Unraveling gluon TMDs in *D*-meson and jet production at EIC

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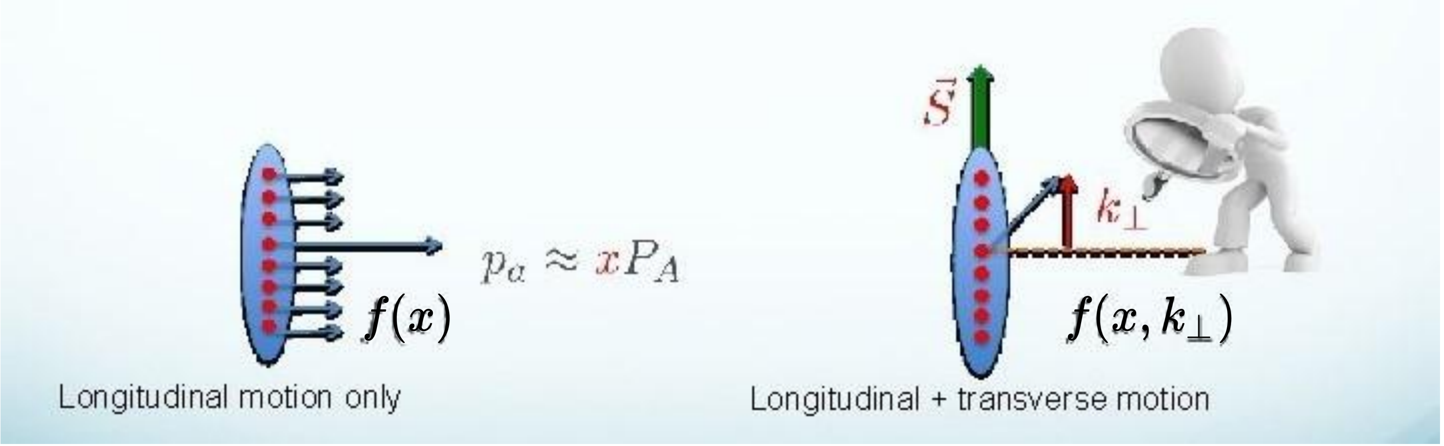
Transverse Momentum Dependent PDFs (TMDs)

PDFs - probability of finding quarks and gluons inside the hadron with given fraction of hadrons momentum.

1D information about the partons

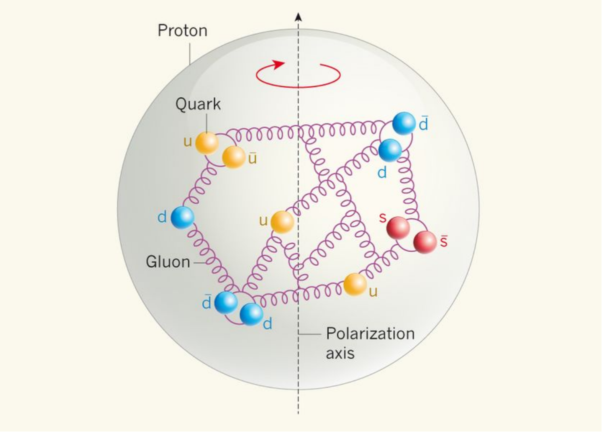


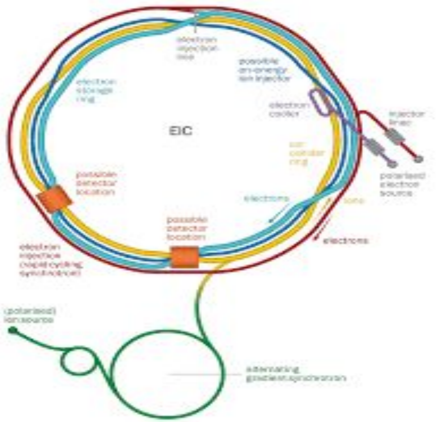
TMD PDFs - gives the distribution of quarks and gluons having longitudinal momentum fraction **x** and transverse momenta $k_\perp$ within the nucleon.

3D information about the partons

Gluon TMDs: A largely unexplored territory

- 3D imaging of the nucleon
- Partons intrinsic transverse motion
- Spin and transverse momentum correlations
- Orbital angular momentum of partons
- Spin of the Proton: core sector of EIC studies





Semi Inclusive Deep Inelastic Scattering (SIDIS)

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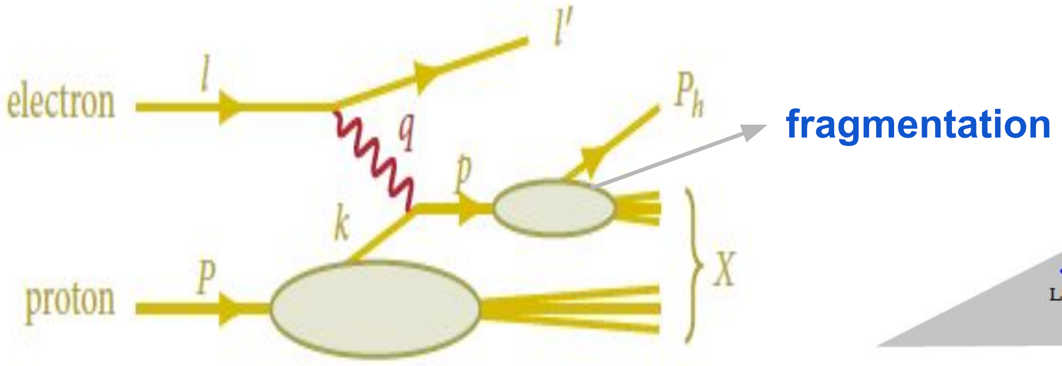
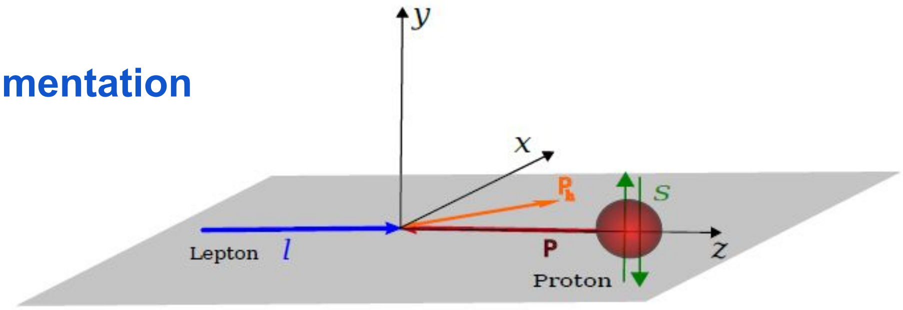
$$l\,p \to l\,h\,X$$





$$\sigma^{SIDIS} \propto f_p(x, k_\perp; Q^2) \otimes \hat{\sigma}^{part} \otimes D_f^h(z, p_\perp; Q^2)$$

$$Q^2 = -q^2$$

TMD PDFs | Hard scattering | TMD FFs

TMD factorization

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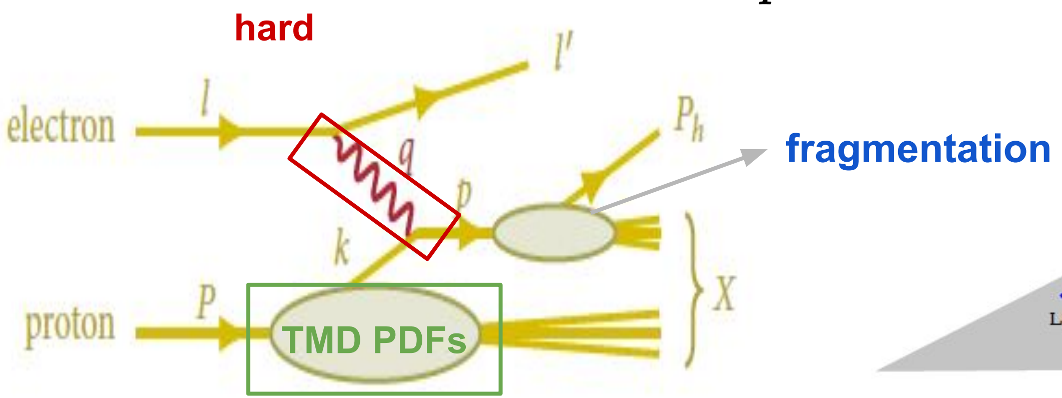

