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Introduction to transverse momentum imaging

lecture 2

International school on probing hadron structure at the EIC

ICTS, Bangalore
January 31, 2024

DIS cross section (polarized nucleon)

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\Bigg\{\left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1-y)\, F_{UU,L} + S_L \lambda_e\, y\left(1 - \frac{y}{2}\right)\, F_{LL}$$
$$+ |\boldsymbol{S}_T| \lambda_e\, y\sqrt{1-y}\, \cos\phi_S\, F_{LT}^{\cos\phi_s}\Bigg\}$$

F...: functions of x, Q

Up to now no partons ...

How do quarks and gluons emerge in this description?

For a summary see e.g. https://inspirehep.net/literature/732275

Plan of these lectures

1. **Breaking hadrons**
2. **Non-collinear partons**
3. **Symmetries & spin**
4. **Factorization, evolution, matching**
5. **Phenomenology**

2. Non-collinear partons

Light cone dominance

See also M. Radici's lectures

$$2\,M\,W_{\mu\nu}(q,P,S) \;=\; \sum_X \int \frac{d^3P_X}{2E_X}\,\delta^4(P+q\,-\,P_X)\,\langle PS|\,J^\dagger_\mu(0)\,|P_X\rangle\,\langle P_X|\,J_\nu(0)|PS\rangle$$

By using :

- properties of delta
- completeness on the intermediate states and
- translating the argument of the current

$$2MW_{\mu\nu}(q,P,S) \;=\; \frac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J^\dagger_\mu(\xi),\,J_\nu(0)\right]\left|PS\right\rangle$$

- Causality implies: $\xi^2 > 0$
- Riemann-Lebesgue lemma implies W is zero unless: $\xi^0 \to 0$

Thus , the W is dominated by what happens at $\xi^2 \simeq 0$ ⇒ We can use the OPE!

Operator Product Expansion (OPE)

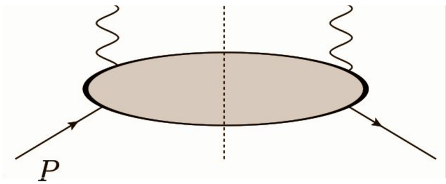
See Muta's book for details

$$A(x)\,B(y) \;=\; \sum_n C_n(x-y)\,O_n(x)\,, \qquad \text{when } |x-y| \text{ is small}$$

operators

Coefficients (lower n : more dominant)

Thanks to the light cone dominance, we can apply this to the hadronic tensor:

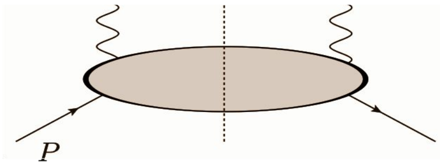


$$2MW_{\mu\nu}(q,P,S) \;=\; \frac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J^\dagger_\mu(\xi),\,J_\nu(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) \;=\; :\,\overline{\psi}(\xi)\,Q\,\gamma_\mu\;\psi(\xi)\,:$$

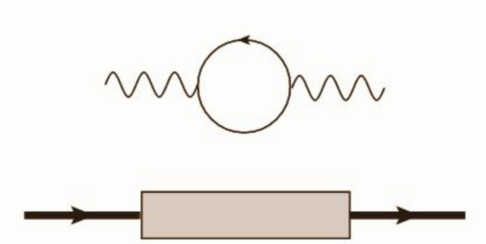
Operator Product Expansion (OPE)

See Muta's book for details



$$2MW_{\mu\nu}(q,P,S) \;=\; \frac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J_\mu^\dagger(x),\,J_\nu(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) \;=\; :\,\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi)\,:$$



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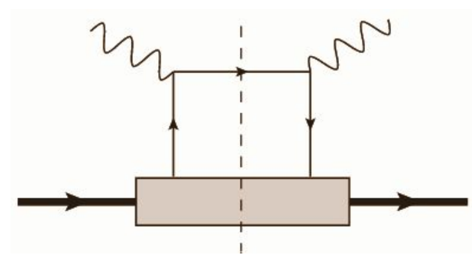

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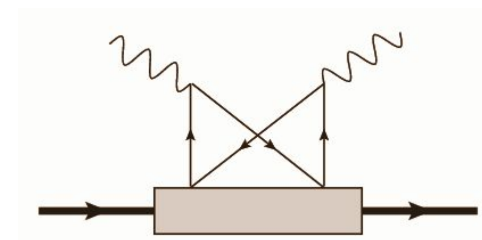
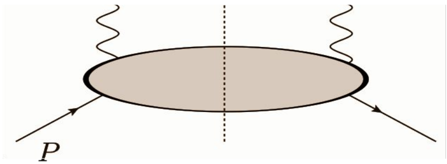


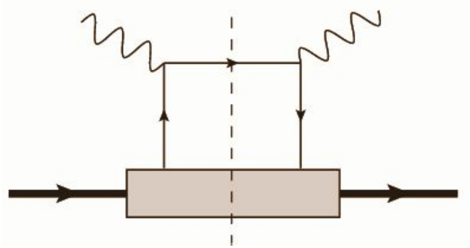
Disconnected	quark-antiquark	higher-twist
irrelevant	dominant	suppressed

Partonic interpretation



$$2MW_{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J_\mu^\dagger(x),\,J_\nu(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) = \;:\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi):$$



quark-antiquark
(handbag diagram)

$$2MW^{\mu\nu}(q,P,S) = \sum_q e_q^2\,\frac{1}{2}\,\mathrm{Tr}\left[\Phi(x,S)\,\gamma^\mu\,\gamma^+\,\gamma^\nu\right]$$

ϕ(x,S) : “collinear” quark correlator

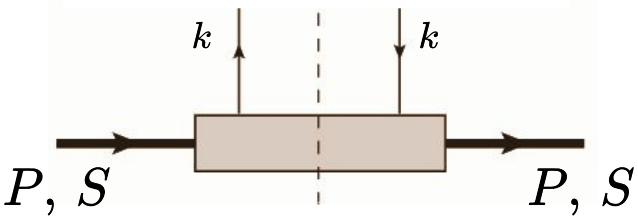
$x_B \simeq x \equiv k^+/P^+$ → measure collinear parton dynamics

The quark transverse momentum is integrated out in DIS

Quark distribution correlator (collinear)

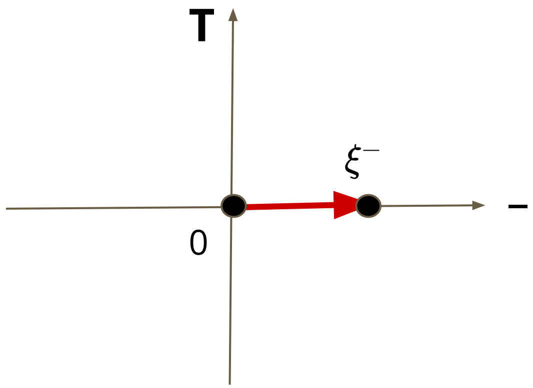
See also M. Radici's lectures

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\, \psi_i(\xi) \big| PS\rangle$$



