Light-front quark models and hadron structure

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Overview

- *Introduction:Internal Structure of the Hadrons*
- *Distribution functions*
  - Parton Distribution Functions (PDFs)
  - Generalized Parton Distributions (GPDs)
  - Transverse Momentum-Dependent Parton Distributions (TMDs)
- *Research Group at NIT Jalandhar*
- *Work done in the Meson Sector*
- *Work done in Baryon Sector*

Outline

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Introduction

- Quantum Chromodynamics (QCD) provides a fundamental description of hadronic structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- **Internal Structure: The knowledge of internal structure provides a basis for understanding more complex, strongly interacting matter.**
- Knowledge has been rather limited because of **confinement** and it is still a big challenge to perform the calculations from the first principles of QCD.

Quantum chromodynamics (QCD): Present Theory of Strong Interactions

- At high energies, **(α_s is small)**, QCD can be used perturbatively.
- At low energies, **(α_s becomes large)**, one has to use other methods such as effective Lagrangian models to describe physics.
- Wide range of applications ranging from the dynamics and structure of hadrons and nuclei to the properties and phases of hadronic matter at the earliest stages of the universe.
- New experimental tools are continually being developed to probe the nonperturbative structure of the theory, for example the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering etc.
- Many fundamental questions have not been resolved. **The most challenging nonperturbative problem in QCD is to determine the structure and spectrum of hadrons in terms of their quark and gluon degrees of freedom.**

Fundamental Questions

- How are the static observable related to each other and how do they emerge?
- How are the sea quarks and their spins, distributed in space and momentum inside the nucleon?
- Role of orbital angular momentum of the quarks and gluons in the non-perturbative regime of QCD.
- The role played by non-valence flavors in understanding the nucleon internal structure.
- How do the quarks and gluons interact with a nuclear medium?

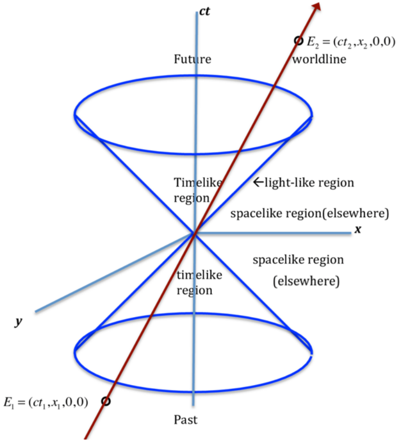
Proton Spin Crisis: The Driving Question

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin).
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)

  "Proton spin crisis"
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

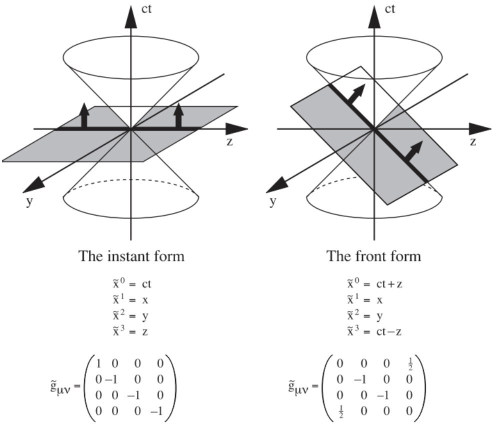
From Special Theory of Relativity:

- Space and time independently are not invariant quantities.
- Space-time is an invariant object.



Instant form v/s Front form

- Instant form
  - All measurements are made at fixed t i.e. at $x^0 = 0$.
  - Energy-momentum dispersion relation $p^0 = \sqrt{\vec{p}^2 + m^2}$.
  - Vacuum is infinitely complex.
- Front form
  - All measurements are made at fixed light-cone time x^+ i.e. at $x^+ = x^0 + x^3 = 0$.
  - Energy-momentum dispersion relation $p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$.
  - Vacuum is simple, as fluctuations are absent.



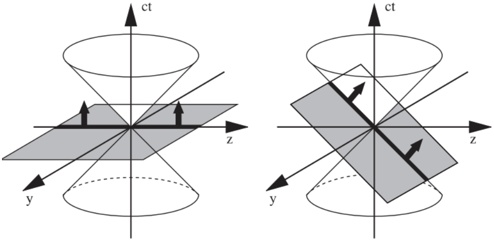
- S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

Why Light Front ?

- NO SQUARE ROOT FACTOR in energy-momentum dispersion relation

$$p^- = \frac{p_\perp^2 + m^2}{p^+},$$

  where
  - p^-: energy
  - p^+: longitudinal momentum
  - $p_\perp$: transverse momentum
- Vacuum expectation value is ZERO.
- Seven out of ten fundamental quantities are KINEMATICAL.
- Ideal Framework to describe theoretically the hadronic structure.



The instant form

$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The front form

$$\tilde{x}^0 = ct+z,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = ct-z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

- S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

Light-Front Coordinates

- A generic four Vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $p^+ = p^0 + p^3$ and light-front energy $p^- = p^0 - p^3$.

- Light Front QCD (LFQCD) is an *ab initio* approach to study the strongly interacting system. It is similar to perturbative and lattice QCD and is directly connected to the QCD Lagrangian.
- It is a Hamiltonian method, formulated in Minkowinski space rather than Euclidean space.
- The theory is quantized at fixed light-cone time $\tau = t + z/c$ rather than ordinary time t.

Outline

Distribution functions

(Mathematical tool to unfold the internal structure of hadrons)

Parton Distribution Functions (PDFs)
Generalized Parton Distributions (GPDs)
Transverse Momentum-Dependent Parton Distributions (TMDs)

Parton Distribution Functions (PDFs) I

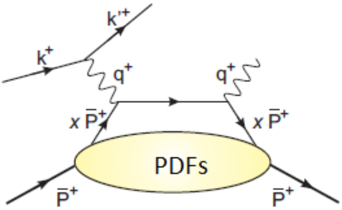
To understand the structure of the hadron in terms of quarks and gluons, different categories of parton distributions are present.

- PDFs were introduced by Feynman in 1969.
- PDFs are the basic ingredient to understand the internal hadron structure.
- From parton densities one can extract the distribution of longitudinal momentum carried by the quarks, antiquarks and gluons and their polarizations.
- PDF $f(x)$ imparts information about the probability of finding a parton carrying a longitudinal momentum fraction x inside the hadron.
- Partonic structure is probed in scattering processes such as Deep Inelastic Scattering (DIS).

Parton Distribution Functions (PDFs) II

- The quark-quark correlation to evaluate proton PDFs are defined as

$$F^{[\gamma^+]}(x) = \frac{1}{2}\int \frac{dz^-}{4\pi} e^{ik^+z^-/2} \langle P|\bar{\Psi}(0)\gamma^+\Psi(z^-)|P\rangle\Big|_{z^+=\mathbf{z}_\perp=0}.$$



How partons are distributed in the plane transverse to the direction in which the hadron is moving, or how important their orbital angular momentum is in making up the total spin of a nucleon?

This missing information is compensated in Generalized Parton Distributions (GPDs). The GPDs are physical observables which can provide deep insight about the internal structure of the hadron and more generally, in non-perturbative QCD.

Generalized Parton Distributions (GPDs) I

- GPDs provide a 3-D picture of the partonic nucleon structure. From 3-D we mean that GPDs encode information on the distribution of partons both in the specific location (transverse plane) and longitudinal direction.

- Generalized Parton Distributions can be accessed through deep exclusive processes such as DVCS or DVMP. DVCS reaction $\gamma^* + p \rightarrow \gamma + p$ has extraordinary senstivity to fundamental features of the hadron's structure.

- GPDs are much richer in content about the hadron structure than ordinary parton distributions.

- GPDs depends on three variables x, ζ, t.
  - x is the fraction of momentum transfer.
  - ζ gives the longitudinal momentum transfer.
  - t is the square of the momentum transfer in the process.

Generalized Parton Distributions (GPDs) II

- Several experiments such as H1 collaboration, ZEUS collaboration and fixed target experiments at HERMES have finished taking data on DVCS.

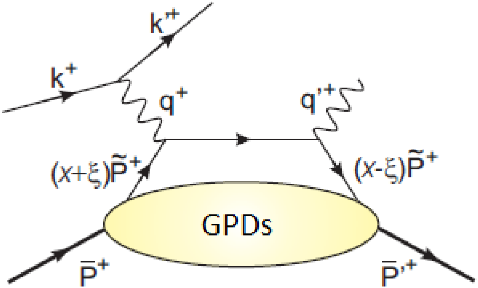
- In the forward limit of zero momentum transfer, the GPDs reduce to ordinary parton distributions.

- One can define the correlation to evaluate unpolarized GPD of hadron $F^{[\Gamma]}(x, \zeta = 0, t)$ as

$$F^{[\Gamma]}(x, 0, t) = \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ixP^+ z^-/2} \langle P_f | \bar{\Psi}(0) \Gamma \Psi(z) | P_i \rangle \Big|_{z^+ = \mathbf{z}_\perp = 0}.$$

Generalized Parton Distributions (GPDs) III

- The GPDs explain through various exclusive processes such as Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP).



Transverse Momentum-Dependent Parton Distributions (TMDs) I

To get the information of hadron structure in momentum space, transverse momentum-dependent parton distributions (TMDs) were introduced.

- TMDs describe the probability to find a parton with longitudinal momentum fraction x and transverse momentum with respect to the direction of the parent hadron momentum in a hadron.

- TMDs $f(x, \vec{k}_\perp)$, are function of longitudinal momentum fraction carried by the active quark $x = \frac{k^+}{P^+}$ and the quark transverse momentum $\vec{k}_\perp$.

- TMDs are also of particular importance because they give rise to single spin asymmetries (SSAs).

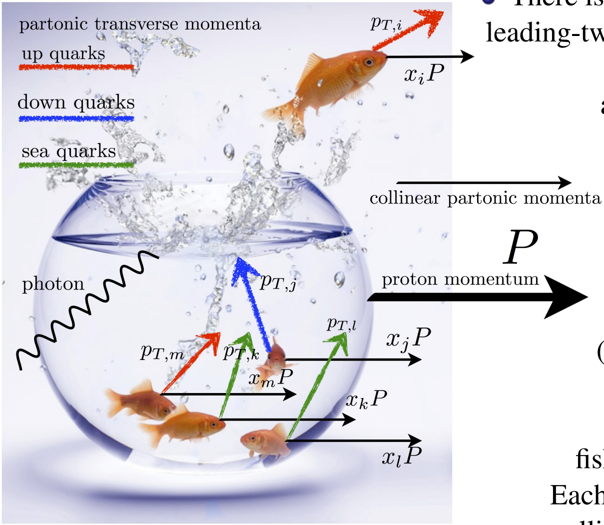
- TMDs represent 3-D hadron picture in *momentum space*.

