



UNIVERSITÀ  
DI TORINO



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# Introduction to transverse momentum imaging

*lecture 5*

*International school on  
probing hadron structure at the EIC*

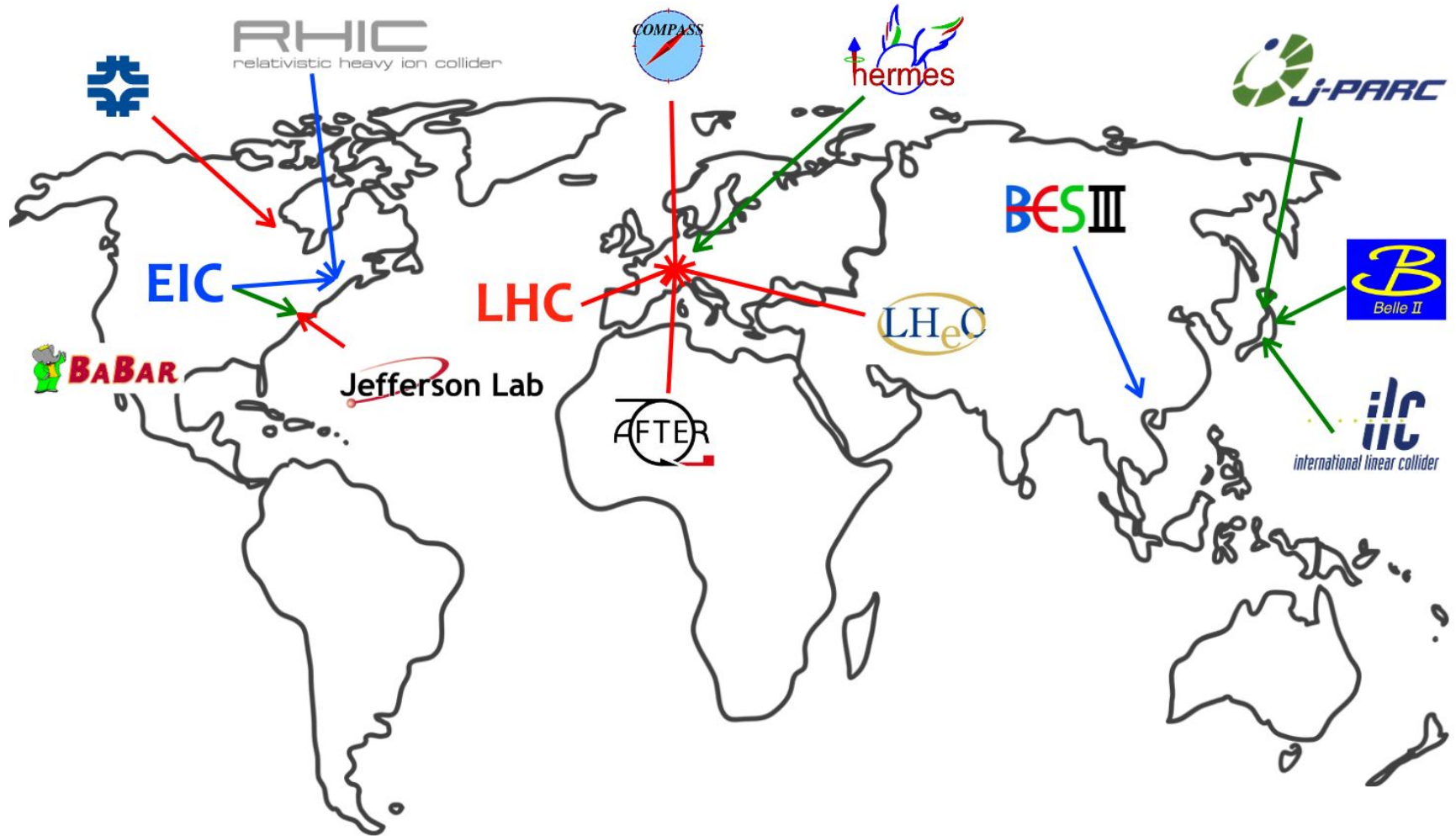
*ICTS, Bangalore  
February 1, 2024*

# Plan of these lectures

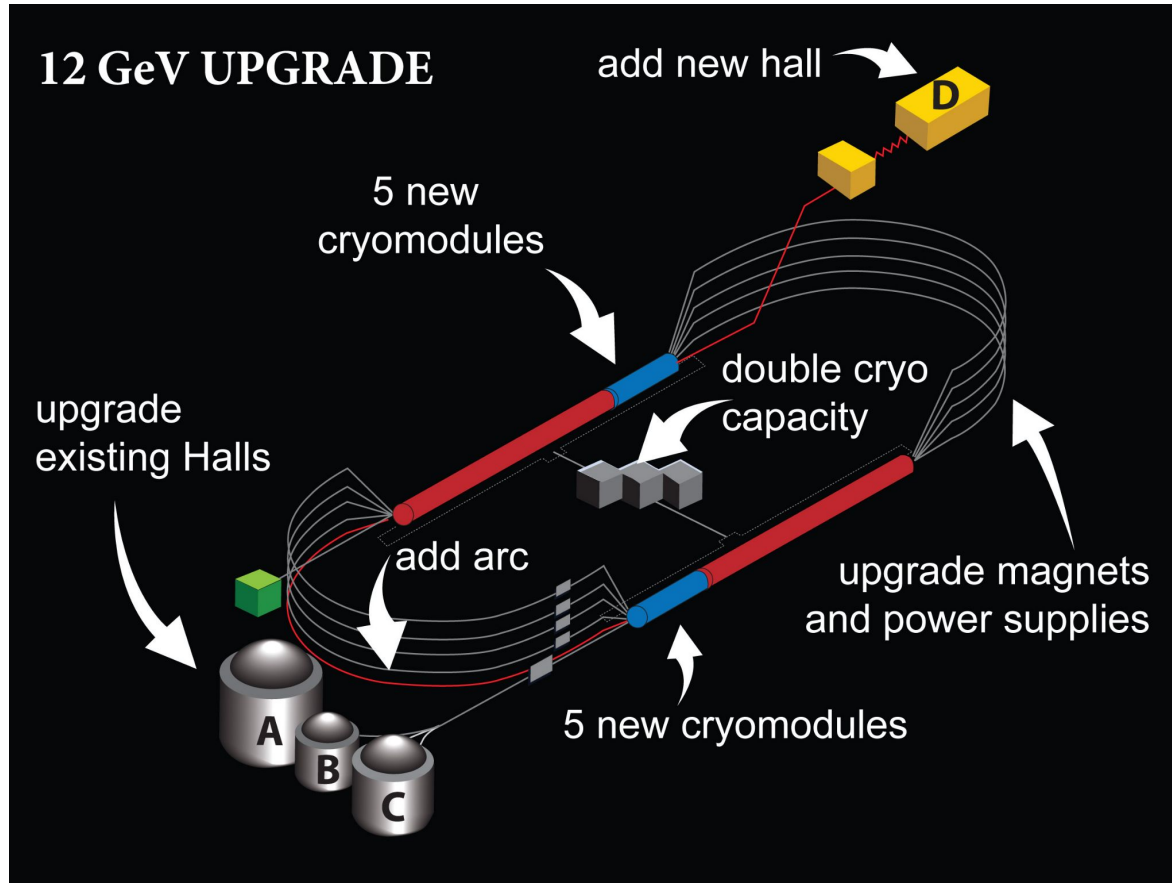
1. **Breaking hadrons**
2. **Non-collinear partons**
3. **Symmetries & spin**
4. **Factorization, evolution,  
matching**
5. **Phenomenology**

# 5.1 Experiments

(a short selection)



# CEBAF at Jefferson Lab



CEBAF:

Continuous Electron  
Beam  
Accelerator Facility

Built in 1984,  
recently completed a  
major upgrade  
from 6 GeV to 12 GeV  
+ one new hall



- Hall A & C: hadron structure, high luminosity
- Hall B: hadron structure,  $4\pi$  coverage
- Hall D: hadron spectroscopy

# The Electron-Ion Collider

- 00 home
- 01 about
- 02 goals
- 03 design
- 04 benefits
- 05 status
- 06 news

about

benefits

goals

status

design

news



The Electron-Ion Collider is a proposed machine for delving deeper than ever before into the building blocks of matter, so that we may better understand the matter within us and its role in the universe around us..

<https://www.jlab.org/eic>



## Precision 3D imaging of protons and nuclei

An Electron-Ion Collider will take three-dimensional precision snapshots of the internal structure of protons and atomic nuclei.

00 home

01 about

02 goals

03 design

04 benefits

05 status

06 news



## Solving the Mystery of Proton Spin

An EIC would reveal how the teeming quarks and gluons inside the proton combine their spins to generate the proton's overall spin.



## Search for Saturation

A unique form of matter, the color glass condensate, may be produced for study for the first time by an EIC, providing deeper insight into gluons and their interactions.



## Quark and Gluon Confinement

Experiments at an EIC would cast fresh light on the mystery of why quarks or gluons can never be observed in isolation but must remain confined within protons and nuclei.



# A fixed-target program at the LHC



Contents lists available at [ScienceDirect](#)

## Physics Reports

journal homepage: [www.elsevier.com/locate/physrep](http://www.elsevier.com/locate/physrep)



## A fixed-target programme at the LHC: Physics case and projected performances for heavy-ion, hadron, spin and astroparticle studies



C. Hadjidakis<sup>1,a</sup>, D. Kikoła<sup>2,a</sup>, J.P. Lansberg<sup>1,\*a</sup>, L. Massacrier<sup>1,a</sup>,  
M.G. Echevarria<sup>3,4,b</sup>, A. Kusina<sup>5,b</sup>, I. Schienbein<sup>6,b</sup>, J. Seixas<sup>7,8,9,b</sup>, H.S. Shao<sup>10,b</sup>,  
A. Signori<sup>11,3,12,b</sup>, B. Trzeciak<sup>13,14,b</sup>, S.J. Brodsky<sup>15</sup>, G. Cavoto<sup>16</sup>, C. Da Silva<sup>17</sup>,  
F. Donato<sup>18</sup>, E.G. Ferreira<sup>19,20</sup>, I. Hřivnáčová<sup>1</sup>, A. Klein<sup>17</sup>, A. Kurepin<sup>21</sup>,  
C. Lorcé<sup>22</sup>, F. Lyonnet<sup>23</sup>, Y. Makdisi<sup>24</sup>, S. Porteboeuf Houssais<sup>25</sup>, C. Quintans<sup>8</sup>,  
A. Rakotozafindrabe<sup>26</sup>, P. Robbe<sup>1</sup>, W. Scandale<sup>27</sup>, N. Topilskaya<sup>21</sup>, A. Uras<sup>28</sup>,  
J. Wagner<sup>29</sup>, N. Yamanaka<sup>1,32,30,31</sup>, Z. Yang<sup>33</sup>, A. Zelenski<sup>24</sup>