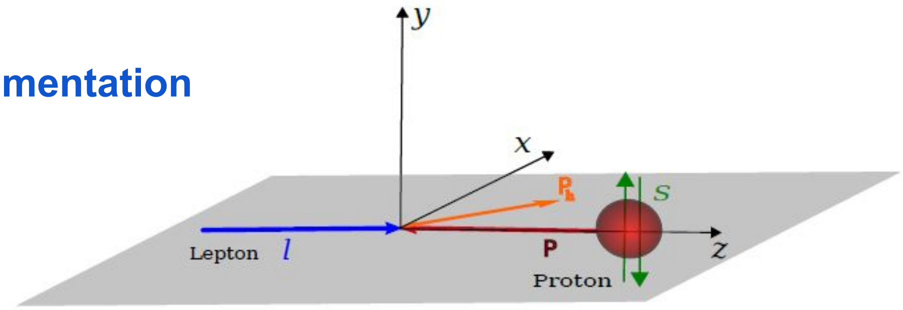

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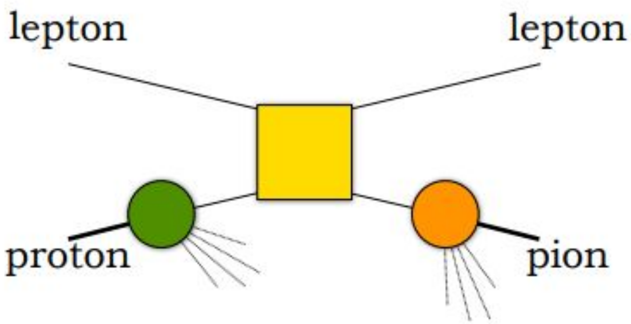
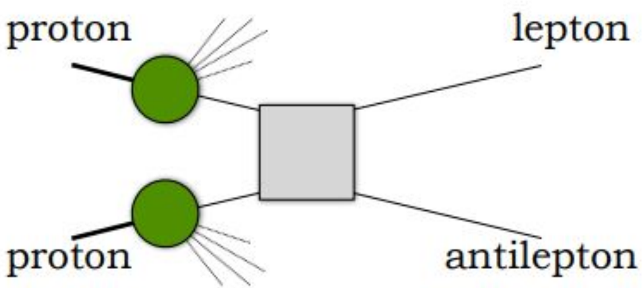
TMD PDFs | Hard scattering | TMD FFs

TMD factorization

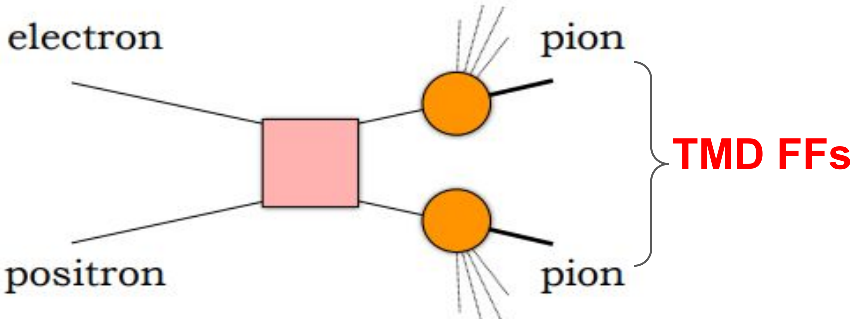
Where we can access the TMD PDFs?



SIDIS



Drell-Yan

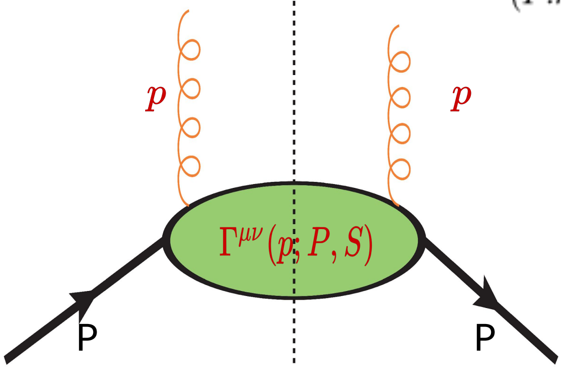


$e^+ + e^- \rightarrow$ **hadrons**

Pic: Celiberto

Gluon Correlator

$$\Gamma_g^{\mu\nu}(p;P,S) = \frac{n_\rho n_\sigma}{(P.n)^2}\int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi}\langle P,S|\mathrm{Tr}[F^{\mu\rho}(0)U_{[0,\xi]}F^{\nu\sigma}(\xi)U'_{[\xi,0]}]|P,S\rangle_{\xi^+=0}$$



The gauge links connecting the two points ensure that the TMD distribution is gauge invariant.

$$U^C = \mathcal{P}\exp[ig\int_C dz^\mu \mathcal{A}_\mu(z)]$$

P. J. Mulders and J. Rodrigues PRD 63 094021(2001)

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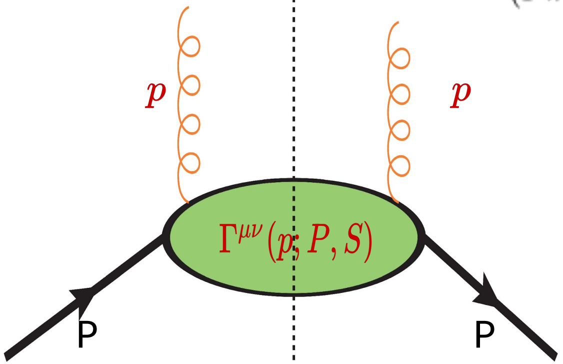

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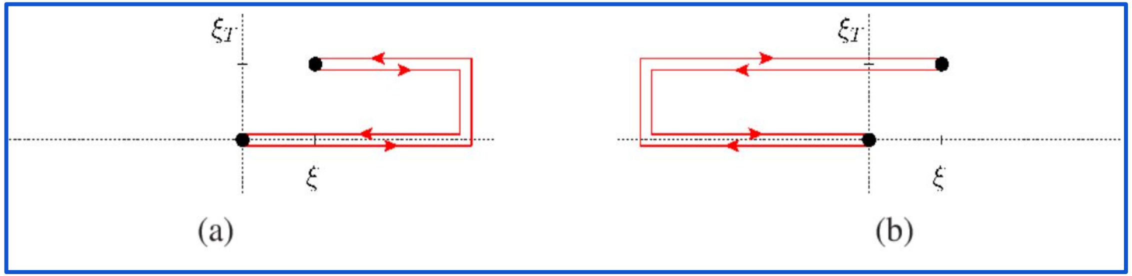

1. ***f-type (WW)***
   a.[+ +]   b.[- -]

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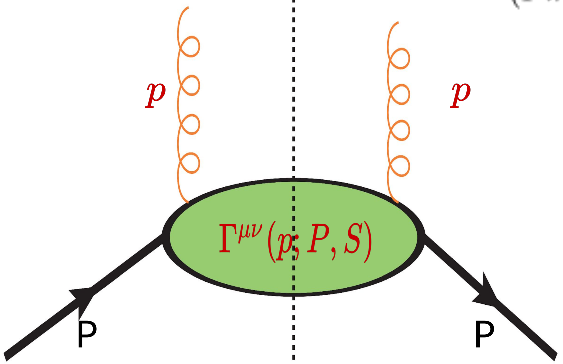

