

Entanglement in trapped-ion experiments

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Institut für
Experimentalphysik

 ÖAW
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Funding:

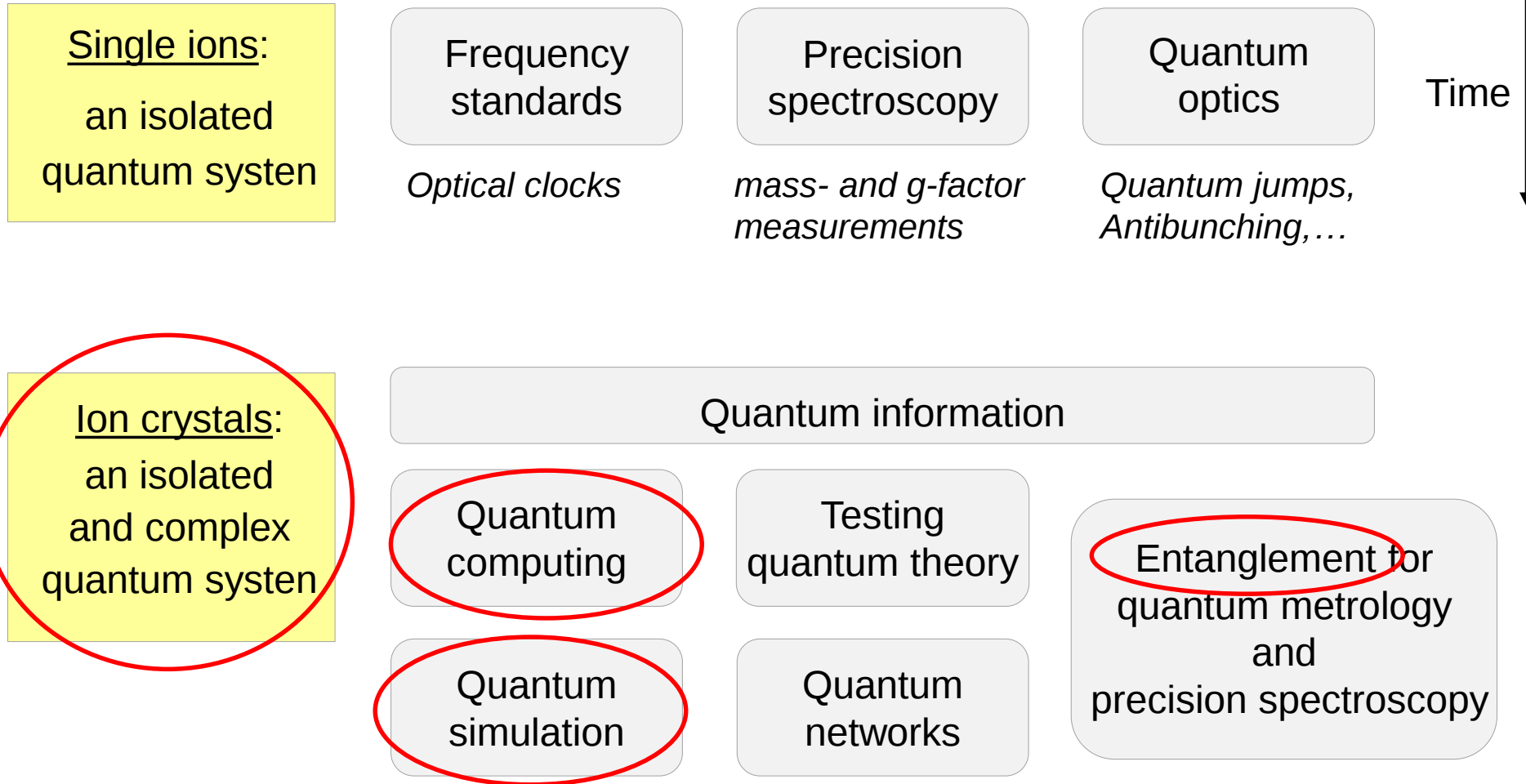


FWF

TIFR-ICTR online school, 17.5.2021

Christian Roos
Institute for Experimental Physics
University of Innsbruck, Austria

Quantum physics with trapped ions

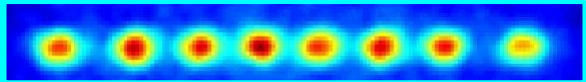


Outline

- Trapped ions as an interacting quantum system
- Properties of ion crystals
- Advanced laser cooling techniques
- Quantum control of long ion strings
- Entangling interactions
- Gate-based quantum computation

Quantum physics with ion crystals

Ion trap



Lasers

- Ion loading ~ minutes
 - Ion storage ~ day(s)
 - Individual experiments ~ 20 ms
(initialization, coherent interaction,
+ measurement)
- ~ $10^5 - 10^6$
quantum measurements per day

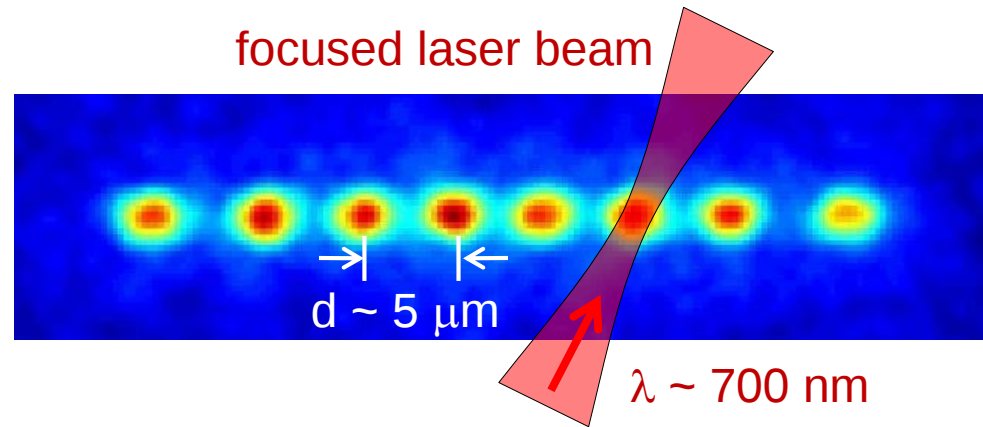
- Ion creation
- Laser cooling
→ quantized ion motion
- Coherent excitation
→ entanglement generation
- Ion detection
→ quantum measurements

Quantum physics with ion crystals

Trap frequencies:

$$\nu_z \propto 1 \text{ MHz}$$

$$\nu_{x,y} \propto 5 \text{ MHz}$$



Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
d	>	λ	>>	z ₀	>>	a ₀
5 μm		700 nm		10 nm		50 pm

A red double-headed arrow is positioned below the table, spanning the width of the first two columns (ion distance and laser wavelength).



- Spatially resolved fluorescence



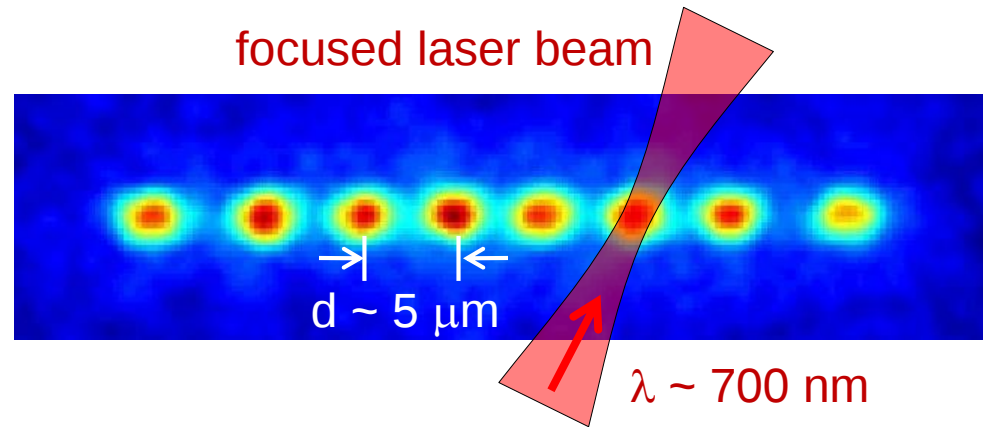
- Individual addressing

Quantum physics with ion crystals

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- Spatially resolved fluorescence

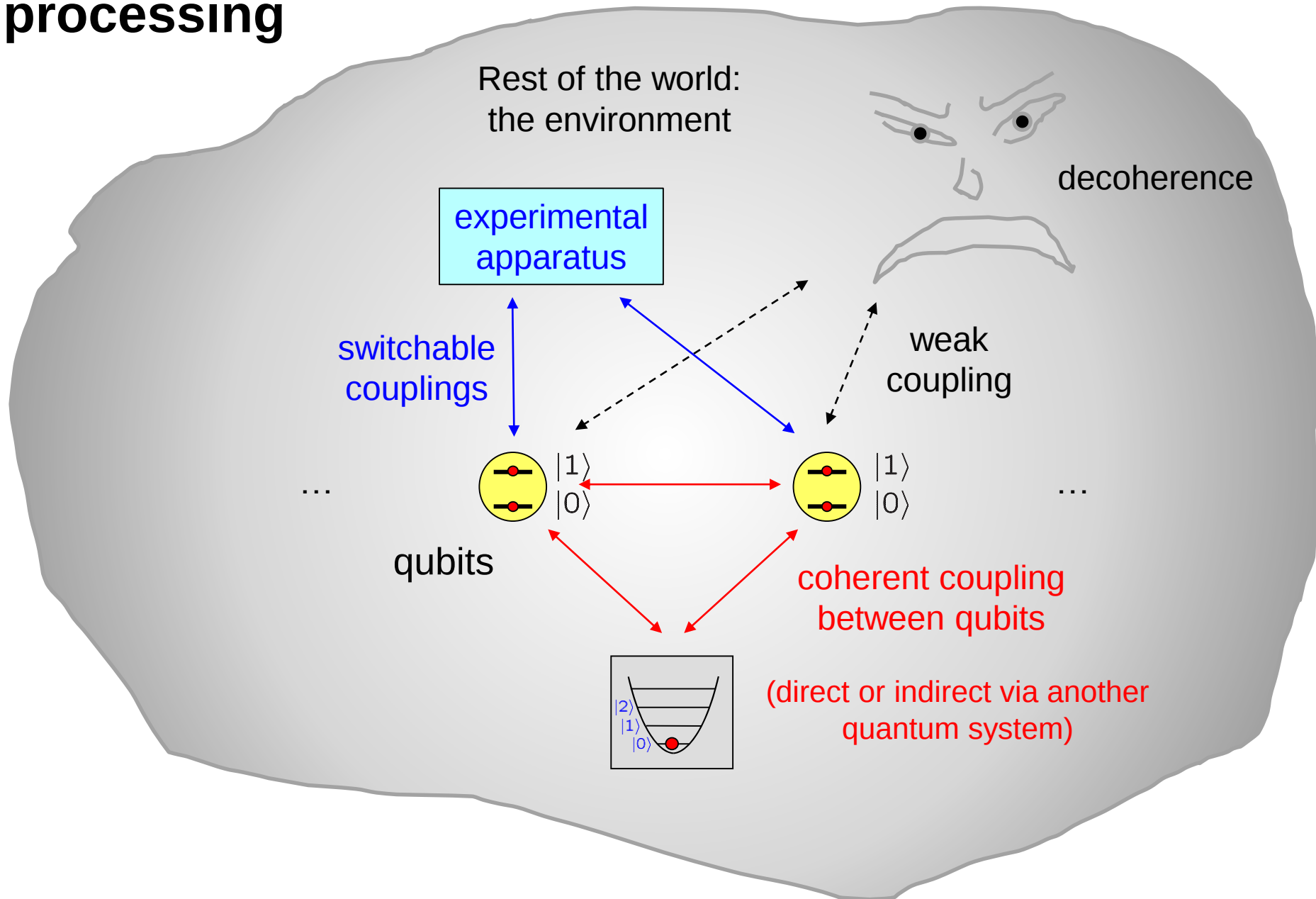


- Individual addressing



- No direct state-dependent interactions between ions

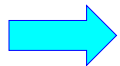
Experimental requirements for quantum information processing



Isolating ions from their environment

In experiments, we want to avoid

- Ion loss by collisions with the background gas
- Motional heating by fluctuating electric fields and motional dephasing
- Decoherence by spontaneously scattered photons
- Dephasing by fluctuating magnetic fields and laser phase noise
- Decoherence by fluctuating control parameters (laser intensity,...)
- Decoherence induced by the ion motion
- Disturbance by the time-varying fields of a radiofrequency trap
- ...

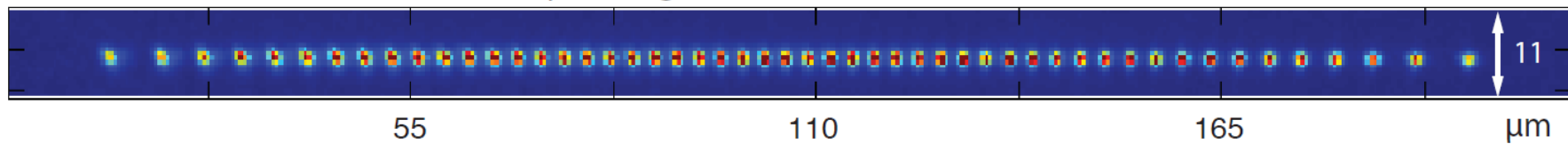


Ultrahigh vacuum, stable lasers, many feedback systems for keeping parameters stable

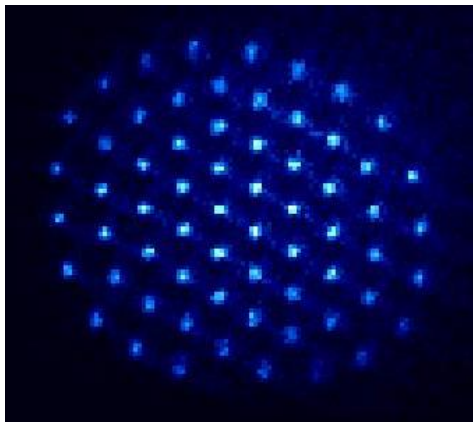
Ion crystals for quantum physics experiments: crystal configurations

Harmonic potential: ratio of trapping frequencies determines the crystal structure

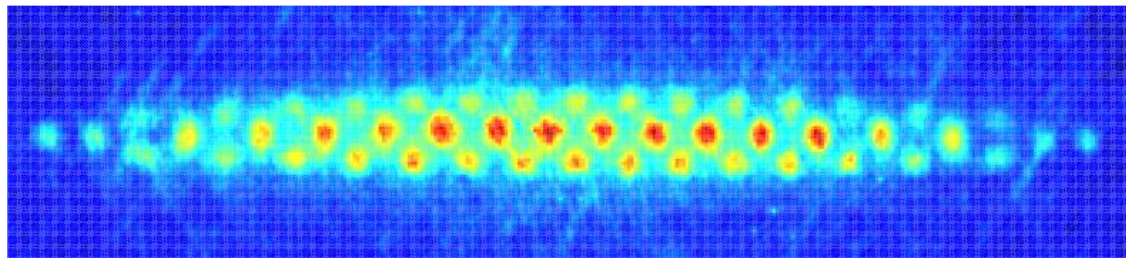
1d: Linear strings



2d: Planar crystals



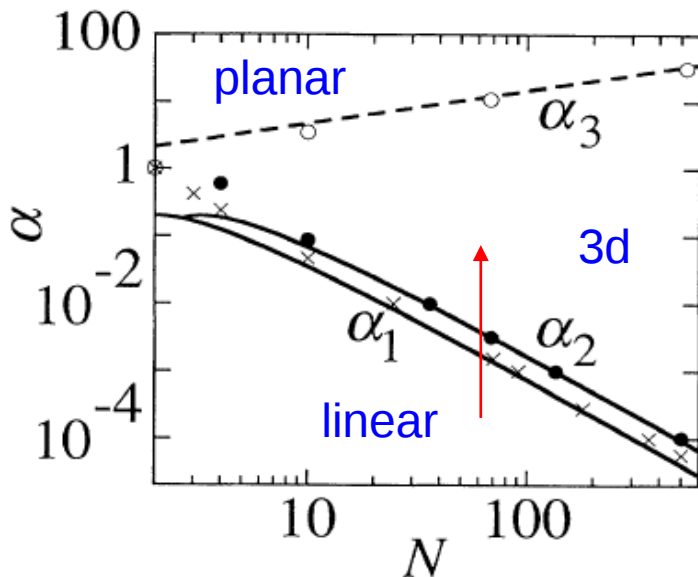
3d: three-dimensional structures



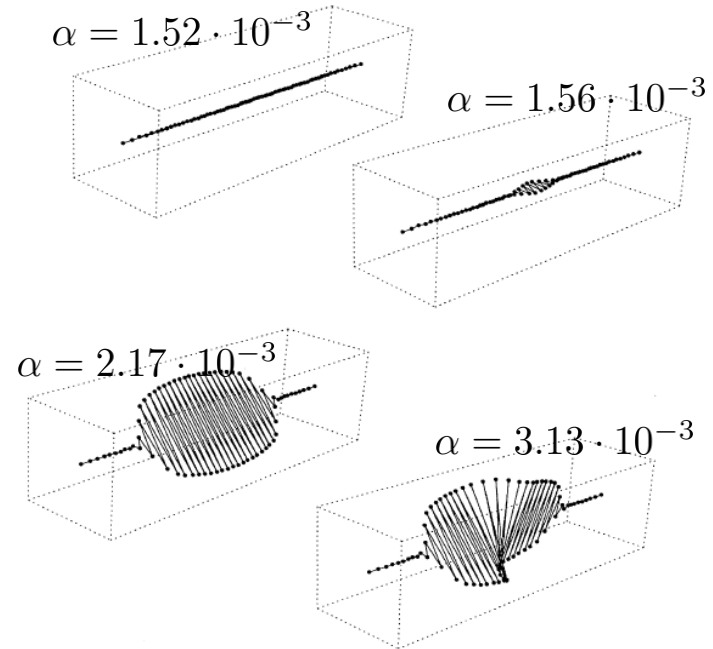
Structural phases of ion crystals

Harmonic potential: ratio of trapping frequencies determines the crystal structure