<https://doi.org/10.1016/j.physrep.2021.01.002>

## 5.2 Collinear PDFs

# PDFs: what do we know

See <https://inspirehep.net/literature/1801417>

2020 PDFLATTICE REPORT

5

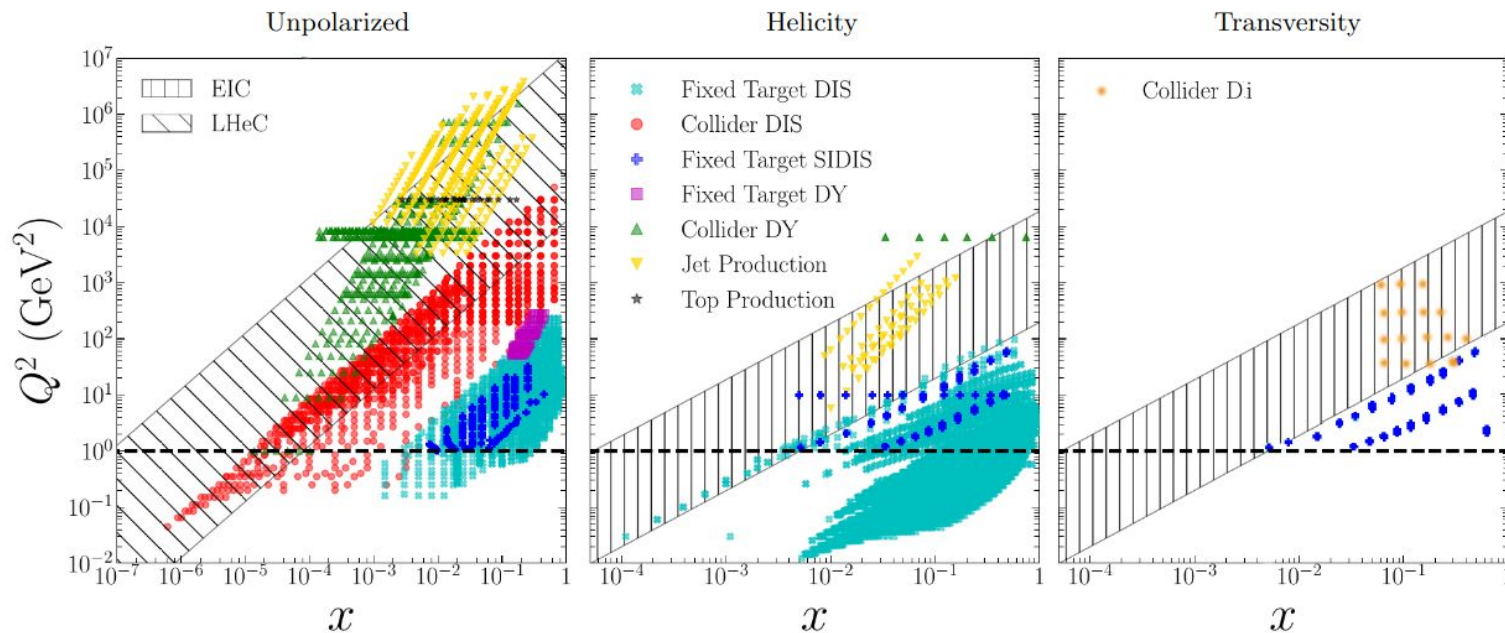


FIG. 1 The kinematic coverage in the  $(x, Q^2)$  plane of the hadronic cross-section data for the processes commonly included in global QCD analyses of collinear unpolarized, helicity, and transversity PDFs. The extended kinematic ranges attained by the LHeC and the EIC are also displayed. See Fig. 1 of Ref. (Ethier and Nocera, 2020) for unpolarized nuclear PDFs.

# PDFs: unpolarized

See <https://inspirehep.net/literature/1801417>

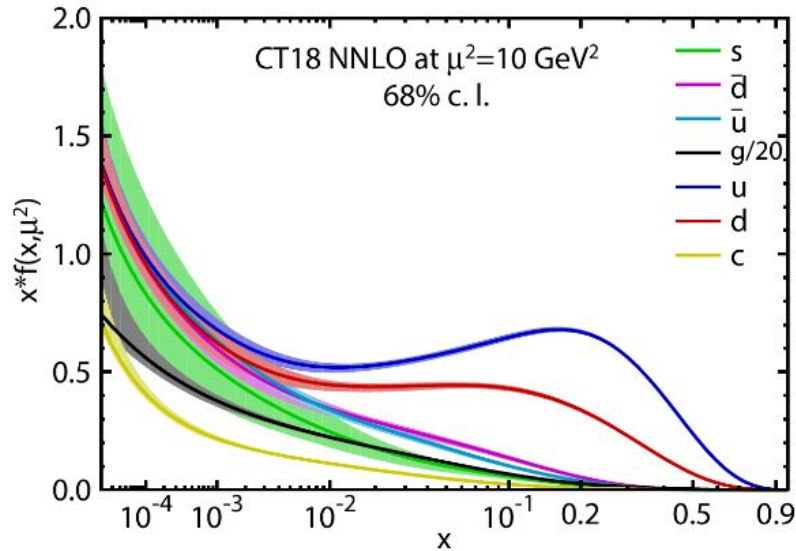


FIG. 2 The CT18 PDFs at  $\mu^2 = 10 \text{ GeV}^2$  for the  $xu$ ,  $x\bar{u}$ ,  $xd$ ,  $x\bar{d}$ ,  $xs = x\bar{s}$ , and  $xg$  PDFs. Error bands correspond to the 68% confidence level. Figure from (Kovářík *et al.*, 2019).

Many extractions available  
See e.g. the LHAPDF library  
<https://lhapdf.hepforge.org/>

A similar repository exists for TMDs:  
<https://tmdlib.hepforge.org/>

# PDFs: helicity

See <https://inspirehep.net/literature/1801417>

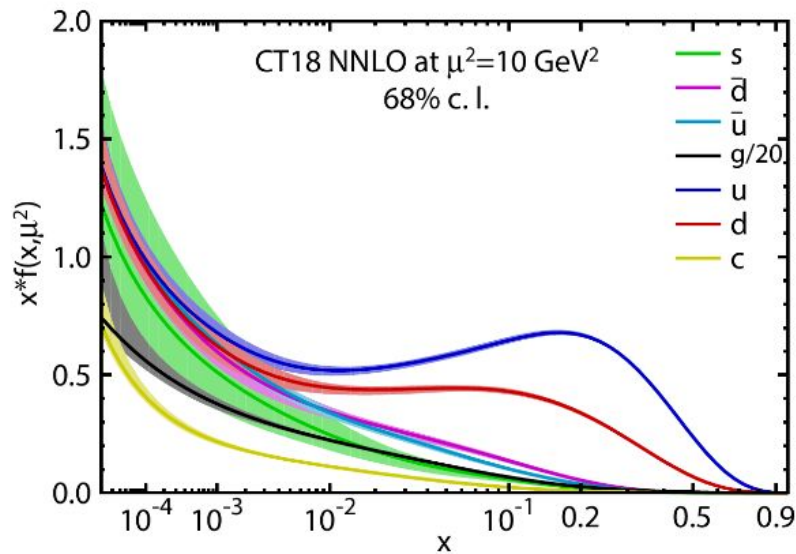


FIG. 2 The CT18 PDFs at  $\mu^2 = 10 \text{ GeV}^2$  for the  $xu$ ,  $x\bar{u}$ ,  $xd$ ,  $x\bar{d}$ ,  $xs = x\bar{s}$ , and  $xg$  PDFs. Error bands correspond to the 68% confidence level. Figure from (Kovářík *et al.*, 2019).

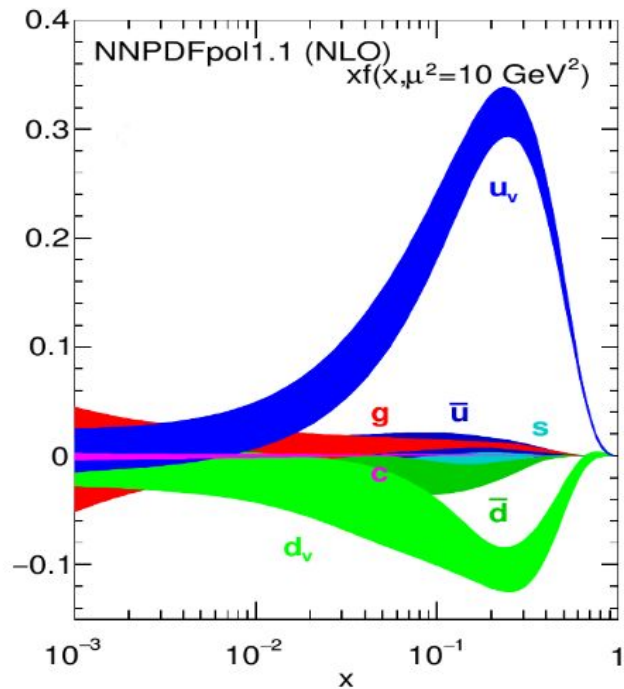


FIG. 5 The helicity PDFs from the NNPDFPOL1.1 parton set at  $\mu^2 = 10 \text{ GeV}^2$ . Figure from (Tanabashi *et al.*, 2018).

# PDFs: transversity

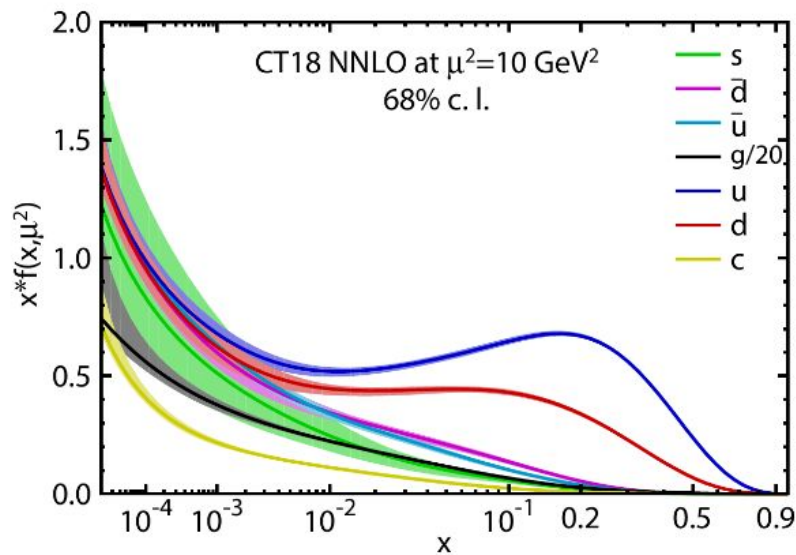


FIG. 2 The CT18 PDFs at  $\mu^2 = 10 \text{ GeV}^2$  for the  $xu$ ,  $x\bar{u}$ ,  $xd$ ,  $x\bar{d}$ ,  $xs = x\bar{s}$ , and  $xg$  PDFs. Error bands correspond to the 68% confidence level. Figure from (Kovářík *et al.*, 2019).