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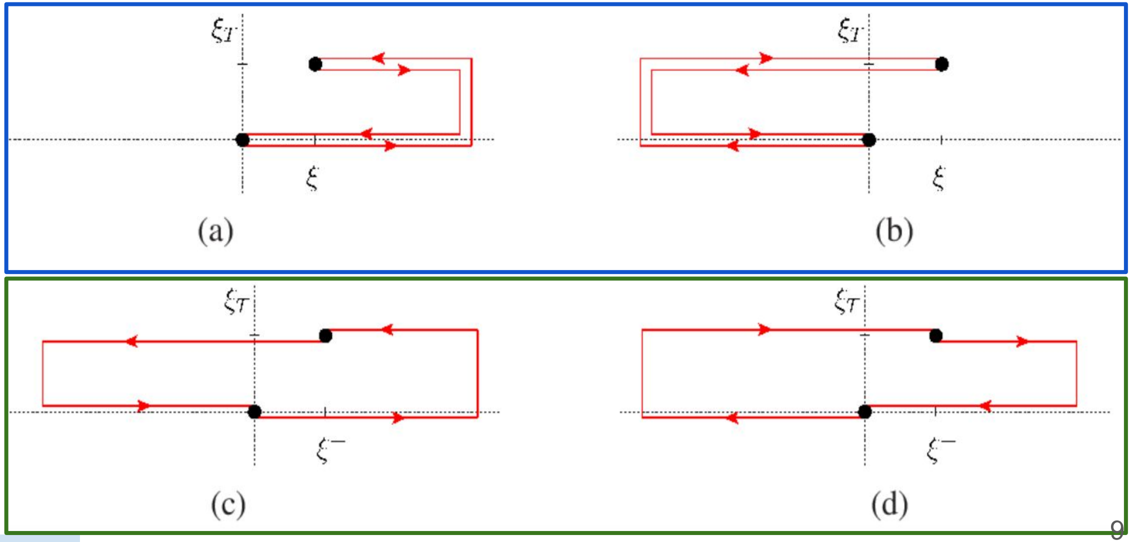

1. ***f-type (WW)***
   a.[+ +] b.[- -]

2. ***d-type (dipole)***
   c.[+ -] d.[- +]

P. J. Mulders and J. Rodrigues PRD 63 094021(2001)

Gluon TMD PDFs at leading twist

$$\Gamma_g^{\mu\nu}(p;P,S) = \frac{n_\rho n_\sigma}{(P.n)^2} \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi} \langle P,S|\mathrm{Tr}[F^{\mu\rho}(0)U_{[0,\xi]}F^{\nu\sigma}(\xi)U'_{[\xi,0]}]|P,S\rangle_{\xi^+=0}$$

		Gluon polarization		
Target polarization		**Unpolarised**	**Circularly**	**Linearly**
	Unpolarised	f_1^g		$h_1^{\perp g}$
	Longitudinal		g_{1L}^g	$h_{1L}^{\perp g}$
	Transverse	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

T-even

T-odd

D. Boer *et al* arXiv 1507.05267

Gluon TMD PDFs at leading twist

Unpolarized TMD PDF

Linearly polarized gluon TMD

Target polarization	Gluon polarization: Unpolarised	Circularly	Linearly
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Helicity TMD PDF

Worm-gear

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Unpolarized TMD PDF

Linearly polarized gluon TMD

Pseudo worm-gear

Sivers function

Helicity TMD PDF

Transversity TMD PDF

Worm-gear

Pretzelosity

D. Boer *et al* arXiv 1507.05267

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Unpolarized TMD PDF (f_1^g)

Linearly polarized gluon TMD ($h_1^{\perp g}$)

Helicity TMD PDF (g_{1L}^g)

Pseudo worm-gear ($h_{1L}^{\perp g}$)

Sivers function ($f_{1T}^{\perp g}$)

Worm-gear (g_{1T}^g)

Transversity TMD PDF (h_{1T}^g)

Pretzelosity ($h_{1T}^{\perp g}$)

T-even

T-odd

D. Boer *et al* arXiv 1507.05267

Linearly polarized gluon TMD

$h_1^{\perp g}$

Azimuthal asymmetries $\cos 2\phi, \cos 4\phi$

WW-type or Dipole type

T-even function

heavy quark pair, dijet, photon pair

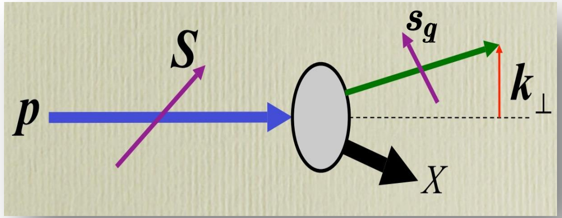
Not extracted from experiments

$$\frac{\mathbf{k}_\perp^2}{2M_h^2}|h_1^{\perp g}(x,\mathbf{k}_\perp^2)| \le f_1^g(x,\mathbf{k}_\perp^2)$$

D. Boer et al. PRL 106 132001 (2011)

Mulders and Rodrigues, PRD 63, 094021 (2001)

Gluon Sivers function



$$\Delta^N f_{g/p^\uparrow}(x, k_\perp) = -2 f_{1T}^{\perp g}(x, k_\perp) \frac{(p \times k_\perp).S}{M_p}$$

Trento convention

$S \cdot (p \times k_\perp)$: Sivers effect

- Sivers function inbeds the correlation between the target spin and gluons transverse momentum.
- Sivers function is Time-reversal odd function.
- Sivers function in DY is equal in magnitude but opposite in sign compared to Sivers function in SIDIS.

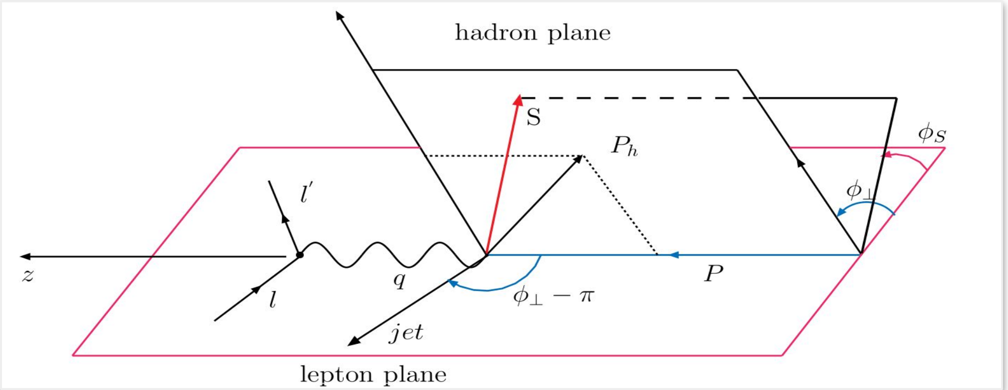
$$\Delta^N f_{g/p^\uparrow}(x, \mathbf{k}_\perp)|_{\text{DY}} = -\Delta^N f_{g/p^\uparrow}^\perp(x, \mathbf{k}_\perp)|_{\text{SIDIS}}$$

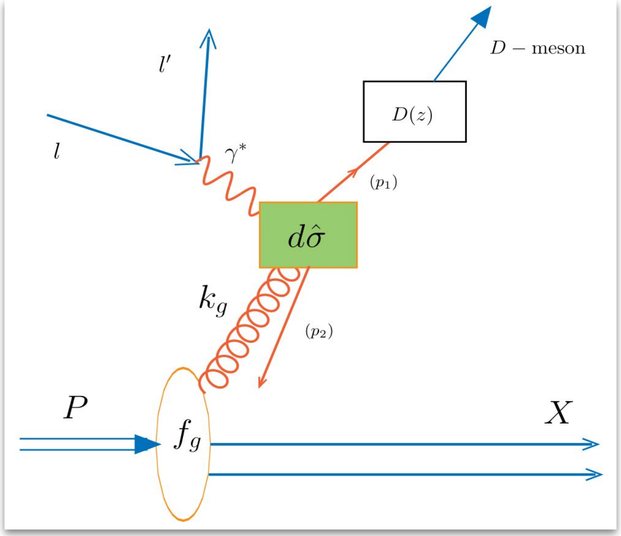
- lepton-pair production, back-to-back jet production in ep and pp collision.
- Azimuthal asymmetry $\sin(\phi_s - \phi_T)$

D. Sivers, PRD 41, 83(1990)

D-meson and jet production at EIC

$$e(l) + p^{\uparrow}(P) \rightarrow e(l') + D(P_h) + \text{jet} + X$$





Kinematic variables:

