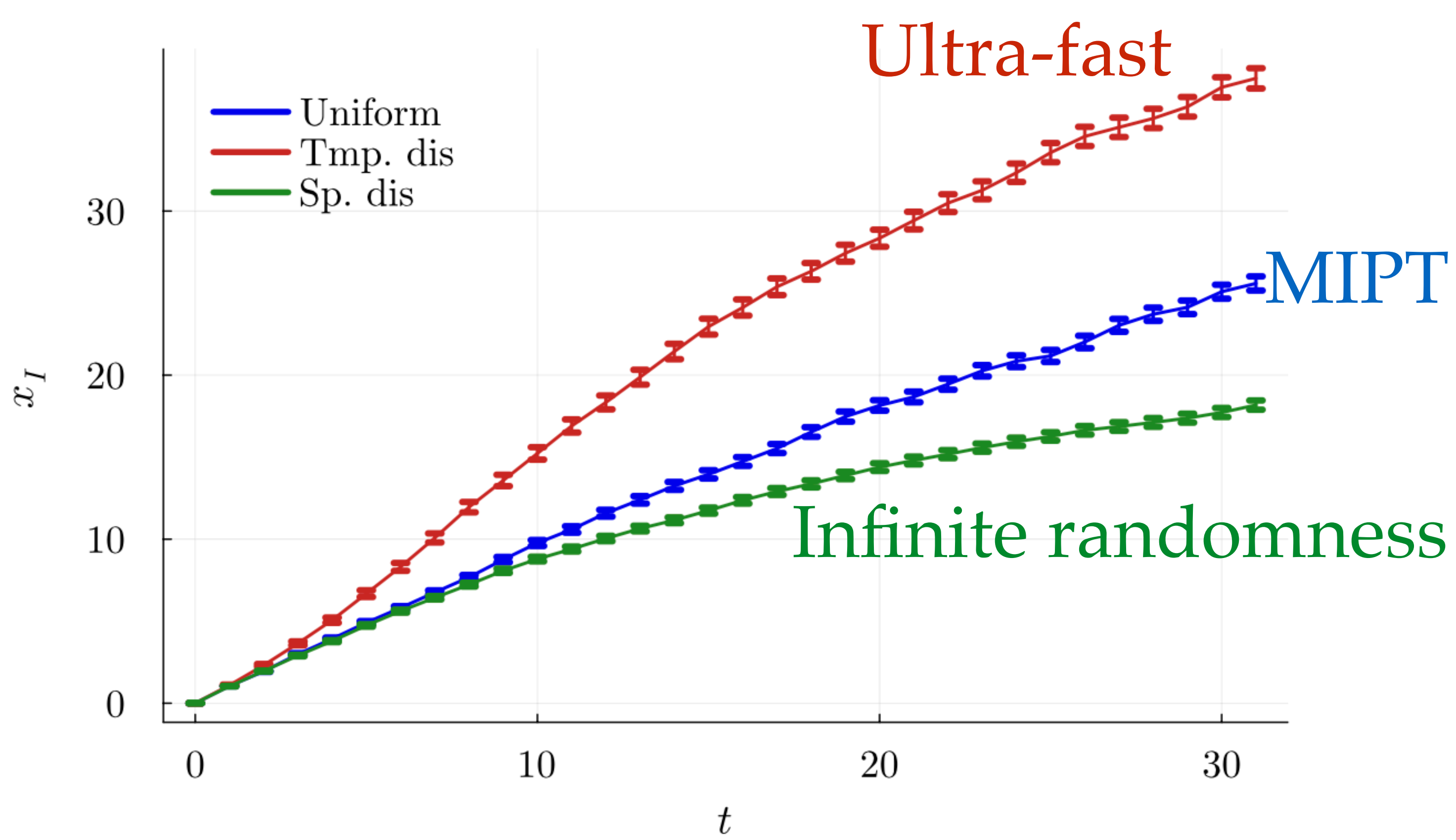

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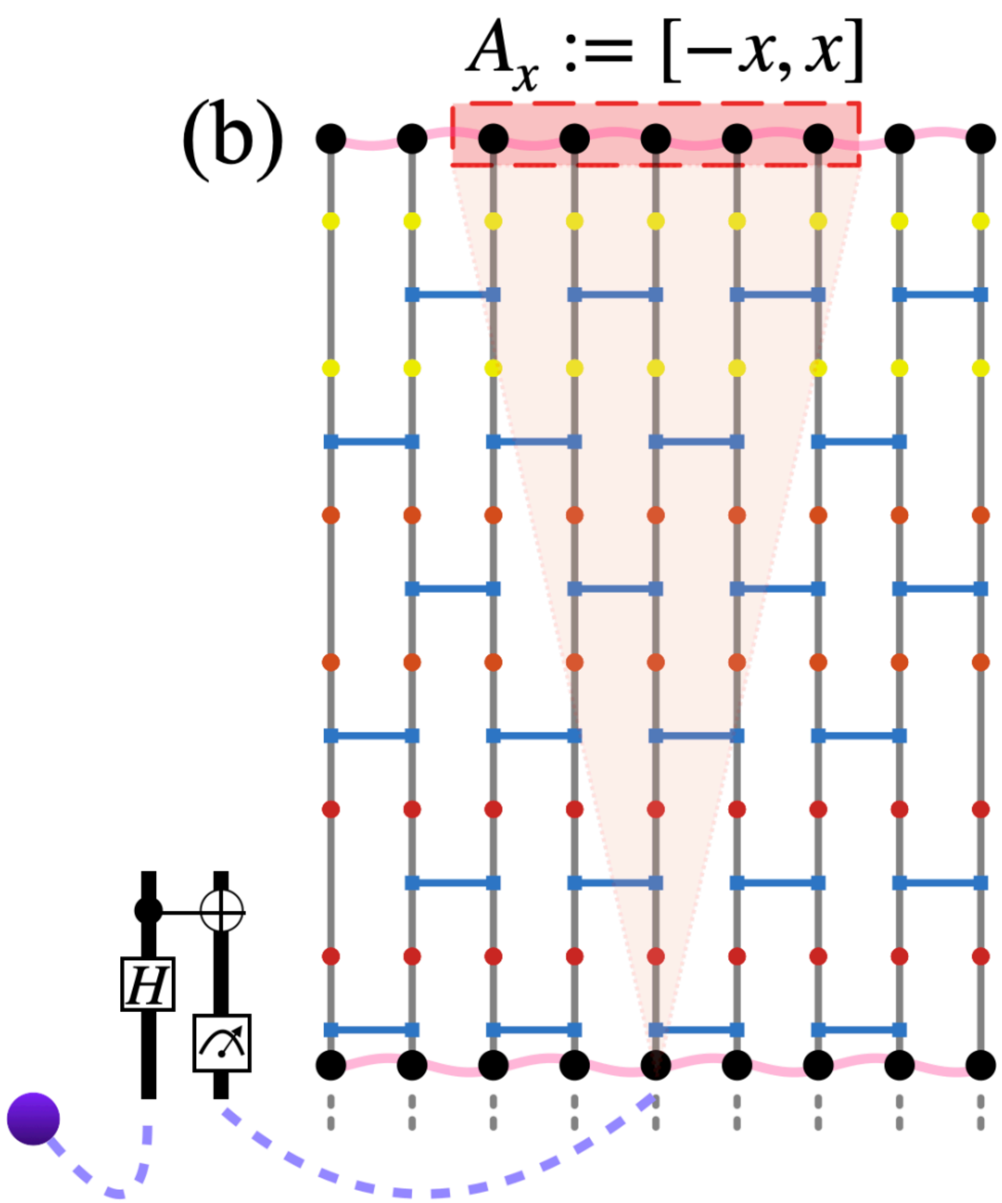


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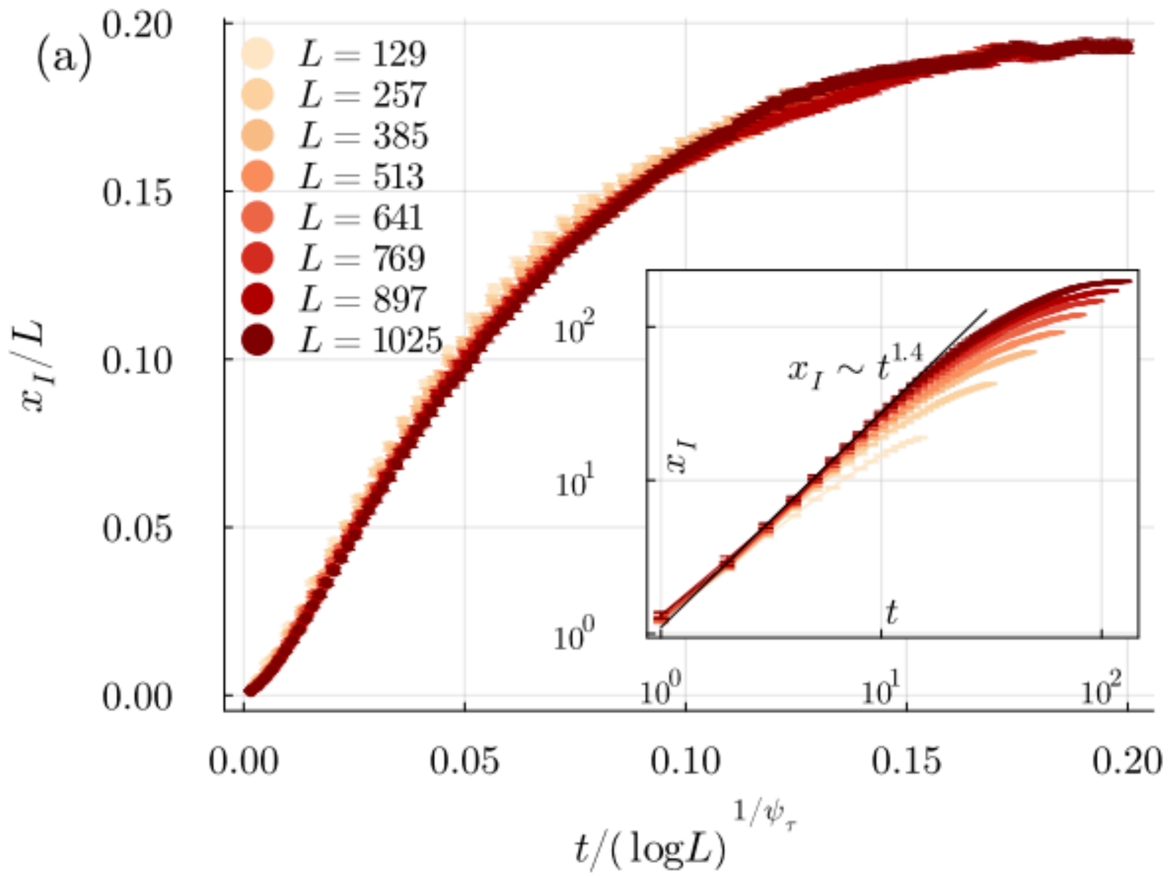
Ultra-fast



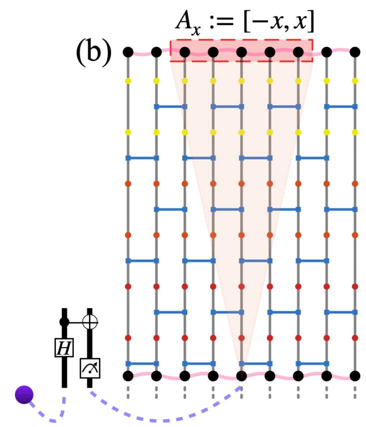

$$t^{\psi_\tau} \sim \log x_I$$

Ultrafast information propagation!