See <https://inspirehep.net/literature/1801417>

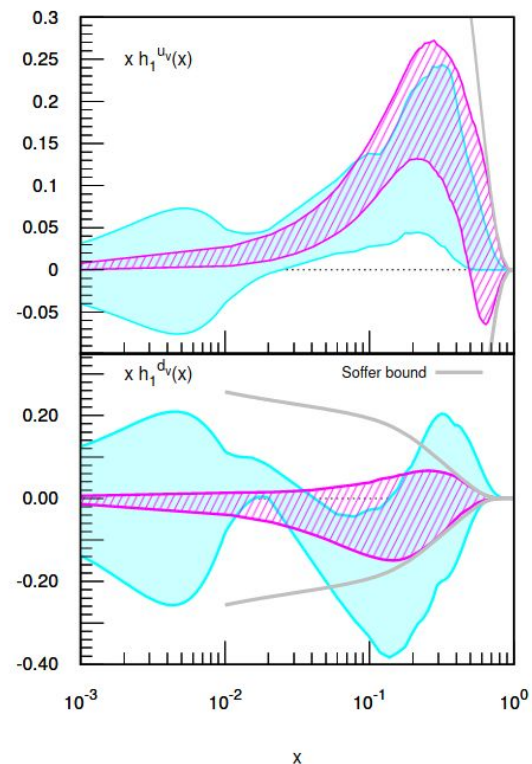
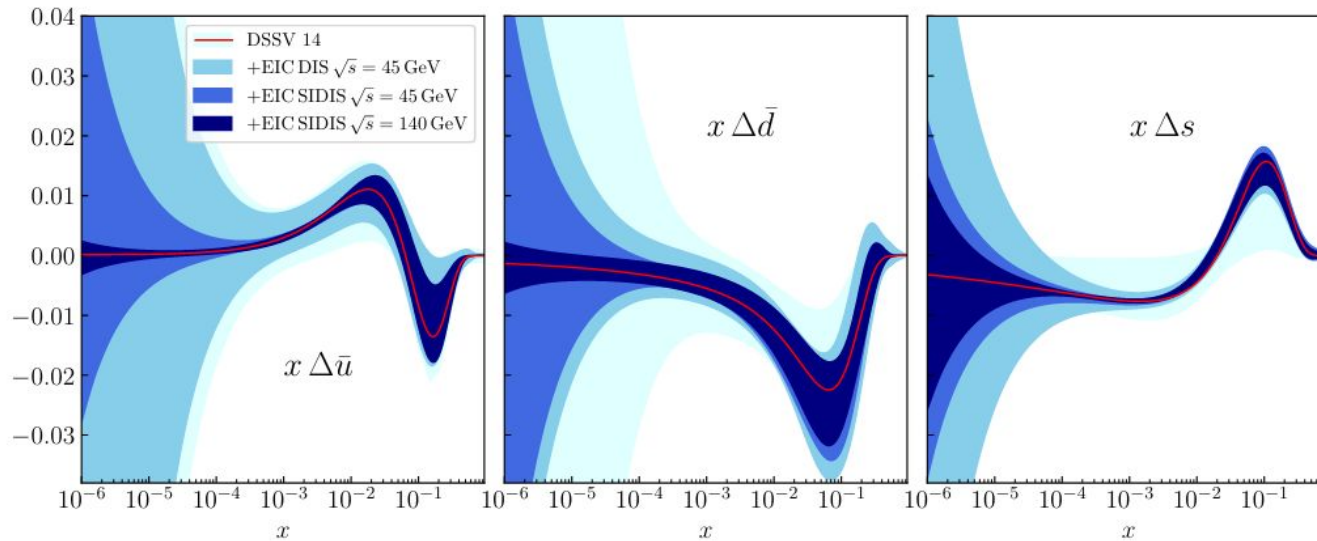


FIG. 6 The transversity  $x h_1(x)$  at 90% CL. Upper (lower) plot for valence up (down) component. Gray lines represent the Softer bound. Darker (pink) band for the PV18 global fit of (Radici and Bacchetta, 2018) at  $Q^2 = 2.4 \text{ GeV}^2$ . Lighter (cyan) band for the MEX19 constrained analysis of (Benel *et al.*, 2020) at the average scale of the data.

# Impact studies

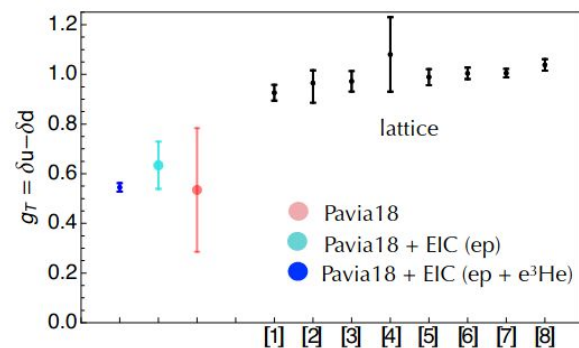
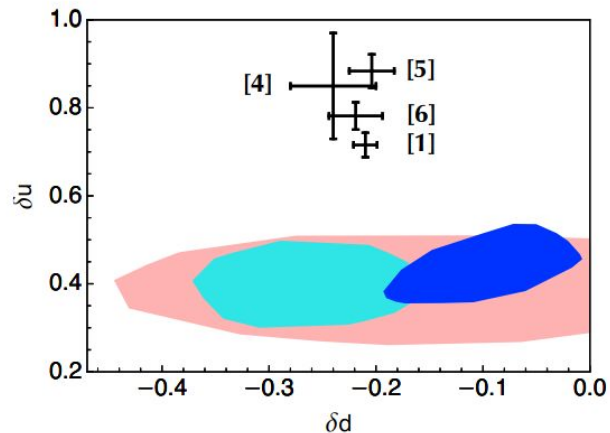
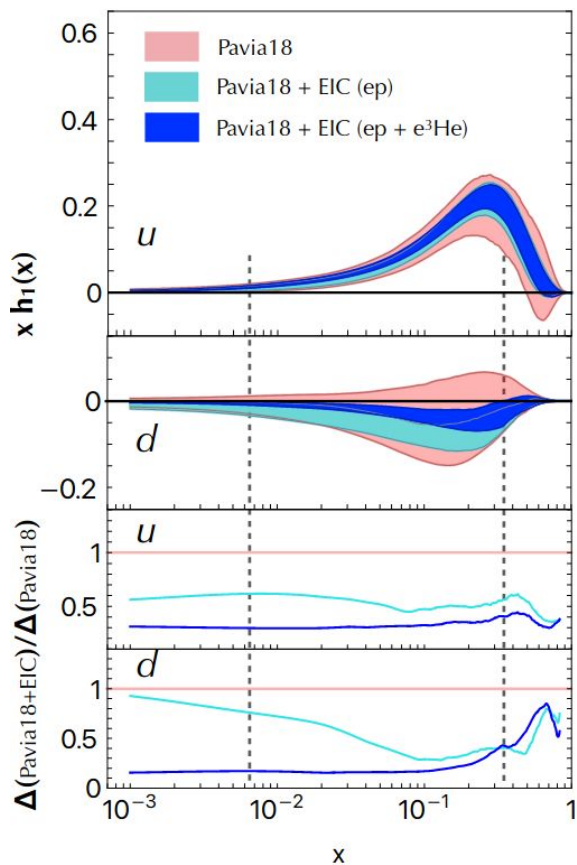
See <https://inspirehep.net/literature/1851258>



**Figure 7.19:** Impact of SIDIS measurements at the EIC on the sea quark helicities  $x\Delta\bar{u}$ ,  $x\Delta\bar{d}$  and  $x\Delta s$  as a function of  $x$  at  $Q^2 = 10 \text{ GeV}^2$ .

# Impact studies

See <https://inspirehep.net/literature/1851258>

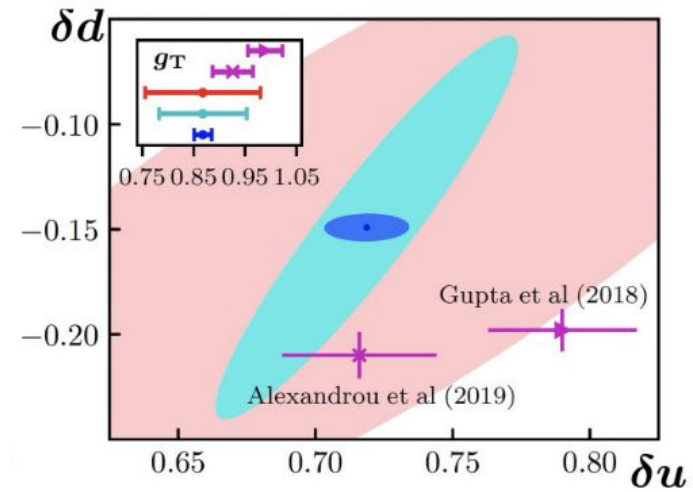
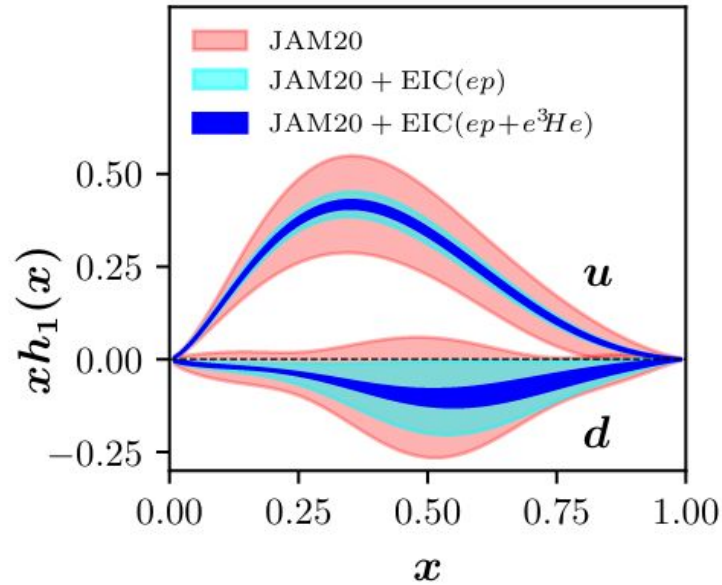