$Q^2 = -q^2$ Virtuality of photon

$x = \frac{Q^2}{2P{\cdot}q}$ Bjorken variable

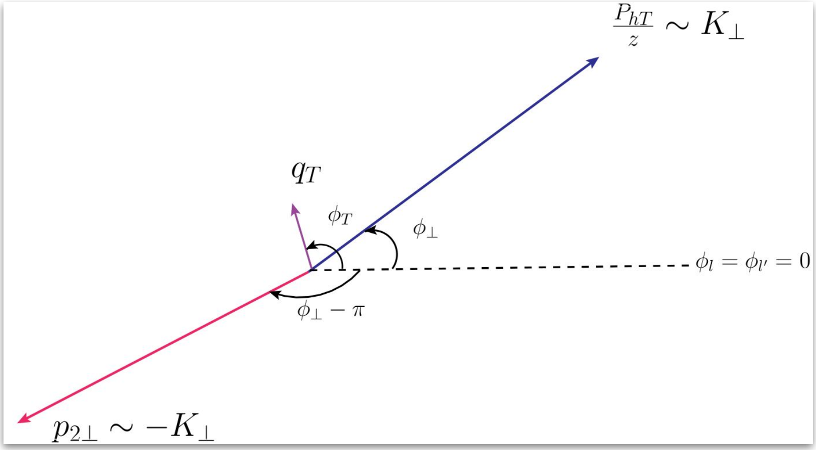
$y = \frac{P{\cdot}q}{P{\cdot}k}$ Inelasticity

$s = (l + P)^2$ Centre of mass energy

$z_h = \frac{P.P_h}{P.q}$ Energy fraction of D-meson

$$\gamma^*(q) + g(k) \rightarrow c(p_1) + \bar{c}(p_2)$$

Differential Scattering Cross-Section



$$\mathbf{q}_T = \frac{\mathbf{P}_{hT}}{z} + \mathbf{p}_{2\perp}$$

$$\mathbf{K}_\perp = \frac{\frac{\mathbf{P}_{hT}}{z} - \mathbf{p}_{2\perp}}{2}$$

- In the case , where the *D*-meson and jet are almost back to back in the transverse plane,

$$|\mathbf{q}_T| \ll |\mathbf{K}_\perp|$$

q_T the transverse imbalance of quark-antiquark pair is equal to the intrinsic transverse momentum of gluon

$$q_T = k_{\perp g}$$

Differential Scattering Cross-Section

Assuming the TMD factorization holds, the total differential scattering cross-section can be written as

$$d\sigma^{ep\to e+D+\bar{c}+X} = \frac{1}{2s}\frac{d^3\mathbf{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2}\int dx_g\, d^2\mathbf{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-p_1-p_2)$$
$$\times\frac{1}{Q^4}L^{\mu\nu}(l,q)\,\Phi_g^{\rho\sigma}(x_g,\mathbf{k}_{\perp g})\,H_{\mu\rho}^{\gamma^* g\to c\bar{c}}\,H_{\nu\sigma}^{*;\gamma^* g\to c\bar{c}}\,D(z)\,J(z)$$

Leptonic tensor

$$L^{\mu\nu} = e^2\frac{Q^2}{y^2}\Big[-(1+(1-y)^2)g_T^{\mu\nu} + 4(1-y)\epsilon_L^{\mu}\epsilon_L^{\nu} + 4(1-y)\left(\hat{l}_\perp^{\mu}\hat{l}_\perp^{\nu}+\frac{1}{2}g_T^{\mu\nu}\right)$$
$$+2(2-y)\sqrt{1-y}\left(\epsilon_L^{\mu}\hat{l}_\perp^{\nu}+\epsilon_L^{\nu}\hat{l}_\perp^{\mu}\right)\Big]$$

Differential Scattering Cross-Section

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The gluon correlator (non-perturbative) for unpolarized proton is given as

$$\Phi_U^{\rho\sigma}(x_g,\mathbf{k}_{\perp g}) = \frac{1}{2x_g}\left[-g_T^{\rho\sigma} f_1^g(x,\mathbf{k}_{\perp g}^2) + \left(\frac{k_{\perp g}^{\rho}k_{\perp g}^{\sigma}}{M_P^2} + g_T^{\rho\sigma}\frac{\mathbf{k}_{\perp g}^2}{2M_P^2}\right) h_1^{\perp g}(x,\mathbf{k}_{\perp g}^2)\right]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

The gluon correlator for transversely polarized proton is given as

$$\Phi_T^{\mu\nu}(x_g,\mathbf{k}_{\perp g}) = \frac{1}{2x_g}\Bigg\{-g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}k_{\perp g\rho}S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x_g,\mathbf{k}_{\perp g}^2) + i\epsilon_T^{\mu\nu}\frac{k_{\perp g}\cdot S_T}{M_p} g_{1T}^g(x_g,\mathbf{k}_{\perp g}^2)$$

$$+\frac{k_{\perp g\rho}\epsilon_T^{\rho\{\mu}k_{\perp g}^{\nu\}}}{2M_p^2}\frac{k_{\perp g}\cdot S_T}{M_p} h_{1T}^{\perp g}(x_g,\mathbf{k}_{\perp g}^2) - \frac{k_{\perp g\rho}\epsilon_T^{\rho\{\mu}S_T^{\nu\}} + S_{T\rho}\epsilon_T^{\rho\{\mu}k_{\perp g}^{\nu\}}}{4M_p} h_{1T}^g(x_g,\mathbf{k}_{\perp g}^2)\Bigg\}$$

Gluon Sivers function

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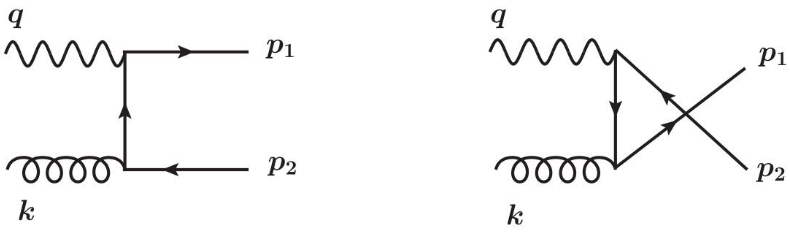
$$\times\frac{1}{Q^4}L^{\mu\nu}(l,q)\,\Phi_g^{\rho\sigma}(x_g,\mathbf{k}_{\perp g})\,H_{\mu\rho}^{\gamma^* g\to c\bar{c}}\,H_{\nu\sigma}^{*;\gamma^* g\to c\bar{c}}\,D(z)\,J(z)$$

Scattering amplitude (perturbative part)

$$\gamma^*(q)+g(k)\to c(p_1)+\bar{c}(p_2)$$



Feynman diagram for D-meson production in SIDIS process

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Jacobian

$$D_{D/c}(z,\mu) = \frac{Nz(1-z)^2}{\left[(1-z)^2+\epsilon z\right]^2}$$

Fragmentation function (non-perturbative part)

$\mu = m_c = 1.5\ \mathrm{GeV}$

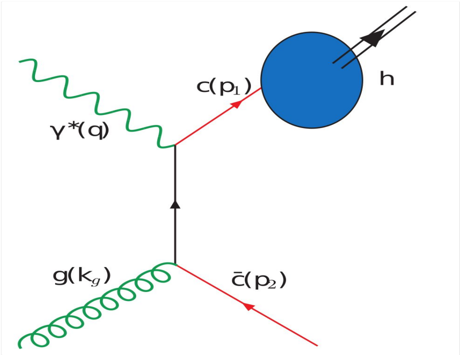
$N = 0.694$

$\epsilon = 0.101$

$k_{D\perp}$: transverse momentum of D-meson w.r.t charm quark

$k_{D\perp} \ll K_\perp$

$\int d^2k_{D\perp}\, D(z,k_{D\perp}) = D(z)$



$$z_1 = \frac{P\cdot p_1}{P\cdot q}$$
$$z_2 = \frac{P\cdot p_2}{P\cdot q}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$z = \frac{P\cdot P_h}{P\cdot p_1} = \frac{z_h}{z_1}$$

Kniehl and Kramer Phys. Rev. D **74**, 037502

Azimuthal asymmetries

The cross-section as the sum of unpolarized and transversely polarized cross-sections,

$$\frac{d\sigma}{dQ^2\,dy\,dz_h\,d^2\mathbf{q}_T\,d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S,\phi_T) = d\sigma^U(\phi_T,\phi_\perp) + d\sigma^T(\phi_S,\phi_T)$$

