UNIVERSITÀ DI TORINO
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Introduction to transverse momentum imaging

lecture 1

International school on probing hadron structure at the EIC

ICTS, Bangalore
January 31, 2024

Introduction

Quantum Chromodynamics (QCD)

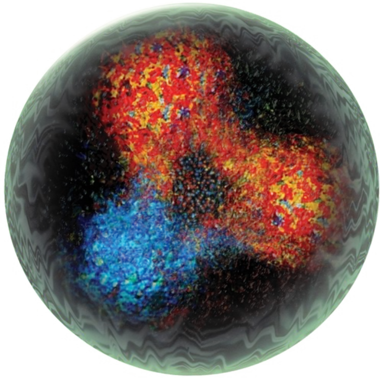
quarks and gluons (partons) are the elementary degrees of freedom in QCD, but they manifest only in bound states (**hadrons**)

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a,\mu\nu} F_{a,\mu\nu} + \overline{\psi}(i \not{D} - m)\psi$$

quark field

$$D_\mu = \partial_\mu - i g T^a A^a_\mu$$

gluon field



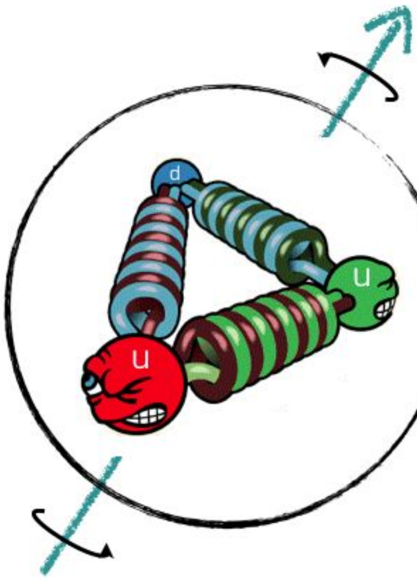
Can we understand the properties of hadrons in terms of quarks and gluons?

Global properties

Can we understand the

mass, spin, size of hadrons

in terms of quarks and gluons?



Confinement

Can we understand

hadron formation
and
confinement

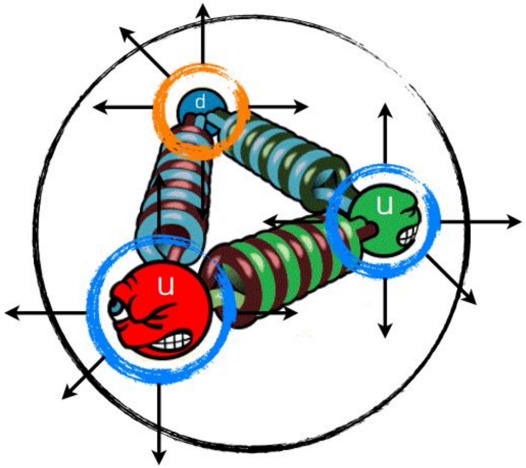
in terms of
quarks and gluons?

Internal structure

Can we understand the

structure of hadrons

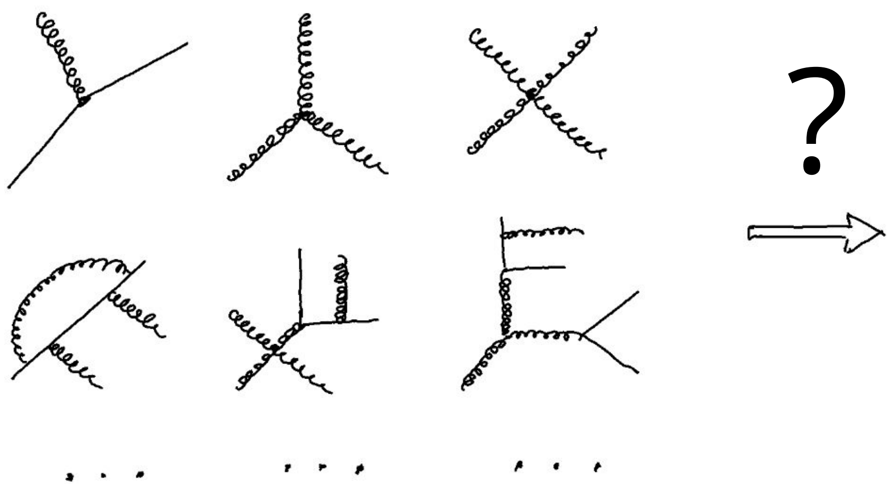
in terms of
quarks and gluons?

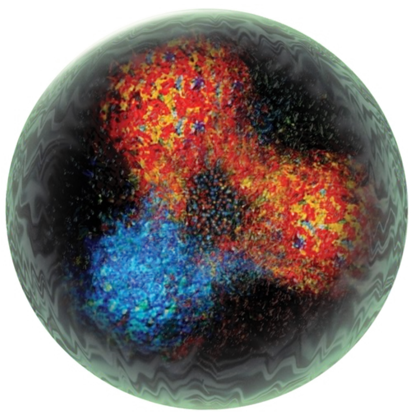


How should we "use" QCD ?

Expansion of observable in powers of the coupling constant α:

$$\mathcal{O}(Q) \sim \mathcal{O}^{(0)} + \alpha_s^1(Q)\,\mathcal{O}^{(1)} + \alpha_s^2(Q)\,\mathcal{O}^{(2)} + \alpha_s^3(Q)\,\mathcal{O}^{(3)} \ldots = ??$$



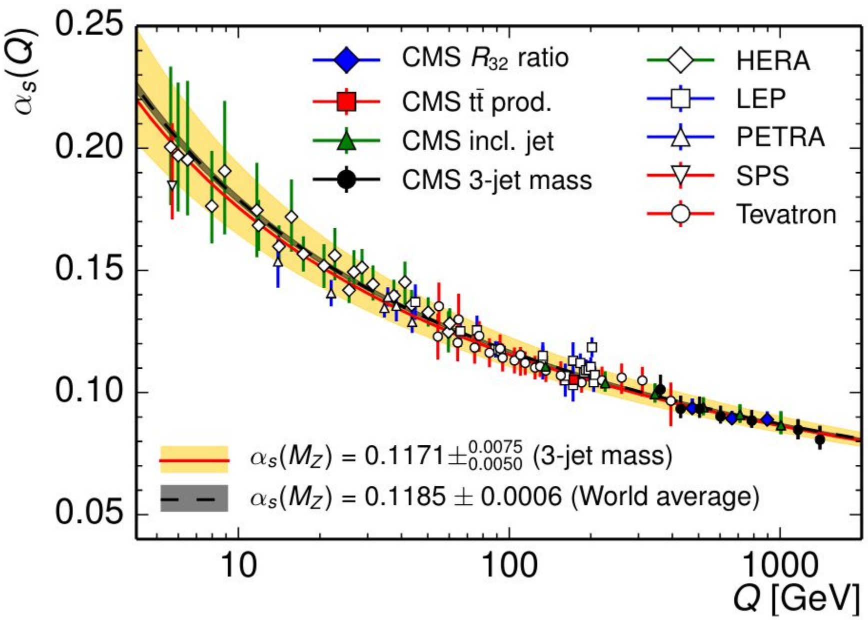


$$Q \sim M_N \sim 1\ \mathrm{GeV}$$

How should we “use” QCD ?

