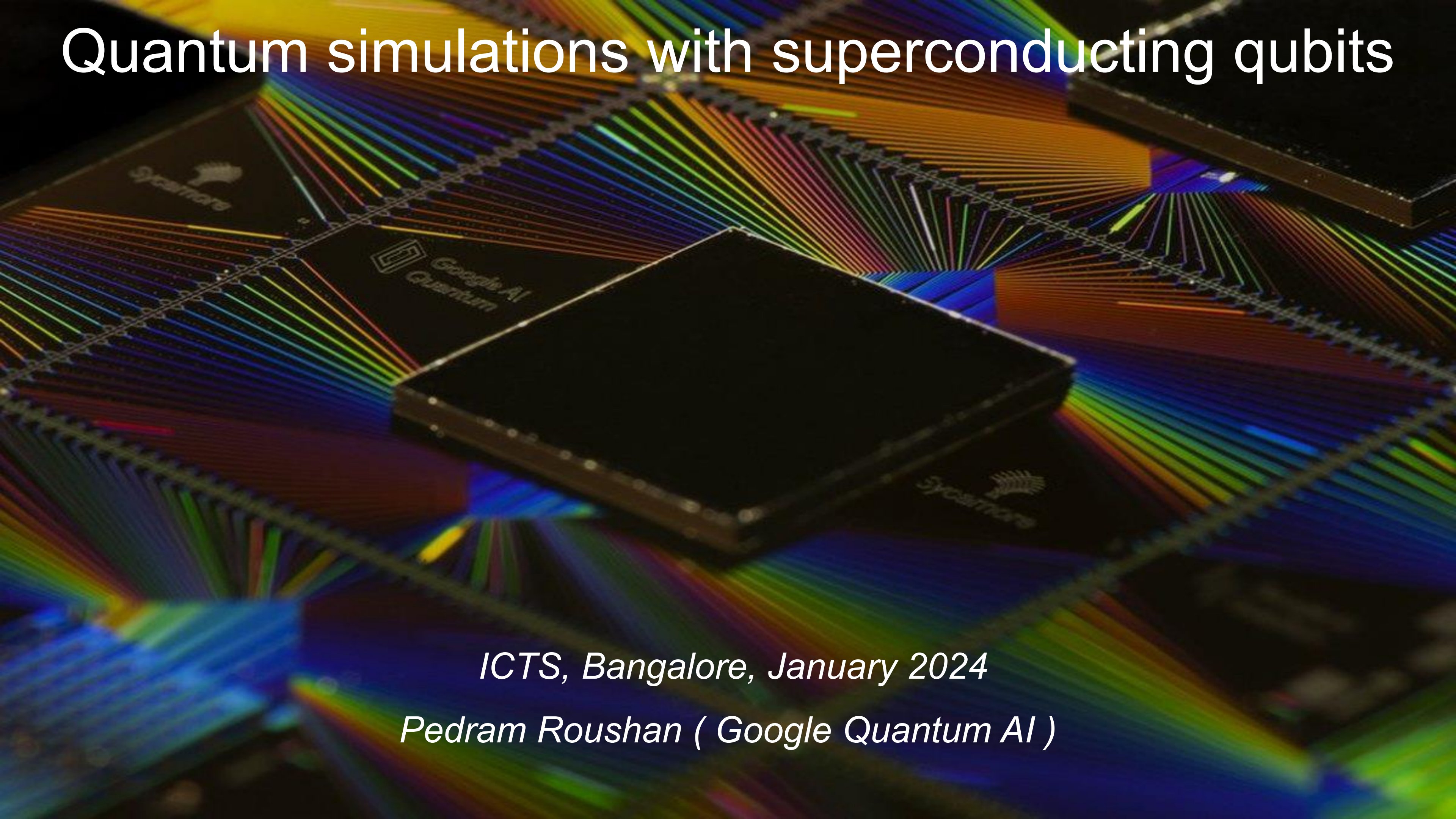
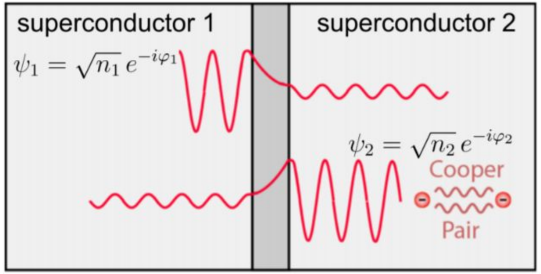
Quantum simulations with superconducting qubits



ICTS, Bangalore, January 2024

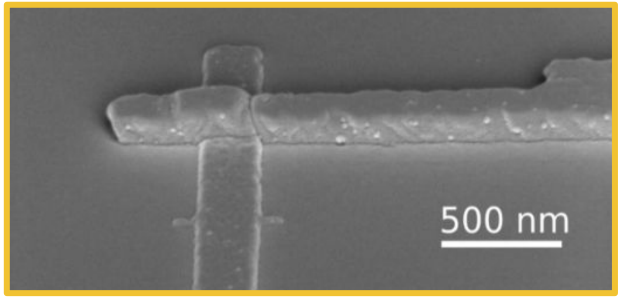
Pedram Roushan (Google Quantum AI)

Josephson junctions → non-linear inductors



$$I = I_0 \sin(\varphi)$$

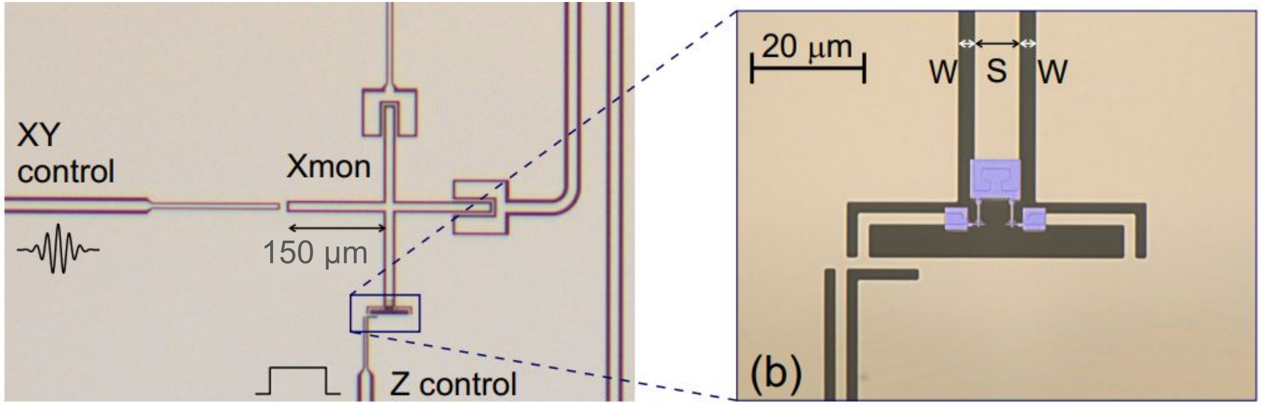
$$V = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt}$$

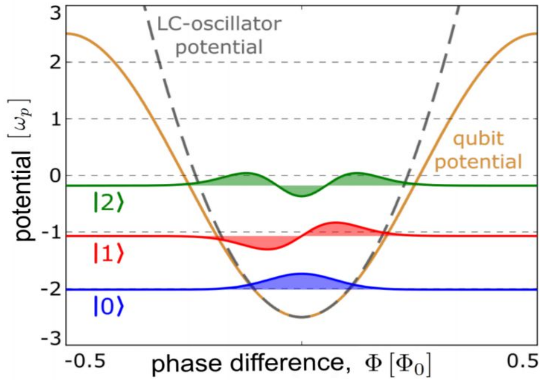


$$V = \frac{\Phi_0}{2\pi}\frac{1}{I_0 \cos(\varphi)}\frac{dI}{dt}$$

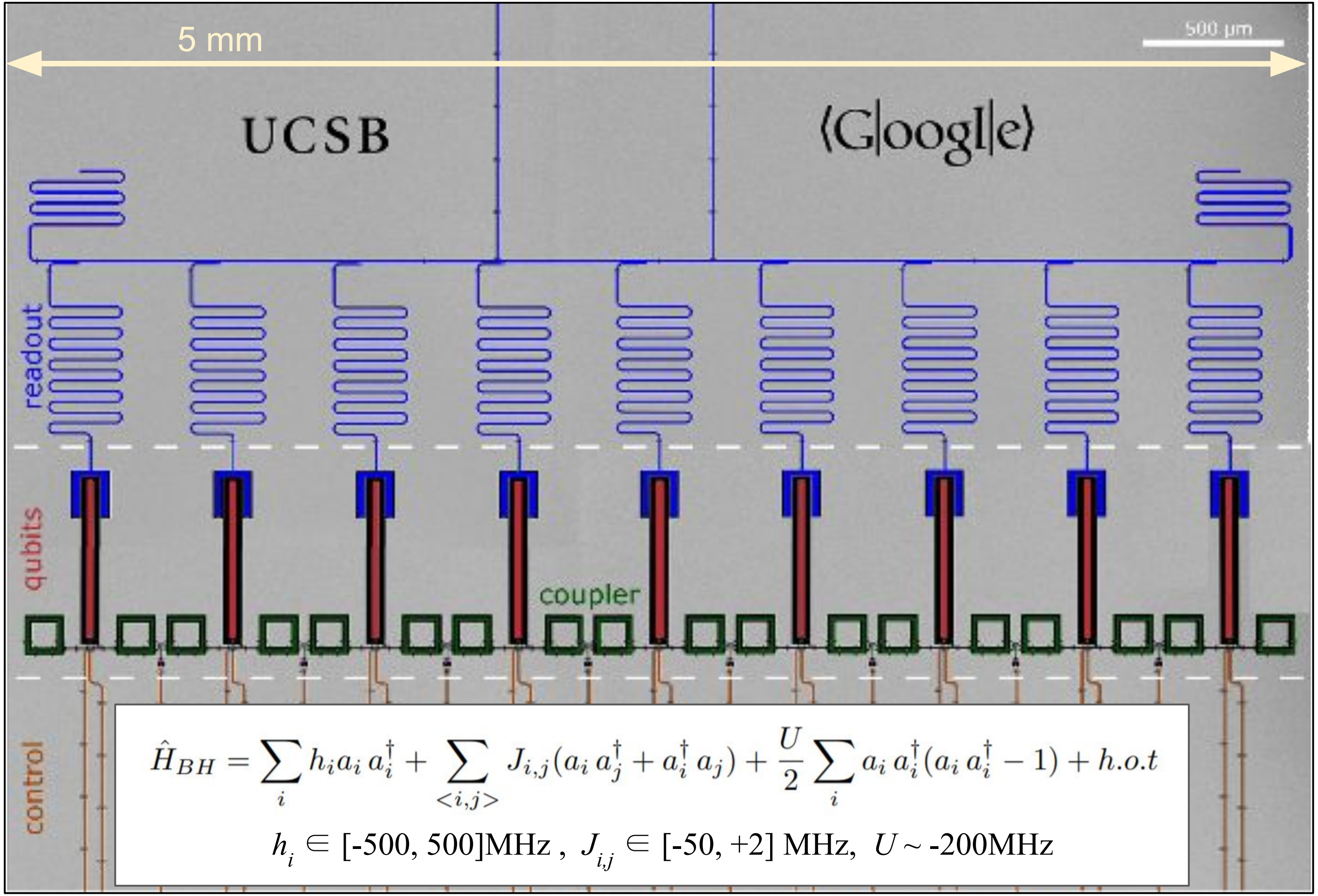
$$L_J \equiv \frac{\Phi_0}{2\pi}\frac{1}{I_0\, \cos(\varphi)}$$

Shunting with a capacitor→ non-linear resonator





Array of coupled non-linear resonators (→ qubits)



$$\hat{H}_{BH} = \sum_i h_i a_i a_i^\dagger + \sum_{<i,j>} J_{i,j}(a_i a_j^\dagger + a_i^\dagger a_j) + \frac{U}{2} \sum_i a_i a_i^\dagger (a_i a_i^\dagger - 1) + h.o.t$$

