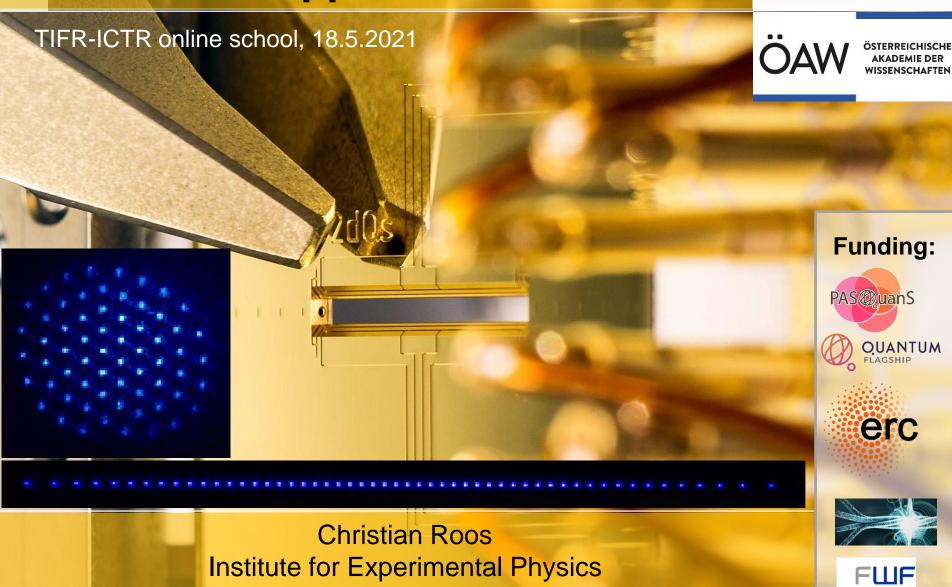
Quantum simulations with trapped ions





University of Innsbruck, Austria

Outline

- Quantum simulation approaches
 - Digital gate-based simulations
 - Crystal geometries
 - Analog simulations: engineering of long-range spin models
 - Detection and characterization of entangled states
 - Variational quantum simulation
 - Scaling quantum simulations to larger particle numbers

Simulating quantum physics

If there are quantum algorithms that run exponentially faster than their classical counterparts:

What stops us from simulating a quantum computer on a classical computer to find a solution in a much shorter time than with the classical algorithm?

Obstacle: There is no solution for simulating general quantum dynamics efficiently on a classical computer.



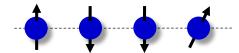
Maybe we should use a quantum processor to simulate the physics of quantum systems which is hard to simulate on classical computers

Quantum simulation

Quantum simulations with trapped ions

Simulating quantum many-body systems

How can we study the physics of quantum many-body systems?



Approaches:

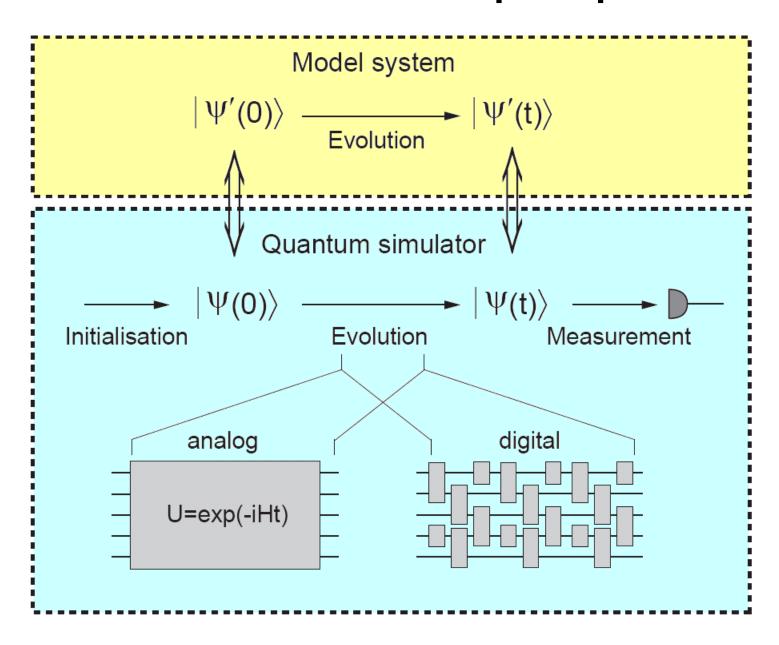
- In some cases: Analytical techniques
- Numerical simulation methods on a computer using approximations

But: Exponential scaling of resources with the system size severely restricts the number of particles that can be exactly simulated.

Interacting spins: exact diagonalization techniques limited to N ~ 40 spins

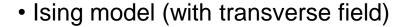
• Feynman (1982), Lloyd (1996): **Quantum simulators**Use a precisely controlled quantum system for simulating a model of interest

Quantum simulation principle

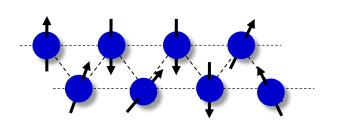


Simulating quantum spin systems

Hamiltonians:



$$H = \frac{1}{2} \sum_{i,j} J_{ij} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z$$



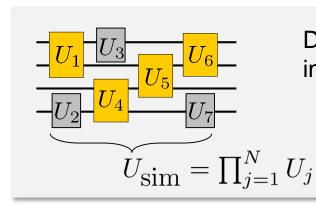
XY model

$$H = \frac{1}{2} \sum_{i,j} J_{ij}^x \sigma_i^x \sigma_j^x + \frac{1}{2} \sum_{i,j} J_{ij}^y \sigma_i^y \sigma_j^y + B \sum_i \sigma_i^z$$

Heisenberg model

$$H = \frac{1}{2} \sum_{i,j} J_{ij}^{x} \sigma_{i}^{x} \sigma_{j}^{x} + \frac{1}{2} \sum_{i,j} J_{ij}^{y} \sigma_{i}^{y} \sigma_{j}^{y} + \frac{1}{2} \sum_{i,j} J_{ij}^{z} \sigma_{i}^{z} \sigma_{j}^{z} + B \sum_{i} \sigma_{i}^{z}$$

Trapped-ion quantum simulations: Gate-based approach



Decompose dynamics induced by system Hamiltonian into sequence of quantum gates

$$U_{\rm sim} \propto U_{\rm sys}$$
 $U_{\rm sys} = e^{-\frac{i}{\hbar}H_{\rm sys} \, \tau}$

Quantum gate toolbox:

- Single qubit-gates
- Entangling two-qubit gates

Building up Hamiltonians

Spin-spin interaction
$$H_{xx} = J \sigma_x^{(1)} \sigma_x^{(2)}$$

Can we add a transverse field?

No... it would perturb the effective Hamiltonian H_{xx}

$$H = J \sigma_x^{(1)} \sigma_x^{(2)} + H_z$$
$$H_z = B \left(\sigma_z^{(1)} + \sigma_z^{(2)}\right)$$

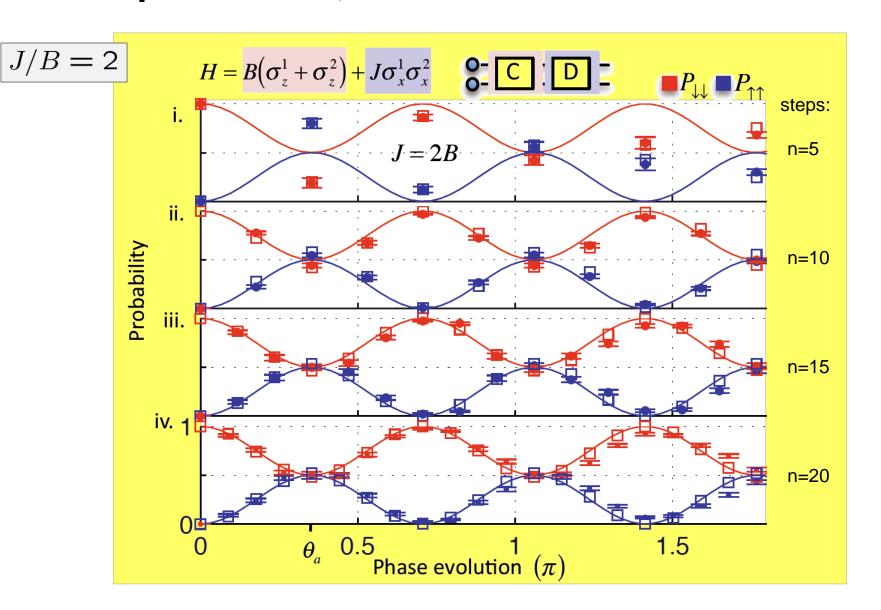
Solution: use ,Trotterization' to generate the dynamics corresponding to $H_{xx} + H_z$

$$U = e^{-i(H_{xx} + H_z)t} \approx e^{-iH_{xx}\Delta t} e^{-iH_z\Delta t} \dots e^{-iH_{xx}\Delta t} e^{-iH_z\Delta t}$$

How many steps?