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$$d\sigma^U = \mathcal{N}\int dz\Big[\big(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp\big) f_1^g(x,\mathbf{q}_T^2) + \big(\mathcal{B}_0\cos 2\phi_T + \mathcal{B}_1\cos(2\phi_T-\phi_\perp) + \mathcal{B}_2\cos 2(\phi_T-\phi_\perp)$$
$$+ \mathcal{B}_3\cos(2\phi_T-3\phi_\perp) + \mathcal{B}_4\cos(2\phi_T-4\phi_\perp)\big)\frac{\mathbf{q}_T^2}{M_p^2}\,h_1^{\perp g}(x,\mathbf{q}_T^2)\Big]\,D(z)$$

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$$\int d\phi_\perp d\sigma^T = 2\pi|\boldsymbol{S}_T|\frac{|\boldsymbol{q}_T|}{M_p}\int dz\left[\mathcal{A}_0\sin(\phi_S-\phi_T)f_{1T}^{\perp g}(x,\boldsymbol{q}_T^2) - \frac{1}{2}\mathcal{B}_0\sin(\phi_S-3\phi_T)\frac{|\boldsymbol{q}_T|^2}{M_p^2}h_{1T}^{\perp g}(x,\boldsymbol{q}_T^2) + \mathcal{B}_0\sin(\phi_S+\phi_T)h_1^g(x,\boldsymbol{q}_T^2)\right]D(z)\,,$$

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$$\frac{d\sigma}{dQ^2\,dy\,dz_h\,d^2\mathbf{q}_T\,d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S,\phi_T) = d\sigma^U(\phi_T,\phi_\perp) + d\sigma^T(\phi_S,\phi_T)$$

$$d\sigma^U = \mathcal{N}\int dz\Big[\big(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp\big) f_1^g(x,\mathbf{q}_T^2) + \big(\mathcal{B}_0\cos 2\phi_T + \mathcal{B}_1\cos(2\phi_T-\phi_\perp) + \mathcal{B}_2\cos 2(\phi_T-\phi_\perp)$$

$$+\,\mathcal{B}_3\cos(2\phi_T-3\phi_\perp) + \mathcal{B}_4\cos(2\phi_T-4\phi_\perp)\big)\frac{\mathbf{q}_T^2}{M_p^2}h_1^{\perp g}(x,\mathbf{q}_T^2)\Big]D(z)$$

$$\int d\phi_\perp d\sigma^T = 2\pi|\boldsymbol{S}_T|\frac{|\boldsymbol{q}_T|}{M_p}\int dz\Big[\mathcal{A}_0\sin(\phi_S-\phi_T)f_{1T}^{\perp g}(x,\boldsymbol{q}_T^2) - \frac{1}{2}\mathcal{B}_0\sin(\phi_S-3\phi_T)\frac{|\boldsymbol{q}_T|^2}{M_p^2}h_{1T}^{\perp g}(x,\boldsymbol{q}_T^2)$$

$$+\,\mathcal{B}_0\sin(\phi_S+\phi_T)h_1^g(x,\boldsymbol{q}_T^2)\Big]D(z)\,,$$

$$A^{W(\phi_S,\phi_T)} \equiv 2\,\frac{\int d\phi_S\,d\phi_T\,d\phi_\perp\,W(\phi_S,\phi_T)\,d\sigma(\phi_S,\phi_T,\phi_\perp)}{\int d\phi_S\,d\phi_T\,d\phi_\perp\,d\sigma(\phi_S,\phi_T,\phi_\perp)}$$

Azimuthal asymmetries

The cross-section as the sum of unpolarized and transversely polarized cross-sections,

$$\frac{d\sigma}{dQ^2\,dy\,dz_h\,d^2\mathbf{q}_T\,d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S,\phi_T) = d\sigma^U(\phi_T,\phi_\perp) + d\sigma^T(\phi_S,\phi_T)$$

$$d\sigma^U = \mathcal{N}\int dz\Big[\big(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp\big) f_1^g(x,\mathbf{q}_T^2) + \big(\mathcal{B}_0\cos 2\phi_T + \mathcal{B}_1\cos(2\phi_T-\phi_\perp) + \mathcal{B}_2\cos 2(\phi_T-\phi_\perp)$$
$$+ \mathcal{B}_3\cos(2\phi_T-3\phi_\perp) + \mathcal{B}_4\cos(2\phi_T-4\phi_\perp)\big)\frac{\mathbf{q}_T^2}{M_p^2}\boxed{h_1^{\perp g}(x,\mathbf{q}_T^2)}\Big]\,D(z)$$

$$\int d\phi_\perp d\sigma^T = 2\pi|\boldsymbol{S}_T|\frac{|\boldsymbol{q}_T|}{M_p}\int dz\Big[\mathcal{A}_0\sin(\phi_S-\phi_T)f_{1T}^{\perp g}(x,\boldsymbol{q}_T^2) - \frac{1}{2}\mathcal{B}_0\sin(\phi_S-3\phi_T)\frac{|\boldsymbol{q}_T|^2}{M_p^2}h_{1T}^{\perp g}(x,\boldsymbol{q}_T^2)$$
$$+ \mathcal{B}_0\sin(\phi_S+\phi_T)h_1^g(x,\boldsymbol{q}_T^2)\Big]\,D(z)\,,$$

$$A^{W(\phi_S,\phi_T)} \equiv 2\,\frac{\int d\phi_S\,d\phi_T\,d\phi_\perp\,W(\phi_S,\phi_T)\,d\sigma(\phi_S,\phi_T,\phi_\perp)}{\int d\phi_S\,d\phi_T\,d\phi_\perp\,d\sigma(\phi_S,\phi_T,\phi_\perp)}$$

- The $h_1^{\perp g}$ gluon TMD could be extracted by studying the following azimuthal asymmetries

$$A^{\cos 2\phi_T} = \frac{\mathbf{q}_T^2}{M_p^2}\,\frac{\int dz\,\mathcal{B}_0\,D(z)\,h_1^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\mathbf{q}_T^2)}$$

$$A^{\cos 2(\phi_T-\phi_\perp)} = \frac{q_T^2}{M_p^2}\,\frac{\int dz\,\mathcal{B}_2\,D(z)\,h_1^{\perp g}(x,q_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,q_T^2)}$$

Azimuthal asymmetries

The cross-section as the sum of unpolarized and transversely polarized cross-sections,

$$\frac{d\sigma}{dQ^2\, dy dz_h\, d^2\mathbf{q}_T\, d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S, \phi_T) = d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_S, \phi_T)$$

$$d\sigma^U = \mathcal{N} \int dz \Big[\big(\mathcal{A}_0 + \mathcal{A}_1 \cos\phi_\perp + \mathcal{A}_2 \cos 2\phi_\perp\big) f_1^g(x, \mathbf{q}_T^2) + \big(\mathcal{B}_0 \cos 2\phi_T + \mathcal{B}_1 \cos(2\phi_T - \phi_\perp) + \mathcal{B}_2 \cos 2(\phi_T - \phi_\perp)$$
$$+ \mathcal{B}_3 \cos(2\phi_T - 3\phi_\perp) + \mathcal{B}_4 \cos(2\phi_T - 4\phi_\perp)\big) \frac{\mathbf{q}_T^2}{M_p^2} h_1^{\perp g}(x, \mathbf{q}_T^2) \Big] D(z)$$

$$\int d\phi_\perp d\sigma^T = 2\pi |\boldsymbol{S}_T| \frac{|\boldsymbol{q}_T|}{M_p} \int dz \Big[\mathcal{A}_0 \sin(\phi_S - \phi_T) f_{1T}^{\perp g}(x, \boldsymbol{q}_T^2) - \frac{1}{2}\mathcal{B}_0 \sin(\phi_S - 3\phi_T) \frac{|\boldsymbol{q}_T|^2}{M_p^2} h_{1T}^{\perp g}(x, \boldsymbol{q}_T^2)$$
$$+ \mathcal{B}_0 \sin(\phi_S + \phi_T) h_1^g(x, \boldsymbol{q}_T^2) \Big] D(z),$$

$$A^{W(\phi_S,\phi_T)} \equiv 2\, \frac{\int d\phi_S\, d\phi_T\, d\phi_\perp\, W(\phi_S,\phi_T)\, d\sigma(\phi_S, \phi_T, \phi_\perp)}{\int d\phi_S\, d\phi_T\, d\phi_\perp\, d\sigma(\phi_S,\phi_T,\phi_\perp)}$$

- The $h_1^{\perp g}$ gluon TMD could be extracted by studying the following azimuthal asymmetries

$$A^{\cos 2\phi_T} = \frac{\mathbf{q}_T^2}{M_p^2} \frac{\int dz\, \mathcal{B}_0\, D(z)\, h_1^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\, \mathcal{A}_0\, D(z)\, f_1^g(x,\mathbf{q}_T^2)}$$

$$A^{\cos 2(\phi_T - \phi_\perp)} = \frac{q_T^2}{M_p^2} \frac{\int dz\, \mathcal{B}_2\, D(z)\, h_1^{\perp g}(x,q_T^2)}{\int dz\, \mathcal{A}_0\, D(z)\, f_1^g(x,q_T^2)}$$

- Sivers asymmetry can be extracted through the azimuthal asymmetry.