$h_i \in$ [-500, 500]MHz , $J_{i,j} \in$ [-50, +2] MHz, $U \sim$ -200MHz

Article

Formation of robust bound states of interacting microwave photons

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For a summary: Tomaž Prosen, News and Views, Nature 612, December 2022

Bound states in XXZ spin chain

Canonical 1D interacting XXZ Hamiltonian model:

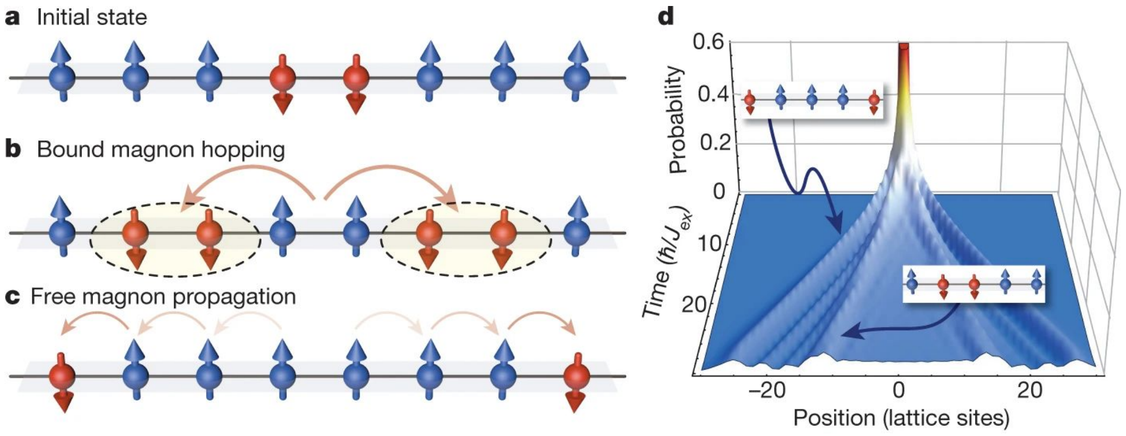
$$\mathcal{H} = \sum_i (X_i X_{i+1} + Y_i Y_{i+1}) + \Delta Z_i Z_{i+1}$$

- Analytical solution and Bound States

  H. Bethe, **Zeitschrift für Physik 71**, 205 (1931)

- Observation of Bound States in XXZ

  Ganahl, Rabel, Essler, Evertz **PRL 108,** 077206 (2012)
  T. Fukuhara *et al.* **Nature 502,** 76-79 (2013)



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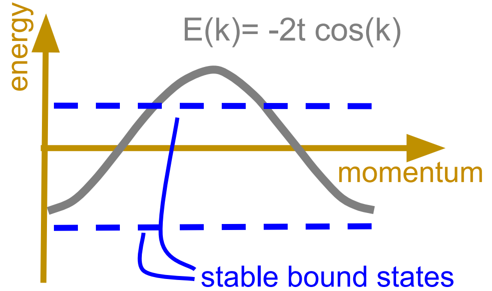
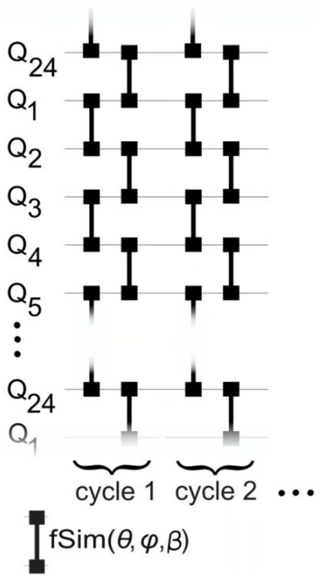
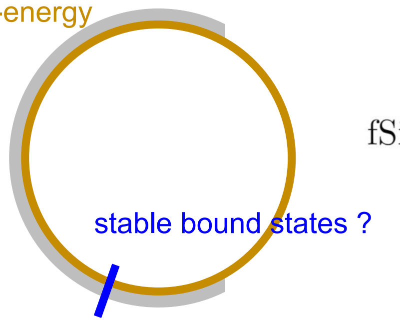




Circuit model: Floquet dynamic

$$\hat{U}_F = \prod_{\text{even bonds}} \text{fSim}(\theta, \phi, \beta) \prod_{\text{odd bonds}} \text{fSim}(\theta$$

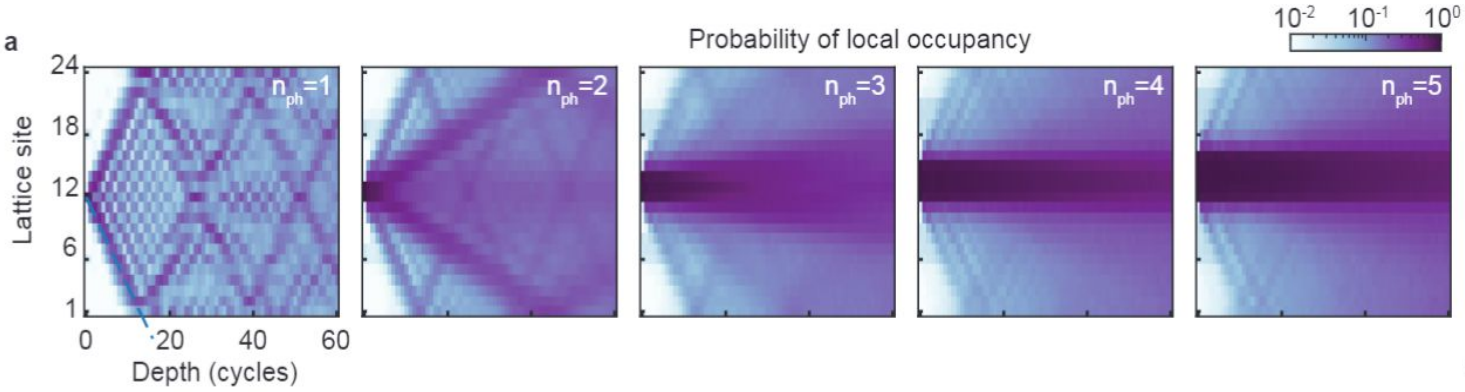
- Floquet XXZ is integrable
  M. Ljubotina *et al.* **PRL 122**, 150605 (2019)
- Analytical solution and Bound States
  I.L. Aleiner Annals of Physics **433**, 168593 (2021)

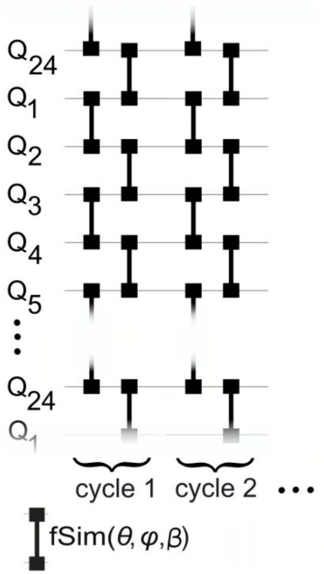


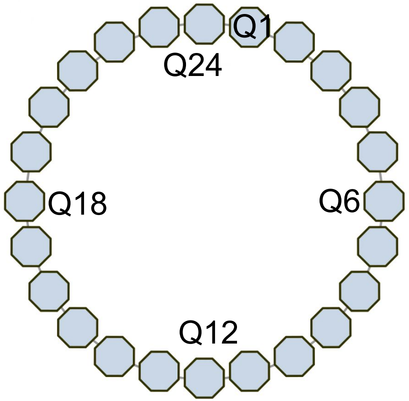
$$\text{fSim}(\theta, \phi, \beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$

kinetic (hopping)
interaction

Trajectory of Bound photons







$$\mathrm{fSim}(\theta,\phi,\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$

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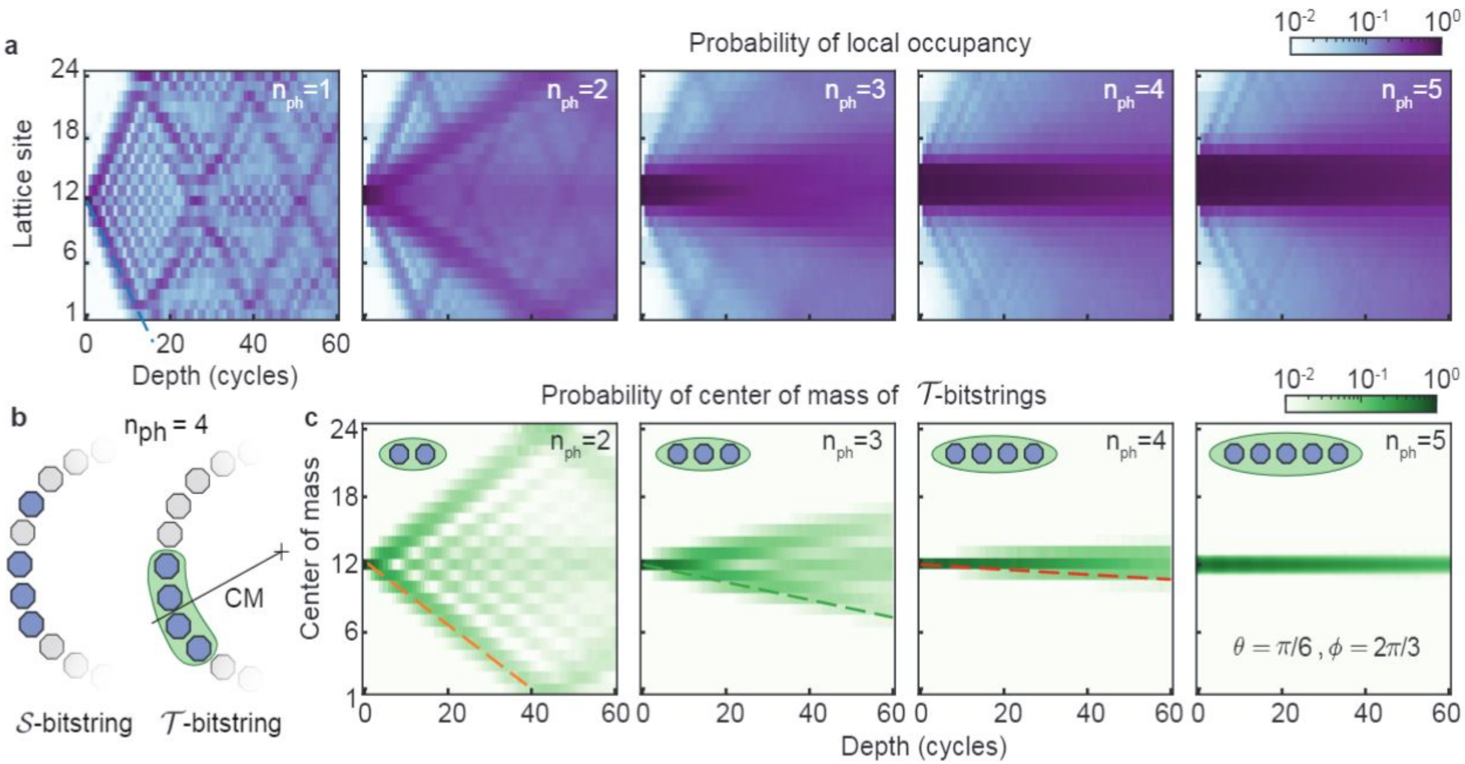
Trajectory of Bound photons



Examples: $\mathcal{T}$-bitsring : ...0000**1111**00000…

$\mathcal{S}$-bitsring: ...00**1**00**111**00000…

...00**0**00**11**00**11**000…

Band structure: few-body spectroscopy method

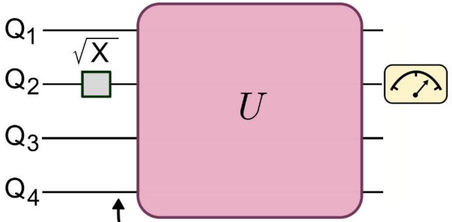
$$\hat{\mathcal{H}}\,|\varphi_n\rangle = \omega_n\,|\varphi_n\rangle$$