Transverse Momentum-Dependent Parton Distributions (TMDs) II

- They can be measured in a variety of reactions in lepton-proton and proton-proton collisions as semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan production where a final-state particle is observed with a transverse momentum.

- The quark-quark correlation to evaluate quark TMDs of hadron is given by

$$\Phi^{[\Gamma]}(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\mathbf{z}_\perp}{(2\pi)^2} e^{ik.z/2} \langle P|\bar{\Psi}(0)\Gamma\Psi(z)|P\rangle\Big|_{z^+=0}.$$

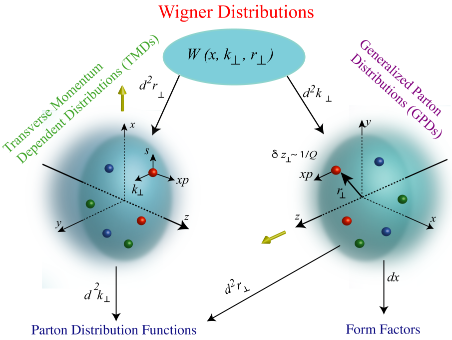


-Image courtesy: A. Signori

- There is one quark TMD at leading-twist in case of kaon, while 8 quark and gluon TMDs at the leading twist in case of nucleon (spin-1/2).
- In the figure, partons (quarks and gluons) are like fishes confined inside a fishbowl (the proton). Each parton has its own collinear and transverse velocity, indicated by black and colored arrow respectively.

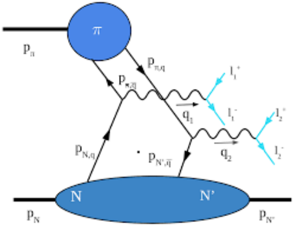
Wigner Distributions (WDs)

- Wigner introduced an idea of phase-space distribution for a quantum mechanical system.
- Give joint position and momentum distributions.

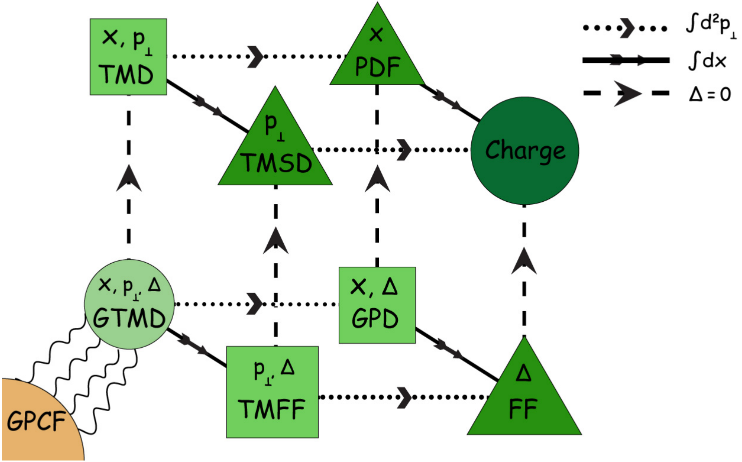


Generalized transverse momentum-dependent parton distributions (GTMDs)

- Natural extension of the concept of TMDs.
- Generic GTMD depends on the $(x, \mathbf{k}_\perp, \xi, \mathbf{\Delta}_\perp)$.
- Accessible through double Drell-Yan processes.



Interconnections



- S. Sharma and H. Dahiya, Eur. Phys. A 59, 235 (2021)

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- *Work done in Baryon Sector*

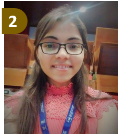
NITJ Research group

1

Dr. Narinder Kumar (Assistant Professor, Doaba College)

- Leading and higher twist distributions
- Mesons and Baryon
- Mechanical properties

2



Dr. Nisha Dhiman

- Beyond SM
- B-meson and their decay

3

Dr. Navdeep Kaur

- Spin 0 particles
- Leading twist distributions

4

Dr. Satvir Kaur (PDF at Inst. Modern Phys., Lanzhou)

- Collaboration with BLFQ
- Mesons and nucleons

5

Mr. Shubham Sharma (Research Scholar, SRF)

- Higher twist distribution functions
- Proton

6

Mr. Satyajit Puhan (Research Scholar, JRF)

- Light and heavy mesons
- Leading and higher twist distributions

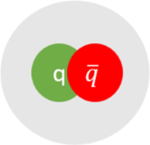
7

Ms. Navpreet Kaur (Research Scholar, JRF)

- Low lying octet baryons
- Leading twist distributions

NITJ Research group

- Explore the internal structure of mesons and baryons
- Meson sector: Pion, Kaon and D-mesons
- Baryon sector: Proton and other members of low-lying octet baryons



Work done so far

In meson sector:

- WDs, GPDs, GTMDs and TMDs at leading twist.
- The leading twist TMDs have been done in different light-front models
  - Light-front holographic model (LFHM)
  - AdS/QCD model
  - Light-cone quark model (LCQM)
- Extended this work to compute twist 4 TMDs for pions and kaons.

In baryon sector:

- GPDs at leading twist
- Twist 3: TMDs for proton
- Twist 4: TMDs, GPDs and GTMDs for proton
- Light-front models used
  - Light-front quark-diquark model (LFQDM)
  - Scalar diquark model

Outline

EIC for pion and kaon

- The EIC, with its high luminosity and wide kinematic range, offers an extraordinary new opportunity to increase our knowledge of the pion and kaon structure. The properties of pions and kaons provide clear windows onto emergent hadronic mass (EHM) and its modulation by Higgs-boson interactions.
- Measurements at the EIC can also address the structure of mesons. Specially, very detailed plans exist to explore pions.

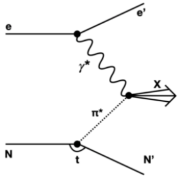


Diagram for the Sullivan process used to probe the structure of the pion.

Yellow paper EIC (2022)

Model description

- The leading twist T-even TMDs in LCQM is calculated with spin improved momentum space wave function along with all possible helicities

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\Big[\big((xM+m)((1-x)M+m) - \mathbf{k}_\perp^2\big)^2 + (M+2m)^2\Big]\frac{|\,\varphi(x,\mathbf{k}_\perp)\,|^2}{\omega_1^2\omega_2^2}.$$

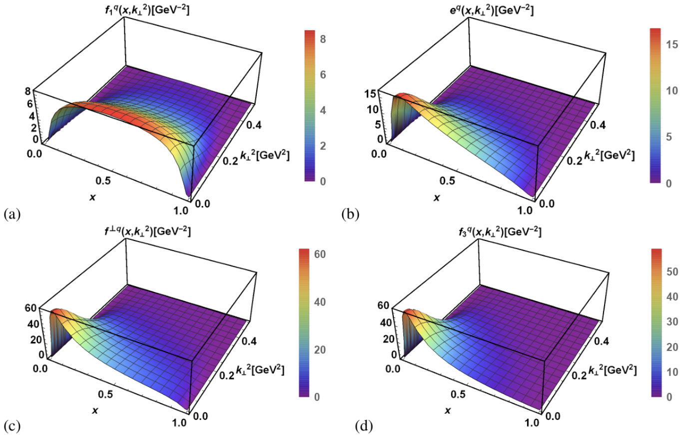
$\omega_1 = [(xM+m_q)^2 + \mathbf{k}_\perp^2]^{\frac{1}{2}}, \omega_2 = [((1-x)M + m_{\bar{q}})^2 + \mathbf{k}_\perp^2]^{\frac{1}{2}}$ and $\varphi(x, \mathbf{k}_\perp)$ being the momentum space wave function.

Model description

- In the LFHM, the leading twist TMD is calculated from the overlap form of quark angular momentum of states of mesons

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{N^2}{2\pi} \frac{[\mathbf{k}_\perp^2 + (m_q + x(1-x)M)^2]}{\kappa^2\, x^3\, (1-x)^3} \exp\left[-\frac{(\mathbf{k}_\perp^2 + (1-x)m_q^2 + xm_{\bar{q}}^2)}{\kappa^2 x(1-x)}\right].$$

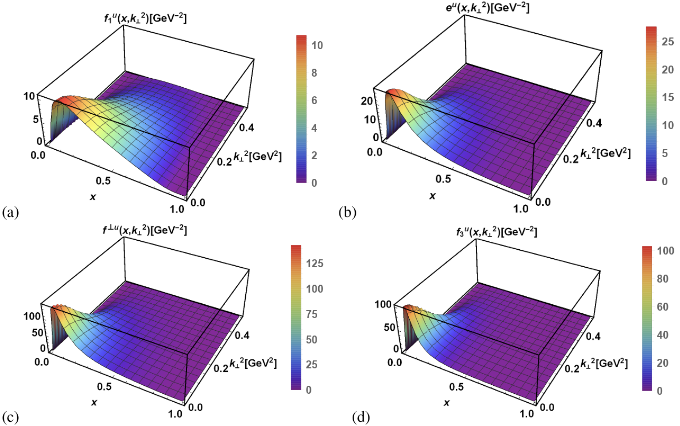
LCQM; Pion; u-Quark



$f_1^q \rightarrow$ twist 2 TMD, e^q and $f^\perp \rightarrow$ twist 3, $f_3 \rightarrow$ twist 4

Unpolarized TMDs

LCQM; Kaon; u-Quark



$$f_1^q \rightarrow \text{twist 2 TMD}, \; e^q \text{ and } f^\perp \rightarrow \text{twist 3}, \; f_3 \rightarrow \text{twist 4}$$

Unpolarized TMDs

Similarities between pion and kaon TMDs

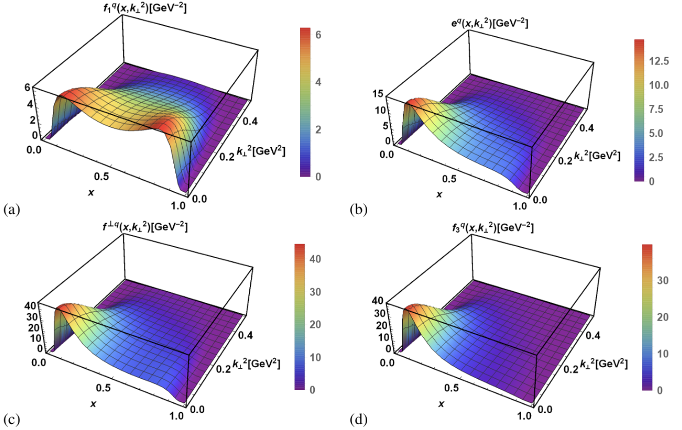
- Leading twist TMD spread over wide range of longitudinal momentum fraction.
- Higher twist TMDs have higher peak values than the leading twist TMDs.
- Higher twist TMDs carry low longitudinal momentum fraction than the leading twist TMDs.