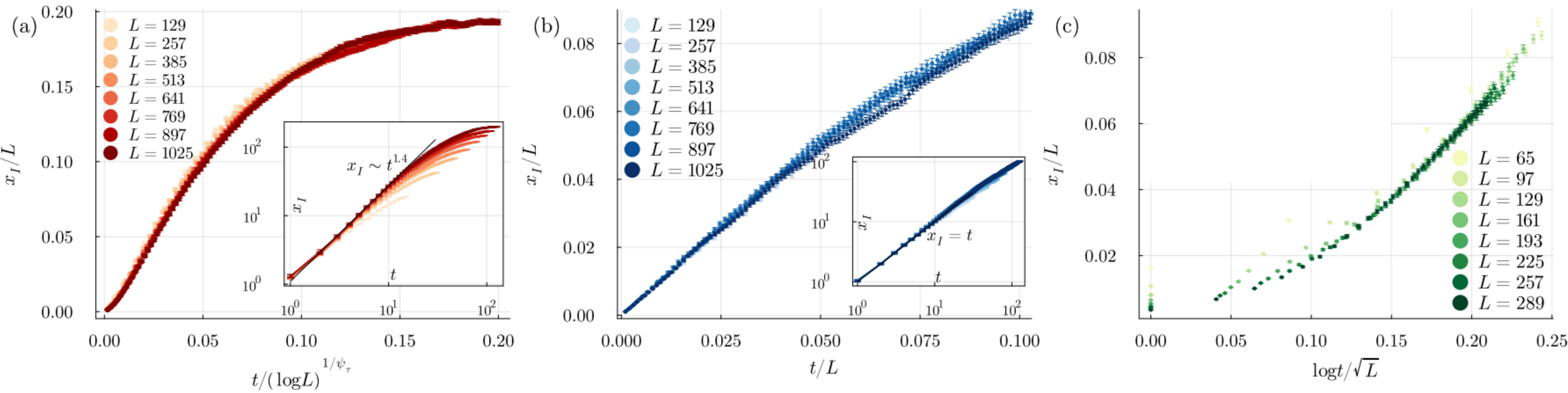
# Comparison of the 3 classes of MIPTs

Compute the mutual information $\mathcal{I}_2(A_x, B)$

find the $x$ where $\mathcal{I}_2(A_x, B) \to 0$ call this $x_I$




**Temporal randomness** | **Space-time randomness** | **Static randomness**




$t^{\psi_\tau} \sim \log x_I$ | $t \sim x$ | $\log t \sim L^{\psi}$

Ultrafast | Lorentz invariant | Ultraslow

$z \to 0$ | $z = 1$ | $z \to \infty$

# Conclusions and Outlook

- The measurement transition is a novel dynamical transition in the structure of the entanglement.
- The transition is described by a logarithmic CFT that is characterized by its *effective central charge*.
- Conservation laws, static, and dynamic perturbations fundamentally alter the dynamics.

How do we overcome this argument?

$$C(x,y) = \mathrm{Tr}\left(\rho_{\mathrm{av}} S_z(x) S_z(y)\right)$$

$$\rho_{\mathrm{av}} = \frac{1}{2^L} \sum_{\mathbf{m}} |\Psi_{\mathbf{m}}(t)\rangle\langle\Psi_{\mathbf{m}}(t)|$$

at late times will approach the infinite temperature Gibbs ensemble **for any *p***

Thank you for your attention!

# Conclusions and Outlook

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How do we overcome this argument?

Consider incorporating **feedback** from the measurement outcome

$$\rho_{\mathrm{av}} = \sum_{\mathbf{m}} A(\mathbf{m}) |\Psi_{\mathbf{m}}(t)\rangle\langle\Psi_{\mathbf{m}}(t)|$$

Now **NOT** equivalent to the infinite temperature Gibbs ensemble

Thank you for your attention!

# Collaborators

<u>Rutgers</u>

*Aidan Zabalo ( ->Prudential)*

*Yicheng Tang*

*Kemal Aziz*

<u>Hebrew University</u>

*Gal Shkolnik*

<u>JQI, NIST</u>

*Michael Gullans*

<u>UGeneva</u>

*Romain Vasseur*

*Pradip Kattel*

<u>UCSB</u>

*Andreas Ludwig*

<u>Princeton</u>

*David Huse*

*Sarang Gopalakrishnan*

<u>Hebrew University</u>

*Snir Gazit*

<u>LSU</u>

*Justin Wilson*

*Ahana Chakraborty*

MIPTs

Zabalo, et al PRB(R) (2020), PRL (2022)
Aziz, Chakraborty, JHP PRB (2024)

U(1) symmetry

Agrawal, Zabalo, et al PRX (2022)

Chakraborty et al PRB (2024)

SWSSB

Tang, Kattel, JHP arXiv (2026)

Static & temporal disorder

Zabalo, et al PRB (2023); Shkolnik et al PRB (2023),
Shkolnik et al PRB (2025)

MBL

Tang, Kattel, Pal,
Yuzbashyan, JHP arXiv (2026)

# Luck Criteria and the Wandering Exponent

Static Quasiperiodic perturbations are irrelevant if

$$\nu \geq \frac{1}{d(1-\beta)}$$

wandering exponent

Luck, Journal of Stat. Phys. (1993)

Grimm and Baake, Journal of Stat. Phys. (1994)

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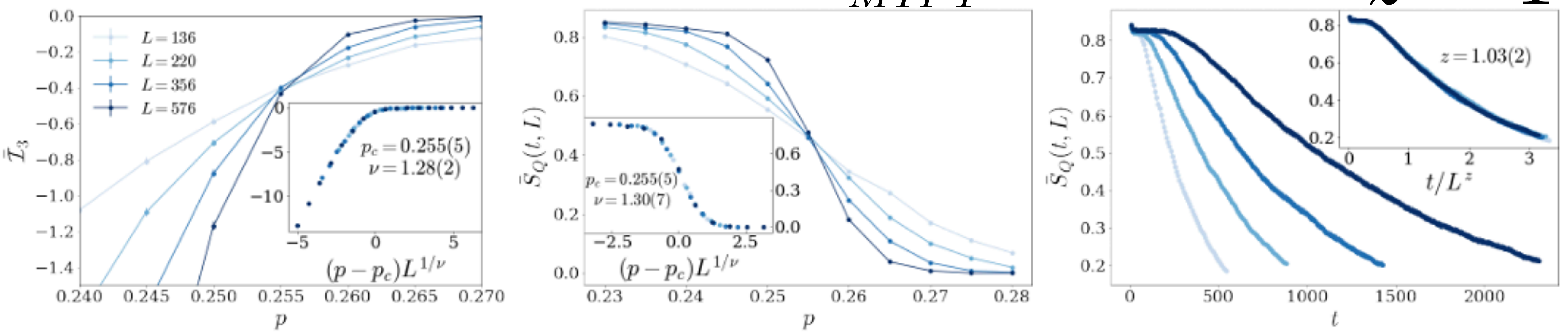
$$\nu \approx 1.28 = \nu_{MIPT}$$

$$z = 1$$



# Quasiperiodic Structures

Static Quasiperiodic perturbations are irrelevant if

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control the wandering exponent
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Move away from smooth structures to discrete (e.g. Fibonacci "words")

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Fibonacci substitution rules $0 \rightarrow 01$

$1 \rightarrow 0$

$$S_n = S_{n-1}S_{n-2}$$

| | |
|---|---|
| $S_0$ | 0 |
| $S_1$ | 01 |
| $S_2$ | 010 |
| $S_3$ | 01001 |
| $S_4$ | 01001010 |
| $S_5$ | 0100101001001 |

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controls the fluctuations

substitution matrix eigenvalues $\lambda_- < \lambda_+$

controls the inflation

# Non-Pisot Quasiperiodic Structures

Non-Pisot structures defined by $|\lambda_-| > 1$ (Pisot $|\lambda_-| \leq 1$)