Expansion of observable in powers of the coupling constant α:

$$\mathcal{O}(Q) \sim \mathcal{O}^{(0)} + \alpha_s^1(Q)\,\mathcal{O}^{(1)} + \alpha_s^2(Q)\,\mathcal{O}^{(2)} + \alpha_s^3(Q)\,\mathcal{O}^{(3)} \ldots = ??$$



High energy → convergence
→ perturbative QCD

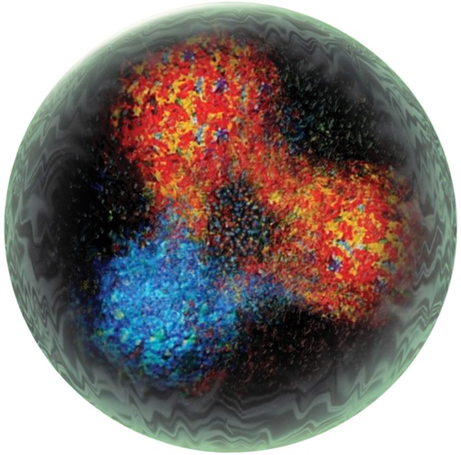
Low energy (hadronic scales)
→ non-perturbative QCD

need alternative techniques

Hadronic physics

Two macro areas to investigate:

1. **Hadron *structure*** : “hadrons → partons” transition



Hadronic physics

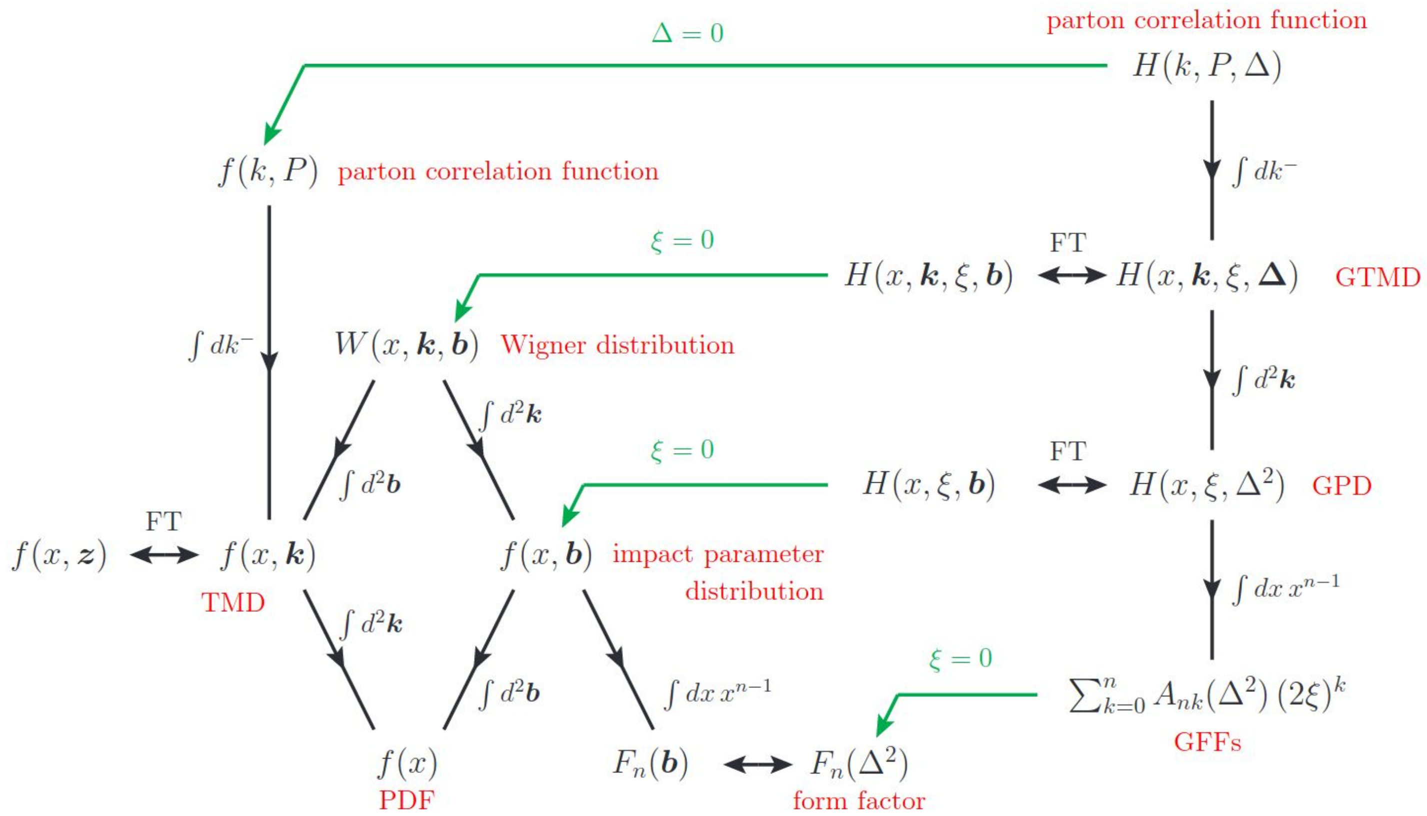
Two macro areas to investigate:

1. **Hadron *structure*** : "hadrons → partons" transition

2. **Hadron *formation*** : "partons → hadrons" transition
   (hadronization)

Credit picture: R. Seidl

The hadron structure landscape



Credit picture: M. Diehl - https://inspirehep.net/literature/1408303

The hadron structure landscape

See also B. Pasquini's lectures

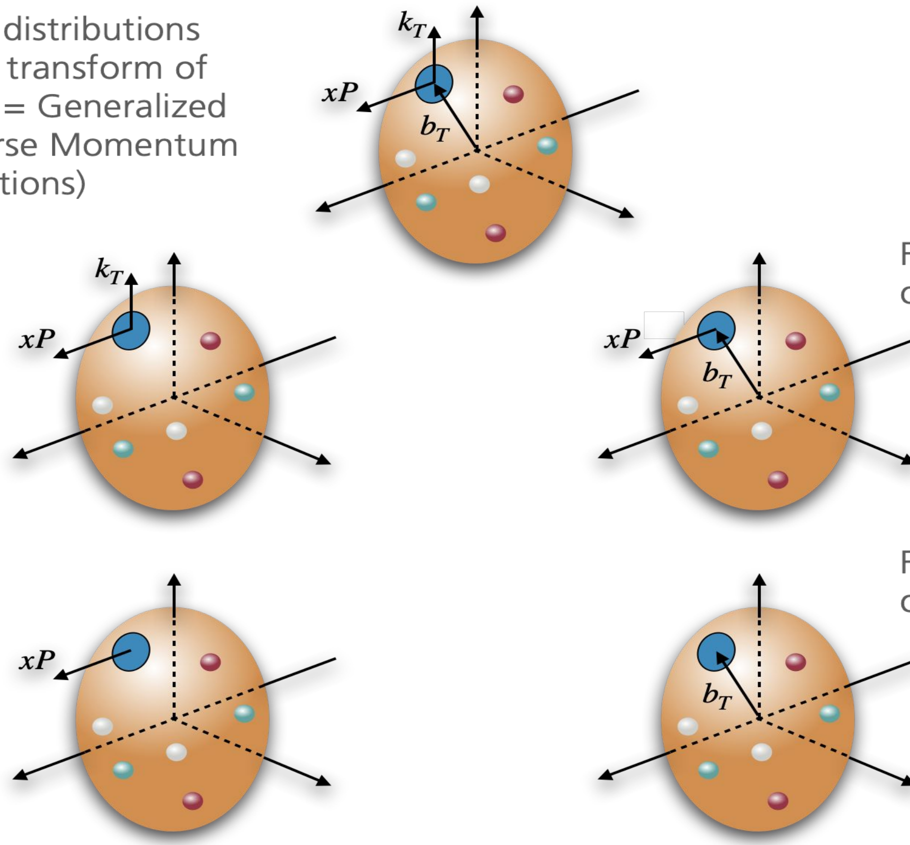
Wigner distributions (Fourier transform of GTMDs = Generalized Transverse Momentum Distributions)