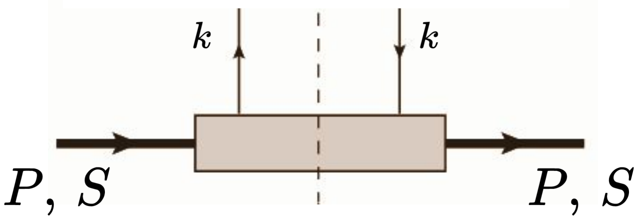
$$\Phi_{ij}(x,S) \;=\; \int dk^+\, dk^-\, d^2\mathbf{k}_T\, \delta\big(k^+ - xP^+\big)\, \Phi(k,P,S) =$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{i\,k\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\, \psi_i(\xi) \big| PS\rangle_{\,\xi^+ = \,\xi_T \,= 0}$$



Parton physics on the light cone

Collinear parton distribution functions



$\Phi_{ij}(k, P, S)$: non-perturbative hadron structure matrix

$$\Phi(x, S, T) = \frac{1}{2} f_1(x) \not{n}_+ +$$ → unpolarized PDF

$$\frac{1}{2} g_1(x) S_L \gamma_5 \not{n}_+ +$$ → longitudinally polarized PDF (helicity)

$$\frac{1}{2} h_1(x) i\sigma_{\mu\nu} \gamma_5 n_+^{\mu} S_T^{\nu} +$$ → transversely polarized PDF (transversity)

$$\frac{1}{2} f_{1\,LL}(x) S_{LL} \not{n}_+$$ → Tensor polarized PDF

"Leading twist" approximation

Ok, but ...

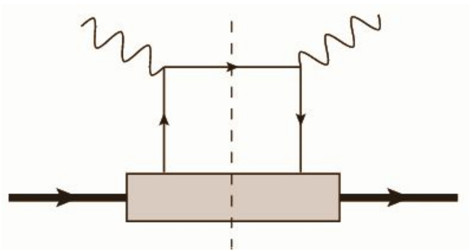
what about transverse momentum?

Where is transverse momentum?

$$\frac{d^3\sigma}{dx_B\,dy\,d\phi_S} = \frac{\alpha^2\,y}{2\,Q^4}\,L_{\mu\nu}(l,l',\lambda_e)\;2MW^{\mu\nu}(q,P,S)$$

INCLUSIVE DIS → differential in xB

We need a process with an **“experimental handle” on transverse momentum**, for example **Semi Inclusive DIS**



quark-antiquark

$$2MW^{\mu\nu}(q,P,S) \;=\; \sum_q e_q^2\,\tfrac{1}{2}\,\mathrm{Tr}\left[\Phi(x,S)\,\gamma^\mu\,\gamma^+\,\gamma^\nu\right]$$