Dissimilarities between pion and kaon TMDs

- For π
  - $\langle x_u \rangle = 0.5$, for leading order TMDs
  - $\langle x_u \rangle < 0.5$, for higher twist TMDs
- For K
  - $\langle x_u \rangle = 0.37$, for leading order TMDs
  - $\langle x_u \rangle < 0.37$, for higher twist TMDs

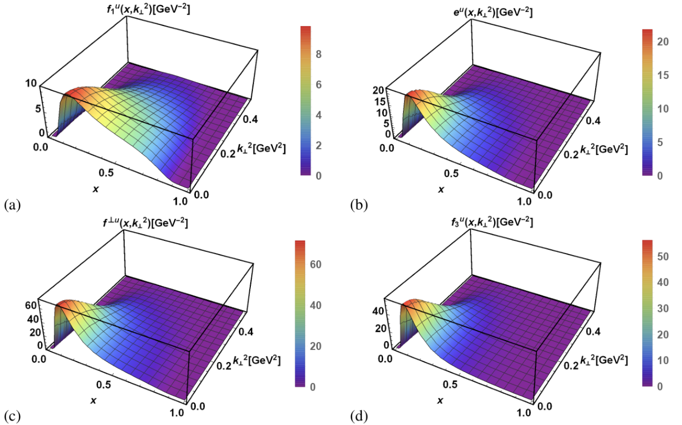
Unpolarized TMDs

LFHM; Pion; u-Quark



$f_1^q \rightarrow$ twist 2 TMD, e^q and $f^\perp \rightarrow$ twist 3, $f_3 \rightarrow$ twist 4

LFHM; Kaon; u-Quark



$f_1^q \to$ twist 2 TMD, e^q and $f^\perp \to$ twist 3, $f_3 \to$ twist 4

Unpolarized TMDs

Difference in the model dependent results

- In LFHM, leading twist TMD has a double peaked structure due to the combination of $L_z = 0$ and $L_z = 1$ states.
- Results of both models give positive distributions. NJL model has few negative TMD distributions.
- The leading twist T-even TMDs in LCQM is calculated with spin improved momentum space wave function along with all possible helicities

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\Big[((xM + m)((1 - x)M + m) - \mathbf{k}_\perp^2)^2 + (M + 2m)^2\Big]\frac{|\,\varphi(x, \mathbf{k}_\perp)\,|^2}{\omega_1^2\omega_2^2}.$$

With $\omega_1 = [(xM + m_q)^2 + \mathbf{k}_\perp^2]^{\frac{1}{2}}, \omega_2 = [((1-x)M + m_{\bar{q}})^2 + \mathbf{k}_\perp^2]^{\frac{1}{2}}$ and $\varphi(x, \mathbf{k}_\perp)$ being the momentum space wave function.

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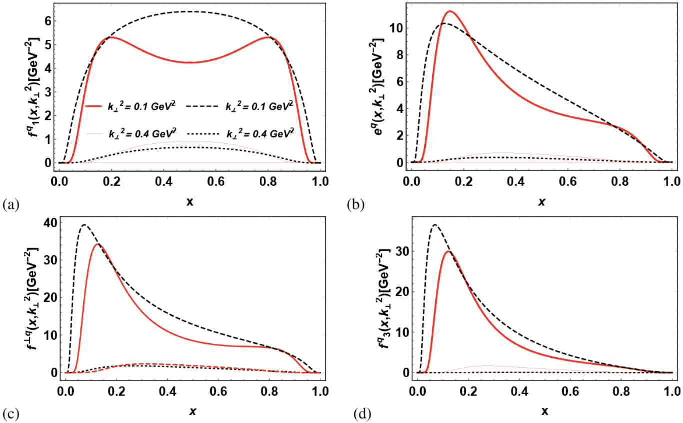
- While in LFHM, the leading twist TMDs is calculated from the overlap form of quark angular momentum of states of mesons

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{N^2}{2\pi} \frac{[\mathbf{k}_\perp^2 + (m_q + x(1-x)M)^2]}{\kappa^2\, x^3\, (1-x)^3} \exp\left[-\frac{(\mathbf{k}_\perp^2 + (1-x)m_q^2 + x m_{\bar{q}}^2)}{\kappa^2 x(1-x)}\right].$$

Pion

TMDs v/s x; fixed values of $k_\perp$

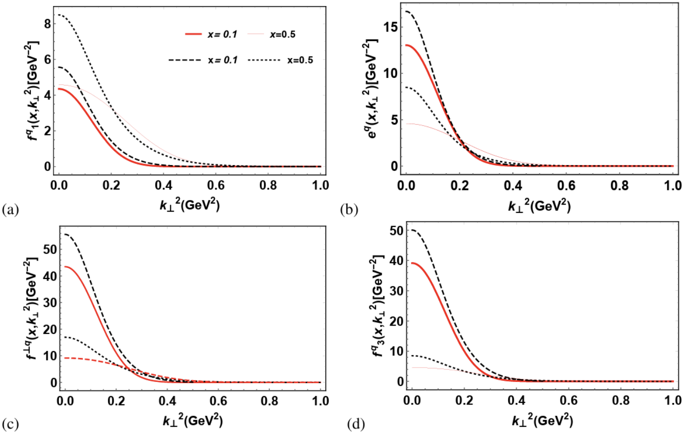


Solid line → LFHM, Dashed lines → LCQM

Comparison of Unpolarized TMDs

Pion

TMDs v/s $k_\perp$; fixed values of x



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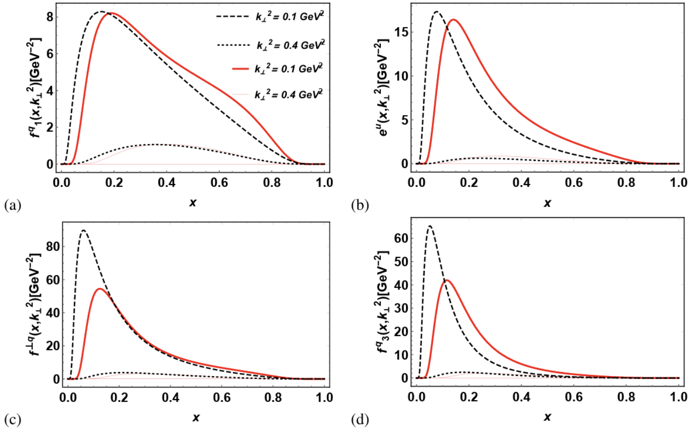
Kaon

TMDs v/s x; fixed values of $k_\perp$



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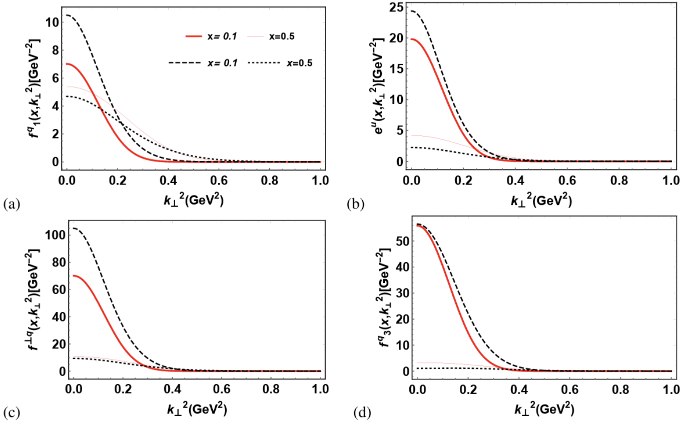
Comparison of Unpolarized TMDs

Kaon

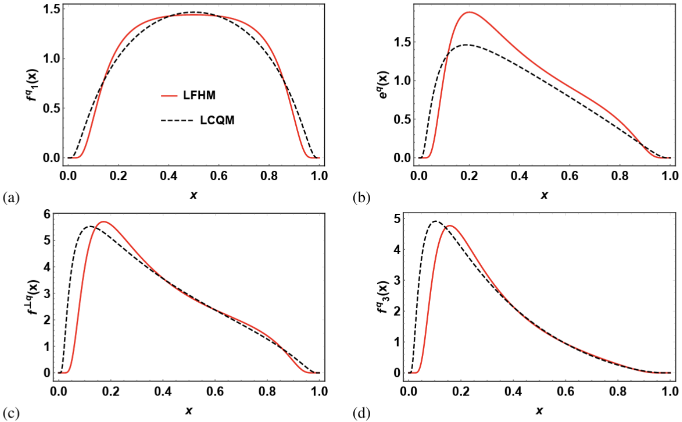
TMDs v/s $k_\perp$; fixed values of x



Solid line → LFHM, Dashed lines → LCQM

Comparison of PDFs

Pion



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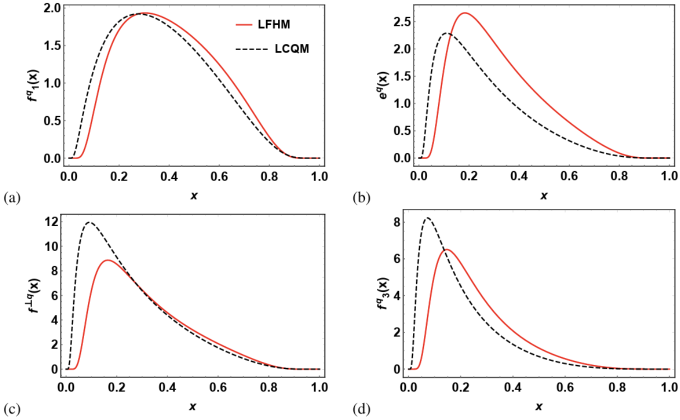
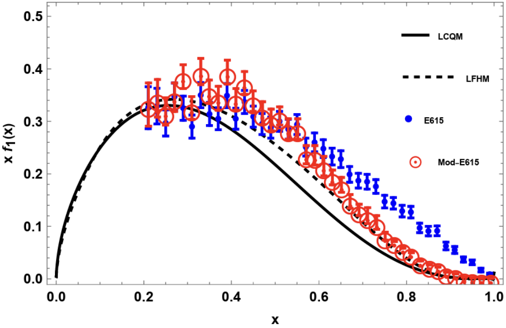
Kaon



Solid line → LFHM, Dashed lines → LCQM

Comparison of evolved PDFs with experimental data, FNAL-E615 and its modified data