**Strong fluctuations in non-Pisot structures**

Count the number of "1s" (or 0s) in the "word" S $= N(S)$

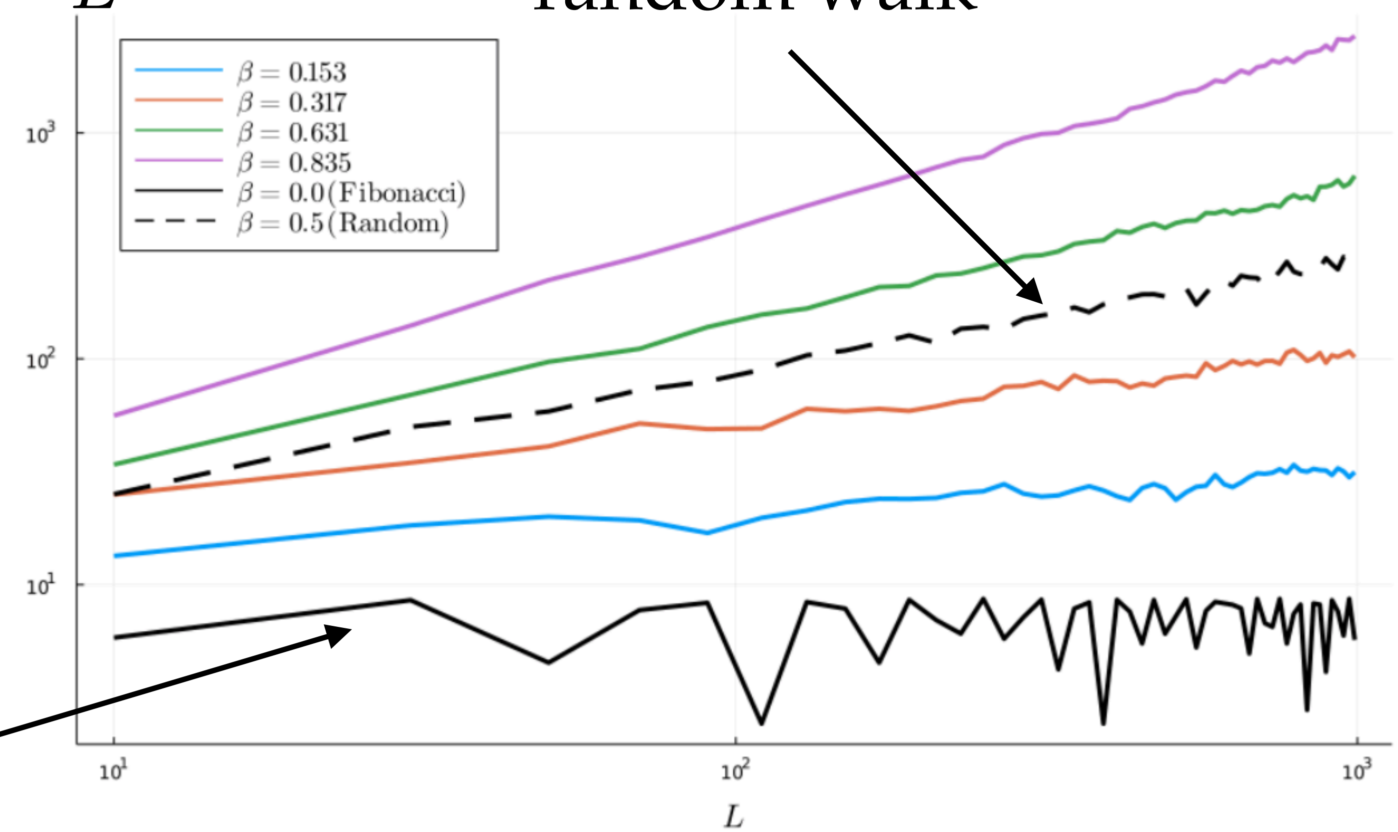
consider a collection of words all of length L

Define a std. dev. over the set of segments of length L

$$\sigma_L = \sqrt{\sum_s N(s)^2 - [\sum_s N(s)]^2} \sim L^\beta$$

$\sigma_L$

random walk




Fibonacci $\beta = 0$

# Summary of these Fixed Points

Transition is controlled by the wandering exponent!

$$S(t \ll L) \sim \log(t) \qquad S(t \to \infty, L) \sim L^{\beta}$$

$0.23 \lesssim \beta < 1/2$

Activated dynamical scaling $t^* \sim e^{cL^{\beta}}$

$$\sigma_L = \sqrt{\sum_s N(s)^2 - [\sum_s N(s)]^2} \sim L^{\beta}$$

Flows to a fixed point that saturates the bound

$$\psi = \beta \qquad \nu = 1/(1-\beta)$$

Shkolnik, Vasseur, Huse, JHP, Gazit PRB (2023)

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Random $\beta = 1/2$

Quasiperiodic, tunable $\beta$

uncovered a whole family of universality classes controlled by the QP structure

Shkolnik, Vasseur, Huse, JHP, Gazit PRB (2023)

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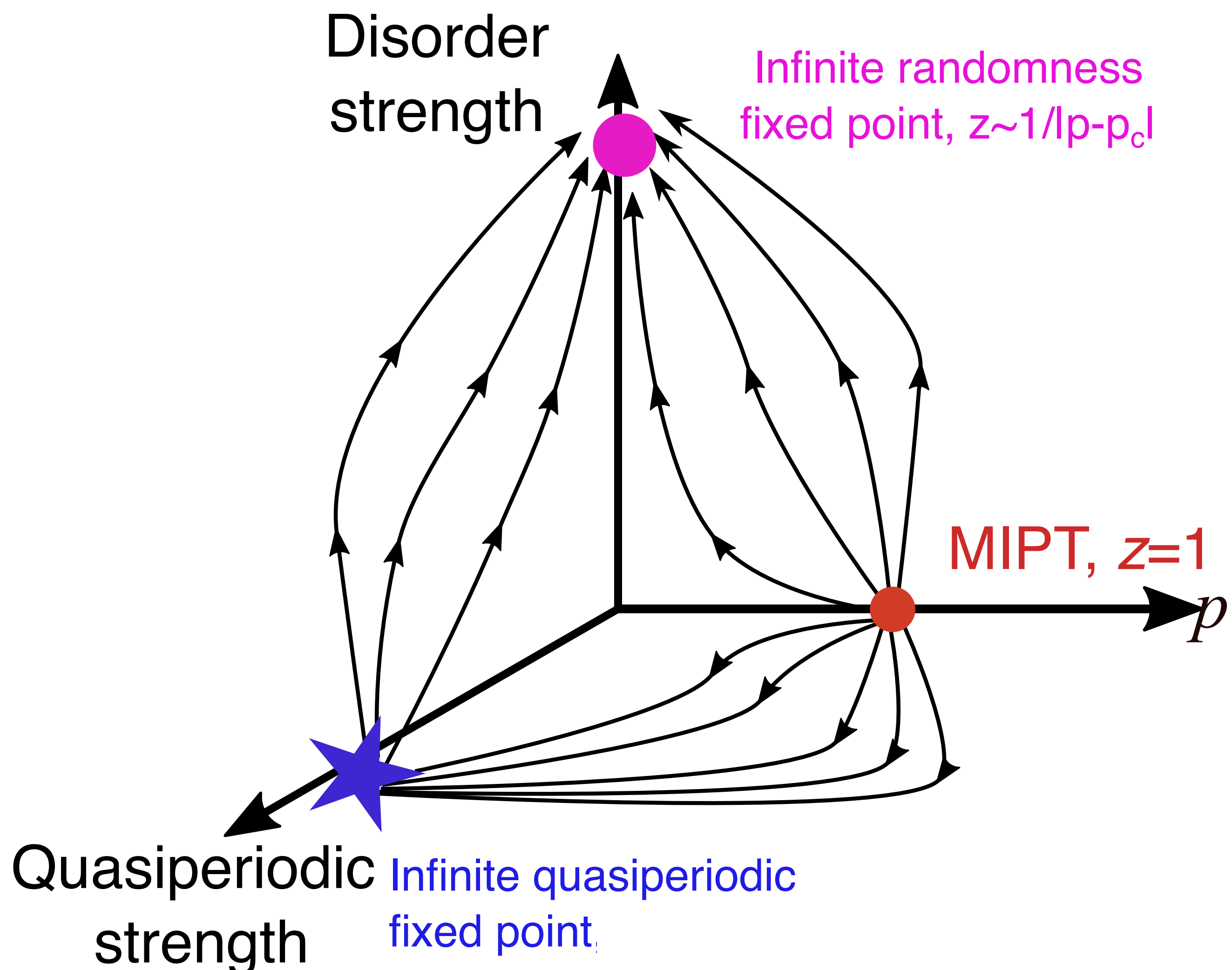
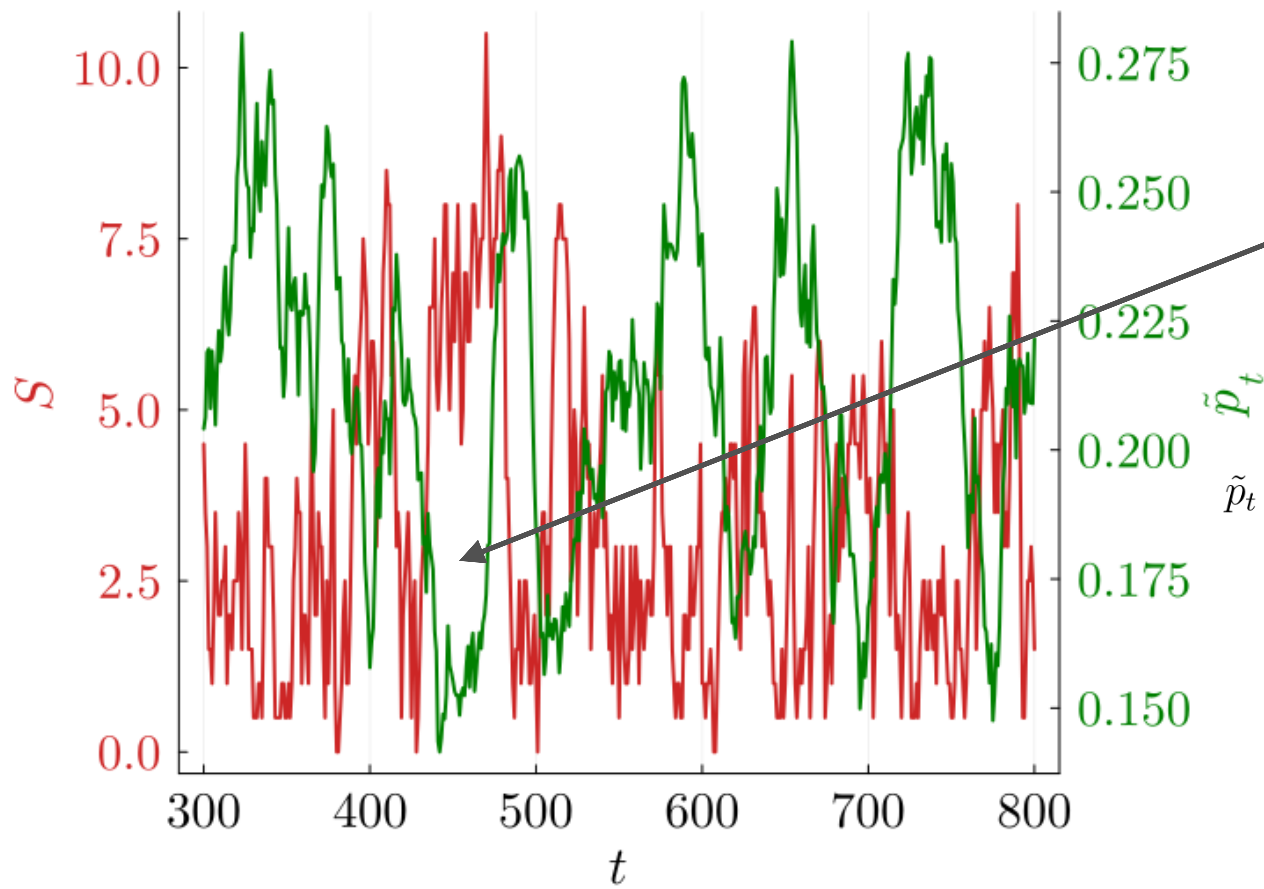
RG Flow: For $0.23 \lesssim \beta < 1/2$