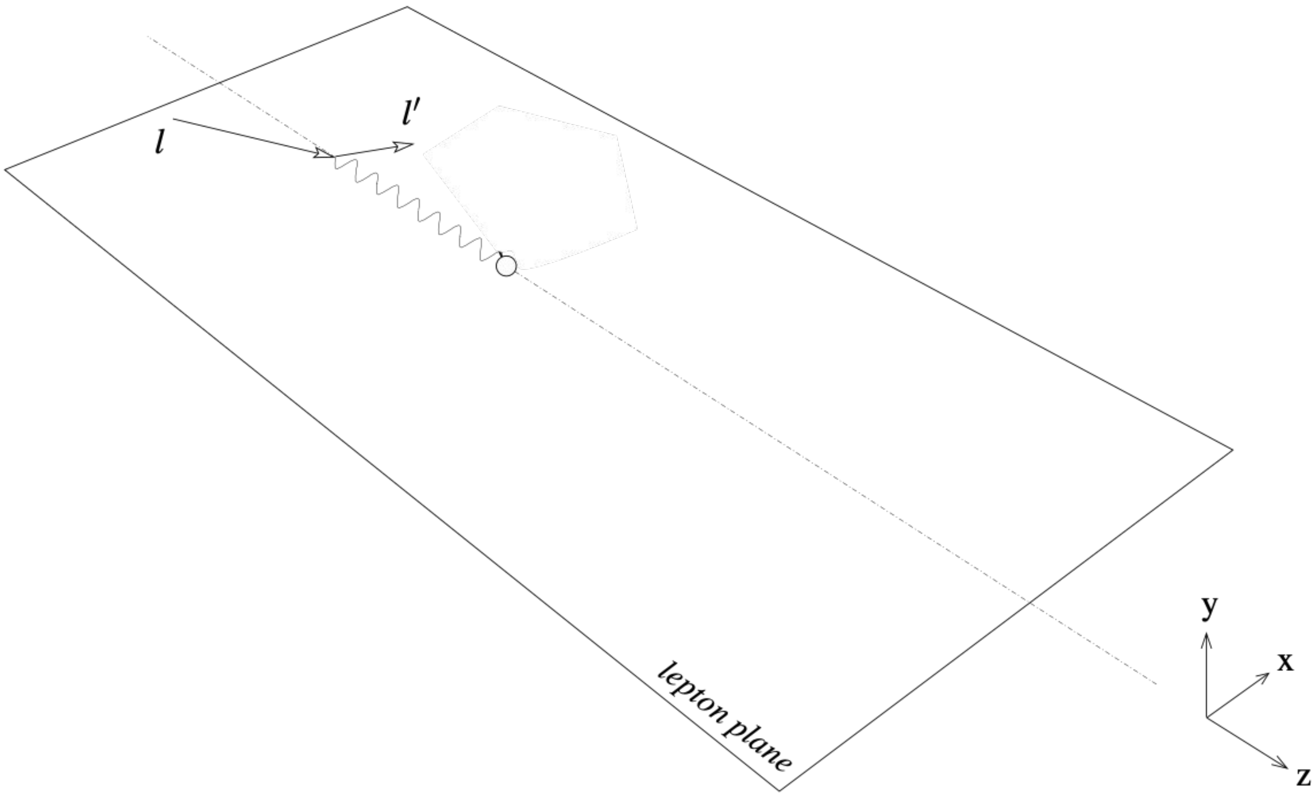
ϕ(x,S) : “collinear” quark correlator

$x_B \simeq x \equiv k^+/P^+$ → measure collinear parton dynamics

The quark transverse momentum is integrated out in DIS

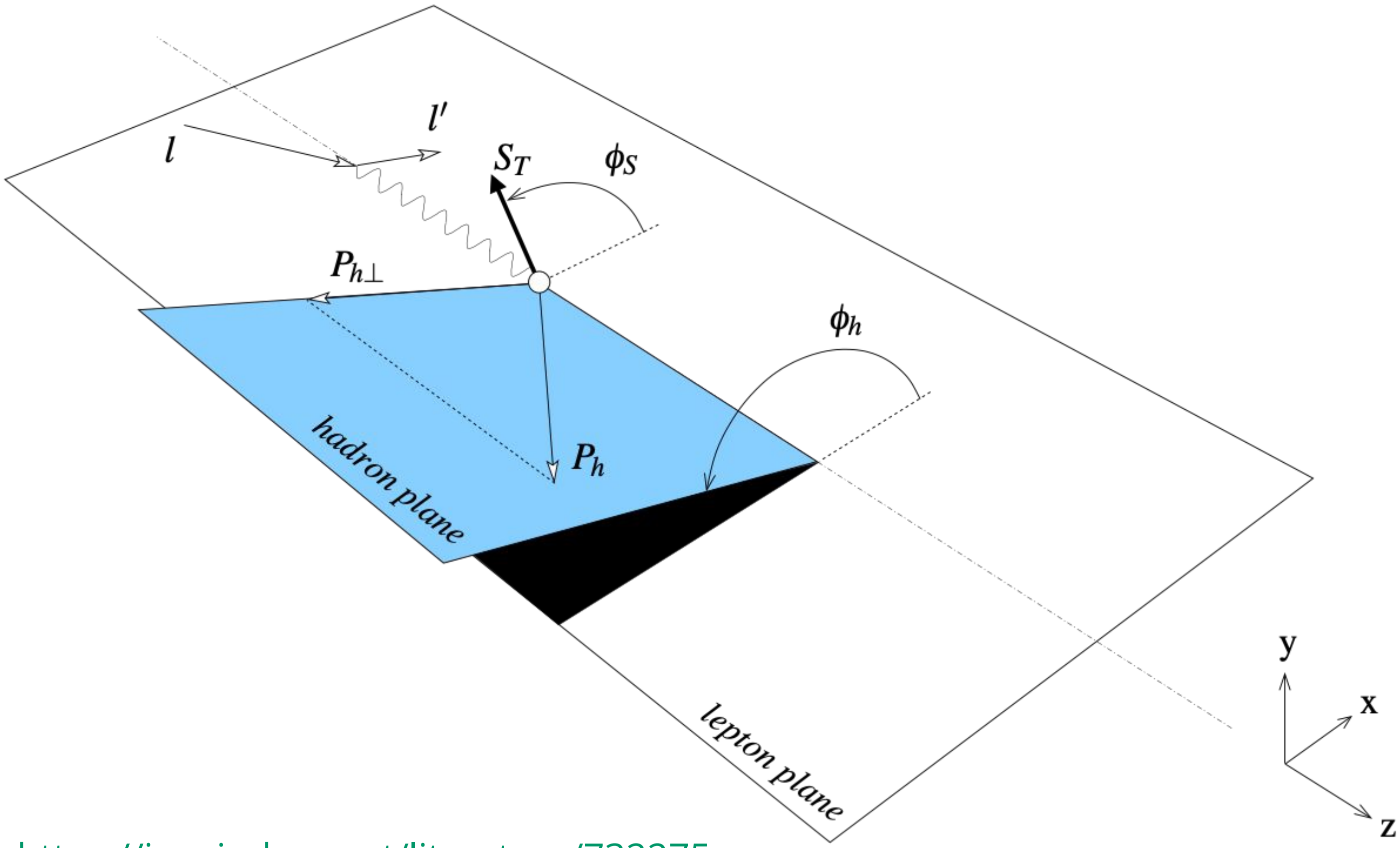
Deep-inelastic scattering

$$l(\ell) + N(P) \rightarrow l'(\ell') + X(P_X)$$



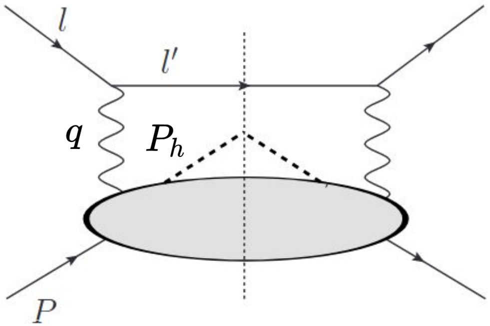
Semi-Inclusive DIS

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$



https://inspirehep.net/literature/732275

Cross section DIS vs SIDIS



$$x_B = \frac{Q^2}{2\,P \cdot q}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}.$$

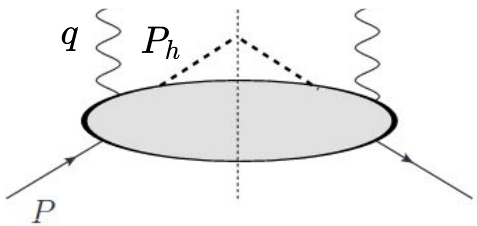
"Handle" on collinear parton dynamics

$$\frac{d^3\sigma}{dx_B\, dy\, d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4} L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$ DIS

"Handle" on transverse parton dynamics too

$$\frac{d^6\sigma}{dx_B\, dy\, dz_h\, d\phi_S\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2\, y}{2\, z_h\, Q^4} L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$ SIDIS

SIDIS hadronic tensor (unpolarized)



Compared to DIS, there are **five** structure functions instead of two for **unpolarized target**

They depend on two extra variables

$$2MW^{\mu\nu}(q,P,S) = \frac{2z_h}{x_B}\Big[-g_\perp^{\mu\nu} F_{UU,T}(x_B,z_h,P_{h\perp}^2,Q^2) + \hat{t}^\mu \hat{t}^\nu F_{UU,L}(x_B,z_h,P_{h\perp}^2,Q^2) + \left(\hat{t}^\mu \hat{h}^\nu + \hat{t}^\nu \hat{h}^\mu\right) F_{UU}^{\cos\phi_h}(x_B,z_h,P_{h\perp}^2,Q^2) + \left(\hat{h}^\mu \hat{h}^\nu + g_\perp^{\mu\nu}\right) F_{UU}^{\cos 2\phi_h}(x_B,z_h,P_{h\perp}^2,Q^2) - \mathrm{i}\left(\hat{t}^\mu \hat{h}^\nu - \hat{t}^\nu \hat{h}^\mu\right) F_{LU}^{\sin\phi_h}(x_B,z_h,P_{h\perp}^2,Q^2)\Big],$$

$$\hat{h} = \frac{P_{h\perp}}{|P_{h\perp}|}$$

SIDIS cross section (unpolarized)