TMDs

PDFs

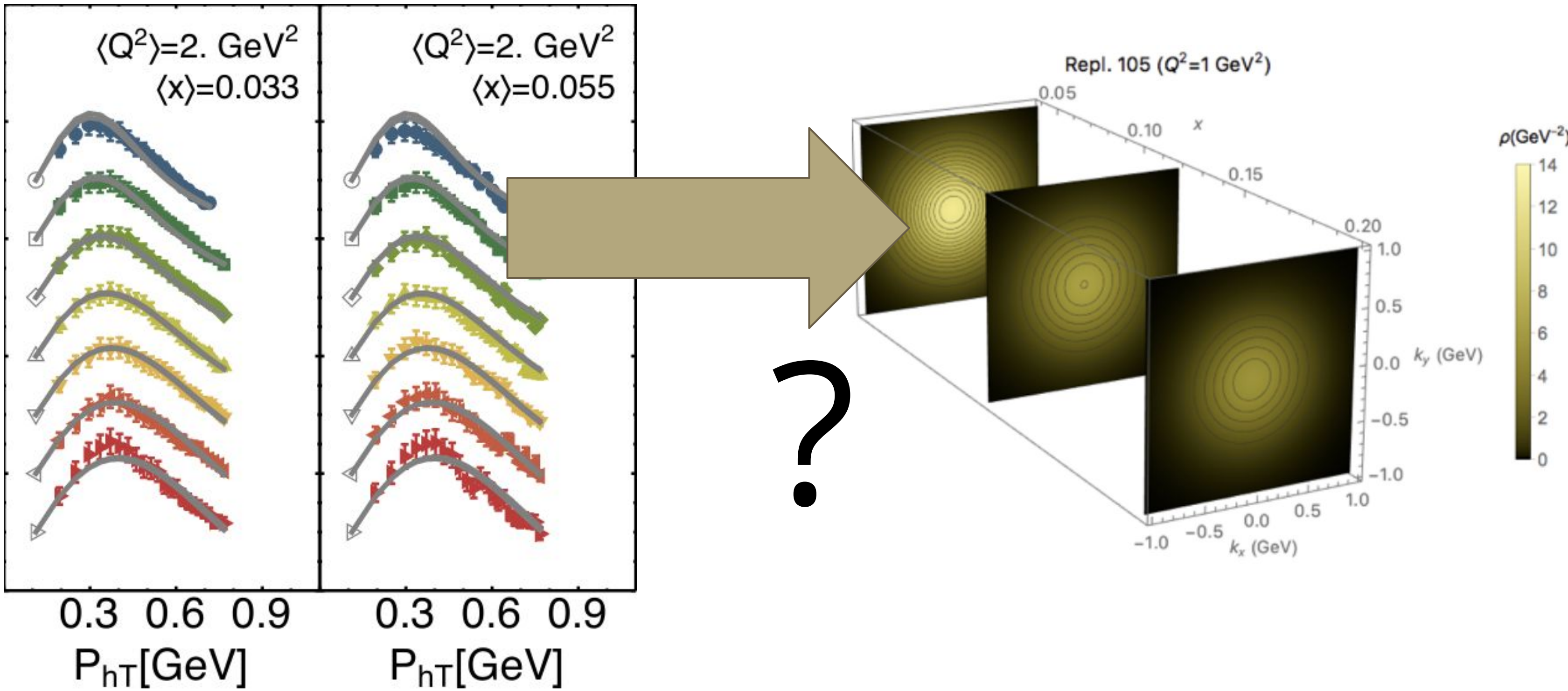
Fourier transform of GPDs

Fourier transform of Form Factors



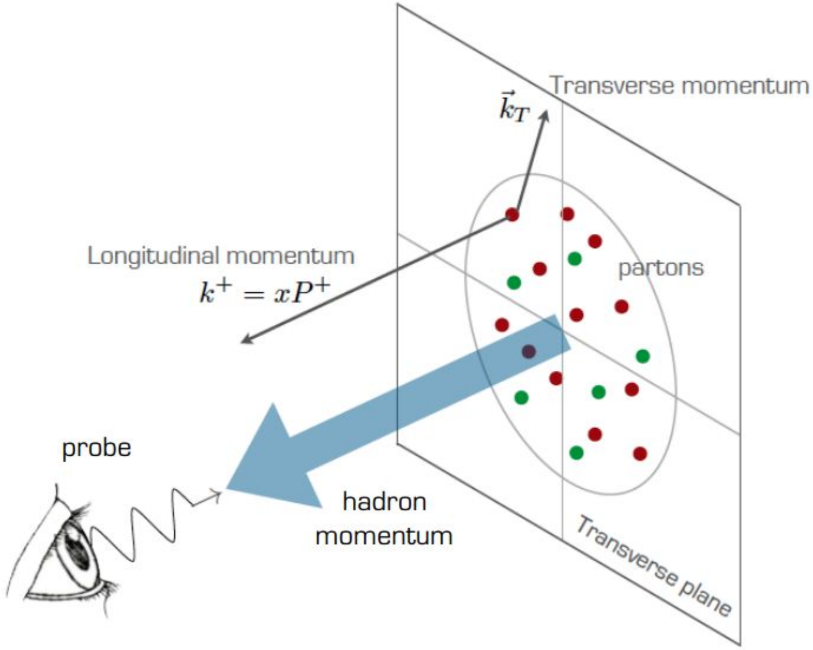
see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

Transverse momentum imaging



Parton distribution functions (PDFs)

“Maps” of hadron *structure* in momentum space



$f_1(x)$

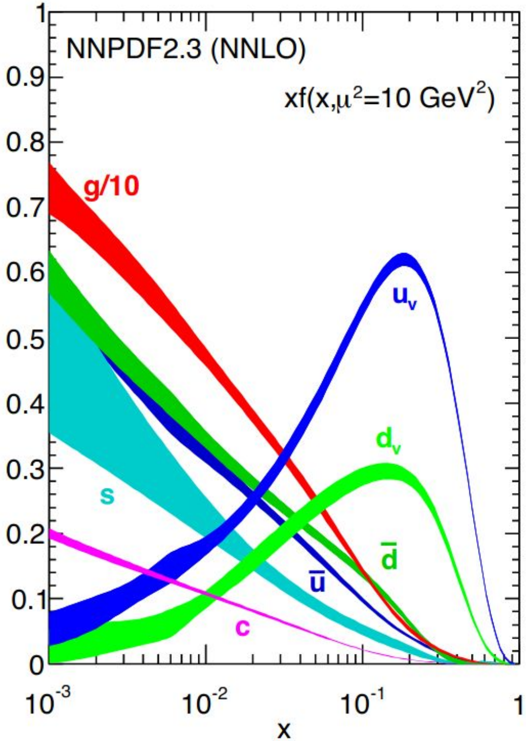
1D structure
in momentum space

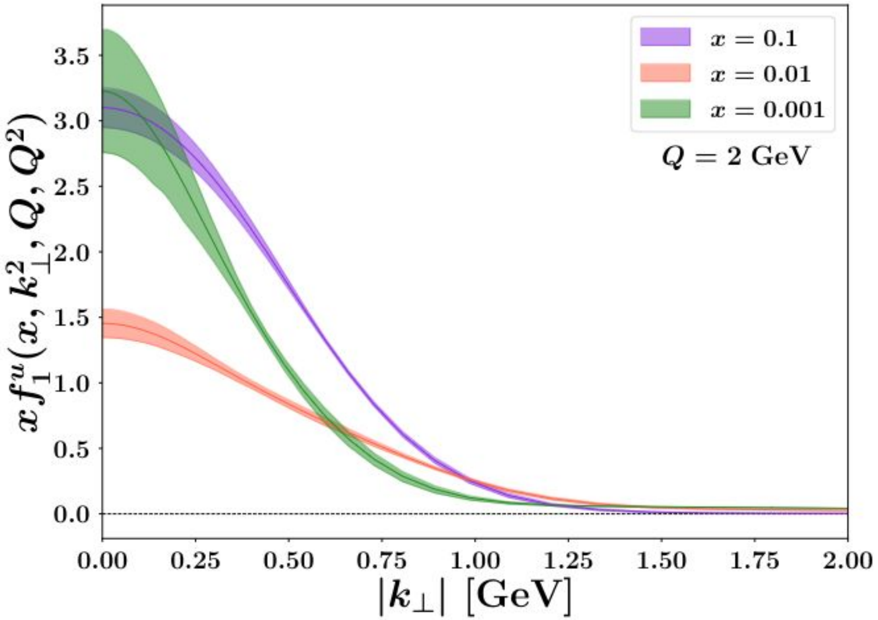
$f_1(x, k_T^2)$

3D structure
in momentum space

Credit picture: A. Bacchetta

collinear & TMD PDFs

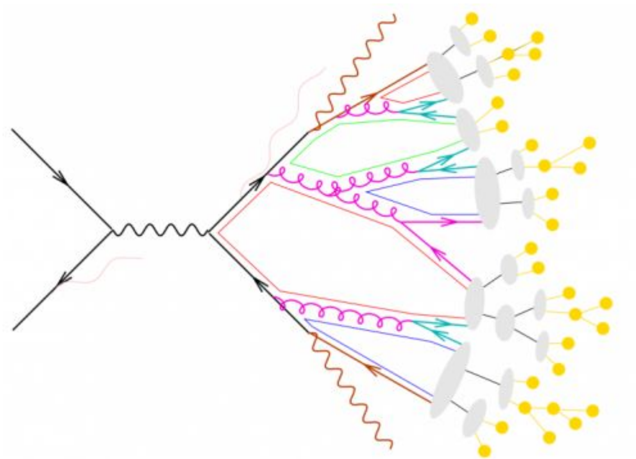




MAP collaboration
(MAPTMD22 extraction)