N=2
$$H_{xx}$$
 H_z H_{xx} H_z

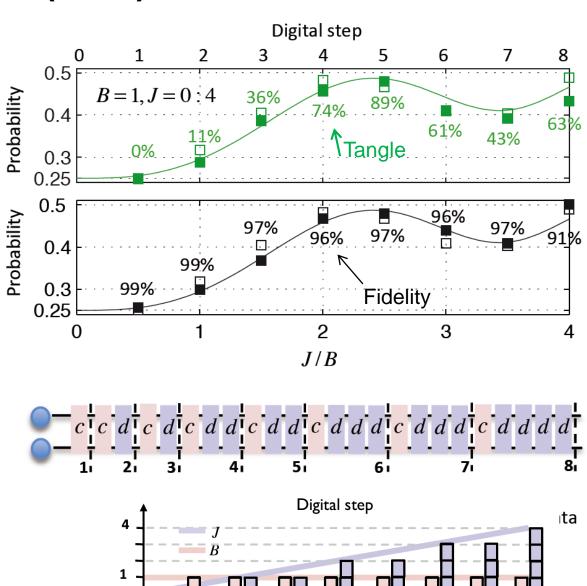
Experimental , Trotterization of $H_{xx} + H_z$



Trotterization of (non-)adiabatic evolution

$$H = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2$$

$$J/B = 0 \rightarrow 4$$



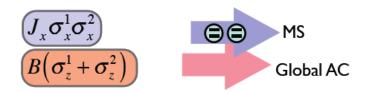
3

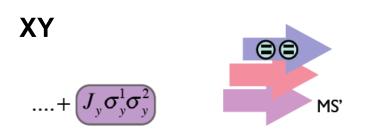
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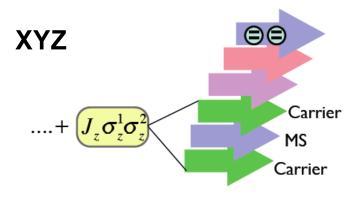
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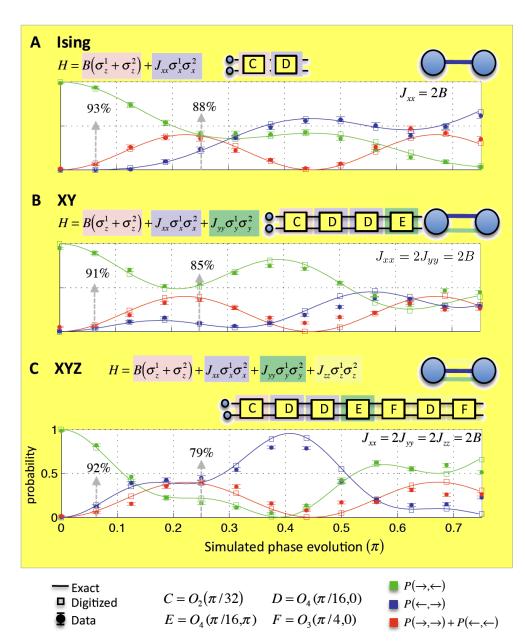
Building new interactions by Trotter technique

Ising



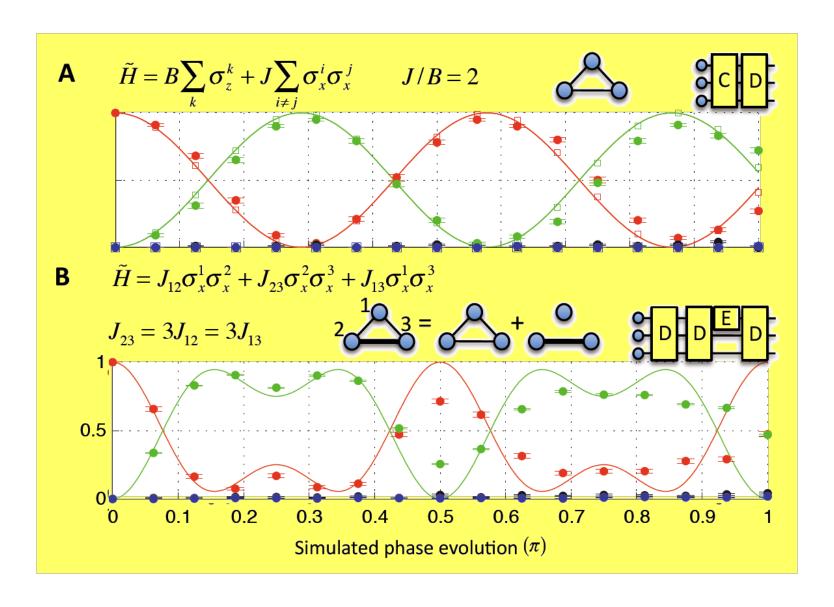






B. Lanyon *et al.*, Science **334**, 57 (2011)

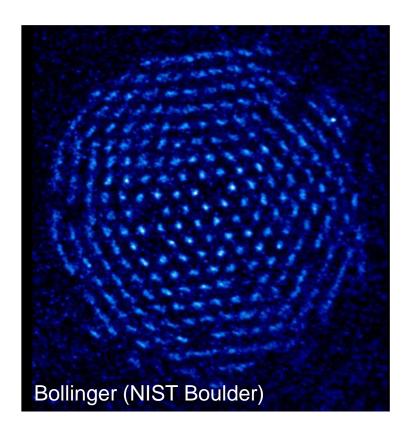
Increasing the number of spins



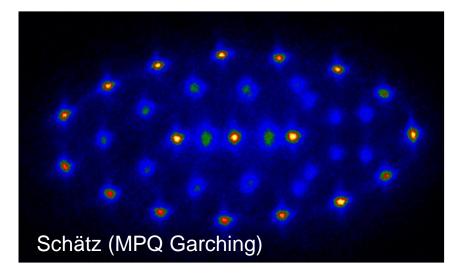
Trapped ions for simulating quantum magnetism

Innsbruck

Challenges:



- Controlling the geometry
- Keeping decoherence low
- Engineering interactions



Trapping geometries: rf traps

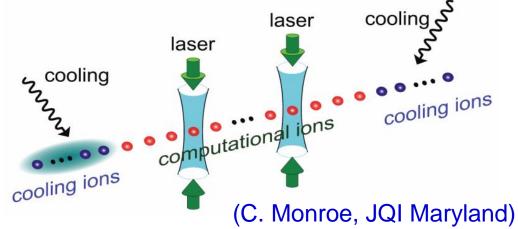
Linear traps: Harmonic anisotropic potentials

N = 2...100(?) ions in a one-dimensional crystal

$$\frac{\omega_r}{\omega_z} > 0.77 \frac{N}{\log N}$$
 longer crystals require very anisotropic potentials

Segmented microtraps: Anharmonic potentials for linear ion strings with equal spacing

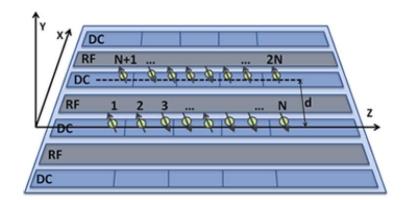
N > 100(?) ions in a one-dimensional crystal



G.-D. Lin, et al., Europhys. Lett. 86, 60004 (2009)

Trapping geometries: rf traps

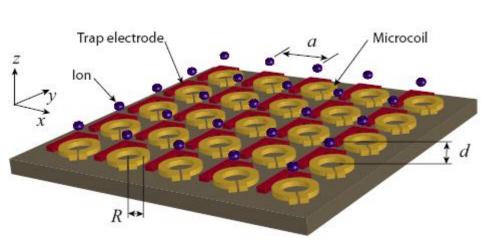
Segmented microtraps for Potentials with multipole trapping sites

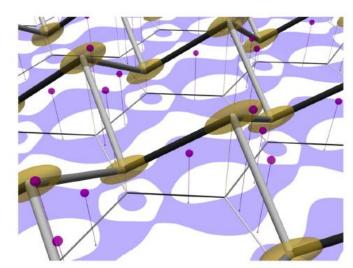


Multiple linear strings in close proximity

J. Welzel et al., EPJD 65, 285 (2011)

2d-lattices of trapping sites





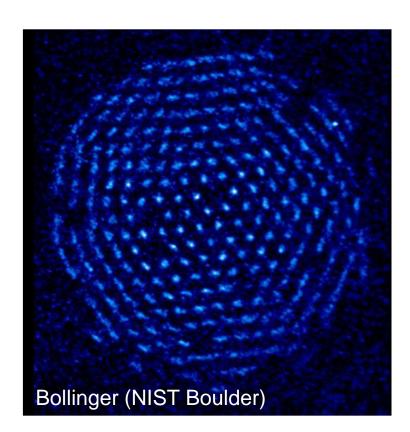
R Schmied et al, PRL 102, 233002 (2009)