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{\int dz\, \mathcal{A}_0\, D(z)\, f_{1T}^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\, \mathcal{A}_0\, D(z)\, f_1^g(x,\mathbf{q}_T^2)}$$

Upper bound

- ➔ Linearly polarized gluon distribution satisfies the positivity bound
- ➔ Upper limit of asymmetry obtained when this bound is saturated

$$\frac{\mathbf{q}_T^2}{2M_p^2}\,|h_1^{\perp\,g}(x,\mathbf{q}_T^2)| \leq f_1^g(x,\mathbf{q}_T^2)$$

$$\frac{\mathrm{q}_t^2}{2M_p^2}\,|h_1^{\perp g}(x,\mathrm{q}_t^2)| = f_1^g(x,\mathrm{q}_t^2)$$

$$A^{\cos 2\phi_T} = \frac{\mathbf{q}_T^2}{M_p^2}\,\frac{\int dz\,\mathcal{B}_0\,D(z)\,h_1^{\perp\,g}(x,\mathbf{q}_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\mathbf{q}_T^2)}$$

$$A^{\cos 2\phi_T} \to U = \frac{2*|\mathbb{B}_0|}{\mathbb{A}_0}$$

$$A^{\cos 2(\phi_T-\phi_\perp)} = \frac{q_T^2}{M_p^2}\,\frac{\int dz\,\mathcal{B}_2\,D(z)\,h_1^{\perp\,g}(x,q_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,q_T^2)}$$

$$A^{\cos 2(\phi_T-\phi_\perp)} \to U = \frac{2*|\mathbb{B}_2|}{\mathbb{A}_0}$$

Parametrization of TMDs

Gaussian Parametrization of TMDs

$$f_1^g(x, \mathbf{q}_T^2) = f_1^g(x,\mu)\frac{e^{-\mathbf{q}_T^2/\langle q_T^2\rangle}}{\pi\langle q_T^2\rangle}$$

Stefano Melis,et al. (2014)

$$h_1^{\perp g}(x, \mathbf{q}_T^2) = \frac{M_p^2 f_1^g(x,\mu)}{\pi\langle q_T^2\rangle^2}\frac{2(1-r)}{r}e^{1-\frac{\mathbf{q}_T^2}{r\langle q_T^2\rangle}}$$

$f_1^g(x,\mu)$ is the collinear gluon PDF

QCD Scale : $\mu = \sqrt{m_D^2 + Q^2}$

$r(0 < r < 1)$ and $\langle \mathrm{q}_T^2\rangle$ are parameters

$r = 1/3$ $\quad \langle \mathrm{q}_T^2\rangle = 1\ \mathrm{GeV}^2$ constant and flavor independent

A. D. Martin, W. J. Stirling, etal., EPJ C 63, 189 (2009)

Sivers function Parameterization

$$\Delta^N f_{g/p^\uparrow}(x, q_T) = \left(-\frac{2|\mathbf{q}_T|}{M_P}\right) f_{1T}^{\perp g}(x, q_T) = 2\frac{\sqrt{2e}}{\pi}\mathcal{N}_g(x)\, f_{g/p}(x)\sqrt{\frac{1-\rho}{\rho}}q_T\frac{e^{-q_T^2/\rho\langle q_T^2\rangle}}{\langle q_T^2\rangle^{3/2}}$$

U. D'Alesio, et al., PRD 99(2019)036013

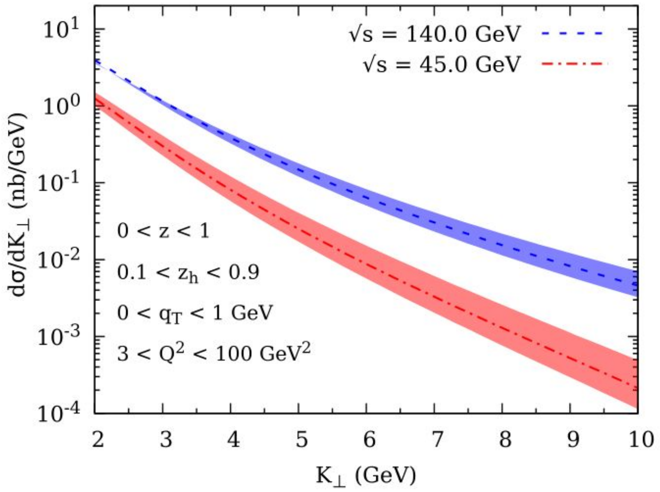
$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

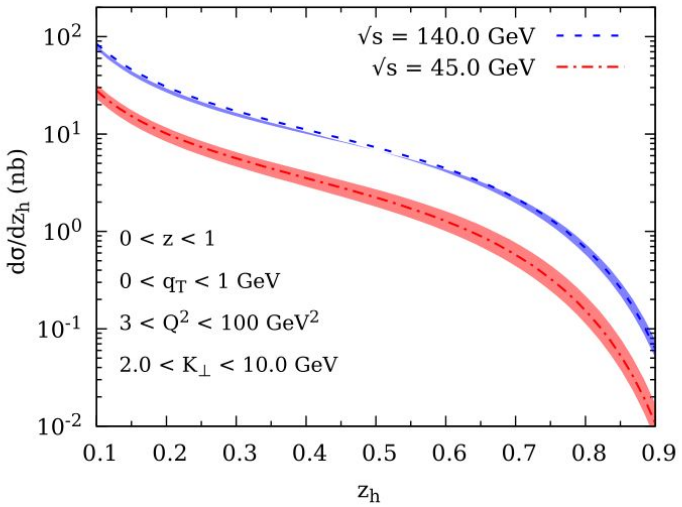
the extracted best fit parameters are (PHENIX Collaboration at RHIC) $N_g = 0.25\,,\quad \alpha = 0.6\,,\quad \beta = 0.6\,,\quad \rho = 0.1$

D. Boer, C. Pisano, PRD 86, 094007 (2012)

Numerical Results

Unpolarized differential scattering cross-section



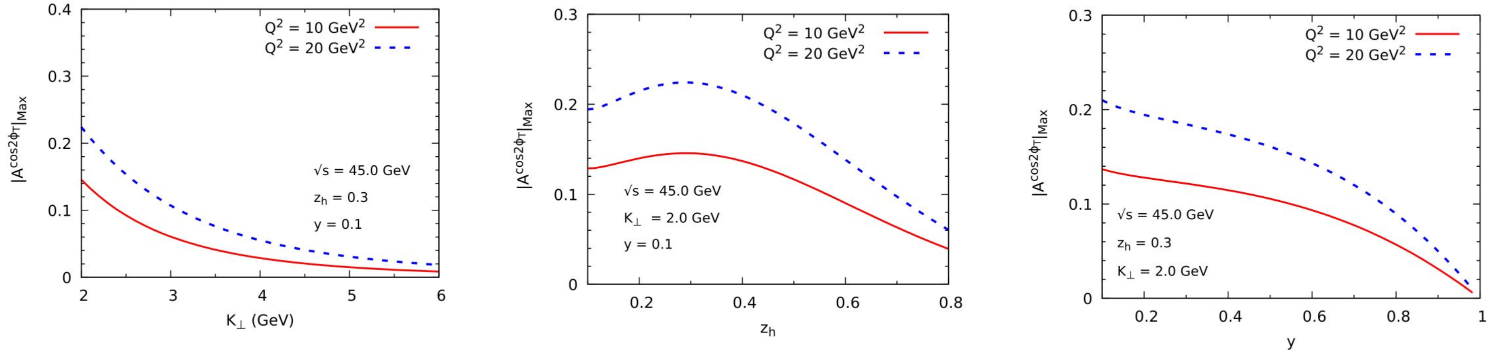


The bands are obtained by varying the factorization scale in the range $\frac{1}{2}\mu < \mu < 2\mu$

