MAX PLANCK INSTITUTE
FOR THE PHYSICS OF COMPLEX SYSTEMS

Engineering Hierarchical Symmetries

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(MPI-PKS)

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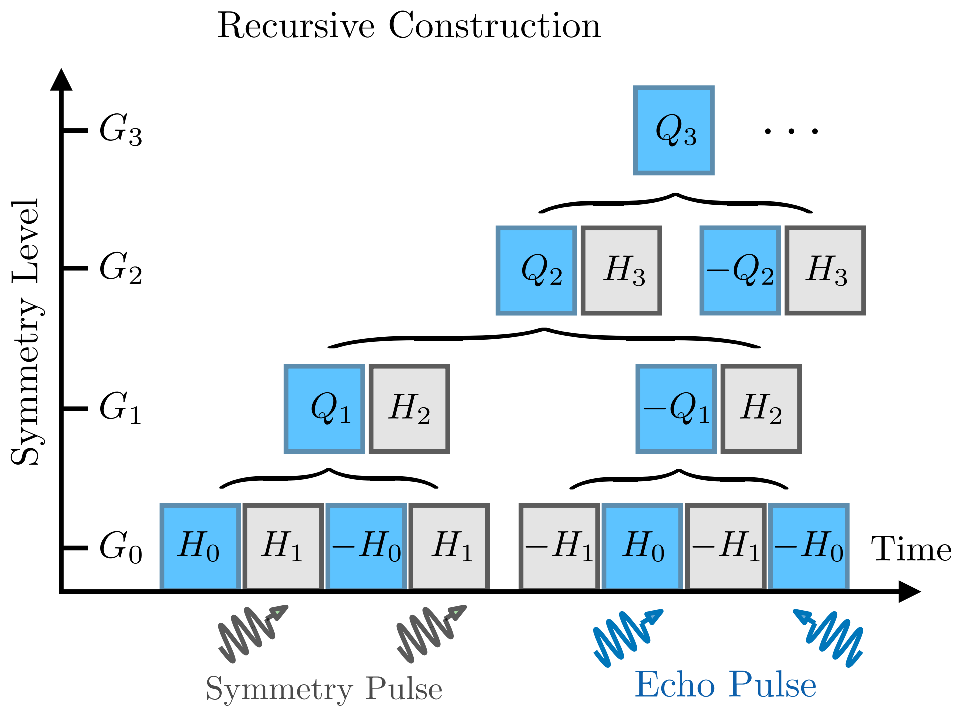
European Research Council
Established by the European Commission

work in progress

Engineering Hierarchical Symmetries

Outline



- Emergent symmetries out of equilibrium
- Engineering Hierarchical Symmetries
- Applications

⟶ abelian & non-abelian symmetry ladder: $\mathrm{SU}(2) \to \mathrm{U}(1) \to \mathbb{Z}_2 \to E$

⟶ nonequilibrium order: discrete time crystals: $\mathbb{Z}_4 \to \mathbb{Z}_2 \to E$

⟶ higher-order topological insulators: $\mathcal{I} \times \mathcal{T} \to \mathcal{I} \to E$

Marin Bukov

MPI-PKS (Dresden)

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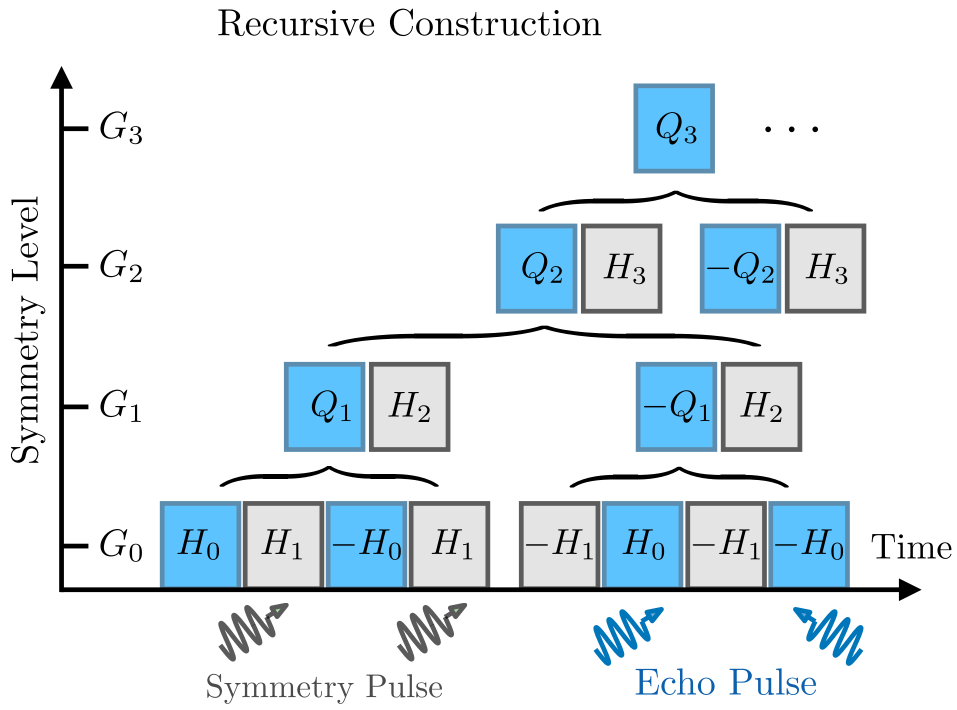

Symmetries in Physics

- why care about symmetries in physics?

  ⟶ determine invariance of physical laws

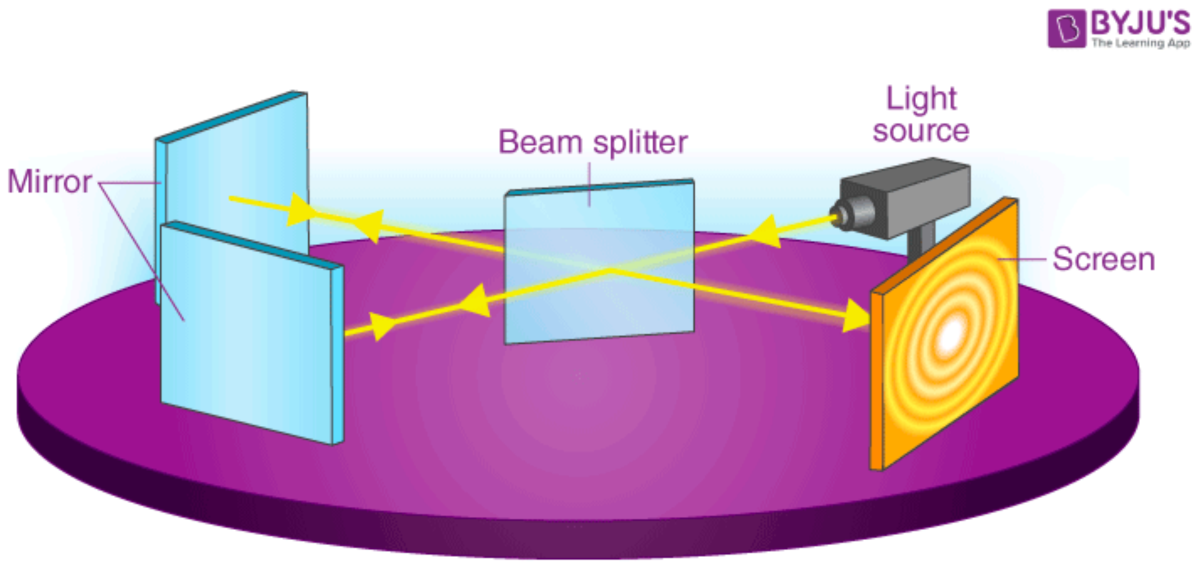

image: BYJU'S

Michelson Morley experiment

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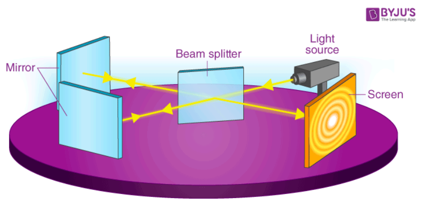


image: BYJU'S

  - ⟶ conservation laws, integrability, phases & phase transitions

Milky way

image: Wikipedia

conservation of angular momentum

phase space tori

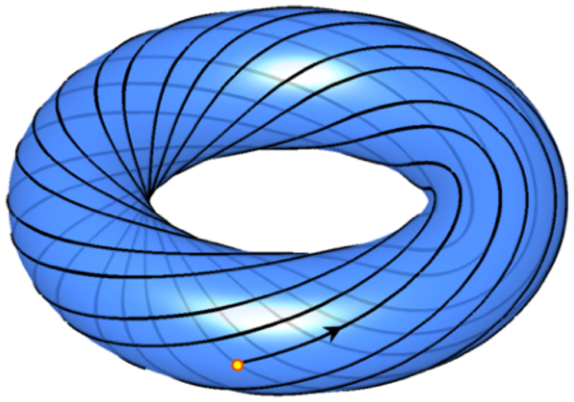
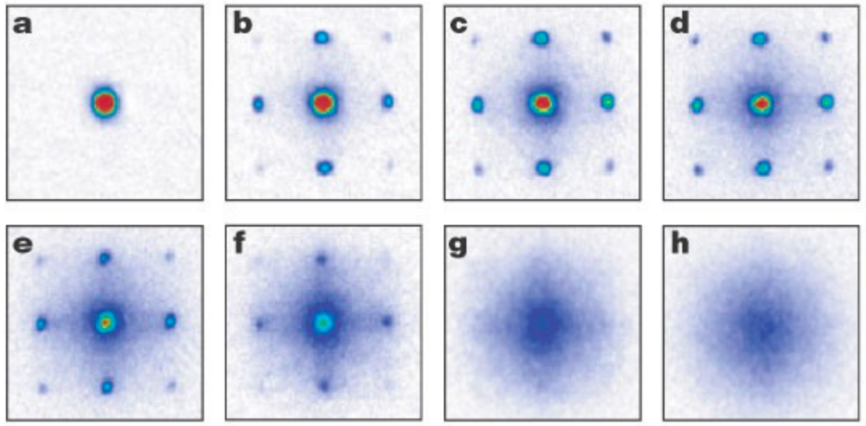


image: Agaoglou et al.

integrability

Mott insulator to superfluid transition

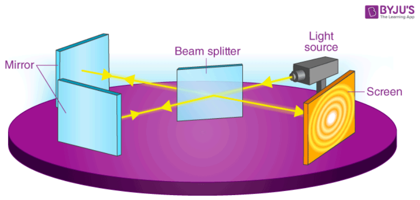


Greiner et al., Nature 415 (2002)

phase transitions

Symmetries in Physics

- why care about symmetries in physics?
  - ⟶ determine invariance of physical laws
  - ⟶ conservation laws, integrability, phases & phase transitions
  - ⟶ build minimal models: Standard model, Landau-Ginzburg theory, etc.



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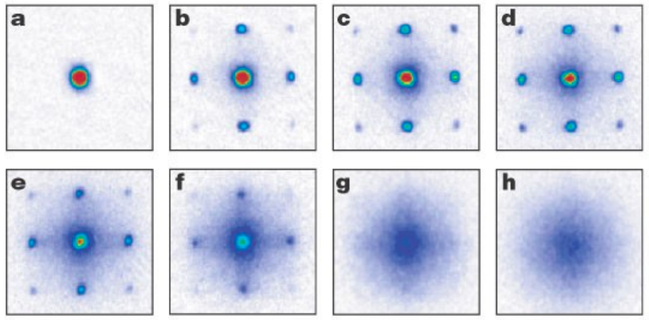

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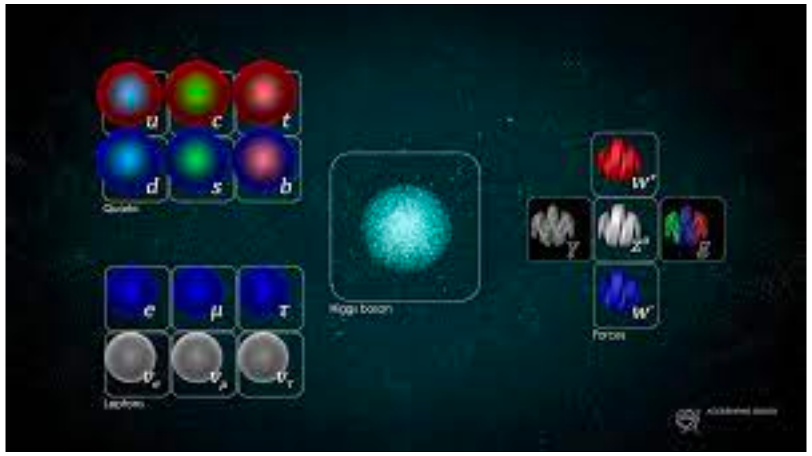

image: CERN

The Standard Model

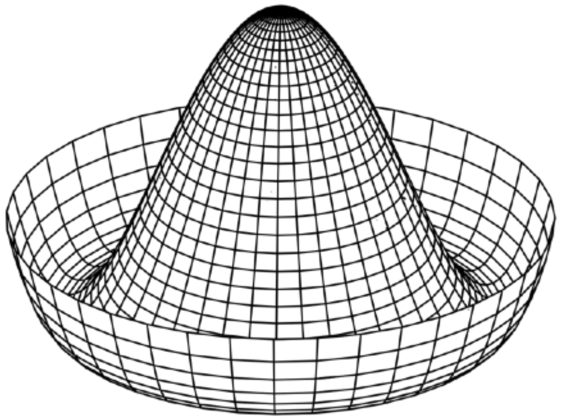

image: Wikipedia

Landau-Ginzburg theory

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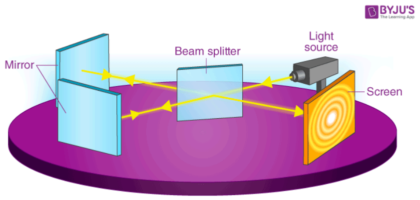

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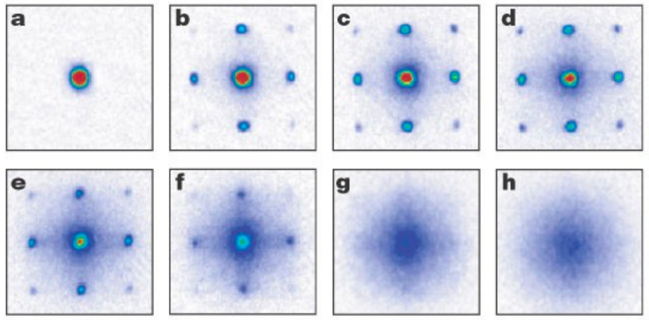

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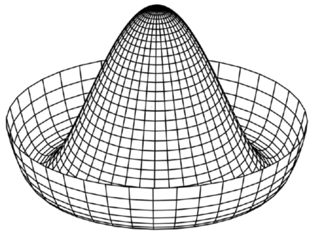

image: Wikipedia

want to simulate symmetries and study their breaking

Symmetries in Physics

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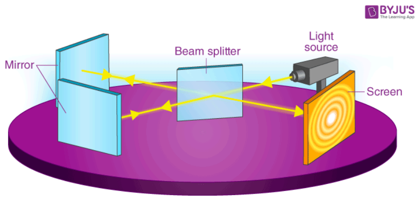

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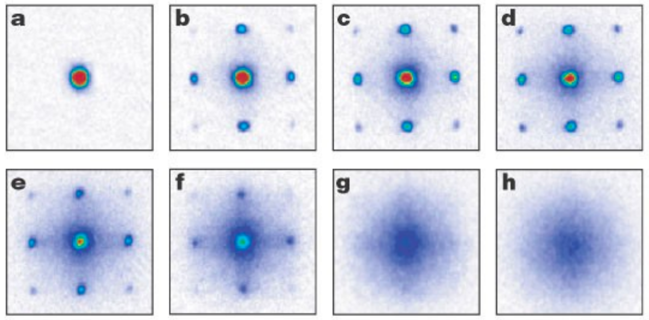

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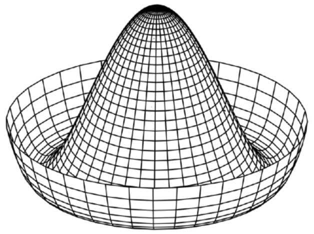

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⟶ determine invariance of physical laws

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want to simulate symmetries and study their breaking

- when are symmetries exact in real-world many-body systems?