Chiaverini and Lybarger, Phys. Rev. A 77, 022324 (2008)

Trapping geometries: Penning traps

Penning trap: anisotropic potential for trapping 2d crystals

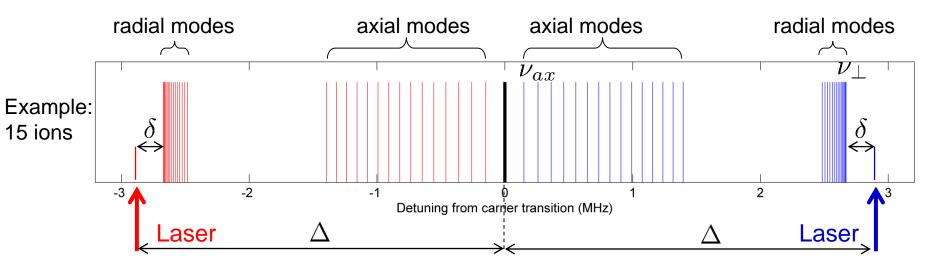
- N≈ 100 300 ions possible
- low internal state decoherence
- challenge: demonstrate same kind of quantum control as in rf-traps

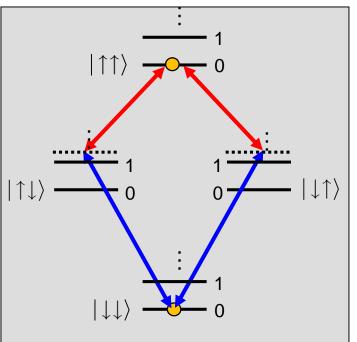


Geometry of laser-ion interaction

Features:

- Long strings \Rightarrow strongly anistropic trapping potentials: $\omega_{\perp}/\omega_{ax} \approx 15-20$
- weak axial confinement \Rightarrow 'hot' axial modes \Rightarrow all laser beams \bot to ion string





$$H=\sum_{i< j}J_{ij}\sigma_i^x\sigma_j^x$$
 with
$$J_{ij}=\Omega^2\frac{(\hbar k)^2}{2m}\sum_m\frac{b_{i,m}b_{j,m}}{\Delta^2-\nu_m^2}$$

Example: 11 ions

vibrational mode



'Tilt'

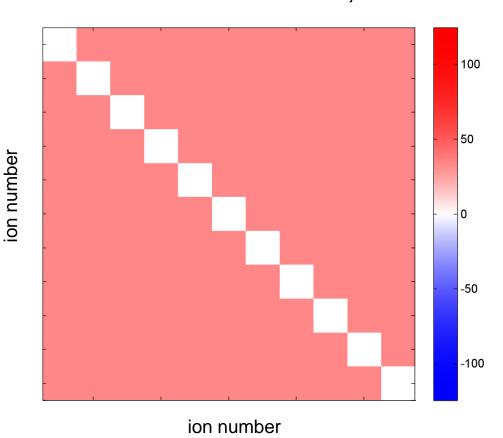
'COM'

:

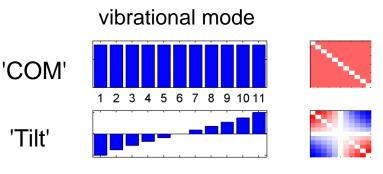
:

$$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_{m} \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

Spin-spin coupling matrix J_{ii} (Hz)



Example: 11 ions

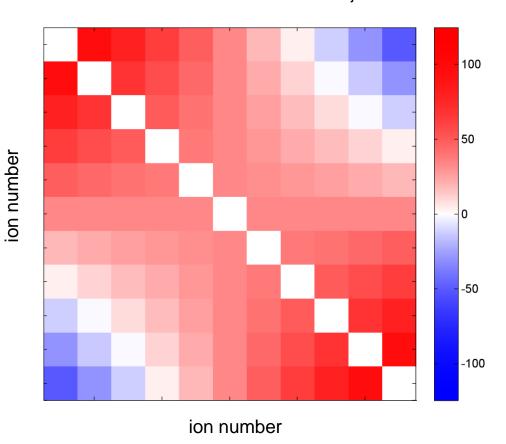


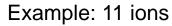
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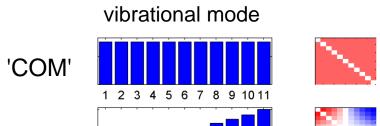
$$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_{m} \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

Spin-spin coupling matrix J_{ii} (Hz)



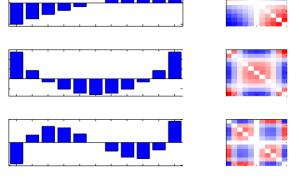


$$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_{m} \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

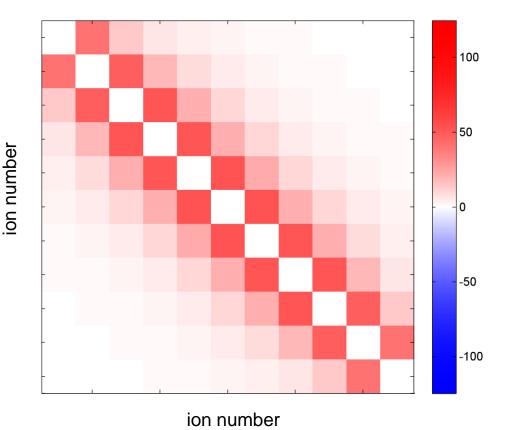


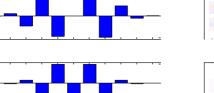




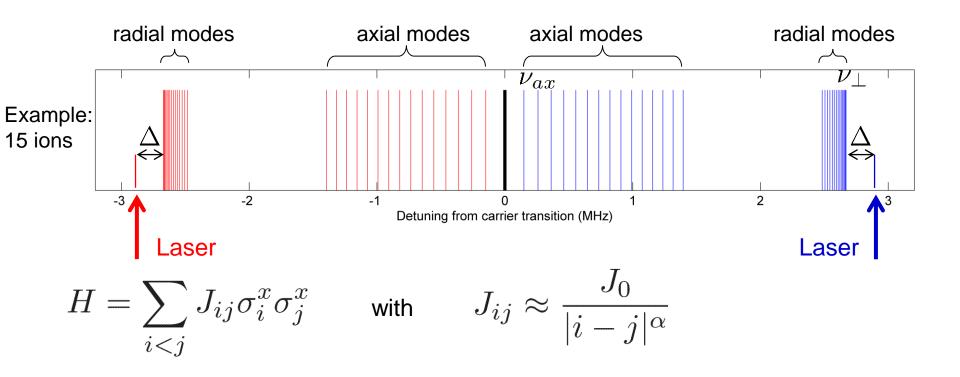












Interaction range: $0<\alpha<3$ couple only to couple to all modes center-of-mass

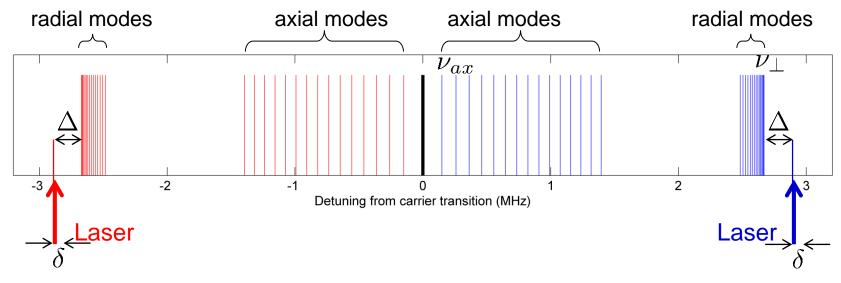
K. Kim et al, PRL **103**, 120502 (2009)

J. Britton et al, Nature **484**, 489 (2012)

Knobs to turn:

- laser detuning Δ
- · spread of radial modes

Ising model with transverse field



$$H = \sum_{i < j} J_{ij} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z \qquad B = \delta/2$$

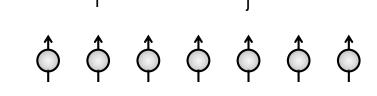
$$pprox \sum_{i < j} J_{ij} (\sigma_i^+ \sigma_j^- + h.c.) + B \sum_i \sigma_i^z$$
 for $B \gg J$