Anisotropy parameter $\alpha = \omega_z^2 / \omega_{\perp}^2$



D. Dubin, PRL 71, 2753 (1993)



J. Schiffer, PRL 70, 818 (1993)

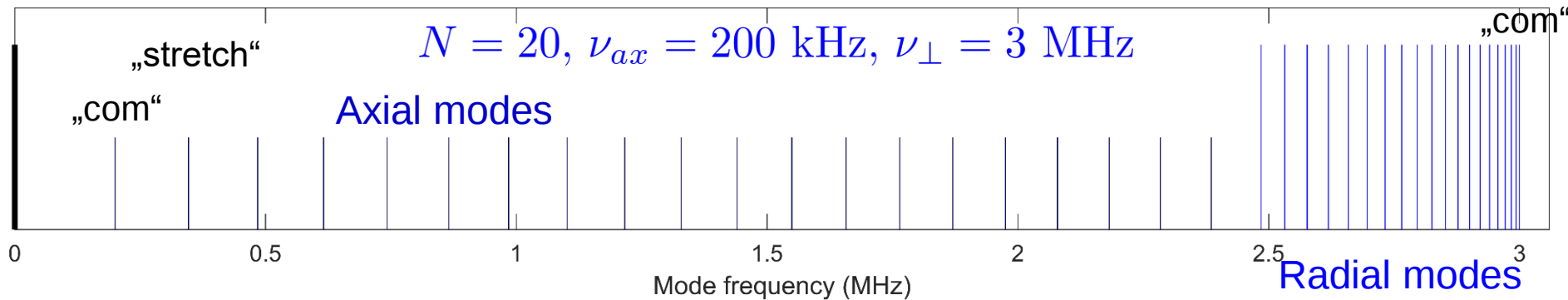
Ion strings in linear radiofrequency traps

(2d-rf quadrupole + axial confinement by dc electric potential)

All ions are at locations that are micromotion-free

Collective motional modes of ion crystals

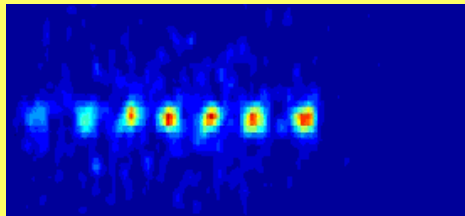
Linear strings: collective modes decouple into axial and transverse modes



- Axial center-of-mass mode spectrally well separated
→ enables coupling to a single oscillator
- Radial modes cluster in frequency space
→ enables coupling to many oscillators

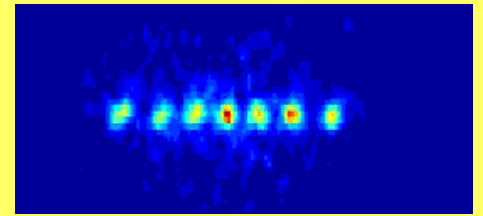
centre-of-mass
mode

$$\nu = \nu_z$$



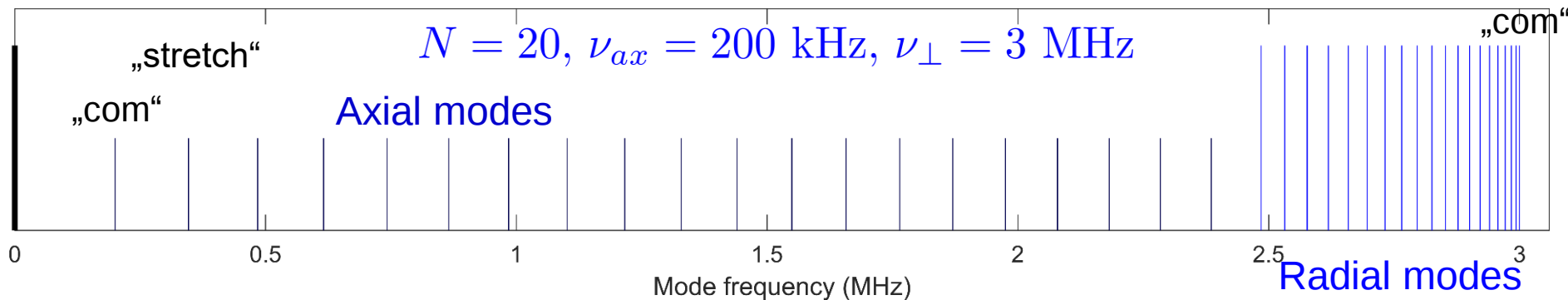
stretch
mode

$$\nu = \sqrt{3} \nu_z$$



Collective motional modes of ion crystals

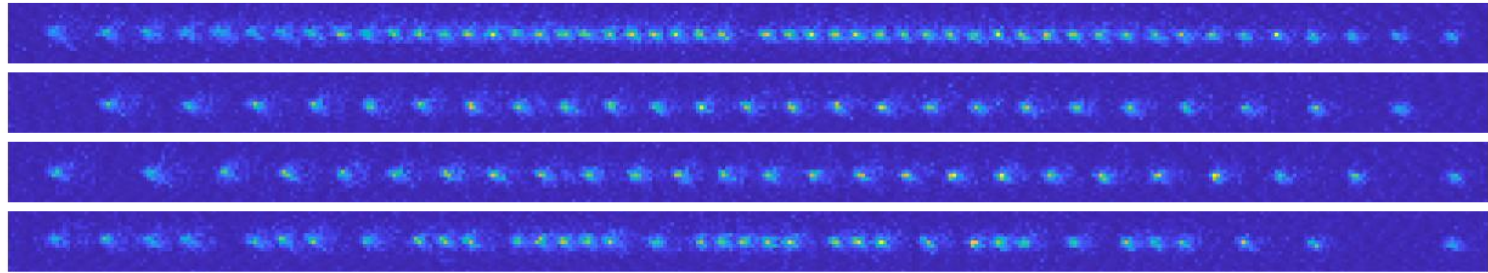
Linear strings: collective modes decouple into axial and transverse modes



- Axial center-of-mass mode spectrally well separated
➡ enables coupling to a single oscillator
- Radial modes cluster in frequency space
➡ enables coupling to many oscillators

Planar crystals: collective modes decouple into $2N$ in-plane and N out-of-plane modes

Quantum control of long ion strings

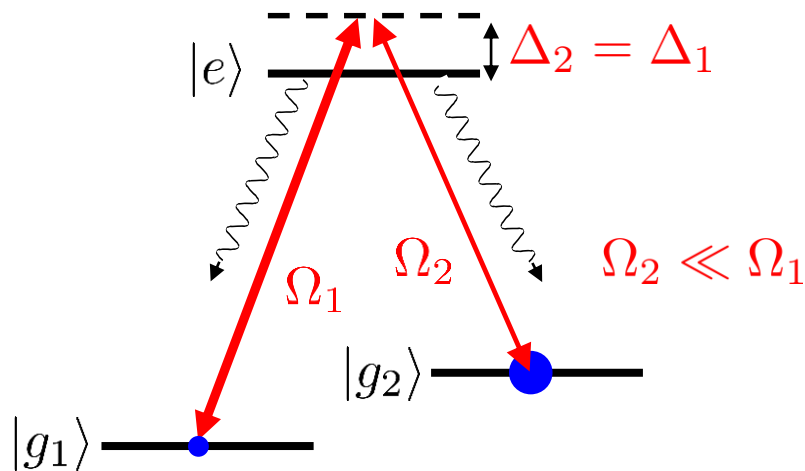


- Laser cooling techniques
 - EIT cooling and polarization gradient cooling
- Controlling individual ion qubits
 - Coherent global control and individual addressing
- Quantum measurements of ion qubits
 - Measurement in different bases; correlation measurements
- Entangling interactions by coupling to motional states
 - Mølmer-Sørensen interactions and other geometric phase gates

Laser cooling: Schemes and performance metrics

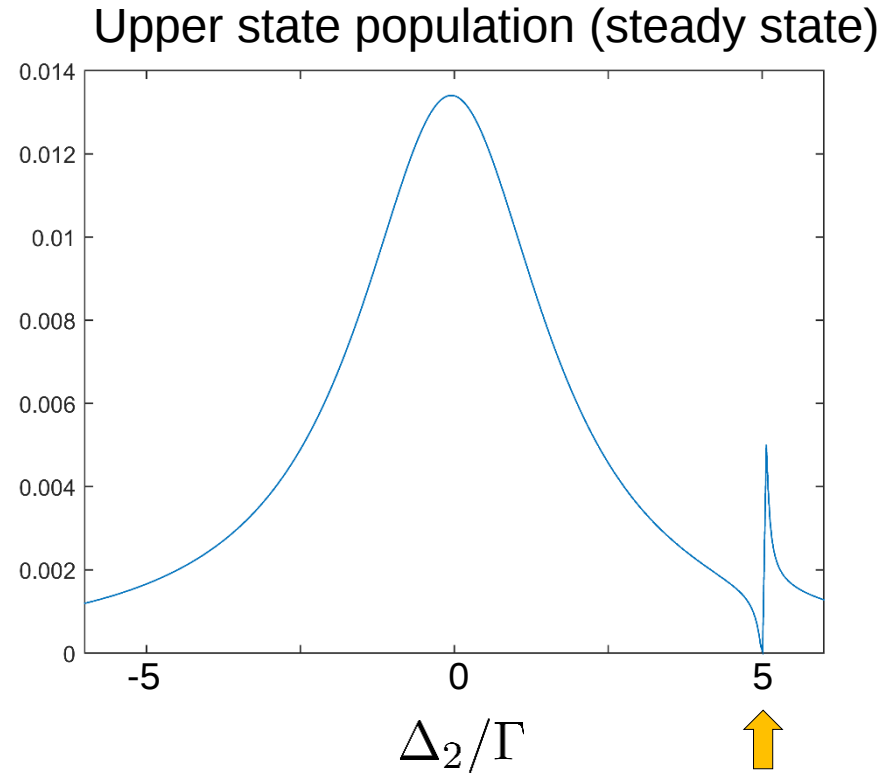
	Final energy	Initial temperature	Cooling speed	Frequency range
Doppler cooling	low	high	high	very high
Resolved Sideband cooling	$\langle n \rangle \approx 0$	low	low	low
EIT cooling	$\langle n \rangle \approx 0$	low	decent	decent
Polarization gradient cooling	sub-Doppler	low	high	high

Cooling by electromagnetically induced transparency (EIT)



Dark state:

$$|g_-\rangle \sim \Omega_2|g_1\rangle + \Omega_1|g_2\rangle$$

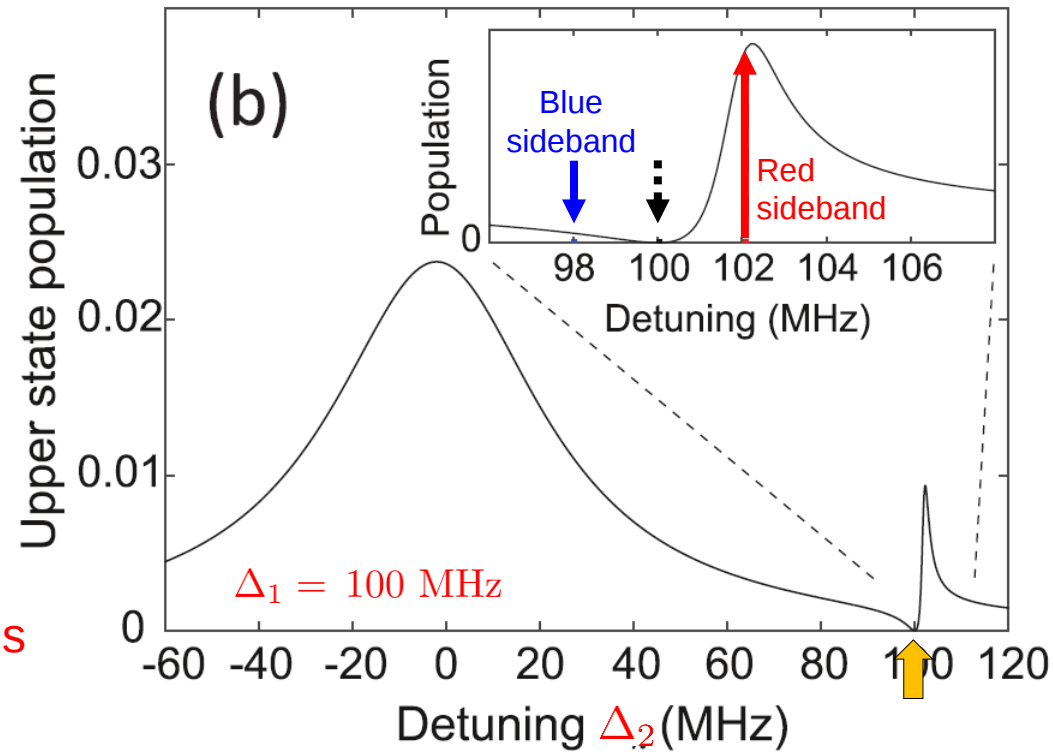
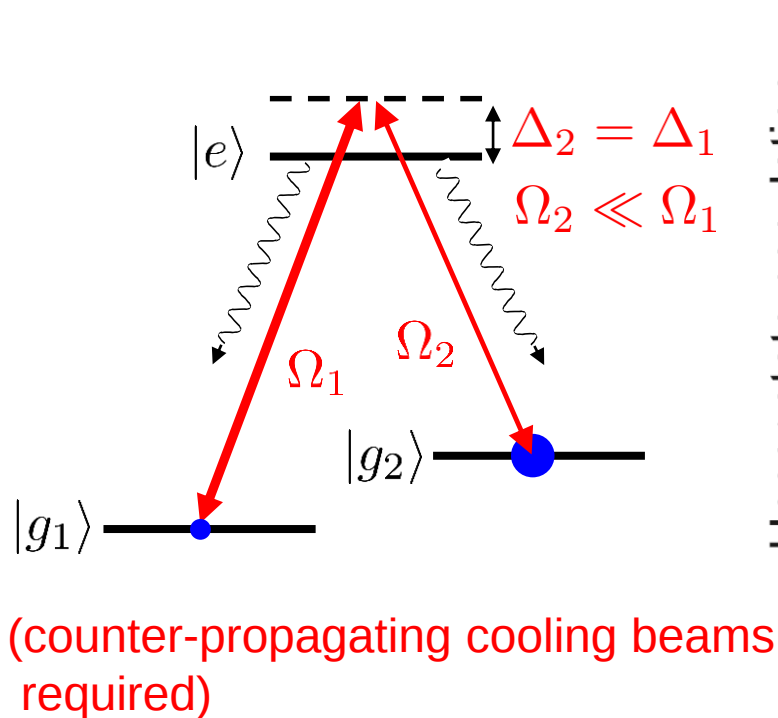


EIT cooling scheme:

A strong blue-detuned “dressing” beam couples levels $|g_1\rangle$ and $|e\rangle$.
 A weak probe beam with detuning $\Delta_2 = \Delta_1$ couples levels $|g_1\rangle$ and $|e\rangle$.
 Together, both beams pump the atom into the dark state $|g_-\rangle$, which has a high overlap with $|g_2\rangle$.

What happens if the atom is free to move in a trap?

EIT cooling scheme: trapped atom

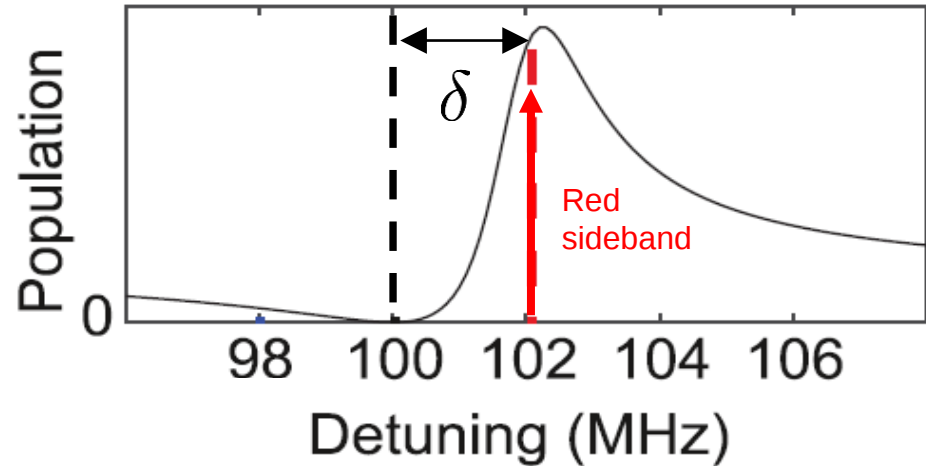
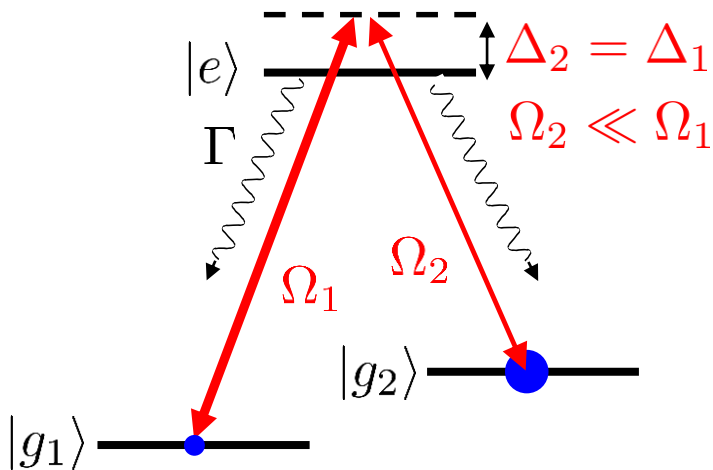


EIT cooling scheme:

An atom in state $|g_-\rangle$ cannot be excited on the carrier transition and very little by a blue sideband process. But it can be excited on the red sideband if the narrow bright resonance is resonant with the sideband.

Ground-state cooling becomes possible!

EIT cooling scheme parameters



The position and width of the bright resonance can be tailored by properly choosing Ω_1 and Δ_1 .

