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Introduction to transverse momentum imaging

lecture 4

International school on probing hadron structure at the EIC

ICTS, Bangalore
February 1, 2024

Plan of these lectures

1. **Breaking hadrons**
2. **Non-collinear partons**
3. **Symmetries & spin**
4. **Factorization, evolution, matching**
5. **Phenomenology**

4.1 TMD factorization

TMD factorization

$q_T \ll Q$

$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\,\bar{l} + X$

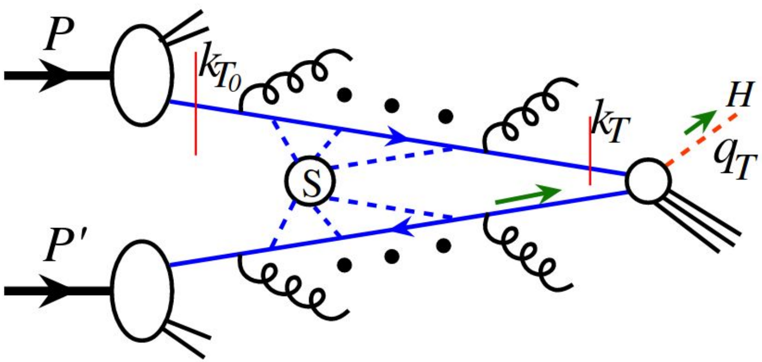
$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}(q_T - k_{T\,a} - k_{T\,b}) \;+ \mathcal{O}(q_T/Q) \;+\; \mathcal{O}(\Lambda/Q)$$

- TMDs & partonic cross section:
  same **IR poles** = same non-perturbative physics

- **observed transverse momentum qT** :
  transverse momenta of **quarks**

- quark transverse momentum :
  radiative (perturbative) and **intrinsic** (non-perturbative) components

- Renormalization = **evolution** equations tell us how to distinguish between the two



TMD factorization

$$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\,\bar{l} + X$$

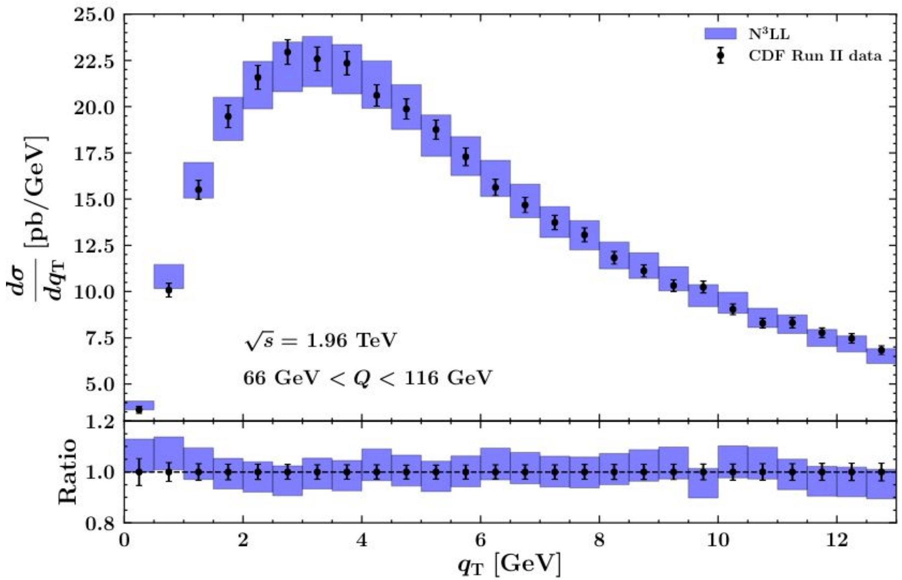
$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\left(q_T - k_{T\,a} - k_{T\,b}\right)$$

Renormalized TMDs

[TMD region, $q_T \ll Q$]

$$+ \mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large q_T and low Q corrections]

Description of data: essential an approach with **predictive power**



Factorization → renormalization (evolution) of TMDs

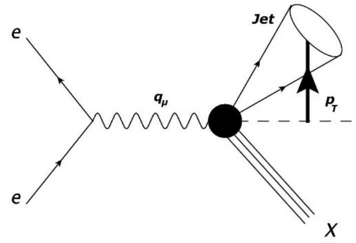
Quarks

Drell-Yan / Z / W production (hh)

Semi-Inclusive DIS (eh)

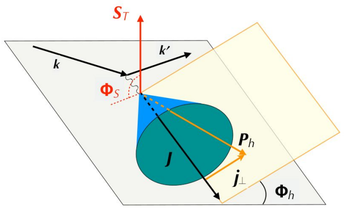
2h-inclusive
e+e- annihilation

Jets:
(e.g. jet SIDIS,
di-jet SIDIS)



TMD factorization

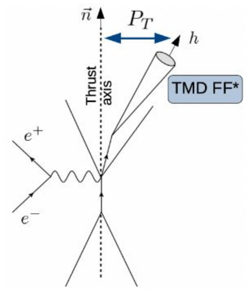
Hadron **"in jet"**:
(eh, hh, e+e-)



Gluons

Higgs production
in hadronic collisions

Y **"+ jet"**:
(e.g. Y = γ, h)



Quarkonium production (e.g. $\eta_{b,c}$
in hadronic collisions)

Quarks

Drell-Yan / Z / W production (hh)

Semi-Inclusive DIS (eh)

2h-inclusive
e+e- annihilation

Global fits (unpolarized)

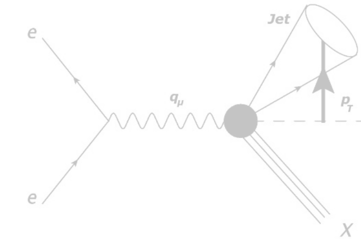
Gluons

Higgs production
in hadronic collisions

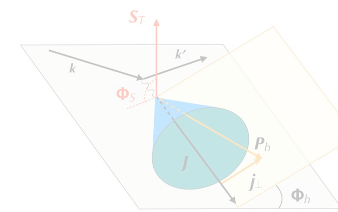
Quarkonium production ($\eta_{b,c}$)
in hadronic collisions

Not enough data (or not at all) for the other processes

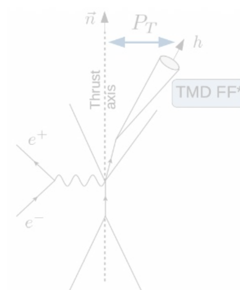
Jets:
(e.g. jet SIDIS,
di-jet SIDIS)