this family of fixed points are unstable to disorder



# Temporal Griffith effects, area law phase




Rare temporal event with low p

Entanglement develops across the chain

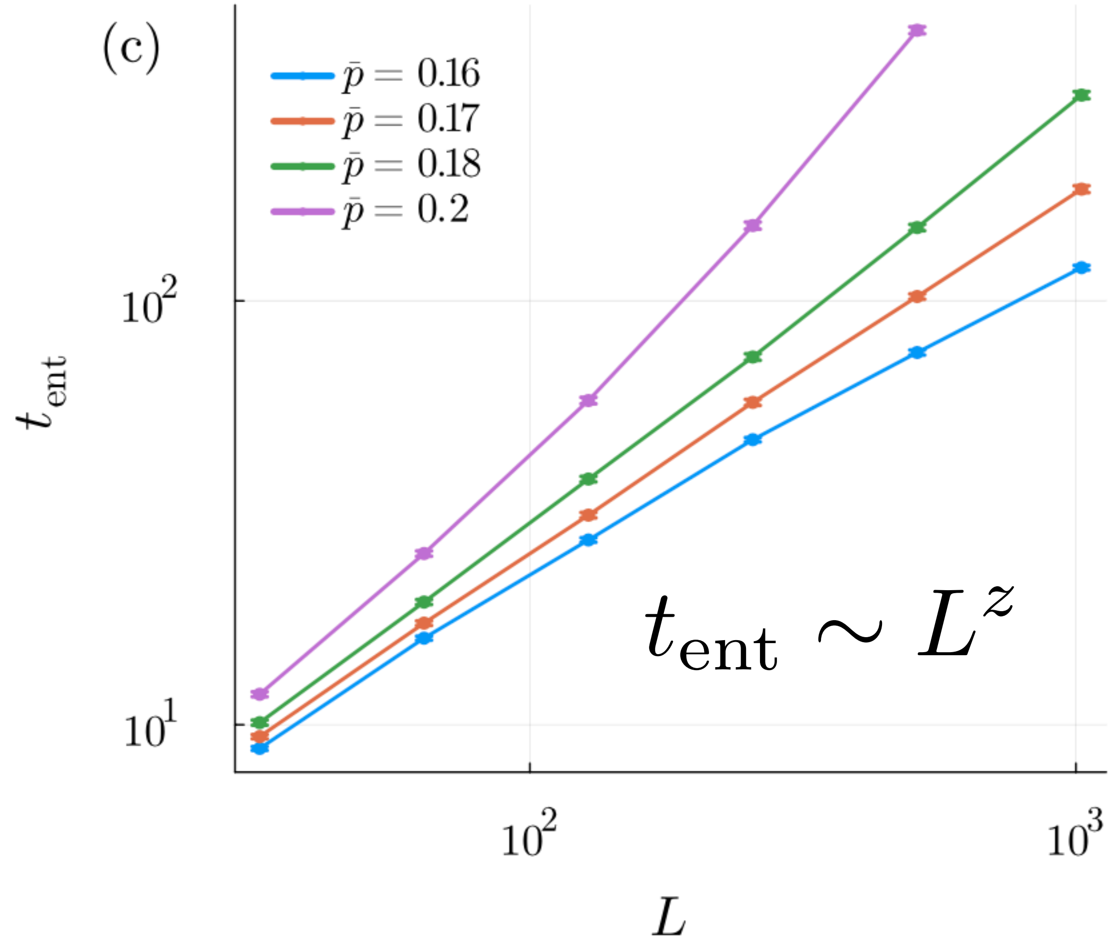
$\tilde{p}_t := \frac{1}{2m+1} \sum_{t' \in [t-m, t+m]} p(t')$

Have to capture the growth of long **range** entanglement

# Temporal Griffith effects, area law phase

Have to capture the growth of long **range** entanglement

Starting from a product state find the earliest time where long range entanglement is generated, ie. find the time where $\mathcal{I}_3 \neq 0$ (searching over cuts)

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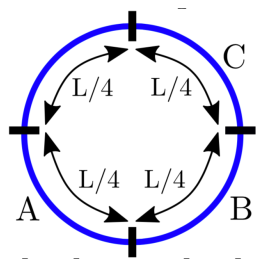

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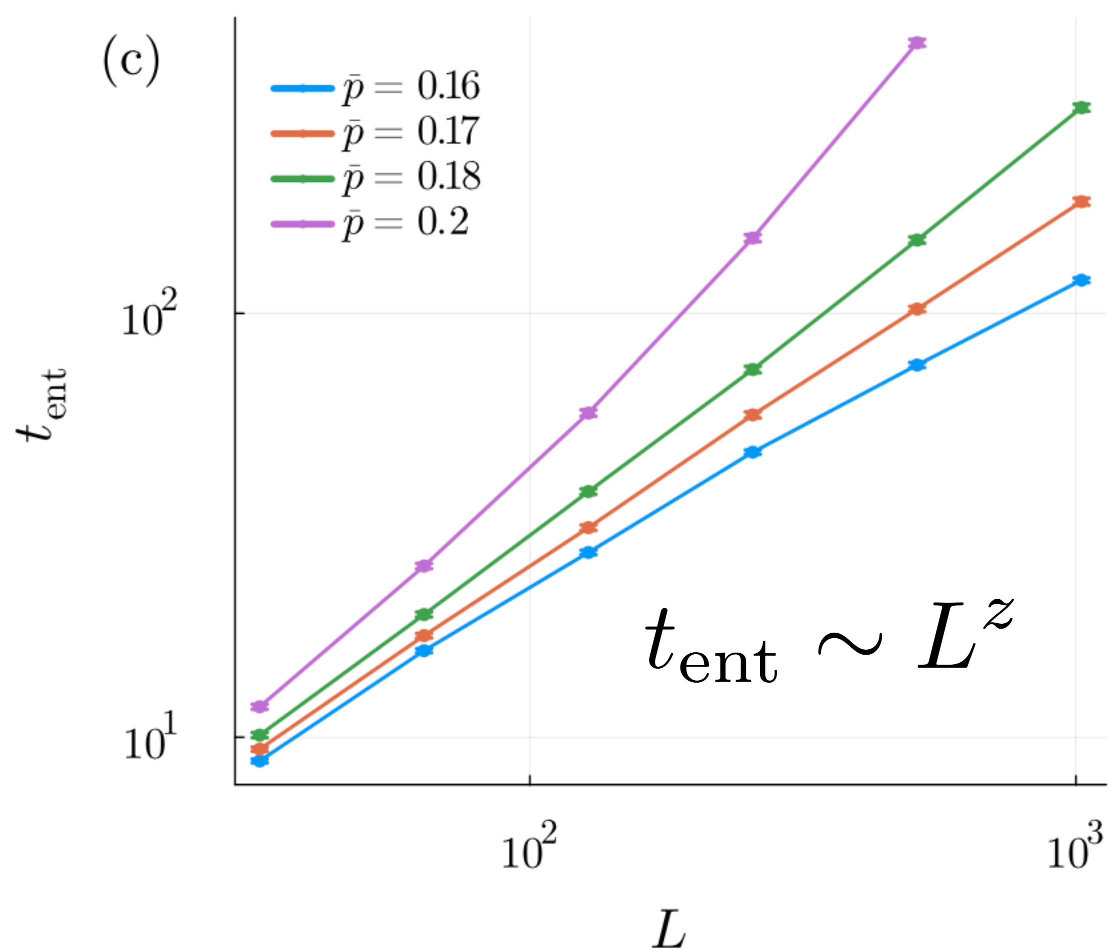


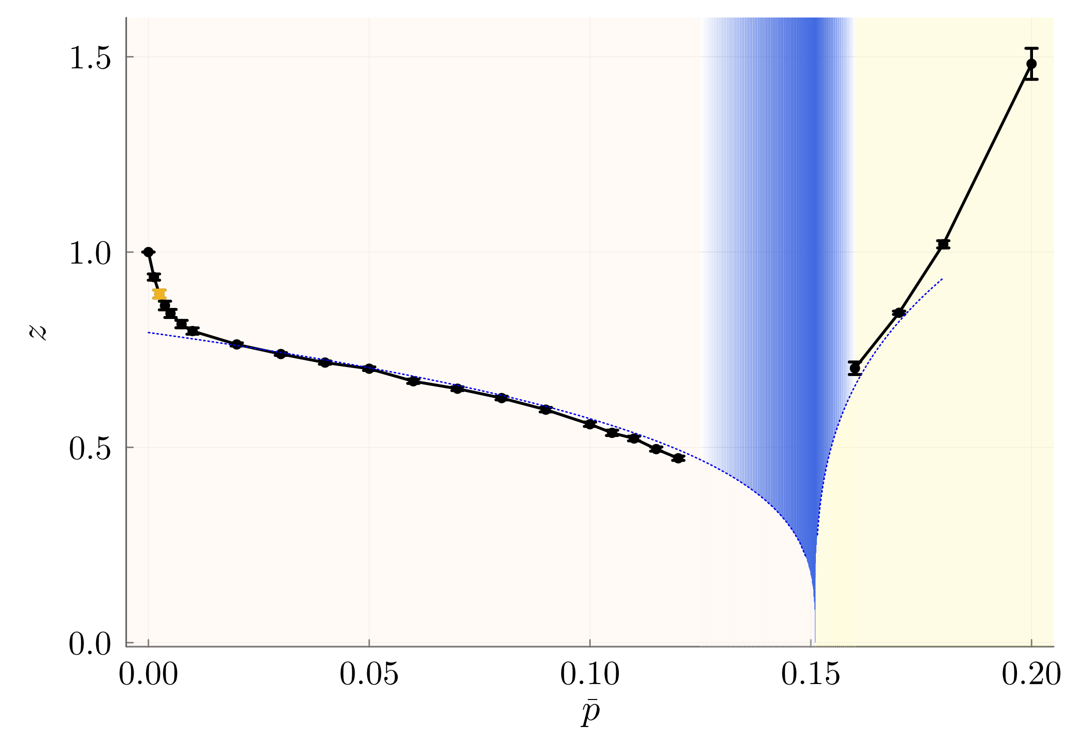


$z \to 0$

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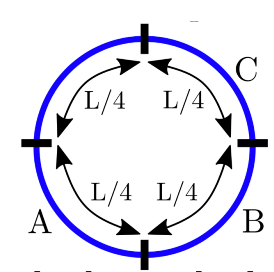