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$\frac{d\sigma}{dx\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2}{x\, y\, Q^2}\, \frac{y^2}{2\,(1-\varepsilon)} \Big\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} \Big\}$$

5 structure functions for unpolarized target

For more details see https://inspirehep.net/literature/732275

SIDIS cross section (polarized nucleon - spin 1/2)

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

18 structure functions
for polarized nucleon target

Dependence on **spin** and
azimuthal **angles**.
One can build
asymmetries to
single out contributions

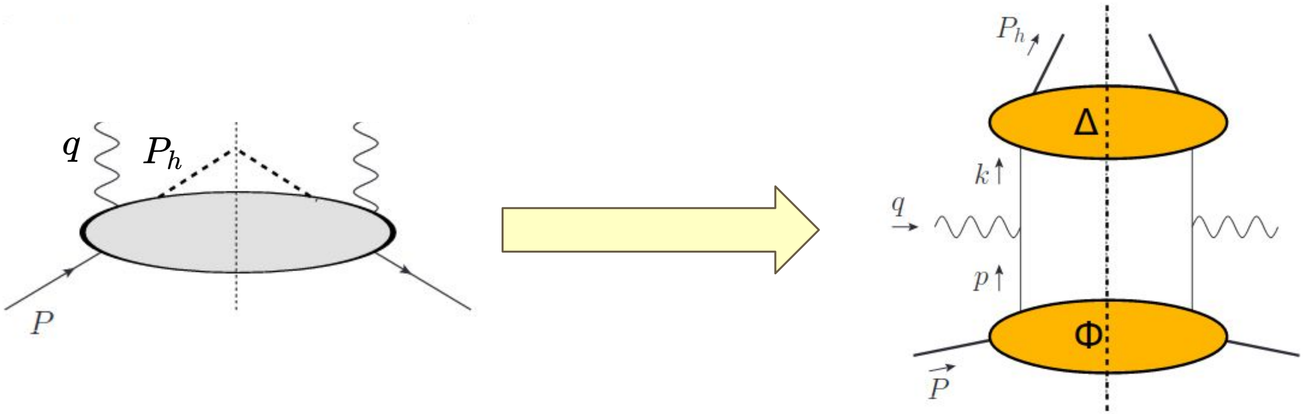
For more details see https://inspirehep.net/literature/732275

SIDIS cross section (polarized deuteron - spin 1)

?

Partonic interpretation: TMD correlators

$$2\,M\,W_{\mu\nu}(q,P,S,P_h) = \sum_X \int \frac{d^3P_X}{2E_X}\,\delta^4(P+q-P_X)\,\langle PS|\,J_\mu^\dagger(0)\,|P_h\,P_X\rangle\,\langle P_h\,P_X|\,J_\nu(0)|PS\rangle$$

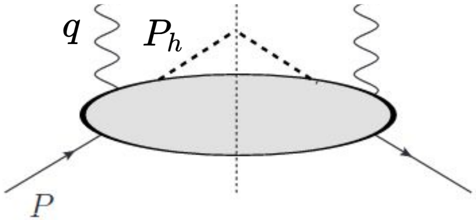


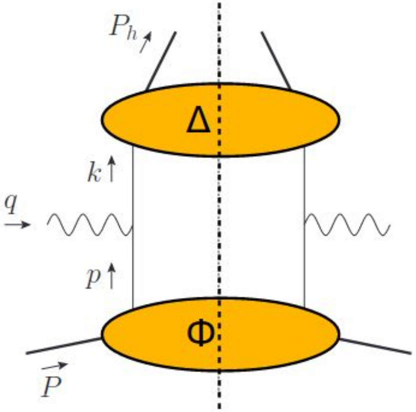
+ higher twist (suppressed)

The presence of an identified hadron does not allow us to use the commutator form
→ **OPE not applicable**

Use “***diagrammatic approach***” → use quark correlation functions for hadron structure and formation : it corresponds to the result in **TMD factorization** (when there is one)

Partonic interpretation: TMD distributions





+ higher twist (suppressed)

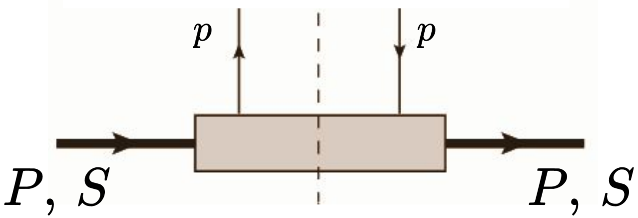
$$2MW^{\mu\nu}(q,P,S,P_h) = \frac{2z_h}{x_B}\,\mathcal{C}\Big[\mathrm{Tr}\big(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu\big)\Big]$$

$$\mathcal{C}[wfD] = \sum x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T,\boldsymbol{k}_T)\, f^a(x,p_T^2)\, D^a(z,k_T^2)$$

We can describe the measured t.m. with the partonic transverse momenta in the target and in the hadronization!

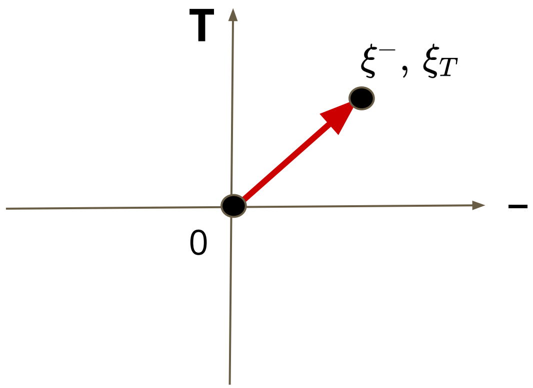
Quark distribution correlator (TMD)

$$\Phi_{ij}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p\cdot\xi} \langle PS | \overline{\psi}_j(0)\, \psi_i(\xi) | PS \rangle$$



$$\Phi_{ij}(x, \mathbf{p}_T, S) = \int dp^+\, dp^-\, \delta(p^+ - xP^+)\, \Phi(p, P, S) =$$

$$= \int \frac{d\xi^-\, d^2\xi_T}{2\pi}\, e^{i\,p\cdot\xi} \langle PS | \overline{\psi}_j(0)\, \psi_i(\xi) | PS \rangle_{\xi^+ = 0}$$



Quark TMD distribution functions (spin ½)

quark pol. / nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

At leading twist: **8** TMD PDFs

(similar classification for gluons)

The **symmetries of QCD** play a crucial role in this classification

- **Black**: time-reversal even AND collinear
- **Blue**: time-reversal even
- **Red**: time-reversal odd (*process dependence*)

Quark inside spin ½ hadron

Quark TMD distribution functions (spin 1)

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$

At leading twist: **18 (!)** TMD PDFs

(similar classification for gluons)

The **symmetries of QCD** play a crucial role in this classification

Quark inside spin 1 hadron

Quark TMD PDFs (spin ½)