PRD 108 (2023) 034005

$\cos 2\phi_T$ Azimuthal Asymmetry

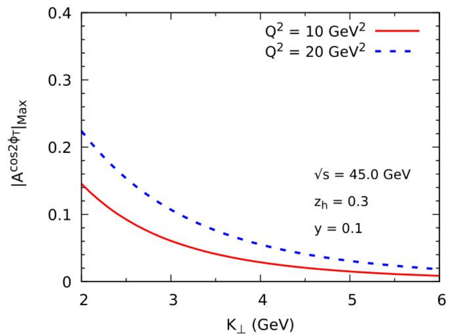
Upper bound

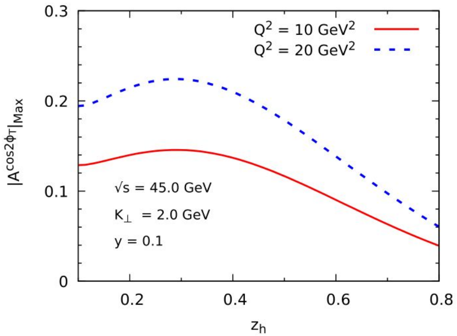


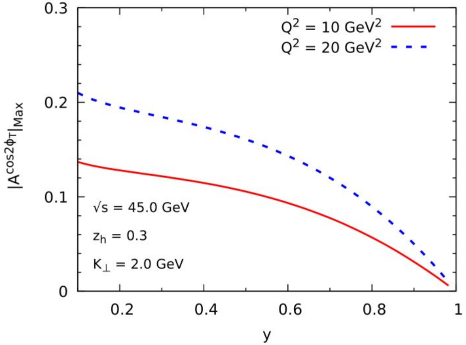
The z is integrated over 0 < z < 1 and q_T is integrated over 0 < q_T < 1 GeV.

$\cos 2\phi_T$ Azimuthal Asymmetry

Upper bound

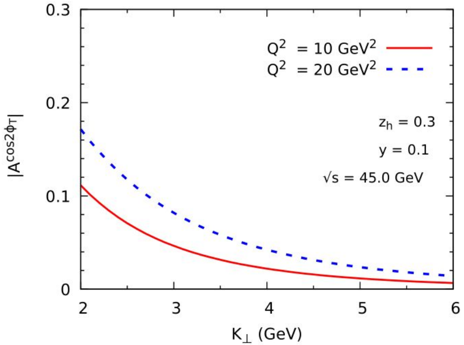


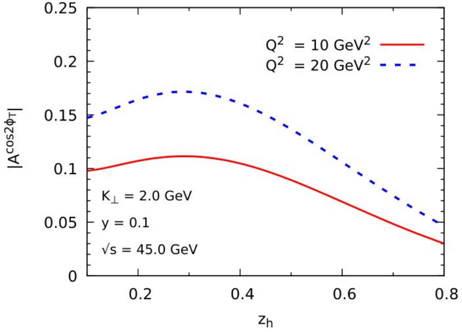


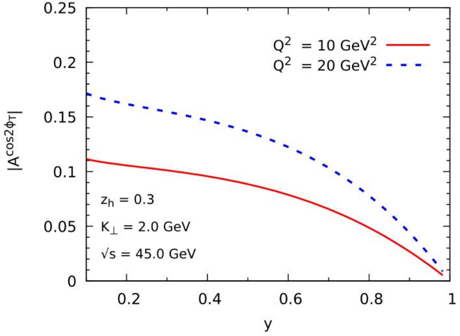


The z is integrated over 0 < z < 1 and q_T is integrated over 0 < q_T < 1 GeV.

Gaussian Parametrization



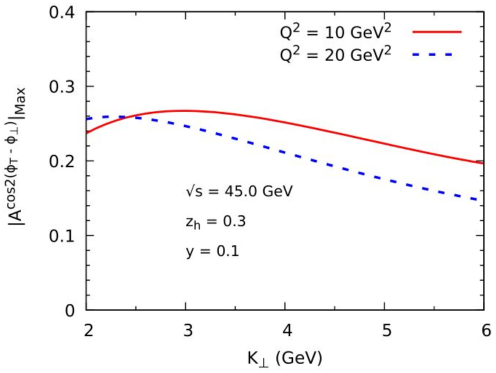


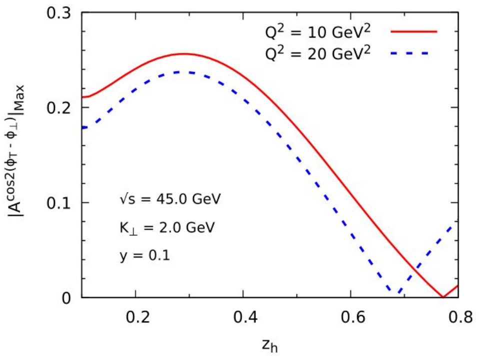


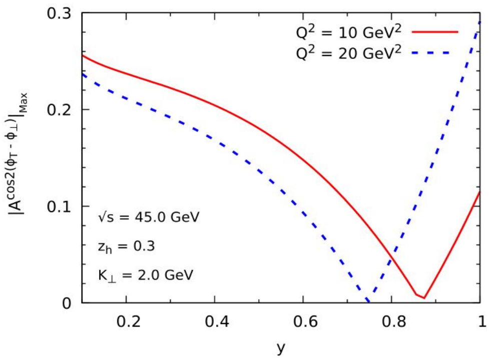
PRD 108 (2023) 034005

$\cos 2(\phi_T - \phi_\perp)$ Azimuthal Asymmetry

Upper bound





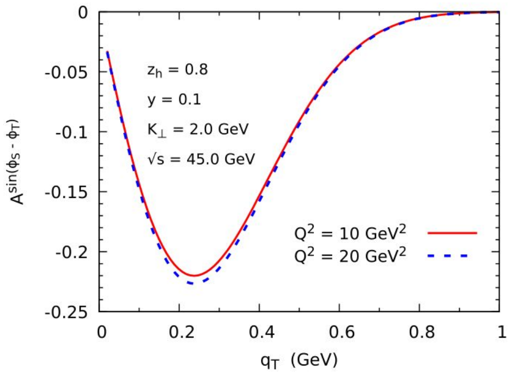


The z is integrated over 0 < z < 1 and q_T is integrated over $0 < q_T < 1$ GeV.

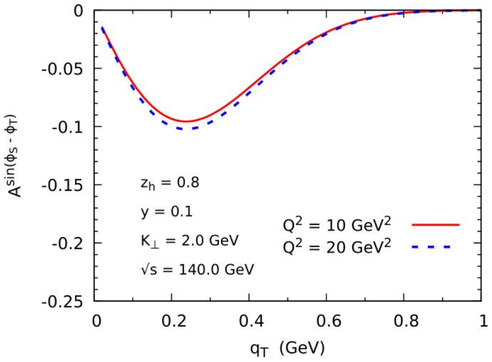
PRD 108 (2023) 034005

Numerical Results- Sivers asymmetry

$\sqrt{s} = 45$ GeV



$\sqrt{s} = 140$ GeV



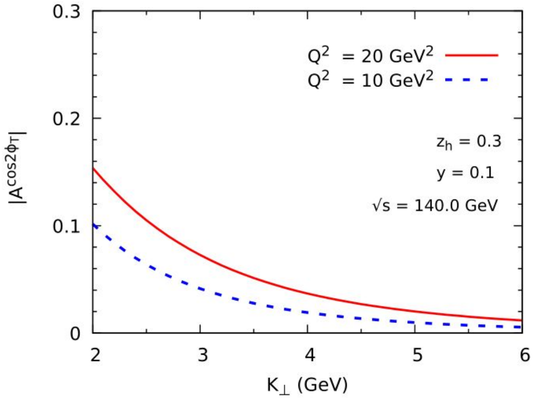
The z is integrated over $0 < z < 1$.