# Area of the Cylinder

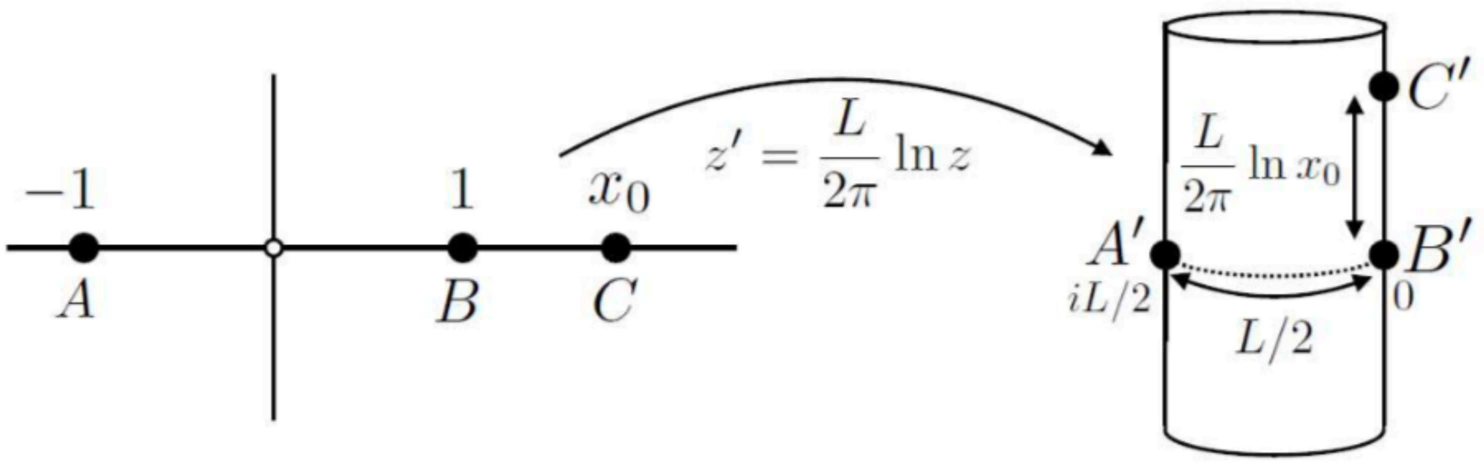
area of the cylinder of the log-CFT $A = \alpha t L$

Handle this 2 ways:

**(1) Compute it directly.**

(2) Choice a convenient set of gates with $\alpha = 1$

conformal mapping from the plane to the cylinder



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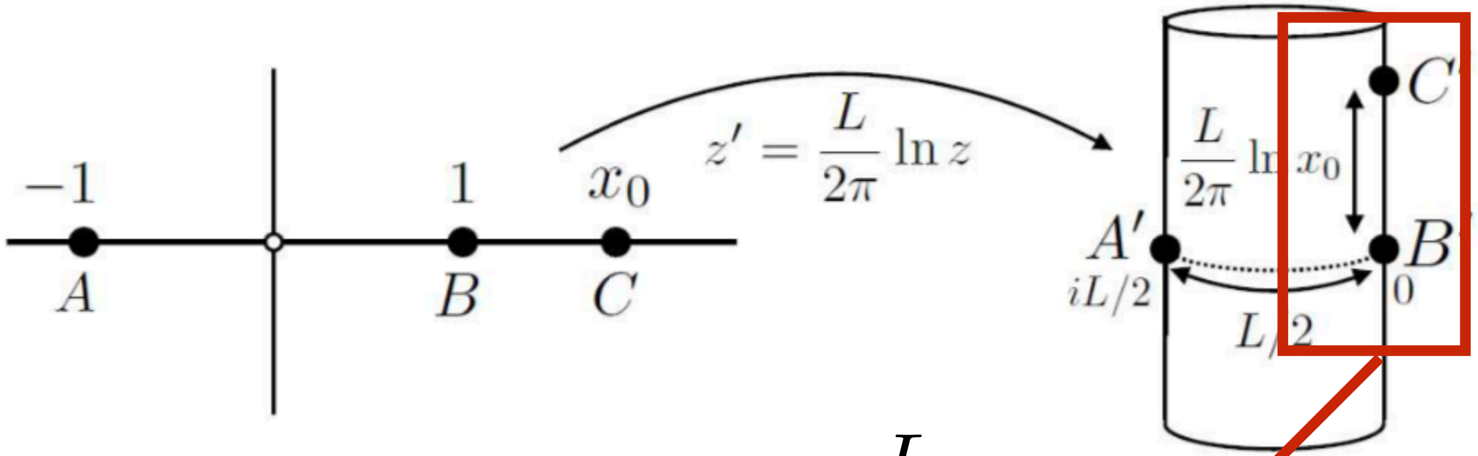


$$\alpha t = x_0' = \frac{L}{2\pi} \log x_0$$

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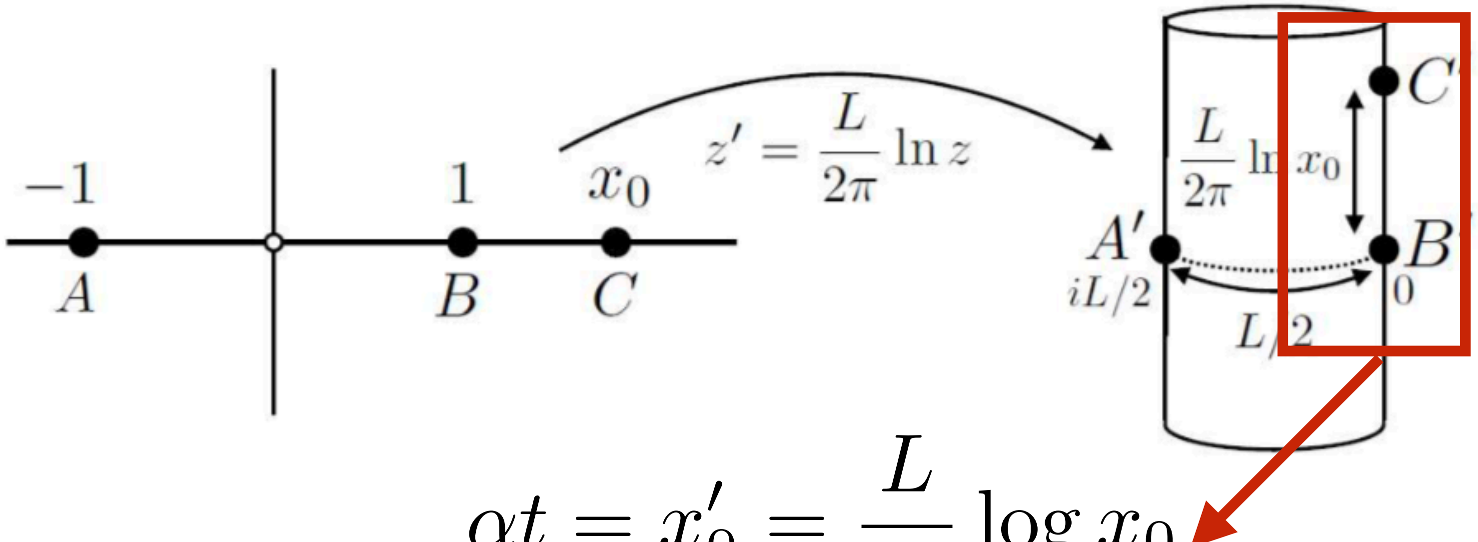


$$g_{AB} \sim \frac{1}{2^{2\Delta}}$$

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$$g_{BC} \sim \frac{1}{|x_0 - 1|^{2\Delta}}$$

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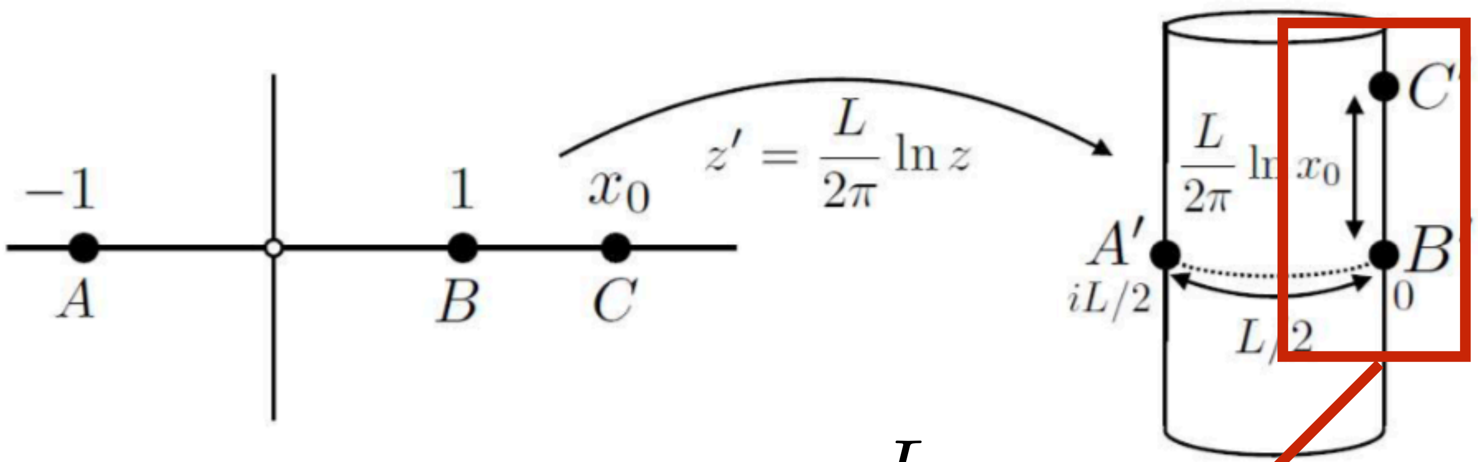


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$$g_{AB} \sim \frac{1}{2^{2\Delta}}$$

$$g_{BC} \sim \frac{1}{|x_0 - 1|^{2\Delta}}$$

$$g' \equiv g(z_1', z_2') = \left|\frac{\partial z_1'}{\partial z_1}\right|^{-\Delta} \left|\frac{\partial z_2'}{\partial z_2}\right|^{-\Delta} g(z_1, z_2)$$