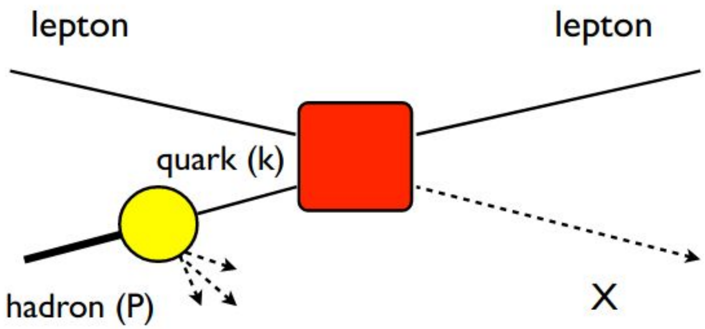
Fragmentation functions (FFs)

"Maps" of hadron *formation* in momentum space



$D_1^h(z)$	single-hadron collinear FF
$D_1^h(z, P_T^2)$	single-hadron TMD FF
$D_1^{h_1\,h_2}(z, \zeta)$	di-hadron FF
$J(s)$	inclusive jet FF
$\mathcal{G}^h(s, z)$	in-jet FF

Operator definition (PDFs)

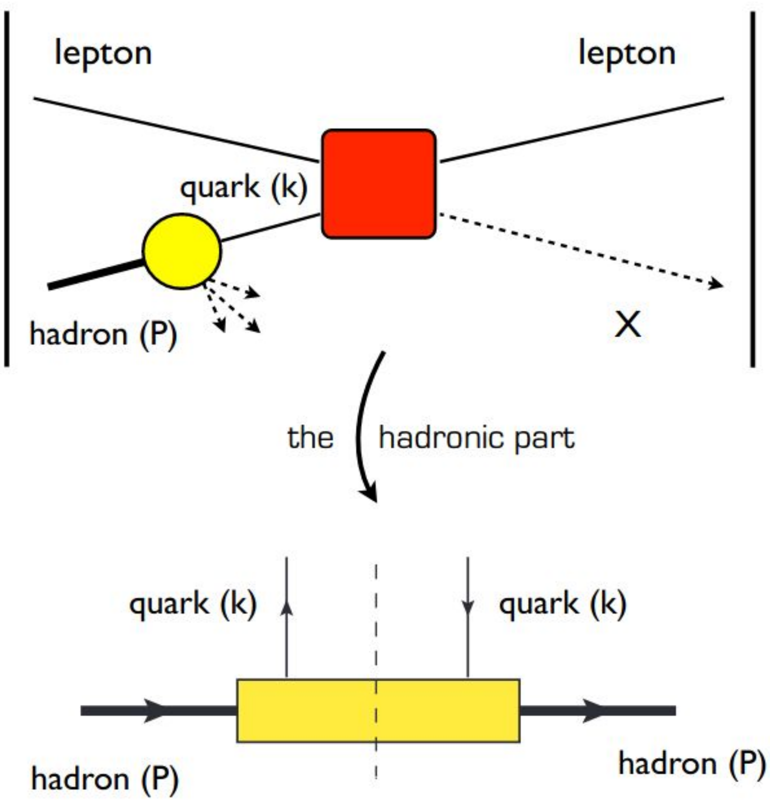


Scattering process with hadron in initial state :
(e.g. Deep Inelastic Scattering - DIS)

need a “hadron → parton” transition

(Parton Distribution Function)

Operator definition (PDFs)



PDFs defined as traces of Φ :

$$F^{[U]}(x, k_T^2) \sim \mathrm{Tr}\,[\Phi\,\Gamma]\;,\;\; \Gamma = \gamma^+\,, \ldots$$

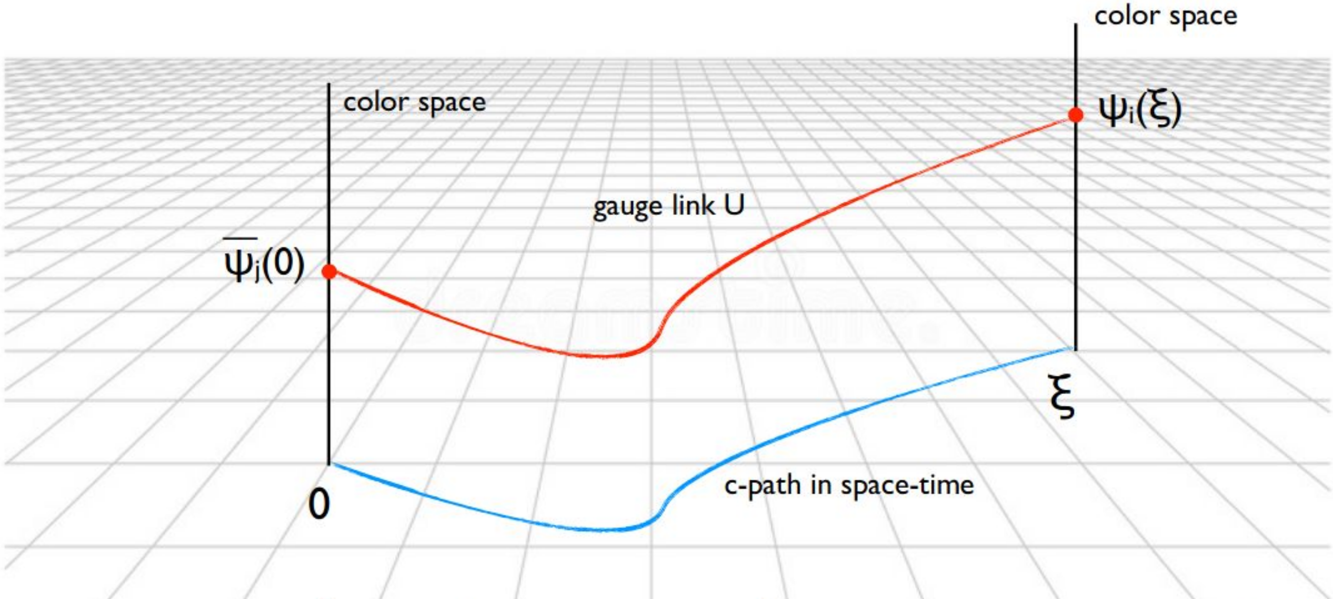
(***8 functions*** that depend on parton kinematics and gauge link U)

Hadronic part described as a ***universal*** "quark-quark correlation function" in space-time

$$\Phi_{ij}(k,P) \;=\; \mathrm{F.T.}\,\langle P\big|\,\overline{\psi_j}(0)\,U\,\psi_i(\xi)\big|P\rangle$$

Geometric structure

$$\Phi(k,P) = \text{F.T.}\langle P|\overline{\psi_j}(0)\ U\ \psi_i(\xi)|P\rangle \longrightarrow f_1^{a\ [U]}(x,k_T^2)\ \not{P} + \cdots$$



$$U(0,\xi) = \mathcal{P}\ \exp\left\{-ig_s\int_{c(0,\xi)} ds_\mu A^{\mu a}(s)t^a\right\}$$