- Position controlled by light shift $\delta = \frac{\Omega_1^2}{4\Delta_1}$ (controls for which oscillation frequency the cooling is optimal)
- Width controlled by bright resonance decay rate (admixture of $|e\rangle$) (control of cooling range)

Advantages of the EIT cooling scheme

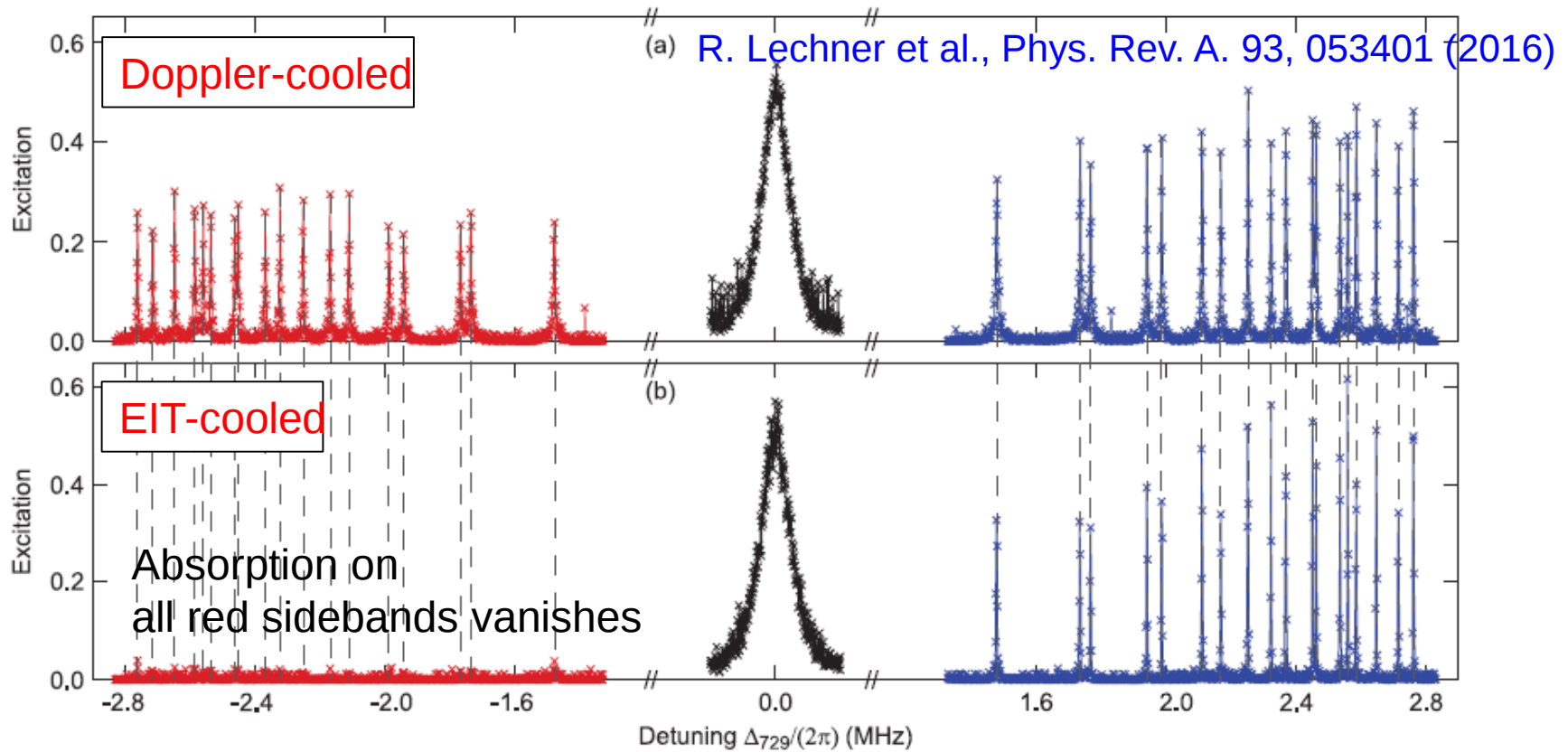
- There is heating by spontaneous emission from carrier absorption. This feature enables ground-state cooling over a wider range of oscillation frequencies
- The absorption profile can be optimally sculpted, resulting in narrow absorption features on a dipole-allowed transition.
- Cooling on a dipole-allowed transition requires less laser power than on a dipole-forbidden transition.
- The frequency stability of the cooling laser is not critical if both beams are derived from the same laser.
- Higher cooling rates can be achieved as compared to sideband cooling.

EIT cooling results

Proposal: Phys. Rev. Lett. 85, 4458 (2000)

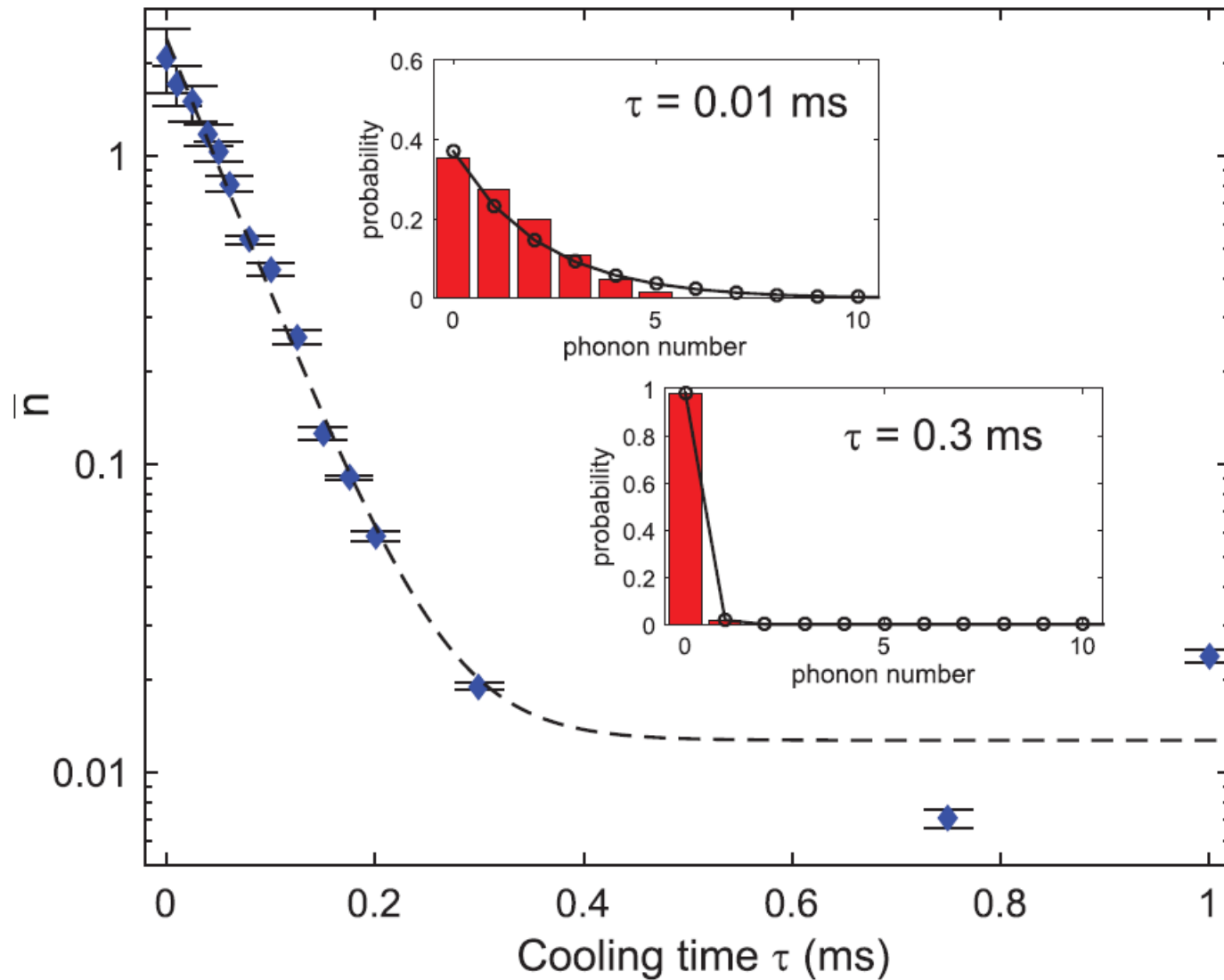
First exp: Phys. Rev. Lett. 85, 5547 (2000)

EIT-cooling of all transverse modes of a 9-ion crystal



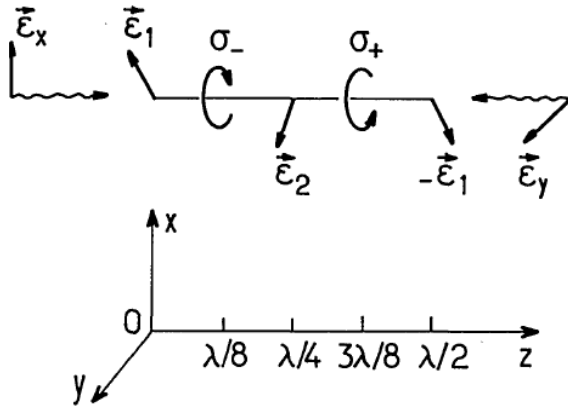
Here, ground-state cooling is confirmed by sideband spectroscopy on a dipole-forbidden transition.

EIT cooling results: 18-ion crystal

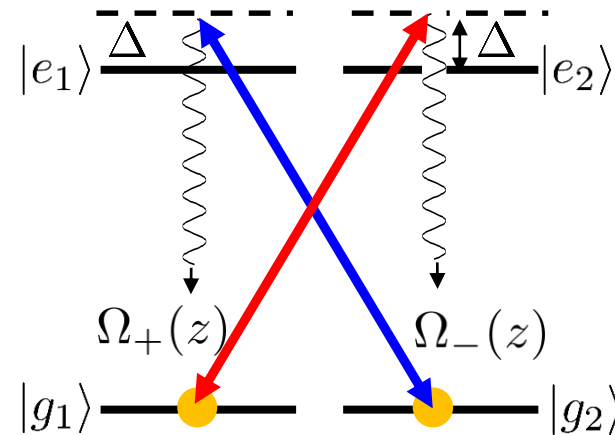


Polarization gradient cooling of ions

Counterpropagating beams with crossed linear polarizations

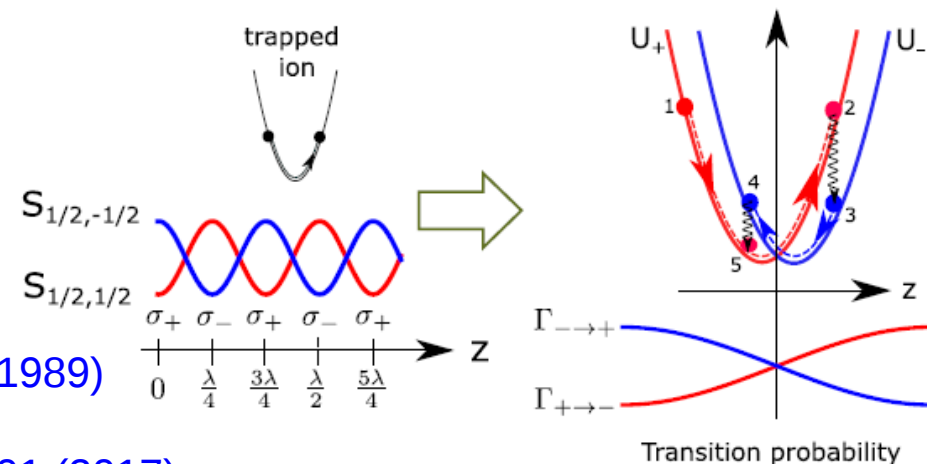


Level scheme and coupled transitions



Cooling principle:

- Position-dependent light shifts
- Position-dependent pumping rates



J. Dalibard, C Cohen-Tannoudji, JOSA B **6**, 2023 (1989)

J. I. Cirac et al., Phys. Rev. A **48**, 1434 (1993)

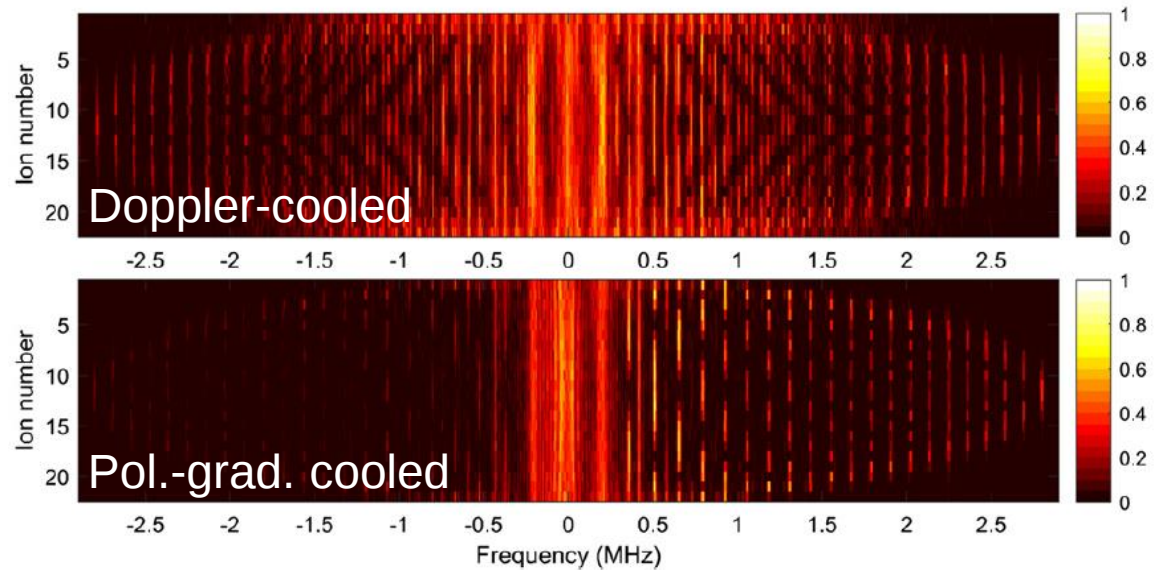
S. Ejtemaee, P. Haljan, Phys. Rev. Lett. **119**, 043001 (2017)

M. K. Joshi et al., NJP **22**, 103013 (2020)

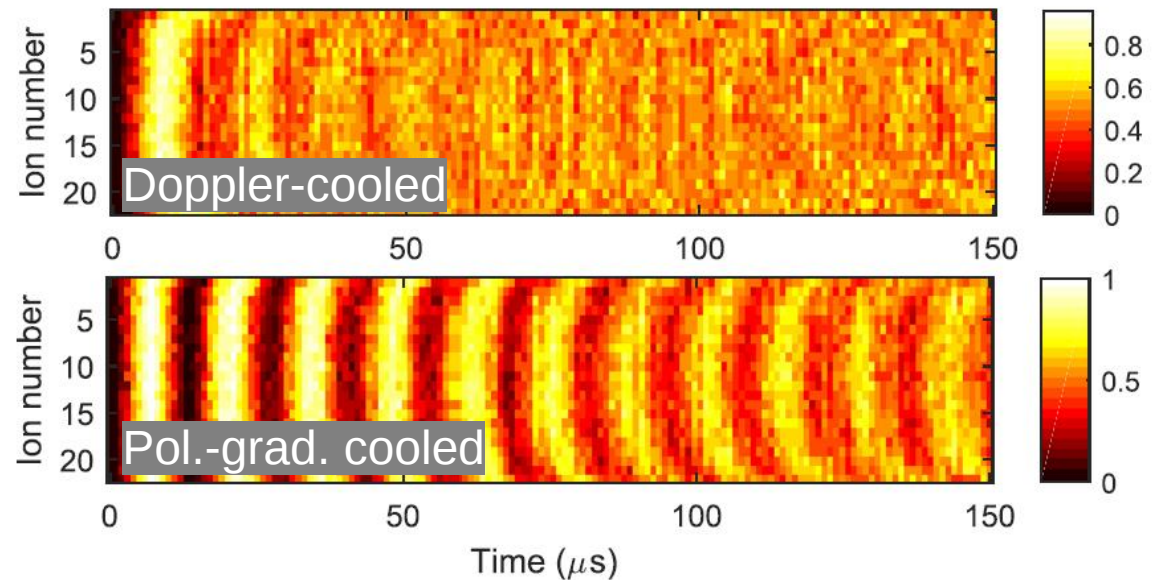
Polarization gradient cooling of ion strings

22-ion string

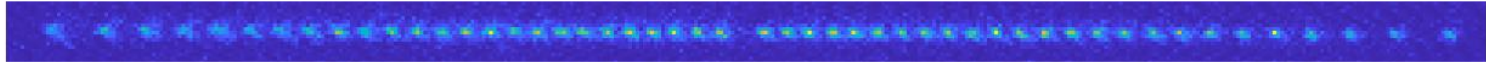
Spatially resolved
Sideband spectrum



Carrier Rabi
oscillations



Experiments with long ion strings



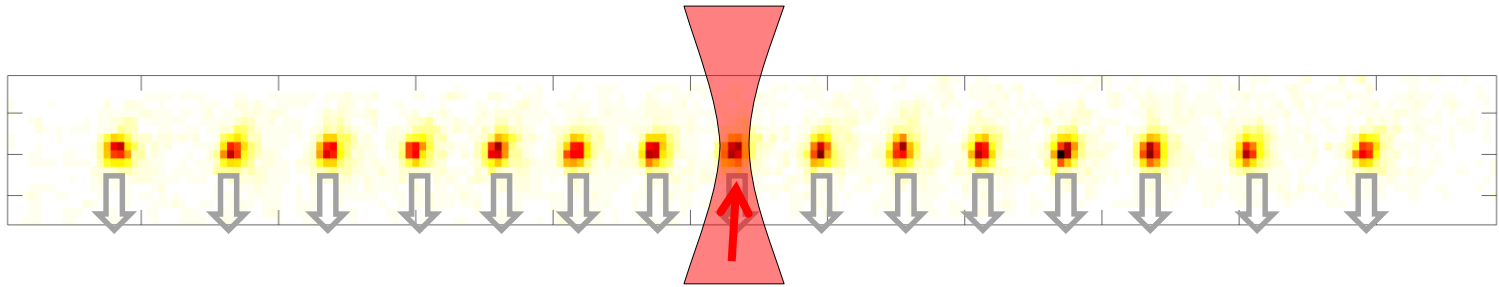
0. Trapping in a very anisotropic potential

(for example: $N = 50$, $\nu_{ax} = 130$ kHz, $\nu_{\perp} = 3$ MHz)

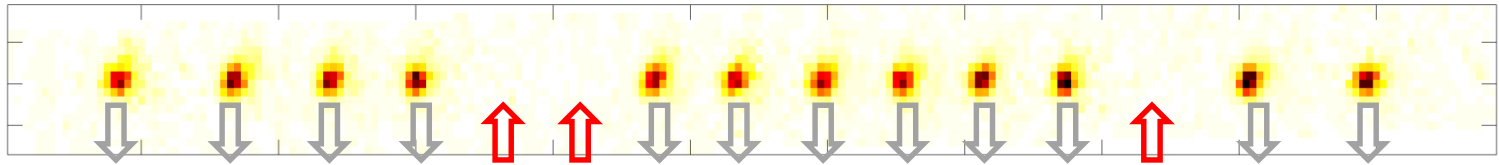
1. Laser cooling:
 1. Doppler cooling into the Lamb-Dicke regime
 2. Sideband cooling of all transverse modes close to their ground state
 3. Polarization gradient cooling of axial modes to sub-Doppler temp.
2. Optical pumping
3. Coherent manipulation by laser pulses
4. Quantum state measurements by spatially resolved fluorescence with CCD camera