⟶ (almost) never: we're forced to deal with approximate symmetries

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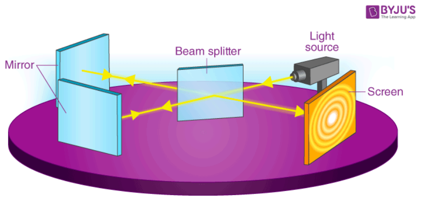

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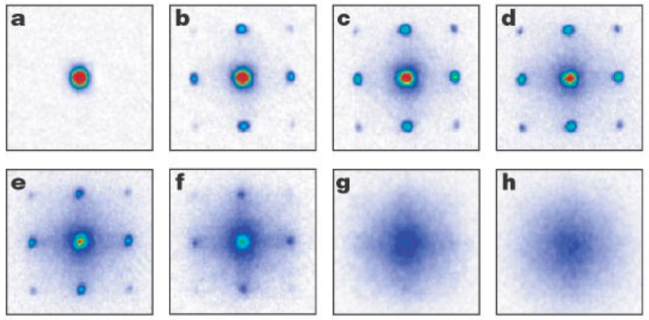

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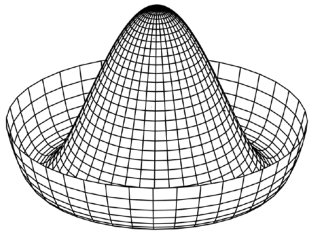

image: Wikipedia

want to simulate symmetries and study their breaking

- when are symmetries exact in real-world many-body systems?
  - → (almost) never: we're forced to deal with approximate symmetries
  - → still: approx. symmetries prove useful to determine hierarchy of phenomena

Emergent Symmetries in Nonequilibrium Drives

periodic/Floquet drives

drive

$$H(t) = H(t+T)$$

evolution

$$U_F = e^{-iTQ}$$

$$U(\ell T, 0) = [U_F]^\ell$$

effective Hamiltonian

$$Q$$

Bukov et al, Adv. Phys. (2015)

Goldman et al, PRX (2014)

Eckardt, Rev. Mod. Phys. (2017)

Emergent Symmetries in Nonequilibrium Drives

	periodic/Floquet drives	**quasi-periodic drives**
drive	$H(t) = H(t+T)$	$H(\omega_1 t, \omega_2 t)$ ω_1, ω_2 ω_1/ω_2 irrational
evolution	$U_F = e^{-iTQ}$ $U(\ell T, 0) = [U_F]^\ell$	$U(t) = \mathcal{T}_t \exp\left(-i\int_0^t H(t')\mathrm{d}t'\right)$
effective Hamiltonian	Q	Q

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Long et al, PRL 126, 106805 (2021)

Emergent Symmetries in Nonequilibrium Drives

	periodic/Floquet drives	**quasi-periodic drives**	**random multipolar drives**
drive	$H(t) = H(t+T)$	$H(\omega_1 t, \omega_2 t)$ ω_1, ω_2 $\omega_1/\omega_2 \text{ irrational}$	$U_\pm = e^{-iTH_\pm}$ *dipolar* $U_A = U_+ U_-$ $U_B = U_- U_+$ *quadrupolar* $U_A = U_+ U_- U_- U_+$ $U_B = U_- U_+ U_+ U_-$
evolution	$U_F = e^{-iTQ}$ $U(\ell T, 0) = [U_F]^\ell$	$U(t) = \mathcal{T}_t \exp\left(-i\int_0^t H(t')\mathrm{d}t'\right)$	$U = U_A U_B U_B U_A U_B U_A U_A U_B \cdots$ *random sequence*
effective Hamiltonian	Q	Q	$Q_A \approx Q_B$

Bukov et al, Adv. Phys. (2015)
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Mori et al, PRL 127, 050602 (2021)

Emergent Symmetries in Nonequilibrium Drives

- periodic, quasi-periodic, & random multipolar drives

⟶ approximate effective Hamiltonian $Q \approx Q^{(0)} + Q^{(1)} + Q^{(2)} + \cdots$

$Q^{(m)} \propto T^m$

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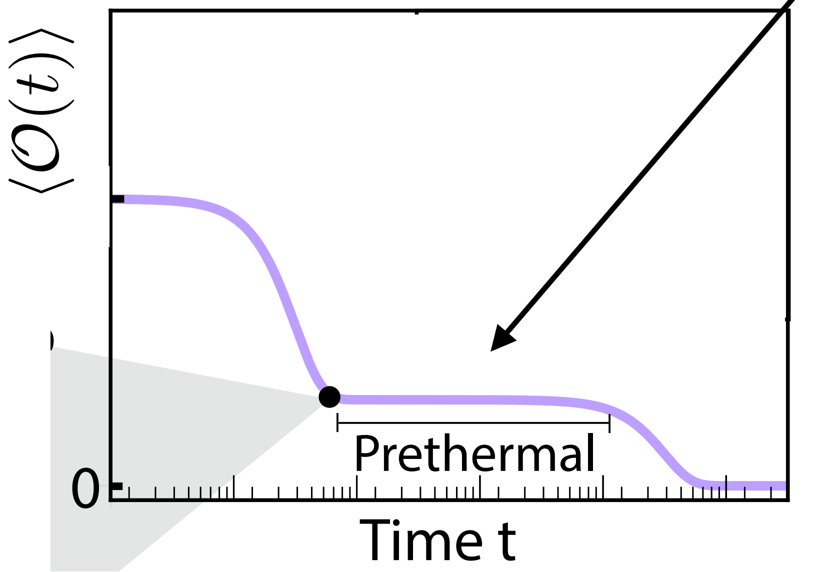
Emergent Symmetries in Nonequilibrium Drives

- periodic, quasi-periodic, & random multipolar drives

$\longrightarrow$ approximate effective Hamiltonian $Q \approx Q^{(0)} + Q^{(1)} + Q^{(2)} + \cdots$ $Q^{(m)} \propto T^m$

- prethermal metastable states

approximate (emergent) symmetry



Ho et al., Ann. of Phys. 454, 169297 (2023)

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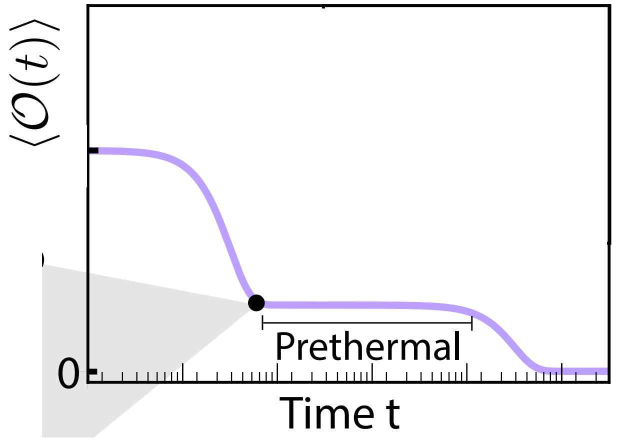

- examples of emergent symmetries:

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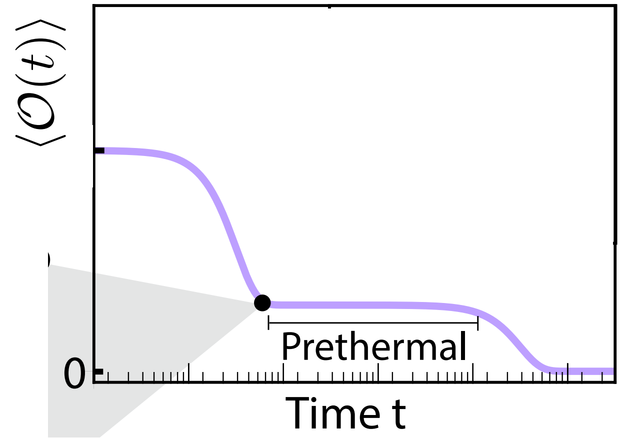

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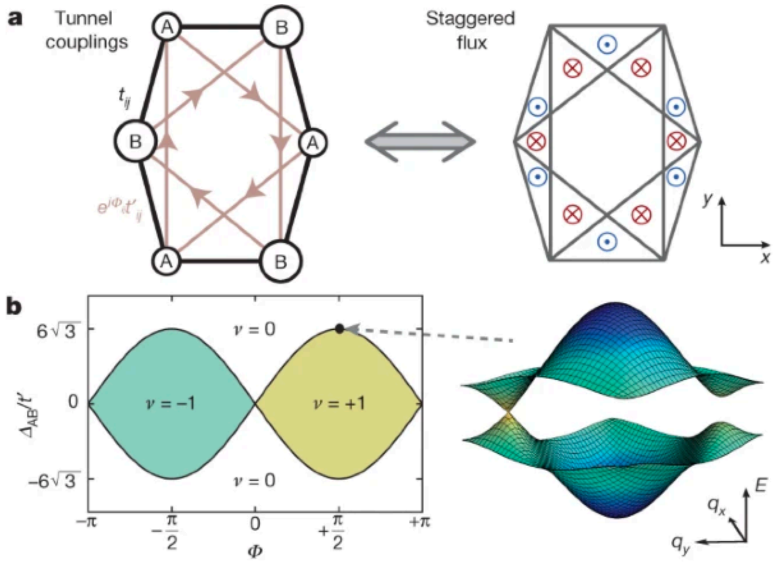

Haldane model: time-reversal breaking

time-rev'sal breaking

$$Q^{(0)} : \mathcal{T}$$

$$Q^{(1)} : \cancel{\mathcal{T}}$$

Kitagawa et al, PRB (2011)

Jotzu et al, Nature 515 (2014)

Emergent Symmetries in Nonequilibrium Drives

- periodic, quasi-periodic, & random multipolar drives

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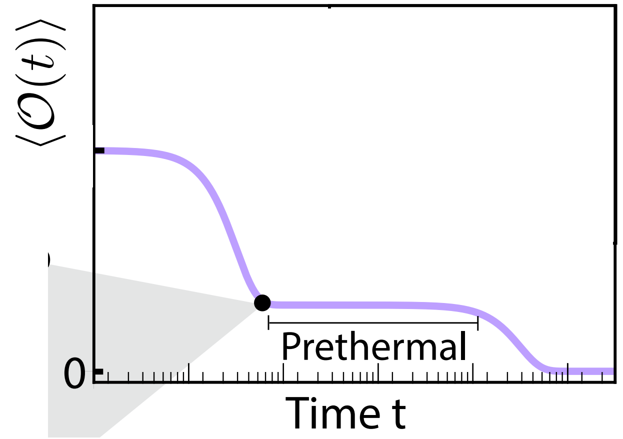

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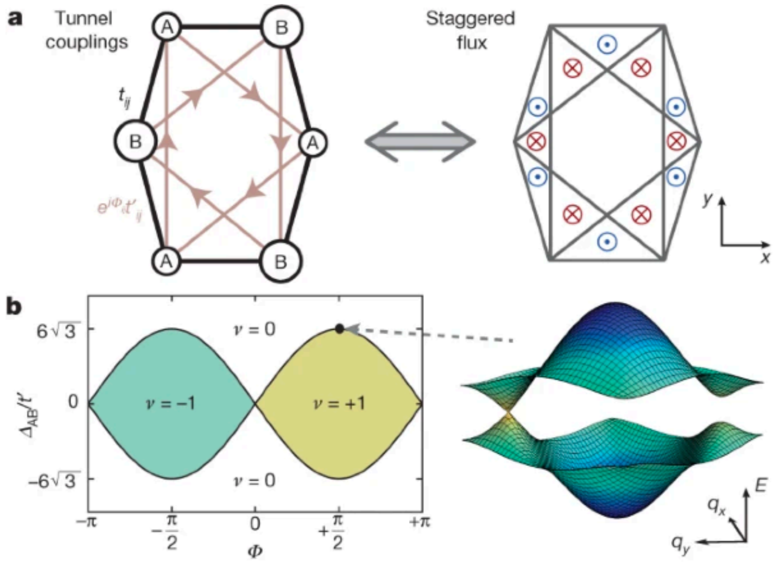

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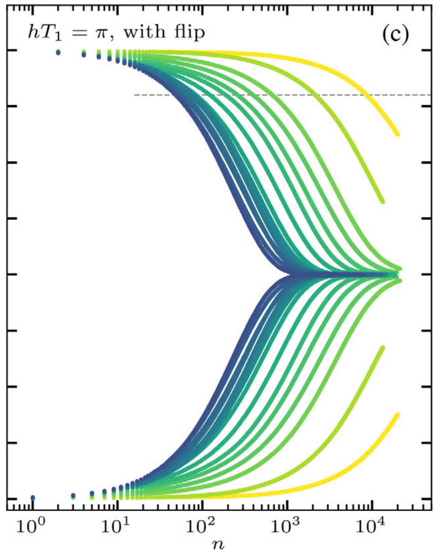

symmetry-'protected' prethermalization w/o temp.

U(1) breaking

$Q^{(0)} : \mathrm{U}(1)$

$Q^{(1)} : \cancel{\mathrm{U}(1)}$

Kitagawa et al, PRB (2011)

Jotzu et al, Nature 515 (2014)

Luitz et al, PRX 10, 021046 (2020)

Ho et al, arXiv 2011.14583

Haldar PRB 97, 245122 (2018)

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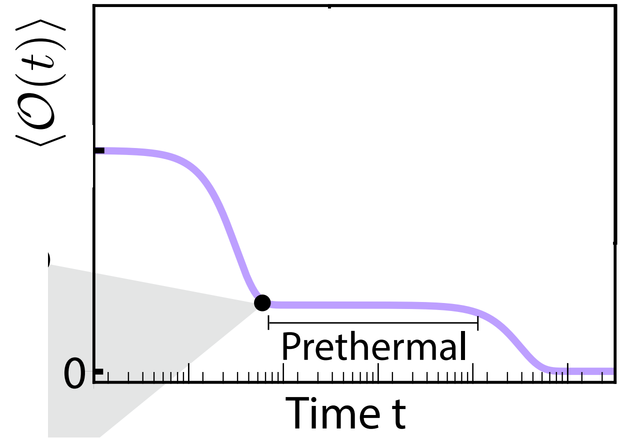
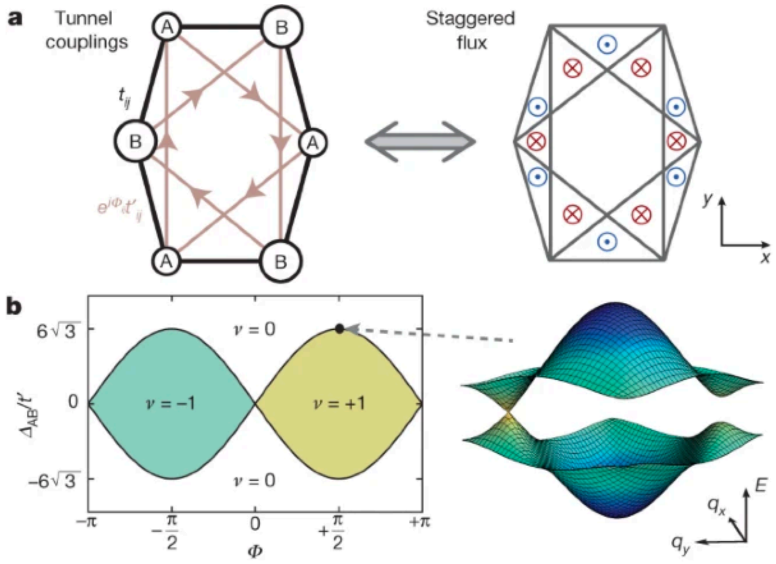