Transversity PDF and tensor charge



# Impact studies

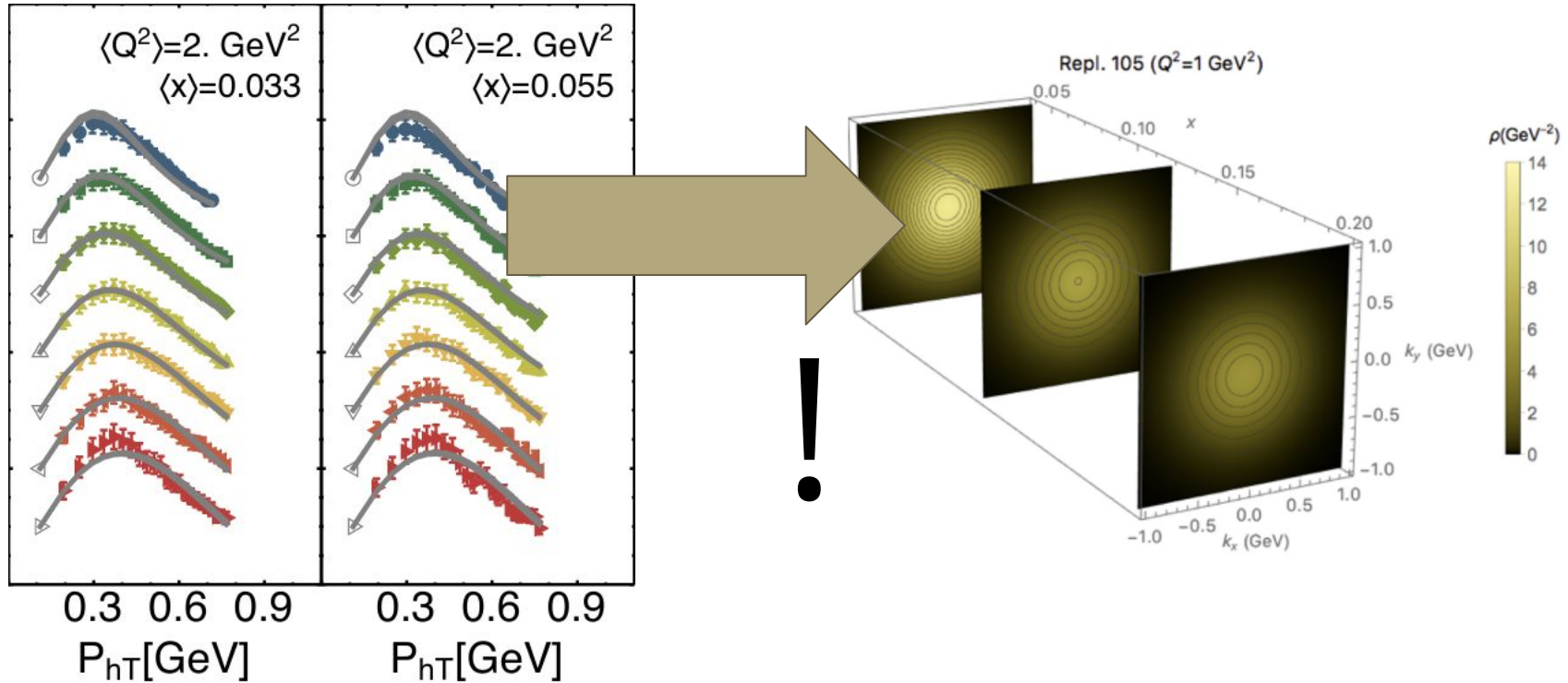
See <https://inspirehep.net/literature/1851258>



Transversity PDF and tensor charge

## 5.3 Unpolarized TMDs

# Transverse momentum imaging



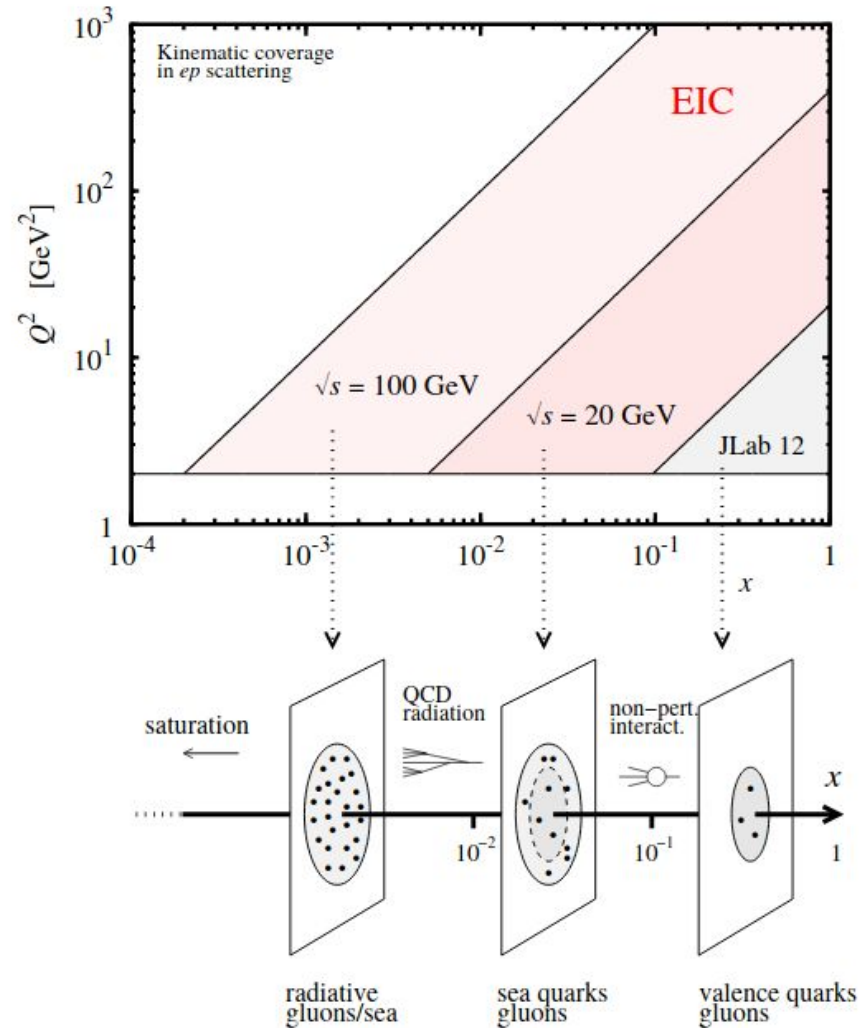
# Imaging from SIDIS

Importance of complementary experiments

from JLab 12 GeV, Hermes, Compass  
to the EIC

**zooming** into hadron structure

Credit picture: C. Weiss



# Available global fits of unpolarized TMDs

	Accuracy	SIDIS	DY	Z production	N of points	$\chi^2/N_{\text{data}}$
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N <sup>3</sup> LL	✓	✓	✓	1039	1.06
<b>MAPTMD22</b>	<b>N<sup>3</sup>LL-</b>	✓	✓	✓	<b>2031</b>	<b>1.06</b>

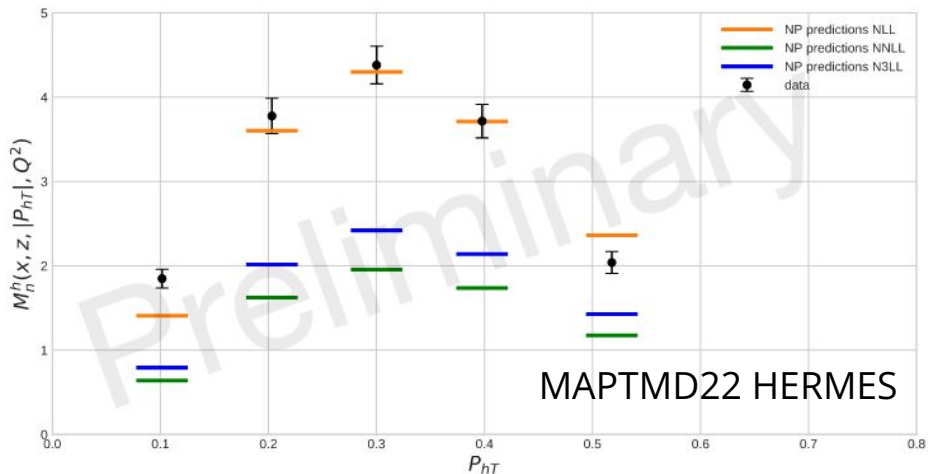
MAP collaboration : <https://github.com/MapCollaboration>

# Other fits of unpolarized TMDs

	Framework	HERMES	COMPASS	DY	Z production	N of points	$\chi^2/N_{\text{points}}$
Pavia 2017 <a href="#">arXiv:1703.10157</a>	NLL	✓	✓	✓	✓	8059	1.55
SV 2017 <a href="#">arXiv:1706.01473</a>	NNLL'	✗	✗	✓	✓	309	1.23
BSV 2019 <a href="#">arXiv:1902.08474</a>	NNLL'	✗	✗	✓	✓	457	1.17
SV 2019 <a href="#">arXiv:1912.06532</a>	NNLL'	✓	✓	✓	✓	1039	1.06
Pavia 2019 <a href="#">arXiv:1912.07550</a>	N <sup>3</sup> LL	✗	✗	✓	✓	353	1.02

# Normalization issues: SIDIS

*Small transverse momentum*



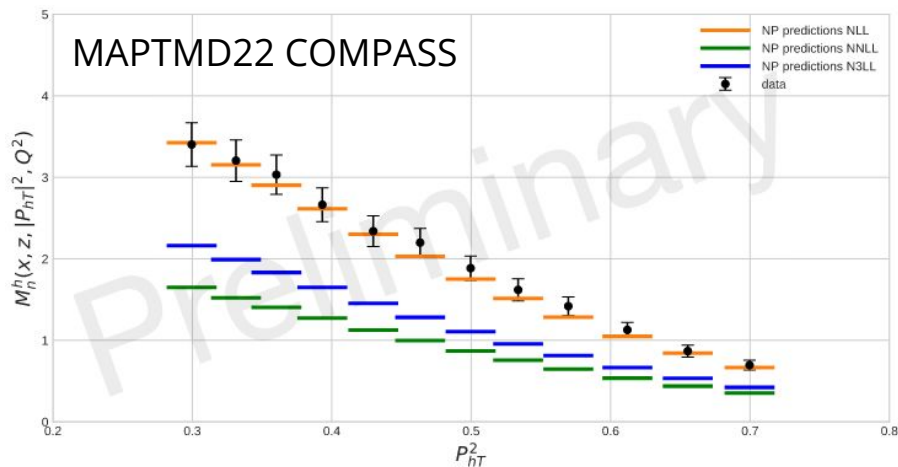
Beyond the NLL,  
the **theoretical** prediction for **SIDIS**  
**undershoots the data**

Who to blame:

- hard function (large coeffs.)
- low Q

But **partial consensus** in literature,  
about the problem

- **SV 19** : *not seen*; power corrections from the start?
- **MAPTMD22** : power corrections from pre-computed normalization coefficients



# Non perturbative components

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0) \quad \rightarrow \text{TMD distribution at initial scales}$$

$$\times \exp \left[ \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F \left( \alpha_s(\mu'), \frac{\zeta}{\mu'^2} \right) \right] \quad \rightarrow \text{evolution in } \mu$$

Calculable in pQCD

$$\times \left( \frac{\zeta}{\zeta_0} \right)^{-D(b_T \mu_0, \alpha_s(\mu_0)) + g_K(b_T; \lambda)} \quad \rightarrow \text{evolution in } \zeta$$

**Non-pert. corrections (large  $b_T$ )**

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes \underline{f_b(x, \mu_0)} F_{NP}(b_T; \lambda) \quad \text{Prior knowledge assumed (?)}$$

See e.g. <https://inspirehep.net/literature/1785810> for more details (but also JCC book, etc.)