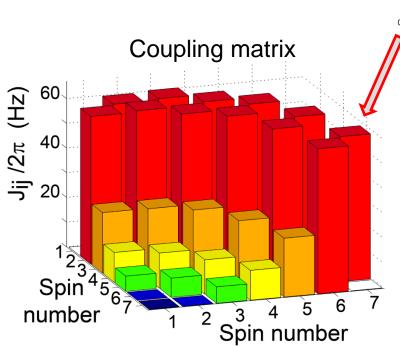
"XY model": hopping of spin excitations

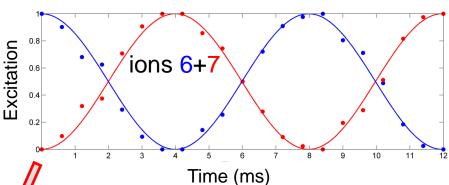
Measurement of the coupling matrix

Protocol:

- 1. Initialize ions in state $|\uparrow\rangle_i|\downarrow\rangle_j$
- 2. Switch on Ising Hamiltonian $|\uparrow\rangle_i|\downarrow\rangle_j\longleftrightarrow |\downarrow\rangle_i|\uparrow\rangle_j$
- 3. Measure coherent hopping rate



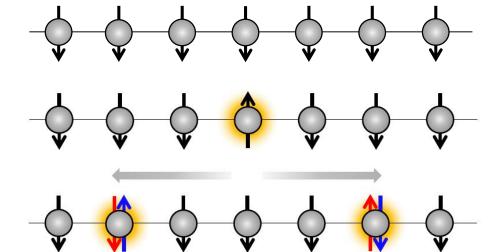




Spread of correlations after local quenches

$$H_{XY} = \sum_{i < j} J_{ij} (\sigma_i^+ \sigma_j^- + h.c.) + B \sum_i \sigma_i^z$$

Ground state: all spins aligned with transverse field

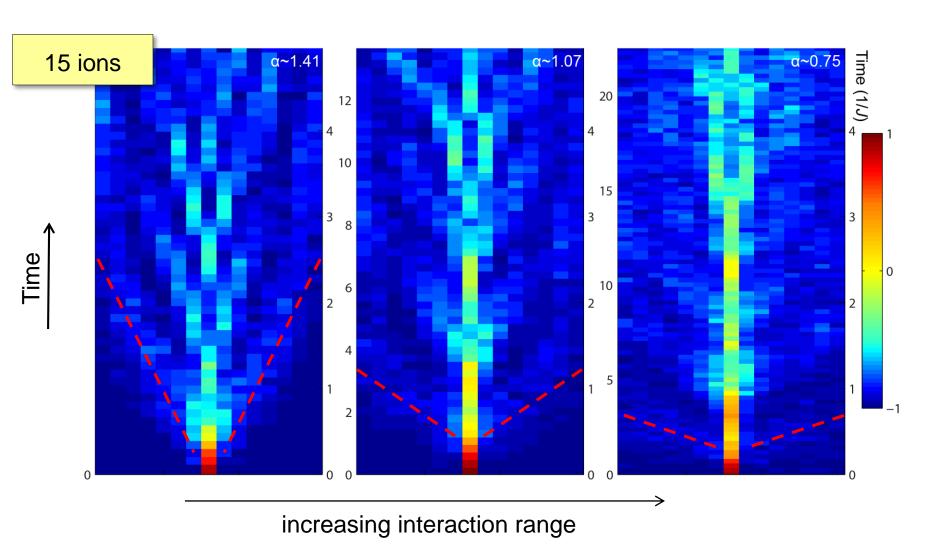


1. Local quench: flip one spin

2. Spread of entanglement

3. Measure magnetization or spin-spin correlations

Magnetization dynamics after a local quench

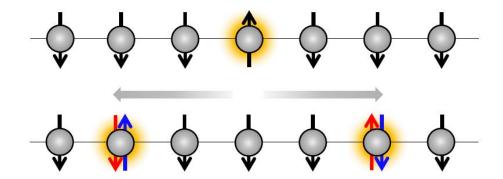


P. Jurcevic et al., Nature **511**, 202 (2014)

 $J_{ij} pprox J_0 rac{1}{|i-j|^{lpha}}$

see also: P. Richerme et al., Nature 511, 198 (2014)

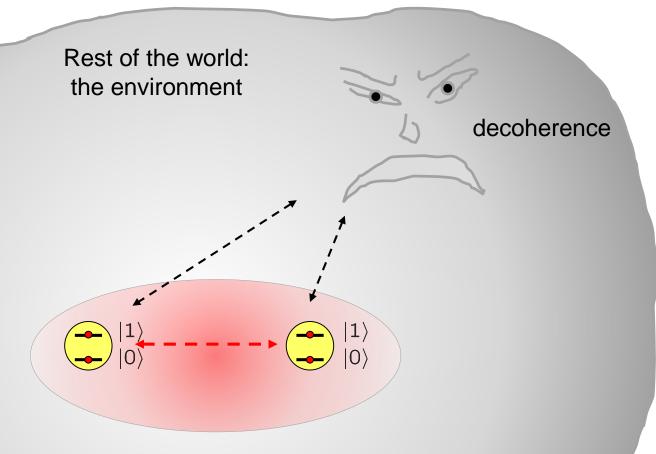
Entanglement generation in many-body dynamics



The quantum dynamics induced by a many-body Hamiltonian creates entanglement.

How can we demonstrates that this really happens in our experiment?

Classical vs quantum correlations



Coupling to the environment

- can create classical correlations
- turns quantum correlations into classical correlations.

Classical vs quantum correlations

Example: Comparison of two different states

$$\Psi_{-} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \qquad \qquad |\uparrow\downarrow\rangle \text{ or } |\downarrow\uparrow\rangle$$
50%

Both state have the same expectation values when measured in z-basis:

$$\langle \sigma_z^{(i)} \rangle = 0$$

 $\langle \sigma_z^{(1)} \sigma_z^{(2)} \rangle = -1$

Only measurements along x or y reveal the difference:

$$\langle \sigma_x^{(1)} \sigma_x^{(2)} \rangle = -1 \qquad \langle \sigma_x^{(1)} \sigma_x^{(2)} \rangle = 0$$
$$\langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle = -1 \qquad \langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle = 0$$

Detecting entanglement

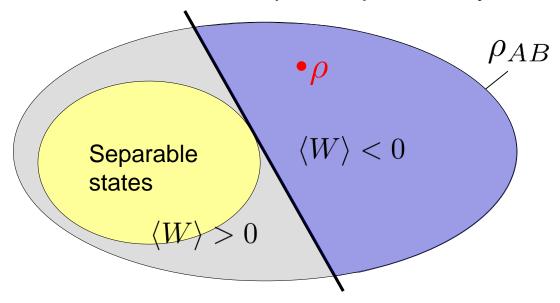
Deciding whether a state is entangled or not is a hard task.

Entanglement detection techniques:

- Positive partial trace (PPT) cryterion: check whether a density matrix after partial transposition has negative eigenvalues.
- Entanglement witnesses: Particular observables that have negative expectation value for some entangled states, but are positive for separable states.
- Entanglement measures (for example: concurrence):
 - Nonlinear functions of the density matrix that are zero for separable mixed states and positive for entangled states.
 - Entanglement measures quantify entanglement but can be hard to calculate even if the density matrix is known; for two qubits, closed expressions exist.

Detecting entanglement by witness operators

Quantum states of a composite quantum system

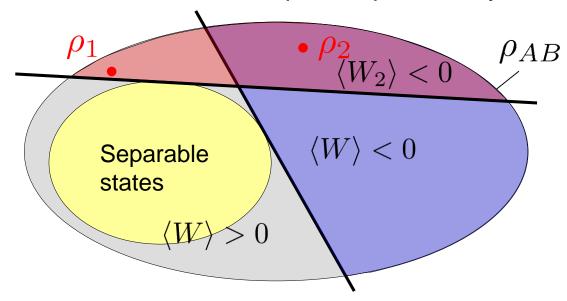


<u>Witness:</u> An observable W whose expectation value is positive for all separable states $\langle W \rangle = {\rm Tr}(\rho W) \geq 0$

If we measure $\langle W \rangle < 0$ in an experiment, we can conclude that the state is entangled.