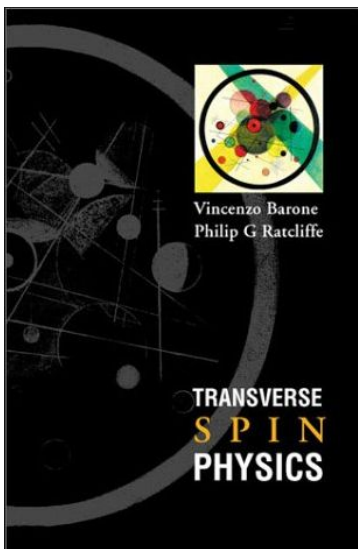
A *selection* of useful references

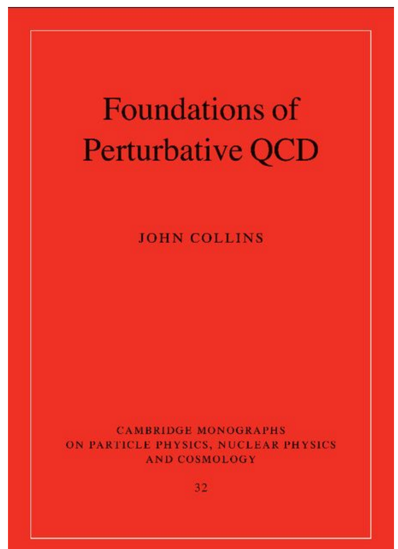
Lecture notes - graduate schools

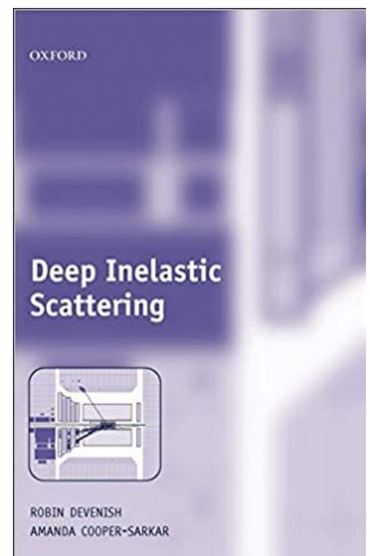
- Barone - Cabeo lecture notes: https://www.fe.infn.it/cabeo_school/2010/cabeo_school_2010.pdf
- Bacchetta - Trento lecture notes: https://www2.pv.infn.it/~bacchett/teaching/Bacchetta_Trento2012.pdf
- Jaffe - Erice lecture notes: https://arxiv.org/pdf/hep-ph/9602236.pdf
- Mulders - GGI lecture notes: http://www.nat.vu.nl/~mulders/tmdreview-vs3.pdf
- ...

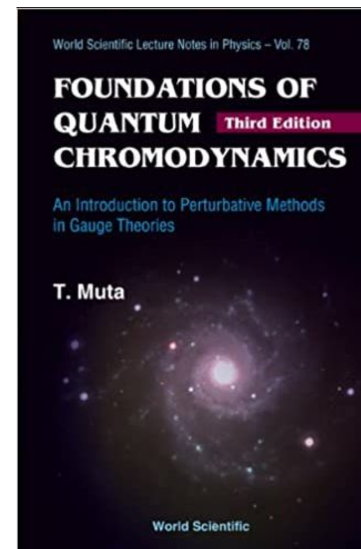
Books

- Barone, Ratcliffe: *Transverse Spin Physics*
- Collins: *Foundations of perturbative QCD*
- Devenish, Cooper-Sarkar: *Deep Inelastic Scattering*
- Muta: *Foundations of Quantum Chromodynamics*
- ...









Papers and reviews

- EPJ-A topical issue: *The 3D structure of the nucleon*
  https://link.springer.com/journal/10050/topicalCollection/AC_628286e999d9a60c9a780398df15f93d
- Diehl: *Introduction to GPDs and TMDs*
  https://inspirehep.net/literature/1408303
- Bacchetta et al.: *Single spin asymmetries: the Trento conventions*
  https://inspirehep.net/literature/660999
- Collins: *Light cone variables, rapidity and all that*
  https://inspirehep.net/literature/443368
- Metz-Vossen*: Parton fragmentation functions*
  https://inspirehep.net/literature/1475000
- *Scimemi: A short review on recent developments in TMD factorization and implementation*
  https://inspirehep.net/literature/1716549
- ...

The HUGS pedagogical page

This is a list of references in preparation for and in support of the HUGS program. Further specific references will be suggested by the speakers. You are also welcome to browse the similarly aimed CTEQ pedagogical page, and to send us your comments and suggestions (hugs@jlab.org).

General texbooks

- Donnelly, Formaggio, Holstein, Milner, Surrow - Foundations of Nuclear and Particle Physics (2017)
  - Short, focused chapters covering practically all past, present, and near future HUGS topics!
- Povh, Rith, Scholz, Zetsche, Rodejohann - Particles and Nuclei (2015)
  - Good introductory level text
- Griffiths - Introduction to Elementary Particles (2008)
  - Another good introductory level text, more focused on the elementary particle aspects
- Halzen, Martin - Quarks and leptons (2008)
  - More advanced, treats QCD in some detail

(Perturbative) QCD

- W. K. Tung, Perturbative QCD and the parton structure of the nucleon
- K. Kovarik, P. M. Nadolski, D. E. Soper, Hadron structure in high-energy collisions
- B. Poetter, Calculational Techniques in Perturbative QCD: The Drell-Yan Process
- Textbooks:
  - J. Collins - Foundations of Perturbative QCD (2011)
  - Kovchegov, Levin - Quantum Chromodynamics at High Energy (2012)
- Check also:
  - The "Suggested QCD literature" list of references by T. Rogers

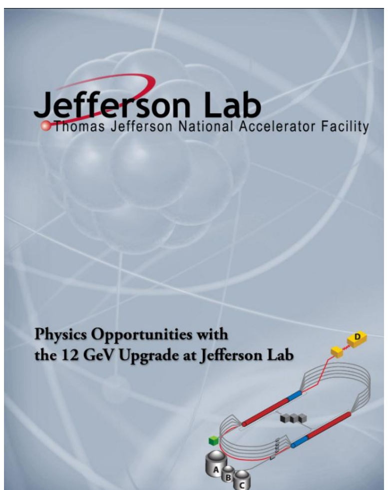
3D Structure of Nucleons

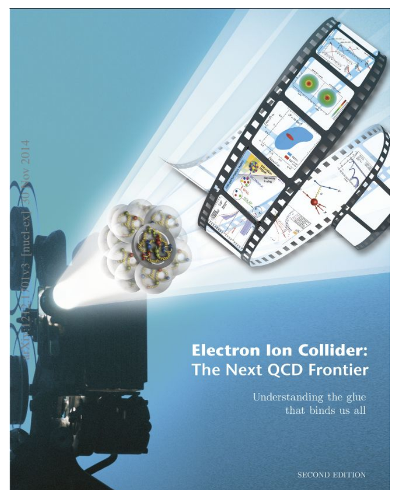
- Introductory:
  - A. Bacchetta, Transverse Momentum DIstributions (a.k.a."Trento lectures", 2012)
  - P. Mulders, Transverse-momentum distributions and beyond: setting up the nucleon tomography, lectures at the Galileo Galilei Institute (2015)
  - M. Diehl, Introduction to GPDs and TMDs, Eur.Phys.J. A52 (2016) 149
- P. Mulders, Transverse momentum dependence in structure functions in hard scattering processes
- M. Diehl, Lectures on GPDs, Varenna (ITA), 2011
- M. Diehl, Generalized Parton Distributions, Phys.Rept. 388 (2003) 41

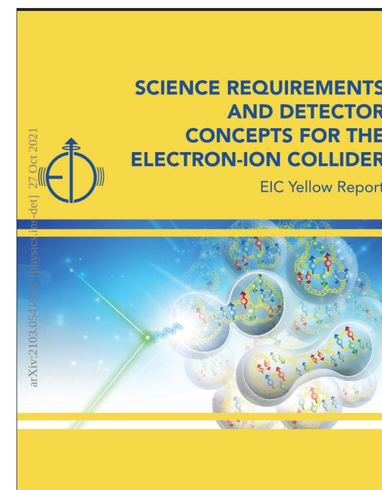
https://www.jlab.org/education/hugs/references

Experimental overviews

- Dudek et al.: *Physics opportunities with the 12 GeV upgrade at Jefferson Lab* https://inspirehep.net/literature/1125972
- Accardi et al.: *Electron Ion Collider: The next QCD Frontier - understanding the glue that binds us all* https://inspirehep.net/literature/1206324
- Abdul Khalek et al.: *The EIC Yellow Report* - https://inspirehep.net/literature/1851258
- ...