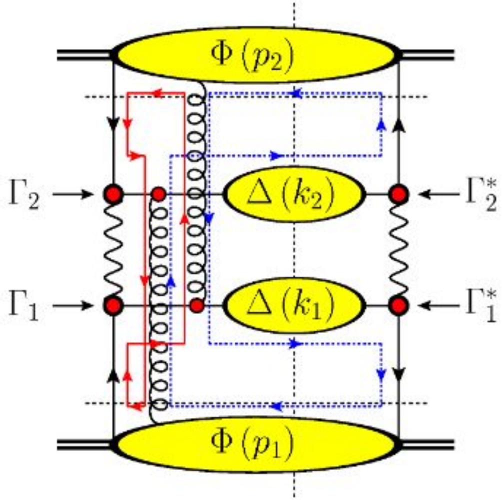
Hadron **"in jet"**:
(eh, hh, e+e-)



Y **"+ jet"**:
(e.g. Y = γ, h)



Non-TMD-factorizable processes



For **pp → h1 h2 X** TMD factorization is violated

See :

- Collins, Qiu (2007) : https://inspirehep.net/literature/750627
- Rogers, Mulders (2010) : https://inspirehep.net/literature/843028

- Buffing (2016) : https://inspirehep.net/literature/1391461 (see figure)

This **endangers** also other processes such as **pp → h X (and similar)**

Quantify factorization breaking effects ? See e.g. :

- Buffing, Kang, Lee, Liu (2018) : https://inspirehep.net/literature/1709823
- Aidala (2019) : https://inspirehep.net/literature/1772224
- LHCb collaboration (2021) : https://inspirehep.net/literature/1901628

Approximations

$$pp \longrightarrow \gamma^{*} / Z \longrightarrow l\bar{l} + X$$

$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\big(q_T - k_{T\,a} - k_{T\,b}\big)$$

[TMD region, $q_T \ll Q$]

$$+\, \mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large q_T and low Q corrections]

Large transverse momentum corrections: **"Matching" to collinear factorization at large qT**

Higher twist corrections: Multiparton correlations (?)

Approximations

See https://inspirehep.net/literature/1732230

Semi-Inclusive Deep-Inelastic Scattering: Nachtmann variable(s)

$$x_{\mathrm{Bj}} = \frac{Q^2}{2P \cdot q}, \qquad x_{\mathrm{N}} = -\frac{q^+}{P^+} = \frac{2x_{\mathrm{Bj}}}{1 + \sqrt{1 + \frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}},$$

Nachtmann and Bjorken-x are the same up to target mass corrections

$$\mathcal{O}(q_T^2/Q^2)$$

And there are also qT-dependent corrections!

$$z_{\mathrm{N}} = \frac{P_{\mathrm{B}}^-}{q^-}$$

$$z_{\mathrm{N}} = \frac{Q^4 x_{\mathrm{N}} z_{\mathrm{h}} \left(1 \pm \sqrt{1 - \frac{4M^2 M_{\mathrm{B}}^2 x_{\mathrm{Bj}}^2 (Q^4 + x_{\mathrm{N}}^2 M^2 \mathbf{q}_{\mathrm{T}}^2)}{Q^8 z_{\mathrm{h}}^2}}\right)}{2x_{\mathrm{Bj}}(Q^4 + x_{\mathrm{N}}^2 M^2 \mathbf{q}_{\mathrm{T}}^2)} \overset{\text{Fixed } x_{\mathrm{N}}, z_{\mathrm{h}}, q_{\mathrm{T}}}{=} z_{\mathrm{h}} \left(1 + O\left(\frac{m^4}{Q^4}\right)\right)$$

Approximations

See https://inspirehep.net/literature/1732230

Semi-Inclusive Deep-Inelastic Scattering: Nachtmann variable(s)

$$x_{\mathrm{Bj}} = \frac{Q^2}{2P \cdot q}, \qquad x_{\mathrm{N}} = -\frac{q^+}{P^+} = \frac{2x_{\mathrm{Bj}}}{1+\sqrt{1+\frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}},$$

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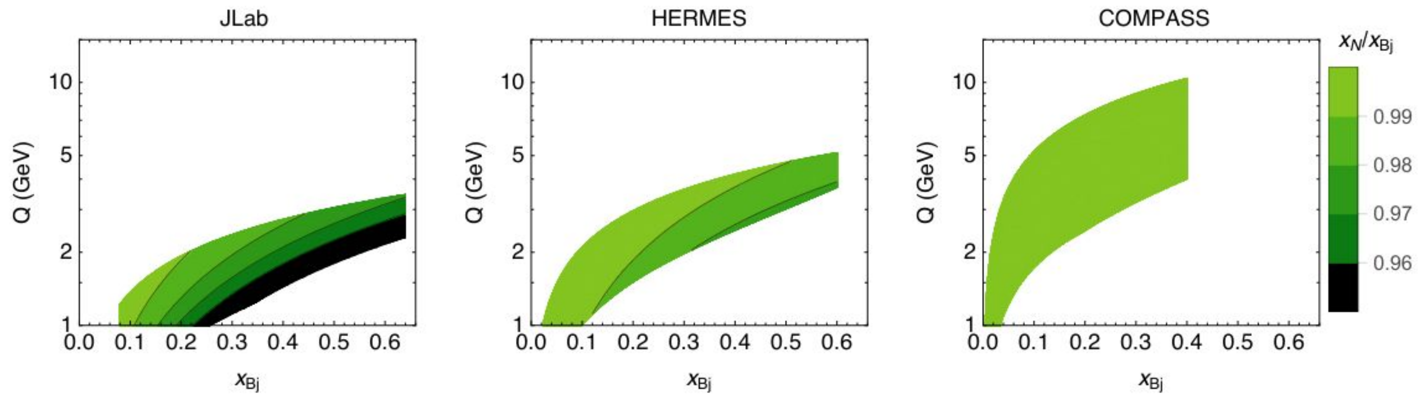


4.2 TMD evolution

Evolution equations

$$F_a(x, b_T^2; \mu, \zeta) = \int d^2\boldsymbol{k}_T \, e^{i\boldsymbol{k}_T \cdot \boldsymbol{b}_T} F_a(x, k_T^2; \mu, \zeta)$$

← Fourier transform of a TMD PDF (bT conjugated to kT)

$$\frac{d \ln F_a(x, b_T^2; \mu, \zeta)}{d \ln \zeta} = -D\left(b_T \mu, \alpha_s(\mu)\right) \,,$$

rapidity

$$\frac{d \ln F_a(x, b_T^2; \mu, \zeta)}{d \ln \mu} = \gamma_F\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right) \,,$$