Consider an initial state $|\psi_0\rangle$ and its evolution $|\psi_t\rangle$

$$|\psi_0\rangle = \sum_n c_n\,|\varphi_n\rangle \rightarrow |\psi_t\rangle = e^{-i\hat{\mathcal{H}}t}\,|\psi_0\rangle = \sum_n c_n\,e^{-i\omega_n t}\,|\varphi_n\rangle$$

$$\langle\psi_0|\,\psi_t\rangle = \sum_n c_n\,\overline{c_n}\,e^{-i\omega_n t}$$

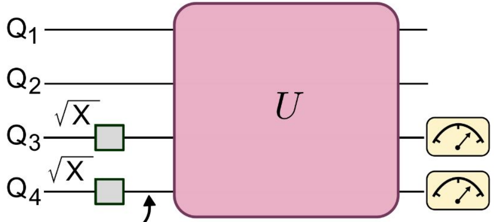
Green function



$$|\psi_0\rangle = |0000\rangle + |0100\rangle$$

$$\sigma_2^- = \hat{X}_2 + i\hat{Y}_2 = |0000\rangle\,\langle 0100|$$

$$+\,|....\rangle\,\langle\text{more than single excitation}|$$



$$|\psi_0\rangle = |0000\rangle + |0011\rangle + |0001\rangle + |0010\rangle$$

b (magnetic field)

$$\sigma_3^-\sigma_4^- = \hat{X}_3\hat{X}_4 - \hat{Y}_3\hat{Y}_4 + i\hat{X}_3\hat{Y}_4 + i\hat{Y}_3\hat{X}_4 = |0000\rangle\,\langle 0011|$$

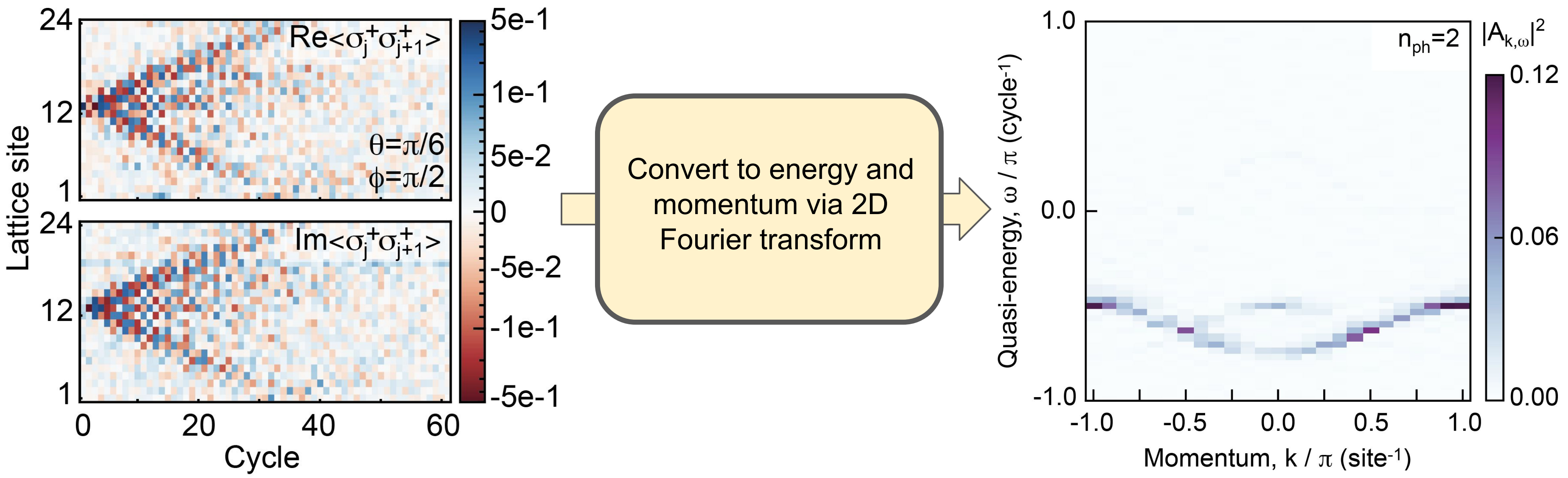
$$+\,|....\rangle\,\langle\text{more than two excitations}|$$

$$\langle C_{i,n_{\mathrm{ph}}}\rangle = \langle \Pi_{j=i}^{i+n_{\mathrm{ph}}-1}\sigma_j^+\rangle = \langle \Pi_{j=i}^{i+n_{\mathrm{ph}}-1}(X_j + iY_j)\rangle$$

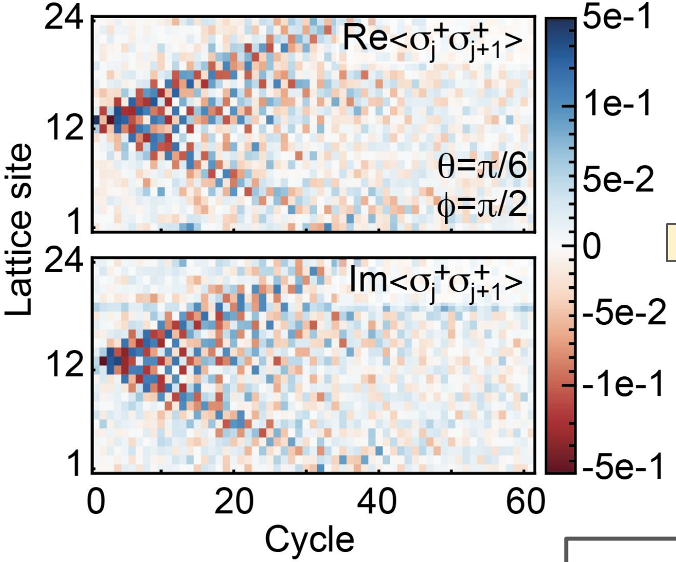
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes n} + \sum_k \alpha_k e^{-i\omega(k)t}\,|k\rangle\right)$$

$$\rightarrow \langle C_{j,n_{\mathrm{ph}}}\rangle = 1/(2\sqrt{n})\sum_k \alpha_k^* e^{i(\omega(k)t - kj)}$$

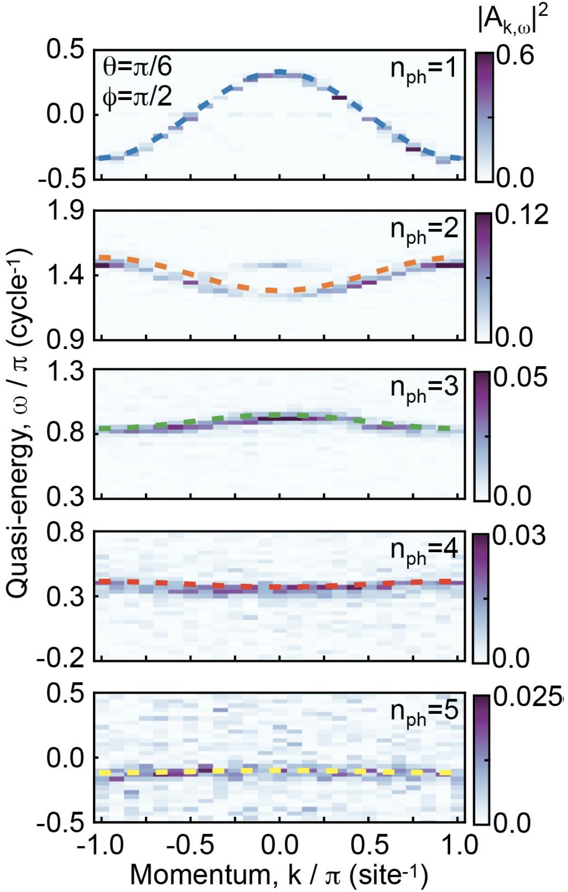
Measuring the bound state band structure



Measuring the bound state band structure



Convert to energy and momentum via 2D Fourier transform



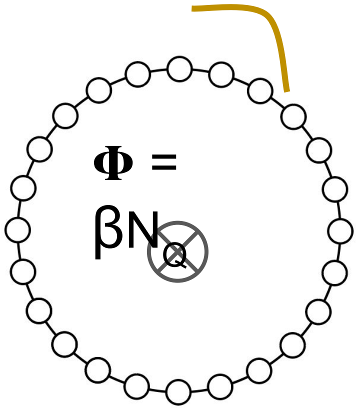
Analytical results:

$$\cos(E(k)-\chi)=\cos^2(\alpha)-\sin^2(\alpha)\cos(k)$$

Extraction of the bound state pseudo-charge

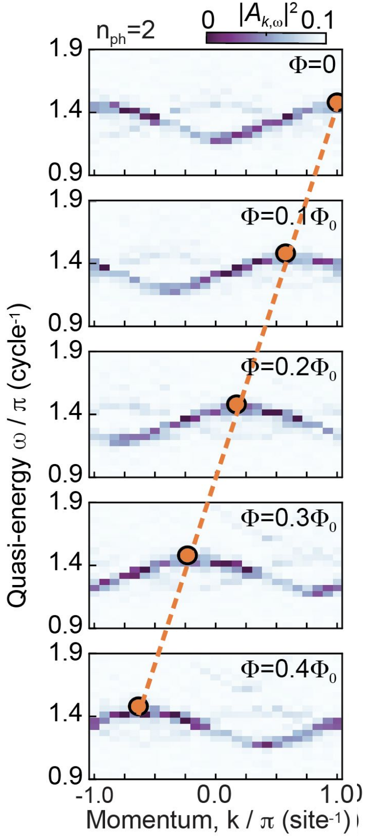
Include SQ rotation before and after fSim:

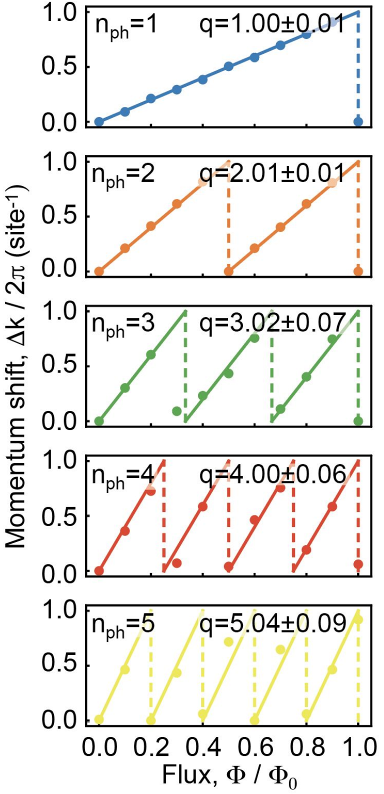
$$\mathrm{fSim}(\theta, \phi, \beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$



-Momentum shift!