Pion

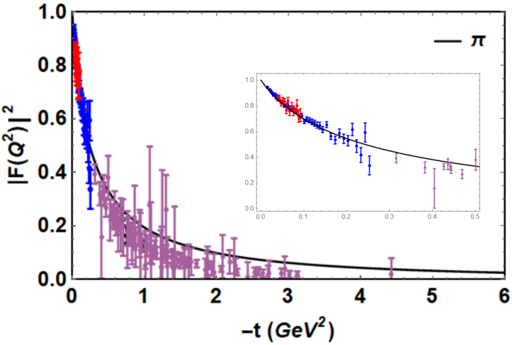


LCQM evolved from $0.19\,GeV^2$ to $16\,GeV^2$
LFHM evolved from $0.20\,GeV^2$ to $16\,GeV^2$

Comparison of form factor with experimental data and simulated data

(Evaluated via 1^{st} Mellin moment of GPD)

Pion



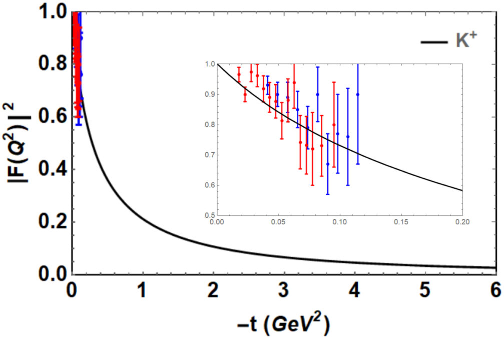
Data points-

NA7 Collaboration, FNAL, Jeefferson Lab Collaboration and Lattice data

Comparison of form factor with experimental data and simulated data

(Evaluated via 1^{st} mellin moment of GPD)

Kaon



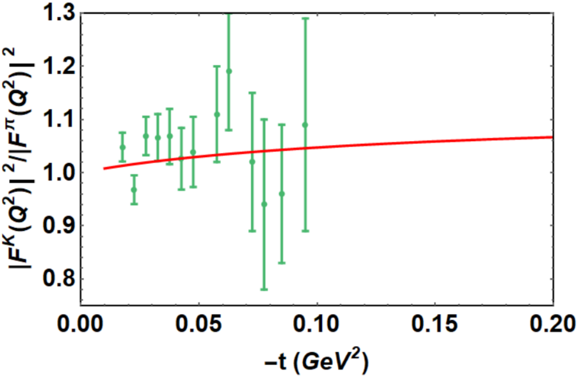
Data points-

PLB178(1986) and PRL 45(1980)

Comparison of form factor's ratio with experimental data

(Evaluated via 1^{st} mellin moment of GPD)

Kaon/Pion



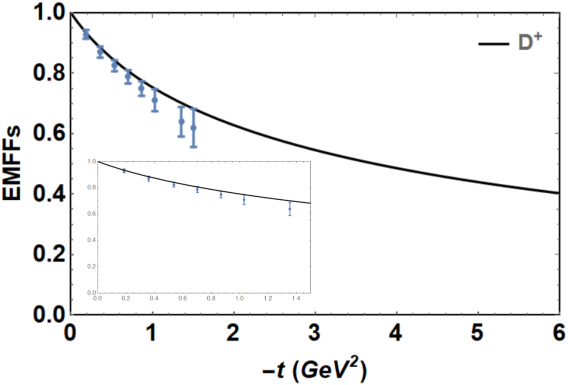
Data points-

PLB178(1986) and Our result

Comparison of form factor with simulated data

(Evaluated via 1^{st} mellin moment of GPD)

D^+



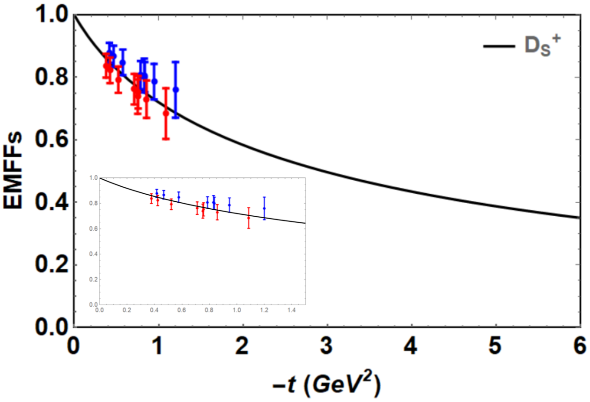
Data points-

Latt. (Can *et al.* (2013))

Comparison of form factor with simulated data

(Evaluated via 1^{st} mellin moment of GPD)

D_s^+



Data points-

Latt. B1 (Li and Wu)(2017) and Latt. C1 (Li and Wu)(2017)

Outline

- *Introduction:Internal Structure of the Hadrons*
- *Distribution functions*
  - Parton Distribution Functions (PDFs)
  - Generalized Parton Distributions (GPDs)
  - Transverse Momentum-Dependent Parton Distributions (TMDs)
- *Research Group at NIT Jalandhar*
- *Work done in the Meson Sector*
- *Work done in Baryon Sector*

Correlator

TMD Correlator

- The unintegrated quark-quark correlator in the light-front formalism for SIDIS is defined as

$$\Phi^{\nu[\Gamma]}_{[\Lambda^{N_i}\Lambda^{N_f}]}(x, \mathbf{p}_\perp; S) = \frac{1}{2}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ip.z} \langle P; \Lambda^{N_f} | \overline{\psi}^{\nu}(0)\Gamma \mathcal{W}_{[0,z]}\psi^{\nu}(z) | P; \Lambda^{N_i}\rangle \Big|_{z^+=0} .$$

- $|P; \Lambda^{N_i}\rangle$ and $|P; \Lambda^{N_f}\rangle$ are the initial and final states of the proton having momentum P with helicities Λ^{N_i} and Λ^{N_f}, respectively.

- S.Meissner, A. Metz and M. Schlegel, JHEP 08, 056 (2009)

Light-Front Quark-Diquark Model

- Proton is described as an aggregate of an active quark and a diquark spectator of definite mass.
- Proton has spin-flavor $SU(4)$ structure and expressed as a made up of isoscalar-scalar diquark singlet $|u\,S^0\rangle$, isoscalar-vector diquark $|u\,A^0\rangle$ and isovector-vector diquark $|d\,A^1\rangle$ states

$$|P;\pm\rangle = C_S|u\,S^0\rangle^{\pm} + C_V|u\,A^0\rangle^{\pm} + C_{VV}|d\,A^1\rangle^{\pm}.$$

Here, the scalar and vector diquark are denoted by S and A respectively. Isospin is represented by the superscripts on them.

- R.Jakob,P.J. Mulders and J. Rodrigues, Nucl. Phys. A 626, 937 (1997)
- A.Bacchetta, F. Conti and M. Radici, Phys. Rev. D 78, 074010 (2008)

Light-Front Quark-Diquark Model

- The light-cone convention $z^{\pm} = z^0 \pm z^3$ has been used.
- The momentum of the proton ($P^i = P^f = P$), struck quark (p) and diquark (P_X) are

$$
\begin{aligned}
P &\equiv \left(P^+, \frac{M^2}{P^+}, \mathbf{0}\right), \\
p &\equiv \left(xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right), \\
P_X &\equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp\right).
\end{aligned}
$$

Light-Front Quark-Diquark Model

$$|\nu\, S\rangle^{\Lambda^N} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\Bigg[\psi_{+}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|+\frac{1}{2},\ s; xP^+,\mathbf{p}_\perp\Big\rangle$$
$$+\psi_{-}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|-\frac{1}{2},\ s; xP^+,\mathbf{p}_\perp\Big\rangle\Bigg],$$
$$|\nu\, A\rangle^{\Lambda^N} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\Bigg[\psi_{++}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|+\frac{1}{2},\ +1; xP^+,\mathbf{p}_\perp\Big\rangle$$
$$+\psi_{-+}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|-\frac{1}{2},\ +1; xP^+,\mathbf{p}_\perp\Big\rangle+\psi_{+0}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|+\frac{1}{2},\ 0; xP^+,\mathbf{p}_\perp\Big\rangle$$
$$+\psi_{-0}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|-\frac{1}{2},\ 0; xP^+,\mathbf{p}_\perp\Big\rangle+\psi_{+-}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|+\frac{1}{2},\ -1; xP^+,\mathbf{p}_\perp\Big\rangle$$
$$+\psi_{--}^{\Lambda^N(\nu)}(x,\mathbf{p}_\perp)\Big|-\frac{1}{2},\ -1; xP^+,\mathbf{p}_\perp\Big\rangle\Bigg].$$

Light-Front Quark-Diquark Model

	λ_q	λ_{Sp}	LFWFs for $\Lambda^N = +1/2$	LFWFs for $\Lambda^N = -1/2$
Scalar	$+1/2$	0	$\psi_{+}^{+(\nu)}(x,\mathbf{p}_\perp) = N_S\ \varphi_1^{(\nu)}$	$\psi_{+}^{-(\nu)}(x,\mathbf{p}_\perp) = N_S\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$
	$-1/2$	0	$\psi_{-}^{+(\nu)}(x,\mathbf{p}_\perp) = -N_S\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{-}^{-(\nu)}(x,\mathbf{p}_\perp) = N_S\ \varphi_1^{(\nu)}$
Vector	$+1/2$	$+1$	$\psi_{++}^{+(\nu)}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{++}^{-(\nu)}(x,\mathbf{p}_\perp) = 0$
	$-1/2$	$+1$	$\psi_{-+}^{+(\nu)}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\ \varphi_1^{(\nu)}$	$\psi_{-+}^{-(\nu)}(x,\mathbf{p}_\perp) = 0$
	$+1/2$	0	$\psi_{+0}^{+(\nu)}(x,\mathbf{p}_\perp) = -N_0^{(\nu)}\sqrt{\frac{1}{3}}\ \varphi_1^{(\nu)}$	$\psi_{+0}^{-(\nu)}(x,\mathbf{p}_\perp) = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$
	$-1/2$	0	$\psi_{-0}^{+(\nu)}(x,\mathbf{p}_\perp) = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{-0}^{-(\nu)}(x,\mathbf{p}_\perp) = N_0^{(\nu)}\sqrt{\frac{1}{3}}\ \varphi_1^{(\nu)}$
	$+1/2$	-1	$\psi_{+-}^{+(\nu)}(x,\mathbf{p}_\perp) = 0$	$\psi_{+-}^{-(\nu)}(x,\mathbf{p}_\perp) = -N_1^{(\nu)}\sqrt{\frac{2}{3}}\ \varphi_1^{(\nu)}$
	$-1/2$	-1	$\psi_{--}^{+(\nu)}(x,\mathbf{p}_\perp) = 0$	$\psi_{--}^{-(\nu)}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$