UV

$$\frac{d\, D\left(b_T \mu, \alpha_s(\mu)\right)}{d \ln \mu^2} = \frac{1}{2}\gamma_K\left(\alpha_s(\mu)\right) \,,$$

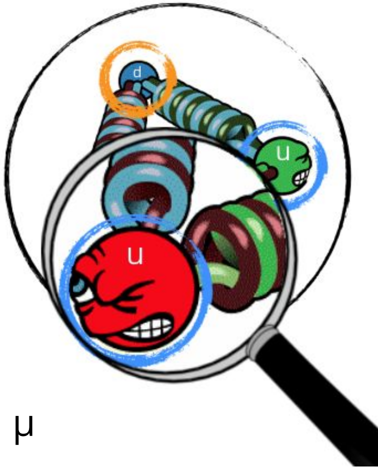
UV

$$\frac{d\, \gamma_F\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right)}{d \ln \zeta} = -\gamma_K\left(\alpha_s(\mu)\right) \,.$$

rapidity

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

QCD evolution of a TMD PDF



$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$ → TMD distribution at initial scales

$$\times \quad \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$ → evolution in μ

Calculable in pQCD

$$\times \quad \left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T \mu_0, \alpha_s(\mu_0)) + g_K(b_T; \lambda)}$$ → evolution in ζ

Non-pert. corrections (large bT)

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes f_b(x, \mu_0) F_{NP}(b_T; \lambda)$$

Prior knowledge assumed (?)

See J.C. Collins' book and many other references, e.g. https://inspirehep.net/literature/1393670

Non-perturbative TMD parts

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$

$$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$

$$\left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T\mu_0, \alpha_s(\mu_0)) + g_K(b_T;\lambda)}$$

Non-pert. corrections (large bT)

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes f_b(x, \mu_0) \, F_{NP}(b_T; \lambda)$$

Intrinsic transverse momentum, potentially flavor dependent!

Predictive power

Small bT → **perturbative (radiative)** contributions to TMD PDF
Large bT → **non-perturbative (intrinsic)** contributions to TMD PDF

Exercise:

In which kinematic regions is the TMD PDF dominated by small / large bT contributions ?

(or, in which kinematic regions is the formalism ***predictive*** and in which regions it is dominated by ***non-perturbative*** contributions?)

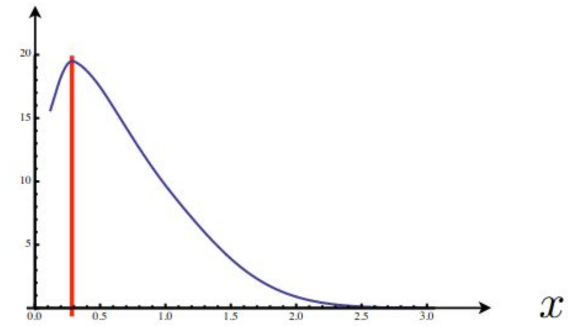
Hint: think about the shape of the TMD PDF in bT space and where it peaks

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

Saddle-point approximation

Function f with a maximum "x_0" in (a,b)

$$I(x_0, A) = \int_a^b dx\; e^{Af(x)} = e^{Af(x_0)} \sqrt{\frac{2\pi}{A(-f''(x_0))}} \left(1 + \mathcal{O}\left(\frac{1}{A}\right)\right)$$



One can apply this approximation to the TMD PDF and plot the position of the peak in bT as a function of x and Q:

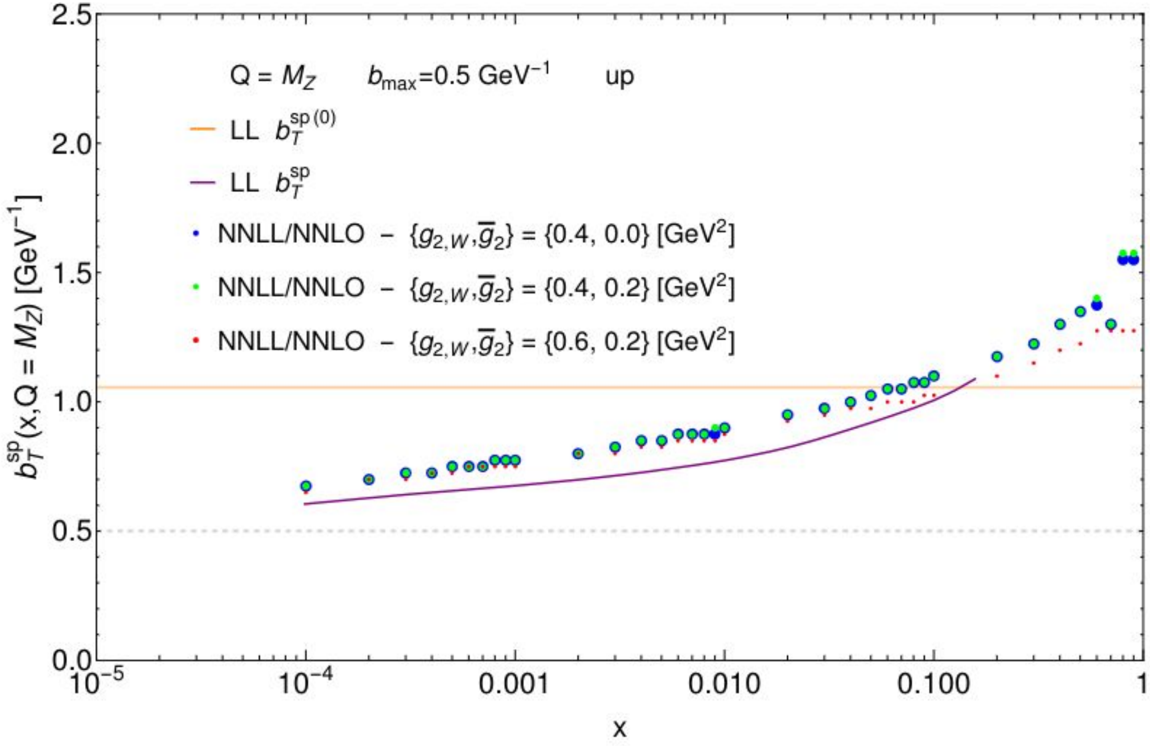
$$\frac{d}{db_T}\left\{ \ln \left[b_T^2\, F_a(x, b_T^2; Q, Q^2) \right] \right\}_{b_T = b_T^{sp}} = 0$$

The TMD PDF is dominated by perturbative contributions at large Q and small x

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

Saddle-point approx.