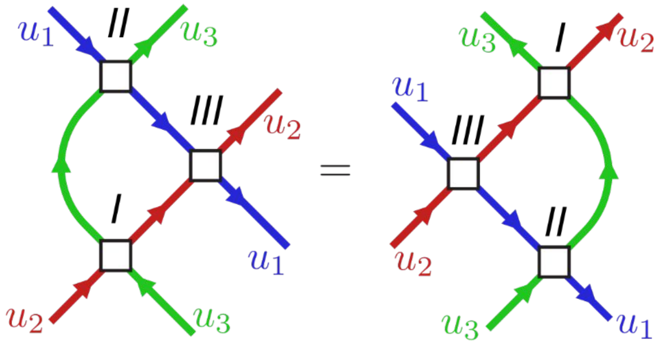
exp(-i*k*j)| j, n_{ph}) → exp(iβn_{ph} j)*exp(-i*k*j) | j, n_{ph})

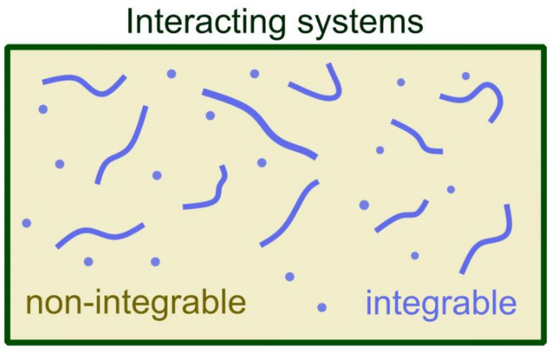


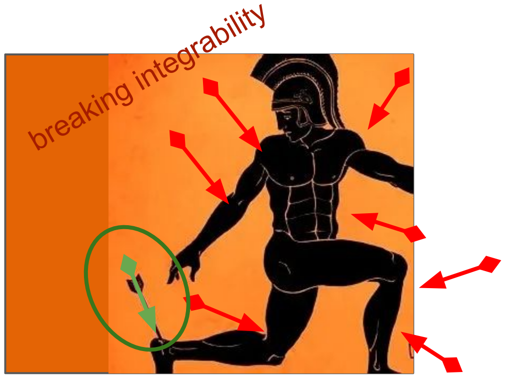


Interaction and integrability

Yang-Baxter relation:





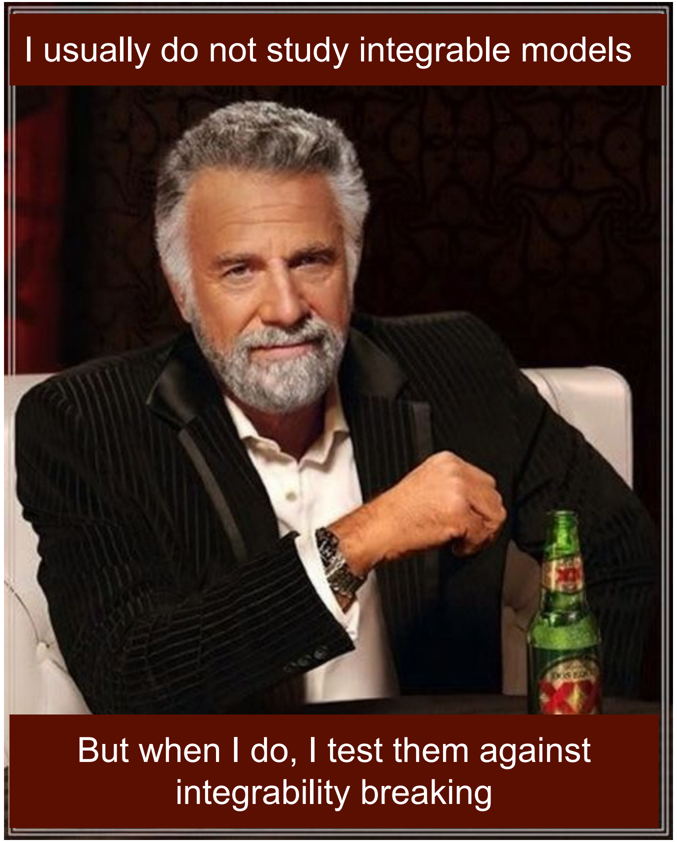


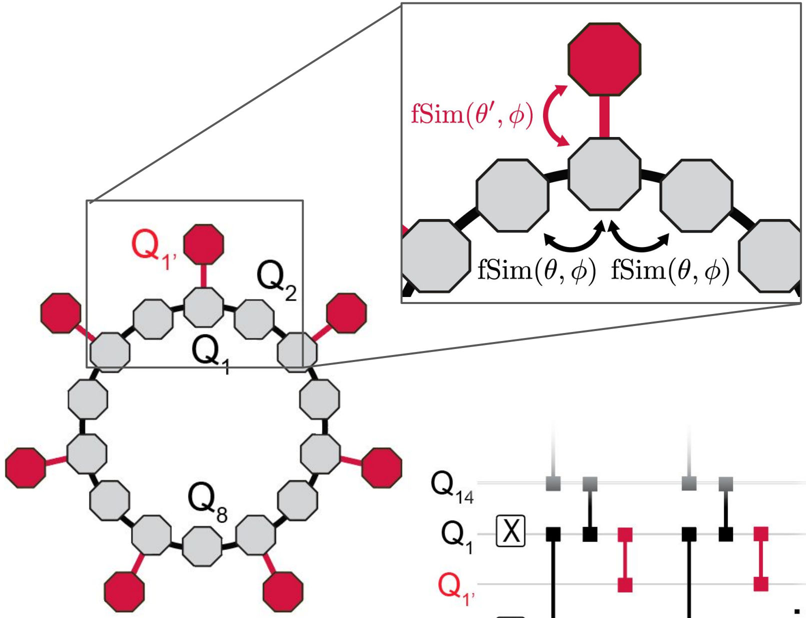
scattering order:

$I \rightarrow II \rightarrow III$ $\qquad$ $III \rightarrow II \rightarrow I$

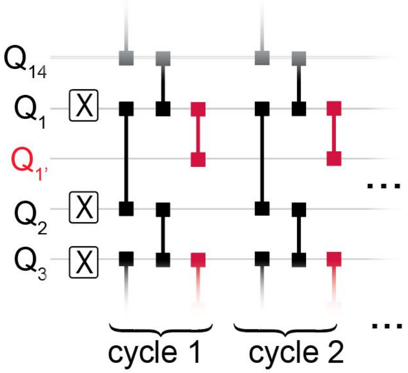
integrable : scattering is factorizable to $2 \rightarrow 2$ scattering processes

Breaking integrability continuously

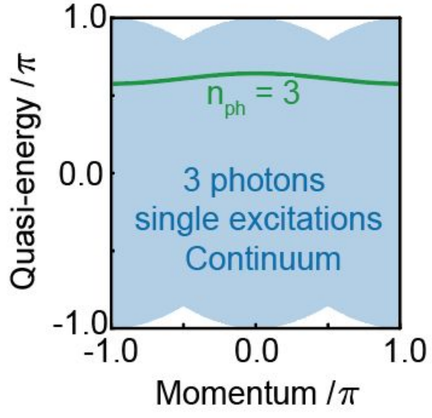


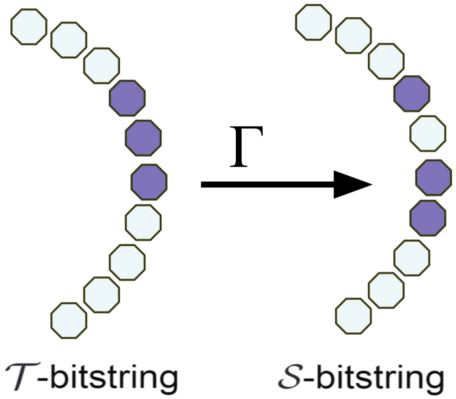


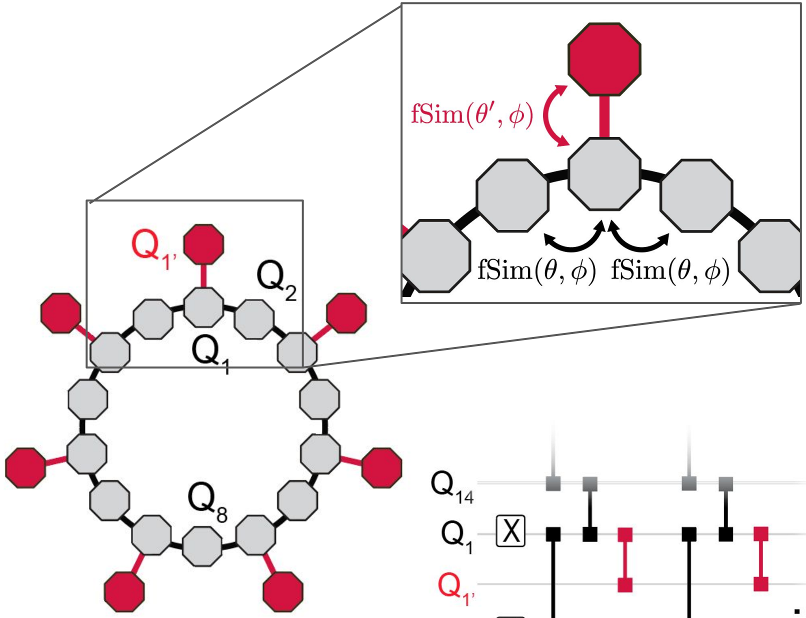
$$\frac{\binom{21}{3}}{\binom{14}{3}} = 3.65$$

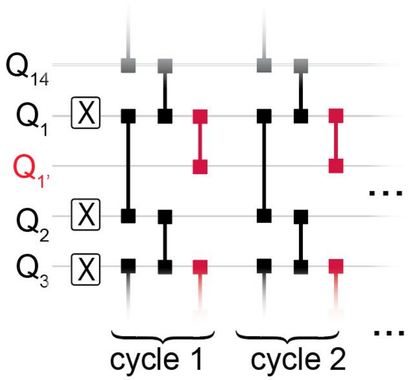


Breaking integrability continuously







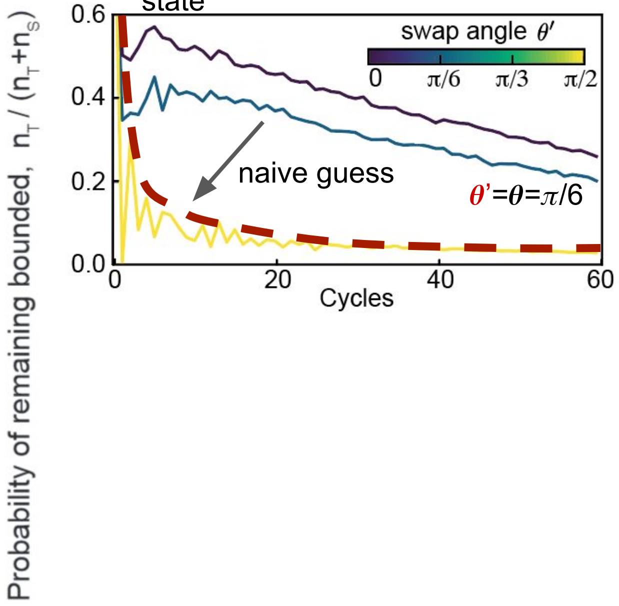


$$\Gamma = \frac{2\pi}{\hbar} \left|\langle BS|H'|Cont.\rangle\right|^2 \rho(E_f)$$