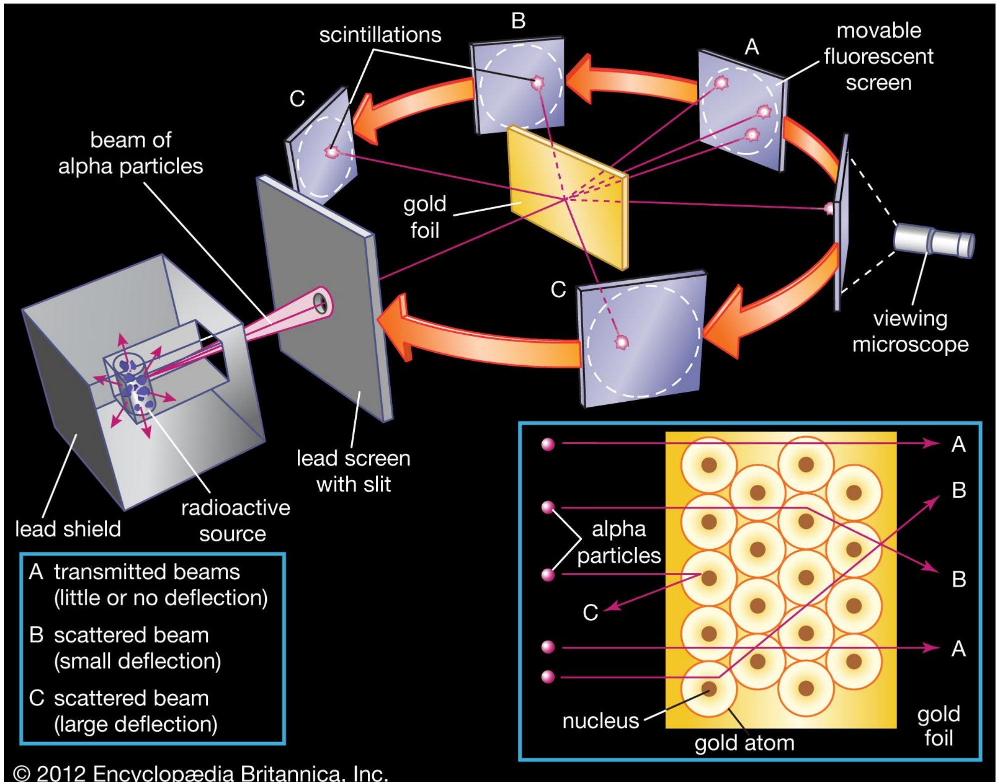


Plan of these lectures

1. **Breaking hadrons**
2. **Non-collinear partons**
3. **Symmetries & spin**
4. **Factorization, evolution, matching**
5. **Phenomenology**

1. Breaking hadrons

Geiger / Marsden / Rutherford experiment



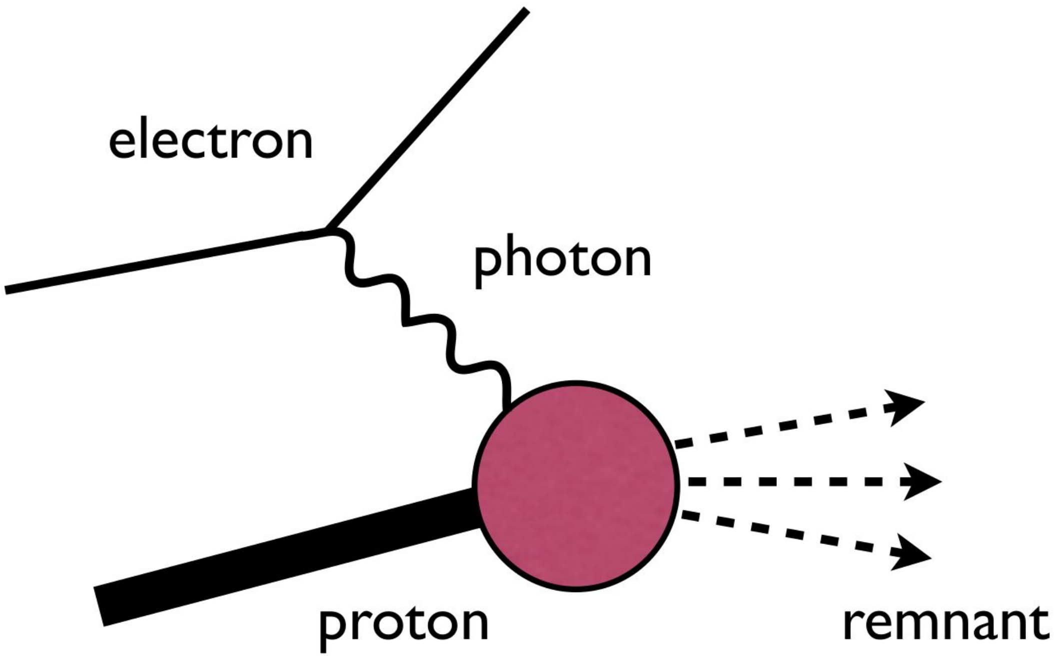
~1910

Scattering of alpha particles on gold:

discovery of the atomic nucleus

Deep-inelastic scattering

$$l(\ell) + N(P) \to l'(\ell') + X(P_X)$$



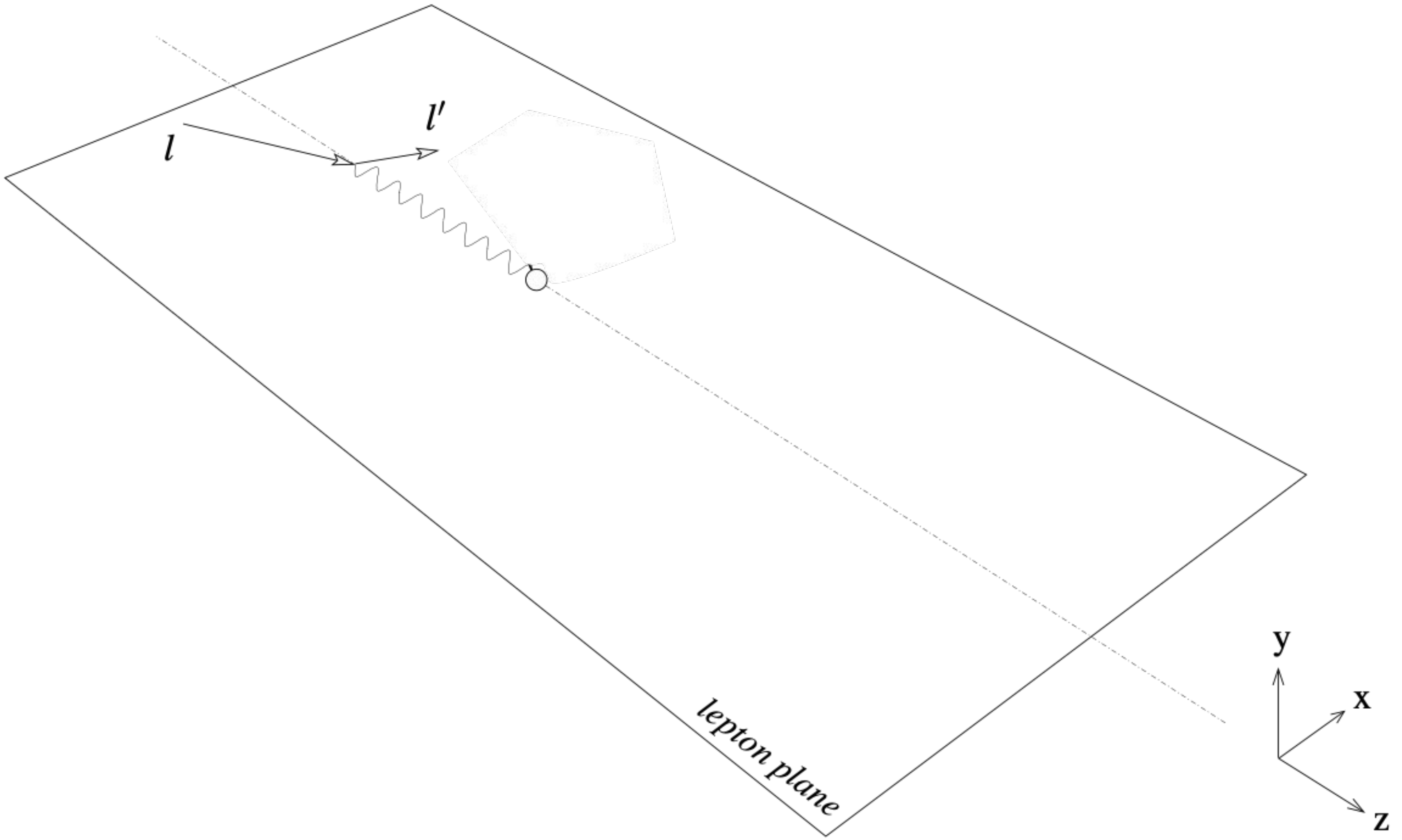
MIT-SLAC experiments ('60-'70)