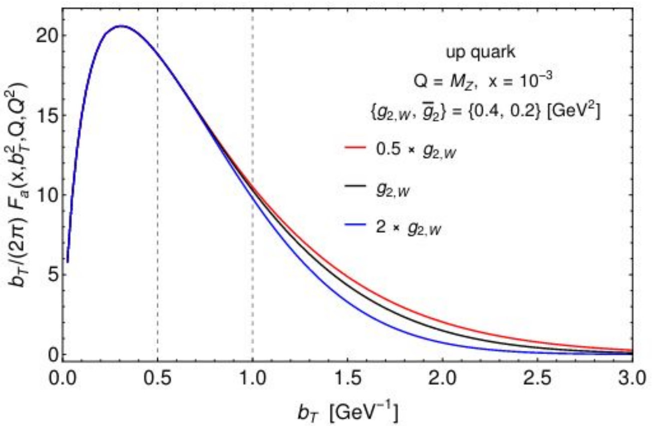
https://inspirehep.net/literature/1785810



Predictive power - quark TMDs

https://inspirehep.net/literature/1785810

Large Q
Small x



(a)

Predictive power - quark TMDs

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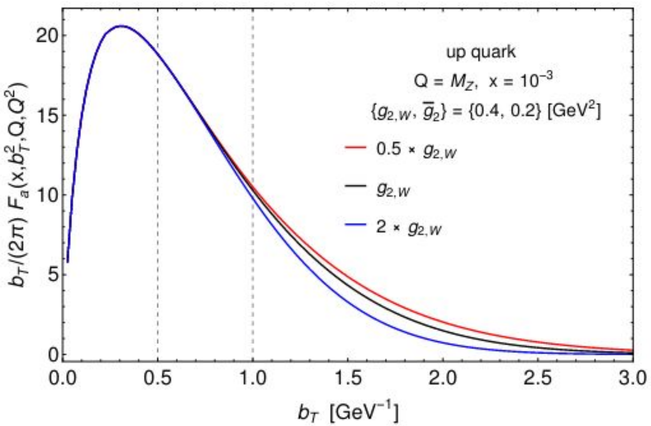
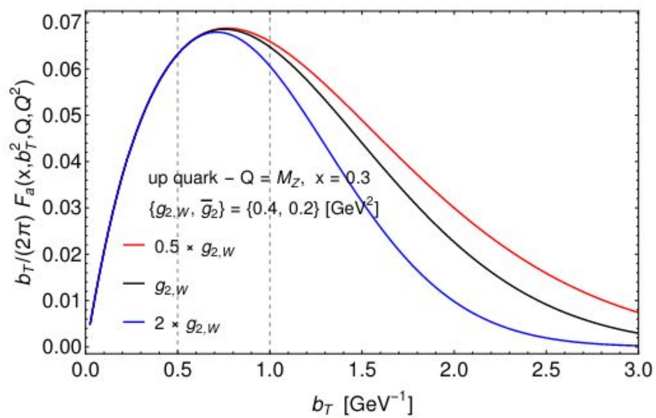
Small x



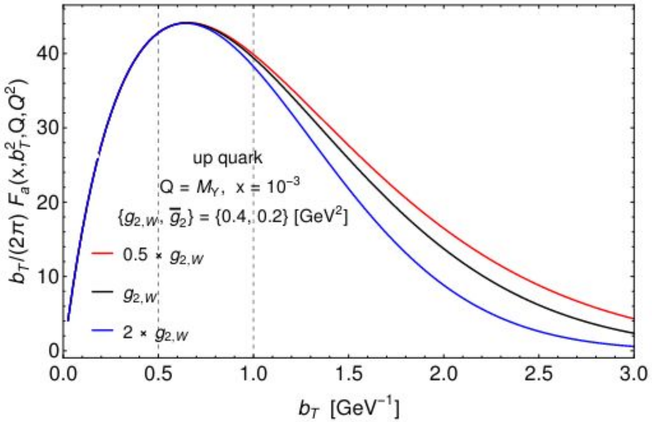
(a)



(b)

Large Q
Large x

Mid Q
Small x



Predictive power - quark TMDs

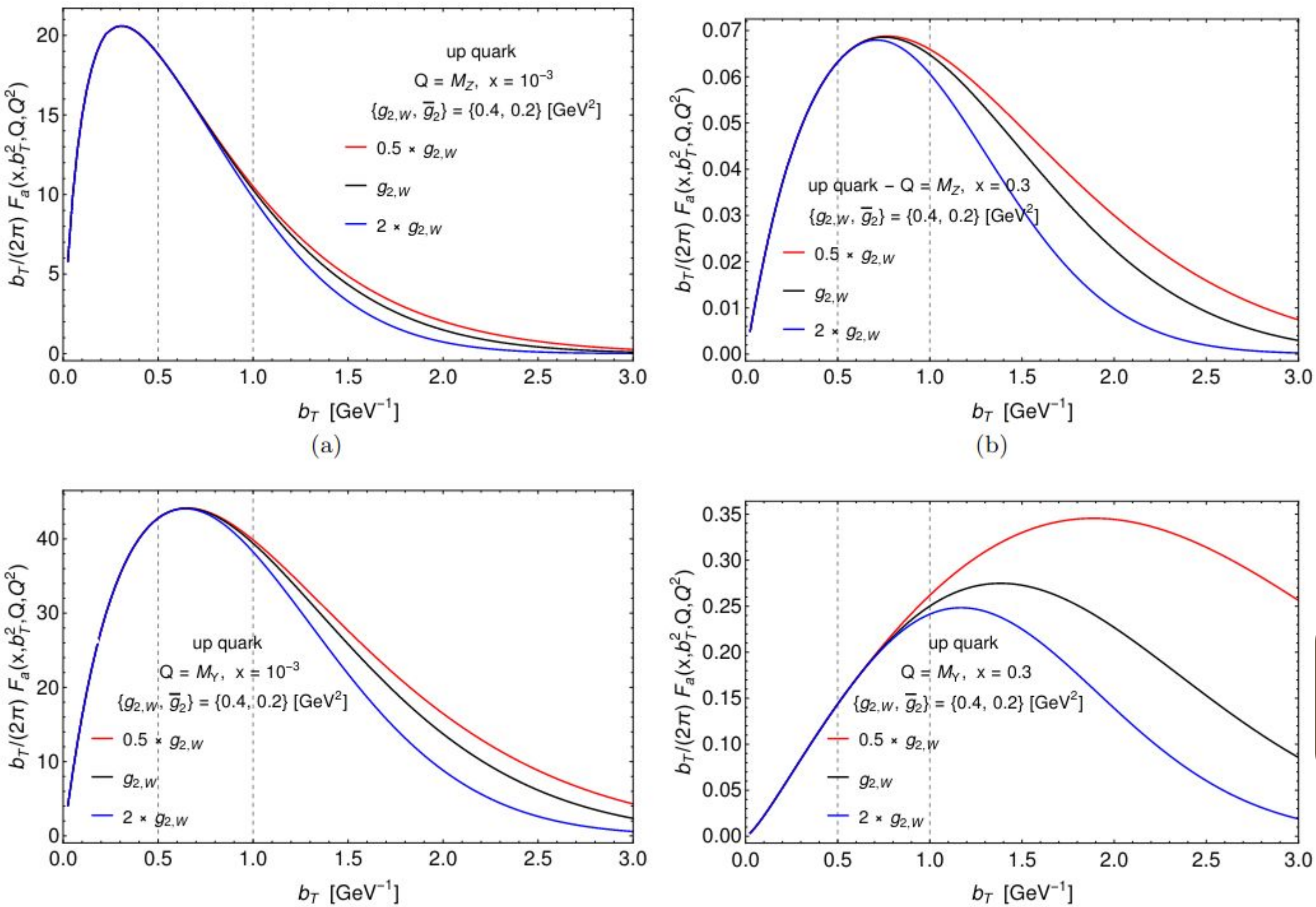
https://inspirehep.net/literature/1785810