$$\frac{\binom{21}{3}}{\binom{14}{3}} = 3.65$$

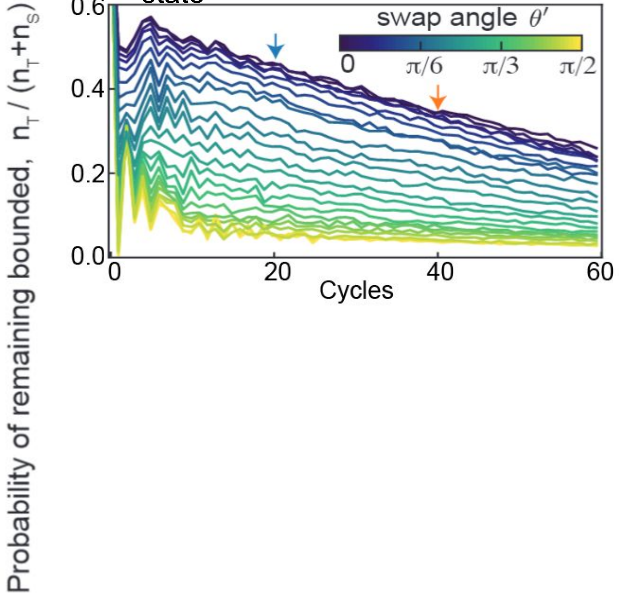
Breaking integrability continuously

Decay of a 3-photon bound state

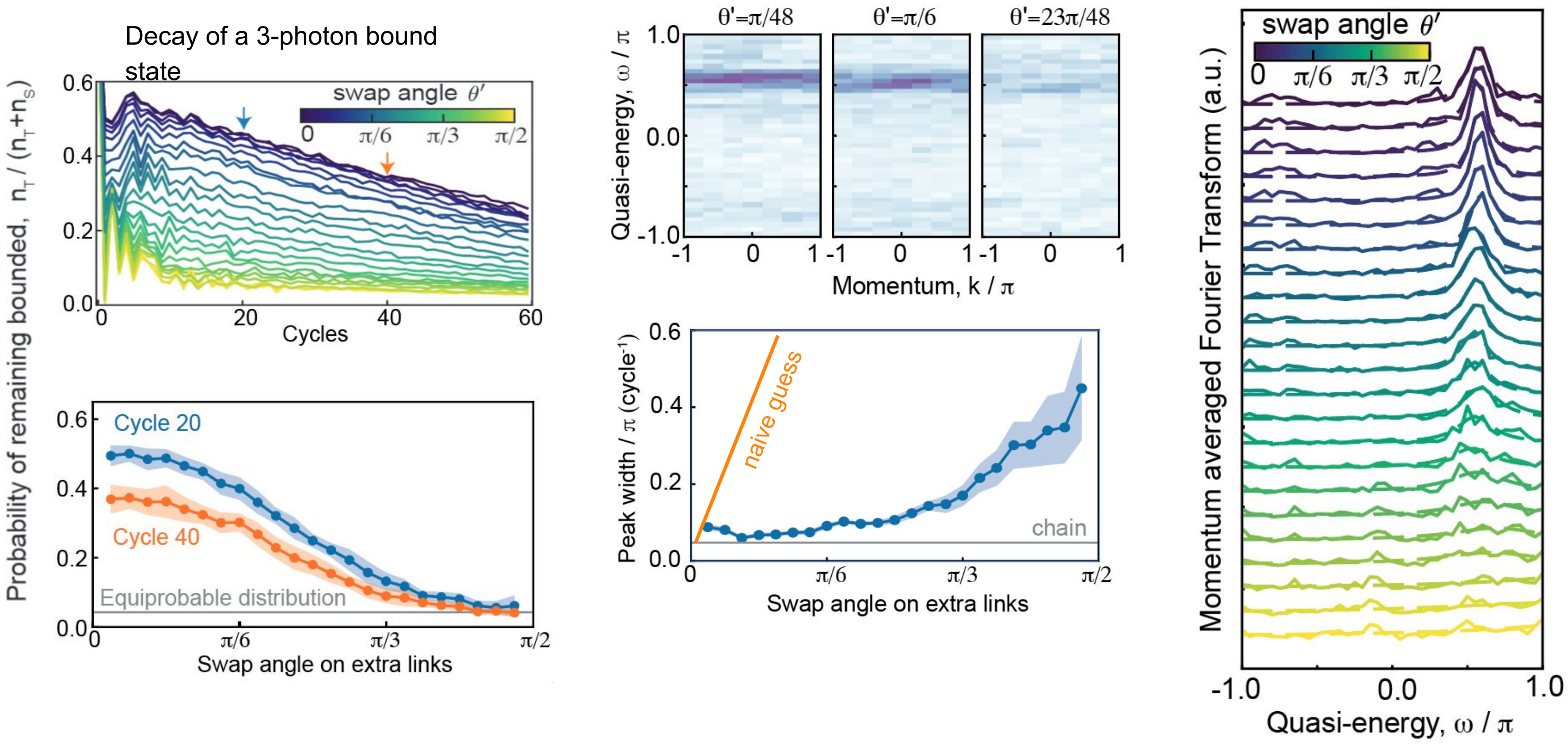


Unexpected resilience to integrability breaking

Decay of a 3-photon bound state



Breaking integrability continuously



Integrability breaking and bound states in Google's decorated XXZ circuits

Ana Hudomal,[1,2] Ryan Smith,[1] Andrew Hallam,[1] and Zlatko Papić[1]

[1] *School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[2] *Institute of Physics Belgrade, University of Belgrade, 11080 Belgrade, Serbia*

(Dated: July 26, 2023)

Recent quantum simulation by Google [Nature **612**, 240 (2022)] has demonstrated the formation of bound states of interacting photons in a quantum-circuit version of the XXZ spin chain. While such bound states are protected by integrability in a one-dimensional chain, the experiment found the

Large but dilute bound states continues to be robust.

Robustness and eventual slow decay of bound states of interacting microwave photons in the Google Quantum AI experiment

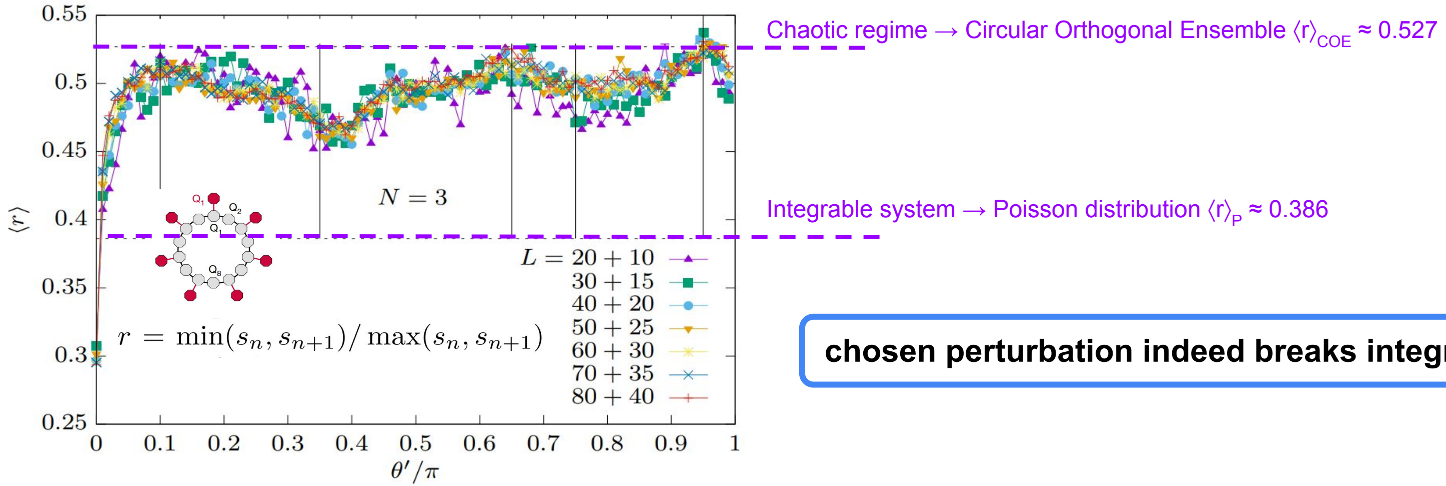
Federica Maria Surace[1] and Olexei Motrunich[1]

[1] *Department of Physics and Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA*

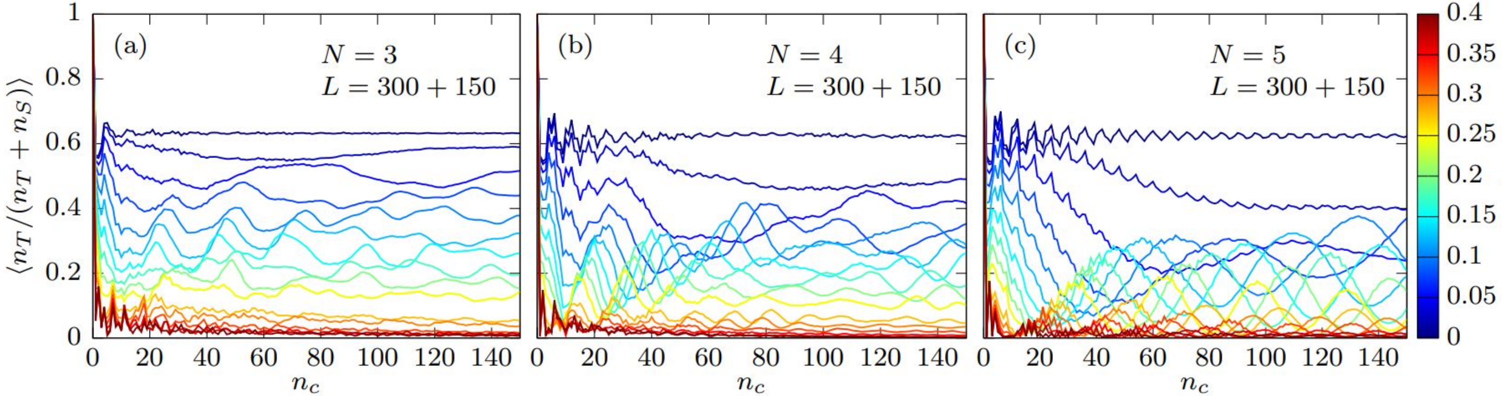
Integrable models are characterized by the existence of stable excitations that can propa indefinitely without decaying. This includes multi-magnon bound states in the celebrated spin chain model and its integrable Floquet counterpart. A recent Google Quantum AI experi [A. Morvan *et al.*, Nature **612**, 240 (2022)] realizing the Floquet model demonstrated the persistence of such collective excitations even when the integrability is broken: this observation is at odds with

It is a few-body physics and most likely will go away at larger sizes

Integrability breaking and bound states in Google's decorated XXZ circuits

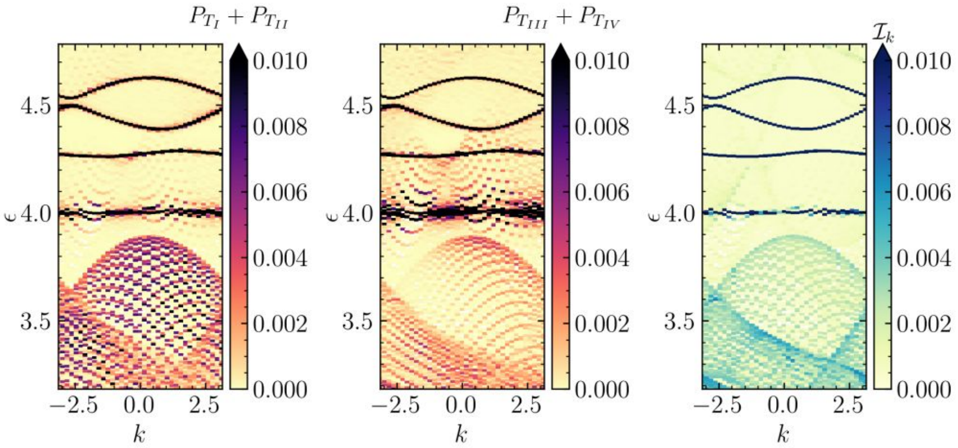


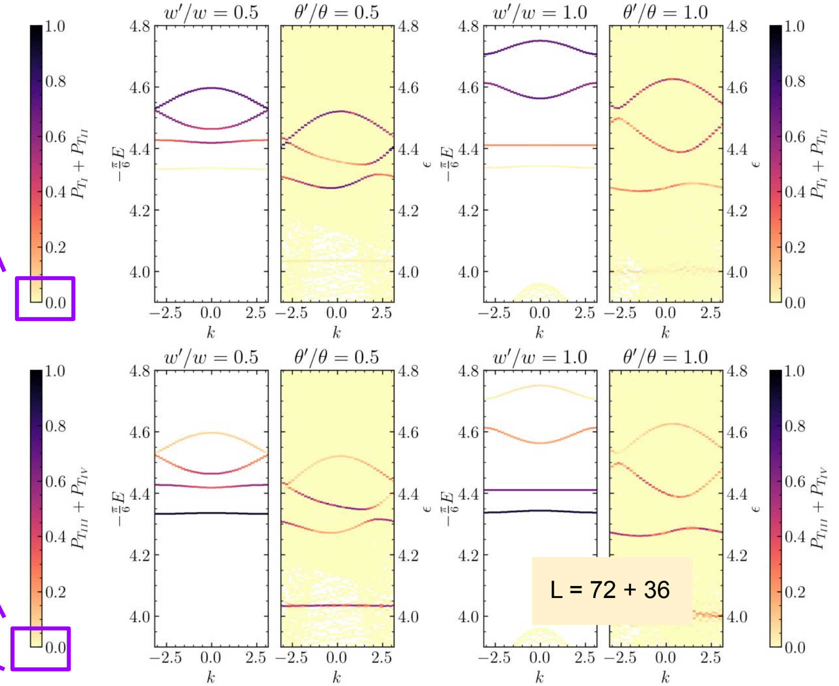
chosen perturbation indeed breaks integrability.