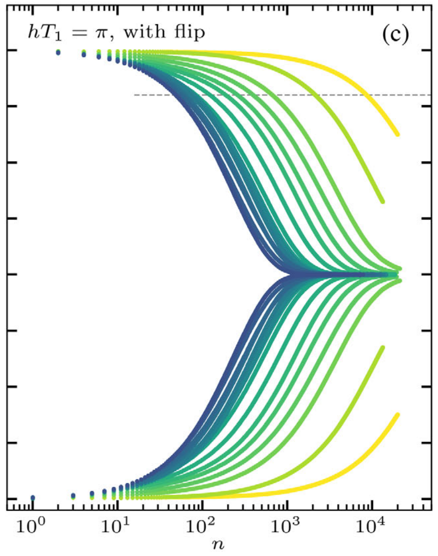
Z2 breaking

$Q^{(0)} : \mathbb{Z}_2$

$Q^{(1)} : \xcancel{\mathbb{Z}_2}$



Haldane model: time-reversal breaking



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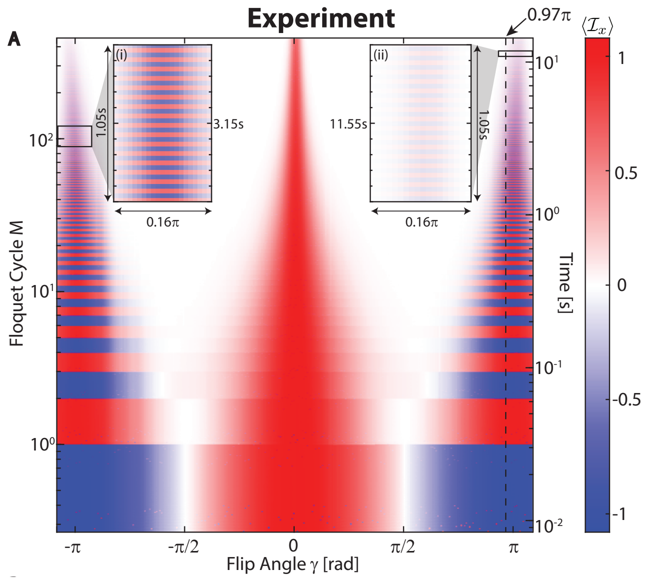

(pre thermal) discrete time crystals

Kitagawa et al, PRB (2011)
Jotzu et al, Nature 515 (2014)

Luitz et al, PRX 10, 021046 (2020)
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Haldar PRB 97, 245122 (2018)

Beatrez et al, Nat. Phys 19 407 (2023)
Sacha et al, Prog. Phys. 81 (2017)
Else et al, Ann Rev Condmat (2019)
Khemani et al, arXiv 1910:10745

Marin Bukov

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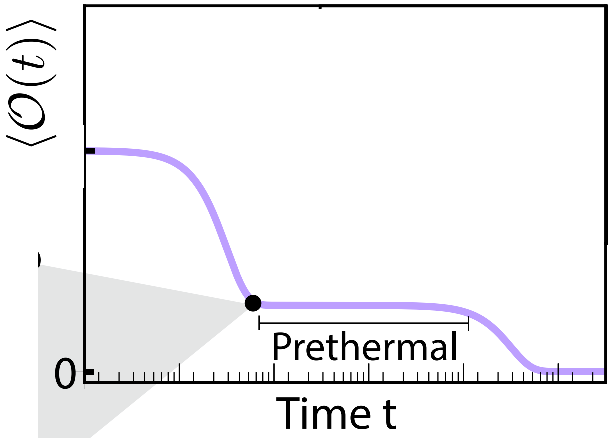
→ approximate effective Hamiltonian $Q \approx Q^{(0)} + Q^{(1)} + Q^{(2)} + \cdots$

- prethermal metastable states

→ approximate (emergent) symmetry

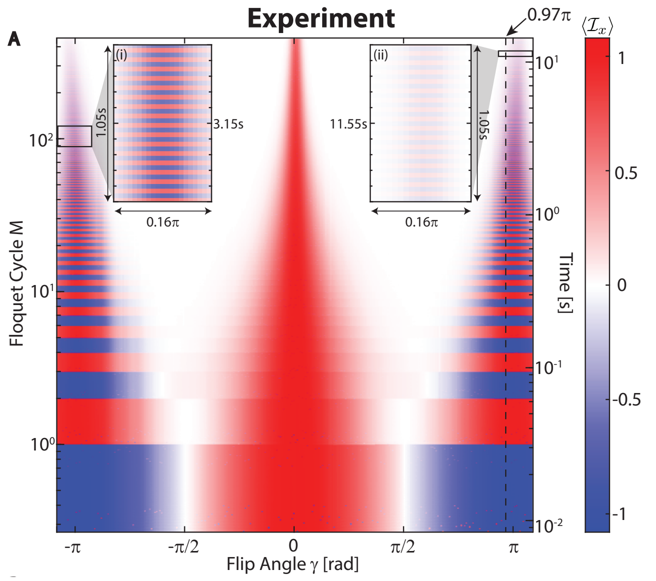


- examples of emergent symmetries:

→ Haldane model: time-reversal breaking

→ symmetry-“protected” prethermalization (without temperature)

→ (prethermal) discrete time crystals



Beatrez et al, Nat. Phys 19 407 (2023)

Q: can we engineer symmetry breaking in a controlled way?

Outline

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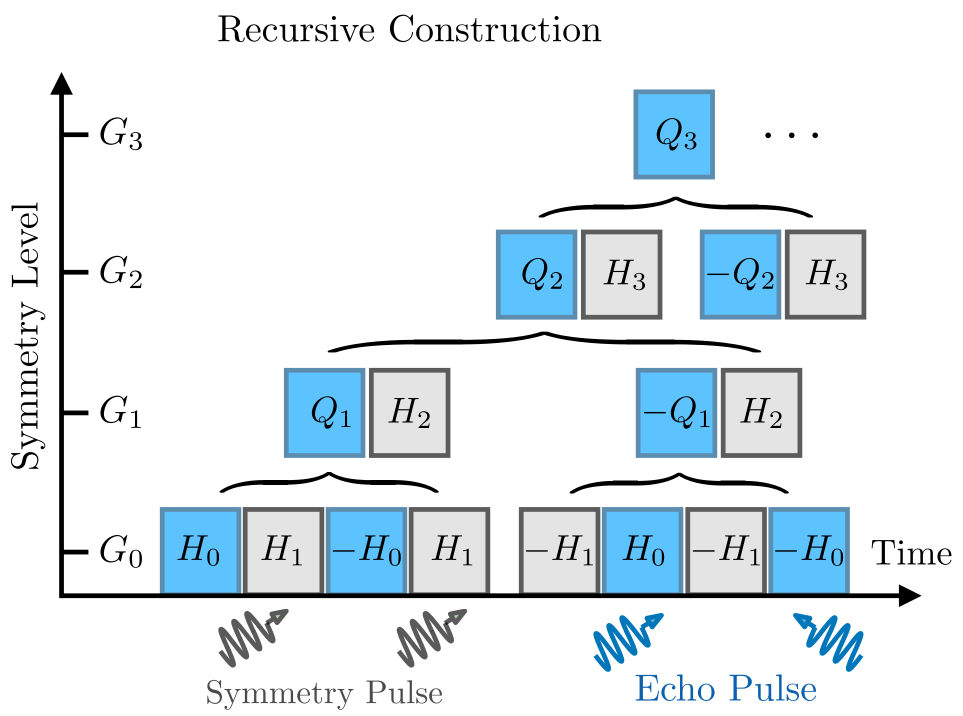
- Applications



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Engineering Hierarchical Symmetries

- objective:

→ given ladder of symmetry groups

$$G_n \supset G_{n-1} \supset \cdots \supset G_0$$

→ construct drive s.t.:

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$$U = e^{-iTQ}$$

$$Q^{(m)} \propto T^m$$

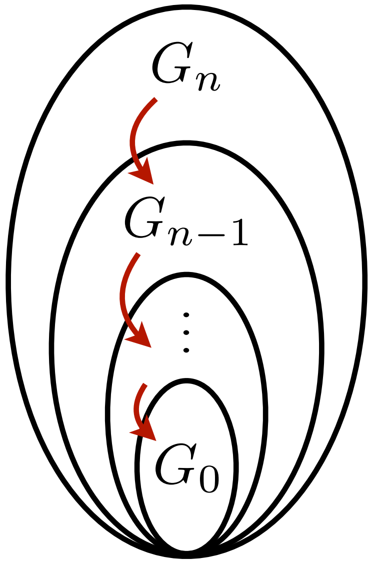
$Q^{(0)}$ has symmetry G_n

$Q^{(1)}$ breaks G_n to G_{n-1}

$Q^{(2)}$ breaks G_{n-1} to G_{n-2}

$\cdots$

$Q^{(n)}$ breaks G_1 to G_0



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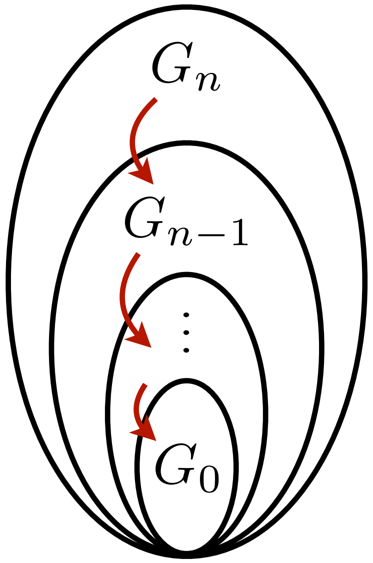
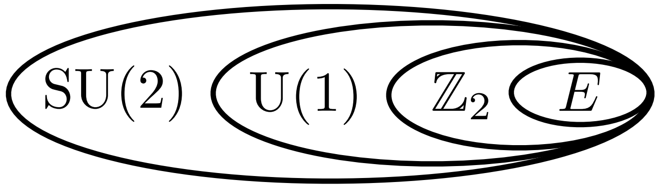
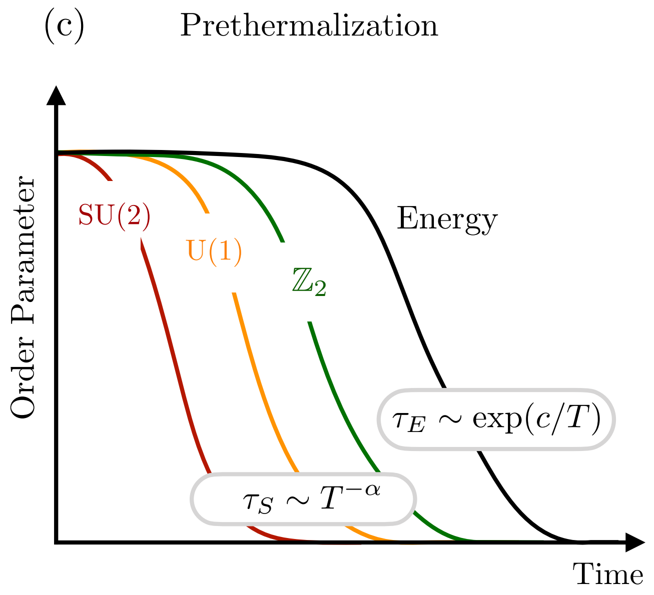


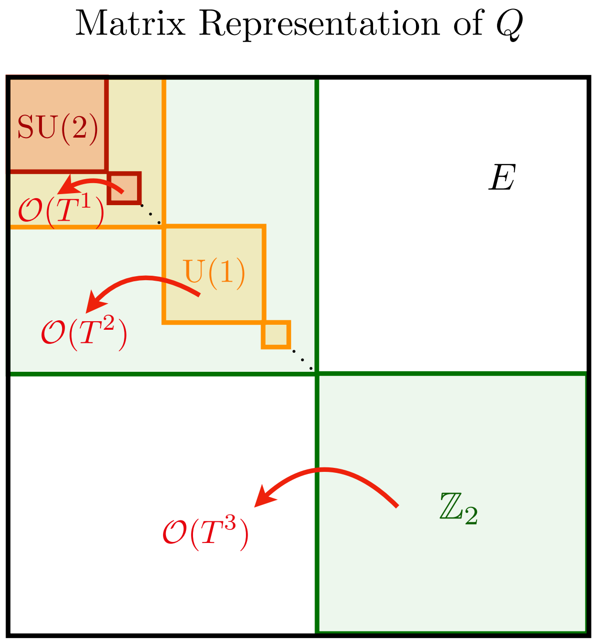
$$U = e^{-iTQ} \qquad Q \approx Q^{(0)} + Q^{(1)} + Q^{(2)} + \cdots$$

- hierarchical structure of prethermal plateaus

e.g.,







Engineering Hierarchical Symmetries

$$Q^{(m)} \propto T^m$$

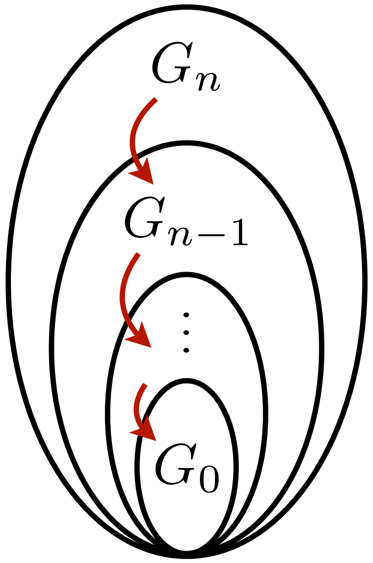
$Q^{(0)}$ has symmetry G_n

$Q^{(1)}$ breaks G_n to G_{n-1}

$Q^{(2)}$ breaks G_{n-1} to G_{n-2}

$\dots$

$Q^{(n)}$ breaks G_1 to G_0



- objective:
  - → given ladder of symmetry groups

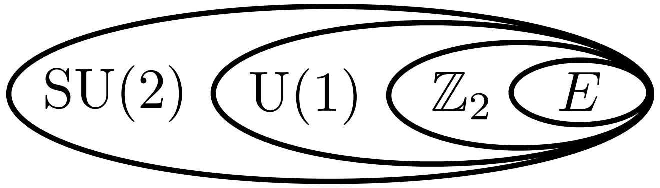
$$G_n \supset G_{n-1} \supset \cdots \supset G_0$$

  - → construct drive s.t.:

$$U = e^{-iTQ} \qquad Q \approx Q^{(0)} + Q^{(1)} + Q^{(2)} + \cdots$$

- hierarchical structure of prethermal plateaus

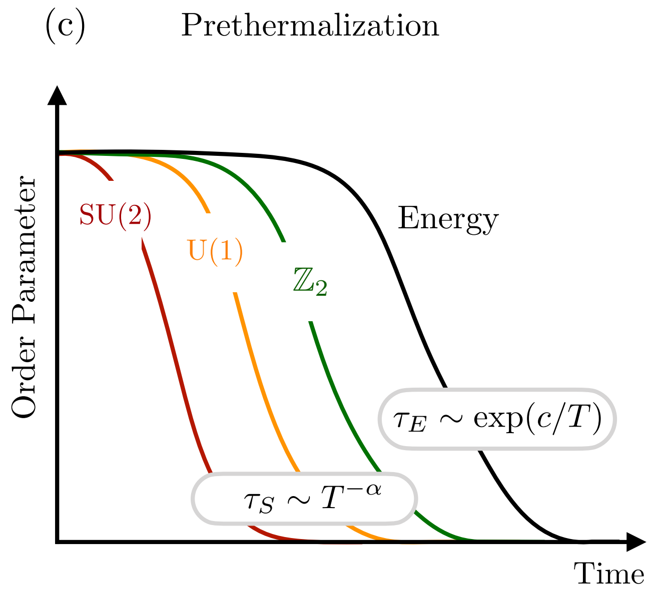
e.g.,

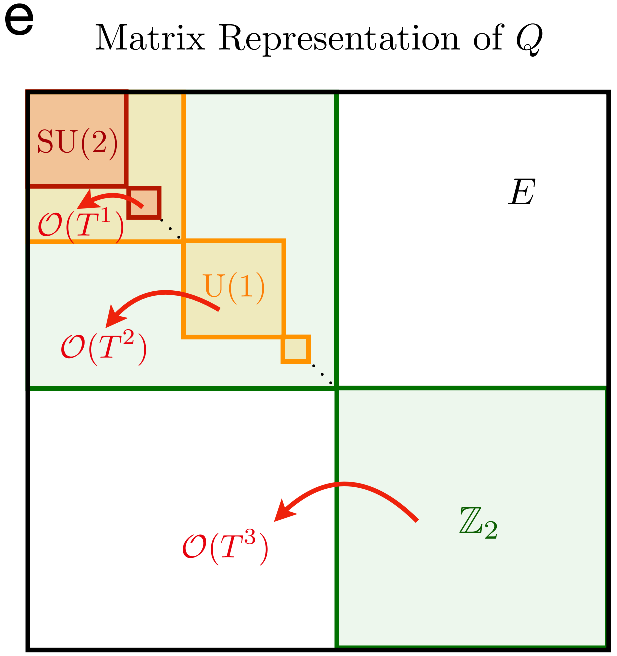


  - → state (in general) described by generalized canonical ensemble

$$\hat{\rho}_{\mathrm{GCE}} \propto \exp\left(- \sum_{\alpha} \lambda_\alpha C_\alpha \right)$$