Large Q
Small x

Large Q
Large x

Mid Q
Small x

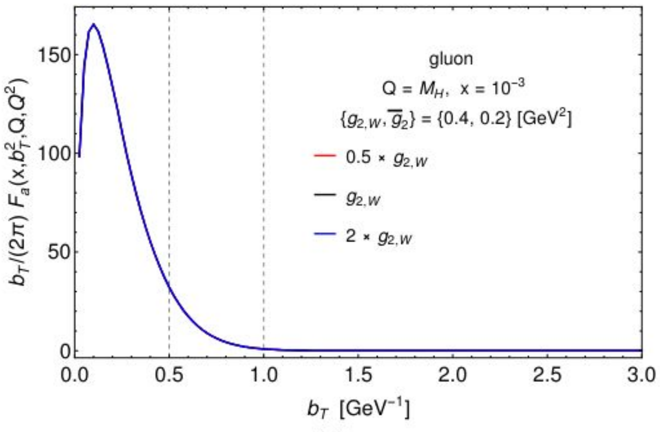
Mid Q
Large x



Predictive power - gluon TMDs

https://inspirehep.net/literature/1785810

Large Q
Small x



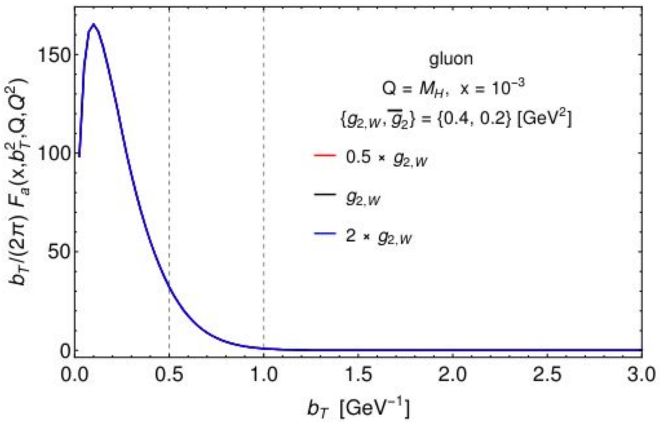
(a)

b_T [GeV^{-1}]

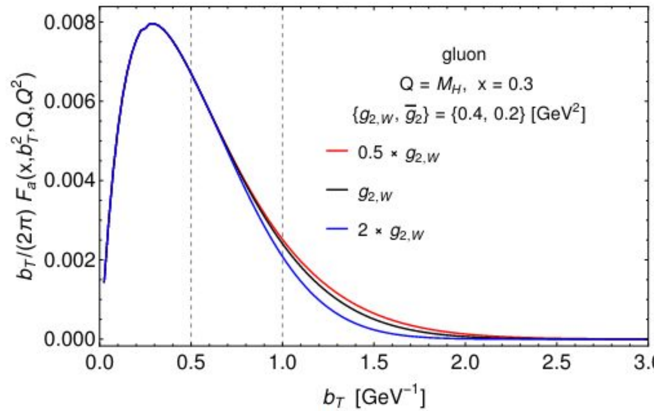
Predictive power - gluon TMDs

https://inspirehep.net/literature/1785810

Large Q
Small x



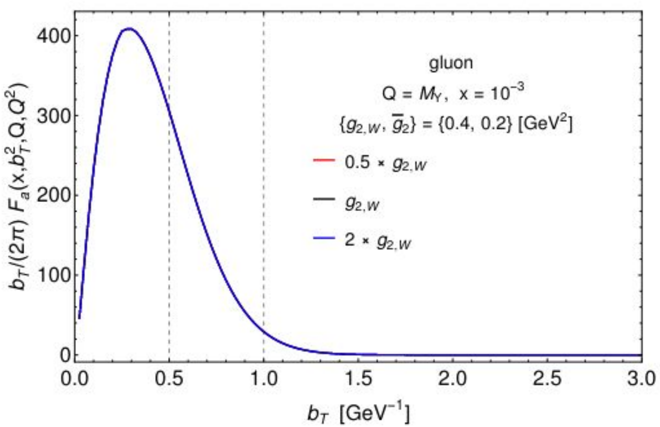
(a)



(b)