Measurement and coherent control of single ions

Spatially resolved fluorescence: **detection of individual spin states**

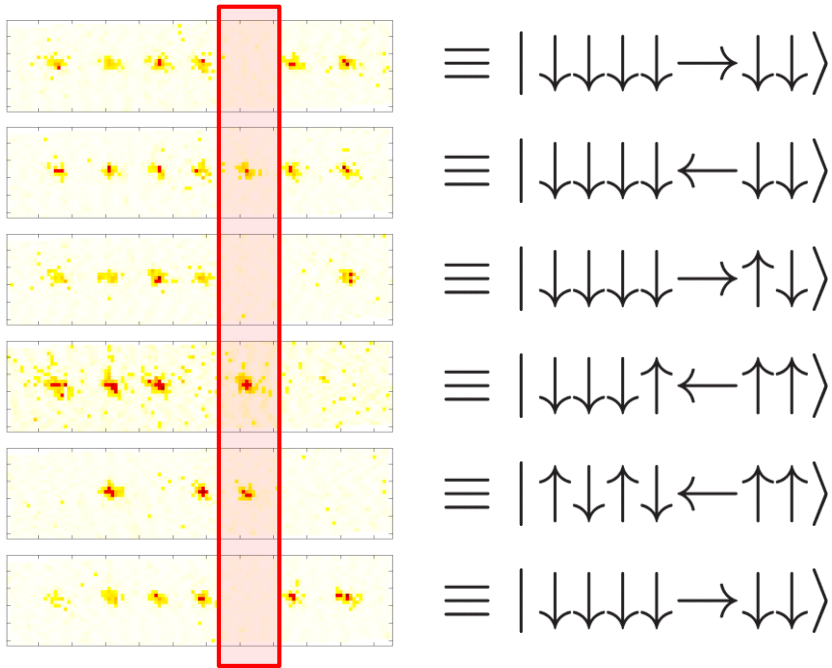


focused steerable laser beam: **coherent single-spin manipulation**



beam switching time $\sim 10 \mu\text{s}$

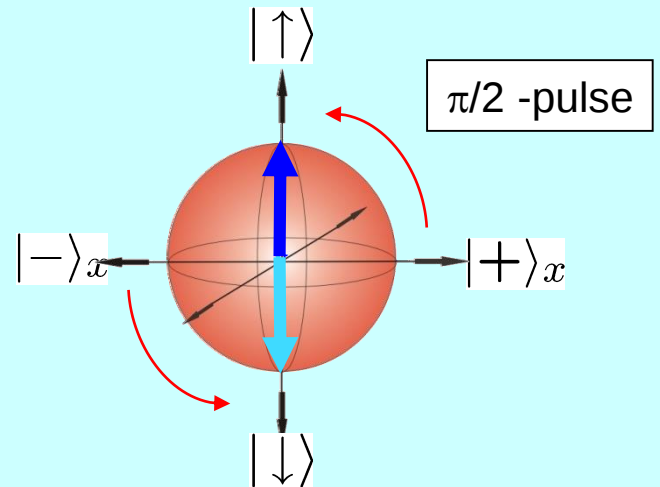
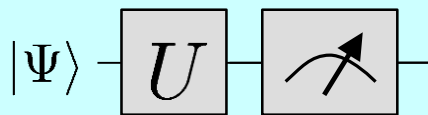
Measuring spins and spin correlations



$\Rightarrow$ any correlation $\langle \sigma_{\alpha_1}^{(1)} \sigma_{\alpha_2}^{(2)} \dots \sigma_{\alpha_N}^{(N)} \rangle$
 can be measured.

Measurement of σ_x

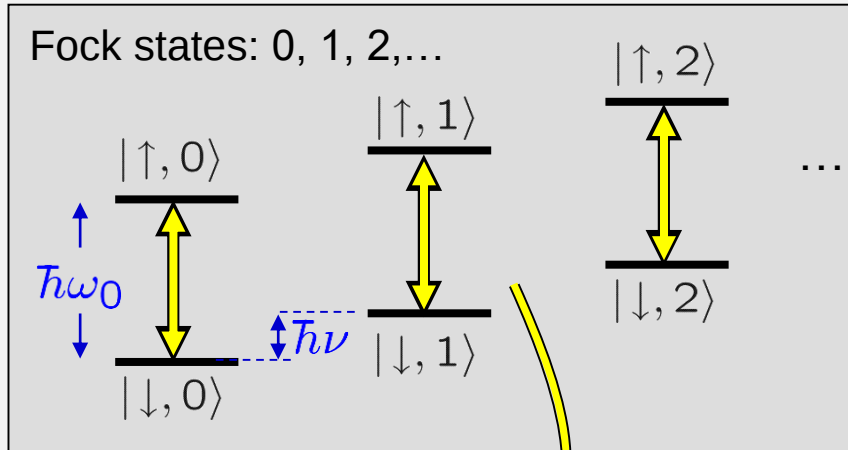
(single)-spin rotation +
fluorescence measurement





Entangling interactions in trapped-ion experiments

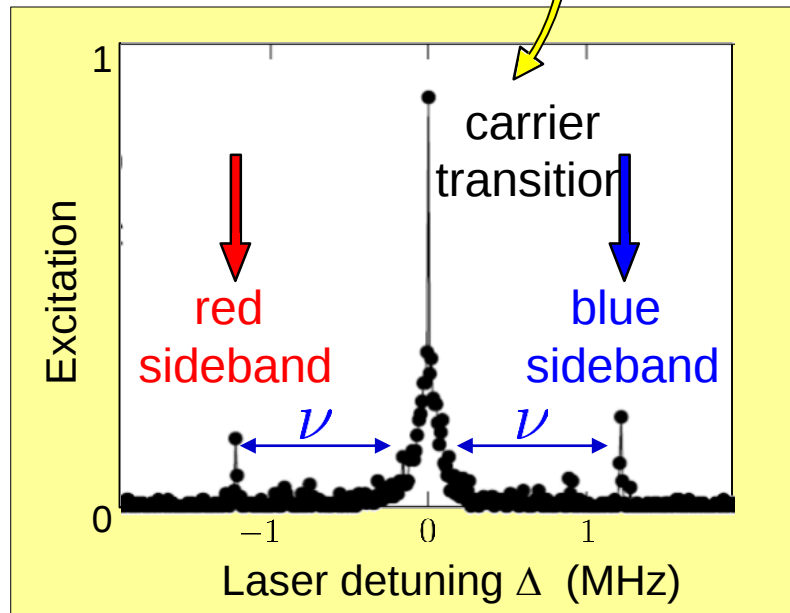
Trapped-ion laser interactions



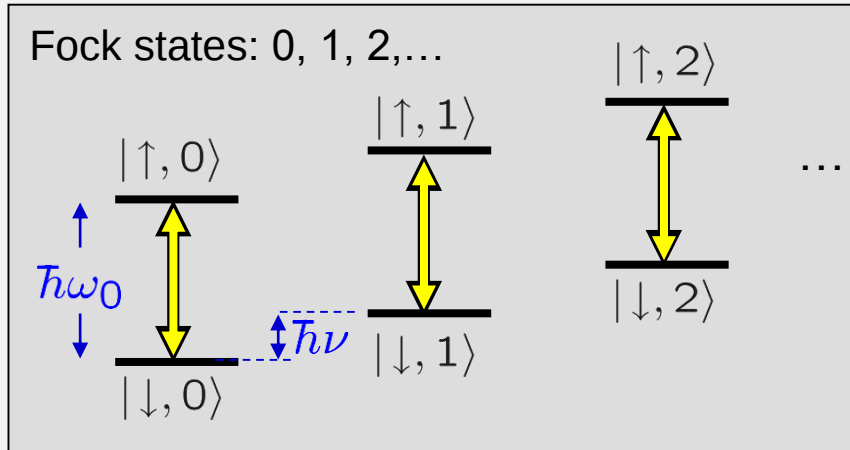
qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x$$



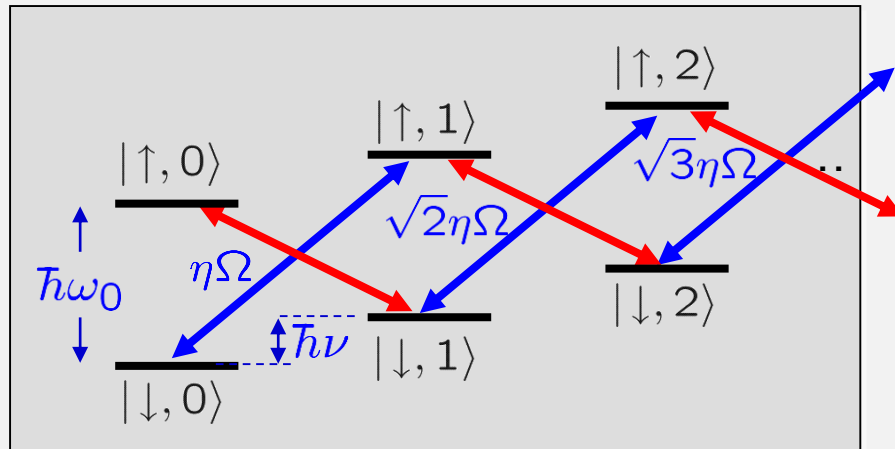
Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

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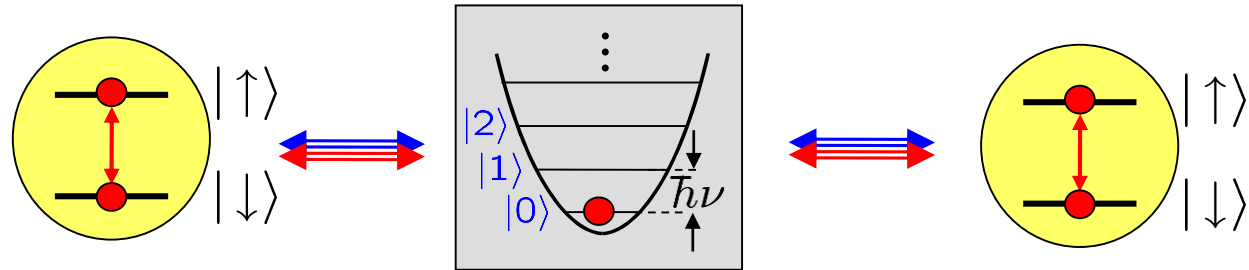
qubit-motion coupling

$$\omega_{laser} = \omega_0 \pm \nu$$

$$H \propto (a + a^\dagger)\sigma_x$$

Entangling a pair of ions

Coupling of internal states
via the ion motion



A. Sørensen, K. Mølmer,
Phys. Rev. Lett. **82**, 1971 (1999)

Effective spin-spin interaction

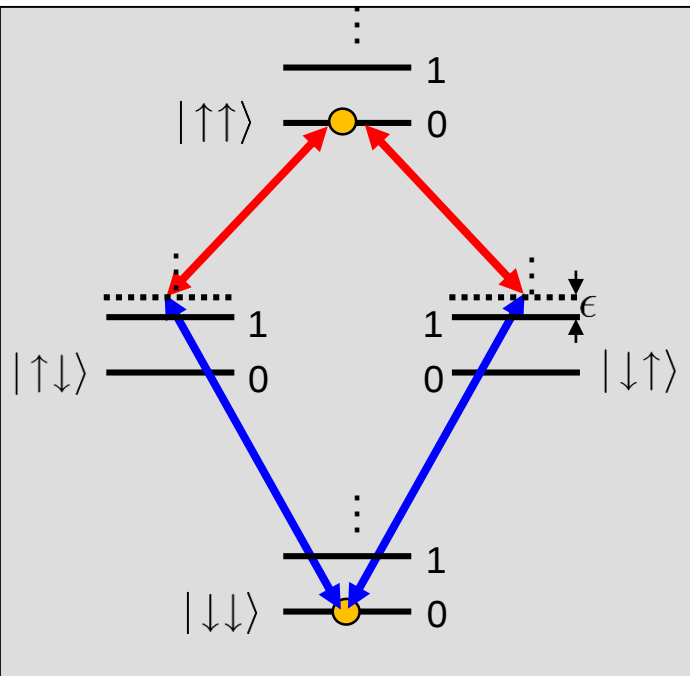
$$|\downarrow\downarrow\rangle \longleftrightarrow |\uparrow\uparrow\rangle$$

$$|\downarrow\uparrow\rangle \longleftrightarrow |\uparrow\downarrow\rangle$$

Creating entanglement:

$$|\downarrow\downarrow\rangle \longrightarrow |\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$$

$$H_{eff} = \hbar J \sigma_1^x \sigma_2^x$$

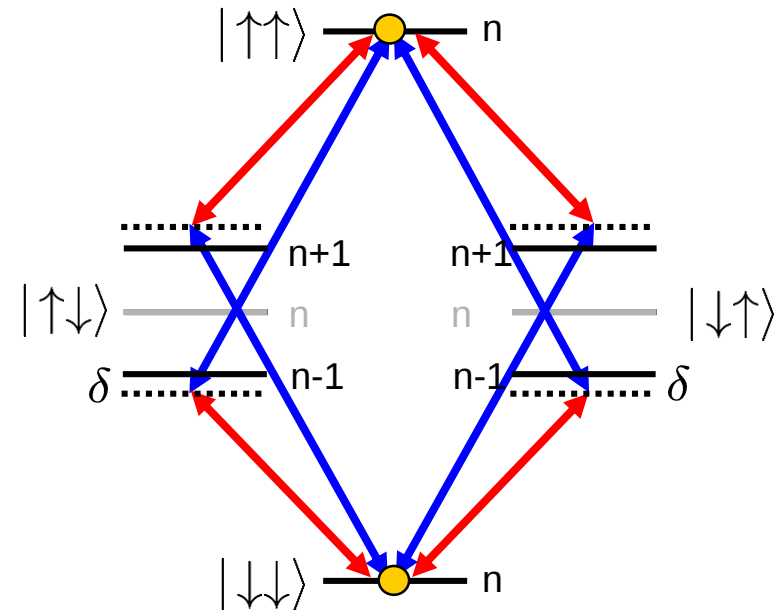


The coupling does not depend on the motional state:

In order to calculate the two-photon coupling strength $\Omega_{\downarrow\downarrow,\uparrow\uparrow}$ between the levels $|\downarrow\downarrow, n\rangle$ and $|\uparrow\uparrow, n\rangle$, we need to sum up four contributions where the coupling is mediated by the intermediate states $|\downarrow\uparrow, n+1\rangle$, $|\uparrow\downarrow, n+1\rangle$, $|\downarrow\uparrow, n-1\rangle$, $|\uparrow\downarrow, n-1\rangle$. We find

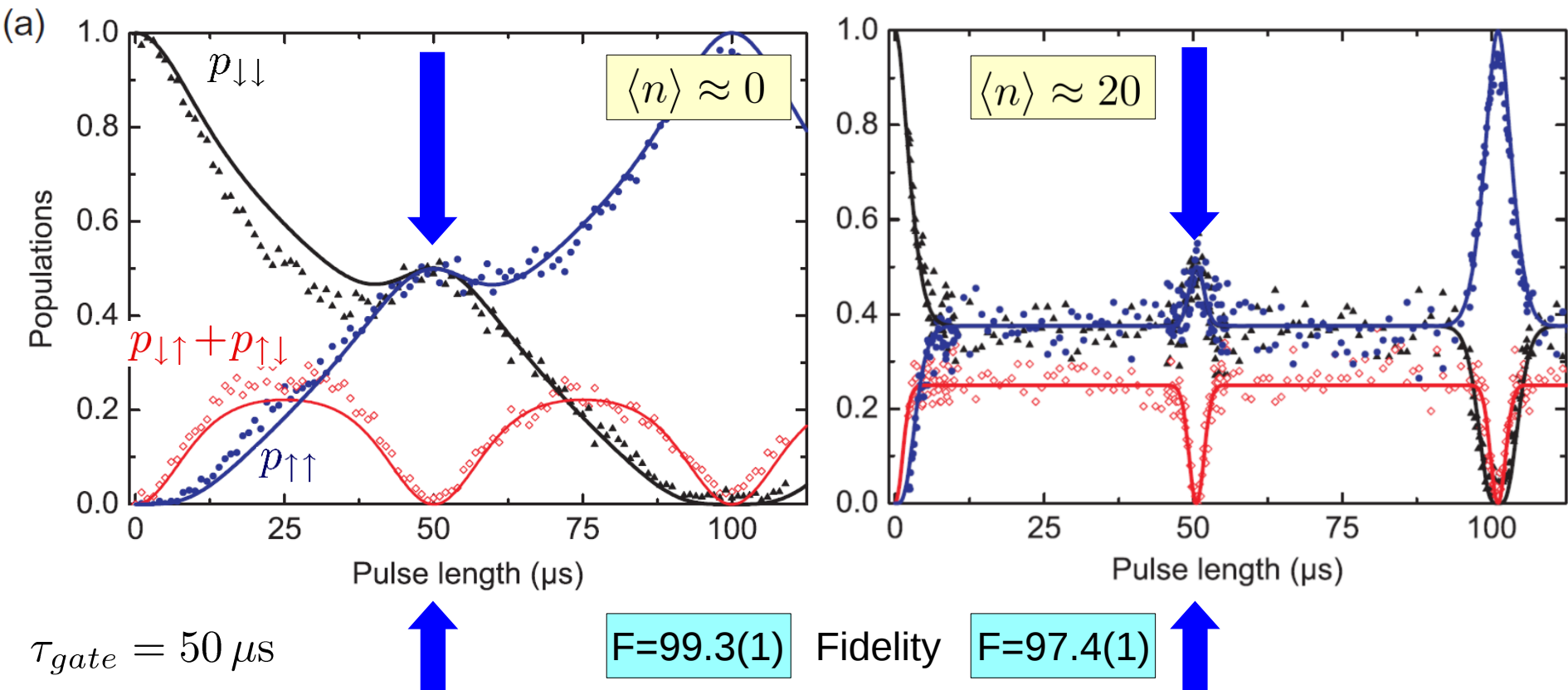
$$\begin{aligned}\Omega_{\downarrow\downarrow,\uparrow\uparrow} &= 2 \left[\frac{(\eta\Omega\sqrt{n})(\eta\Omega\sqrt{n})}{\delta} + \frac{(\eta\Omega\sqrt{n+1})(\eta\Omega\sqrt{n+1})}{-\delta} \right] \\ &= -4 \frac{\eta^2 \Omega^2}{\delta}\end{aligned}$$

where we took into consideration that the detuning of the lasers from the intermediate levels is opposite when coupling to levels with $n+1$ and $n-1$ phonons and that the coupling strength on the second step of the transition needs to be calculated for a vibrational phonon number of $n+1$ and $n-1$ respectively.