Detecting entanglement by witness operators

Quantum states of a composite quantum system



Not every entangled state is detected by a witness

$$\longrightarrow \rho_1$$

$$\operatorname{Tr}(\rho_1 W) \geq 0$$

• An entangled state can be detected by more than one witness $\longrightarrow \rho_2$

$$\operatorname{Tr}(\rho_2 W) < 0 \quad \operatorname{Tr}(\rho_2 W_2) < 0$$

Tomographic reconstruction of the density matrix

Representation of ρ as a sum of orthogonal observables A_i :

$$\rho = \sum_{i} \lambda_{i} A_{i} \text{ with } Tr(A_{i} A_{j}) = \delta_{ij}$$

 ρ is completely determined by the expectation values $<A_i>$:

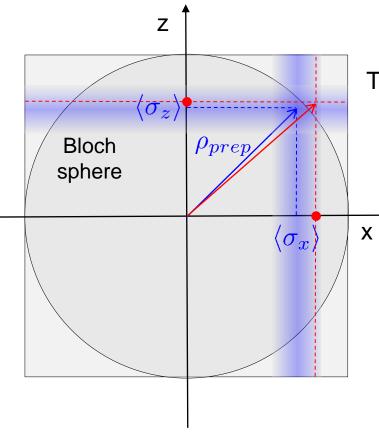
$$\langle A_j \rangle = Tr(\rho A_j) = \sum_i \lambda_i Tr(A_i A_j) = \lambda_j$$

For a two-ion system : $A_i \in \{\sigma_i^{(1)} \otimes \sigma_j^{(2)}, \sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}\}$

Joint measurements of all spin components $\sigma_i^{(1)} \otimes \sigma_j^{(2)}$

$$\rho_R = \sum_{i=1}^{16} \langle A_i \rangle A_i$$

Example: Tomography of a qubit



The experimental procedure prepares the state ρ_{prep}

$$\rho_{prep} = \frac{1}{2} (I + \langle \sigma_x \rangle \sigma_x + \langle \sigma_y \rangle \sigma_y + \langle \sigma_z \rangle \sigma_z)$$

Reconstruction by estimation of $\langle \sigma_x \rangle, \langle \sigma_y \rangle, \langle \sigma_z \rangle$ using a finite number of copies of the state:

$$s_z = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$
, $s_x = \dots$, $s_y = \dots$

$$\rho_{tomo} = \frac{1}{2}(I + s_x \sigma_x + s_y \sigma_y + s_z \sigma_z) \neq \rho_{prep}$$

Ptomo might not be within the Bloch sphere!

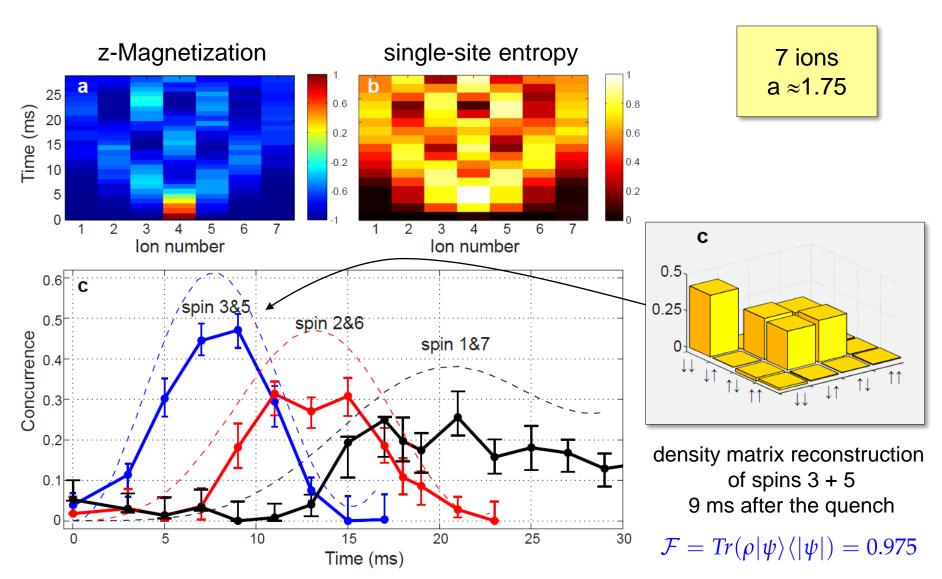


DISASTER !!!



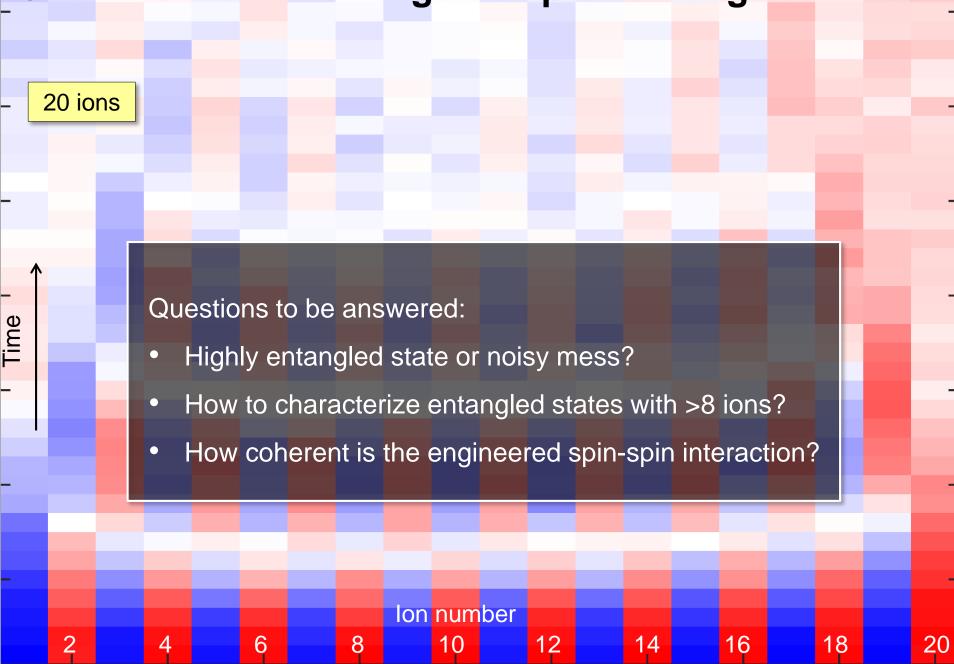


Spread of entanglement after a local quench

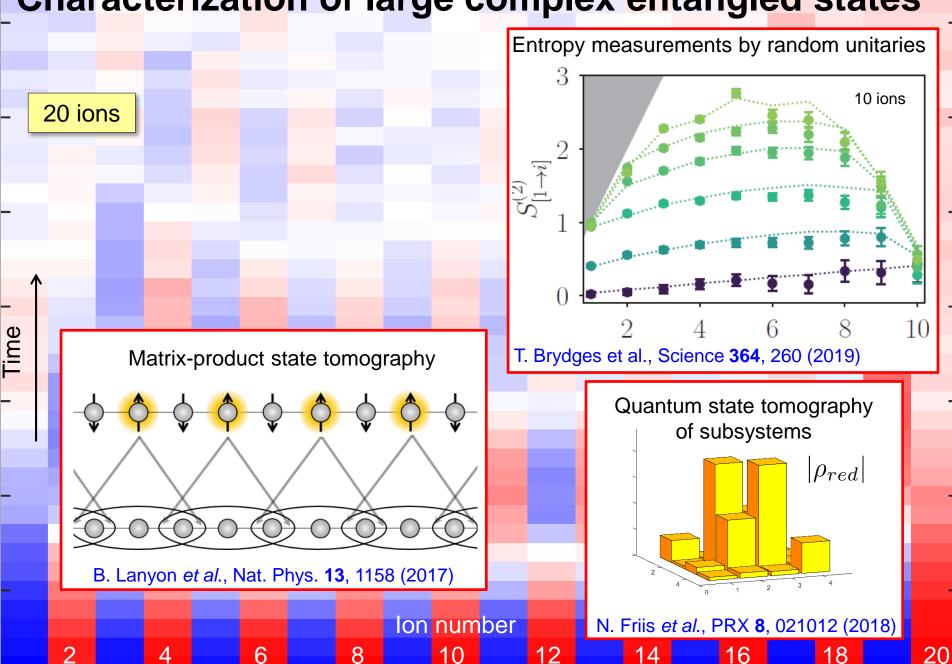


P. Jurcevic et al., Nature **511**, 202 (2014)

Characterization of large complex entangled states



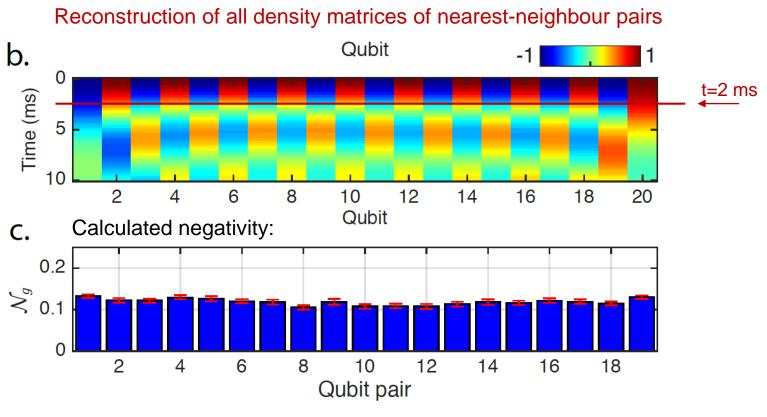
Characterization of large complex entangled states