# Separating small and large $b_T$

One needs to “separate” the small (perturbative)  $b_T$  region from the large (non-perturbative)  $b_T$  region:

$$\alpha_s(\mu = \mu_b \sim 1/b_T) \longrightarrow b_T < b_{max}$$

Avoid the Landau pole of QCD

$$\int_{\mu_b \sim 1/b}^Q \gamma_F, \mu_b < Q \longrightarrow b_T > b_{min}$$

Otherwise gluon “absorption” instead of “emission”

# Separating small and large $b_T$

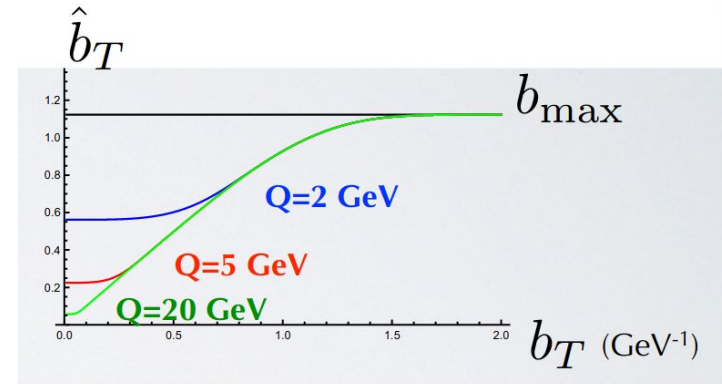
One needs to “separate” the small (perturbative)  $b_T$  region from the large (non-perturbative)  $b_T$  region:

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max} \left( \frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right) \begin{array}{l} \nearrow b_{\max}, \quad b_T \rightarrow +\infty \\ \searrow b_{\min}, \quad b_T \rightarrow 0 \end{array}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

These choices guarantee that for  $Q=1$  GeV the TMD coincides with the NP model

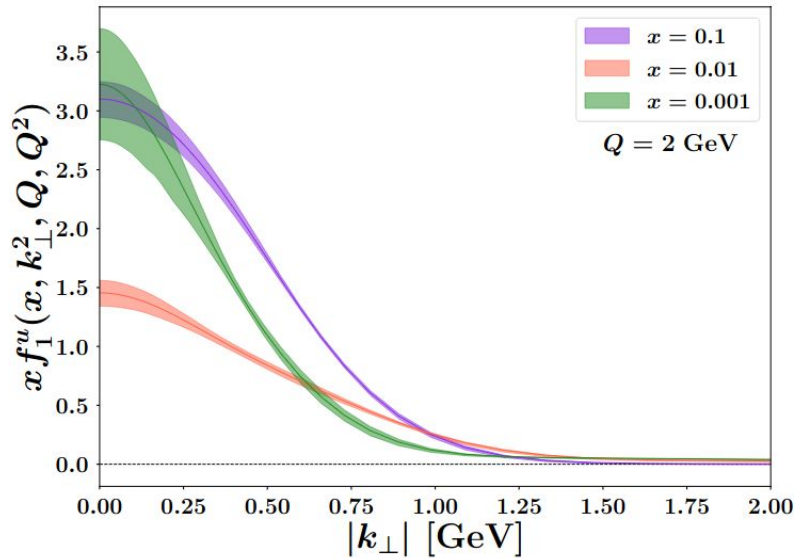


For more details see <https://inspirehep.net/literature/1520011>

# MAP extraction

See : <https://inspirehep.net/literature/2096333>

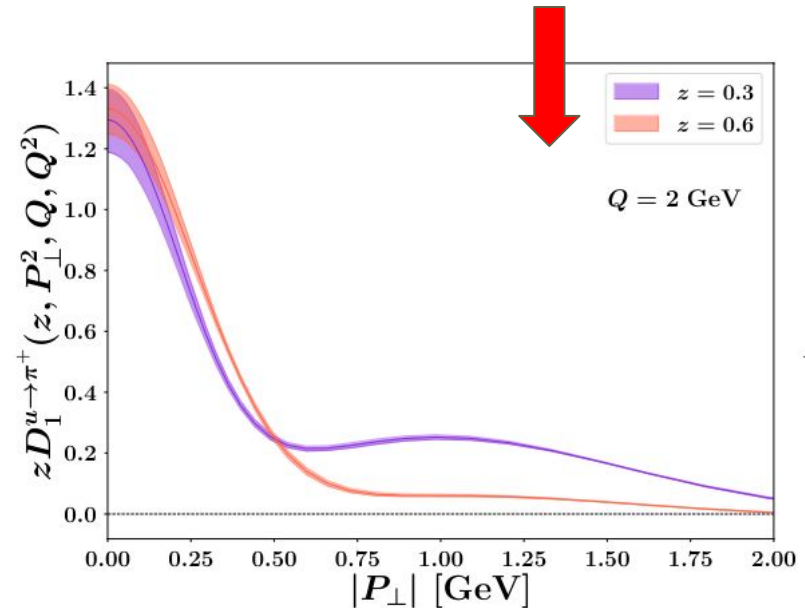
Imaging from **SIDIS** data (Hermes and Compass) **and Drell-Yan** data (low and high energy)



Combining SIDIS and Drell-Yan:  
Possibility to disentangle  
hadron structure and formation

← Unpolarized TMD PDF

Unpolarized TMD FF



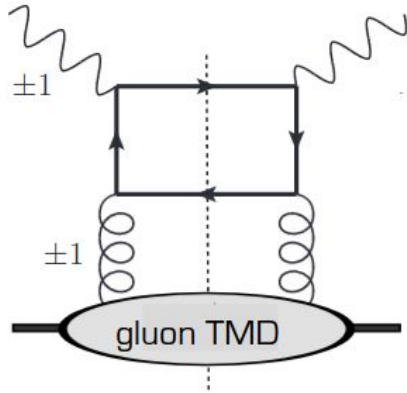
# Some open questions

A non-exhaustive *personal* list of open questions:

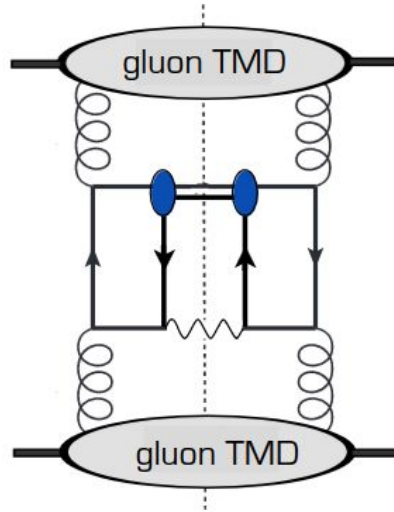
- deepen our understanding of **sea** quarks
- **flavor structure** of TMDs
- experimental confirmation of **sign change** relation
- **gluon** observables and **spin-1** effects
- what can **hadronization** teach us about **confinement**?
- interplay between **nuclear/hadron** and **high-energy** physics (e.g. **W mass**)
- ..

# Access to gluon distributions

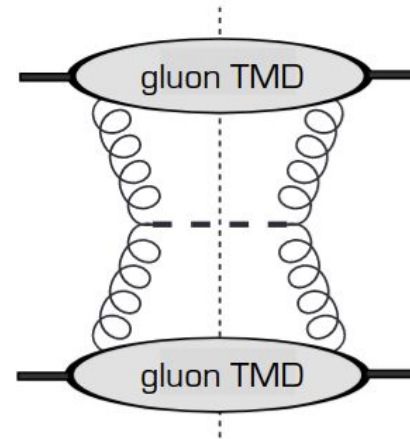
$$e p \rightarrow e \text{ jet jet } X$$



$$p p \rightarrow J/\psi \gamma X$$



$$p p \rightarrow \eta_c X$$



...

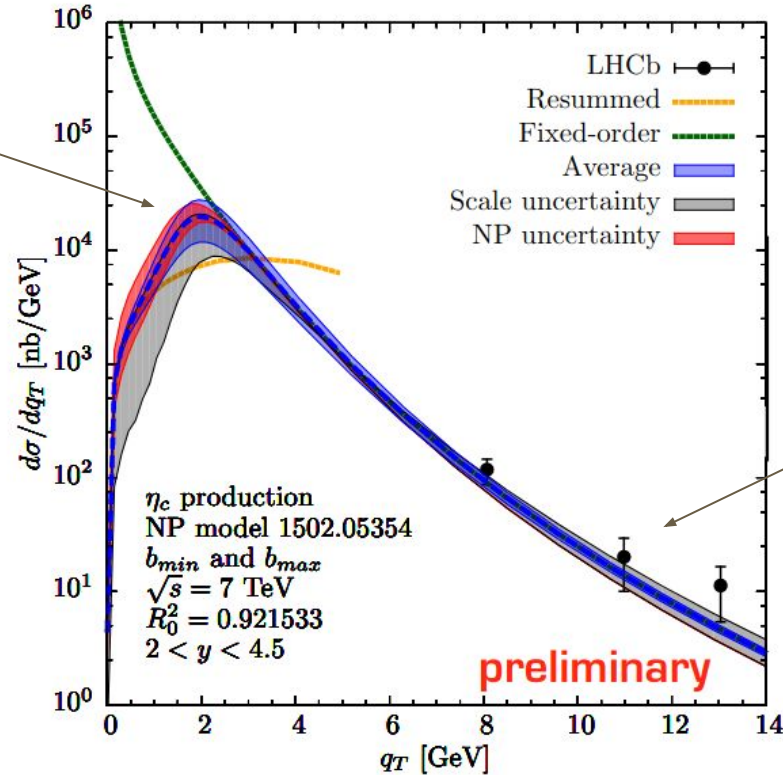
See e.g.:

- <https://inspirehep.net/literature/1962932> (di-jet case)
- <https://inspirehep.net/literature/1279490> (ccbar-photon case)

This is going to be discussed also next week during the workshop.

# eta b,c production : access to gluon TMD PDFs

The **low transverse momentum** region gives access to gluon TMD PDFs



LHCb measurement at **large transverse momentum**: collinear gluon PDFs

Lansberg et al. - ongoing work

# 5.4 Polarized TMDs

# Spin asymmetries

$$A_{UT} \sim \frac{d\sigma(\uparrow) - d\sigma(\downarrow)}{d\sigma(\uparrow) + d\sigma(\downarrow)} \quad \longrightarrow \quad \text{Polarized structure functions / unpolarized one}$$

$$\downarrow \\ f_1 \otimes D_1$$

Asymmetries in general have the benefits to :