∴ large but dilute bound states continue to be robust.

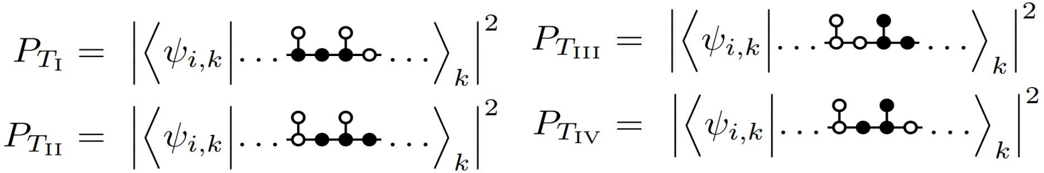
Robustness and eventual slow decay of bound states of interacting microwave photons in the Google Quantum AI experiment





1110000 → 110010000 (no energy overlap)

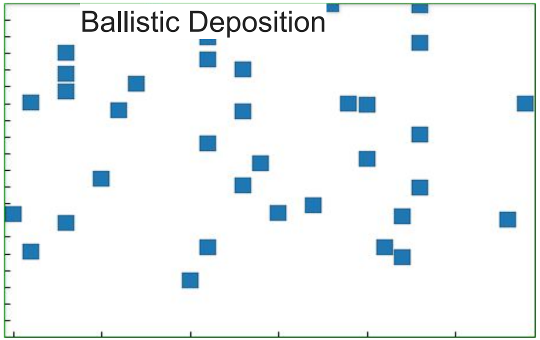
1110000 → 1001010000 (small matrix element)

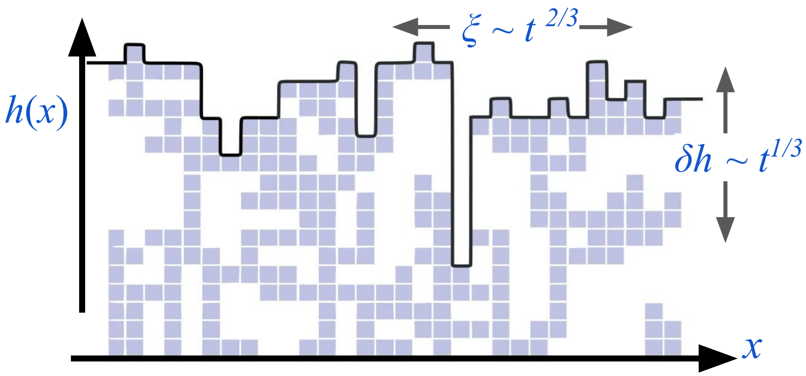


Kardar-Parisi-Zhang (KPZ) Universality Class

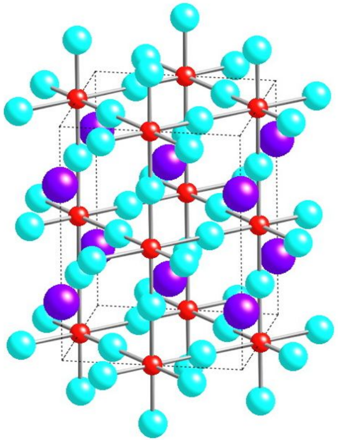
$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t)$$

diffusion growth noise





Spin dynamics of a 1D Heisenberg antiferromagnet



KCuF3

Kardar-Parisi-Zhang (KPZ) Universality Class

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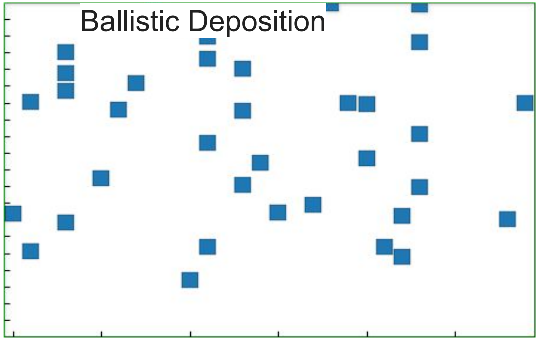
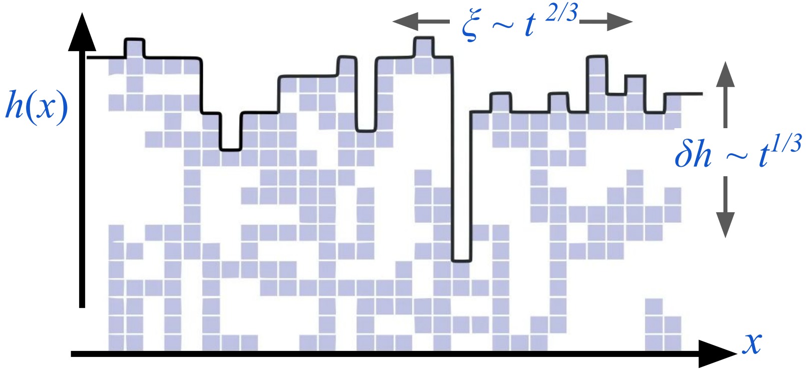




The KPZ conjecture :

KPZ		Heisenberg spin chains
$\partial h/\partial x$	$\rightarrow$	$S^z(x,t)$ Magnetization profile
$2h(0,t) - h(-L/2,t) - h(L/2,t)$	$\rightarrow$	$\mathcal{M}(t)/2 = N_{R,1}(b_t) - N_{R,1}(b_i)$
Relative height at the center		Transferred magnetization

Baik-Rains $\boldsymbol{\mu} = 0$ TW-GUE $\boldsymbol{\mu} \neq 0$

Numerical: Ljubotina, Žnidaric, Prosen, PRL 122, 210602 (2019)
Experimental: D. Wei *et al.*, Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion, Science 376, 716 (2022).

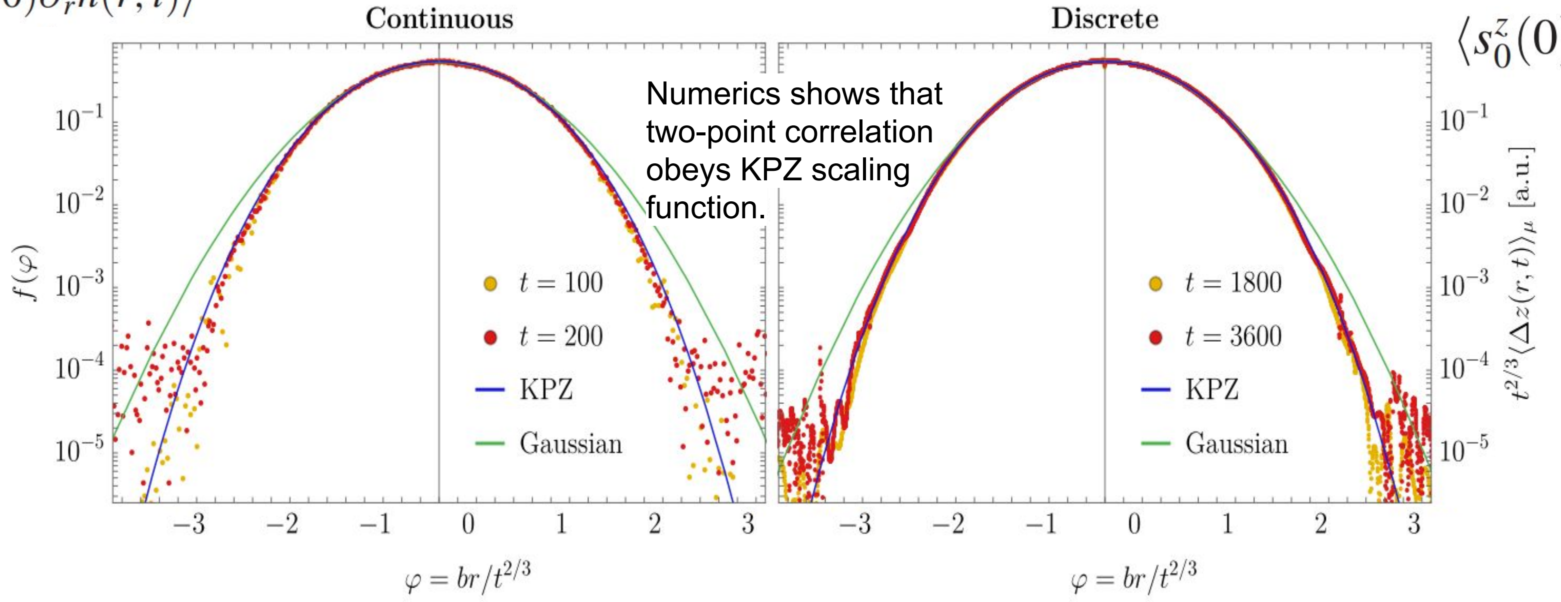
Evidence for being in the KPZ universality class

KPZ prediction

$\langle \partial_r h(0,0) \partial_r h(r,t) \rangle$

Spin chain numerics (red and yellow)

$\langle s_0^z(0) s_r^z(t) \rangle$



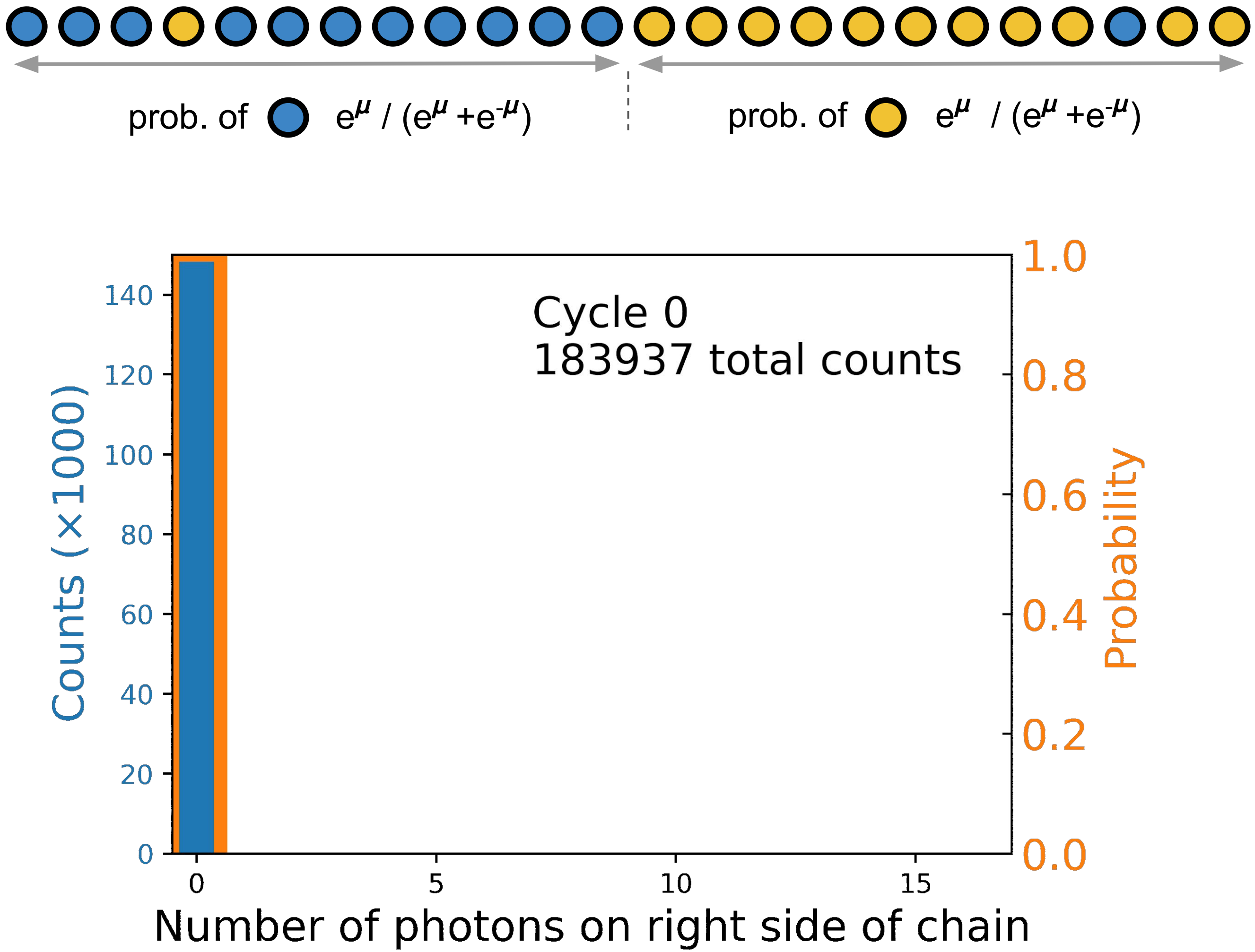
Numerics shows that two-point correlation obeys KPZ scaling function.