C_α : conserved quantities assoc. with G_k

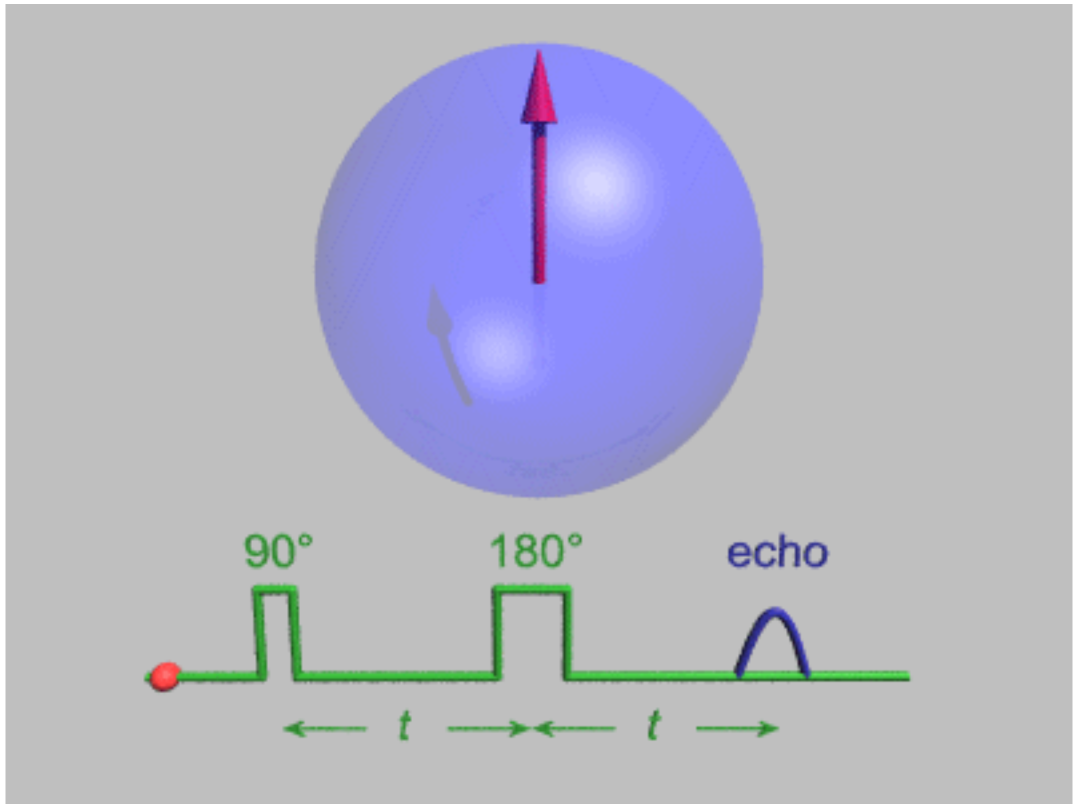




Engineering Hierarchical Symmetries

- basic idea: generalization of spin-echo

$$U_F = e^{\overset{\downarrow}{-}iTQ}e^{-iTH}e^{\overset{\downarrow}{+}iTQ}e^{-iTH} \approx e^{-i2TH}$$



gif: Wikipedia

experts

Engineering Hierarchical Symmetries

- iterative construction

step 1

step 2

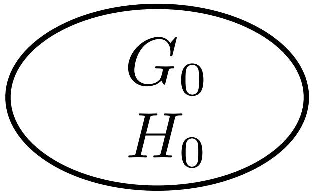
experts

Engineering Hierarchical Symmetries

- iterative construction

step 1

→ start with generator H_0 with symmetry G_0



step 2

experts

Engineering Hierarchical Symmetries

- iterative construction

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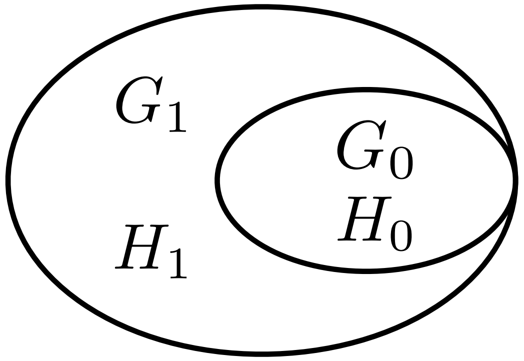
→ start with generator H_0 with symmetry G_0

→ take new generator H_1 with symmetry $G_1 \supset G_0$



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Engineering Hierarchical Symmetries

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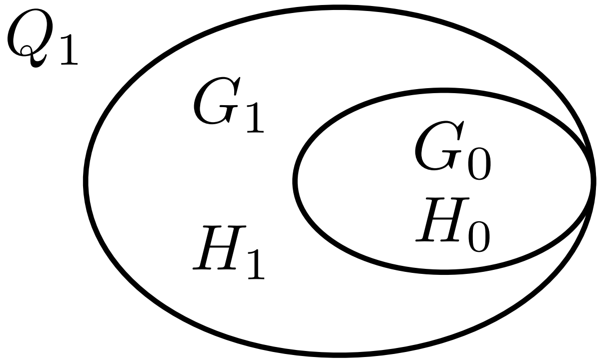

$\ell_n = 3 \times 2^n - 2$

→ consider Floquet unitary: $U_{F,1} = \overset{\downarrow}{e^{-i\ell_0 T Q_0}} e^{-iTH_1} \overset{\downarrow}{e^{+i\ell_0 T Q_0}} e^{-iTH_1} \equiv e^{-i\ell_1 T Q_1}$

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experts

Engineering Hierarchical Symmetries

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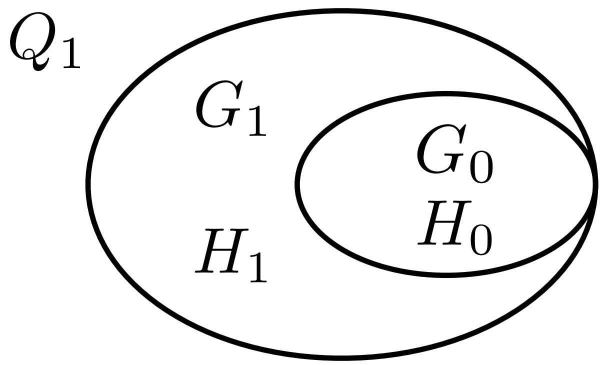

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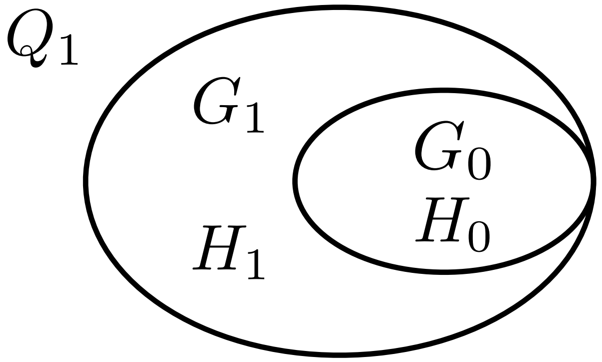
step 2

experts

Engineering Hierarchical Symmetries

- iterative construction



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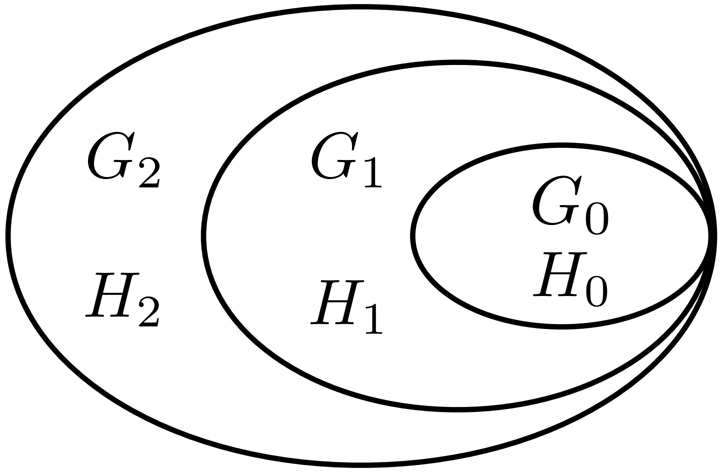

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step 2

→ take new generator H_2 with symmetry $G_2 \supset G_1 \supset G_0$

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experts

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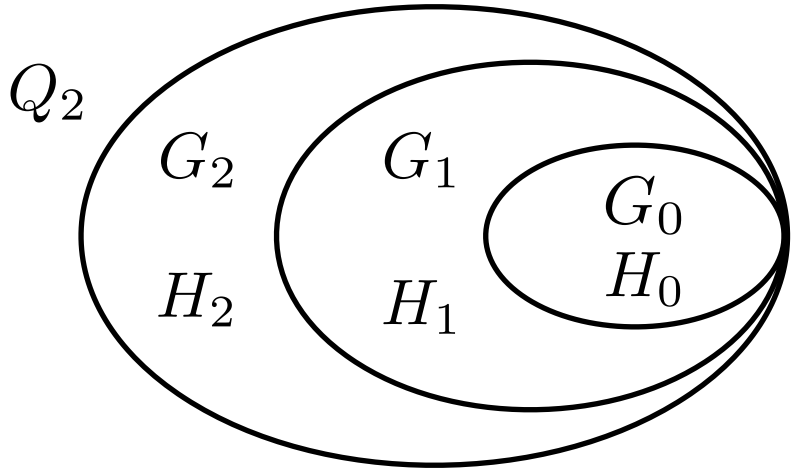

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obeys G_1 obeys G_0 obeys G_0 obeys G_0

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Engineering Hierarchical Symmetries

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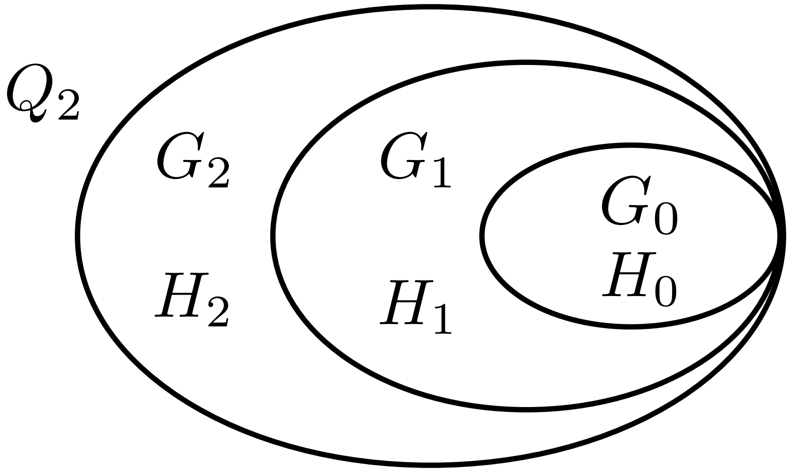

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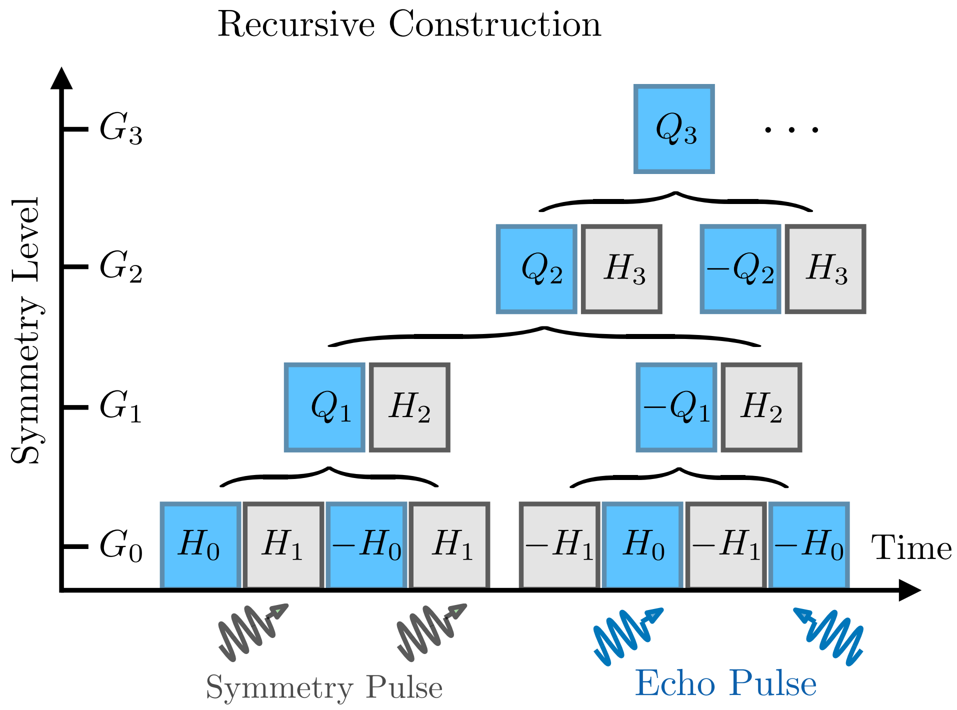
$$= \frac{2}{\ell_2}H_2 + i\frac{2}{\ell_2}T\,[H_2,H_1] - T^2\left(\frac{\ell_0}{\ell_2}[H_2,[H_1,H_0]] + \frac{1}{\ell_2}\left([H_2,[H_2,H_1]] + 2[H_1,[H_1,H_2]]\right)\right) + \cdots$$

obeys G_2 obeys G_1 obeys G_0 obeys G_1 obeys G_1

Engineering Hierarchical Symmetries

- iterative construction $U_{F,n} = e^{-i\ell_{n-1}TQ_{n-1}} e^{-iTH_n} e^{+i\ell_{n-1}TQ_{n-1}} e^{-iTH_n} \equiv e^{-i\ell_n TQ_n}$



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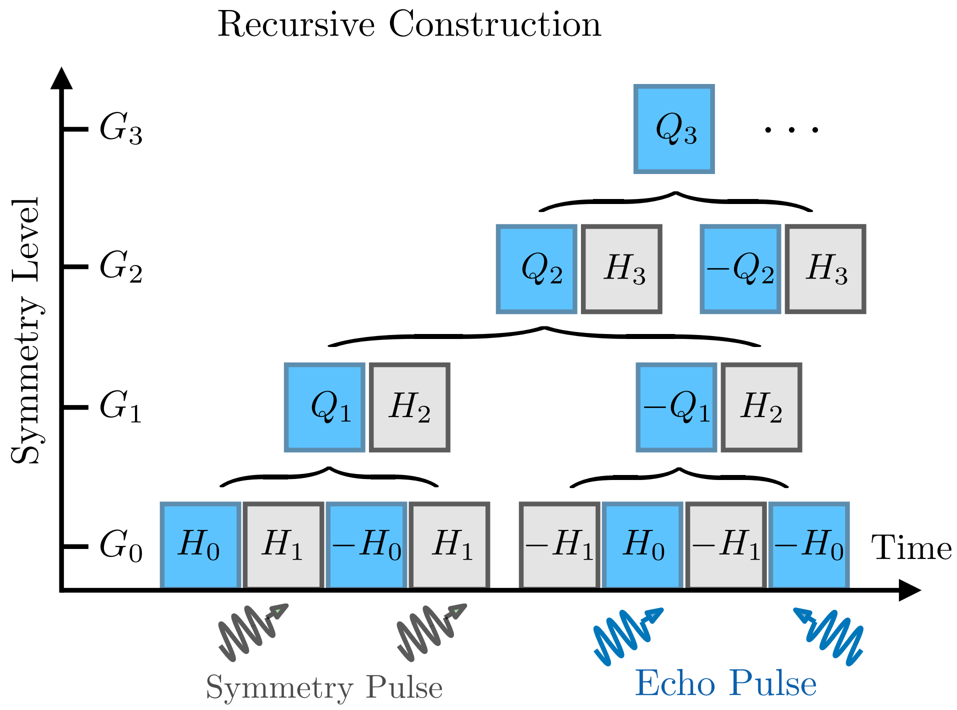
→ sequence length $\ell_n = 3 \times 2^n - 2$
exponential in # of unitaries n