Large Q
Large x

Mid Q
Small x



Predictive power - gluon TMDs

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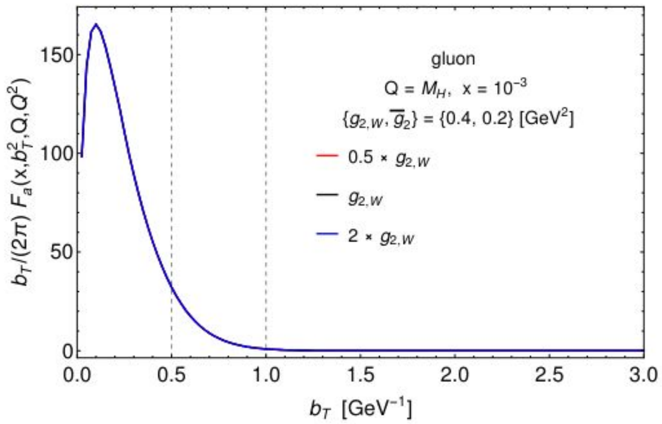
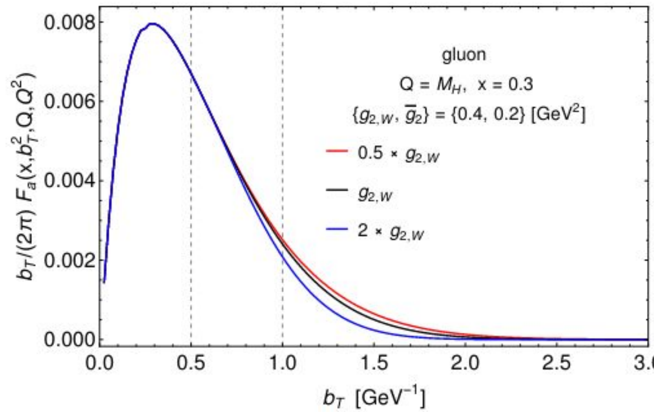
Large Q
Small x



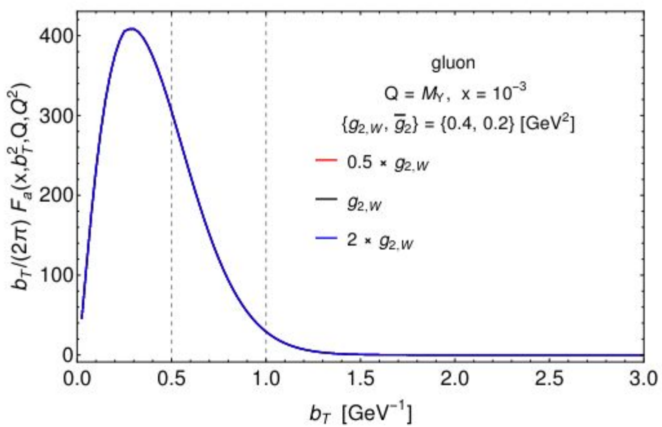
(a)

Large Q
Large x

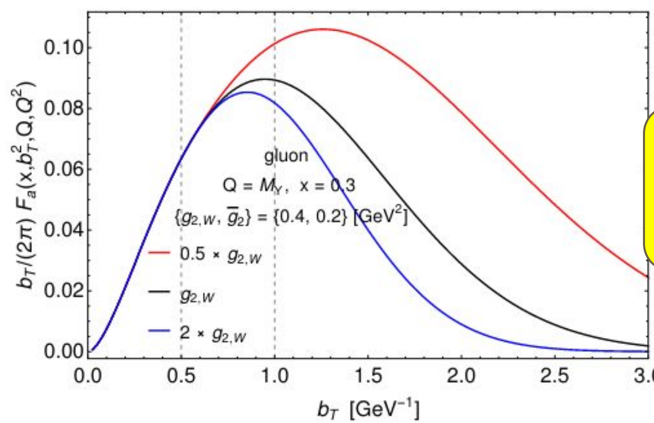


(b)