# Area of the Cylinder

area of the cylinder of the log-CFT $A = \alpha t L$

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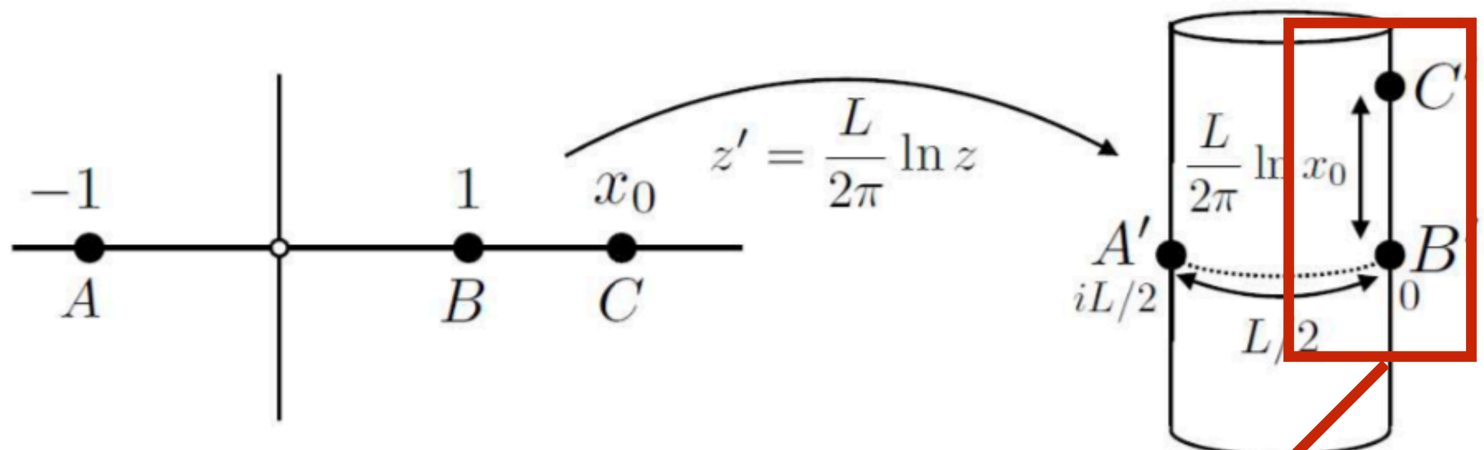


$$g_{AB} \sim \frac{1}{2^{2\Delta}} \qquad g'_{AB} \sim \left(\frac{2\pi}{L}\right)^{2\Delta} \frac{1}{2^{2\Delta}}$$

$$g_{BC} \sim \frac{1}{|x_0 - 1|^{2\Delta}} \qquad g'_{BC} \sim \left(\frac{2\pi x_0}{L}\right)^{2\Delta} \frac{1}{|x_0 - 1|^{2\Delta}}$$

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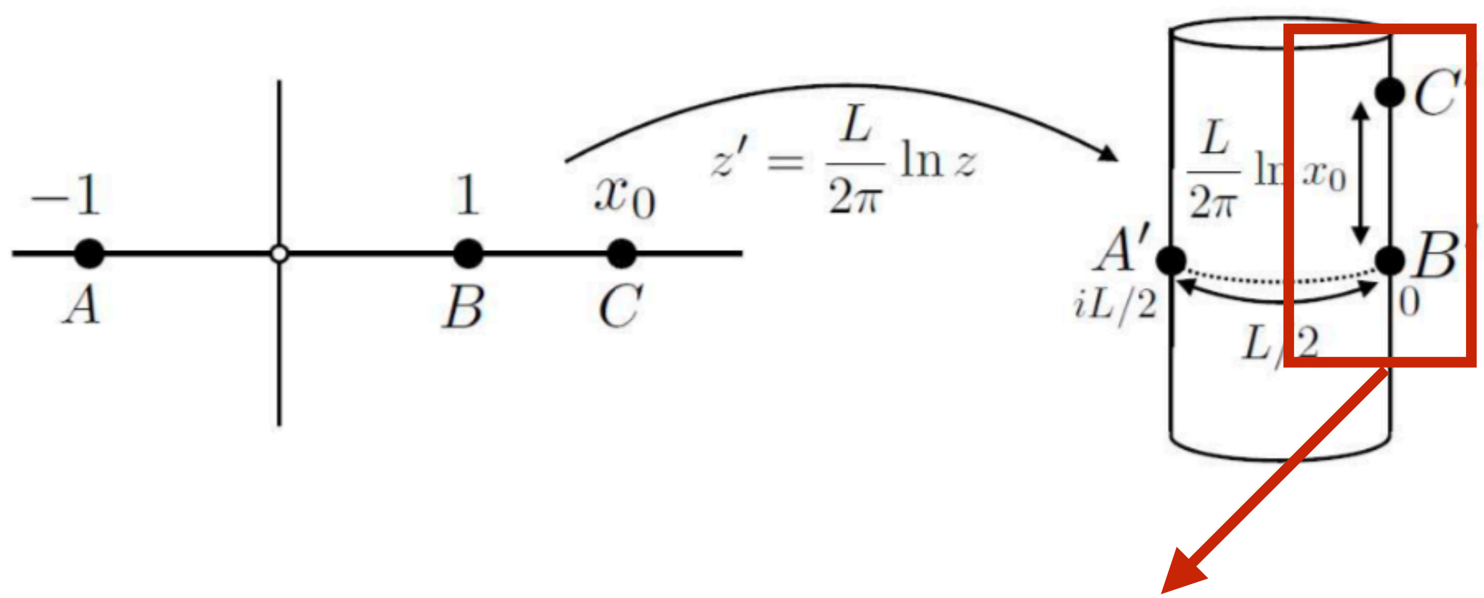


$$\left(\frac{g'_{BC}}{g'_{AB}}\right)^{1/\Delta} = \frac{4x_0}{(x_0 - 1)^2}$$

$$\alpha t = x'_0 = \frac{L}{2\pi} \log x_0$$

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area of the cylinder of the log-CFT $A = \alpha t L$

Handle this 2 ways:

**(1) Compute it directly.**

(2) Choice a convenient set of gates with $\alpha = 1$

setting

$$\left(\frac{g'_{BC}}{g'_{AB}}\right)^{1/\Delta} = \frac{4x_0}{(x_0-1)^2} = 1$$ eliminates dependence on $\Delta$

Now must find the time, $t^*$ where these correlation functions match!

$$\alpha t^* = \frac{L}{2\pi}\log(3+\sqrt{8})$$

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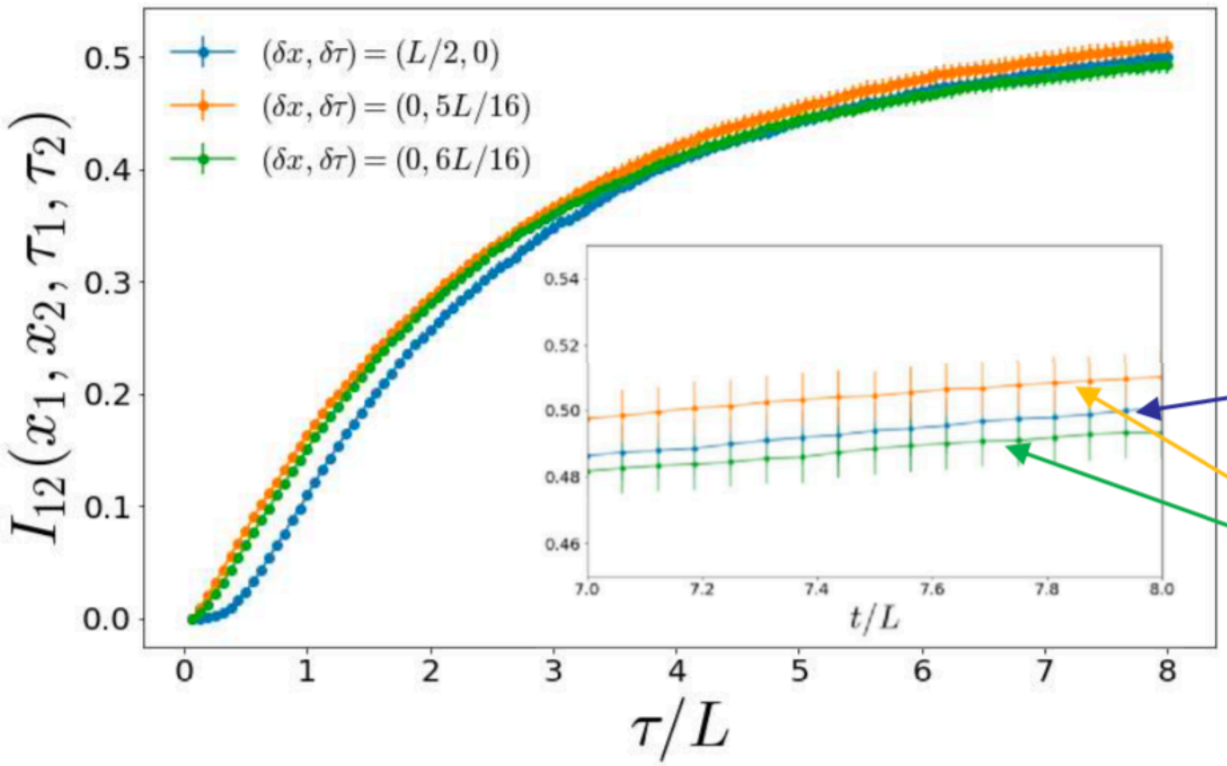
area of the cylinder of the log-CFT $A = \alpha t L$

Now must find the time, $t^*$ where these correlation functions match!




$$\alpha t^* = \frac{L}{2\pi} \log(3 + \sqrt{8})$$

$$g'_{AB} = \left(\frac{2\pi}{L}\right)^{2\Delta} \frac{1}{2^{2\Delta}}$$

$$g'_{BC} = \left(\frac{2\pi}{L}\right)^{2\Delta} \frac{x_0^{\Delta}}{(x_0 - 1)^{2\Delta}}$$