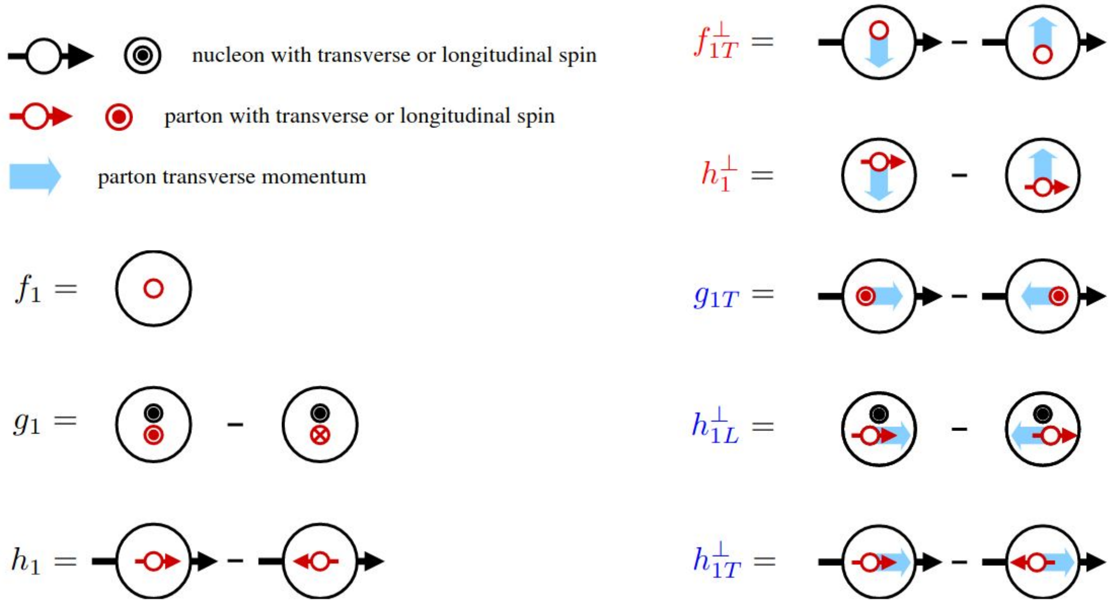
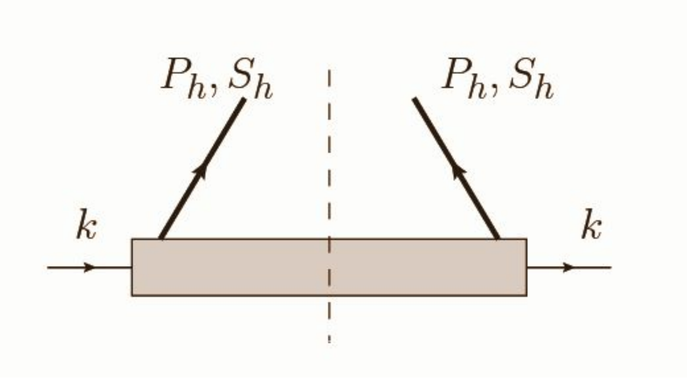


Figure 3.5: Probabilistic interpretation of twist-2 transverse-momentum-dependent distribution functions. To avoid ambiguities, it is necessary to indicate the directions of quark's transverse momemtum, target spin and quark spin, and specify that the proton is moving out of the page, or alternatively the photon is moving into the page.

From Bacchetta's "Trento" lecture notes

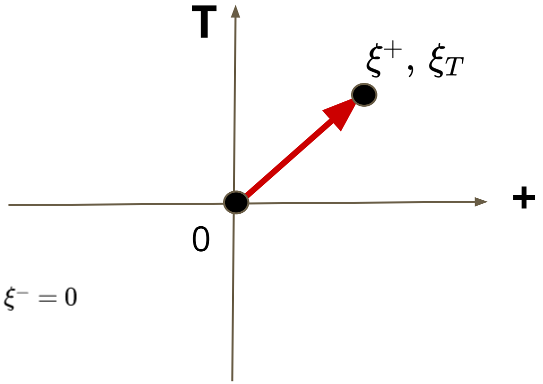
Quark fragmentation correlator (TMD)

$$\Delta^h_{ij}(k, P_h, S_h) \; = \; \sum_X \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle 0 \big| \psi_i(\xi) | X P_h S_h \rangle \langle X P_h S_h | \overline{\psi}_j(0) \big| 0 \rangle$$



$$\Delta_{ij}(z, \mathbf{k}_T, S_h) \; = \; \int dk^- \, dk^+ \, \delta\left(k^- - \frac{1}{z} P_h^-\right) \Delta_{ij}(k, P_h, S_h) =$$

$$= \sum_X \int \frac{d\xi^+ \, d^2\xi_T}{2\pi}\, e^{i\,k\cdot\xi}\, \langle 0 \big| \overline{\psi}_i(\xi) \big| X P_h S_h \rangle \langle X P_h S_h \big| \overline{\psi}_j(0) \big| 0 \rangle_{\;\xi^- = 0}$$



Quark TMD fragmentation functions

quark pol.

hadron pol.	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	G_{1T}	H_1, $H_{1T}^{\perp}$

Collins function

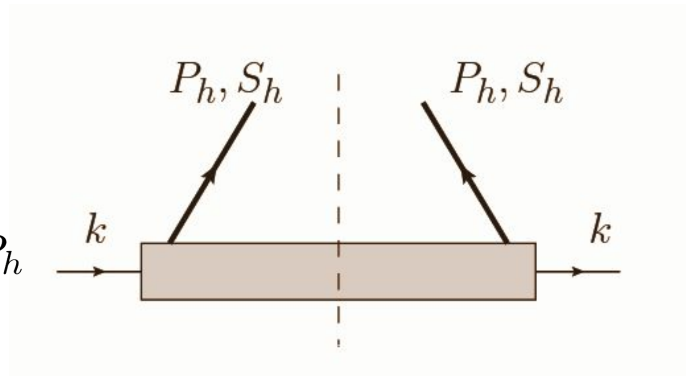
At leading twist:
8 TMD FFs and
3 collinear FFs (diagonal)