Creating Bell states with Mølmer-Sørensen gates

$$|\downarrow\downarrow\rangle \longrightarrow |\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle$$



$$p_{\downarrow\downarrow} + p_{\uparrow\uparrow} = 0.9965(4) \quad 13,000 \text{ measurements}$$

J. Benhelm *et al.*, Nature Phys. **4**, 463 (2008)

G. Kirchmair *et al.*, New J. Phys. **11**, 023002 (2009)

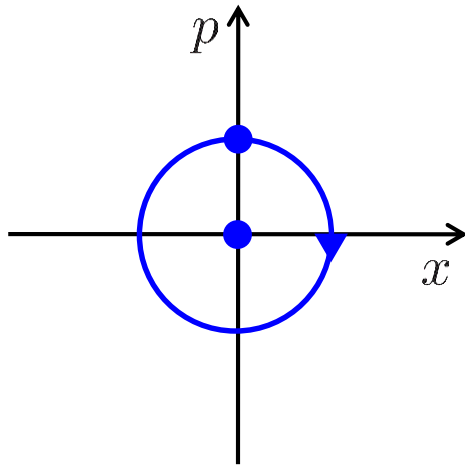
Geometric phases:

Another way to understand Mølmer-Sørensen and related gates

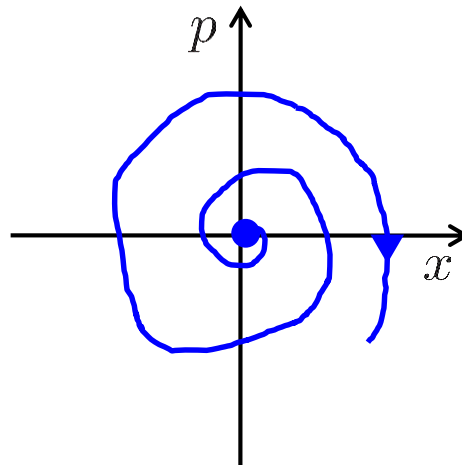
1. Driven harmonic oscillator: phase space picture
2. Driven harmonic oscillator with qubit state-dependent coupling

Classical harmonic oscillator in phase space

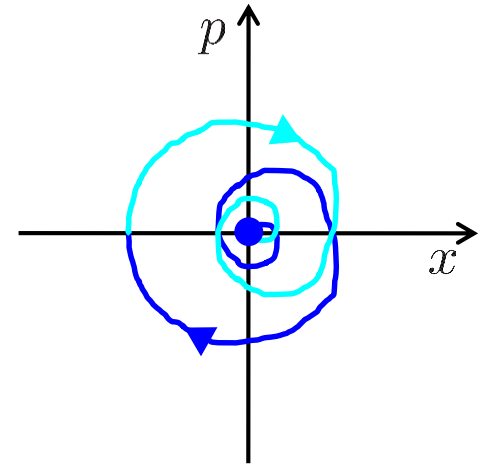
Phase space trajectory
of harmonic oscillator



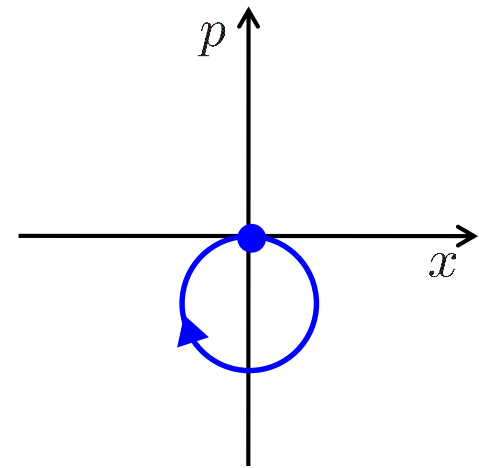
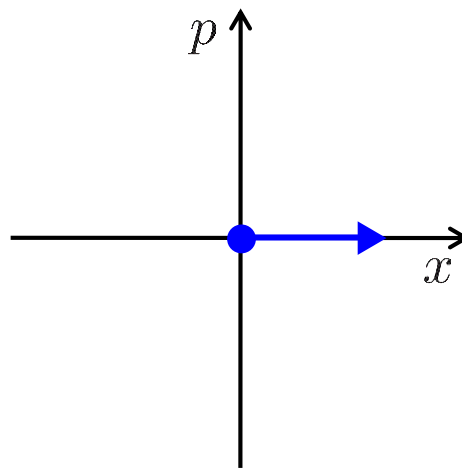
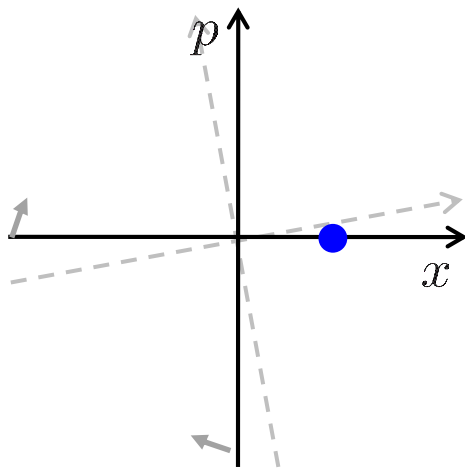
Resonantly driven
harmonic oscillator



Off-resonantly driven
harmonic oscillator

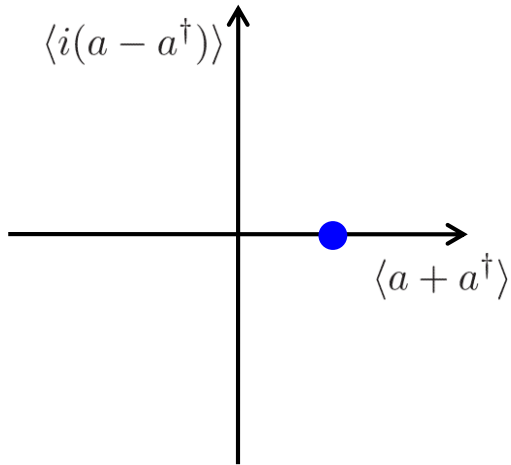


Rotating frame: get rid of the boring oscillation

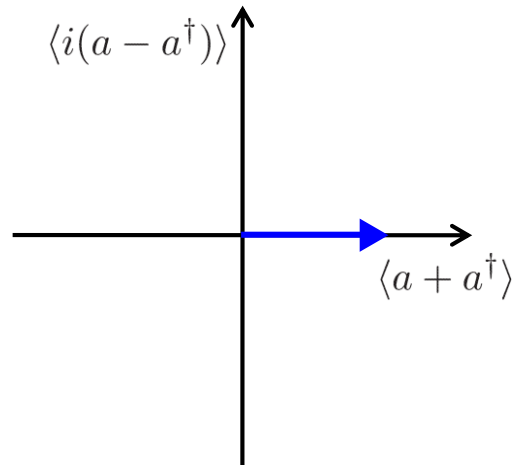


Quantum harmonic oscillator in phase space

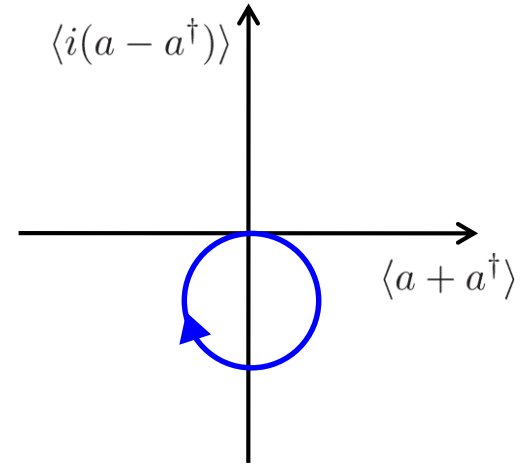
Free harmonic oscillator



Resonantly driven harmonic oscillator



Off-resonantly driven harmonic oscillator



Hamiltonian driving the motion:

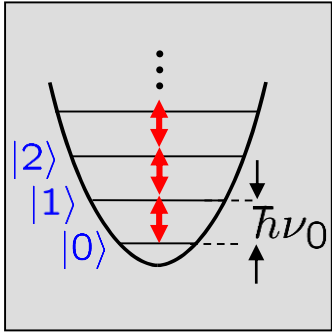
$$H = 0$$

$$H = \hbar\Omega i(a - a^\dagger)$$

$$H = \hbar\Omega i(ae^{i\delta t} - a^\dagger e^{-i\delta t})$$

Driven quantum harmonic oscillator

Harmonic oscillator



$$H(t) = \hbar\nu_0 a^\dagger a + \hbar\Omega i(a^\dagger e^{i\nu t} - a e^{-i\nu t})$$

Interaction picture:

Drive frequency ν

$$H_{int} = \hbar\Omega i(a^\dagger e^{i\delta t} - a e^{-i\delta t}), \quad \delta = \nu - \nu_0$$

Time evolution:

$$U(t) = \lim_{n \rightarrow \infty} \prod_{k=1}^n \exp\left(-\frac{i}{\hbar} H(t_k) \Delta t\right) \quad \begin{aligned} \Delta t &= t/n \\ t_k &= k\Delta t \end{aligned}$$

Displacement operator:

$$\hat{D}(\gamma) = e^{\gamma a^\dagger - \gamma^* a}$$

Baker-Campbell-Hausdorff formula:

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]} \quad \text{if } [A, [A, B]] = [B, [A, B]] = 0$$

Driven quantum harmonic oscillator

Time evolution:

$$\begin{aligned} U(t) &= \lim_{n \rightarrow \infty} \prod_{k=1}^n \exp\left(-\frac{i}{\hbar} H(t_k) \Delta t\right) = \lim_{n \rightarrow \infty} \prod_{k=1}^n \hat{D}(\Omega e^{i\delta t_k} \Delta t) \\ &= \hat{D}(\alpha(t)) e^{i\Phi(t)} \quad \text{with} \quad \alpha(t) = i \left(\frac{\Omega}{\delta}\right) (1 - e^{i\delta t}) \\ &\quad \Phi(t) = \left(\frac{\Omega}{\delta}\right)^2 (\delta t - \sin \delta t) \end{aligned}$$

Displacement operator:

$$\hat{D}(\gamma) = e^{\gamma a^\dagger - \gamma^* a}$$

$$\hat{D}(\alpha) \hat{D}(\beta) = \hat{D}(\alpha + \beta) e^{i\text{Im}(\alpha\beta^*)}$$

Baker-Campbell-Hausdorff formula:

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]} \quad \text{if } [A, [A, B]] = [B, [A, B]] = 0$$

Driven quantum harmonic oscillator

Time evolution:

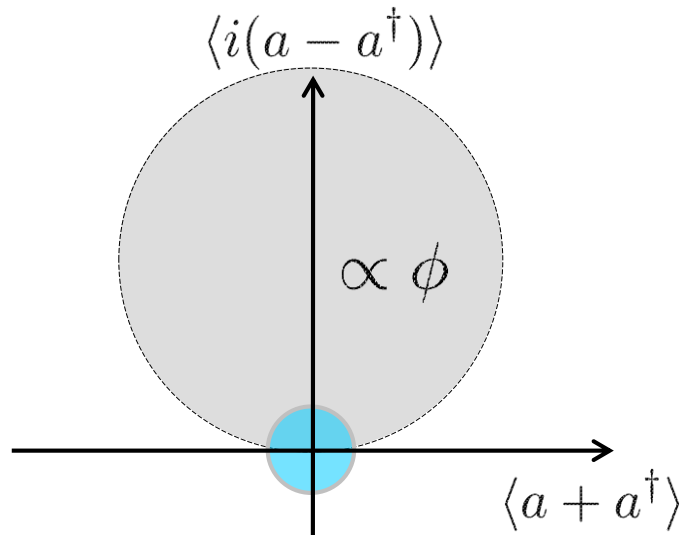
$$\begin{aligned} U(t) &= \lim_{n \rightarrow \infty} \prod_{k=1}^n \exp\left(-\frac{i}{\hbar} H(t_k) \Delta t\right) = \lim_{n \rightarrow \infty} \prod_{k=1}^n \hat{D}(\Omega e^{i\delta t_k} \Delta t) \\ &= \hat{D}(\alpha(t)) e^{i\Phi(t)} \quad \text{with} \quad \alpha(t) = i \left(\frac{\Omega}{\delta}\right) (1 - e^{i\delta t}) \\ &\quad \Phi(t) = \left(\frac{\Omega}{\delta}\right)^2 (\delta t - \sin \delta t) \end{aligned}$$

Time evolution for the ground state: $\psi(t=0) = |0\rangle$

$$\begin{aligned} U(t)|0\rangle &= \hat{D}(\alpha(t)) e^{i\Phi(t)} |0\rangle \\ &= e^{i\Phi(t)} |\alpha(t)\rangle \quad |\alpha(t)\rangle : \text{coherent state} \end{aligned}$$

Geometric phases in the harmonic oscillator

$$H = \hbar\Omega i(ae^{i\delta t} - a^\dagger e^{-i\delta t})$$



Geometric phase by
cyclic quantum evolution:

$$|\psi\rangle \longrightarrow e^{i\phi} |\psi\rangle$$

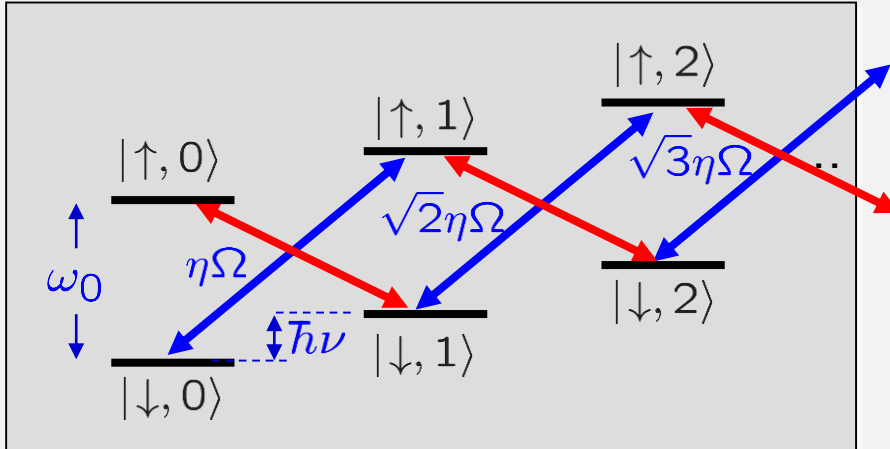
irrelevant global phase

If the phase could be made dependent on the quantum state, the phase would matter:

$$|\psi_1\rangle + |\psi_2\rangle \longrightarrow e^{i\phi_1} |\psi_1\rangle + e^{i\phi_2} |\psi_1\rangle$$

$\phi \propto \text{enclosed area} \propto \Omega^2 \longrightarrow$ we need a state-dependent force

State-dependent forces for entangling gates

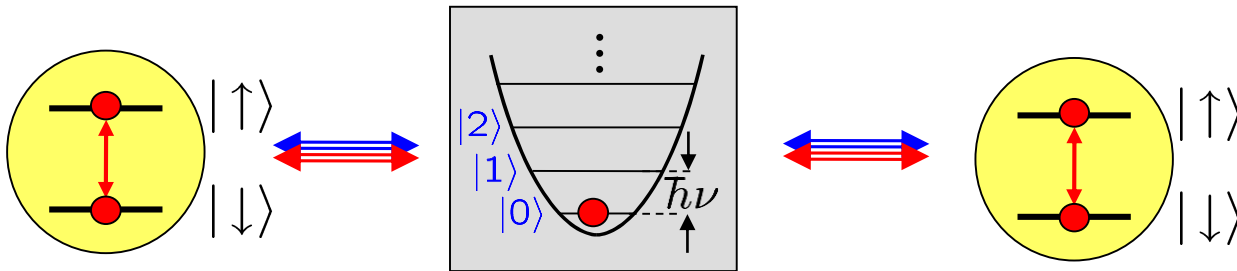


Bichromatic excitation

$$\omega_{laser} = \omega_0 \pm (\nu + \delta)$$

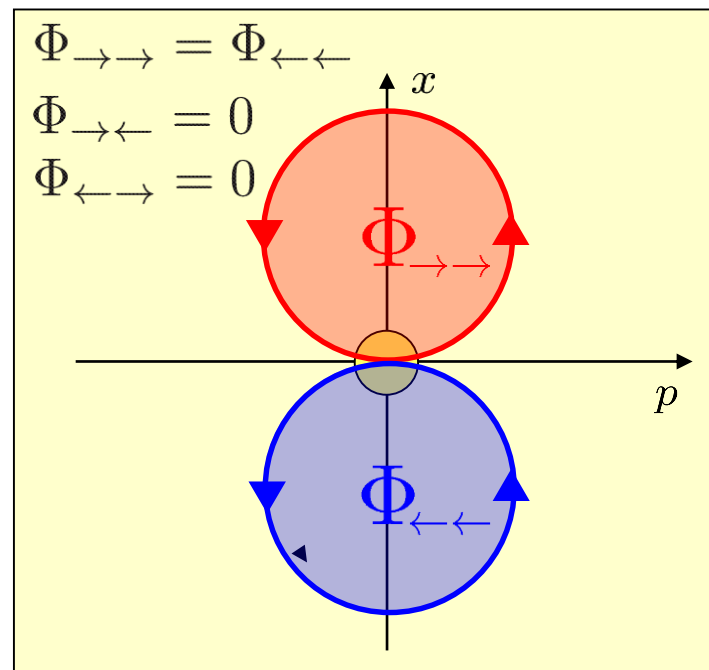
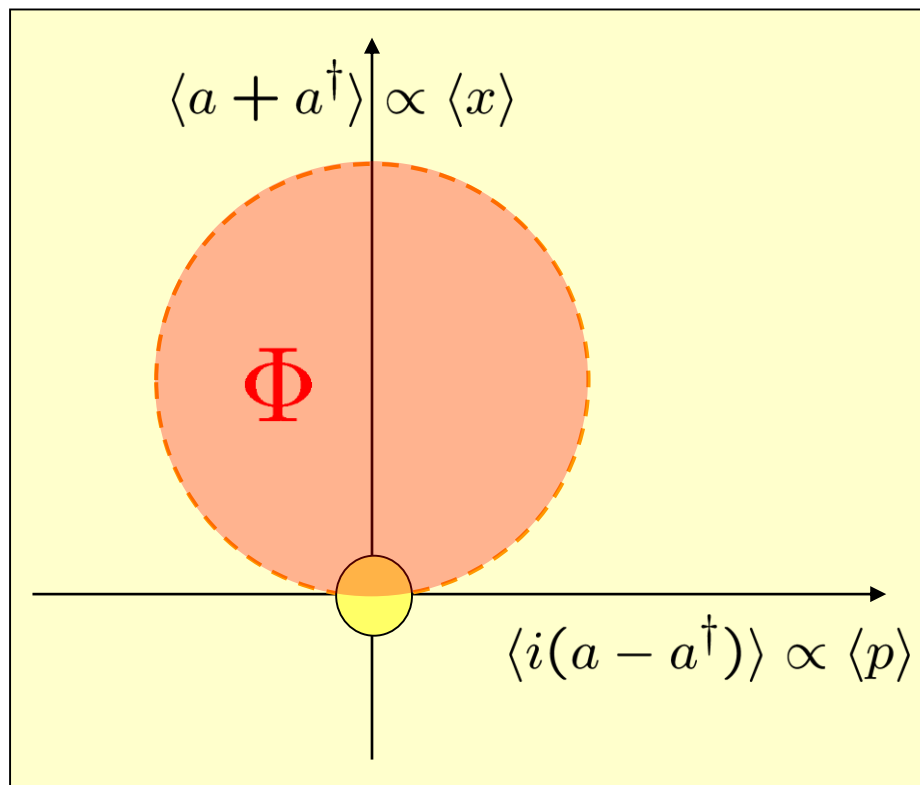
$$H \propto (ae^{i\delta t} + a^\dagger e^{-i\delta t})\sigma_x$$