  - not a problem for real-world systems with physical symmetry groups



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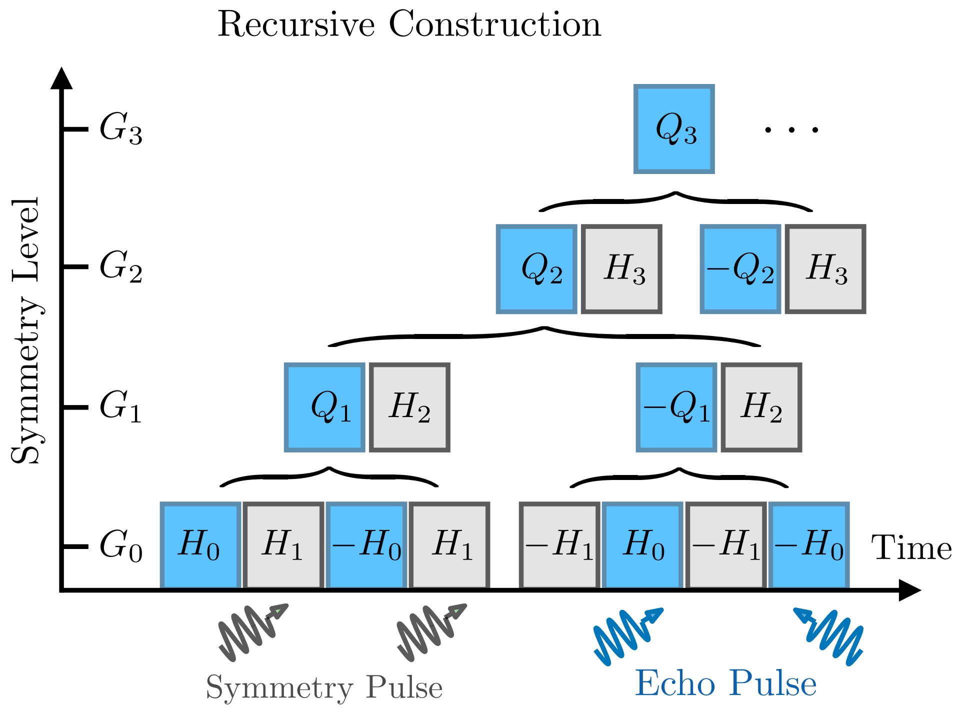


→ shorter sequences exist when additional structure is present

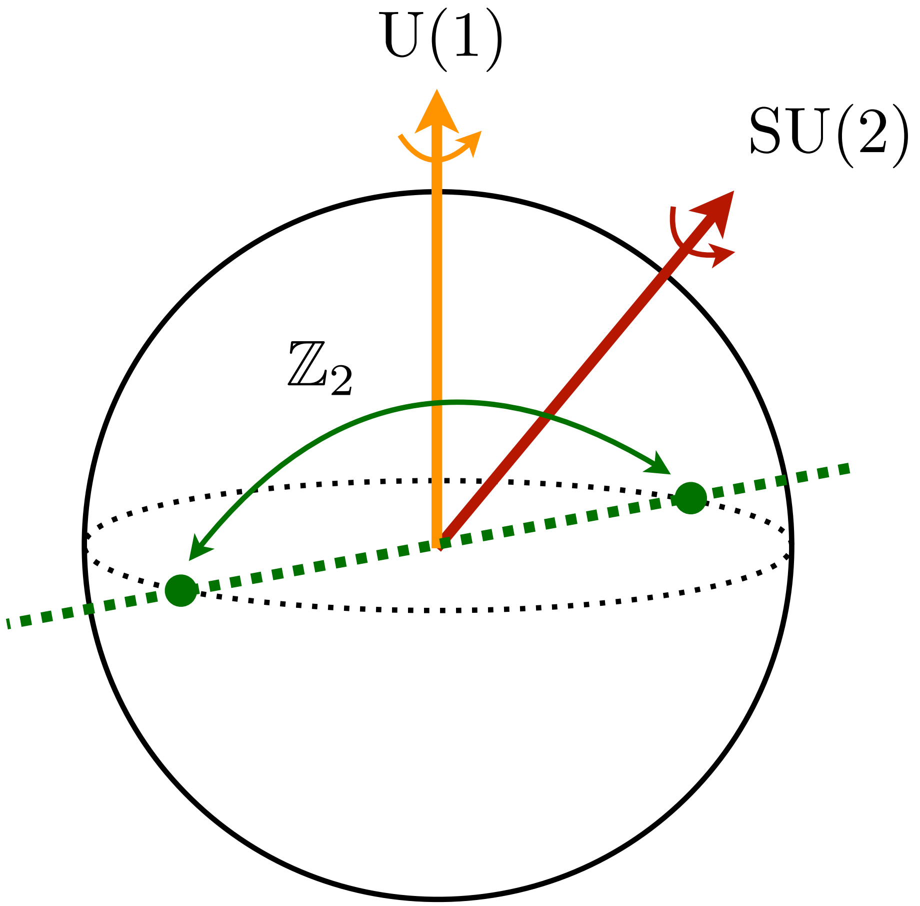
- physical example: $G_2 \supset G_1 \supset G_0$ extra structure: $[H_0, H_1 + H_2] = 0$

$H_2 \quad H_1 \quad H_0$

$$U_F{=}(e^{-iH_0T}e^{-iH_1T})e^{-iH_2T}(e^{iH_0T}e^{iH_1T})e^{-iH_2T}{=}e^{-iTQ}$$

$\ell{=}6 < \ell_2{=}10$

Outline



- Emergent symmetries out of equilibrium

- Engineering Hierarchical Symmetries

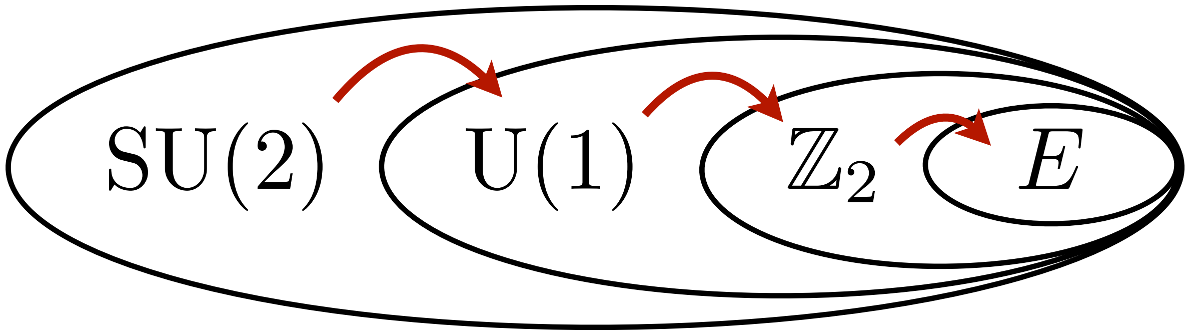
- Applications

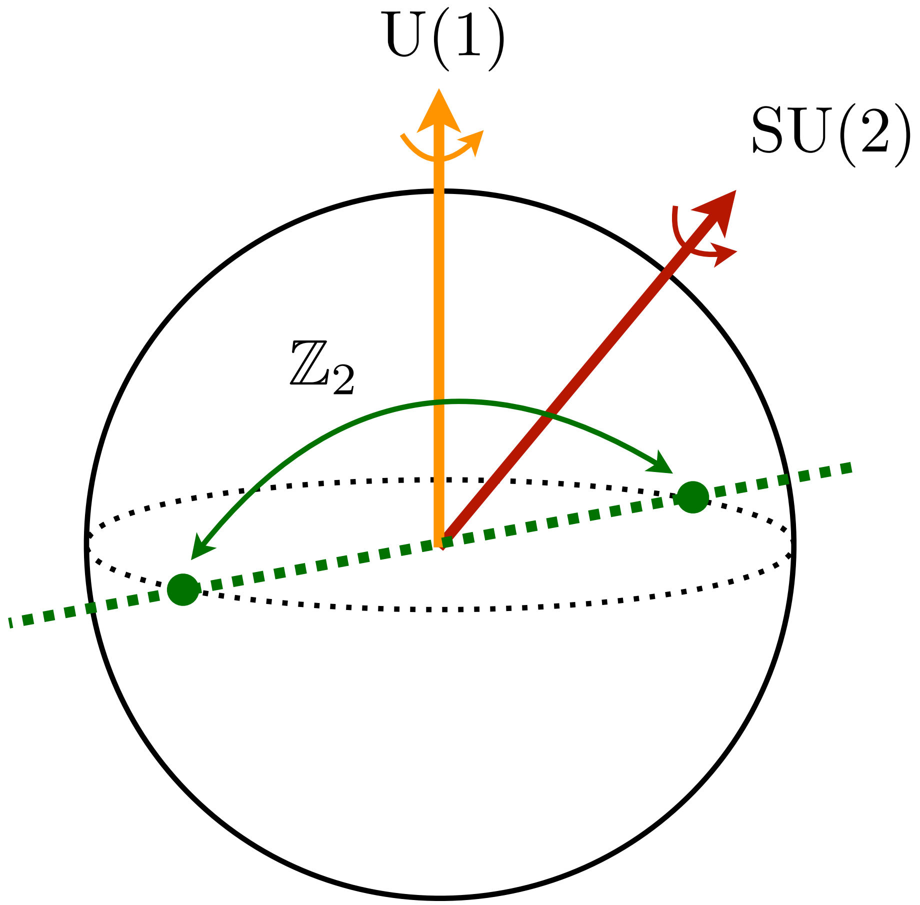
  → abelian & non-abelian symmetry ladder: $\mathrm{SU}(2) \to \mathrm{U}(1) \to \mathbb{Z}_2 \to E$

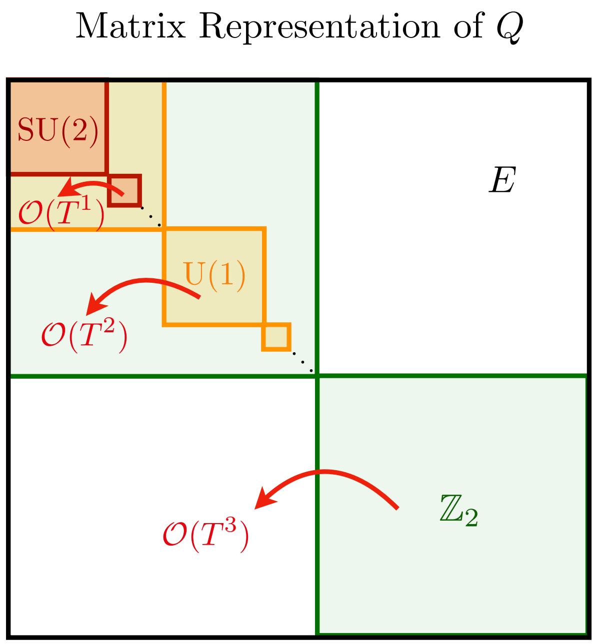
  → nonequilibrium order: discrete time crystals: $\mathbb{Z}_4 \to \mathbb{Z}_2 \to E$

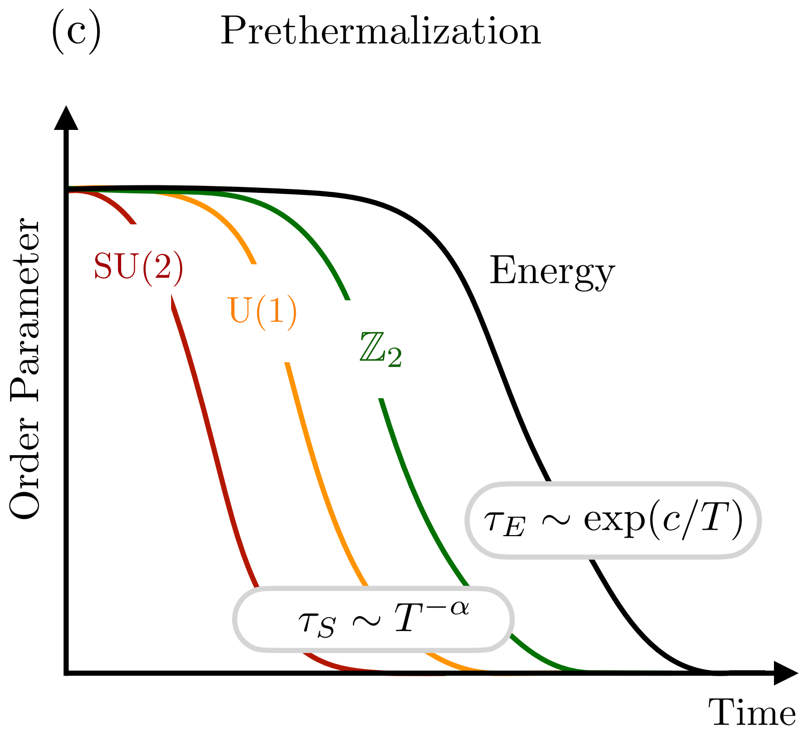
  → higher-order topological insulators: $\mathcal{I} \times \mathcal{T} \to \mathcal{I} \to E$

Hierarchical breaking of abelian & non-abelian symmetries









Hierarchical breaking of abelian & non-abelian symmetries

$$\mathrm{SU}(2) \to \mathrm{U}(1) \to \mathbb{Z}_2 \to E$$

- HSB protocol:

$$U(D_1, ..., D_l|T) = e^{-iD_1T} \cdots e^{-iD_lT}$$

$$\begin{aligned} U_F =& U(-H_0, H_1, H_2, H_0, -H_1, H_2, H_3|T/14) \times \\ & U(-H_2, H_1, -H_0, -H_2, -H_1, H_0, H_3|T/14) \end{aligned}$$

$$\begin{aligned} H_3 &= J \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y + \sigma_i^z \sigma_j^z, \\ H_2 &= J' \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y - \sigma_i^z \sigma_j^z, \\ H_1 &= -J' \sum_{\langle i,j \rangle} \sigma_i^y \sigma_j^y - \sigma_i^z \sigma_j^z, \\ H_0 &= \delta_x \sum_i \sigma_i^x \end{aligned}$$

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effective Hamiltonian	symmetry	order parameter
$Q^{(0)} \propto H_3$:	$\mathrm{SU}(2)$	$\vec{S} = \sum_j \vec{\sigma}_j$
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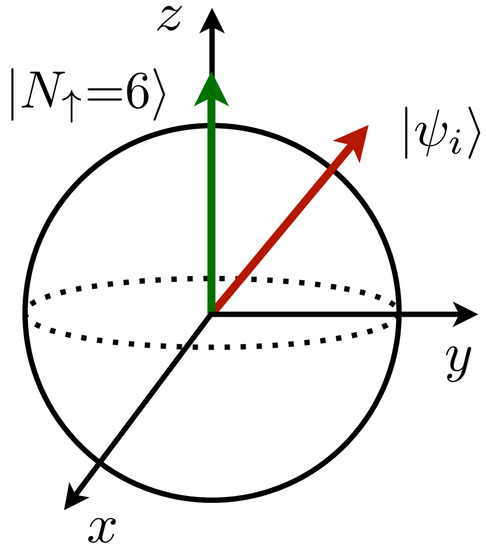
→ initial state

$$|\psi_i\rangle = R_x\left(\frac{\pi}{16}\right)|N_\uparrow = 6\rangle$$

→ normalize obs.

$$\overline{\mathcal{O}} = \langle \mathcal{O}(t)\rangle / \langle \mathcal{O}(0)\rangle$$