The **symmetries of QCD** play
a crucial role in this classification

Quark fragmentation and propagation

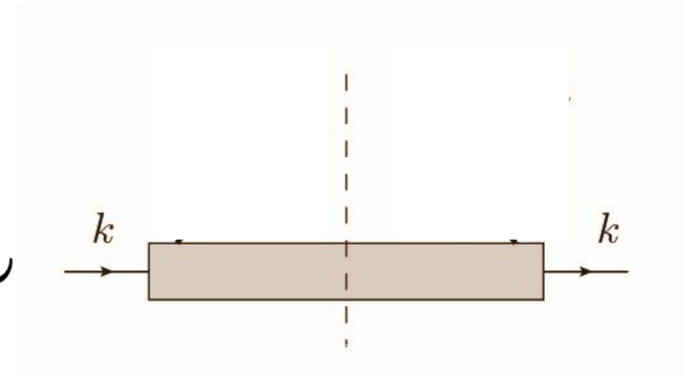
See https://inspirehep.net/literature/1797479

$$\sum_{h,S_h} \int d^4 P_h$$



Fragmentation correlator

$$\sim$$



Cut quark propagator

The **hadronization** mechanism, and thus fragmentation functions, is connected to the **dynamical content of the propagator**

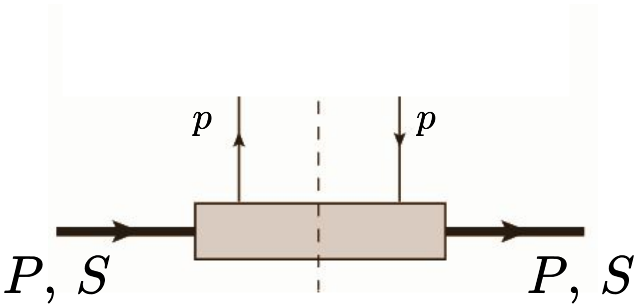
E.g. connection between **twist 3 fragmentation functions** and **dynamical quark mass**

hadronization ⟷ **chiral symmetry breaknig**

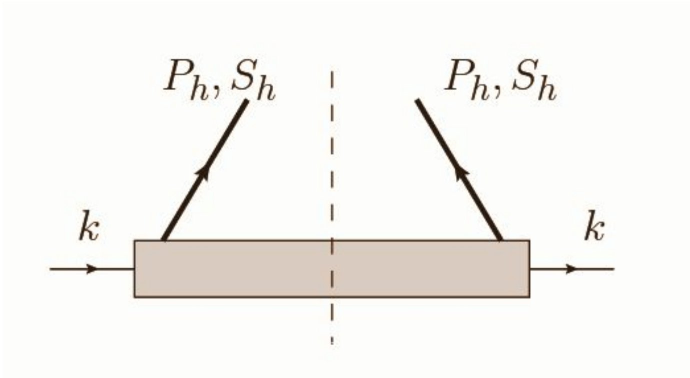
Gluon correlators

See also https://inspirehep.net/literature/534393

$$\Gamma^{\mu\nu;\rho\sigma}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\xi}\, \langle PS|F^{\mu\nu}(0)\, F^{\rho\sigma}(\xi)|PS\rangle$$



$$\hat{\Gamma}^{\mu\nu;\rho\sigma}(k, P_h, S_h) = \sum_X \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\xi} \times$$
$$\langle 0|F^{\mu\nu}(0)|P_h S_h X\rangle\langle P_h S_h X|F^{\rho\sigma}(\xi)|0\rangle$$



Quark and gluon TMD PDFs - spin 1/2

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$

Gluons	$-g_T^{ij}$	$i\epsilon_T^{ij}$	k_T^i, k_T^{ij}, etc.
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

See https://inspirehep.net/literature/1505204 for more details

Quark and gluon TMD PDFs - spin 1

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$

Gluons	$-g_T^{ij}$	$i\epsilon_T^{ij}$	k_T^i, k_T^{ij}, etc.
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	$\boldsymbol{h_{1TT}}$, $h_{1TT}^\perp$, $h_{1TT}^{\perp\perp}$

See https://inspirehep.net/literature/1505204 for more details

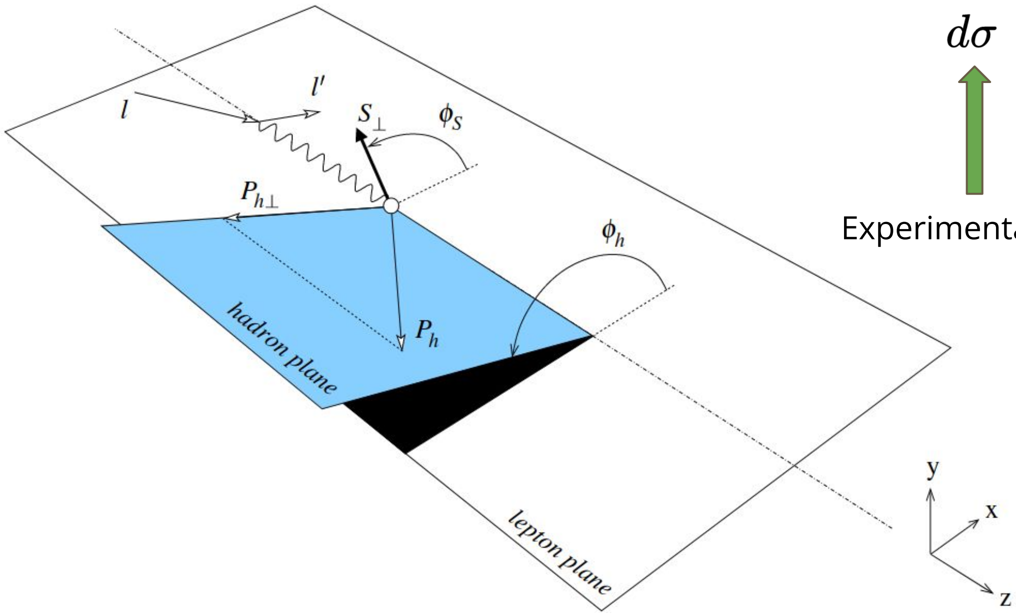
Transverse momentum imaging

Calculable
in perturbation theory

$$d\sigma \sim \mathcal{H} \; \boxed{\text{TMD PDF} \otimes \text{TMD FF}}$$

Experimental data

The partonic “maps”, to be extracted from data



$$\ell\, N \to \ell'\, h\, X$$

hermes COMPASS JLab, EIC, ...

DIS: from structure functions to PDFs

$$F_T = x_B \sum_q e_q^2 f_1^q(x_B),$$

$$F_L = 0,$$

$$F_{LL} = x_B \sum_q e_q^2 g_1^q(x_B),$$

$$F_{UT}^{\sin \phi_S} = 0,$$

$$F_{LT}^{\cos \phi_S} = -x_B \sum_q e_q^2 \frac{2M}{Q} \left(x_B g_T^q(x_B) + \frac{M_q - m_q}{M} h_1^q(x_B) \right)$$

Leading twist PDFs

Higher twist PDF

LHS: measurable
RHS: partonic quantities

→ **"Collinear" imaging**

DIS on a spin ½ hadron: structure functions at leading order in perturbation theory
(at higher orders: convolution with perturbative coefficients)

For a summary see e.g. https://inspirehep.net/literature/1603092 and https://inspirehep.net/literature/732275

SIDIS: from structure functions to TMDs

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\Big[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^\perp H_1^\perp\Big]$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^\perp D_1\Big],$$

Etc. ...