Entanglement detection in multi-ion experiments

Local characterization of the beginning of entanglement spreading

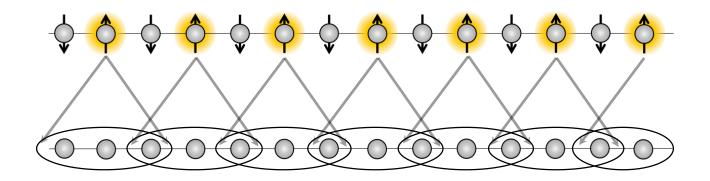
Option 1: Quantum state tomography to reconstruct the density matrix of subsystems



→ After 2 ms of time evolution, all ions are entangled with their neighbours

Entanglement detection in multi-ion experiments

Local characterization of the beginning of entanglement spreading

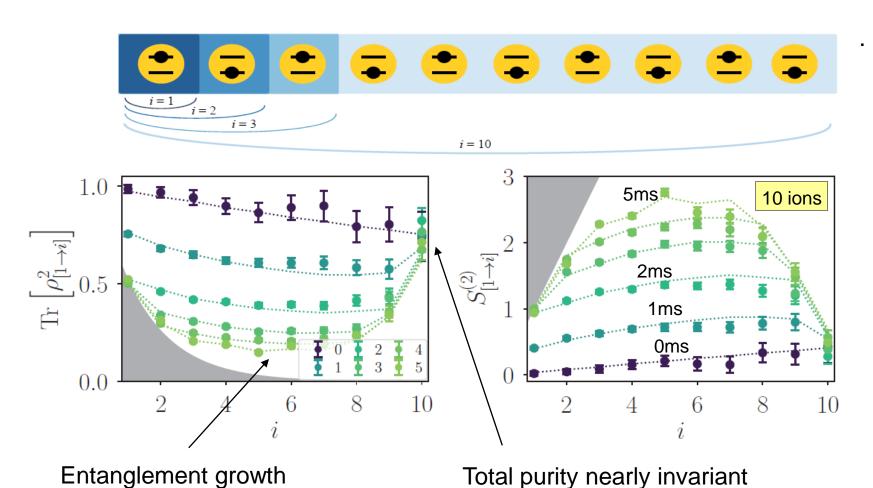


Option 2: Measure all correlation functions between groups of neighbouring ions (pairs, triplets,...) and try to build up a global representation of the quantum state using a suitable parametrization of the state

Matrix product state tomography

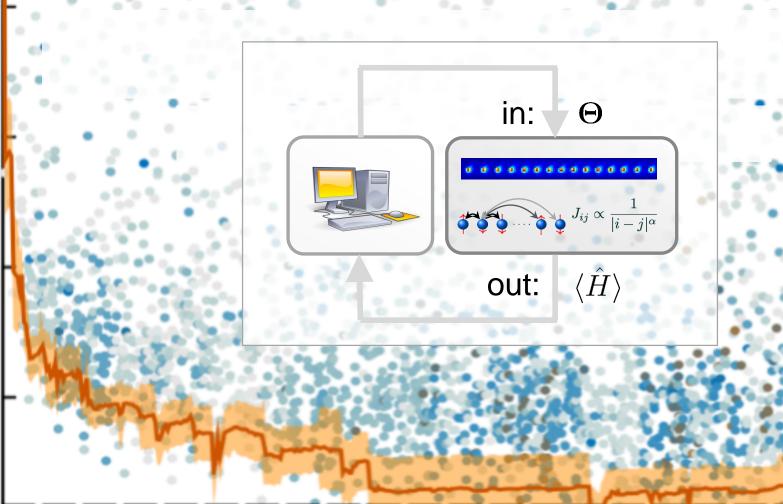
Entanglement detection in multi-ion experiments

Option 3: Compare the purity of density matrices describing subsystems to the purity of the overall density matrix



T. Brydges, A. Elben et al., Science 364, 260 (2019)

Entanglement as a computational resource: Variational quantum simulation



Target |



spin Hamiltonian

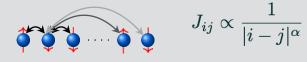
$$\hat{H}_T = \sum_{n=1}^M \hat{h}_n$$

$$\hat{h}_n = \frac{\mathbf{a_n}}{\mathbf{a_n}} \hat{\sigma}_i^x \hat{\sigma}_j^y \hat{\sigma}_k^z \cdots$$

→ sums of Pauli products:

Quantum Resource



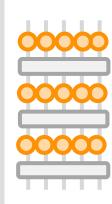


$$J_{ij} \propto rac{1}{|i-j|^{lpha}}$$

$$U_1(\Theta) = \exp\left(-i\Theta\sum_{i < j} J_{ij}(\sigma_i^+ \sigma_j^- + \text{h.c.})\right)$$

$$U_{2,i}(\Theta) = \exp\left(-\mathrm{i}\Theta\sigma_i^z\right)$$

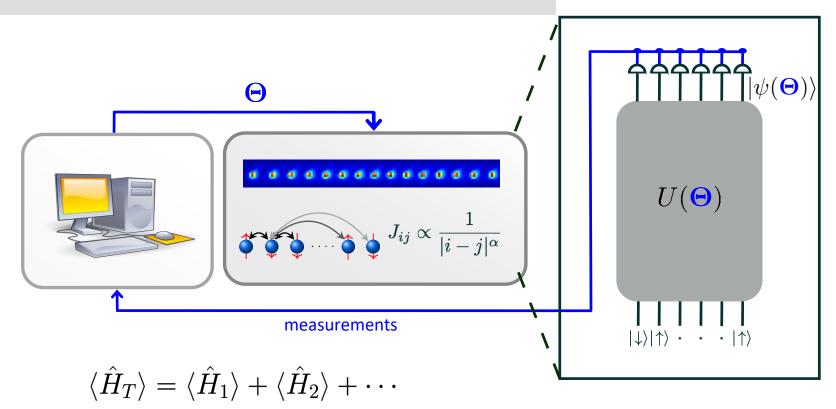
$$|\psi(\Theta)\rangle = \hat{U}_N(\Theta_N) \cdots \hat{U}_2(\Theta_2) \hat{U}_1(\Theta_1) |\psi_0\rangle$$





The goal of Variational Quantum Simulation

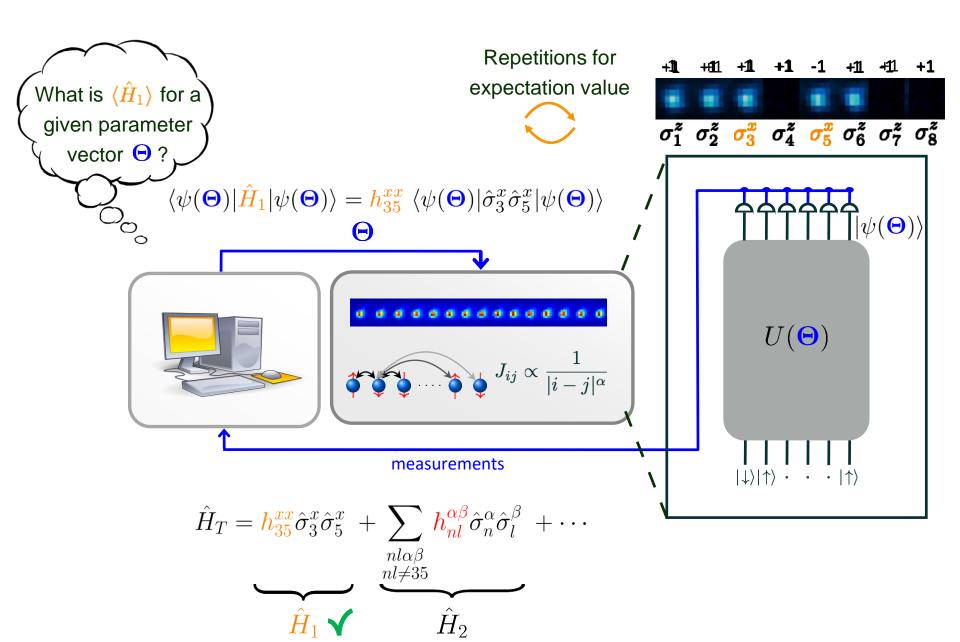
Prepare ground state of \hat{H}_T by minimising $\langle \psi(\mathbf{\Theta})|\hat{H}_T|\psi(\mathbf{\Theta})\rangle$