Light-Front Quark-Diquark Model

- The flavor index $\nu = u$ (for the case of scalar) and $\nu = u, d$ (for the case of vector)).
- We have $|\lambda_q,\ \lambda_{Sp}; xP^+, \mathbf{p}_\perp\rangle$ representing the two particle state with quark helicity of $\lambda_q = \pm\frac{1}{2}$ and spectator diquark helicity of λ_{Sp}. The helicity of spectator for scalar diquark is $\lambda_{Sp} = \lambda_S = 0$ (singlet) and that for vector diquark is $\lambda_{Sp} = \lambda_D = \pm 1, 0$ (triplet).
- N_S, N_0 and N_1 are the normalization constants.
- Generic ansatz of LFWFs $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ is being adopted from the soft-wall AdS/QCD prediction and the parameters a_i^ν, b_i^ν and δ^ν are established as

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a_i^\nu}(1-x)^{b_i^\nu} \exp\left[-\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

TMD Parameterization for proton at twist-3

$$\Phi^{[1]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[e^{\nu}\left(x,\mathbf{p}_\perp^2\right) - \frac{i\sigma^{i+}p_T^i}{P^+}e_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[-\frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}e_T^{\nu}\left(x,\mathbf{p}_\perp^2\right) - i\sigma^{+-}\gamma_5 e_L^{\nu}\left(x,\mathbf{p}_\perp^2\right)\right] u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[\gamma^j]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{p_T^j}{M}f^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\,i\sigma^{j+}}{P^+}f_T^{\prime\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{k+}p_T^k}{M\,P^+}f_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{i\sigma^{ij}p_T^i}{M}f_L^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})\,,$$

TMD Parameterization for proton at twist-3

$$\Phi^{[\gamma^j\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\Big[\frac{i\epsilon_T^{ij}p_T^i}{M}g^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\, i\sigma^{j+}\gamma_5}{P^+}g_T'^{\nu}\left(x,\mathbf{p}_\perp^2\right)$$

$$+\frac{p_T^j\, i\sigma^{k+}\gamma_5 p_T^k}{M\,P^+}g_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\, i\sigma^{+-}\gamma_5}{M}g_L^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\Big]u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[i\sigma^{ij}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = -\frac{i\epsilon_T^{ij}}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\Big[-h^{\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{i\sigma^{k+}p_T^k}{P^+}h_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\Big]u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\Big[\frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}h_T^{\nu}\left(x,\mathbf{p}_\perp^2\right) + i\sigma^{+-}\gamma_5 h_L^{\nu}\left(x,\mathbf{p}_\perp^2\right)\Big]u(P,\Lambda^{N_i})\,.$$

Explicit Expressions of TMDs

For proton, the twist-3 TMDs can be given as

$$xe^{\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\frac{m}{M}\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big],$$

$$xf^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big],$$

$$xg_L^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{32\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(-\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\Big[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\Big],$$

$$xg_T^{\prime\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\Big)\frac{m}{M}\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big],$$

$$xg_T^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{8\pi^3}\Big(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\Big)\Big[\frac{1}{x}|\varphi_1^\nu||\varphi_2^\nu| - \frac{m}{x^2M}|\varphi_2^\nu|^2\Big],$$

S-Wave P-Wave D-Wave

Explicit Expressions of TMDs

$$xh_L^\nu(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(-\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\right)\right)\frac{1}{M}\left[m\left(|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right) + \frac{2p_\perp^2}{xM}|\varphi_1^\nu||\varphi_2^\nu|\right],$$

$$xh_T^\nu(x,\mathbf{p}_\perp^2) = \frac{1}{8\pi^3}\left(-C_S^2 N_s^2 + \frac{1}{3}C_A^2|N_0^\nu|^2\right)\left[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\right],$$

$$xh_T^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\right)\left[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right].$$

- ***S. Sharma, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)***

Equation of Motion

$$xe^q(x,\mathbf{p}_\perp^2) = x\tilde{e}^q(x,\mathbf{p}_\perp^2) + \frac{m}{M} f_1^q(x,\mathbf{p}_\perp^2),$$

$$xf^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{f}^{\perp q}(x,\mathbf{p}_\perp^2) + f_1^q(x,\mathbf{p}_\perp^2),$$

$$xg_L^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_L^{\perp q}(x,\mathbf{p}_\perp^2) + g_1^q(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_{1L}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xg_T'^q(x,\mathbf{p}_\perp^2) = x\tilde{g}_T'^q(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_1^q(x,\mathbf{p}_\perp^2) - \frac{m}{M}\frac{\vec{p}_T^{\,2}}{2M^2} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xg_T^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_T^{\perp q}(x,\mathbf{p}_\perp^2) + g_{1T}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$\frac{xh_L^q(x,\mathbf{p}_\perp^2)}{2} = x\tilde{h}_L^q(x,\mathbf{p}_\perp^2) - \frac{\vec{p}_T^{\,2}}{M^2} h_{1L}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} g_1^q(x,\mathbf{p}_\perp^2),$$

$$\frac{xh_T^q(x,\mathbf{p}_\perp^2)}{2} = x\tilde{h}_T^q(x,\mathbf{p}_\perp^2) - h_1^q(x,\mathbf{p}_\perp^2) - \frac{\vec{p}_T^{\,2}}{2M^2} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{2M} g_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xh_T^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{h}_T^{\perp q}(x,\mathbf{p}_\perp^2) + h_1^q(x,\mathbf{p}_\perp^2) - \frac{\vec{p}_T^{\,2}}{2M^2} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2).$$

Gaussian Ansatz

- TMD from Guassian Ansatz, average transverse momentum and Gaussian transverse dependence ratio is defined as

$$\Upsilon^{\nu}_{Gauss}(x, \mathbf{p}_\perp^2) = \frac{\Upsilon^\nu(x)}{\pi\langle \mathbf{p}_\perp^2(\Upsilon)\rangle^\nu}\, e^{\frac{-\mathbf{p}_\perp^2}{\langle \mathbf{p}_\perp^2(\Upsilon)\rangle^\nu}},$$

$$\langle \mathbf{p}_\perp^r(\Upsilon)\rangle^\nu = \frac{\int dx \int d^2p_\perp p_\perp^r \Upsilon^\nu(x, \mathbf{p}_\perp^2)}{\int dx \int d^2p_\perp \Upsilon^\nu(x, \mathbf{p}_\perp^2)},$$

$$R_G(\Upsilon)^\nu = \frac{2}{\sqrt{\pi}} \frac{\langle \mathbf{p}_\perp^1(\Upsilon)\rangle^\nu}{\langle \mathbf{p}_\perp^2(\Upsilon)\rangle^{\nu\,1/2}}.$$

-

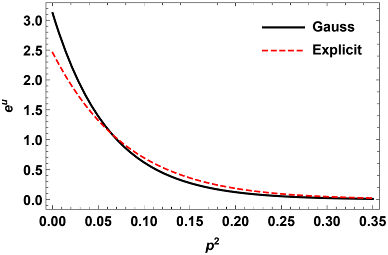
TMD Υ	e^u	$f^{\perp u}$	h_L^u	$g_L^{\perp u}$	$g_T^{\perp u}$	$g_T'^u$	$h_T^{\perp u}$	h_T^u
$\langle p_\perp\rangle^u$	0.22	0.22	0.28	0.28	0.20	0.22	0.22	0.28
$\langle p_\perp^2\rangle^u$	0.06	0.06	0.09	0.09	0.05	0.06	0.06	0.09
$R_G(\Upsilon)^\nu$	1.02	1.02	1.04	1.09	1.00	1.02	1.02	1.09

- *S. Sharma, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)*

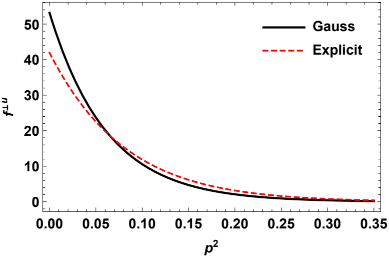
Key Points

- For u quark, TMDs which have Gaussian transverse dependence ratio R_G^{ν} less than 1.04 are successfully demonstrated by Gaussian factorization.
- There is no direct connection between the TMD associated waves being S, P and D with the applicability of Gaussian Ansatz.
- No direct connection between the equation of motion being quadratic in transverse momenta and the validity of Gaussian Ansatz.
- This approach successfully describes a vast body of data at leading twist and is useful for estimating the outcome of experimental measurements at higher twist.
- Factorization of collinear and transverse momentum dependence is certainly violated in full TMD evolution.

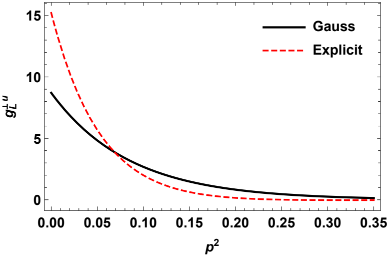
TMD vs $p_\perp^2$ at $x = 0.3$



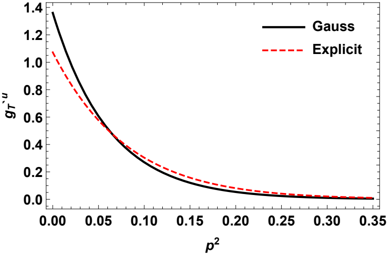
(a)



(b)

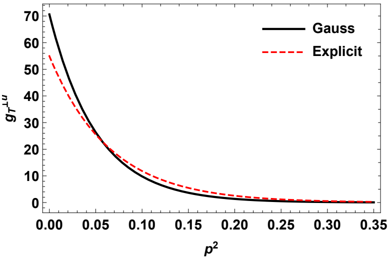


(c)

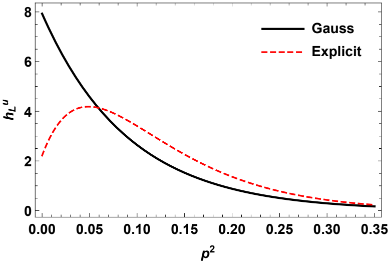


(d)

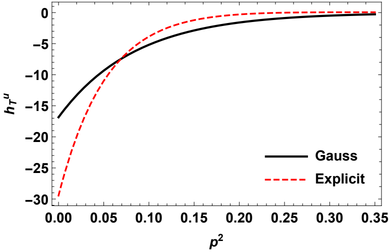
$TMD\ vs\ p_\perp^2\ at\ x = 0.3$



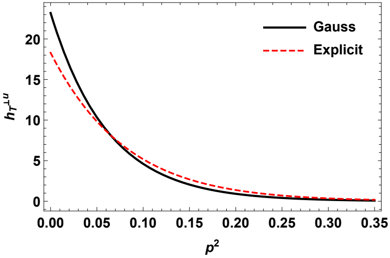
(e)