The **LHS** of these equations can be **measured** and the **RHS** is expressed in terms of **partonic quantities (TMDs)**

→ **transverse momentum imaging:**

- azimuthal modulations (Boer-Mulders and Collins)

- build spin asymmetries (Sivers)

$$\mathcal{C}\big[w f D\big] = \sum x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

structure functions at leading order in perturbation theory
(at higher orders: convolution with perturbative coefficients)

SIDIS: from structure functions to TMDs

Leading twist TMDs

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right],$$

"Collins effect"

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\, h_1^\perp H_1^\perp\right]$$

"Sivers effect"

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\, f_{1T}^\perp D_1\right],$$

Etc. ...

Higher twist TMDs

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(x e\, H_1^\perp + \frac{M_h}{M}\, f_1 \frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(x g^\perp D_1 + \frac{M_h}{M}\, h_1^\perp \frac{\tilde{E}}{z}\right)\right]$$

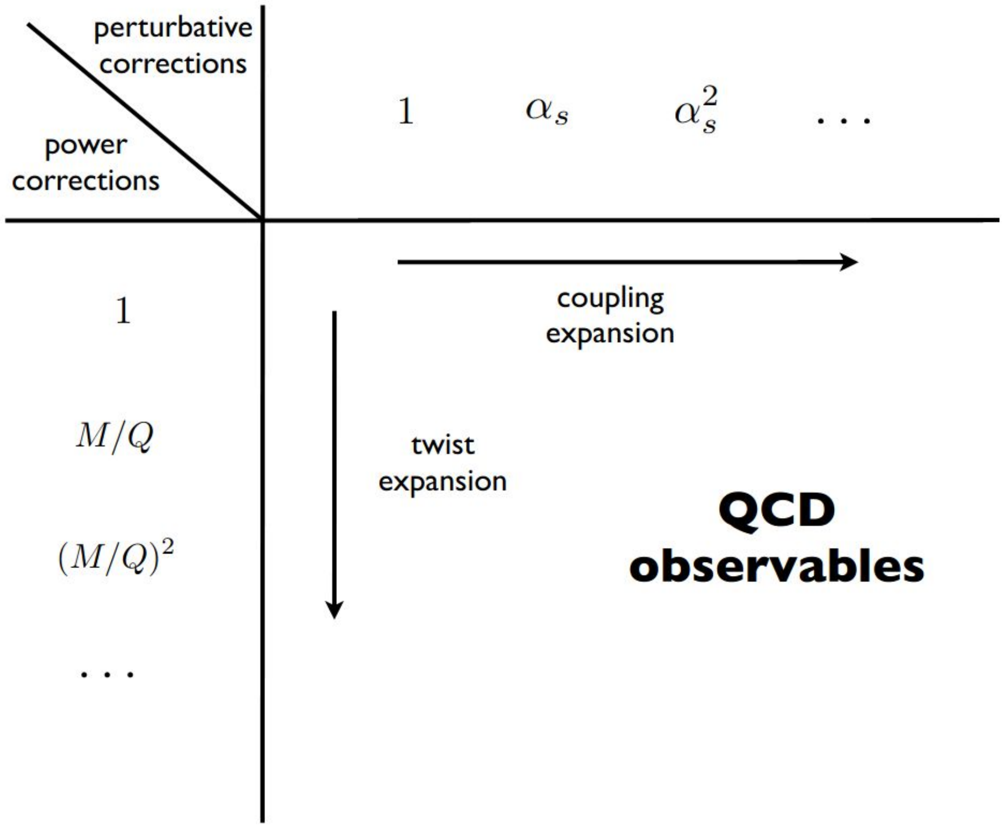
For a summary see https://inspirehep.net/literature/732275

Higher twist

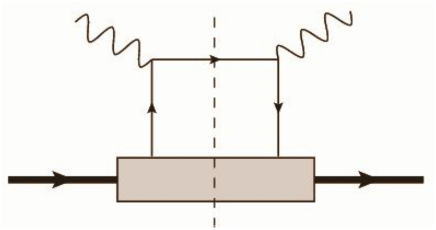
See also M. Radici's lectures

Twist t : $\left(\frac{M}{P^+}\right)^{t-2}$

(or power corrections)



Higher twist PDFs

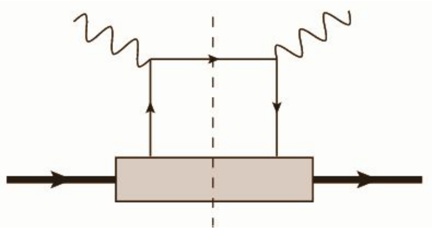


Twist 2 $$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\, \not{n}_+ + \lambda\, g_1(x)\, \gamma_5\, \not{n}_+ + h_1(x)\, \frac{\gamma_5\, [\not{S}_T, \not{n}_+]}{2} \right\}$$

Twist t : $\left(\frac{M}{P^+}\right)^{t\,-\,2}$

For more details on the definition of twist, see Jaffe's "Erice" lecture notes

Higher twist PDFs



Twist 2

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\, \not{n}_+ + \lambda\, g_1(x)\, \gamma_5\, \not{n}_+ + h_1(x)\, \frac{\gamma_5\, [\not{S}_T, \not{n}_+]}{2} \right\}$$

Twist 3

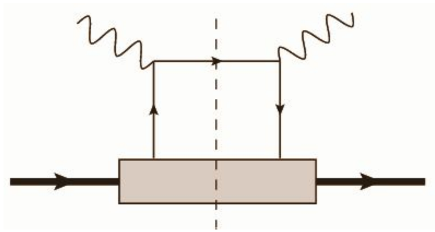
$$+ \frac{M}{2P^+}\left\{ e(x) + g_T(x)\, \gamma_5\, \not{S}_T + \lambda\, h_L(x)\, \frac{\gamma_5\, [\not{n}_+, \not{n}_-]}{2} \right\}$$

$$+ \frac{M}{2P^+}\left\{ -\lambda\, e_L(x)\, i\gamma_5 - f_T(x)\, \epsilon_T^{\rho\sigma} \gamma_\rho S_{T\sigma} + h(x)\, \frac{i\, [\not{n}_+, \not{n}_-]}{2} \right\}$$

Twist t : $\left(\frac{M}{P^+}\right)^{t\,-\,2}$

For more details on the definition of twist, see Jaffe's "Erice" lecture notes

Higher twist PDFs



Twist 2

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\, \not{n}_+ + \lambda\, g_1(x)\, \gamma_5\, \not{n}_+ + h_1(x)\, \frac{\gamma_5\, [\not{S}_T, \not{n}_+]}{2} \right\}$$