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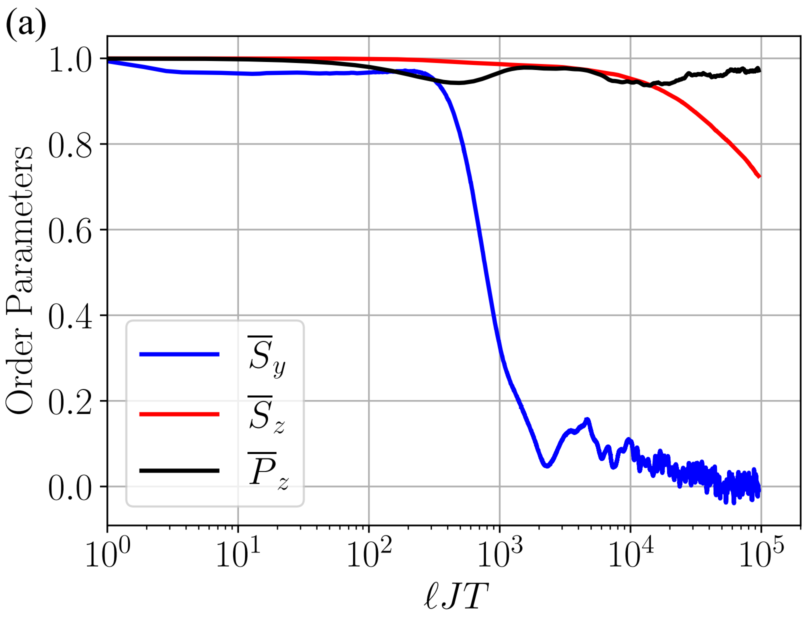

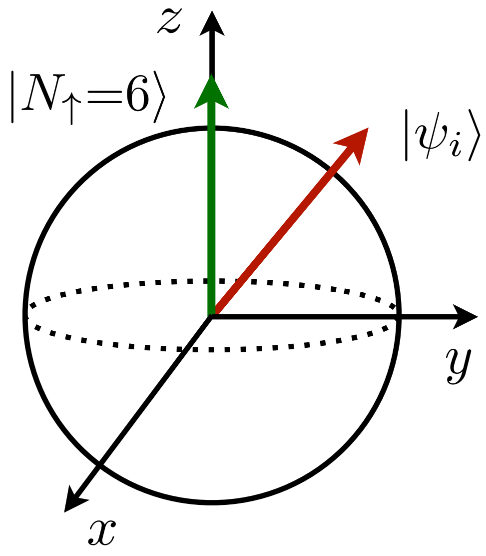

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$$H_0 = \delta_x\sum_i \sigma_i^x$$

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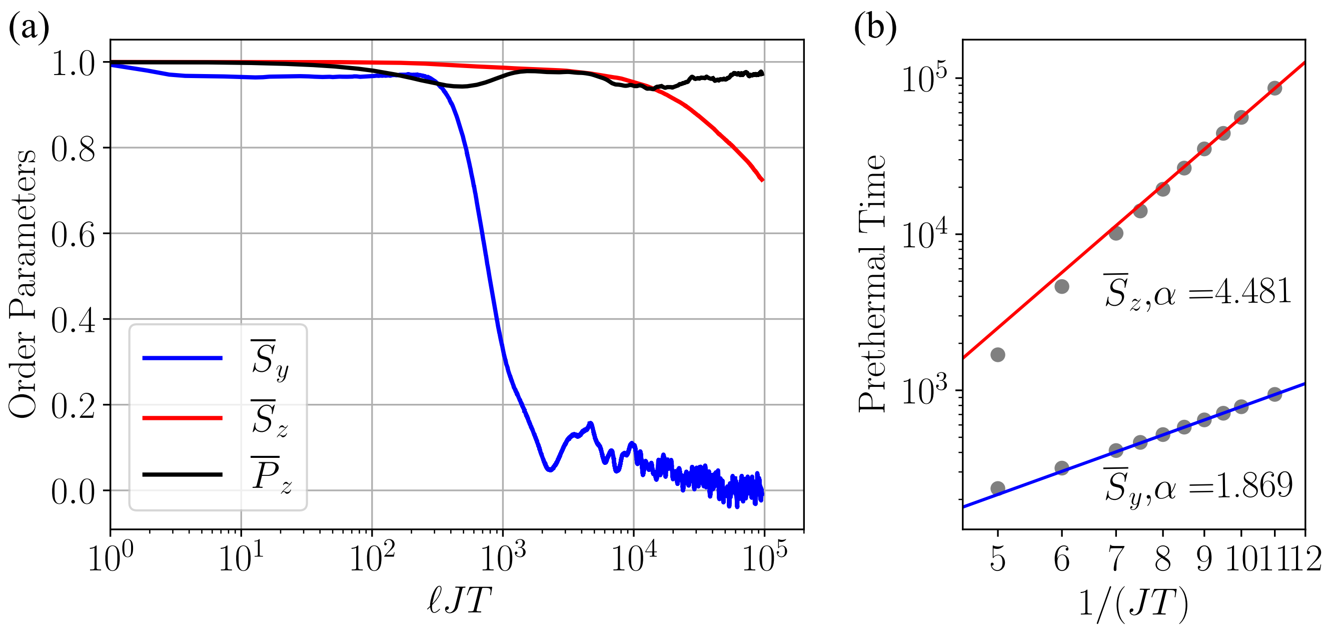
  - → initial state

$$|\psi_i\rangle = R_x\left(\frac{\pi}{16}\right)|N_\uparrow = 6\rangle$$

  - → normalize obs.

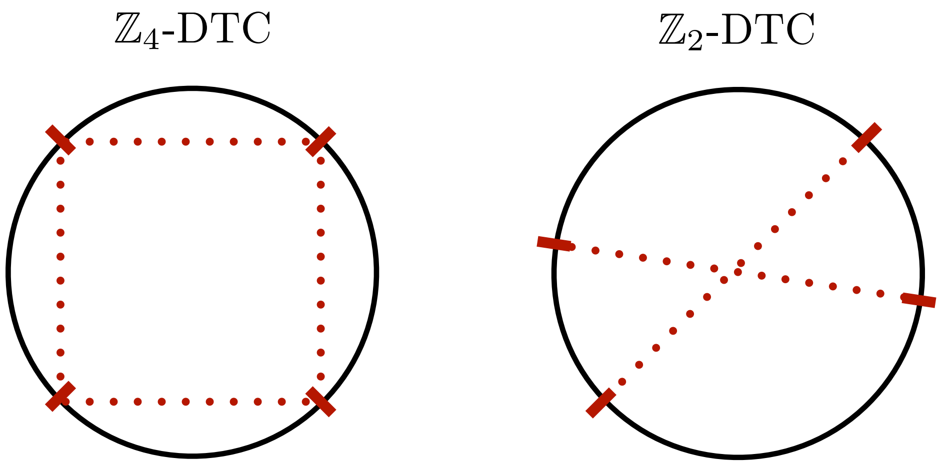
$$\overline{\mathcal{O}} = \langle\mathcal{O}(t)\rangle/\langle\mathcal{O}(0)\rangle$$

Fermi's Golden Rule scaling



Outline

- Emergent symmetries out of equilibrium



- Engineering Hierarchical Symmetries

- Applications

  - ⟶ abelian & non-abelian symmetry ladder: $\mathrm{SU}(2) \to \mathrm{U}(1) \to \mathbb{Z}_2 \to E$
  - ⟶ nonequilibrium order: discrete time crystals: $\mathbb{Z}_4 \to \mathbb{Z}_2 \to E$
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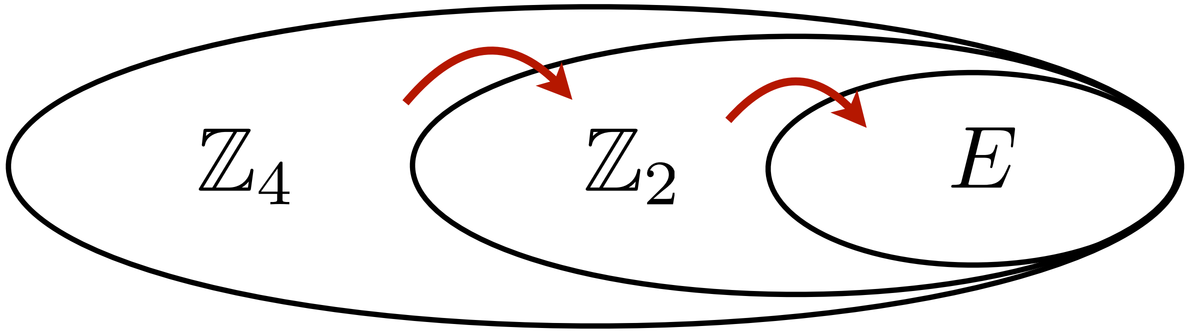
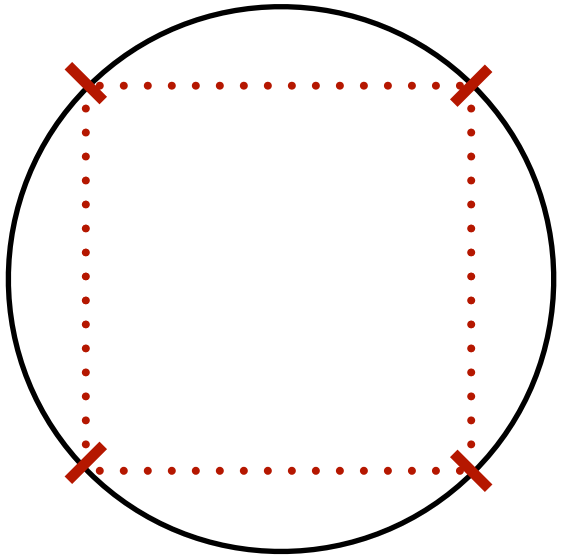
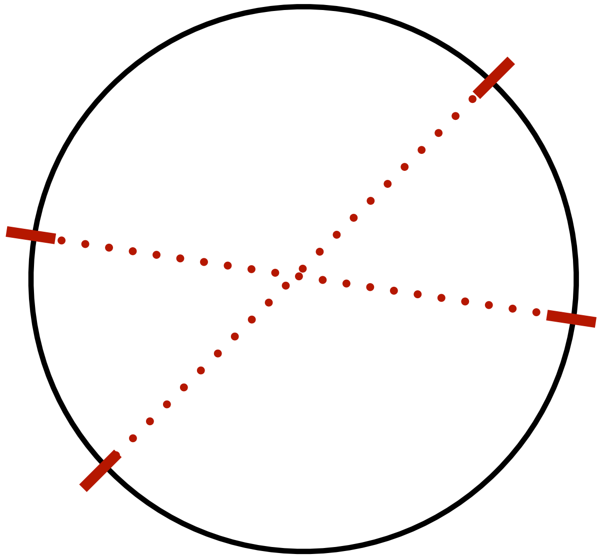
Nonequilibrium order: discrete time crystals



$\mathbb{Z}_4$-DTC



$\mathbb{Z}_2$-DTC



Nonequilibrium order: discrete time crystals

$$\mathbb{Z}_4 \to \mathbb{Z}_2 \to E$$

$$Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \qquad X = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

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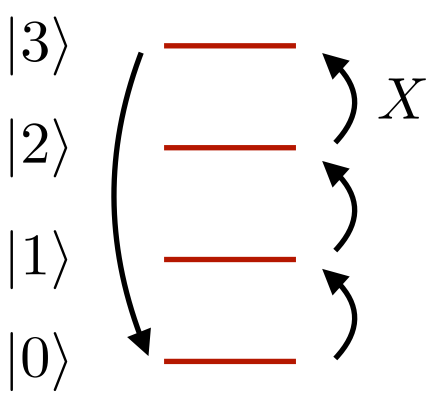
$$\boxed{\begin{aligned} H_0 &= \sum_{i=1}^{L} b_i (Z_i + Z_i^\dagger) \\ H_1 &= \sum_{\langle i,j \rangle} J_{ij} \left(Z_i^2 Z_j^2 - \eta (e^{i\phi} Z_i^\dagger Z_j + \text{h.c.}) \right) \\ &\quad + \sum_i h_i \left(Z_i^2 - \frac{1}{2} \left(X_i + X_i^\dagger \right) \right) + \sum_i g_i X_i^2 \end{aligned}}$$

4-level system



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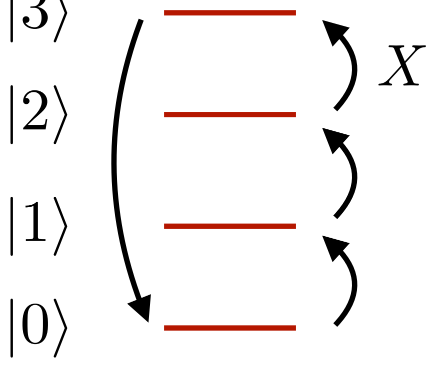

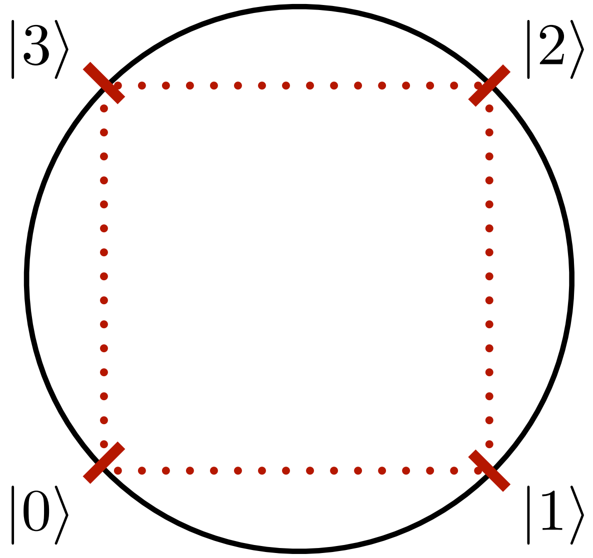

Marin Bukov

Surace et al, PRB 99 104303 (2019)

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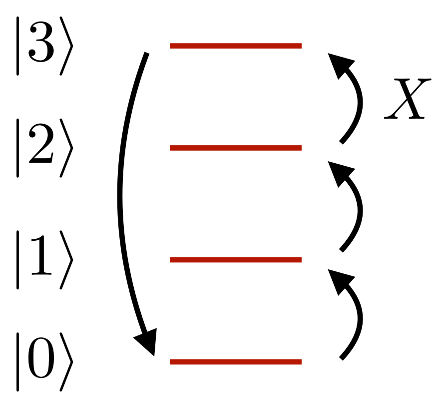

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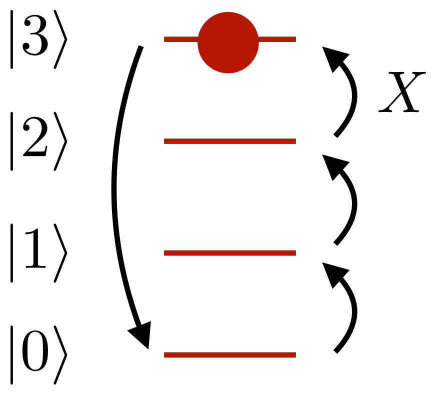

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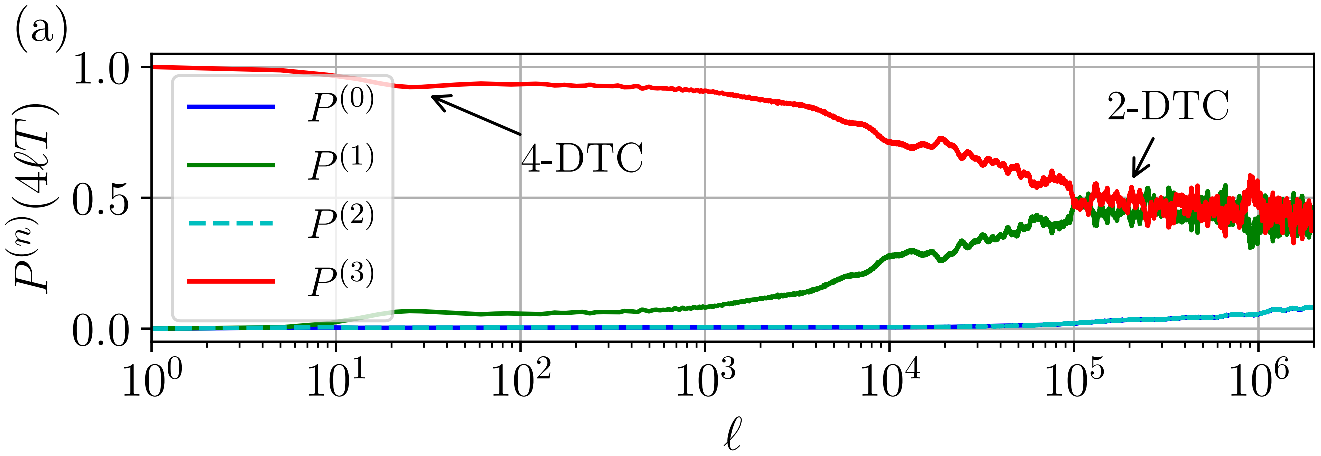