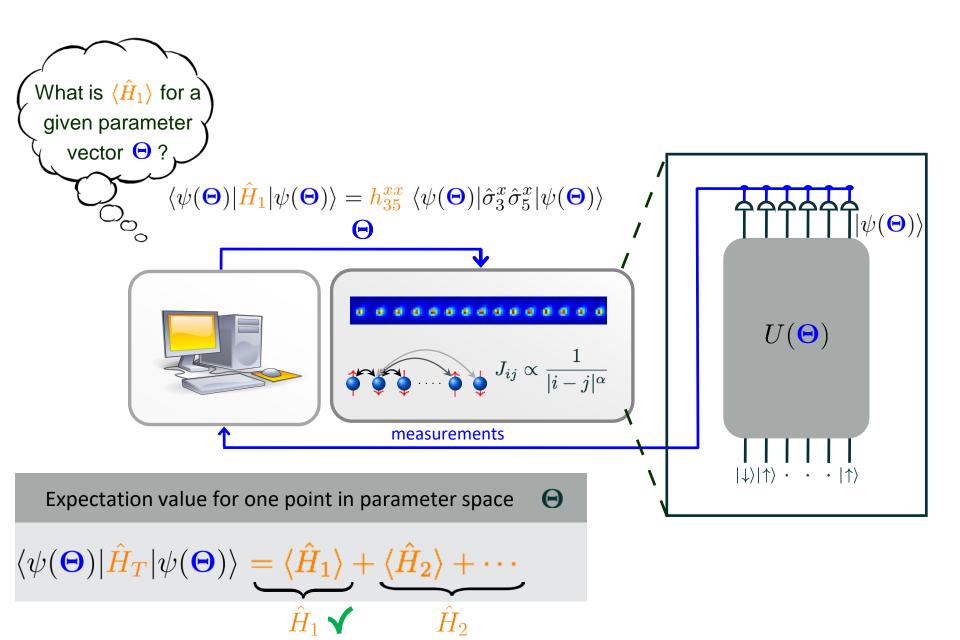


Peruzzo et al., Nature Comm. 5, 4213 (2014)

Farhi et al., arXiv:1411.4028 (2014)

McClean et al., NJP 18, 023023 (2016)





Quantum resources for variational search: ground state energy of Schwinger lattice model

Entangling operations

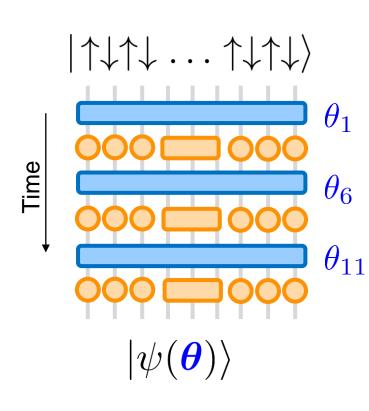
$$U(\theta) = \exp(i\theta \sum_{i < j} J_{ij} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+))$$

Single-qubit rotations

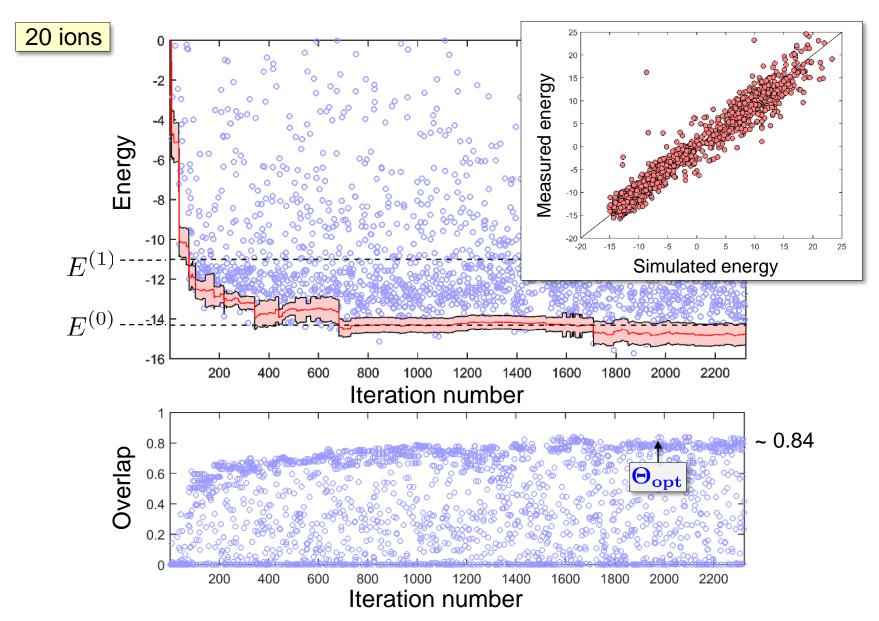
$$U(\theta) = \exp(i\theta \,\sigma_i^z)$$

Collective qubit rotations

$$U(\theta) = \exp(i\theta \sum_{i} \sigma_{i}^{x})$$



Experimental results: energy minimization



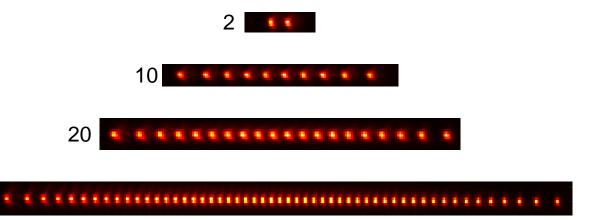
C. Kokail, C. Maier et al., Nature **569**, 355 (2019)

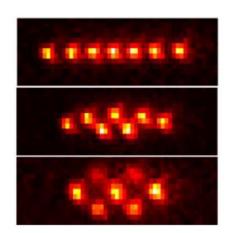
Scaling up trapped-ion quantum simulations

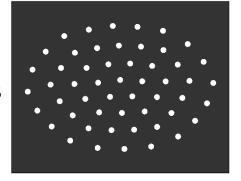
Options for experimenting with larger ion crystals:

Longer linear ion strings

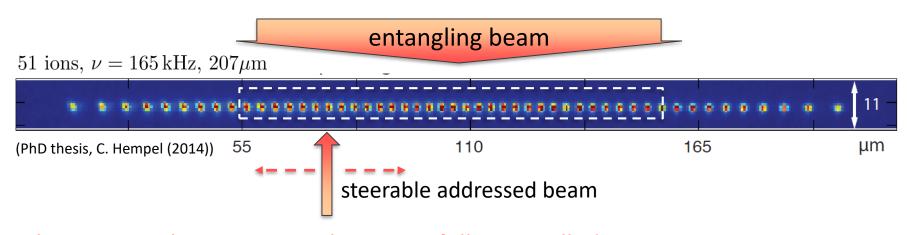
Two-dimensional crystals







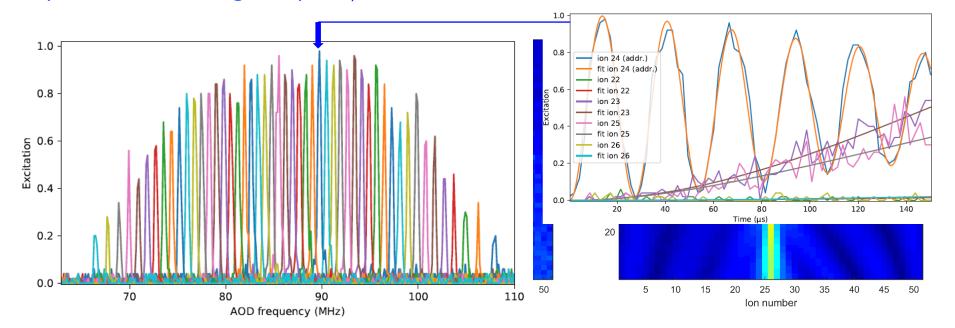
Ion strings with larger ion numbers:



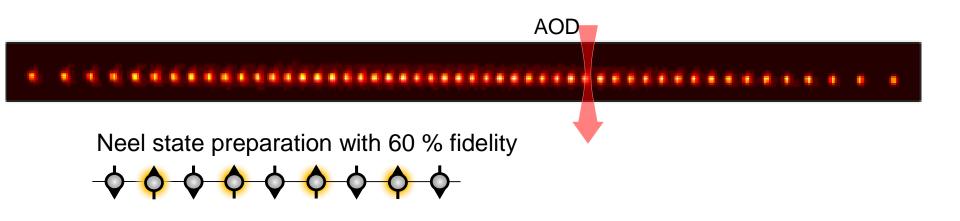
Until very recently:

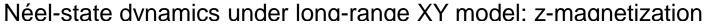
Only 20 ions fully controlled ions

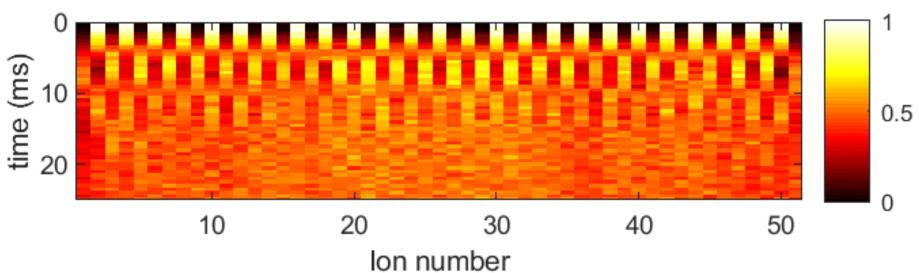
Improved addressing setup 50 addressable ions