(f)

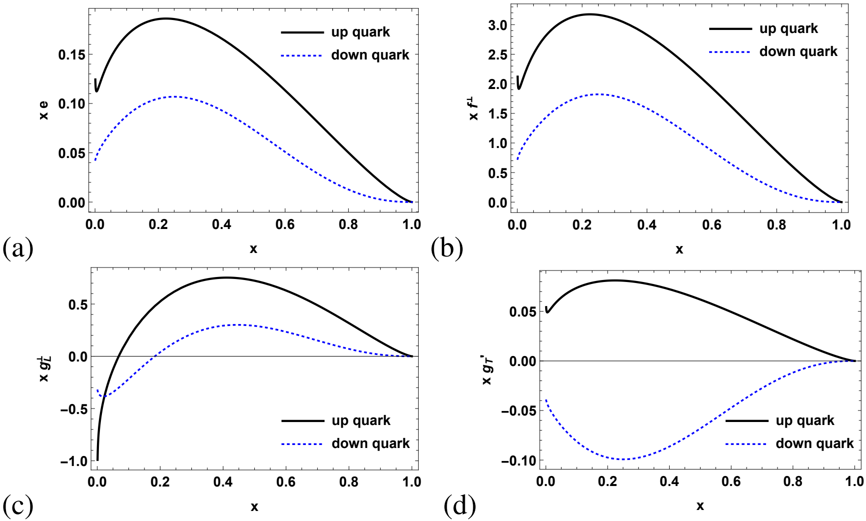


(g)



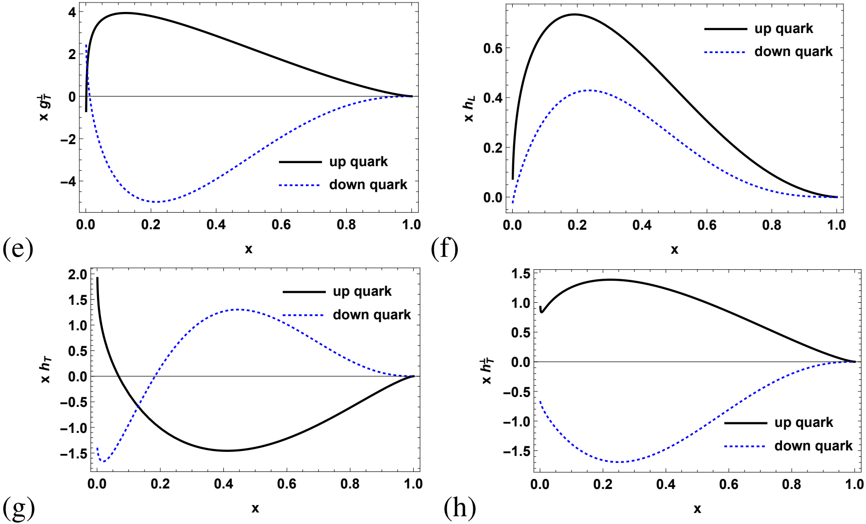
(h)

TMDPDF vs x



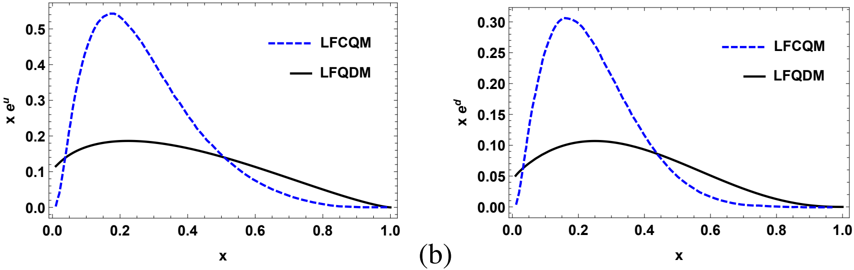
Plots of $xe^{\nu}(x)$, $xf^{\perp\nu}(x)$, $xg_L^{\perp\nu}(x)$ and $xg_T^{'\nu}(x)$ PDFs with respect to x.

TMDPDF vs x



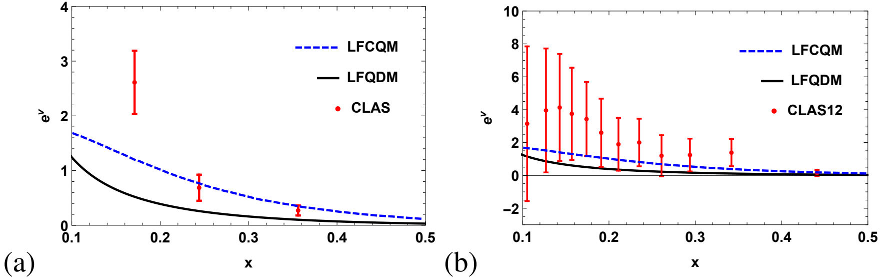
Plots of $xg_T^{\perp\nu}(x)$, $xh_L^\nu(x)$, $xh_T^\nu(x)$ and $xh_T^{\perp\nu}(x)$ PDFs with respect to x.

Comparison with LFCQM



(a) (b)

$xe^u(x)$ and $xe^d(x)$ (mass-term contribution only) plotted with respect to x for LFQDM (at model scale $0.09GeV^2$) and LFCQM (at model scale $1GeV^2$). The left and right column correspond to u and d quarks respectively.



Flavor combination $e^v(x)$ (mass-term contribution only) plotted with respect to x in LFQDM (evolved to $1GeV^2$) and LFCQM (at model scale $1GeV^2$) for CLAS and CLAS12 extracted data respectively.

LFCQM: S.Rodini and B.Pasquini, Nuovo Cim. C 42, 2-3, 112 (2019)
CLAS: A.Courtoy,arXiv: 1405.7659, HEP-PH (2014)
CLAS12: A.Courtoy, A. S. Miramontes, H. Avakian, M. Mirazita and S. Pisano, Phys. Rev. D 106, 014027 (2022)

Discussion

- CLAS data provides a good access to scalar PDF $e(x)$ by means of fragmentation functions (FFs) at both twist-2 and twist-3 level.
- Courtoy *et al.* extracted $e(x)$ from the beam spin asymmetry data from CLAS collaboration.
- The flavor combination $e^{v}(x) = \frac{4}{9}e^{u}(x) - \frac{1}{9}e^{d}(x)$ comparison of model results with the old CLAS data and updated CLAS12 data has been performed.
- This comparison corresponds to only Wandzura-Wilczek approximation and shows that model results are in good agreement at higher values of x. However, at lower values of x, the results differ in magnitude.
- The model results can further be improved by considering the contribution from the quark-gluon interaction term for $e(x)$.

S. Sharma, N. Kumar and H. Dahiya, Nucl. Phys. B 992, 116247 (2023)

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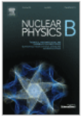
Nuclear Physics B
Volume 992, July 2023, 116247



High Energy Physics – Phenomenology

Sub-leading twist transverse momentum dependent parton distributions in the light-front quark-diquark model

Shubham Sharma [a], Narinder Kumar [a b], Harleen Dahiya [a]

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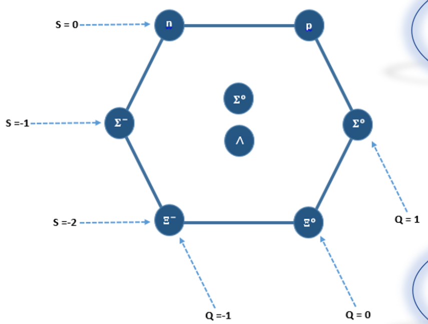
$Spin-\frac{1}{2}$; *Strange baryons*

p = uud
n = ddu

Ξ^{-} = uss
Ξ^{o} = dss

Λ = uds



$\Sigma^{+} = uus$
$\Sigma^{o} = uds$
$\Sigma^{-} = dds$

Quark content of isospin partners

Quark-scalar diquark model

Hadron state; Quark-Scalar diquark
on complete basis of Fock-states

- S.J.Brodsky, D. S. Hwang, B. Q. Ma and I. Schmidt, Nucl. Phys. B 593, 311 335 (2001).

$$|\psi_A(P^+, \mathbf{P}_\perp)\rangle = \sum_{\mathcal{N}} \prod_{i=1}^{\mathcal{N}} \frac{dx\, d^2\mathbf{k}_{\perp i}}{2(2\pi)^3 \sqrt{x_i}}\, 16\pi^3\, \delta\Big(1 - \sum_{i=1}^{\mathcal{N}} x_i\Big)\, \delta^{(2)}\Big(\sum_{i=1}^{\mathcal{N}} \mathbf{k}_{\perp i}\Big)$$
$$\times\, \psi_{\mathcal{N}}(x_i, \mathbf{k}_{\perp i}, \lambda_i) |\mathcal{N}; x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle\,,$$

- $\mathbf{k}_{\perp i}$- intrinsic transverse momentum
- λ_i- helicity

$J^z = +\frac{1}{2}$,

$$\psi^{\uparrow X}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \Big(M_X + \frac{m_q}{x}\Big)\varphi_X\,,$$
$$\psi^{\uparrow X}_{-\frac{1}{2}}(x, \mathbf{k}_\perp) = -\frac{(k^1 + \imath k^2)}{x}\varphi_X\,.$$

$J^z = -\frac{1}{2}$,

$$\psi^{\downarrow X}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \frac{(k^1 - \imath k^2)}{x}\varphi_X\,,$$
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$$\varphi_X = \varphi_X(x, \mathbf{k}_\perp) = \frac{\frac{g}{\sqrt{1-x}}}{M_X^2 - \frac{\mathbf{k}_\perp^2 + m_q^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda_n^2}{1-x}}\,.$$

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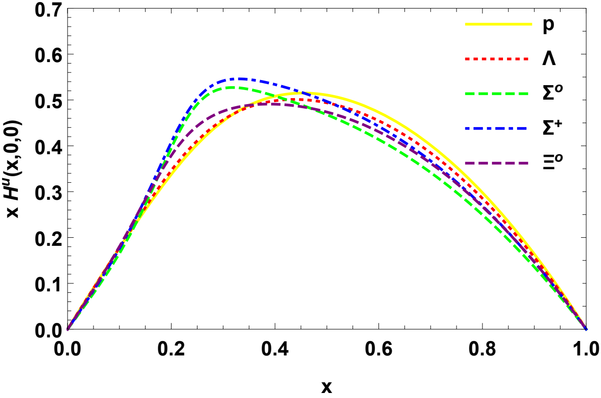
Unpolaized GPD; Active u-Quark