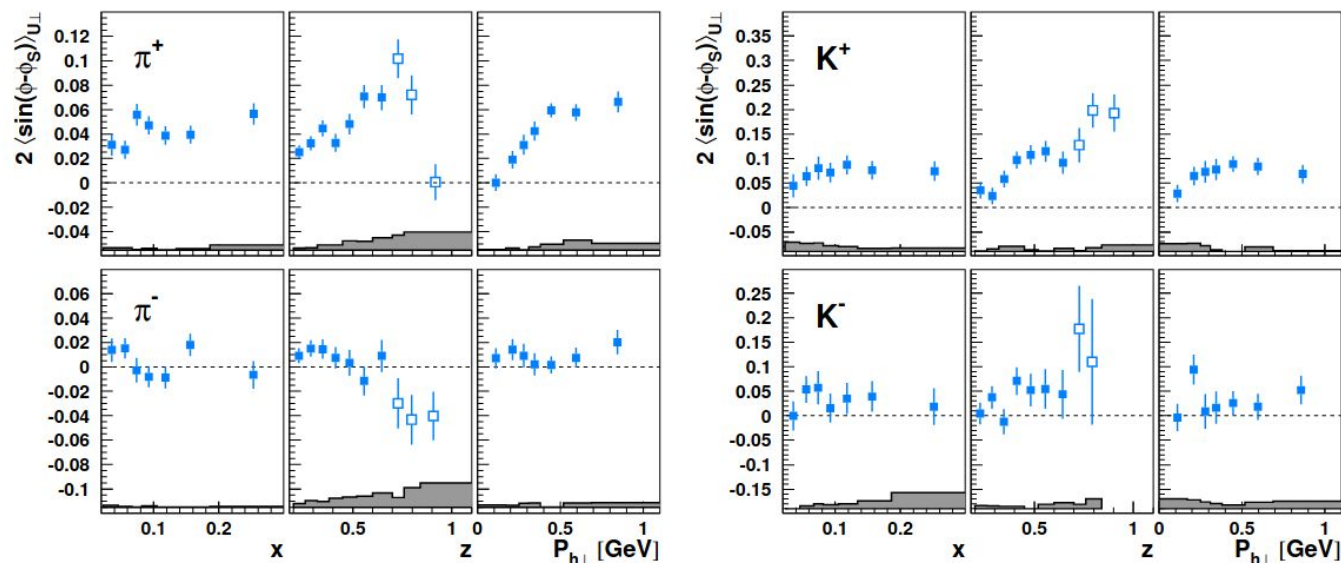
- Single out **specific structure functions** (using Fourier analysis too)
- **Reduce** the effect of **systematic uncertainties** common to denominator and numerator (e.g. acceptance effects)
- But ... **knowledge** of the **unpolarized** cross section (denominator) is required in order to study the numerator



# Sivers from SIDIS

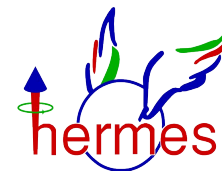
see <https://inspirehep.net/literature/1806922>

Sivers TMD PDF



$$\frac{f_{1T}^\perp \otimes D_1}{f_1 \otimes D_1}$$

**Figure 12.** Sivers SFA for charged mesons (left: pions; right: kaons) presented either in bins of  $x$ ,  $z$ , or  $P_{h\perp}$ . Data at large values of  $z$ , marked by open points in the  $z$  projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.



# Chiral-odd structures

spin-1/2 hadron (Nucleon)

quark polarization

	U ●	L →	T ↑
U ○	<b>f<sub>1</sub></b>		$h_{1\perp}$
L →		<b>g<sub>1</sub></b>	$h_{1L\perp}$
T ↑	$f_{1T\perp}$	$g_{1T}$	<b>h<sub>1</sub></b> $h_{1T\perp}$

unpolarized PDF  
helicity PDF  
transversity PDF

Z polarization

$$\Gamma = \gamma^+ \rightarrow$$

$$\Gamma = \gamma^+ \gamma_5 \rightarrow$$

$$\Gamma = i \sigma^{i+} \gamma_5 \rightarrow$$

$$\Gamma = \gamma^+ \rightarrow \bar{\psi} \gamma^+ \psi \rightarrow R^\dagger R + L^\dagger L \quad \text{sum of right-handed / left-handed quark densities} \quad f_1 = \odot$$

$$\Gamma = \gamma^+ \gamma_5 \rightarrow \bar{\psi} \gamma^+ \gamma_5 \psi \rightarrow R^\dagger R - L^\dagger L \quad \text{difference " " " } g_1 = \odot \rightarrow - \odot \rightarrow$$

$$\Gamma = i \sigma^{i+} \gamma_5 \rightarrow \bar{\psi} i \sigma^{i+} \gamma_5 \psi \rightarrow L^\dagger \gamma_i R - R^\dagger \gamma_i L \quad \text{not diagonal on helicity basis "chiral-odd" structure} \quad h_1 = \odot \uparrow - \odot \downarrow$$

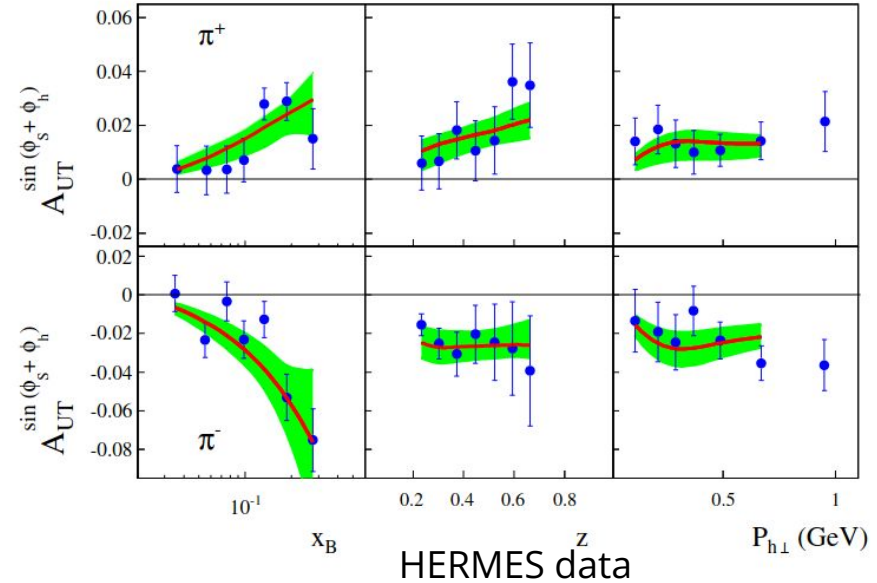
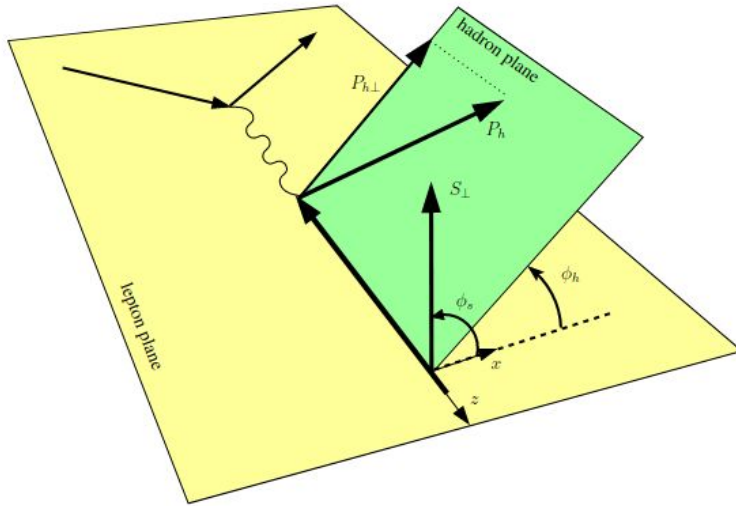
Slide from M. Radici's lectures

# Transversity and Collins

One possible “partnership”  
between chiral-odd structures

$$d\sigma^{lN \rightarrow lhX} \sim h_1(x, k_T^2) \otimes H_1^\perp(z, P_T^2)$$

See <https://inspirehep.net/literature/1372084>

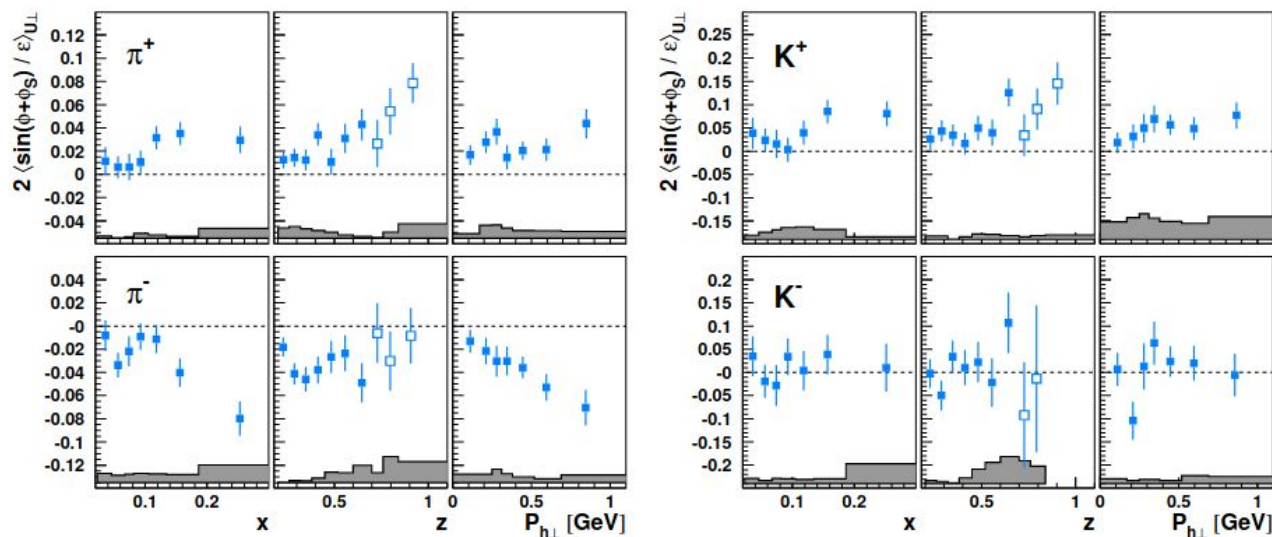


Correlation between **transverse spin** of the quark and **transverse momentum** of the quark (similar to spin-orbit effect!)

# Other TMDs from SIDIS

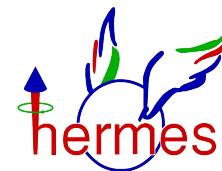
see <https://inspirehep.net/literature/1806922>

Transversity TMD PDF & Collins TMD FF



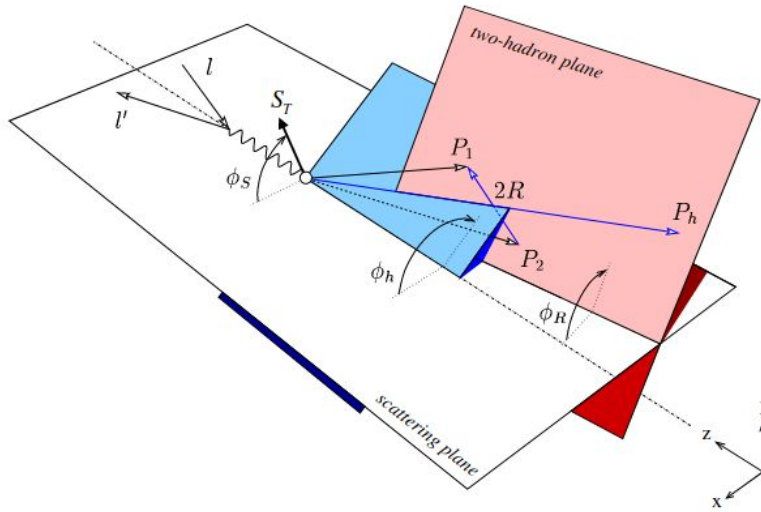
$$\frac{h_1 \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

**Figure 8.** Collins SFA for charged mesons (left: pions; right: kaons) presented either in bins of  $x$ ,  $z$ , or  $P_{h\perp}$ . Data at large values of  $z$ , marked by open points in the  $z$  projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.