Twist 3

$$+\frac{M}{2P^+}\left\{ e(x) + g_T(x)\, \gamma_5\, \not{S}_T + \lambda\, h_L(x)\, \frac{\gamma_5\, [\not{n}_+, \not{n}_-]}{2} \right\}$$

$$+\frac{M}{2P^+}\left\{ -\lambda\, e_L(x)\, i\gamma_5 - f_T(x)\, \epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} + h(x)\, \frac{i\, [\not{n}_+, \not{n}_-]}{2} \right\}$$

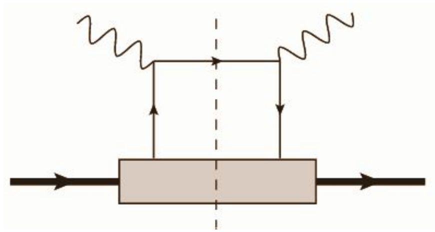
Twist 4

$$+\frac{M^2}{2(P^+)^2}\left\{ f_3(x)\, \not{n}_- + \lambda\, g_3(x)\, \gamma_5 \not{n}_- + h_3(x)\, \frac{\gamma_5\, [\not{S}_T, \not{n}_-]}{2} \right\},$$

Twist t : $\left(\frac{M}{P^+}\right)^{t\,-\,2}$

For more details on the definition of twist, see Jaffe's "Erice" lecture notes

Higher twist TMD PDFs



Twist 2

$$\Phi(x,\boldsymbol{k}_T) = \frac{1}{2}\Bigg\{ f_1(x,\boldsymbol{k}_T)\,\not{n}_+ + f_{1T}^\perp(x,\boldsymbol{k}_T)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_T^\rho S_T^\sigma}{M} + g_{1s}(x,\boldsymbol{k}_T)\,\gamma_5\,\not{n}_+$$

$$+h_{1T}(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\not{S}_T,\not{n}_+]}{2} + h_{1s}^\perp(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\not{k}_T,\not{n}_+]}{2M} + h_1^\perp(x,\boldsymbol{k}_T)\,\frac{i\,[\not{k}_T,\not{n}_+]}{2M}\Bigg\}$$

Twist 3

$$+\frac{M}{2P^+}\Bigg\{ e(x,\boldsymbol{k}_T) + f^\perp(x,\boldsymbol{k}_T)\,\frac{\not{k}_T}{M} - f_T(x,\boldsymbol{k}_T)\,\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma}$$

$$-\lambda\, f_L^\perp(x,\boldsymbol{k}_T)\,\frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M} - e_s(x,\boldsymbol{k}_T)\,i\gamma_5$$

$$+g_T'(x,\boldsymbol{k}_T)\,\gamma_5\,\not{S}_T + g_s^\perp(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,\not{k}_T}{M} + h_T^\perp(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\not{S}_T,\not{k}_T]}{2M}$$

$$+h_s(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\not{n}_+,\not{n}_-]}{2} + h(x,\boldsymbol{k}_T)\,\frac{i\,[\not{n}_+,\not{n}_-]}{2}\Bigg\}. \qquad (3.44)$$

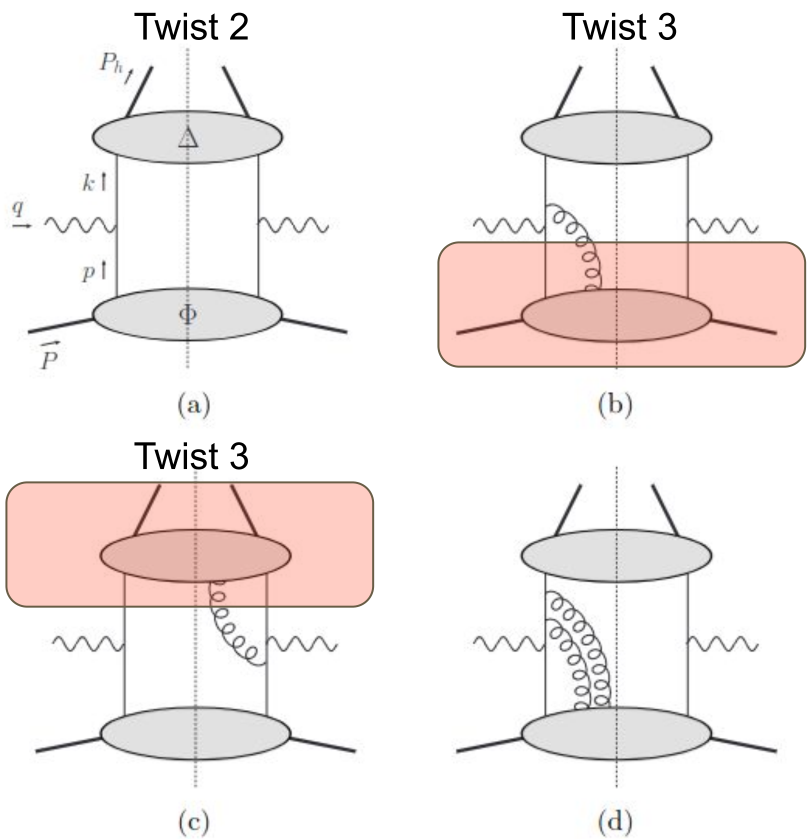
Derived within the “**diagrammatic approach**” : https://inspirehep.net/literature/400866

Interpretations in **TMD factorization** too:

- https://inspirehep.net/literature/2514090
- https://inspirehep.net/literature/1991138
- https://inspirehep.net/literature/2669575

SIDIS

$$2MW^{\mu\nu} = 2z\sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\ \mathrm{Tr}\Big\{\Phi^a(x,p_T)\gamma^\mu\Delta^a(z,k_T)\gamma^\nu - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu \tilde{\Phi}^a_{A\alpha}(x,p_T)\gamma^\mu\Delta^a(z,k_T) + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Delta}^a_{A\alpha}(z,k_T)\gamma^\nu\Phi^a(x,p_T) + \text{h.c.}\Big]\Big\},$$



"**Dynamical**" distributions

See https://inspirehep.net/literature/732275

Equations of motion

Use the Dirac equation for the quark fields in the correlators

Scalar PDF

$$x\, e(x) = x\, \tilde{e}(x) + \frac{m_q}{M_h}\, f_1(x)$$

q-g-q

Twist-2 x mass ratio

WW approx.

Scalar FF

$$E(z) = \tilde{E}(z) + \frac{m_q}{M_h}\, z D_1(z)$$

q-g-q

Twist-2 x mass ratio

WW approx.

Dressed quark mass

$$M_q = \tilde{m}_q + m_q$$

q-g-q

(**dynamical mass**)

current mass

WW approx.

See https://inspirehep.net/literature/732275