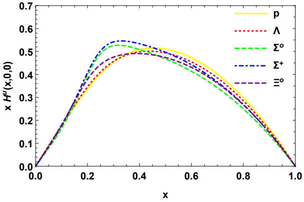


Capacity to carry longitudinal momentum fraction, $p > \Lambda > \Xi^o > \Sigma^+ > \Sigma^o$

Amplitudes, $\Sigma^+ > \Sigma^o > p > \Lambda > \Xi^o$

GPDs

Unpolaized GPD; Active u-Quark

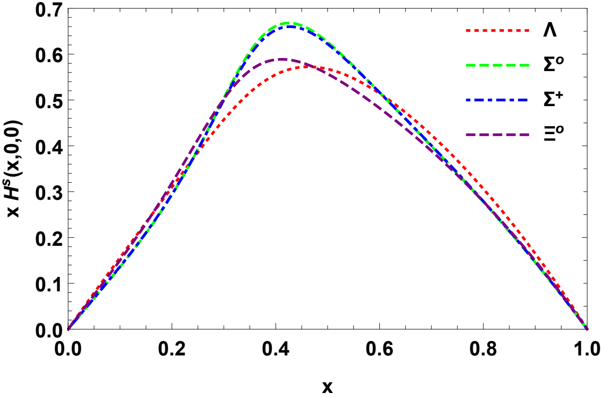


Capacity to carry longitudinal momentum fraction, $p > \Lambda > \Xi^o > \Sigma^+ > \Sigma^o$

Reason:

- Momentum carried by an object $\propto$ its mass
- Moving from $p \rightarrow \Lambda, \Sigma^+, \Sigma^o \rightarrow \Xi^o$, diquark becomes heavier and tends to carry more x

Unpolaized GPD; Active *s*-Quark

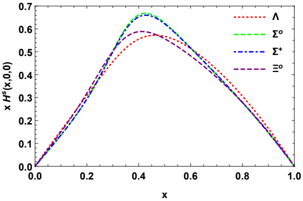


Capacity to carry longitudinal momentum fraction $\Lambda > \Sigma^o > \Sigma^+ > \Xi^o$

Amplitudes $\Sigma^o > \Sigma^+ > \Xi^o > \Lambda$

GPDs

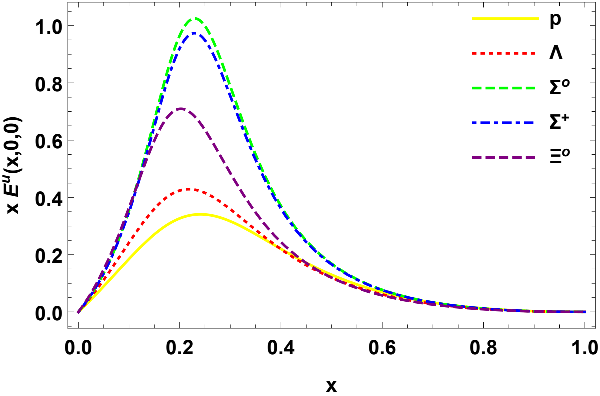
Unpolaized GPD; Active *s*-Quark



Comparison among an active *u* and *s* quarks

- Amplitude is more for an active *s* quark
- An active *s* quark carries smaller longitudinal momentum fraction

Transversely polaized quark in unpolarized baryon; GPD E; Active u-Quark

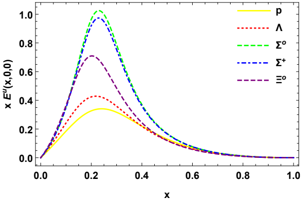


Capacity to carry longitudinal momentum fraction $p > \Sigma^o > \Sigma^+ > \Lambda > \Xi^o$

Amplitudes $\Sigma^o > \Sigma^+ > \Xi^o > \Lambda > p$

GPDs

Transversely polaized quark in unpolarized baryon; GPD E; Active u-Quark



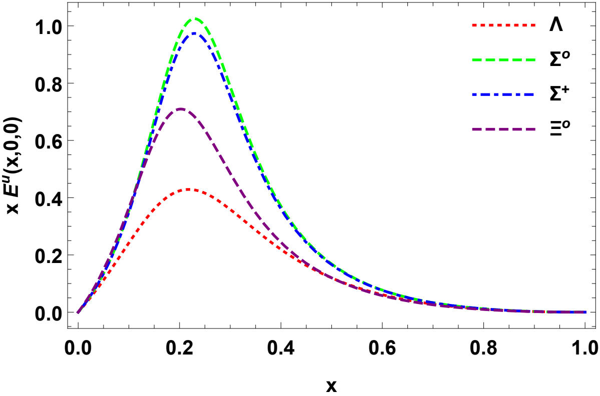
Amplitudes, $\Sigma^o > \Sigma^+ > \Xi^o > \Lambda > p$

Reason:

- GPD $E \propto$ baryon mass
- More the mass of baryon, more the amplitude
- Exceptional case: Ξ^o

GPDs

Transversely polaized quark in unpolarized baryon; GPD E; Active s-Quark

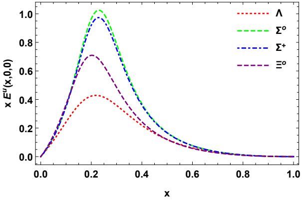


Capacity to carry longitudinal momentum fraction $\Sigma^o > \Sigma^+ > \Lambda > \Xi^o$

Amplitudes $\Sigma^o > \Sigma^+ > \Xi^o > \Lambda$

GPDs

Transversely polaized quark in unpolarized baryon; GPD E; Active s-Quark



Comparison among an active u and s quarks

- Amplitude is more for an active u quark
- An active s quark carries smaller longitudinal momentum fraction

Charge distribution

In coordinate space,
Charge distribution in overlap form

$$P_X^q(x, \mathbf{r}_\perp) = \Big[\tilde{\psi}^{\uparrow X*}_{+\frac{1}{2}}(x, \mathbf{r}_\perp)\,\tilde{\psi}^{\uparrow X}_{+\frac{1}{2}}(x, \mathbf{r}_\perp) + \tilde{\psi}^{\uparrow X*}_{-\frac{1}{2}}(x, \mathbf{r}_\perp)\,\tilde{\psi}^{\uparrow X}_{-\frac{1}{2}}(x, \mathbf{r}_\perp)\Big],$$

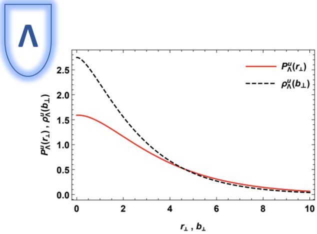
and its explicit expression,

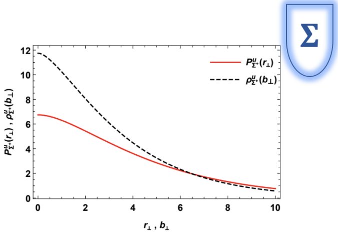
$$P_X^q(x, \mathbf{r}_\perp) = \frac{g^2 M_X^4}{(4\pi)^2}(1-x)^3\mathbf{r}_\perp^2\Big[\frac{(x\,M_X + m_q)^2}{\mathcal{M}_X}K_1^2\Big(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\Big) + K_0^2\Big(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\Big)\Big].$$

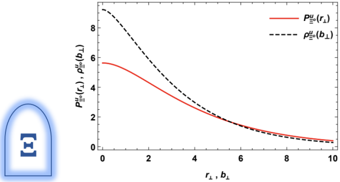
In impact parameter space,

$$\rho_X^q(x, b_\perp) = \int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\,e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}H_X^q(x, 0, \mathbf{\Delta}_\perp) = \frac{1}{(1-x)^2}\,P_X^q\Big(x, \frac{\mathbf{b}_\perp}{-1+x}\Big).$$

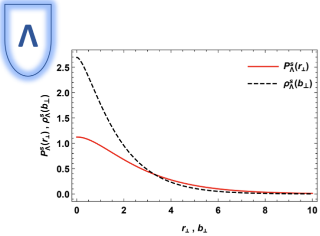
Active *u*-Quark

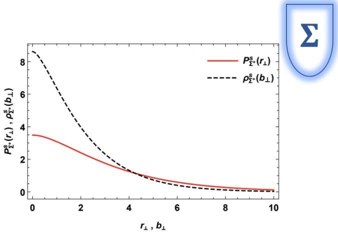


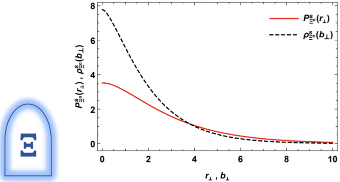




Active s-Quark







Charge distribution

Similarity among an active u and s quark,

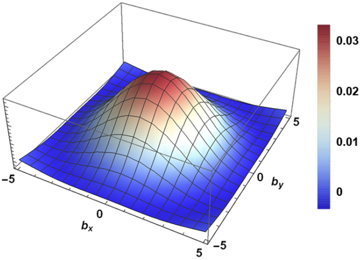
- Difference in $\mathbf{r}_\perp$ and $\mathbf{b}_\perp$ space arises due to the presence of $\frac{1}{-1+x}$ factor in the expression of charge distribution in space.

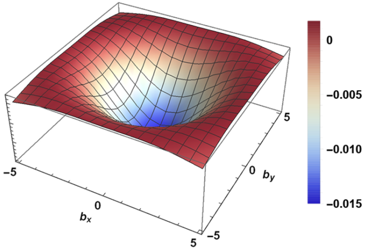
Dissimilarity among an active u and s quark,

- Smaller amplitude of an active s quark distributions $\Rightarrow$ Smaller contribution.
- Distributions fall rapidly wrt $\mathbf{r}_\perp$ ($\mathbf{b}_\perp$) $\Rightarrow$ Localized at smaller region.

Intrinsic magnetization moment distribution

$$\mu_M^{X_q}(\mathbf{b}_\perp) = \frac{e_q\hbar}{2M_X}\int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}\Big(F_{1X_q}(\mathbf{\Delta}_\perp)+F_{2X_q}(\mathbf{\Delta}_\perp)\Big).$$





Distributions of μ_M for an active u Quark and s Quark

Amplitudes $p > \Sigma^+ > \Sigma^o \approx \Lambda > \Xi^o$

Reason: Amplitude is inversely proportional to baryon mass.