# 2h interference FF

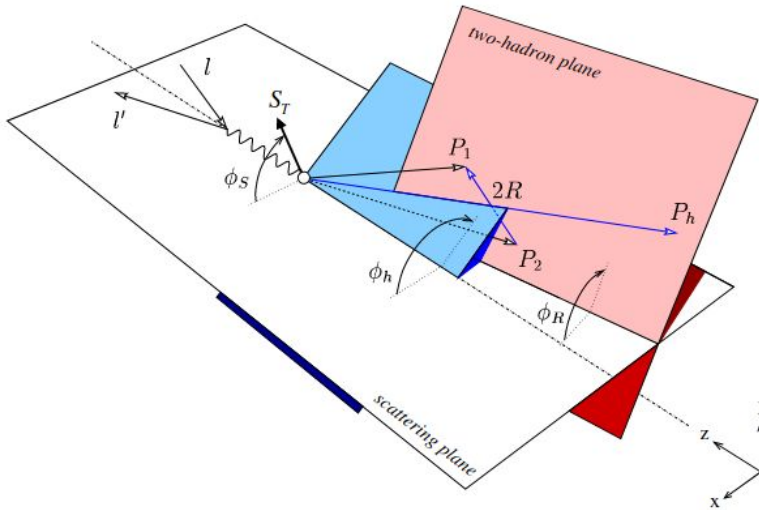
$$d\sigma^{lN \rightarrow l h_1 h_2 X} \sim h_1(x) \otimes H_1^{\triangleleft}(z)$$



hep-ph/0311173 - Bacchetta et al.

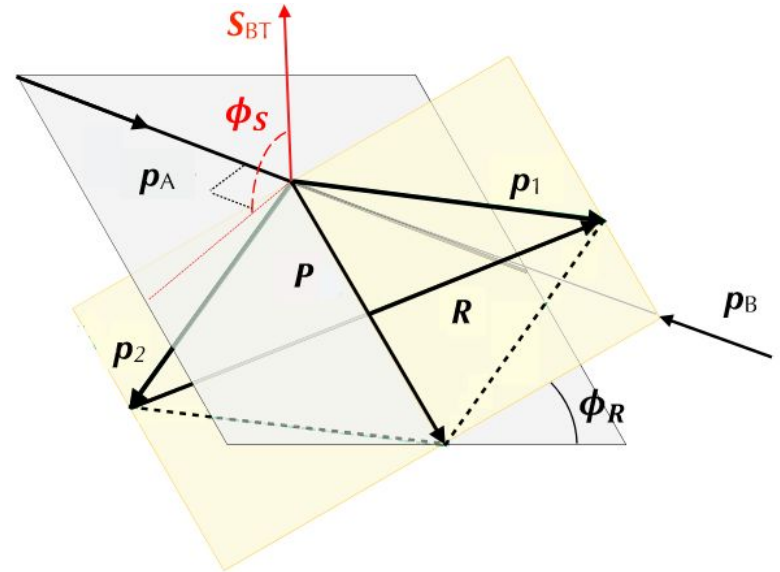
# 2h interference FF

$$d\sigma^{lN \rightarrow l h_1 h_2 X} \sim h_1(x) \otimes H_1^{\triangleleft}(z)$$



hep-ph/0311173 - Bacchetta et al.

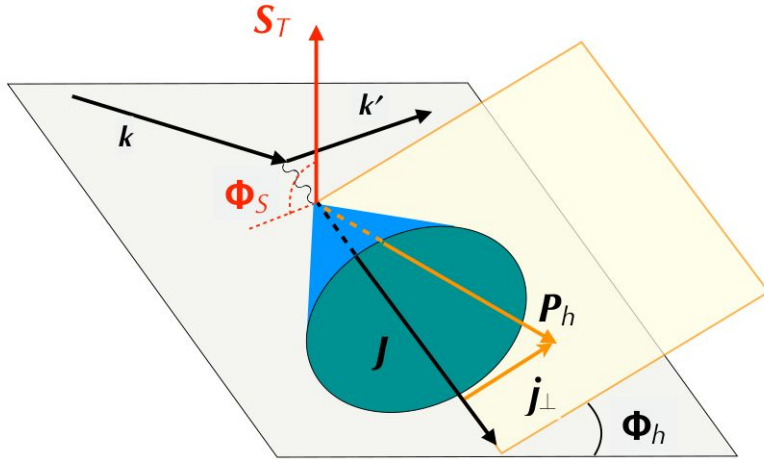
$$d\sigma^{pp \rightarrow h_1 h_2 X} \sim f_1(x_a) h_1(x_b) H_1^{\triangleleft}(z)$$



arXiv 1604.06585 - Radici et al.

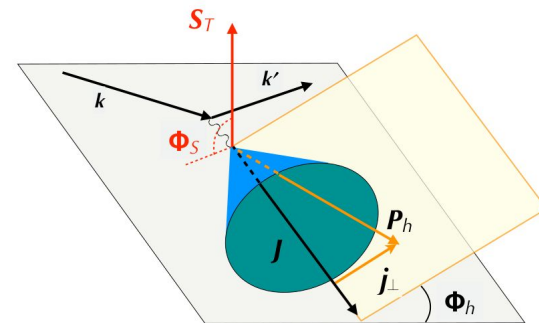
# In-jet Collins TMD FF

$$d\sigma^{lN \rightarrow ljet(h)X} \sim h_1(x, k_T^2) \otimes H_1^\perp(z_h, j_T^2)$$

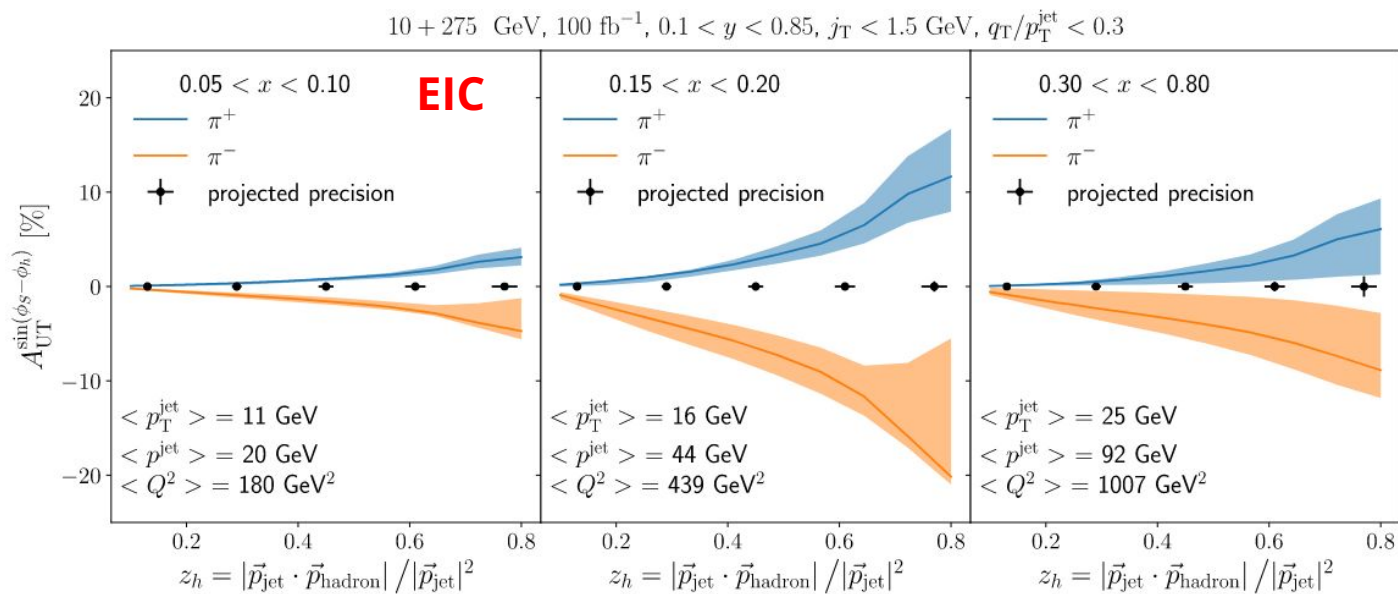


Credit picture: M. Radici

# In-jet Collins TMD FF



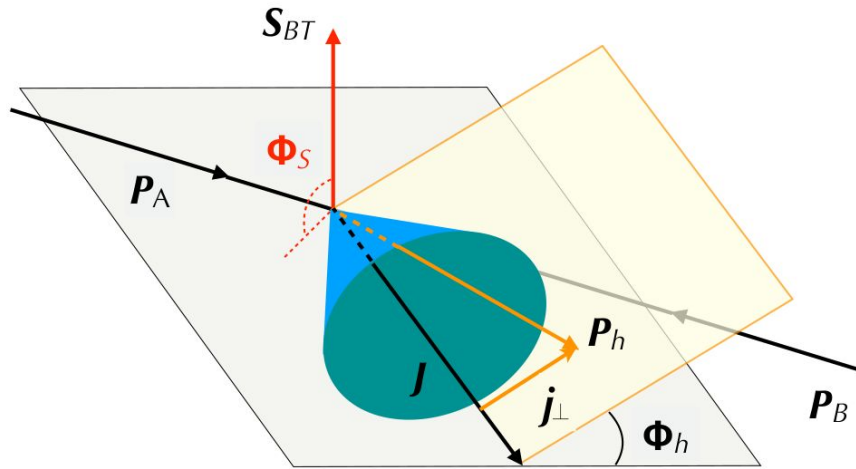
$$d\sigma^{lN \rightarrow ljet(h)X} \sim h_1(x, k_T^2) \otimes H_1^\perp(z_h, j_T^2)$$