Scattering of electrons off protons to test hadrons' substructure:

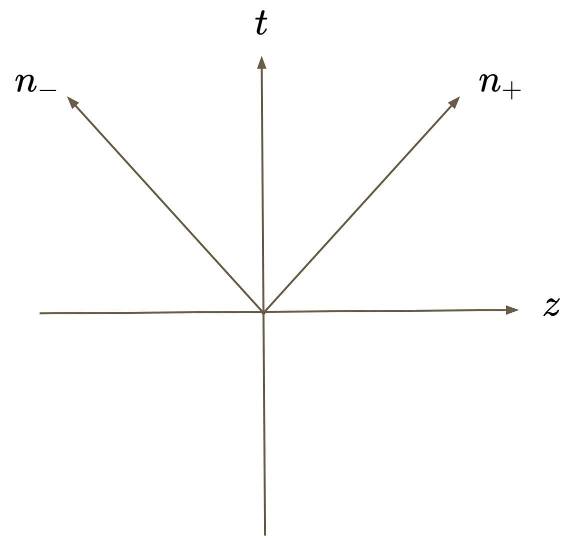
First evidence of free point-like spin-½ **constituents (partons) inside the proton**

Deep-inelastic scattering

$$l(\ell) + N(P) \to l'(\ell') + X(P_X)$$



Light cone variables



Choice of a basis :

$$\{n_+,\, n_-\}$$

$$n_+^2 = n_-^2 = 0$$

$$n_+ \cdot n_- = 1$$

Projectors on the transverse space:

$$g_T^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu - n_-^\mu n_+^\nu$$

$$\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\, n_-^\rho\, n_+^\sigma$$

$$V^\mu = \left(V^0,\, V^1,\, V^2,\, V^3\right) = \left[V^+,\, V^-,\, \mathbf{V}_T\right]$$

$$V^\pm = \frac{1}{\sqrt{2}}\left(V^0 \pm V^3\right), \quad \mathbf{V}_T = \left(V^1, V^2\right)$$

$$V^2 = 2V^+V^- - |\mathbf{V}_T|^2$$

Kinematics

With a nucleon target, we have four "external" vectors at our disposal: "spin", $P,\ \ell,\ \ell'$

We can build the following invariants

$$s\ =\ (P+\ell)^2$$

$$W^2 = (P+q)^2$$

$$Q^2 = -q^2 = -\left(\ell-\ell'\right)^2$$

... and variables :

$$x_B = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot \ell}$$

Deep-inelastic regime (*Bjorken limit*):

$$Q^2,\ P\cdot q \to +\infty \qquad \left(\gg\ M^2\ \ \text{in practice}\right)$$

$$x_B\quad \text{fixed}$$

Spin 1/2

The spin is described by means of a **density operator** (matrix, standard Quantum Mechanics)

Spin J → (up to) rank 2J tensor (e.g.: spin ½ → rank 1 tensor = spin vector)

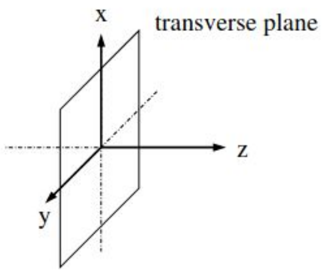
Spin 1/2

$$\rho = \frac{1}{2}\left(1 \, + \, S^i \, \sigma^i\right)$$

→ identity operator 1 and Pauli matrices

→ spin 3-vector S $S^i \, = \, \left(S_T^x, \, S_T^y, \, S_L\right)$

(z chosen as longitudinal direction)



Covariant spin vector

$$S^\mu \, = \, \left(0, \, S^i\right)$$

Spin 1

see https://inspirehep.net/literature/530045 for more details

The spin is described by means of a **density operator** (matrix, standard Quantum Mechanics)

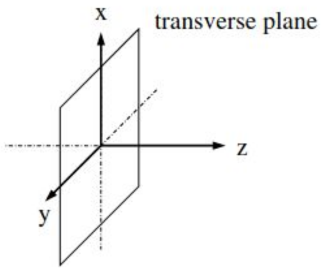
Spin J → (up to) rank 2J tensor (e.g.: spin 1 → rank 2 tensor = spin matrix)

Spin 1

$$\rho = \frac{1}{3}\left(1 + \frac{3}{2}\, S^i\, \Sigma^i + 3\, T^{ij}\, \Sigma^{ij}\right)$$

→ spin 3-vector "S" and spin matrix "T"

→ identity 1, 3D Pauli matrices Σ and their generalization to rank-2



$$\boldsymbol{S} = \left(S_T^x, S_T^y, S_L\right),$$

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}$$

Spin - take home message

Spin 1/2

$$\rho = \frac{1}{2}\left(1 \; + \; S^i\, \sigma^i\right)$$

→ spin 3-vector "S"

Spin 1

$$\rho \; = \; \frac{1}{3}\;\left(1 + \; \frac{3}{2}\, S^i\, \Sigma^i \; + \; 3\, T^{ij}\, \Sigma^{ij}\,\right)$$

→ spin 3-vector "S" and spin matrix "T"

A polarized deuteron (spin 1) has more "degrees of freedom" compared to a polarized nucleon (spin ½) .

This leads to a richer spin structure.

Cross section

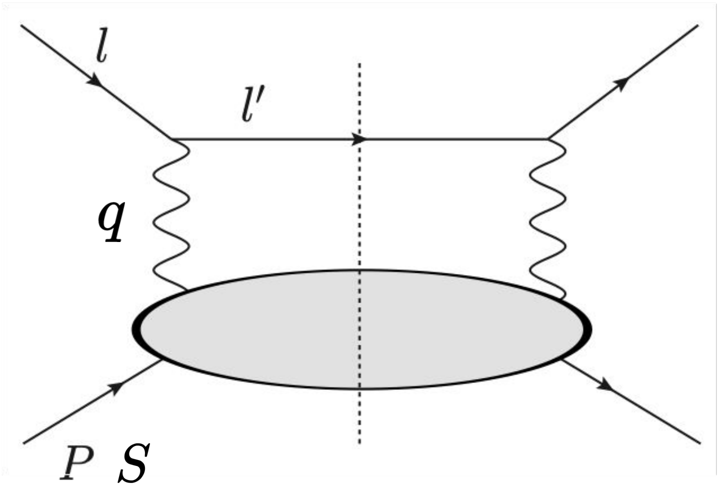
See also previous lectures

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\,Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

CUT DIAGRAM notation
for one-photon exchange approximation.