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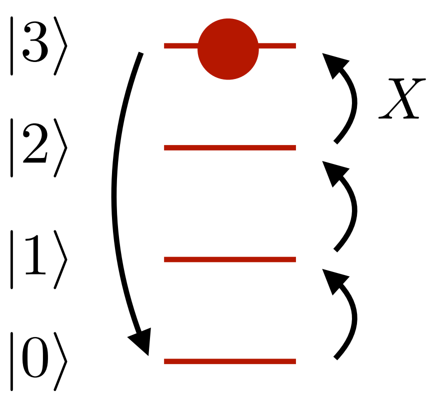

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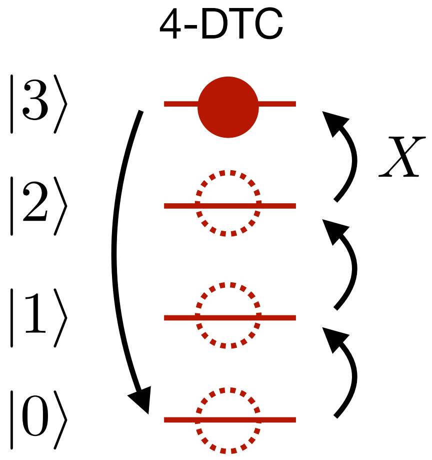

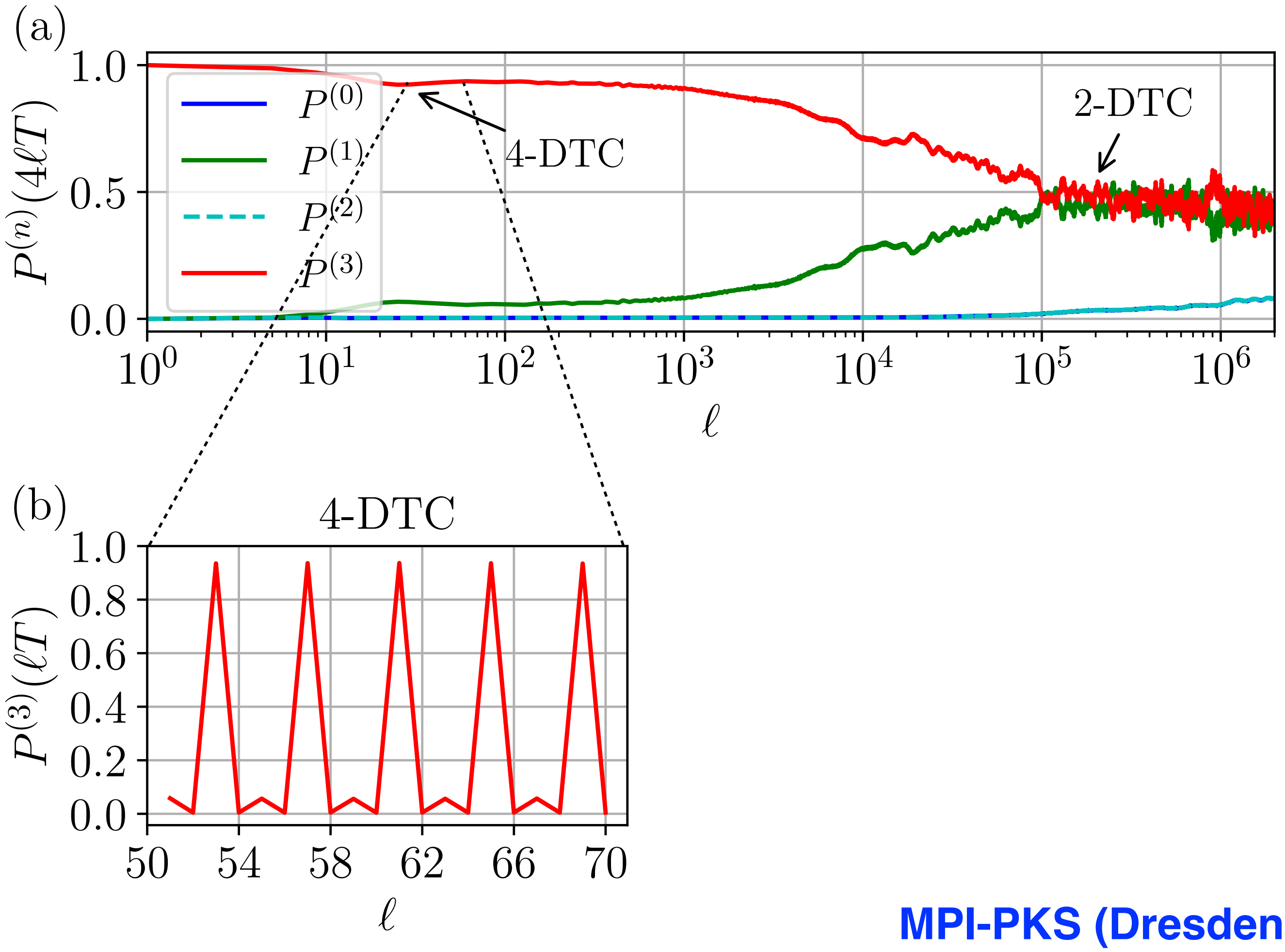

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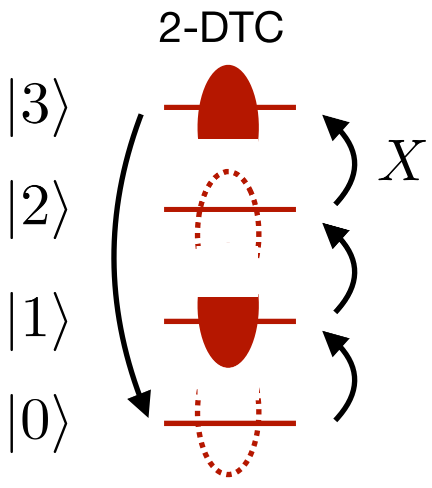

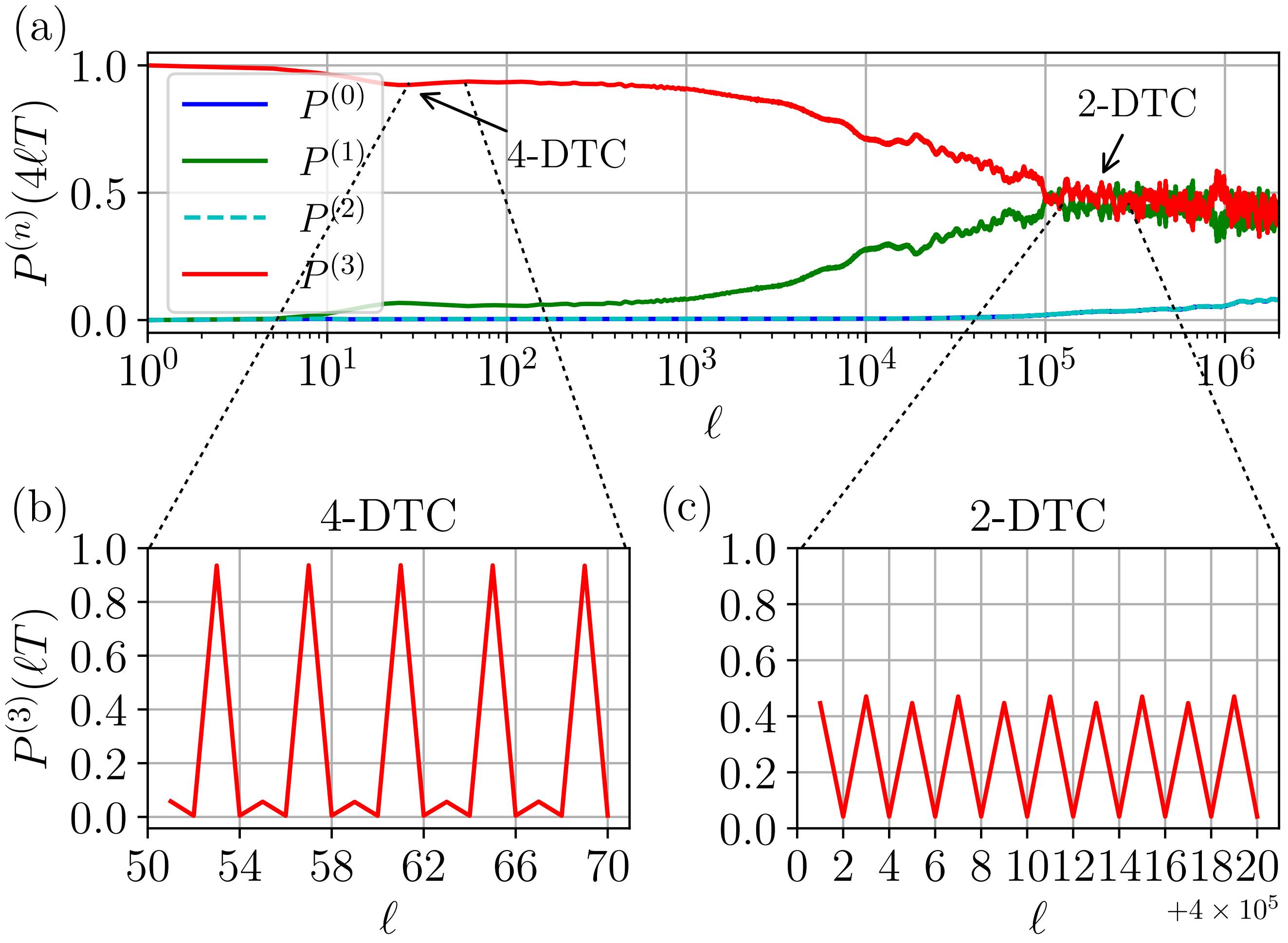

Outline

- Emergent symmetries out of equilibrium

- Engineering Hierarchical Symmetries

- Applications

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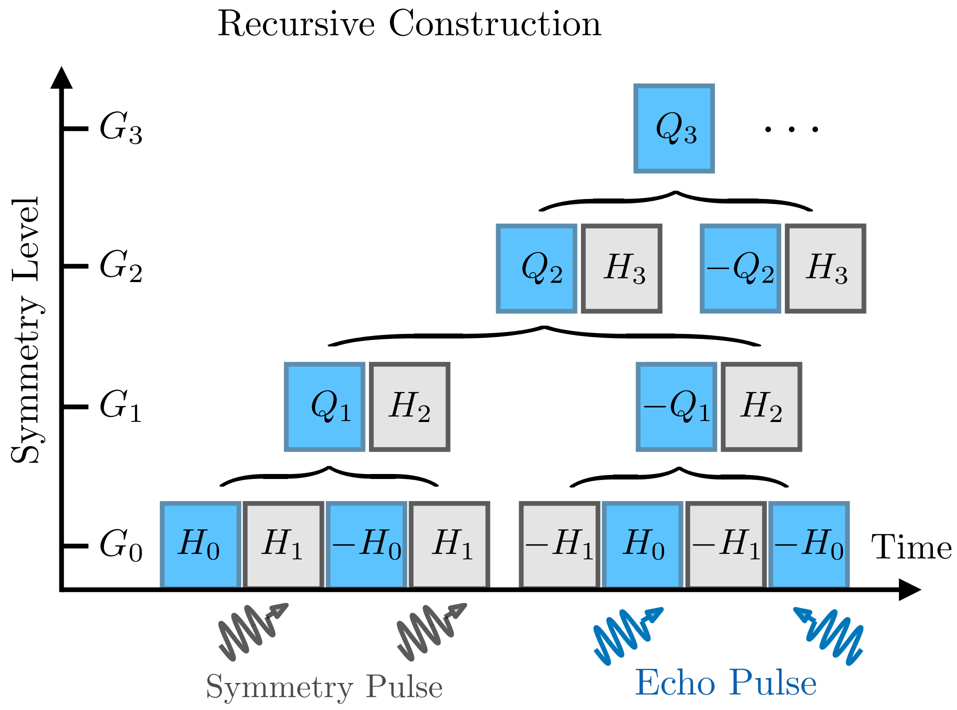
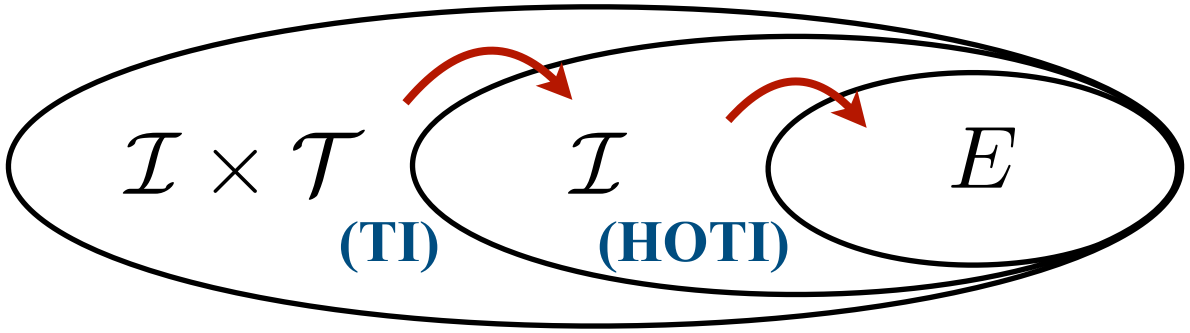
  → higher-order topological insulators: $\mathcal{I} \times \mathcal{T} \to \mathcal{I} \to E$



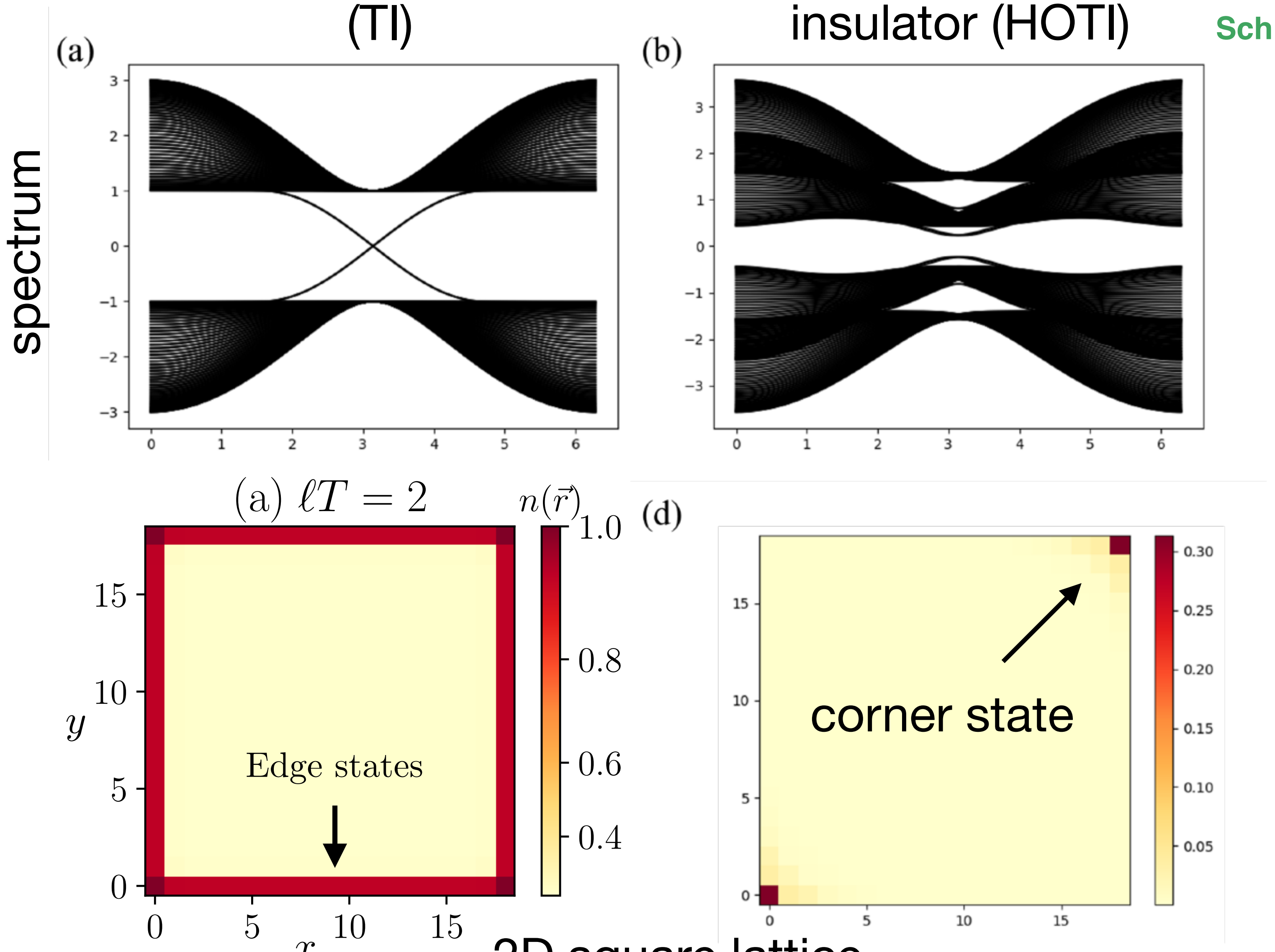
Higher order topological insulators



topological insulator (TI)

higher-order topological insulator (HOTI)