Summary

- *We have discussed the TMDs which are important to explain the 3D structure of hadrons.*
- *We estimate the* $\cos 2\phi_T$ *and Sivers asymmetry in almost back to back D-meson and jet electroproduction at the future EIC.*
- *At the LO the gluon channel only contribute to the partonic subprocess.*
- *We have used fragmentation function to describe the production of D-meson.*
- *The sizeable asymmetry is obtained for both the asymmetries.*
- *Back to back production of D-meson and jet can be a promising channel to access the linearly polarized gluon TMD and the gluon Sivers TMD at the upcoming EIC.*

Thank you for attention

Backup slides

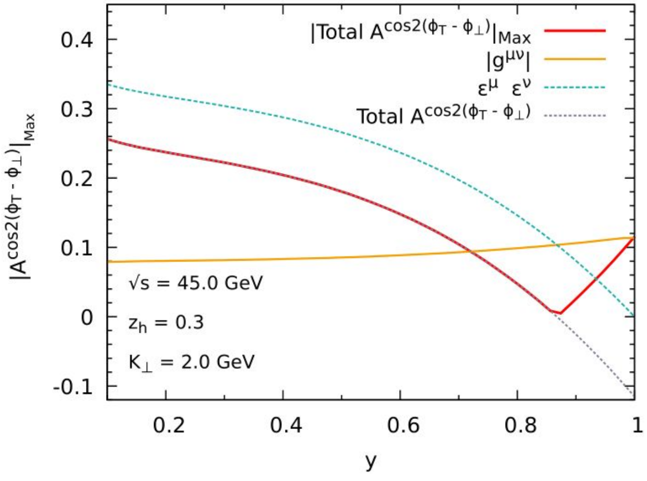


$$x_g = \frac{(m_c^2 + K_\perp^2)}{z_1(1-z_1)ys} + x_B$$

$$x_g \to 1, \ \text{pdf} \to 0$$

Reason for discontinuities

Virtual photon polarizations



transversely polarized Longitudinally polarized

$$L^{\mu\nu} = e^2 \frac{Q^2}{y^2}\Big[-(1+(1-y)^2) g_T^{\mu\nu} + 4(1-y)\epsilon_L^\mu \epsilon_L^\nu + 4(1-y)\left(\hat{l}_\perp^\mu \hat{l}_\perp^\nu + \frac{1}{2} g_T^{\mu\nu}\right) + 2(2-y)\sqrt{1-y}\left(\epsilon_L^\mu \hat{l}_\perp^\nu + \epsilon_L^\nu \hat{l}_\perp^\mu\right)\Big]$$

Linearly polarized

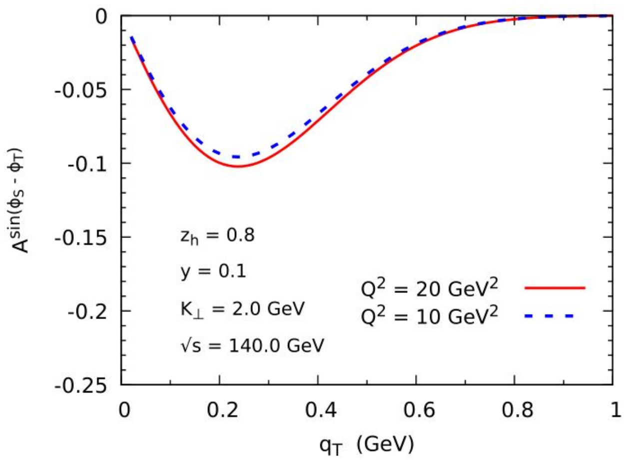
Interference

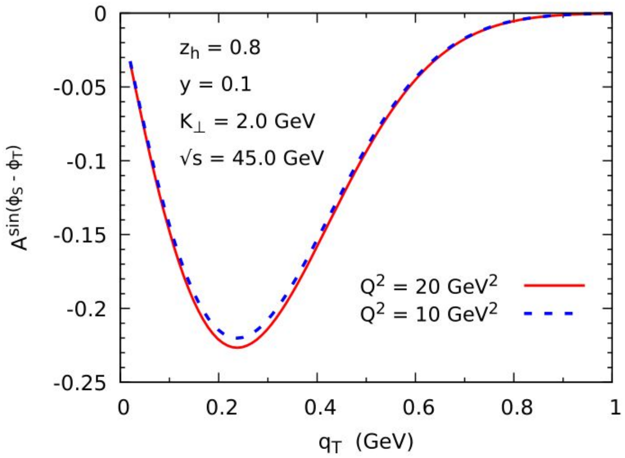
$$\mathrm{U} + \mathrm{L} \rightarrow \mathcal{B}_2$$

$$\text{Linearly polarized} \rightarrow \mathcal{B}_0, \mathcal{B}_4$$

$$\mathrm{I} \rightarrow \mathcal{B}_1, \mathcal{B}_3$$

Sivers asymmetry explanation





$$A^{\sin(\phi_S-\phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{\int dz\, \mathcal{A}_0\, D(z)\, f_{1T}^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\, \mathcal{A}_0\, D(z)\, f_1^g(x,\mathbf{q}_T^2)}$$

$$f_{1T}^{\perp g}\left(x, q_T\right) = \frac{\sqrt{2e}}{\pi}\mathcal{N}_g\left(x\right) f_{g/p}\left(x\right) \sqrt{\frac{1-\rho}{\rho}} \frac{e^{-q_T^2/\rho\left\langle q_T^2\right\rangle}}{\left\langle q_T^2\right\rangle^{3/2}}$$

$$\mathcal{N}_g\left(x\right) = N_g x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta} \qquad x_g = \frac{(m_c^2+K_\perp^2)}{z_1(1-z_1)ys} + x_B$$

Sivers Asymmetry

The z is integrated over $0 < z < 1$.

Positivity bound

Gluon Helicity $|+\rangle$ $|+\rangle$ $|-\rangle$ $|-\rangle$

$\Phi^{\mu\nu}$

$$
\begin{array}{c}
\langle +| \\ \langle +| \\ \langle -| \\ \langle -|
\end{array}
\quad
\begin{pmatrix}
f_1^g + g_{1L}^g & \frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g - i f_{1T}^{\perp g}) & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} + i h_{1L}^{\perp g}) & -i\frac{q_T e^{-3i\phi}}{M_p} h_{1T}^{\perp g} \\
\frac{q_T e^{i\phi}}{M_p}(g_{1T}^g + i f_{1T}^{\perp g}) & f_1^g - g_{1L}^g & -i\frac{q_T e^{-i\phi}}{M_p} h_1^g & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} - i h_{1L}^{\perp g}) \\
-\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} - i h_{1L}^{\perp g}) & i\frac{q_T e^{i\phi}}{M_p} h_1^g & f_1^g - g_{1L}^g & -\frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g + i f_{1T}^{\perp g}) \\
i\frac{q_T e^{3i\phi}}{M_p} h_{1T}^{\perp g} & -\frac{q_T^2 e^{2i\phi}}{M_p^2}(h_1^{\perp g} + i h_{1L}^{\perp g}) & -\frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g - i f_{1T}^{\perp g}) & f_1^g + g_{1L}^g
\end{pmatrix}
$$

$$M_p^2\ \Phi^{ii} = 2(P^+)^2 \int_0^1 dx_g x_g f_1^g(x_g, Q^2) = \langle P, S|T^{++}|P, S\rangle$$

Mulders and Rodrigues, PRD 63, 094021 (2001)

Positivity bound for h1g

Gluon Helicity $|+\rangle$ $|+\rangle$ $|-\rangle$ $|-\rangle$

$\Phi^{\mu\nu}$

$$
\begin{array}{c} \langle+| \\ \langle+| \\ \langle-| \\ \langle-| \end{array}
\quad
\begin{Bmatrix}
f_1^g + g_{1L}^g & \frac{q_T e^{-i\phi}}{M_p} g_{1T}^g & -\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} & 0 \\
\frac{q_T e^{i\phi}}{M_p} g_{1T}^g & f_1^g - g_{1L}^g & 0 & -\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} \\
-\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} & 0 & f_1^g - g_{1L}^g & -\frac{q_T e^{-i\phi}}{M_p} g_{1T}^g \\
0 & -\frac{q_T^2 e^{-2i\phi}}{M_p^2} h_1^{\perp g} & -\frac{q_T e^{-i\phi}}{M_p} g_{1T}^g & f_1^g + g_{1L}^g
\end{Bmatrix}
$$