# In-jet Collins TMD FF

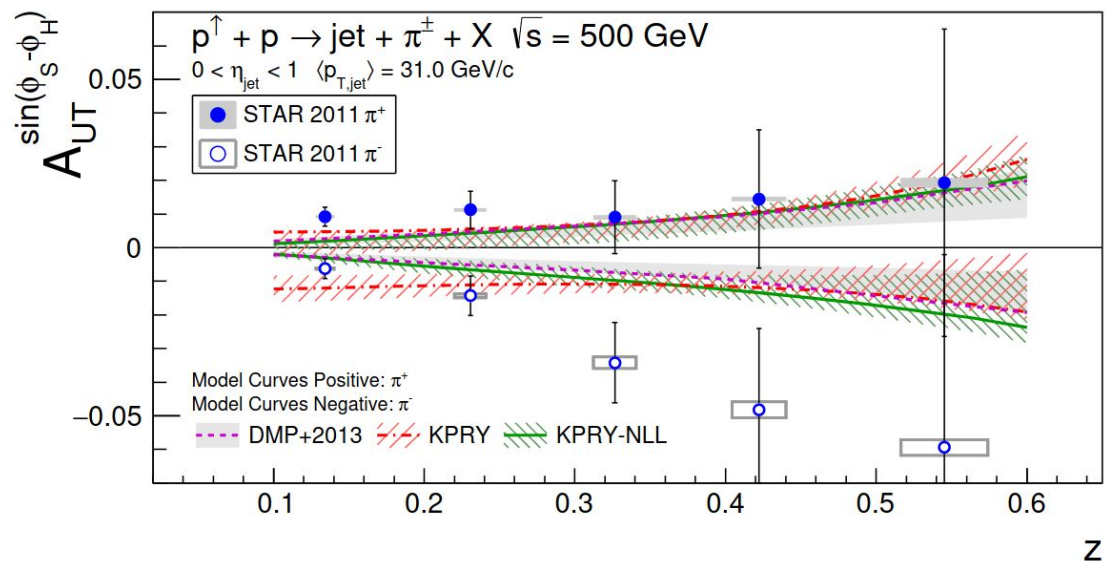
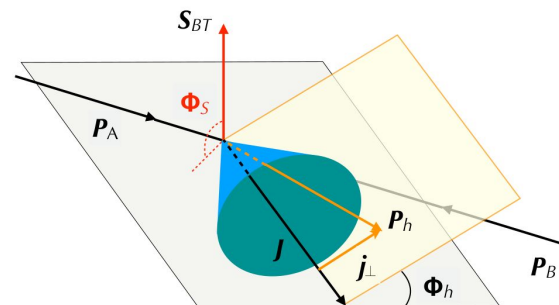
$$d\sigma^{pp \rightarrow jet(h) X} \sim f_1(x_a) \otimes h_1(x_b) \otimes H_1^\perp(z_h, j_T^2)$$



Credit picture: M. Radici

# In-jet Collins TMD FF

$$d\sigma^{pp \rightarrow jet(h) X} \sim f_1(x_a) \otimes h_1(x_b) \otimes H_1^\perp(z_h, j_T^2)$$

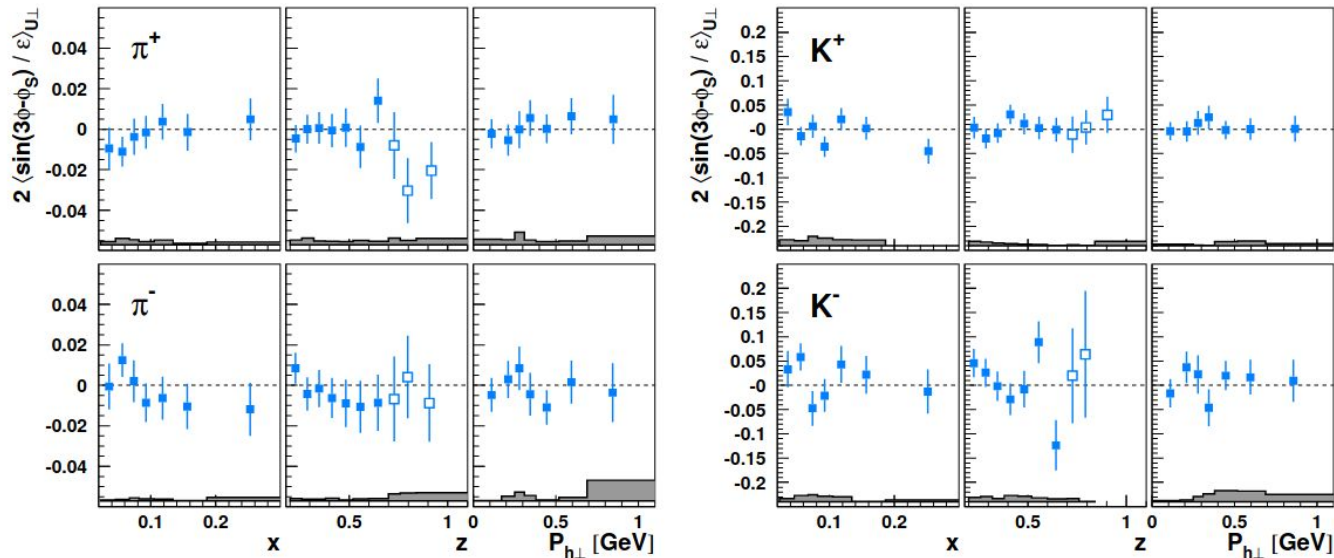


# Other TMDs from SIDIS

see <https://inspirehep.net/literature/1806922>

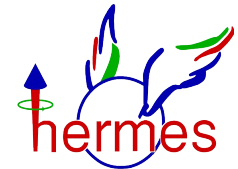
Pretzelosity TMD PDF & Collins TMD FF

(vanishing signal ..?)



$$\frac{h_{1T}^{\perp} \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

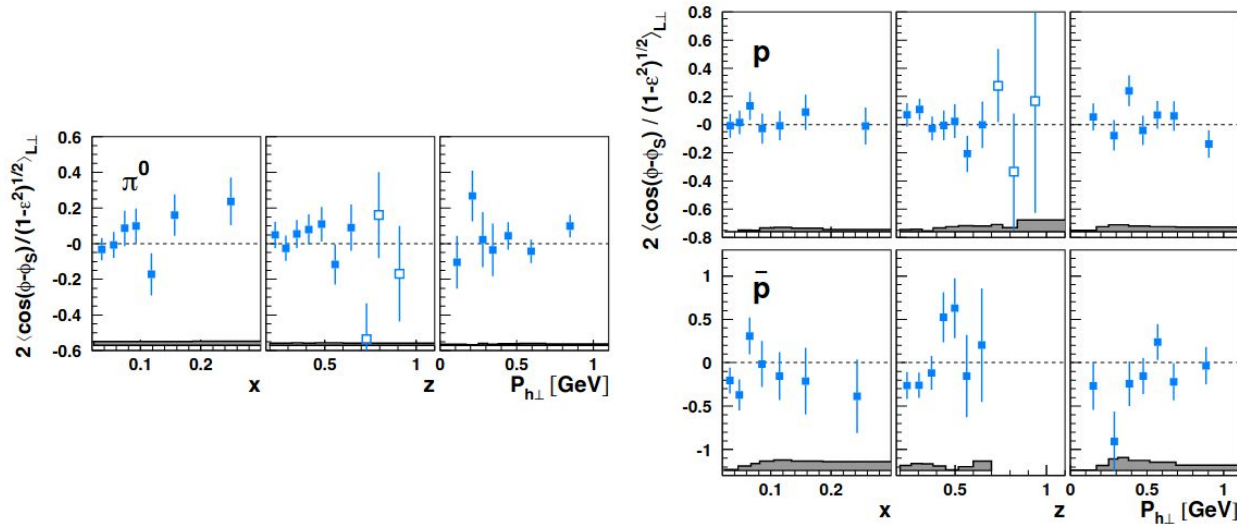
**Figure 19.** Pretzelosity SFA for charged mesons (left: pions; right: kaons) presented either in bins of  $x$ ,  $z$ , or  $P_{h\perp}$ . Data at large values of  $z$ , marked by open points in the  $z$  projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.



# Other TMDs from SIDIS

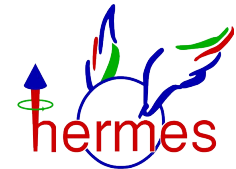
see <https://inspirehep.net/literature/1806922>

Worm-gear TMD PDF



**Figure 22.** The  $2\langle \cos(\phi - \phi_S) / \sqrt{1 - \epsilon^2} \rangle_{L\perp}^h$  amplitudes for  $\pi^0$  (left), protons, and antiprotons (right) presented either in bins of  $x$ ,  $z$ , or  $P_{h\perp}$ . Data at large values of  $z$ , marked by open points in the  $z$  projection, are not included in the other projections (no such high- $z$  points are available for antiprotons due to a lack of precision). Systematic uncertainties are given as bands, not including the additional scale uncertainty of 8.0% due to the precision in the determination of the target and beam polarizations.

$$\frac{g_{1T} \otimes D_1}{f_1 \otimes D_1}$$



# Other TMDs from SIDIS

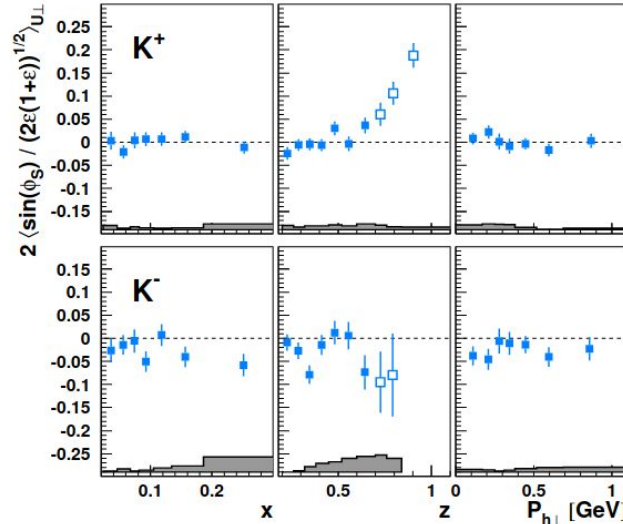
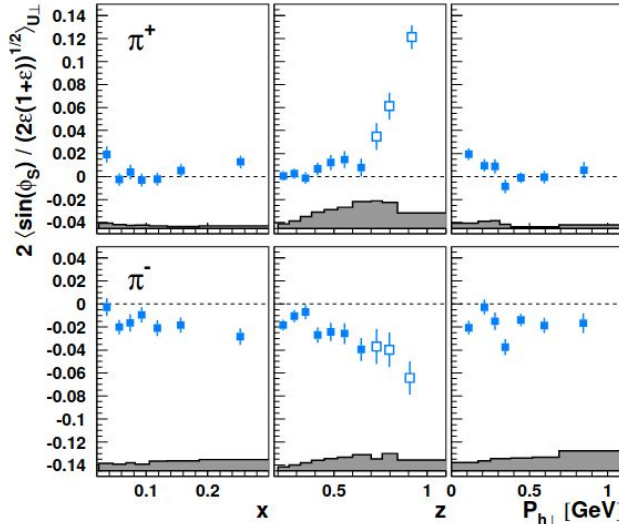
see <https://inspirehep.net/literature/1806922>

**Twist 3** TMD PDFs and TMD FFs

$$\frac{f_T \otimes D_1}{f_1 \otimes D_1}$$

$$\frac{h_1 \otimes \tilde{H}}{f_1 \otimes D_1}$$

$$\frac{h_T \otimes H_1^\perp}{f_1 \otimes D_1}$$



$$\frac{g_{1T} \otimes \tilde{G}^\perp}{f_1 \otimes D_1}$$

$$\frac{h_T^\perp \otimes H_1^\perp}{f_1 \otimes D_1}$$

$$\frac{f_{1T}^\perp \otimes \tilde{D}^\perp}{f_1 \otimes D_1}$$

**Figure 25.** The  $2\langle \sin(\phi_S) / \sqrt{2\epsilon(1+\epsilon)} \rangle_{U\perp}^h$  amplitudes for charged mesons (left: pions; right: kaons) presented either in bins of  $x$ ,  $z$ , or  $P_{h\perp}$ . Data at large values of  $z$ , marked by open points in the  $z$  projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.