The KPZ conjecture :

In the long time limit : $\lim_{\mu \to 0} \mathcal{M}(t) \longleftrightarrow 2h(0,t) - h(-\infty,t) - h(\infty,t)$

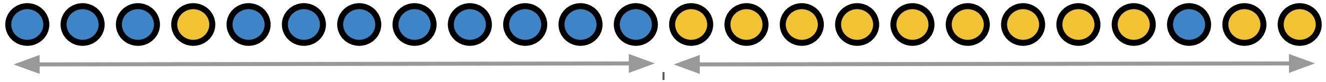
Numerical: Ljubotina, Žnidaric, Prosen, PRL 122, 210602 (2019)

Experimental: D. Wei *et al.*, Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion, Science 376, 716 (2022).



Arxiv 2306.09333

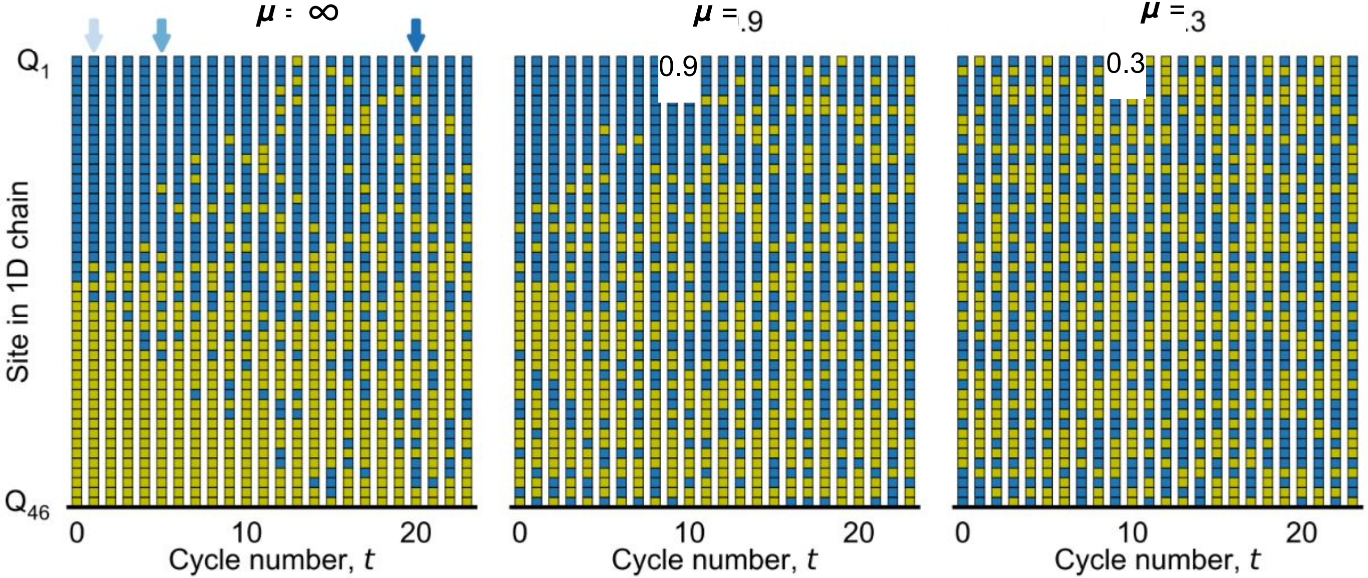
Magnetic domain wall dynamics in a XXZ spin chain



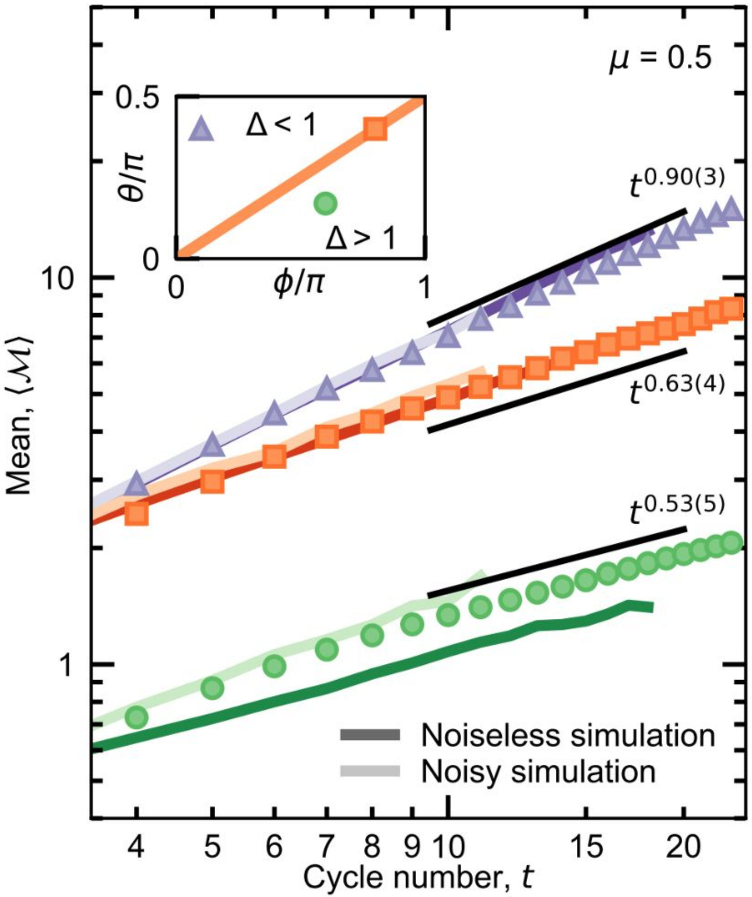
prob. of (blue) $e^{\mu} / (e^{\mu} + e^{-\mu})$ prob. of (yellow) $e^{\mu} / (e^{\mu} + e^{-\mu})$

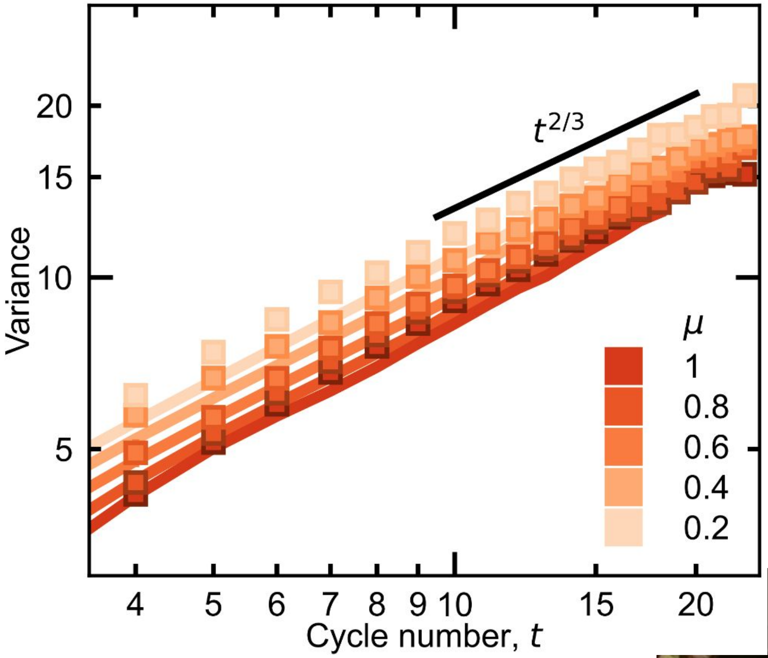
initial state: $\rho(t=0) \propto (e^{2\mu S^z})^{\otimes N_Q/2} \otimes (e^{-2\mu S^z})^{\otimes N_Q/2}$