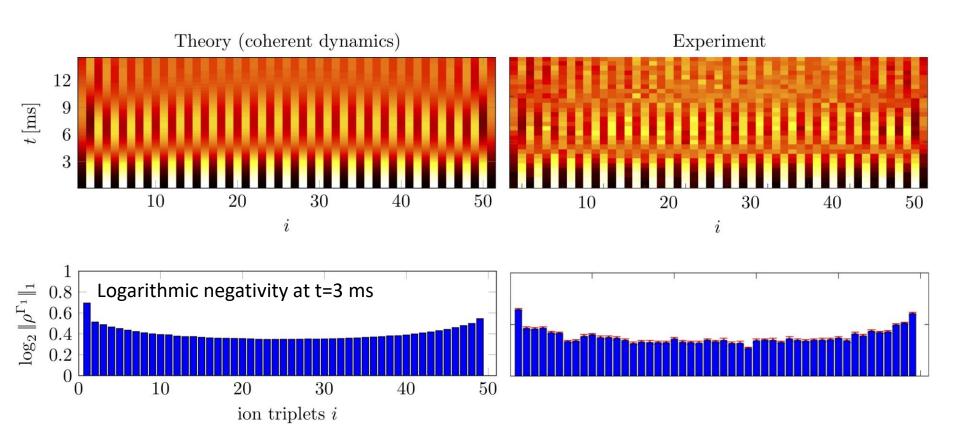
Entanglement in 50-ion strings







Entanglement in 50-ion strings



in collaboration with P. Zoller and co-workers

Scaling up trapped-ion quantum simulations

1d ion crystals:

 Very anistropic trapping potentials needed for keeping the ion string linear:

$$\nu_{\perp}/\nu_{axial} > N \log N$$

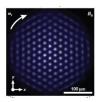
- → low axial confinement
 - tricky to control axial motion
 - length of string complicates addressing

u_{axial}

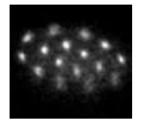
2d ion crystals:



rf linear trap → micromotion





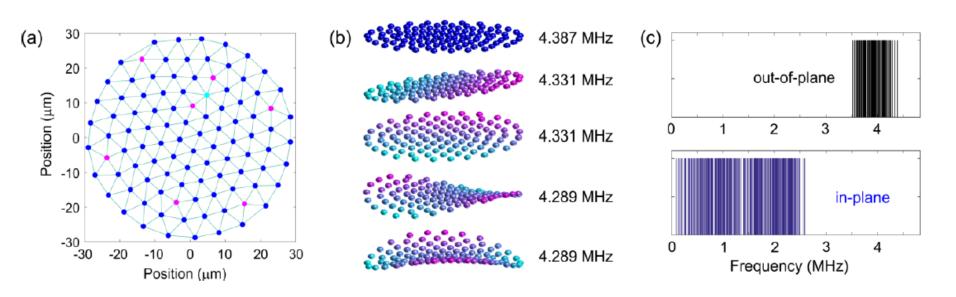


Planar ion crystals

(Campbell group, UCLA)

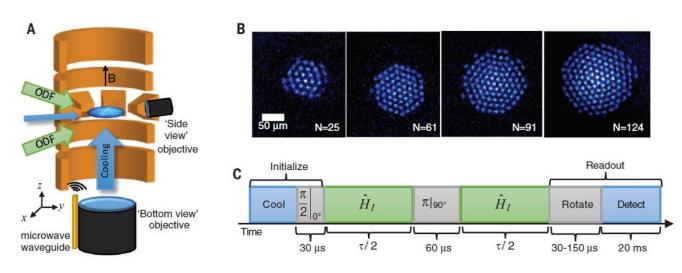
M. Block et al.J. Phys. B **33**, L375 (2000)

lons form triangular lattices with defects



Effective long-range spin-spin interactions by lasers coupling to the out-of-plane modes of vibration

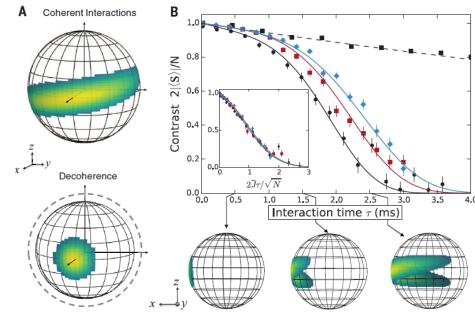
Penning trap: spin dynamics in planar crystals



Experiment at NIST, Boulder (USA):

Quantum spin dynamics with >100 ions:

- Optical dipole force inducing spin squeezing
- Measurement of collective spin operators (rotating crystals)



J. G. Bohnet et al, Science 253, 1297 (2016)

ig. 2. Depolarization of the collective spin from spin-spin interactions and decoherence. (A) The

Planar ion crystals in rf-traps: Micromotion



in-plane 2

250

300

Z

20

10

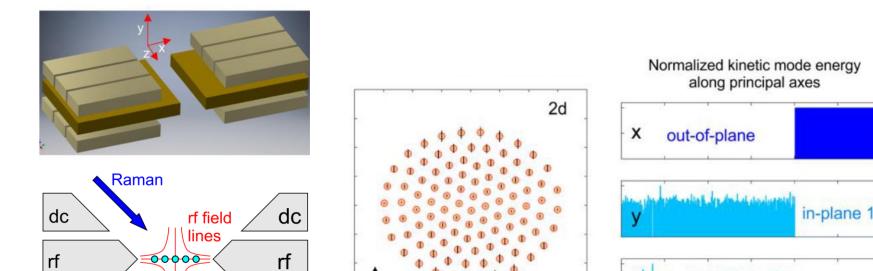
30

50

150

Mode number

200



no micromotion in direction of out-of-plane normal modes

-30

-20

-10

Doppler cooling normal to image

plane

Raman

dc

dc

for certain geometries, laser cooling of in-plane modes not affected by MM

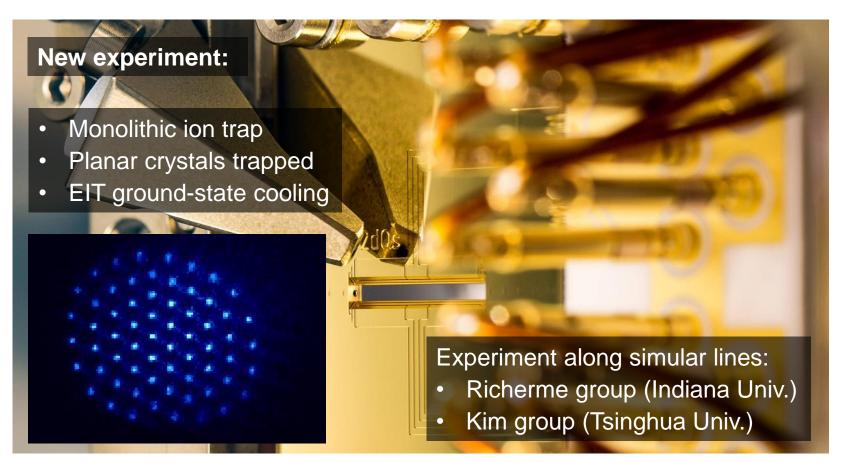
Position (µm)

Spin-spin simulations with planar crystals in rf-traps



Challenges:

- melting of crystals by background gas collisions + recrystallization
- Laser cooling of in-plane modes
- Motional heating



Summary and outlook

Trapped-ion quantum simulations

- Quantum simulation approaches: digital vs analog
- Realization of long range spin models in trapped ions
- Entanglement creation and characterization in multi-ion strings
- Scaling up quantum simulations to larger ion numbers
- Variational quantum simulation

Outlook:

- Exploration of non-equilibrium quantum dynamics in systems with >50 qubits
- Experiments with planar ion crystals with single-ion control
- J. I.Cirac, P. Zoller, "Goals and opportunities in quantum simulation", Nat. Phys. 8, 264 (2012)
- R. Blatt, C. F. Roos, "Quantum simulations with trapped ions", Nat. Phys. 8, 277 (2012)
- C. Monroe et al., "Programmable quantum simulations of spin systems with trapped ions" Rev. Mod. Phys. 93, 025001 (2021)

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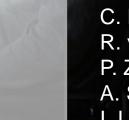
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A. Schuckert

I. Lovas

M. Knap

R. Blatt

B. Lanyon

Experiment

Theory

Theory

(Munich)

(Innsbruck)