- The correlation functions match at $t_* \in [5,6]$

# Area of the Cylinder

area of the cylinder of the log-CFT $\quad A = \alpha t L$

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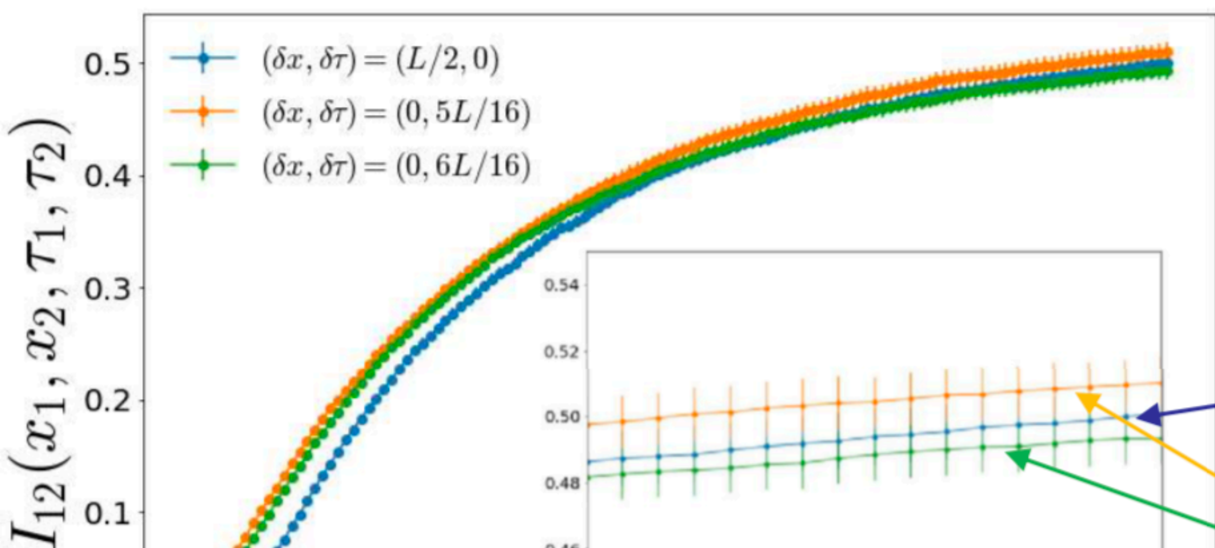


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$$\left(\frac{2\pi}{L}\right)^{2\Delta} \frac{x_0^{\Delta}}{(x_0 - 1)^{2\Delta}}$$

,6]

| Haar | Clifford |
|---|---|
| $p_c = 0.170$ | $p_c = 0.1596$ |
| $\alpha = 0.808(90)$ | $\alpha = 0.616(25)$ |

# “Easy Way out”: Dual Unitaries

area of the cylinder of the log-CFT $A = \alpha t L$

Handle this 2 ways:

(1) Compute it directly.

**(2) Choice a convenient set of gates with** $\alpha = 1$

Use unitary gates that act in the same manner in space and time.

Use **dual unitary gates**:  $V[J] = \exp\left[-i\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right)\right]$

Bertini, Kos, and Prosen, Phys. Rev. Lett. (2019)

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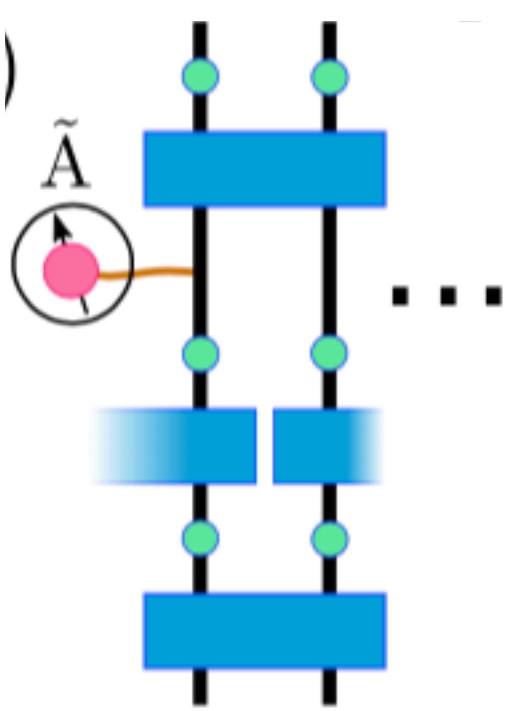
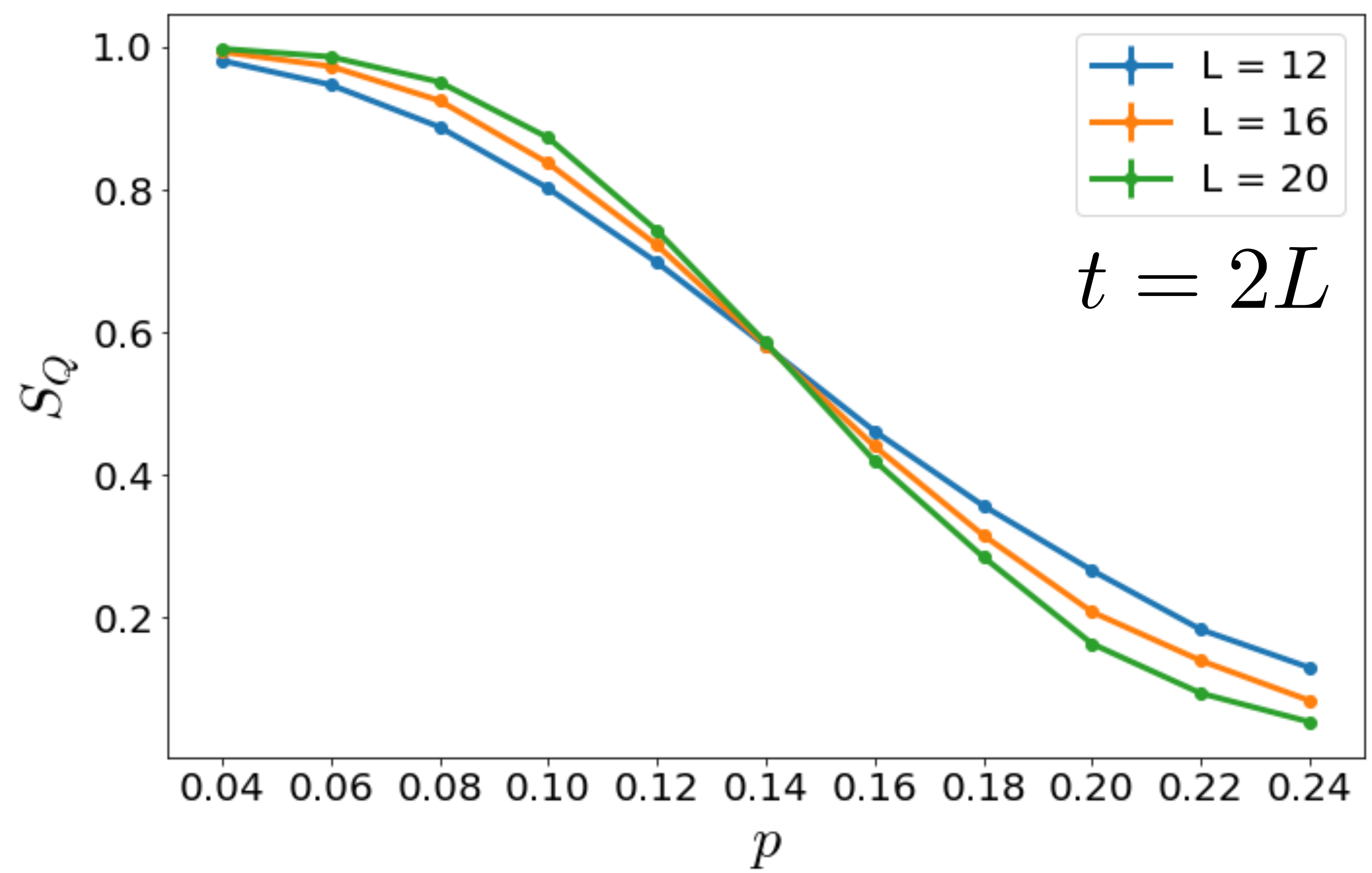
Order parameter dynamics



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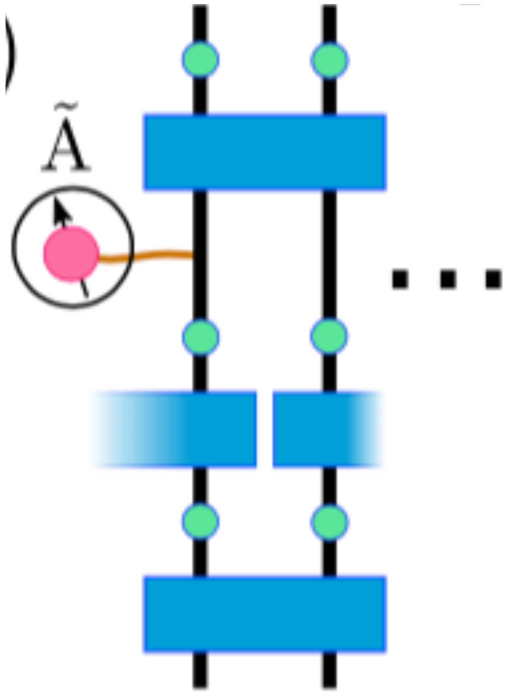