Two ions: $H \propto (ae^{i\delta t} + a^\dagger e^{-i\delta t})(\sigma_x^{(1)} + \sigma_x^{(2)})$



Geometric phase gate

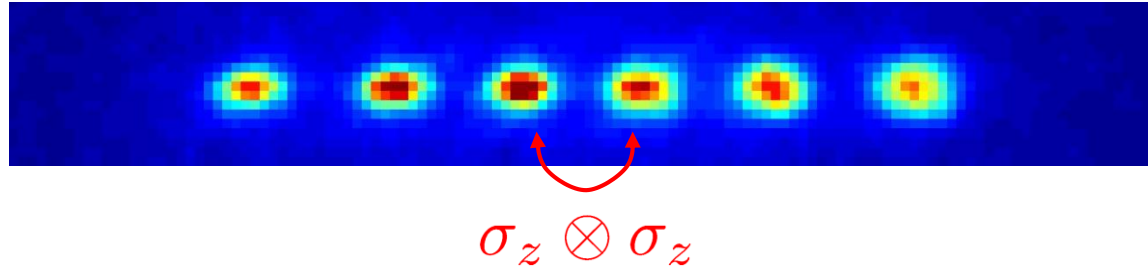
$$H(t) \propto (ae^{i\delta t} + a^\dagger e^{-i\delta t})(\sigma_x^{(1)} + \sigma_x^{(2)})$$



$$H_{eff} = \hbar\Omega\sigma_x^{(1)}\sigma_x^{(2)}$$

—→ The phase Φ depends nonlinearly on the internal states of the ions

Spin-spin interactions by different approaches



- Direct state-dependent forces between the ions (as in molecules) ?

Reduce the ion-ion distance to a_0 ? Impossible!

Make the ions bigger ? Difficult, but possible. Rydberg ions !

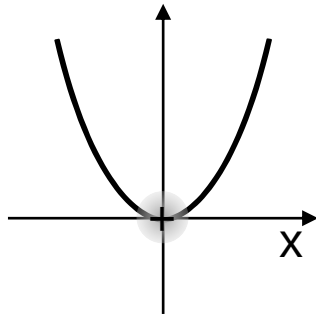
- Use external fields for engineering state-dependent forces

Spin-spin interactions mediated by Coulomb interaction

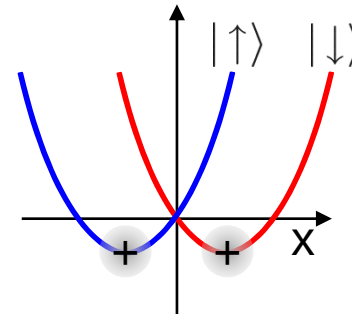
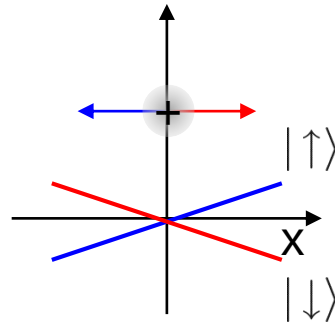
- (a) Laser interactions: Absorption and stimulated emission transfer momentum!
- (b) Magnetic field gradients: Position-dependent Zeeman shifts

Spin-spin couplings by magnetic field gradients

Trapping potential



Zeeman energy

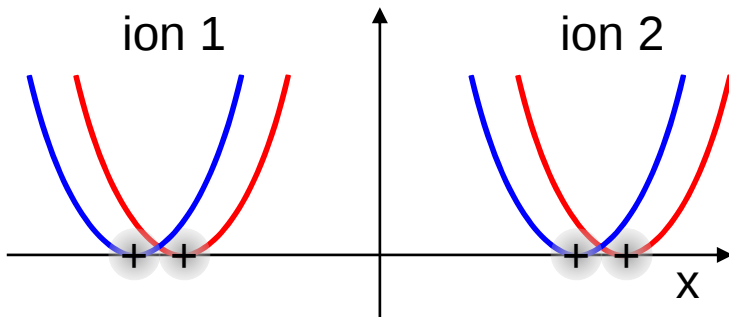


$$H = \hbar\nu a^\dagger a + \underbrace{\mu B' \hat{x} \sigma_z / 2}_{\propto (a + a^\dagger) \sigma_z} \quad \text{spin-dependent force}$$

In a magnetic field gradient, any microwave-induced spin flip also couples to the vibrational state.

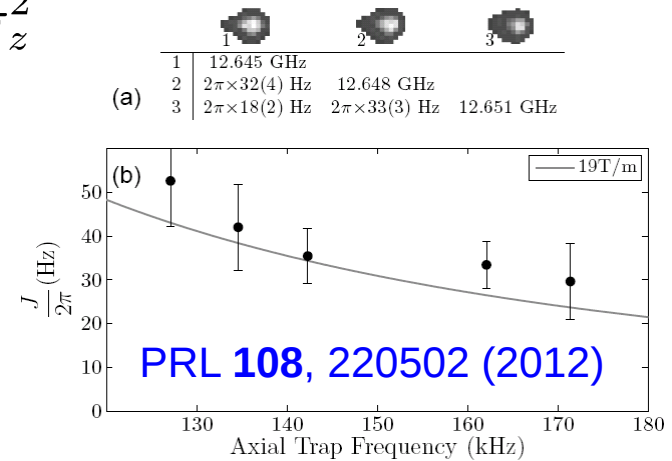
Spin-spin couplings by magnetic field gradients

(a) Ion crystals in a static field gradient



Potential energy (trap + Coulomb energy) depends on both internal states

$$H = J\sigma_z^1\sigma_z^2$$

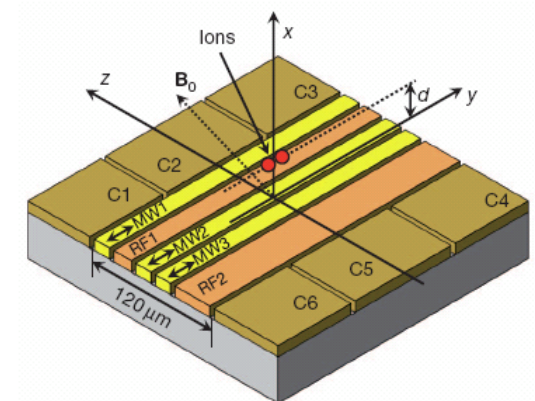


Wunderlich (Siegen)

(b) Ion crystals in a oscillating field gradient

$$H(t) \propto (ae^{-i\delta t} + a^\dagger e^{i\delta t})\sigma_z \longrightarrow H_{eff} = J\sigma_z^1\sigma_z^2$$

Off-resonant excitation of a vibrational mode with a state-dependent potential



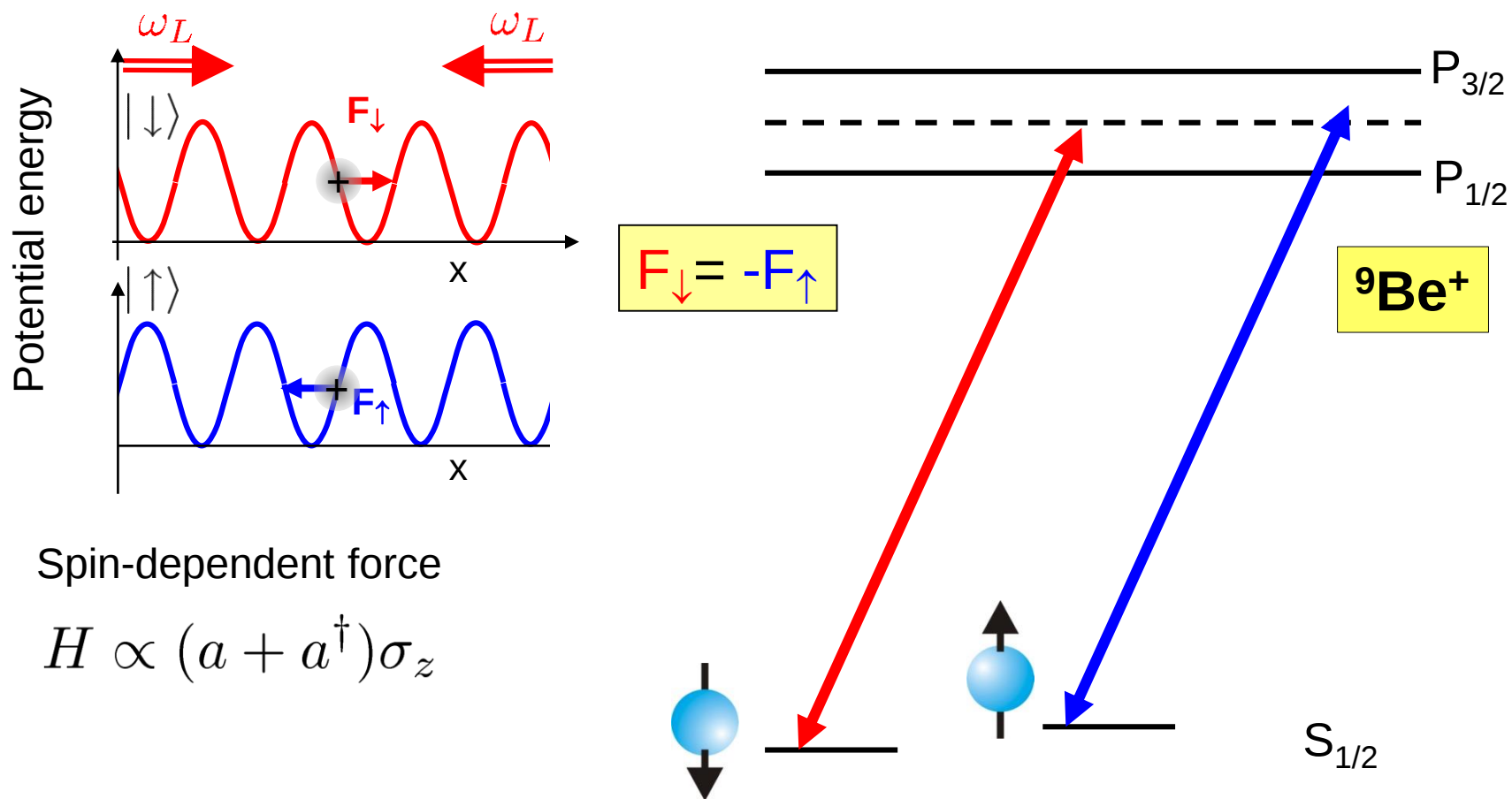
Wineland (NIST Boulder)

C. Ospelkaus et al., Nature **476**, 181 (2011)

Spin-spin couplings by laser-induced potentials

1. Conditional phase shift gate

Spatial light shifts by off-resonant coupling to P-states in a standing wave



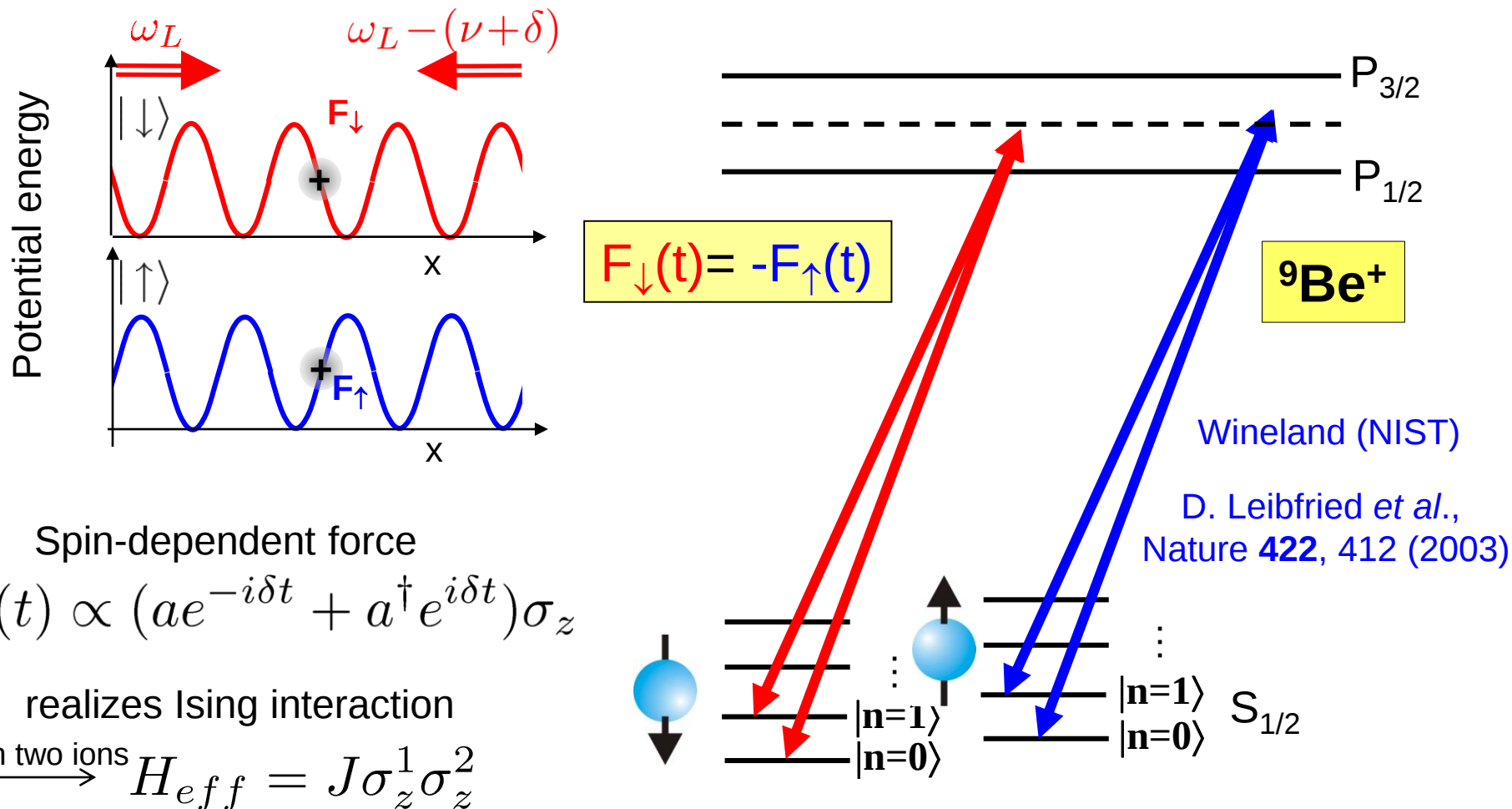
Spin-dependent force

$$H \propto (a + a^\dagger)\sigma_z$$

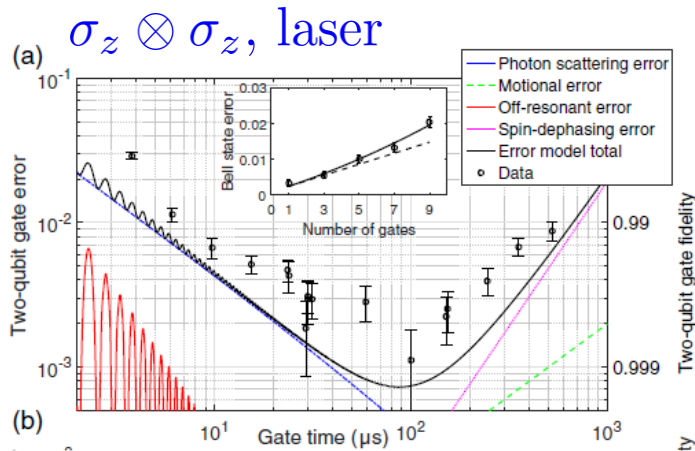
Spin-spin couplings by laser-induced potentials

Conditional phase shift gate

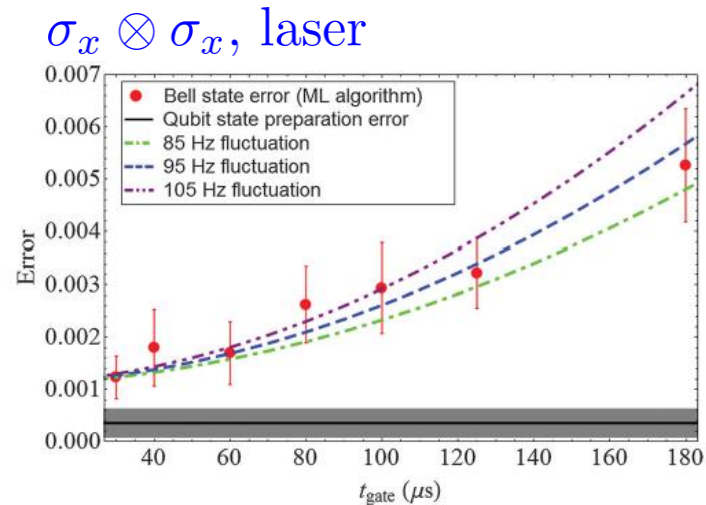
Spatial light shifts by off-resonant coupling to P-states in a standing wave
Frequency shifting one beam creates a moving standing wave.