Mid Q
Small x

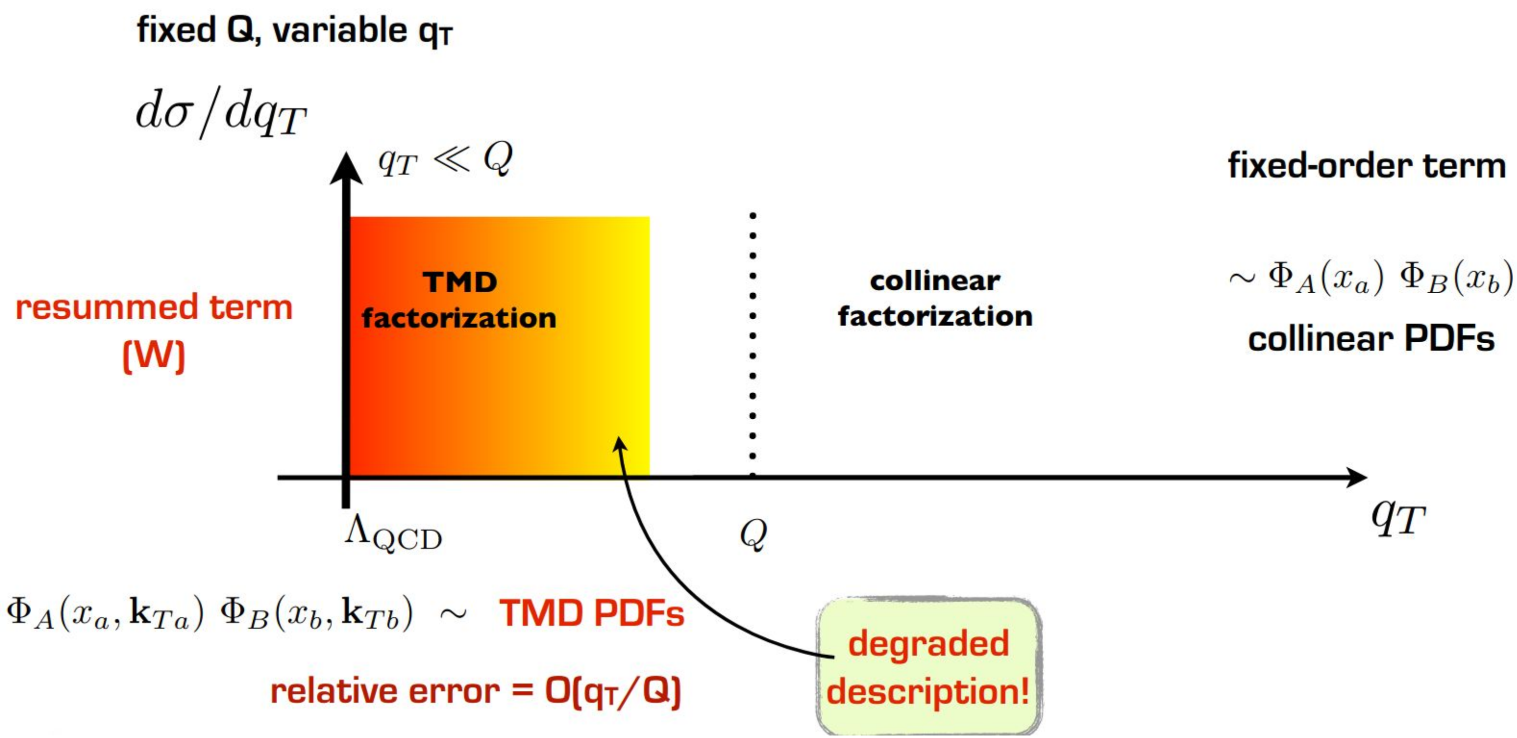


Mid Q
Large x

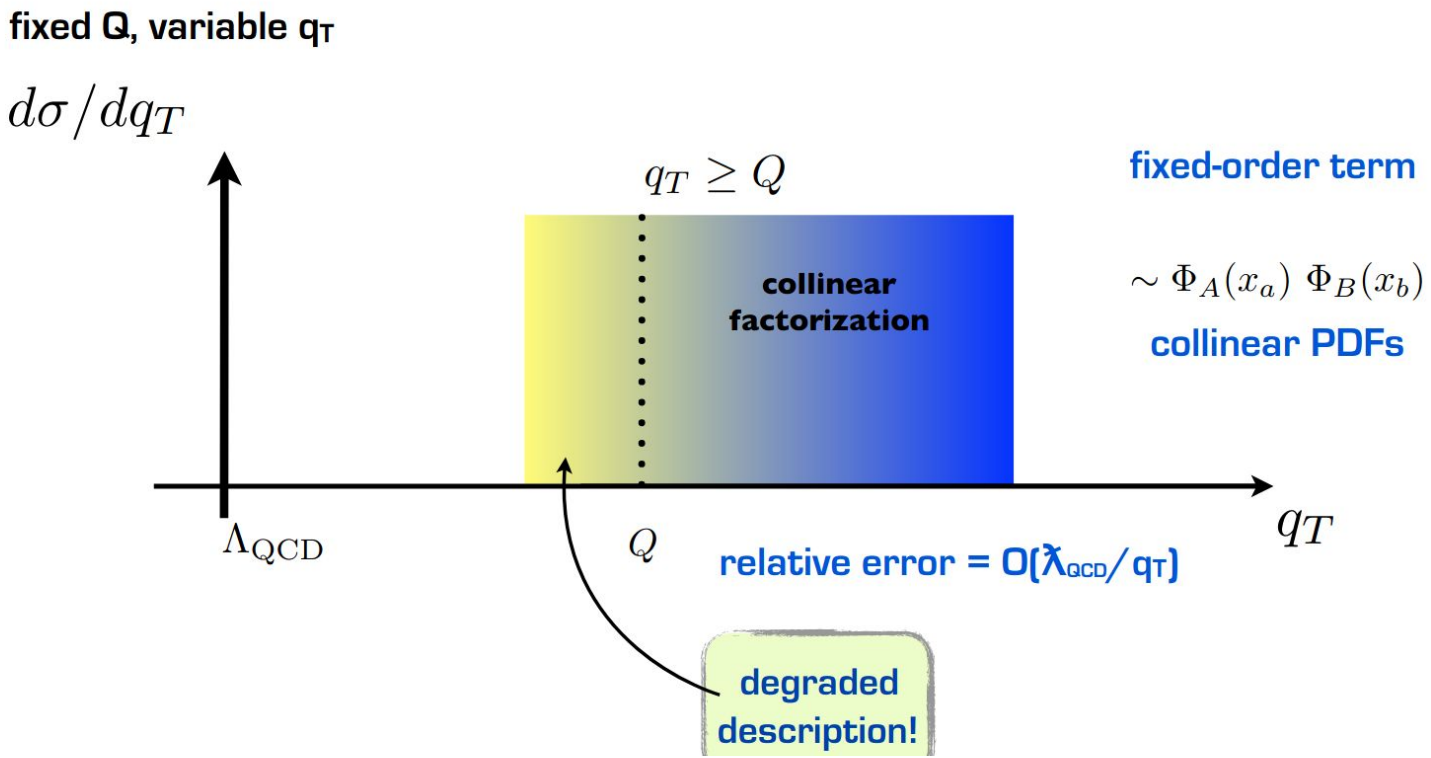


4.3 Matching

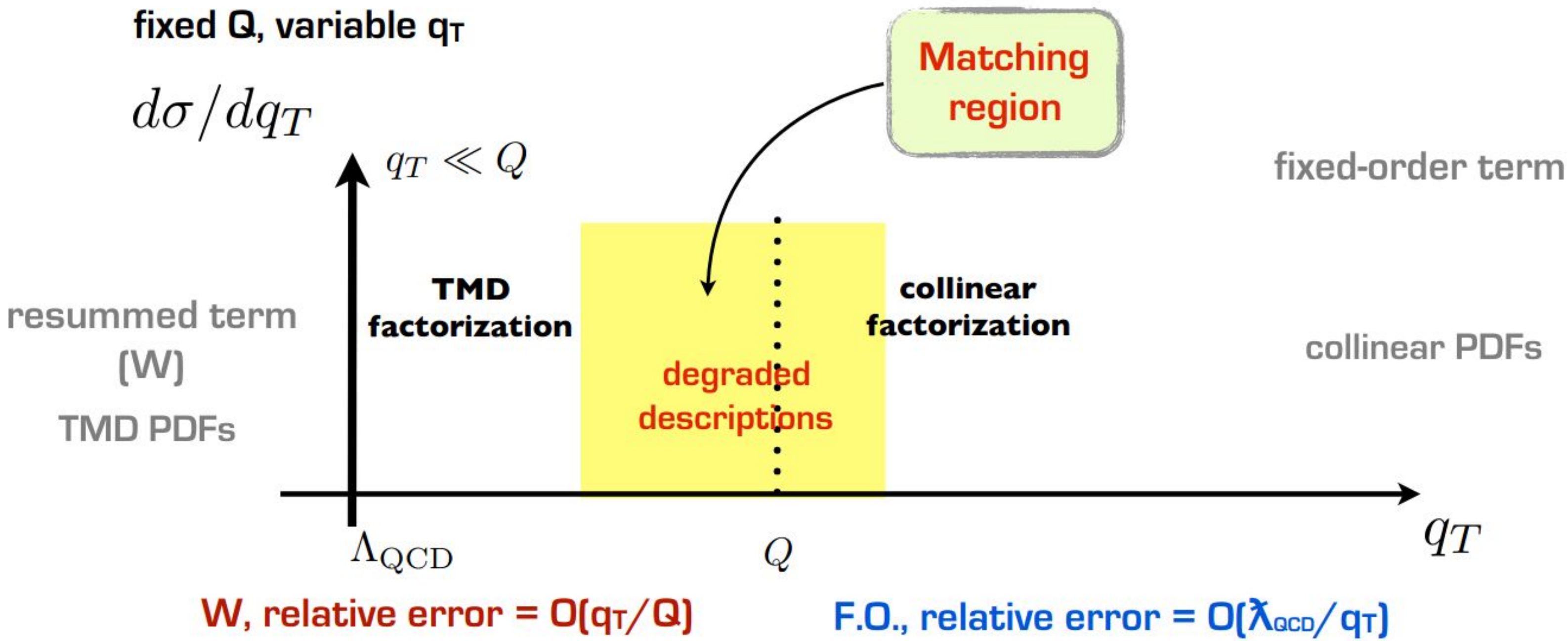
Small transverse momentum



Large transverse momentum



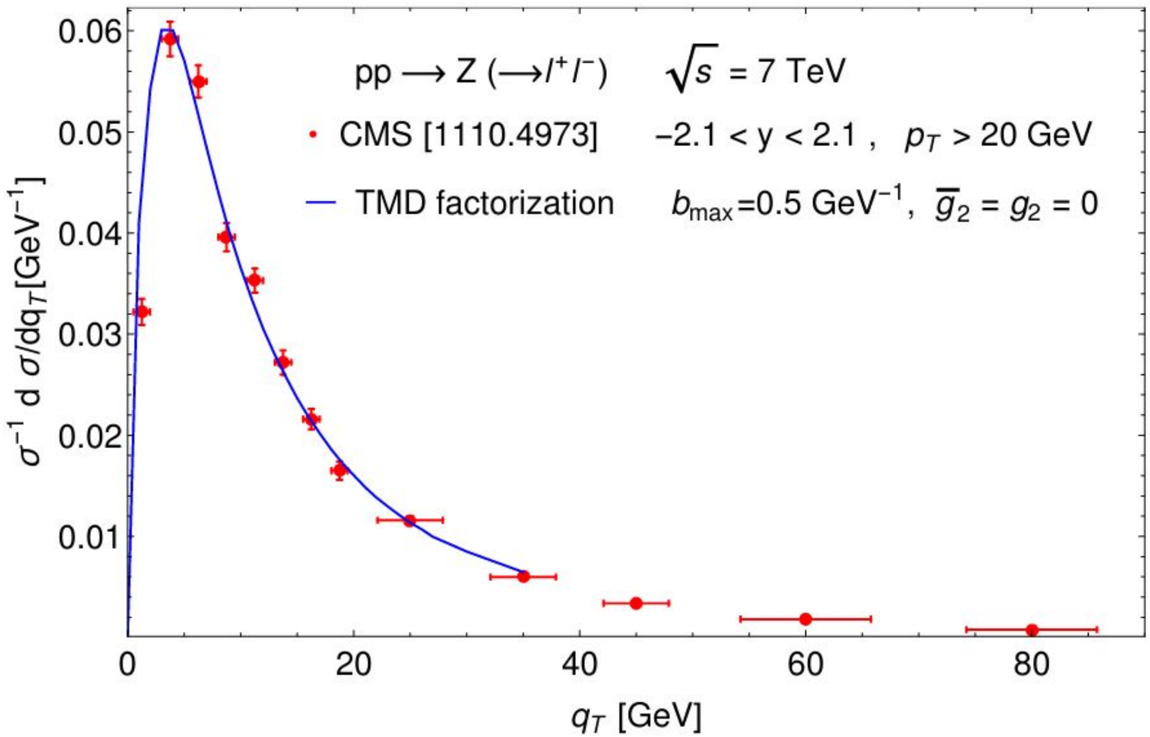
Matching region



Relevance for phenomenology

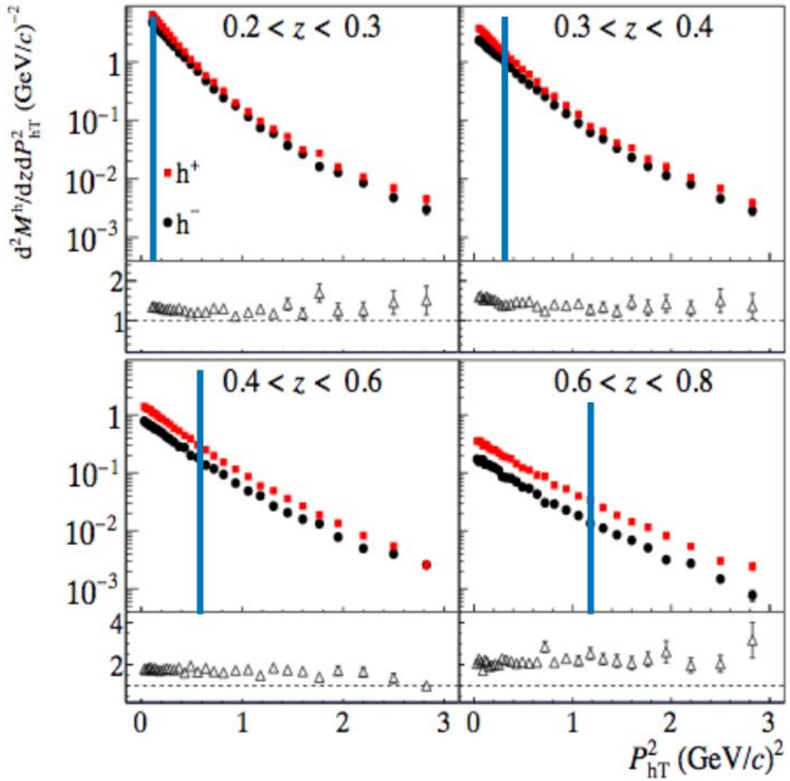
https://inspirehep.net/literature/1785810

$q_T \ll Q$



Relevance for phenomenology

$$q_T \ll Q$$



COMPASS unpolarized SIDIS multiplicities - arxiv 1709.07374

SIDIS - TMD region

$$P_{hT}^2/z^2 \ll Q^2$$

Let's highlight

$$P_{hT}^2/z^2 \sim 0.25\ Q^2$$

One of the bins with highest Q:

$$\langle Q^2 \rangle = 9.78\ \mathrm{GeV}^2$$
$$\langle x \rangle = 0.149$$