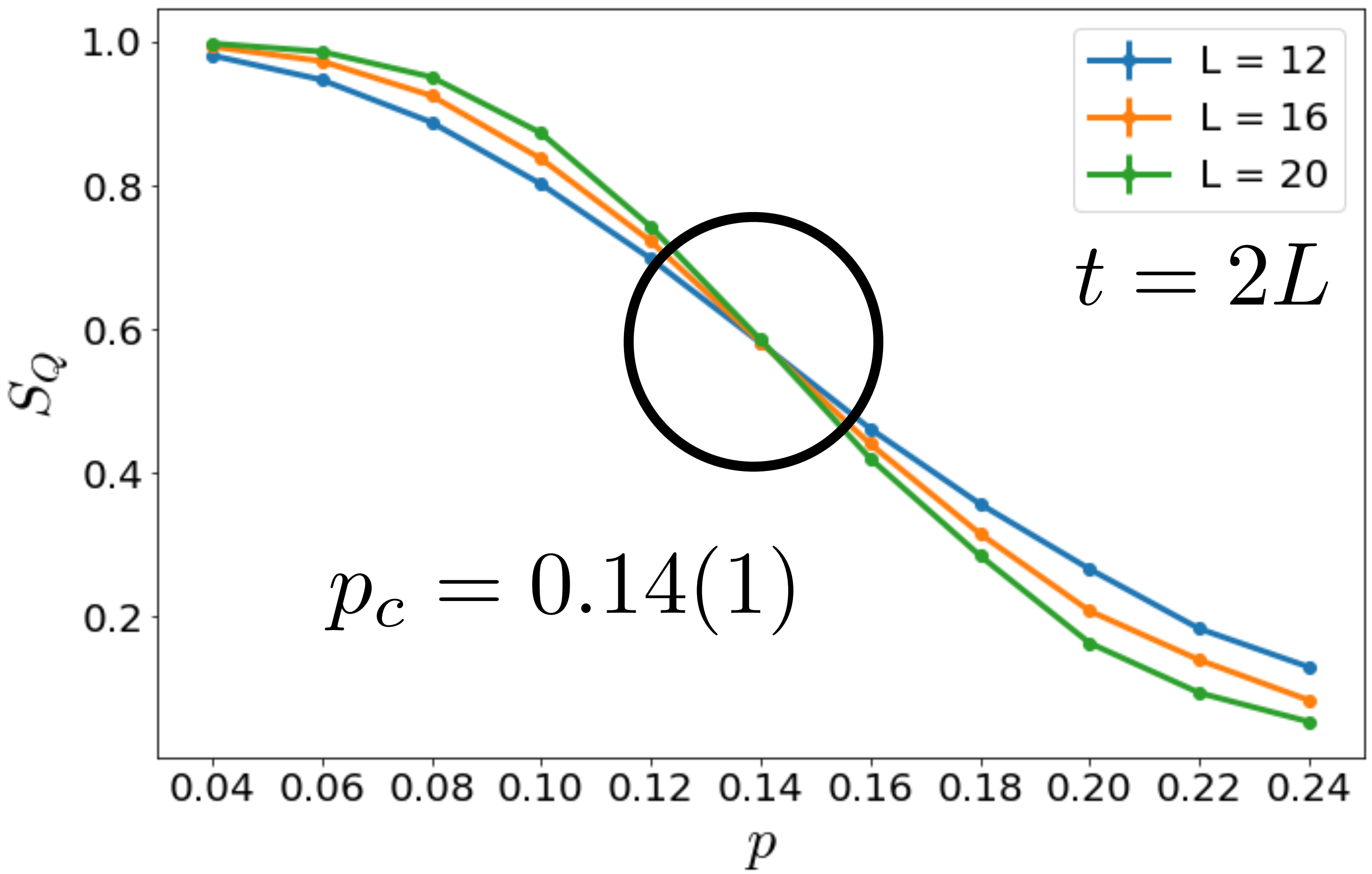

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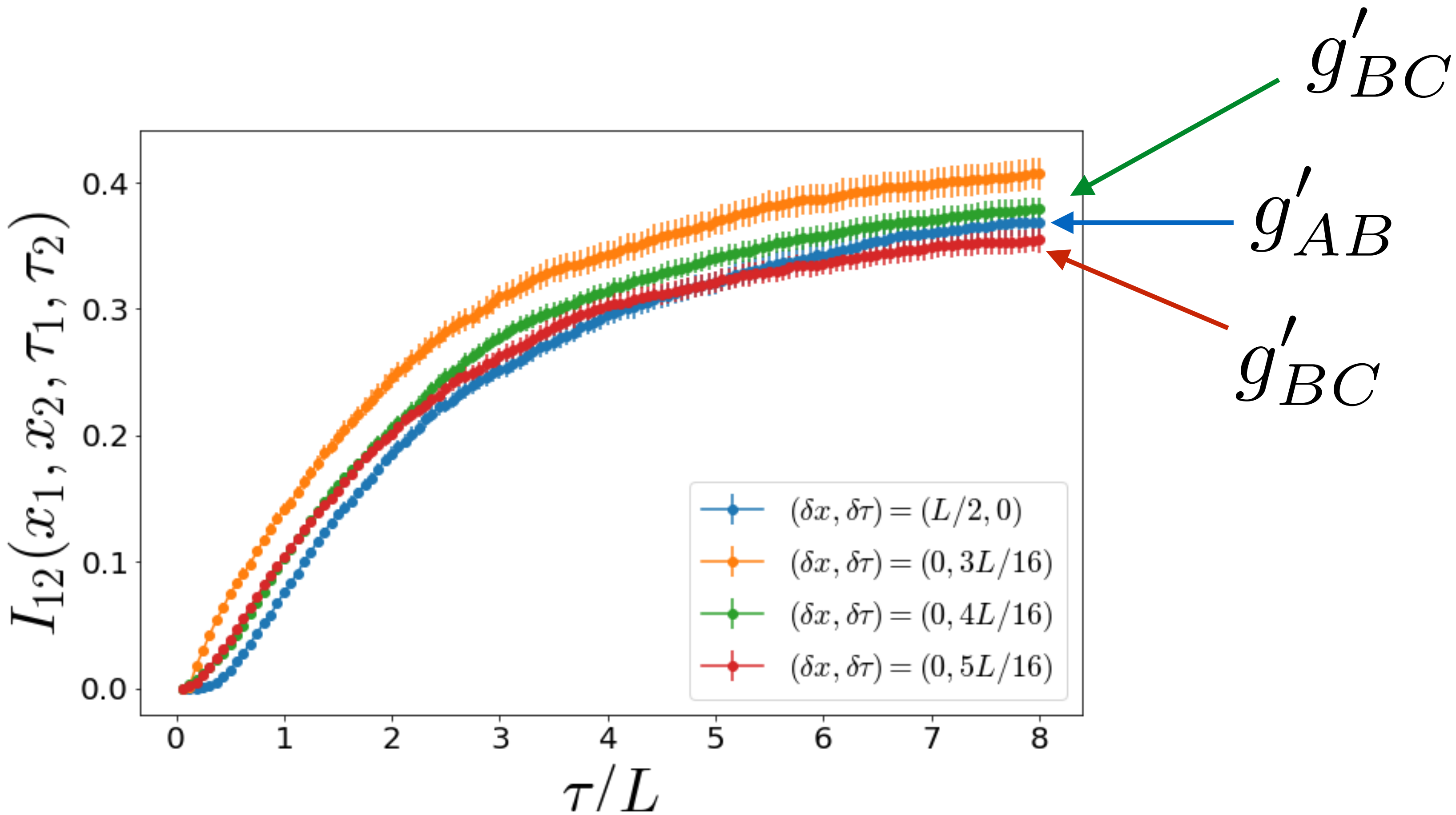
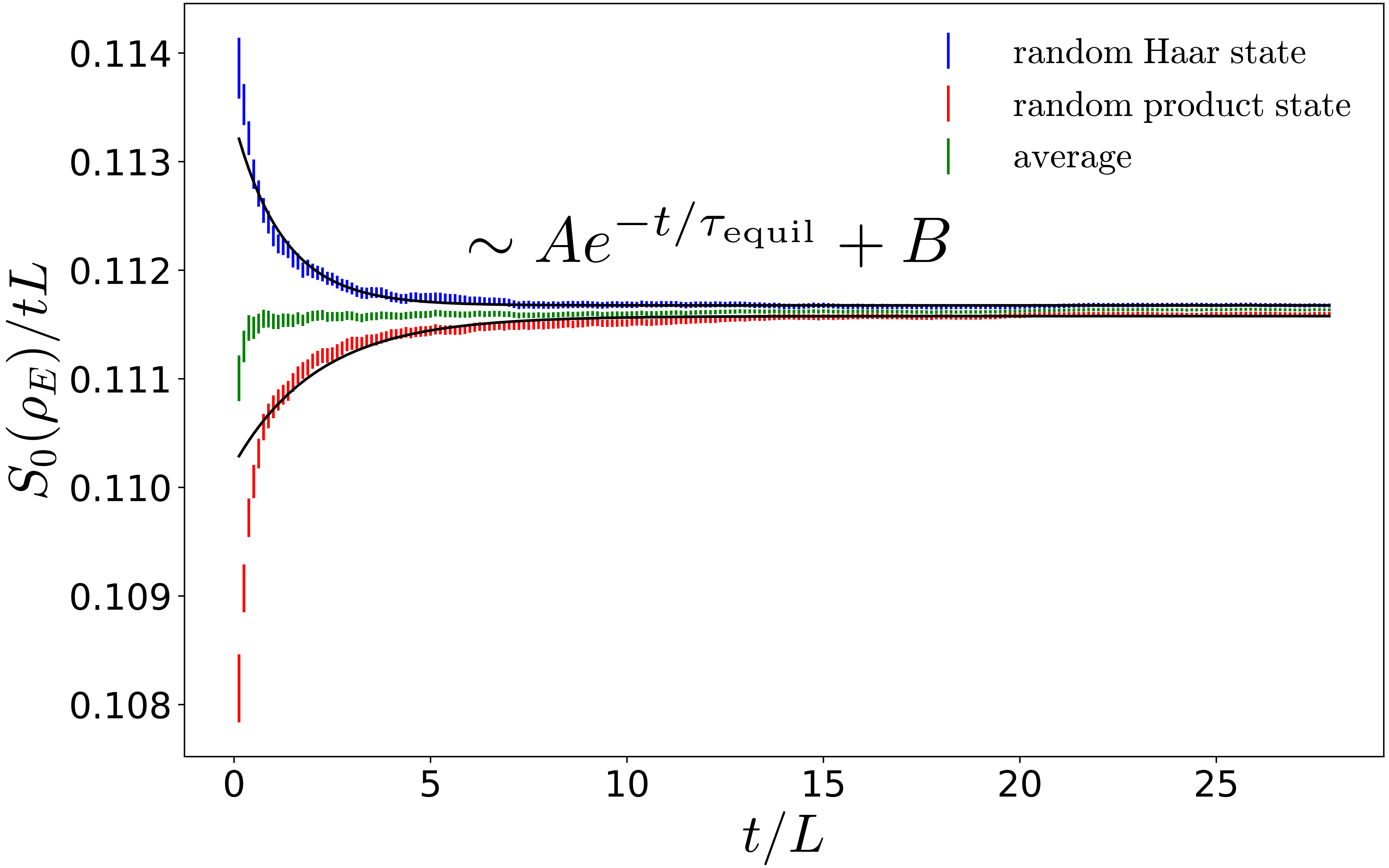



$\alpha = 1.01(11)$

# Entropy of the Measurement Record

$$S(\rho_E) = -\sum_{\mathbf{m}} \langle \log p_{\mathbf{m}} \rangle = -\sum_{i=1}^{N_{\text{meas}}} \langle \log p(m_i | m_{i-1}, \ldots, m_1) \rangle$$



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