Represents the product of two Feynman amplitudes (→ cross section), one at the left and one at the right of the “cut”

The dashed line represents the “cut”:
particles that go to the final state (on-shell)

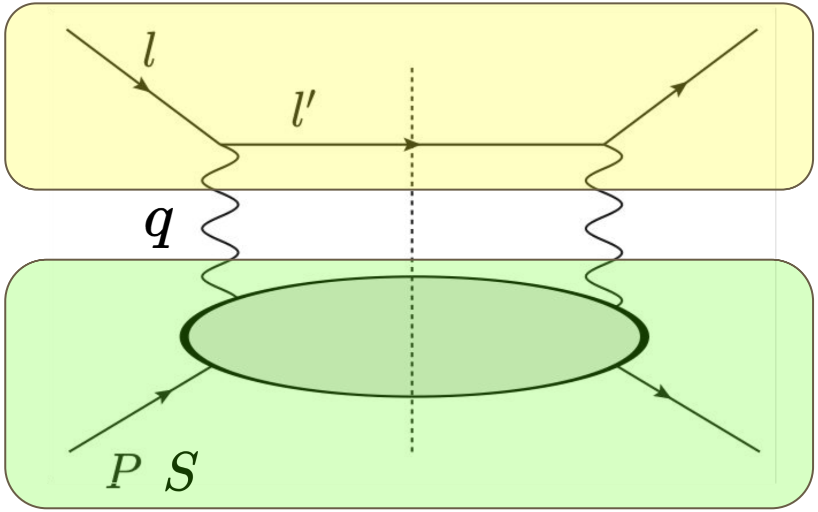


Cross section

See also previous lectures

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4} L_{\mu\nu}(l, l', \lambda_e)\, 2MW^{\mu\nu}(q, P, S)$$

Leptonic tensor - **QED** (completely calculable)



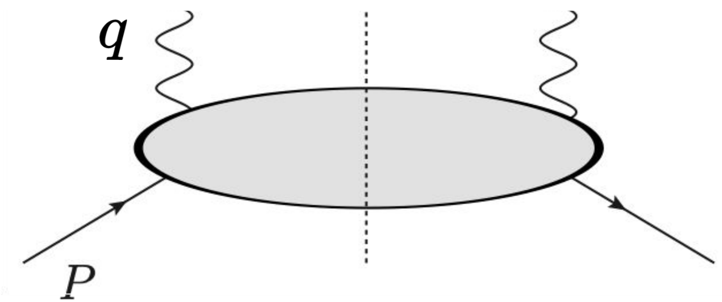
Hadronic tensor - **QCD** **(NOT completely calculable)**

The hadronic tensor (unpolarized)

$$2\,M\,W_{\mu\nu}(q,P) \;=\; \sum_X \int \frac{d^3P_X}{2E_X}\,\delta^4(P+q\,-\,P_X)\,\langle P|\,J^\dagger_\mu(0)\,|P_X\rangle\,\langle P_X|\,J_\nu(0)|P\rangle$$

The scattering electron “feels” the electromagnetic current J in the target

$J_\mu(\xi) \;=\; :\,\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi)\,:$ (in case of weak interaction (W,Z) the current is different)

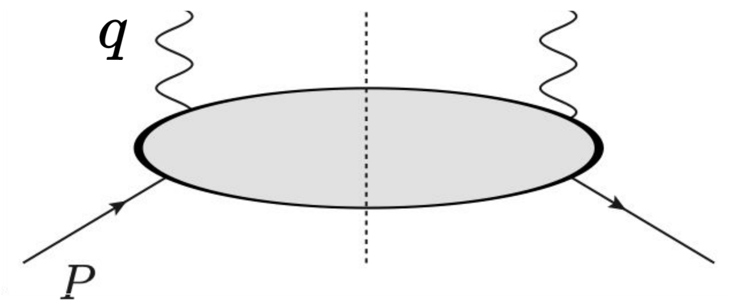


The hadronic tensor (unpolarized)

We can **parametrize W** using the available vectors and the symmetries of the theory:

$$2\,M\,W_{\mu\nu}(q,P) \,=\, \left[A\,g_{\mu\nu} \,+\, B\,\frac{q_\mu q_\nu}{q^2} \,+\, C\,\frac{P_\mu\,P_\nu}{M^2} \,+\, D\,\frac{P_\mu\,q_\nu + P_\nu\,q_\mu}{M^2}\right]$$

Conditions: **parity** invariance, **time-reversal** invariance, **gauge** invariance and *hermiticity*



Weak interaction: no parity → additional terms!
Spin → additional terms!

Structure functions

See also Ravindran's lectures

$$M\,W^{\mu\nu}(q,P) \;=\; \Big(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\Big)\,\boxed{F_1\big(x_B,\,Q^2\big)} + \frac{1}{P\cdot q}\Big(P^\mu - \frac{P\cdot q}{q^2}q^\mu\Big)\Big(P^\nu - \frac{P\cdot q}{q^2}q^\nu\Big)\boxed{F_2\big(x_B,\,Q^2\big)} \;=$$

$$= -g_\perp^{\mu\nu}\boxed{F_{UU,T}\big(x_B, Q^2\big)} + \hat{t}^{\,\mu}\,\hat{t}^{\,\nu}\boxed{F_{UU,L}\big(x_B,\,Q^2\big)}$$

F1, F2: "standard" unpolarized DIS structure functions

FT, FL: structure functions in the {t, z} basis (direct connection with the photon polarization)

Orthogonal and normalized basis

$$z^\mu \;=\; -\,\hat{q}^\mu \equiv q^\mu\,/\,Q$$

$$\hat{t}^{\,\mu} \;=\; \frac{2\,x_B}{Q\sqrt{1+\gamma^2}}\Big(P^\mu - \frac{P\cdot q}{q^2}q^\mu\Big)$$

Transverse projectors

$$g_\perp^{\mu\nu} \;=\; g^{\mu\nu} + \hat{q}^{\,\mu}\,\hat{q}^{\,\nu} \;-\; \hat{t}^{\,\mu}\,\hat{t}^{\,\nu}$$

$$\epsilon_\perp^{\mu\nu} \;=\; \epsilon^{\mu\nu\rho\sigma}\,\hat{t}_{\,\rho}\,\hat{q}_{\,\sigma}$$

Polarized case - spin 1/2

$$W^{\mu\nu}(q,P,S) \sim -g_\perp^{\mu\nu} F_{UU,T} + \hat{t}^{\,\mu}\hat{t}^{\,\nu} F_{UU,L}$$

$$+ i\, S_L \epsilon_\perp^{\mu\nu} \boxed{F_{LL}} + i\left(\hat{t}^{\,\mu}\epsilon_\perp^{\nu\rho} - \hat{t}^{\,\nu}\epsilon_\perp^{\mu\rho}\right) S_\rho \boxed{F_{LT}^{\cos\phi}}$$

Two additional structure functions for the nucleon:
longitudinal and **transverse** target polarization → related to "standard" g1 and g2 functions

Transverse beam polarization is proportional to electron mass and thus suppressed

Polarized case - spin 1

See also https://inspirehep.net/literature/262935

$$W^{\mu\nu}(q,P,S) \sim \; -g_\perp^{\mu\nu}\, F_{UU,T} + \hat{t}^{\,\mu}\hat{t}^{\,\nu}\, F_{UU,L}$$

$$+ i\, S_L \epsilon_\perp^{\mu\nu}\, F_{LL} + i\Big(\hat{t}^{\,\mu}\epsilon_\perp^{\nu\rho} - \hat{t}^{\,\nu}\epsilon_\perp^{\mu\rho}\Big) S_\rho\, F_{LT}^{\cos\phi}$$

$$+ \boxed{\{b_1\,,\, b_2\,,\, b_3,\, b_4\}} \longrightarrow T^{\mu\nu} \text{ tensor polarized terms}$$

Two additional structure functions for the nucleon:
longitudinal and **transverse** target polarization → related to “standard” g1 and g2 functions

Transverse beam polarization is proportional to electron mass and thus suppressed

For a deuteron there are four additional structures associated with the **tensor polarization**!

Cross section (polarized nucleon)

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2} \Bigg\{ \left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1-y)\, F_{UU,L} + S_L \lambda_e\, y \left(1 - \frac{y}{2}\right) F_{LL} + |\boldsymbol{S}_T| \lambda_e\, y \sqrt{1-y}\, \cos\phi_S\, F_{LT}^{\cos\phi_s} \Bigg\}$$

F...: functions of x, Q

Up to now no partons ...

How do quarks and gluons emerge in this description?

For a summary see e.g. https://inspirehep.net/literature/732275

Tools for the exercises

- Analytic

- Numeric:
  - Google Colaboratory: https://colab.research.google.com/
  - Wolfram Mathematica: https://www.wolfram.com/mathematica/trial/

(many of the things that you can do with Mathematica can be done also in python, in particular using the SymPy library: https://www.sympy.org/)