Transverse distortion

- M.Burkardt and D.S.Hwang, Phys. Rev. D 69, 074032 (2004)

For a transversely polarized baryon

$$|Y\rangle = \frac{1}{\sqrt{2}}[|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + i|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle].$$

A non-zero spin-flip GPD $\Rightarrow$ transverse distortion

$$q_{\hat{y}}^{X_q}(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \left[H_X^q(x, 0, t) + i\frac{\mathbf{\Delta}_\perp^x}{2M} E_X^q(x, 0, t) \right]$$
$$= \mathcal{H}_X^q(x, \mathbf{b}_\perp) + \frac{1}{2M}\frac{\partial}{\partial b^x}\mathcal{E}_X^q(x, \mathbf{b}_\perp)\,.$$

Transverse distortion

- M.Burkardt and D.S.Hwang, Phys. Rev. D 69, 074032 (2004)

Left-right asymmetry $\Rightarrow$ Single spin asymmetry.
In accordance to "$SSA = GPD * FSI$"

$$\mathcal{P}_y^{X(q)} = \frac{1}{C}\left(i\left(\mathcal{A}_X^q(\Uparrow\to\uparrow)^* \mathcal{A}_X^q(\Downarrow\to\uparrow) - \mathcal{A}_X^q(\Uparrow\to\uparrow)\mathcal{A}_X^q(\Downarrow\to\uparrow)^*\right)\right.$$
$$\left. + \; i\left(\mathcal{A}_X^q(\Uparrow\to\downarrow)^* \mathcal{A}_X^q(\Downarrow\to\downarrow) - \mathcal{A}_X^q(\Uparrow\to\downarrow)\mathcal{A}_X^q(\Downarrow\to\downarrow)^*\right)\right),$$

where the normalization C for the unpolarized cross section is

$$C = |\mathcal{A}_X^q(\Uparrow\to\uparrow)|^2 + |\mathcal{A}_X^q(\Downarrow\to\uparrow)|^2 + |\mathcal{A}_X^q(\Uparrow\to\downarrow)|^2 + |\mathcal{A}_X^q(\Downarrow\to\downarrow)|^2.$$

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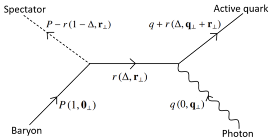
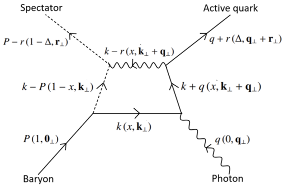
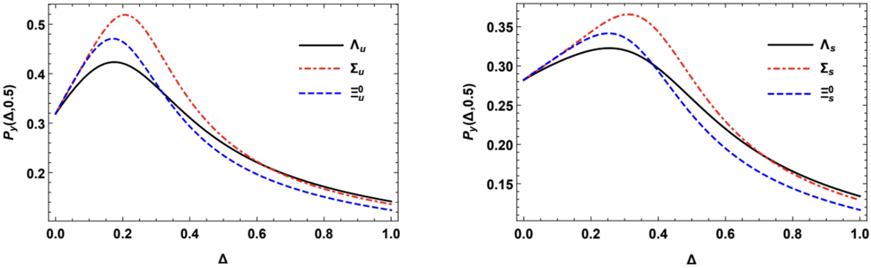




Single-spin asymmetry

Dependence of SSA on the quark light-front momentum fraction Δ

$$\mathcal{P}_y^{X(q)} = -\frac{e_1 e_2}{8\pi} \frac{2\left(\Delta\, M_X + m_q\right) r_1}{\left(\Delta\, M_X + m_q\right)^2 + \mathbf{r}_\perp^2} \left[\mathbf{r}_\perp^2 + \mathcal{M}_X^{un}(\Delta)\right] \frac{1}{\mathbf{r}_\perp^2} \ln \frac{\mathbf{r}_\perp^2 + \mathcal{M}_X^{un}(\Delta)}{\mathcal{M}_X^{un}(\Delta)}.$$



For u quark,

- SSA is recognizable at smaller values of quark light front momentum fraction (Δ).
- Σ with lighter diquark has the most SSA, gains a peak at $\Delta = 0.206$ and then decreases slowly.
- Peak comparatively at smaller value of $\Delta = 0.171$ and fast decrement has been observed for Ξ baryon due to the dominance of a heavy ss diquark.
- Smaller mass of Λ baryon attributes a decrement in the amplitude, shifting of a peak towards smaller value of $\Delta = 0.174$ and slow reduction with Δ.

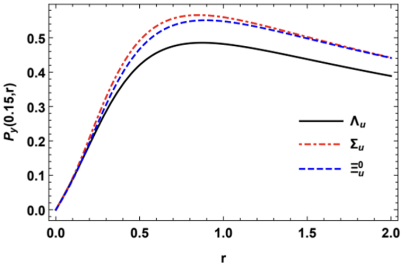
Single-spin asymmetry

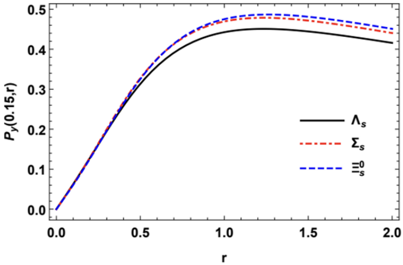
For s quark,

- Σ baryon on the top of the list with the maximum amplitude and a peak at $\Delta = 0.311$.
- Next, there is a Ξ^o baryon for which amplitude is reduced and peak shifted to Δ at 0.252 due to the effect of comparatively heavier diquark us than uu in Σ.
- At last, Λ with smallest amplitude and peak at $\Delta = 0.254$ because of its mass.

Dependence of SSA on the magnitude of an active quark momentum jet $\mathbf{r}_\perp$ relative to the virtual photon direction at $\Delta = 0.15$ momentum fraction Δ

$$\mathcal{P}_y^{X(q)} = -\frac{e_1 e_2}{8\pi} \frac{2\left(\Delta M_X + m_q\right) r_1}{\left(\Delta M_X + m_q\right)^2 + \mathbf{r}_\perp^2} \left[\mathbf{r}_\perp^2 + \mathcal{M}_X^{un}(\Delta)\right] \frac{1}{\mathbf{r}_\perp^2} \ln \frac{\mathbf{r}_\perp^2 + \mathcal{M}_X^{un}(\Delta)}{\mathcal{M}_X^{un}(\Delta)}.$$





Single-spin asymmetry

Dependence of SSA on the magnitude of an active quark momentum jet $\mathbf{r}_\perp$ relative to the virtual photon direction at $\Delta = 0.15$

- For u quark,
  - SSA in Σ shows minimal difference as compared to SSA in Ξ^o.
  - There is a significant difference from the value for Λ.
- For s quark
  - Small distinction among all three baryons with less amplitude of SSA.
  - Queued according to the decreasing order of the masses of baryons.

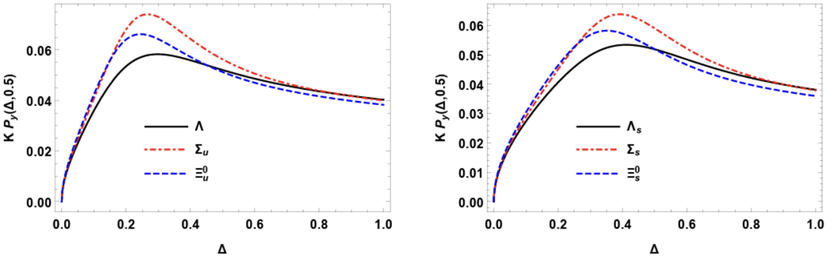
Single-spin asymmetry

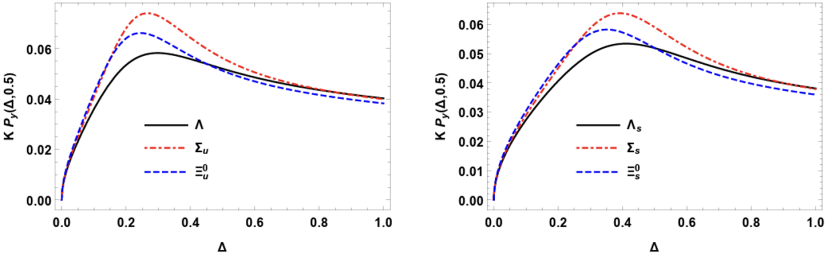
The transverse azimuthal spin asymmetry $\mathcal{P}_y$ can be compared with experimentally measured HERMES transverse asymmetry $A_{UT}^{\sin\phi}$ by introducing a kinematical factor

$$K = \frac{Q}{\nu}\sqrt{1-y} = \sqrt{\frac{2\,x\,M_X}{E}}\sqrt{\frac{1-y}{y}},$$

and the longitudinal asymmetry can be can be obtained as

$$A_{UL}^{\sin\phi} = K\,A_{UT}^{\sin\phi}.$$





Outcome: Queue of the strange baryons and the trend of peak positioning of the longitudinal SSA as a function of Δ is same as in the case of transverse SSA.

For $E = 27.6\ GeV$

(K. Ackerstaff *et al.* [HERMES], Nucl. Instrum. Meth. A 417, 230-265 (1998))

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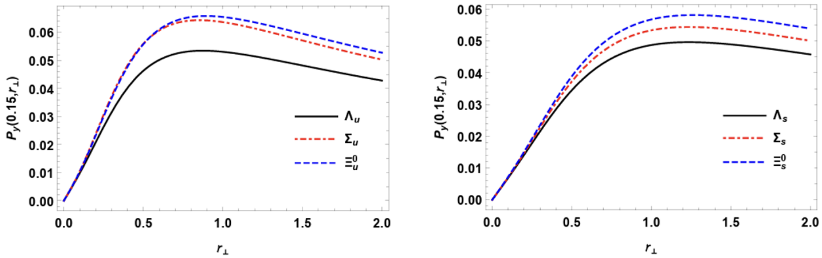
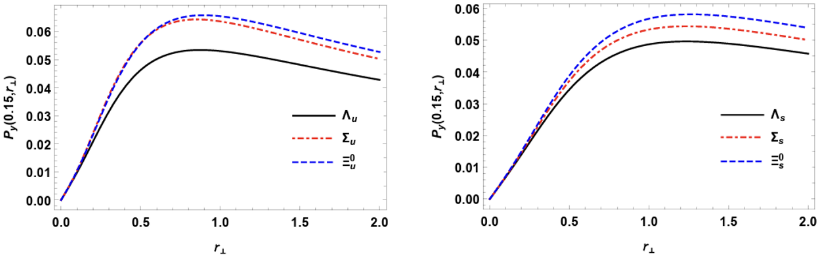


Single-spin asymmetry



Outcome: Decrement of the amplitude of longitudinal SSA as the mass of a baryon increases and this mass effect attributes to the presence of a baryon mass term in kinematical factor K.

Thank you!