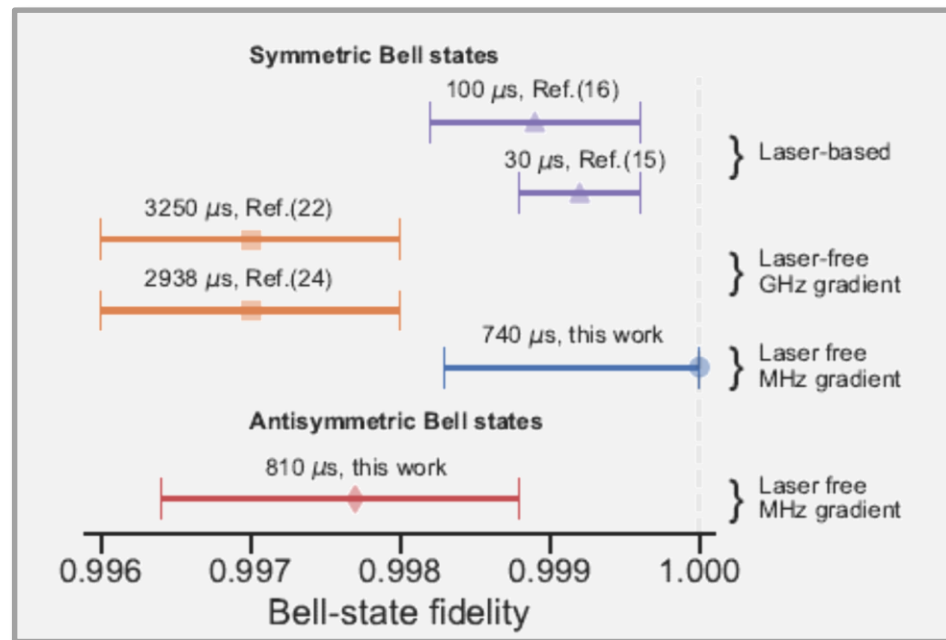
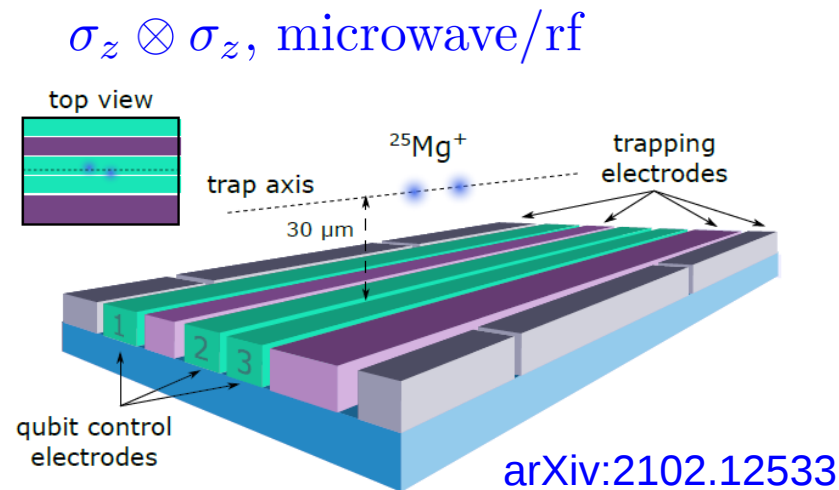
High-fidelity entangling quantum gates



PRL 117, 060504 (2016)

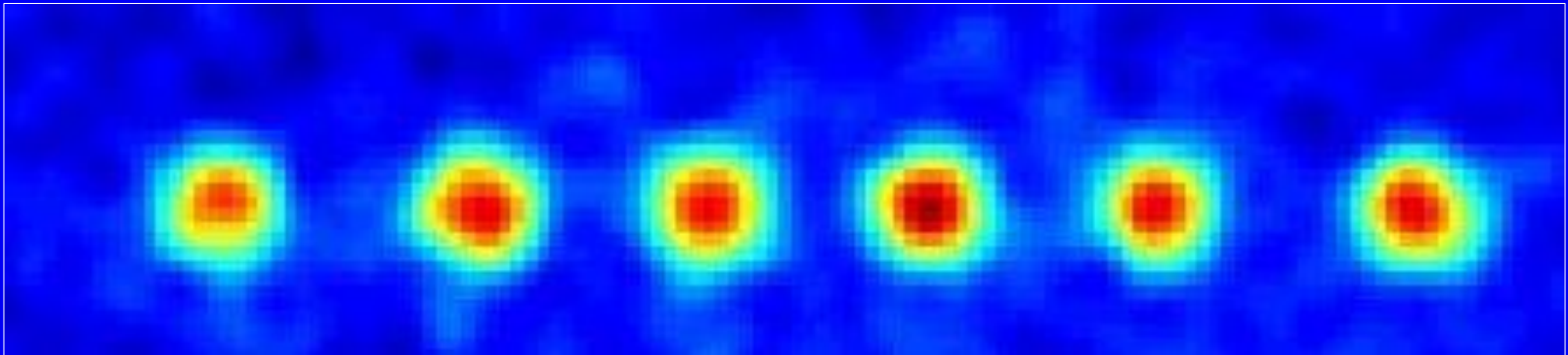


PRL 117, 060505 (2016)

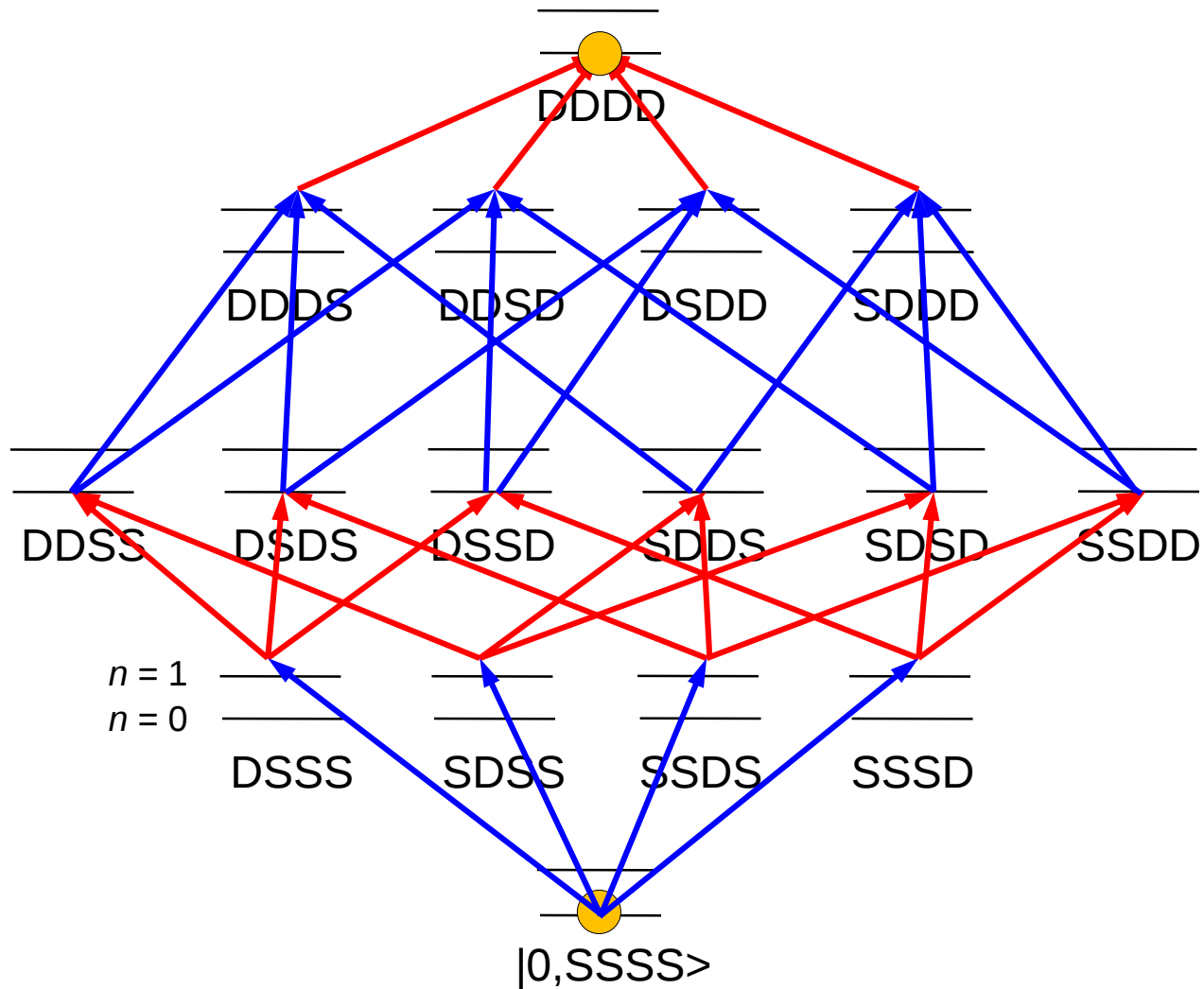


Quantum physics with more than 2 ions

**Scaling the system up for
gate-based quantum computation**

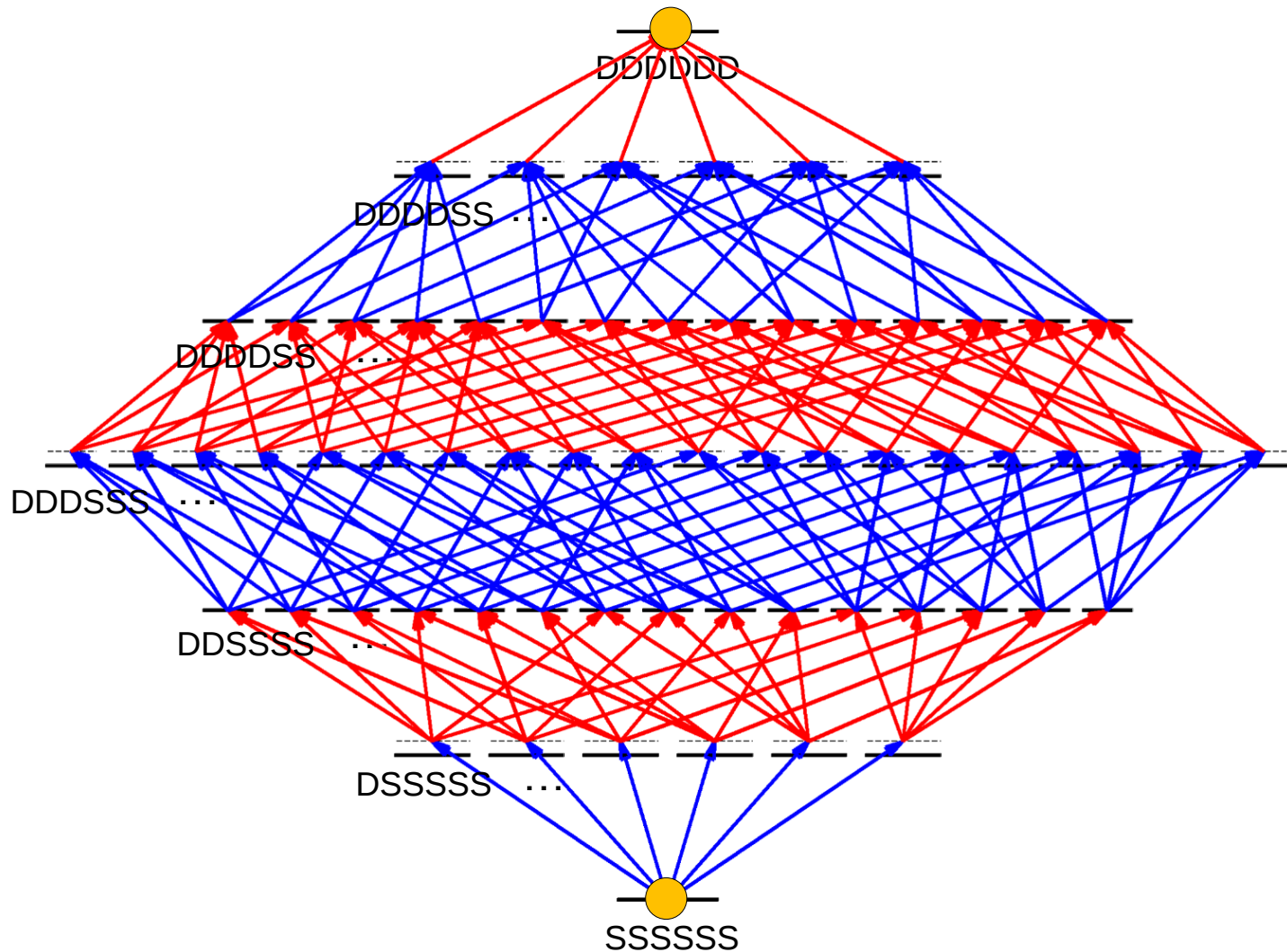


Creating GHZ-states with 4 ions



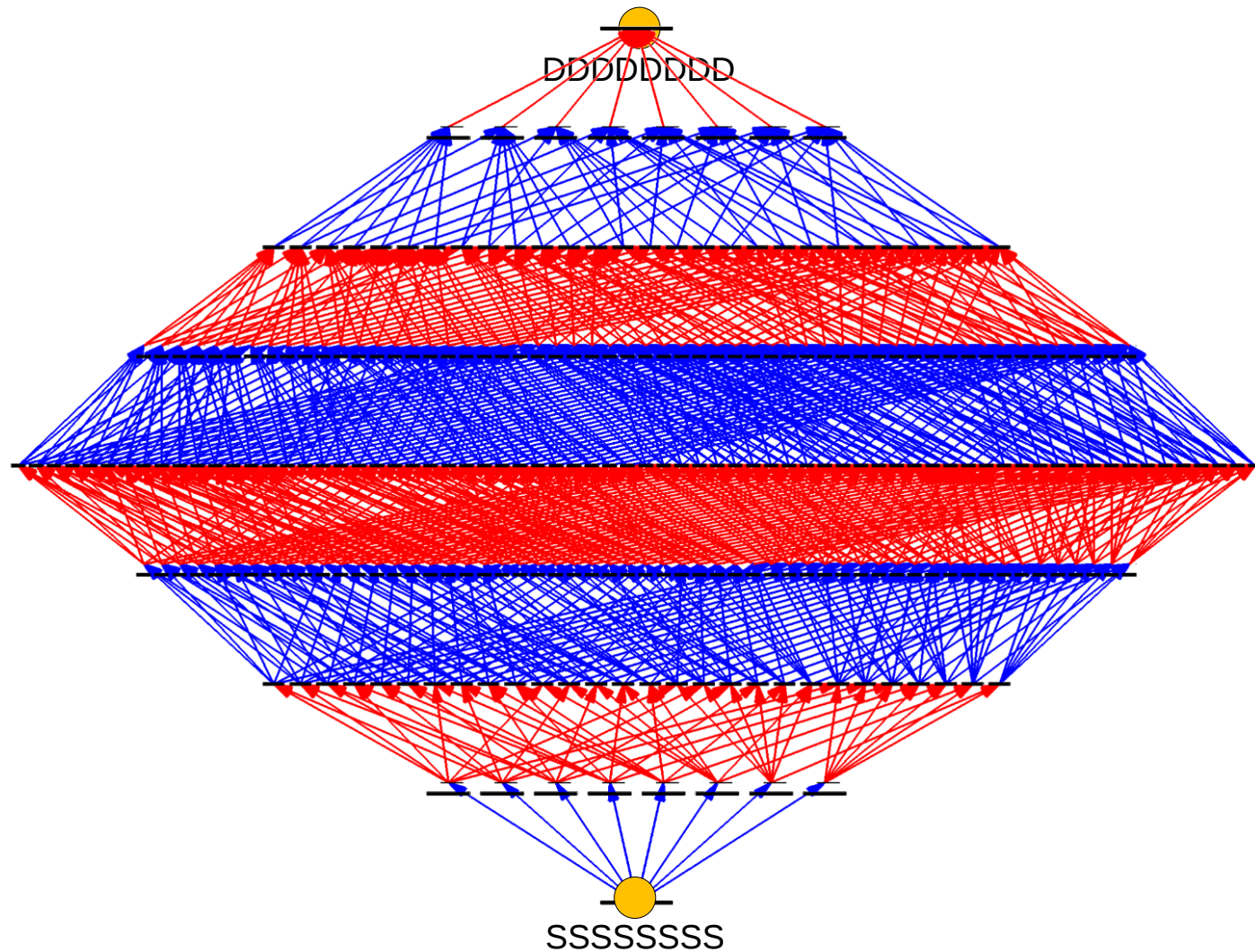
$$|SSSS\rangle \longrightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

Creating GHZ-states with 6 ions



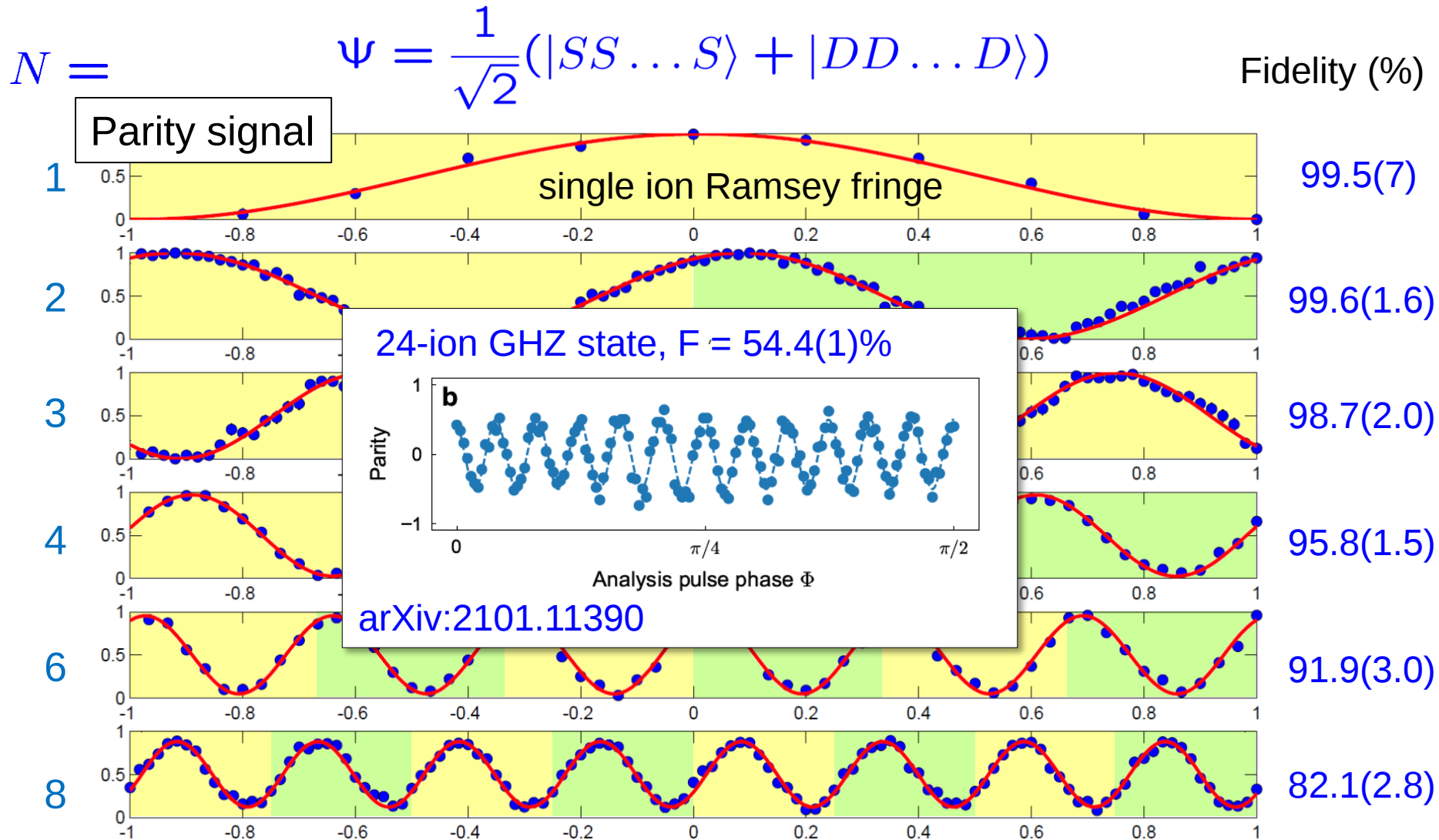
$$|SSSSSS\rangle \longrightarrow (|SSSSSS\rangle + |DDDDDD\rangle)/\sqrt{2}$$