Schindler et al, Science Adv. 4 (2018)



spectrum

corner state

2D square lattice

Higher order topological insulators

$$\mathcal{I}\times\mathcal{T}\to\mathcal{I}\to E$$

2D square lattice

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$$U_F = U\left(H_0, \frac{H_1}{2}, \frac{H_1'}{2}, -H_0, \frac{H_1}{2}, \frac{H_1'}{2}, H_2 \middle| \frac{T}{10}\right)\times$$
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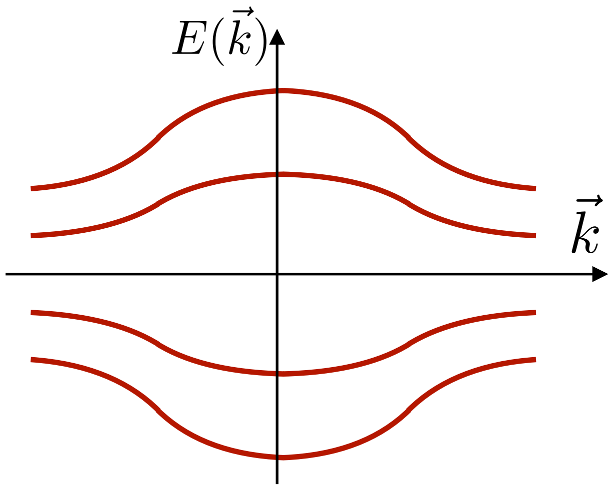

4-band TI model

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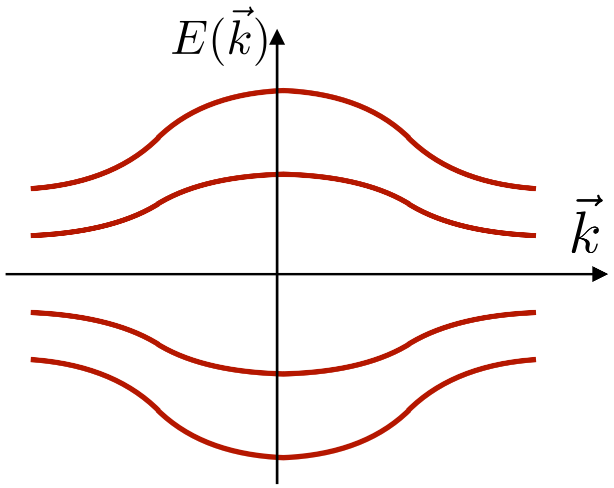

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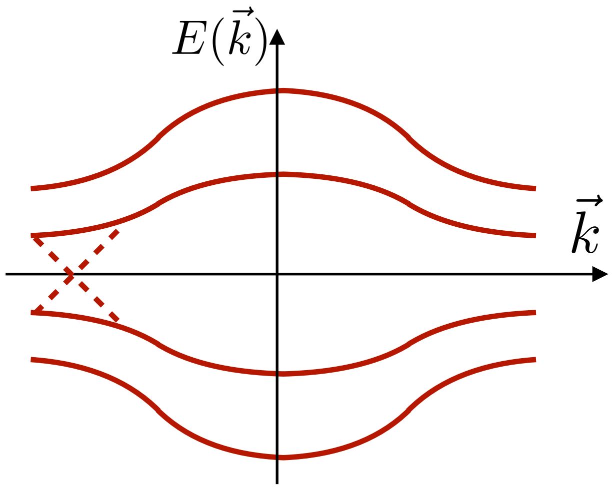

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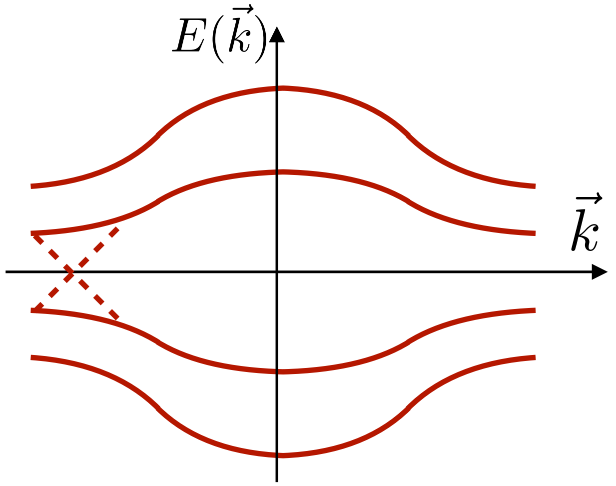

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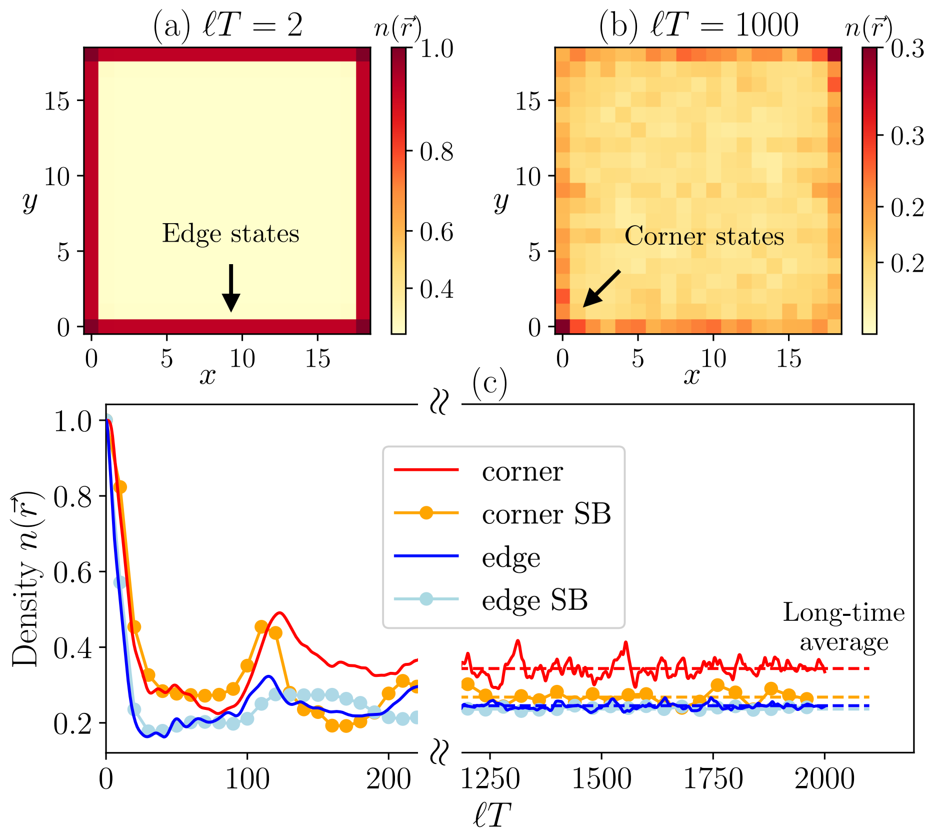

Marin Bukov

Hunting for emergent (wannabe) Goldstone modes

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Hunting for emergent (wannabe) Goldstone modes

- symmetry ladder: $\boxed{\mathrm{SU}(2) \to \mathrm{U}(1)} \to \mathbb{Z}_2 \to E$ ⟶ focus on continuous symmetry breaking
- break continuous symmetry: gapless Goldstone bosons ⟶ magnons (CM)

  ⟶ but we don't have an exact symmetry… $Q^{(1)} : \mathrm{SU}(2) \to \mathrm{U}(1)$

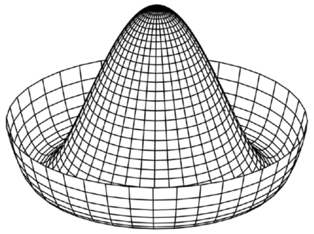
image: Wikipedia

Hunting for emergent (wannabe) Goldstone modes

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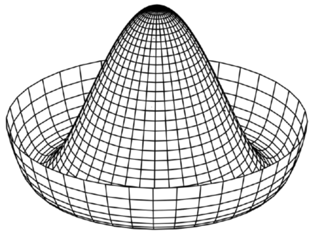

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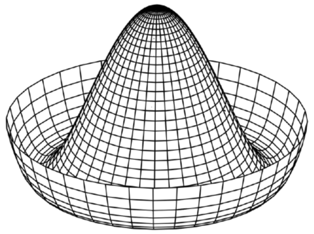
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$$\begin{aligned} U_F =& U(-H_0, H_1, H_2, H_0, -H_1, H_2, H_3|T/14)\times \\ & U(-H_2, H_1, -H_0, -H_2, -H_1, H_0, H_3|T/14) \end{aligned}$$

$$\begin{aligned} H_3 &= J \sum_{\langle i,j \rangle} \sigma^x_i\sigma^x_j + \sigma^y_i\sigma^y_j + \sigma^z_i\sigma^z_j, \\ H_2 &= J' \sum_{\langle i,j \rangle} \sigma^x_i\sigma^x_j + \sigma^y_i\sigma^y_j - \sigma^z_i\sigma^z_j, \\ H_1 &= -J' \sum_{\langle i,j \rangle} \sigma^y_i\sigma^y_j - \sigma^z_i\sigma^z_j, \\ H_0 &= \delta_x \sum_i \sigma^x_i \end{aligned}$$

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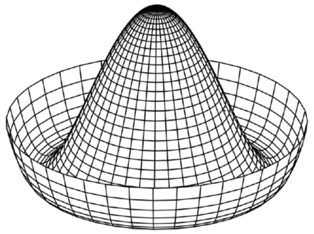

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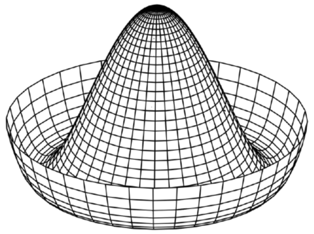

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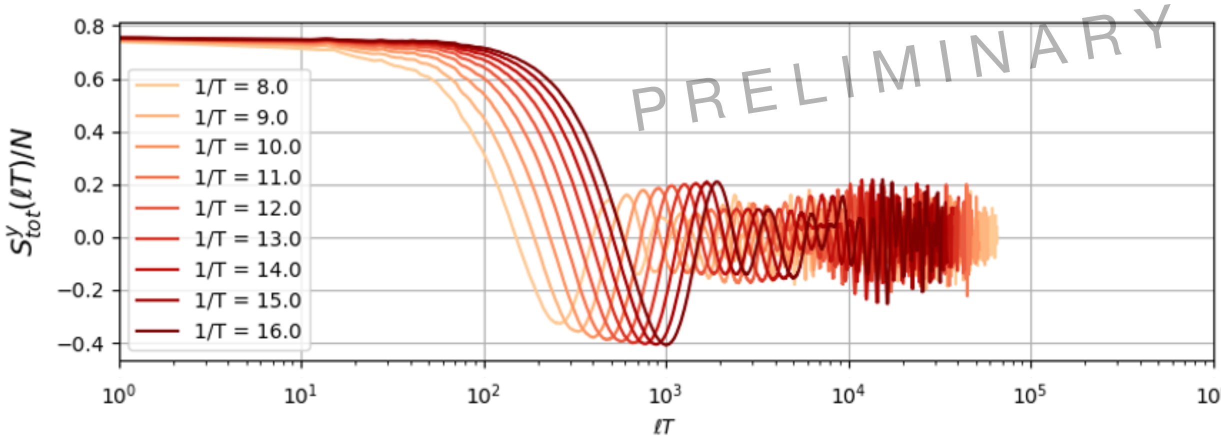
⟶ what sets lifetime of the pseudo Goldstone?

  - intrinsic decay due to finite gap $\tau \propto T$
  - external decay due to finite duration of U(1) prethermal plateau $\tau \propto T^2$



Outlook


MPI-PKS, Dresden

$$U_{F,n} = e^{-i\ell_{n-1}TQ_{n-1}} e^{-iTH_n} e^{+i\ell_{n-1}TQ_{n-1}} e^{-iTH_n} \equiv e^{-i\ell_n TQ_n}$$

work in progress

- engineering hierarchical symmetries:
  - ⟶ continuous/discrete, abelian/non-abelian, topological symmetries
  - ⟶ easy to implement in experiment: mechanism akin to spin echo / dynamical decoupling
    - allows to investigate equilibrium/nonequilibirum order in a single time evolution
  - ⟶ controllable lifetime of pre thermal plateaus: Fermi's Golden rule scaling

- generic construction:
  - ⟶ any species: fermions, boson, spins (interacting & noninteracting)
  - ⟶ classical & quantum systems; *open direction:* open systems
  - ⟶ quasi-periodic & random-multipolar drives (not limited to Floquet systems)
    - sufficient condition: existence of approximate effective Hamiltonian
  - ⟶ continuous drives (Magnus expansion generalizes BCH formula)
  - ⟶ *open direction:* local gauge symmetries

thanks for the attention!

www.pks.mpg.de/nqd

Higher order topological insulators

$$\mathcal{I}\times\mathcal{T}\to\mathcal{I}\to E$$

$$H_j = \sum_{\vec{k}} \psi^\dagger_{\vec{k}} H_j(\vec{k}) \psi_{\vec{k}}$$

$$H_2(\vec{k}) = [M + J(\cos k_x + \cos k_y)]\ \tau_z\sigma_0$$

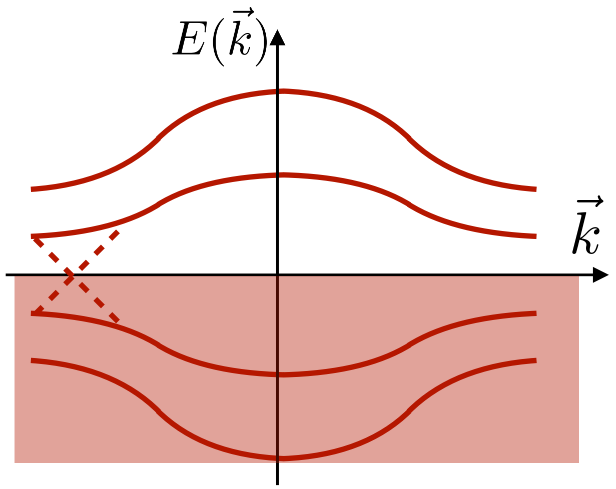
$$H_1(\vec{k}) = \Delta_1\tau_z(\sigma_x+\sigma_y),$$

$$H_1'(\vec{k}) = \Delta_1\tau_z\sigma_z,$$

$$H_0(\vec{k}) = \Delta_2\tau_x\sigma_y$$

time-rev'sal $\mathcal{T}: \vec{k}\to-\vec{k},\ \vec{\sigma}\to-\vec{\sigma}$

inversion $\mathcal{I}: \vec{k}\to-\vec{k},\ \tau_{x/y}\to-\tau_{x/y}$



4-band TI model

- dynamics:

  → initial state: corner state

$$|\psi_i\rangle = |\mathrm{GS}(H_2)\rangle$$