$$\mathcal{H} = \sum_i (X_i X_{i+1} + Y_i Y_{i+1}) + \Delta Z_i Z_{i+1}$$



Mean and Variance of M $\rightarrow$ consistent with KPZ

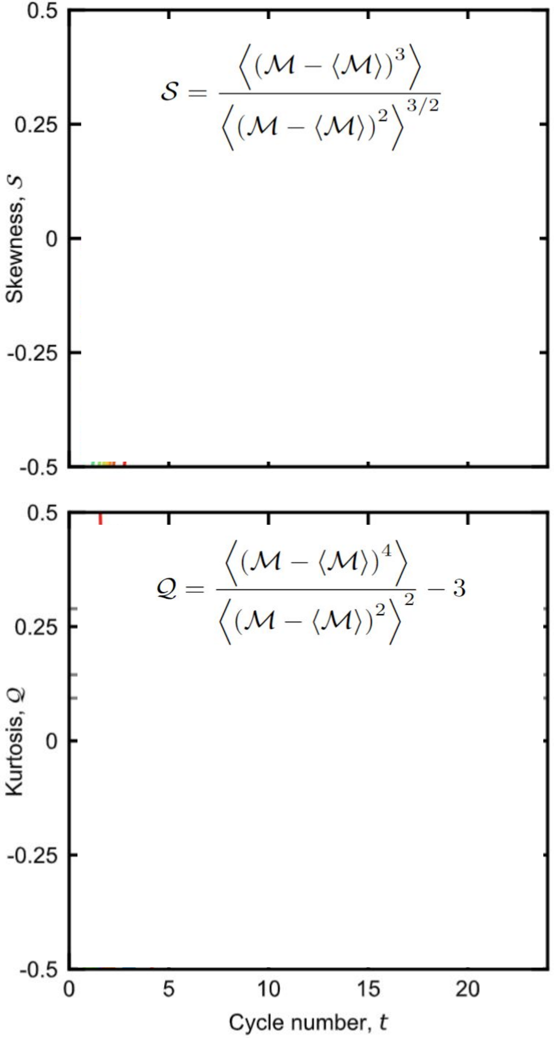


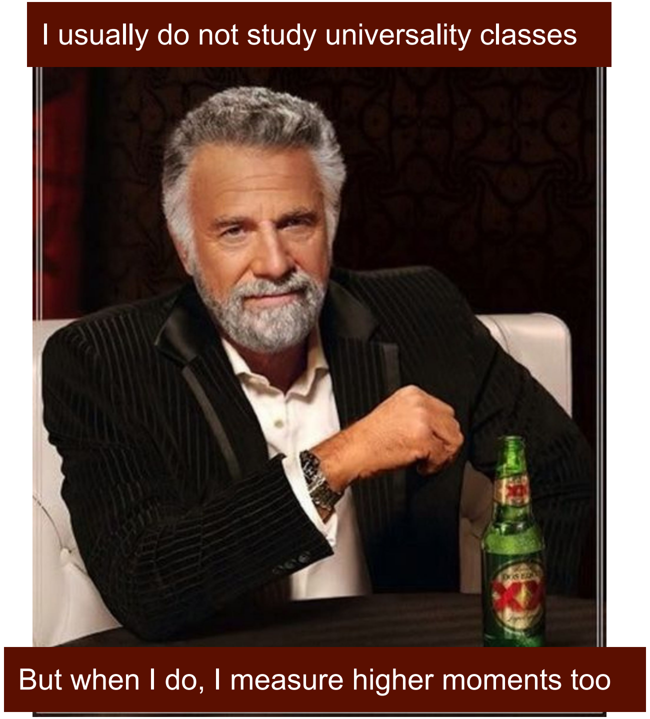


agreement with
cold atom experiments

Wei *et al.*, Science 376, 2022

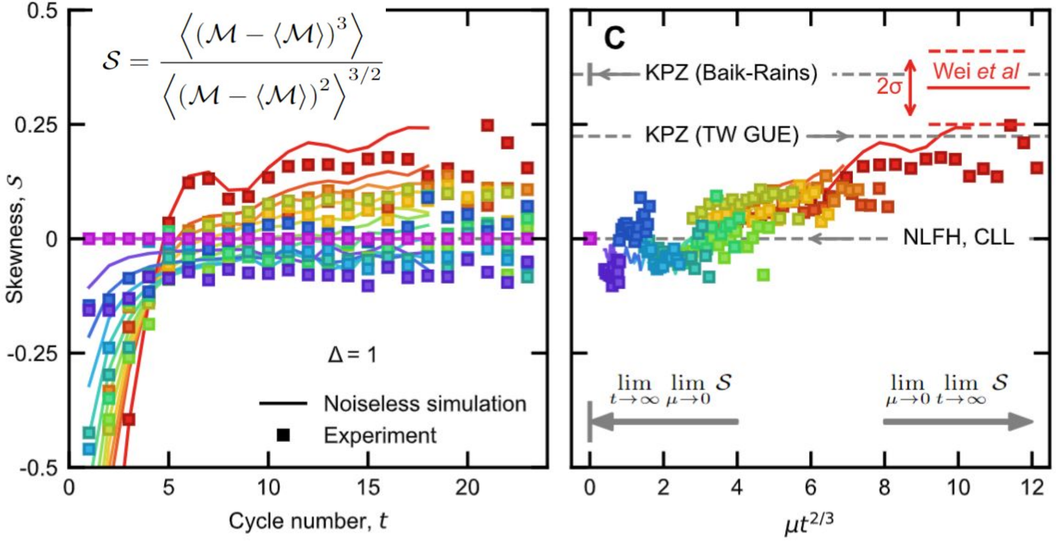
Higher moments of the transferred magnetization

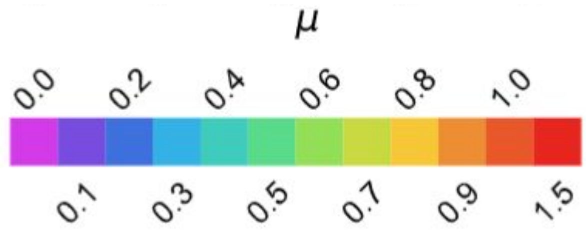




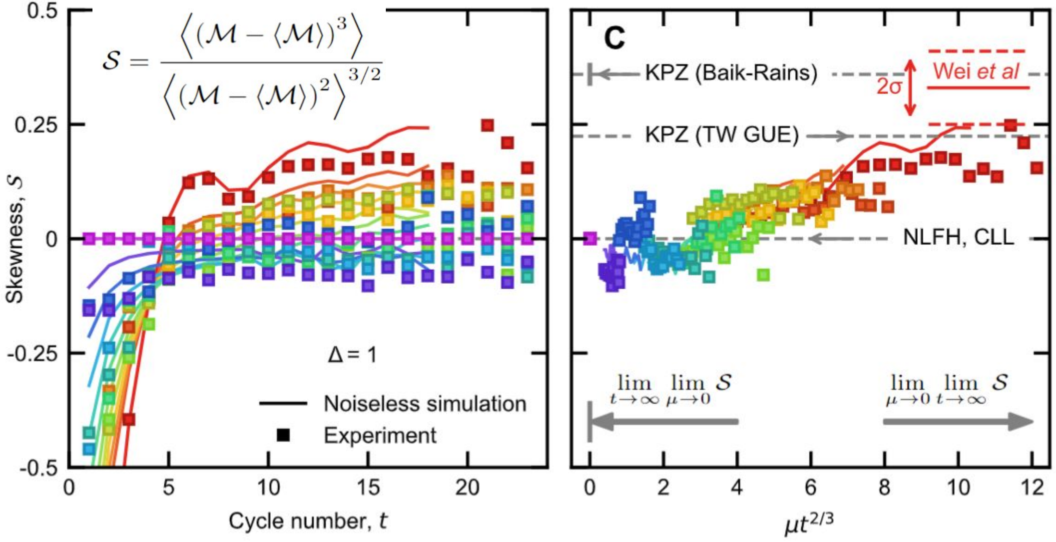
The importance of studying higher moments in determining dynamic universality classes.

Skewness of transferred magnetization

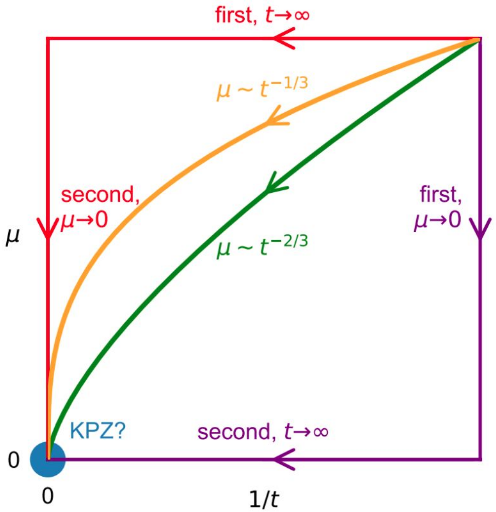


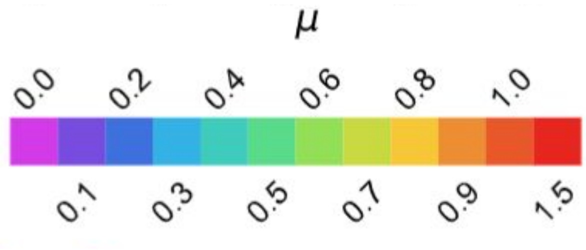


Skewness of transferred magnetization



	$\langle \mathcal{M} \rangle$	σ^2	$\mathcal{S}$
Experiment	$t^{2/3}$	$t^{2/3}$	0 *
KPZ (Baik-Rains)	$t^{2/3}$	$t^{2/3}$	0.36
NLFH	$t^{2/3}$	$t^{2/3}$	0
CLL	$t^{2/3}$	$t^{2/3}$	0

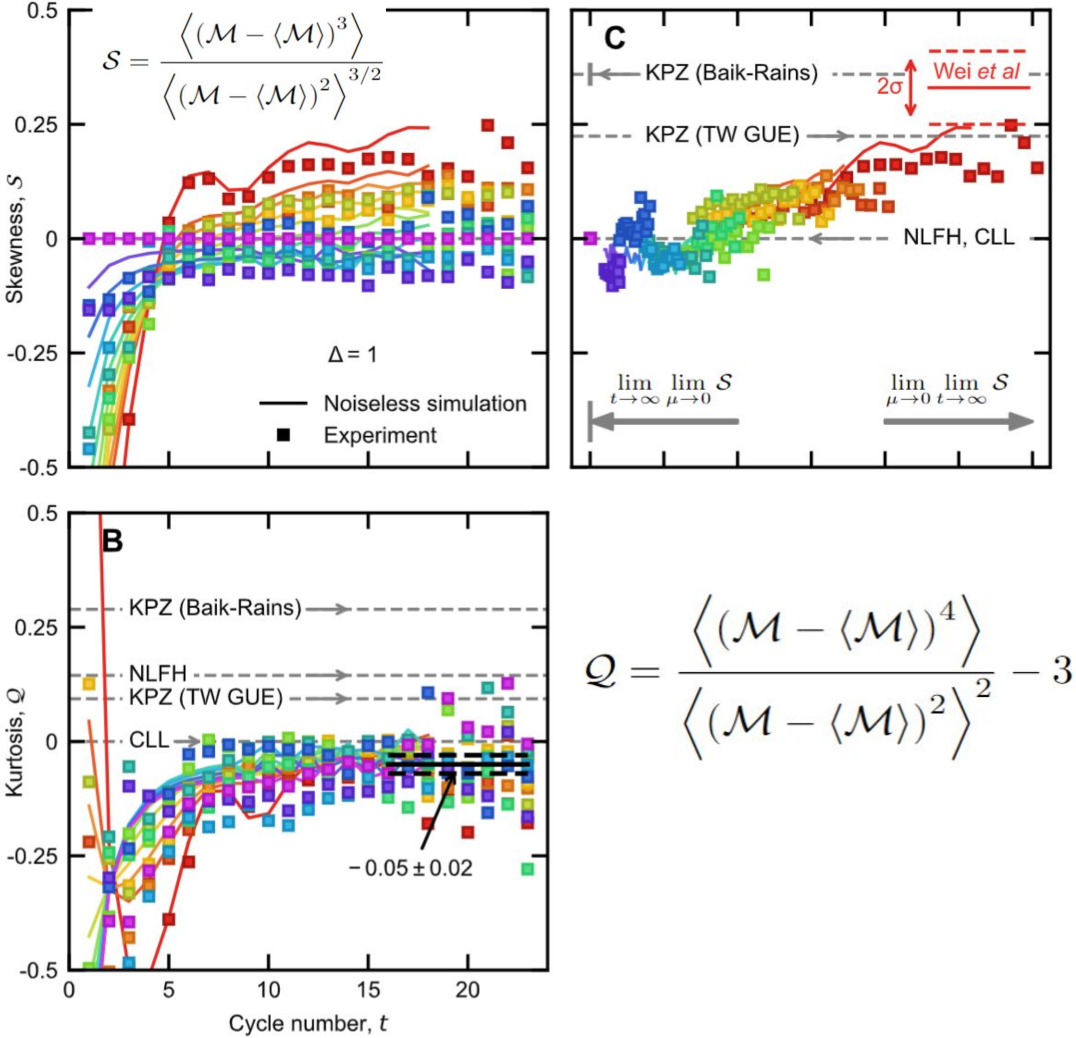




NLFH = non-linear fluctuating hydrodynamics
(De Nardis, **Gopalakrishnan**, and Vasseur, 2023)

CLL = classical Landau-Lifshitz
(Krajnik, Ilievski, and **Prosen**, 2022)

Higher moments of the transferred magnetization



	$\langle \mathcal{M} \rangle$	σ^2	$\mathcal{S}$	$\mathcal{Q}$
Experiment	$t^{2/3}$	$t^{2/3}$	$0\,*$	-0.05 ± 0.02
KPZ (Baik-Rains)	$t^{2/3}$	$t^{2/3}$	0.36	0.29
NLFH	$t^{2/3}$	$t^{2/3}$	0	0.14
CLL	$t^{2/3}$	$t^{2/3}$	0	$\in [-0.03, 0.03]$

$$\mathcal{Q} = \frac{\left\langle (\mathcal{M} - \langle \mathcal{M} \rangle)^4 \right\rangle}{\left\langle (\mathcal{M} - \langle \mathcal{M} \rangle)^2 \right\rangle^2} - 3$$

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