Creating GHZ-states with 8 ions

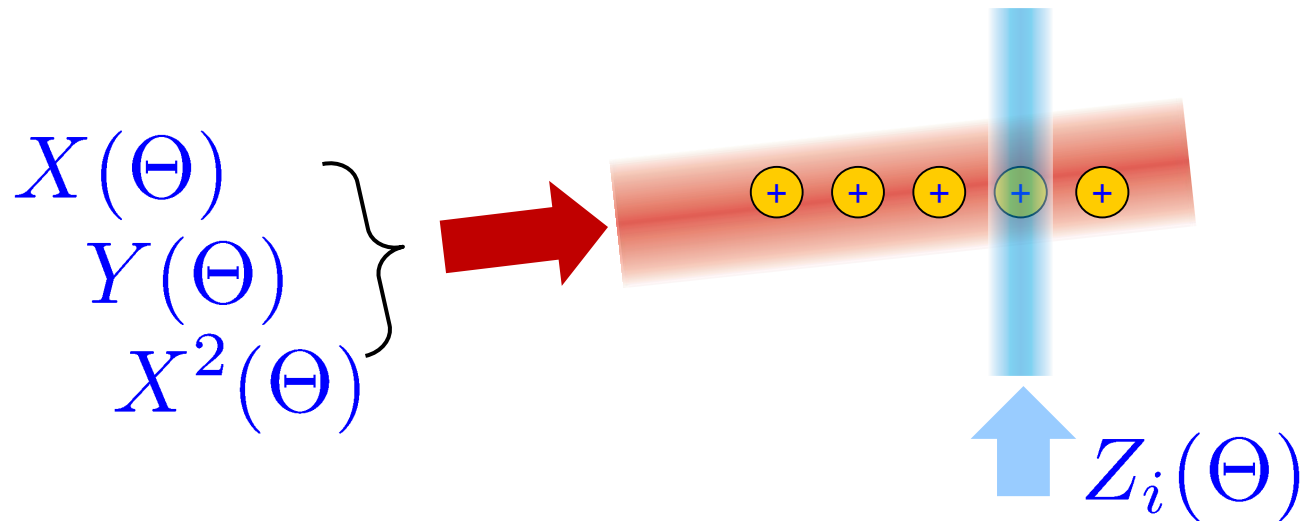


$$|SSSSSSSSS\rangle \longrightarrow (|SSSSSSSSS\rangle + |DDDDDDDDDD\rangle)/\sqrt{2}$$

N - qubit GHZ state generation



Quantum gate operations: universal toolbox



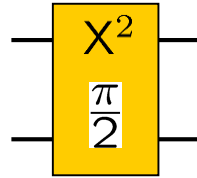
$X(\Theta)$ } collective local operations
 $Y(\Theta)$ }

$Z_i(\Theta)$ single-qubit z-rotations

$X^2(\Theta)$ Mølmer-Sørensen entangling operation

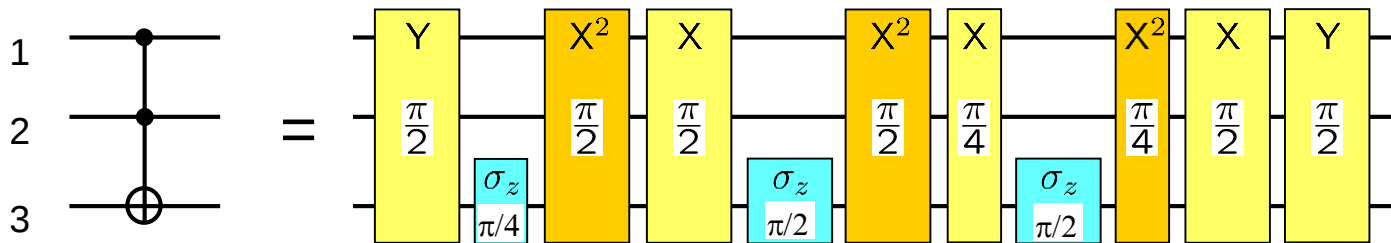
Entangling gates for quantum algorithms

Entangling gate



Building block for realizing quantum algorithms

Example: quantum Toffoli gate



V. Nebendahl *et al.*, PRA **79**, 012312 (2009)

Current experiments: 2 to 11 qubits, > 100 gate operations

K. Wright *et al.*, Nat. Commun. 10, 5464 (2019), A. Erhardt *et al.*, Nature 589, 220 (2021)

Scaling the ion trap quantum processor ...

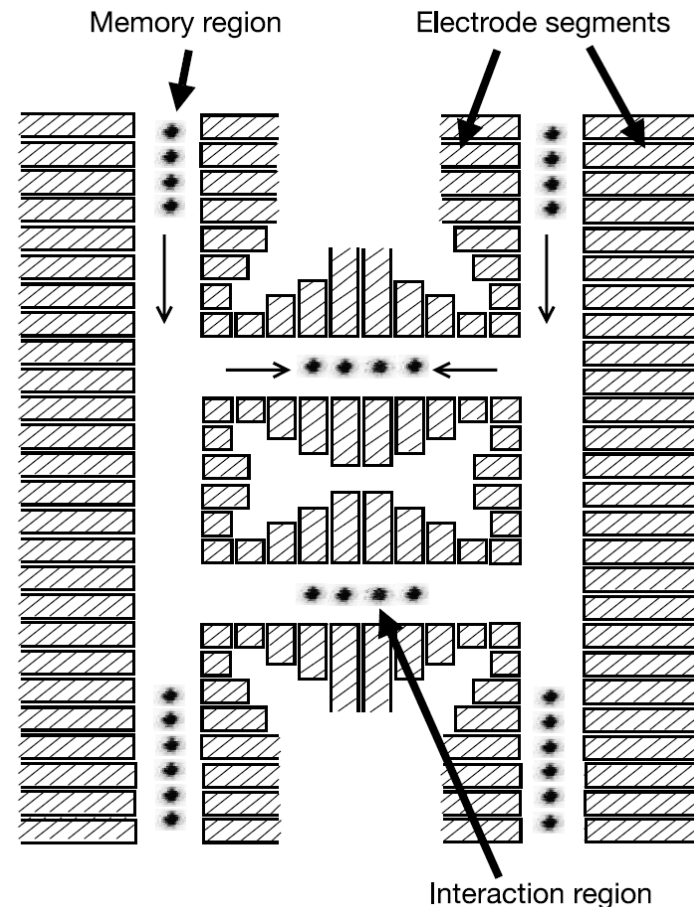
Making ion strings longer and longer is infeasible. Alternatives:

- put ions into arrays of traps, ions couple by exchange of phonons
- move ions, carry quantum information around

Kielpinski et al.,
Nature **417**, 709 (2002)

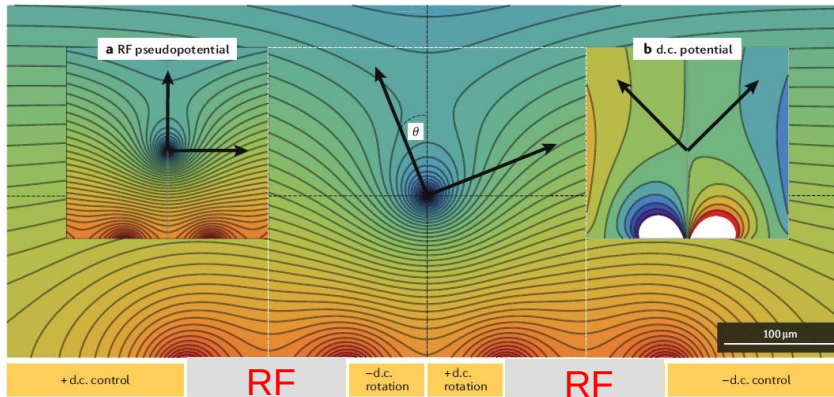
requires small,
integrated trap structures,

miniaturized optics
and electronics



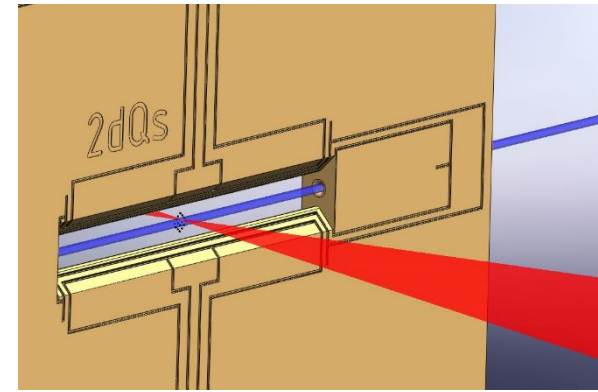
Microfabricated ion traps

Surface ion traps



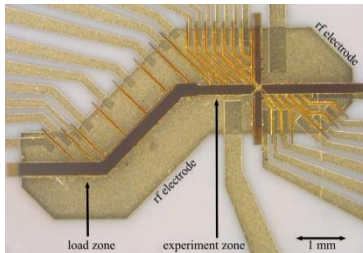
Nature Review Physics 2, 285 (2020)

Monolithic 3d-trap



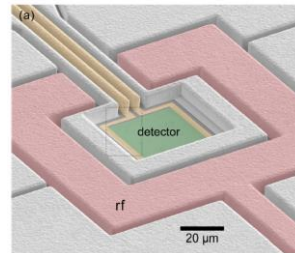
Innsbruck, 2020

Traps with junctions



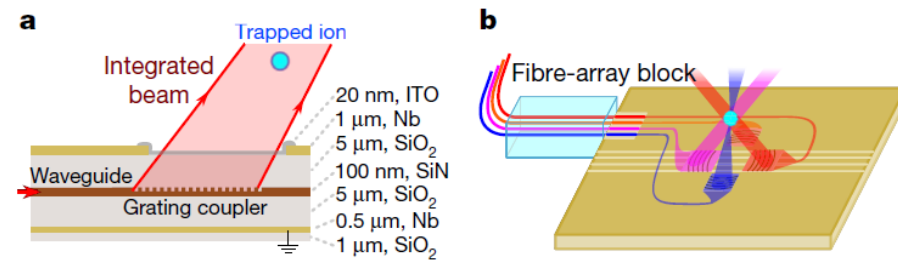
NIST Boulder

Integrated photon detectors



Opt. Express 25, 8705 (2017)

Integrated optics



Nature 586, 538 (2020)

Further features:

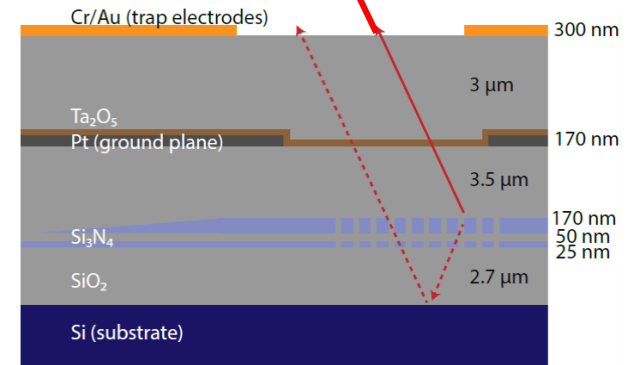
High optical access, thick metal layers, CMOS-compatible traps, backside loading,...

Optical wiring for trapped-ion quantum computers

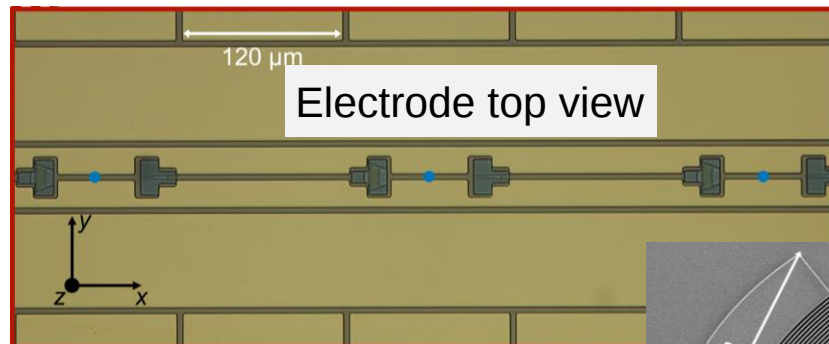
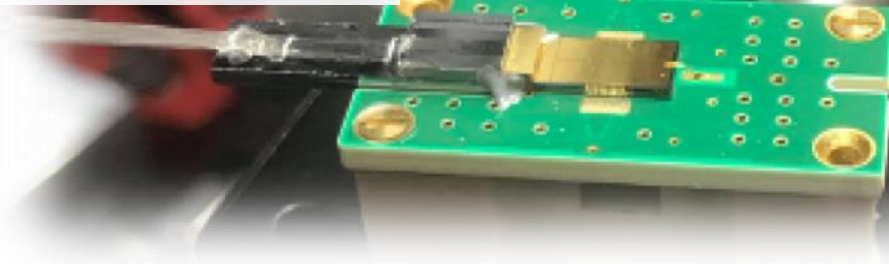
- MIT/Lincoln labs: Single-qubit gates: K. Mehta et al. Nature Nano 11, 1066 (2016)
- ETH Zürich: Two-qubit gates: K. Mehta et al. Nature 586, 533 (2020)



Light to ion
50 micron above
surface

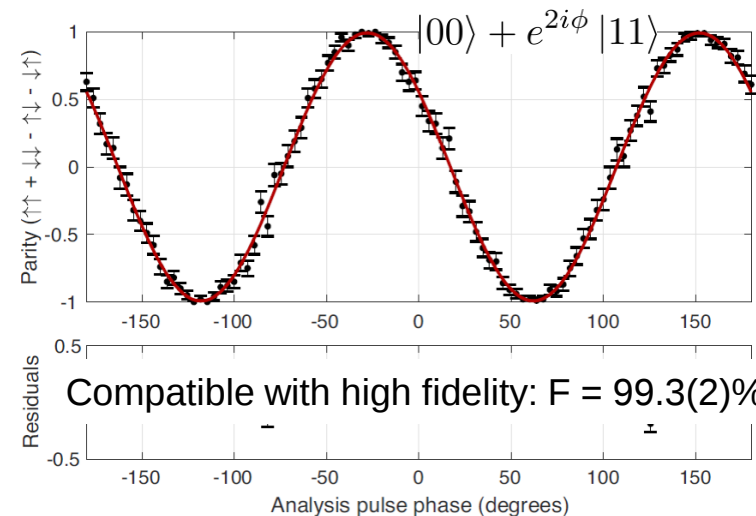
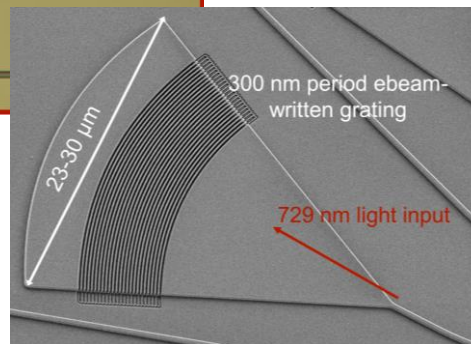


8-channel fibre array



Electrode top view

e-beam written gratings
for focussed light



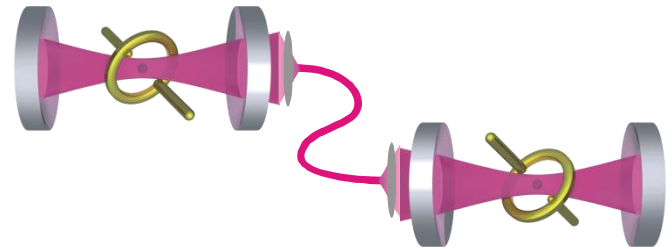
Scaling the ion trap quantum computer: Coupling ions to photons

Cavity QED: atom – photon interface, use photons for networking

J. I. Cirac et al., PRL **78**, 3221 (1997)

T. Northup, B. Lanyon, Univ. Innsbruck

More experiments: JQI, Sussex, Bonn, Duke, Sandia,
ETH, Saarbrücken, SK telecom...



- Idea:
- Spontaneously emitted photons can become entangled in polarization/frequency with internal states of an ion
 - Putting an ion into a high-finesse cavity can give rise to highly directional spontaneous emission
 - Remote entanglement between ions by (i) joint measurements of emitted photons, or (ii) coupling the photon into a second ion-cavity system

References

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Ion Coulomb crystals

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Laser cooling

S. Stenholm, *Rev. Mod. Phys.* 56, 699 (1986)

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Quantum physics with trapped ion

D. Wineland et al., *J. Res. Natl. Inst. Stand. Technol.* 103, 259 (1998)

D. Leibfried et al., *Rev. Mod. Phys.* 75, 281 (2003)

R. Blatt, D. J. Wineland, *Nature* 453, 1008 (2008)

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J. I. Cirac, P. Zoller, *Phys. Rev. Lett.* 74, 4091 (1995)

A. Sorensen, K. Molmer, *Phys. Rev. A* 62, 022311 (2000)

D. Leibfried et al., *Nature* 422, 412 (2003)

Trapped-ion quantum computing

N. R. Brown et al, *npj Quantum Information* 2, 16034 (2016)

A. Bermudez et al., *Phys. Rev. X* 7, 041061 (2017)