Formation of Kerr black holes from self-gravitating Bose-Einstein condensates of ultra-light dark bosons

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Introduction

- Galaxies harbour supermassive black holes (SMBHs) in their nuclear regions.
- Central Engines of bright quasars, at redshifts $\gtrsim 6$, have SMBHs of mass $\gtrsim 10^9 M_\odot$ at the time when the universe was barely $\sim 10^9$ yrs old.
- The ultra-luminous quasar SDSS J0100+2802 (redshift, z=6.3):

  Estimated mass of the SMBH $\cong 1.2 \times 10^{10}\ M_\odot$

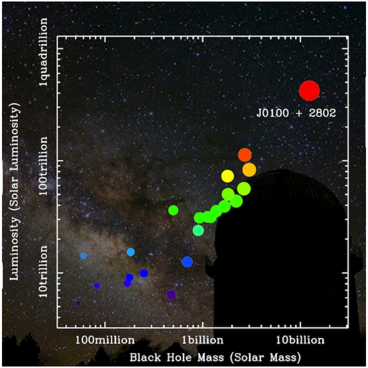

Figure: Image Credit:arstechnica.com/science/2015/03/black-hole-is-the-most-massive-discovered-in-the-early-universe/?comments=1&post=28592469/ Zhaoyu Li (Shanghai Astronomical Observatory) Background image: Yunnan Observatories

- Scenarios for SMBH formation:
- Black hole (BHs) seeds from remnants of Pop III stars

  (Carr, Bond & Arnett, 1984; Madau & Rees, 2001; Volonteri & Rees, 2005; ...)

- Direct collapse models

  (Haehnelt & Rees, 1993; Loeb & Rasio, 1994; Bromm & Loeb, 2003;...)

- Collapse of rotating gas clouds in dark matter halos, bar-instability and cooling by neutrino emission

  (Begelman, Volonteri & Rees, 2006;...)

  Caveats: Disposal of angular momentum; Metal free gas; Suppression of the formation of molecular H;...

- Astrophysical Structures from Self-gravitating BECs of light scalars/pseudo-scalars:

  (Kaup, 1968; Ruffini & Bonazzola, 1969; Khlopov, Malomed & Zeldovich,1985; Kolb & Tkachev, 1993, 1994;...)

- A natural question: Can such structures lead to formation of BHs?

  A likely path: CDM halos made of very light axion-like particles (ALPs) $\Rightarrow$

  A fraction of light ALPs of the DM halo that have undergone Bose-Einstein condensation (BEC) $\Rightarrow$

  Gravitational collapse $\Rightarrow$ SMBHs

- We had explored this possibility: (PDG & Eklavya Thareja, 2017)

- Dynamical evolution of self-gravitating BEC was studied using Gross-Pitaevskii equation (PDG, 2015; Chakrabarty, Enomoto, Han, Sikivie, & Todarello, 2018)

- Self-gravitating BEC of ultra-light ALPs with rest mass m indeed can lead to Schwarzschild SMBHs:

$$m \; M_{SMBH} \gtrsim 0.64 \; m_{Pl}^2 \;\; \Rightarrow \;\; m \gtrsim 8.5 \times 10^{-21} \left(\frac{M_{SMBH}}{10^{10} \; M_{\odot}} \right) \text{eV}$$

- What about generating Kerr SMBHs from ultra-light ALP-BEC?

(PDG & Fazlu Rahman, 2018; Fazlu Rahman & PDG, 2018)

- Event Horizon size:

$$R_{BH} = \frac{R_s}{2} + \sqrt{\left(\frac{R_s}{2}\right)^2 - \left(\frac{L}{Mc}\right)^2} \, .$$

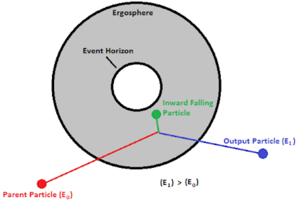

Figure: Image
Credit:http://large.stanford.edu/courses/2011/ph240/nagasawa2/

Simple estimates:

A condensate of mass M can form on a scale R_h provided:

$$\lambda_{DB} \sim \frac{h}{p} \gtrsim \left(\frac{3N}{4\pi R_h^3}\right)^{-1/3} = R_h \left(\frac{3M}{4\pi m}\right)^{-1/3}$$

The condensate can have an initial angular momentum $\Leftarrow$ Tidal torques on the DM halo so that the energy of a single boson:

$$E \sim \frac{p^2}{2m} + \frac{l^2}{2mR_h^2} - \frac{GMm}{R_h} \sim \frac{\hbar^2}{8mR_h^2} + \frac{n^2\hbar^2}{2mR_h^2} - \frac{GMm}{R_h}$$

where $p \sim \Delta p \sim \hbar/(2R_h)$ and $l \sim n\hbar, \; n = 1, 2, \ldots$

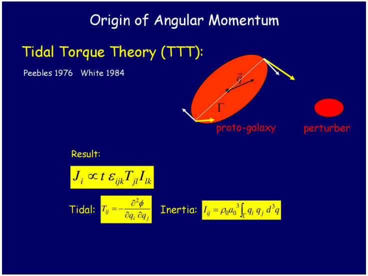

Figure: Image Credit:https://slideplayer.com/slide/13795823/

Minimum energy configuration $\Rightarrow$

$$R_{h0} = \left(n^2 + \frac{1}{4}\right)\frac{\hbar^2}{GMm^2} \cong 86 \left(n^2 + \frac{1}{4}\right)\left(\frac{10^9 M_\odot}{M}\right)\left(\frac{10^{-22}\ \text{eV}}{m}\right)^2 \ \text{pc}$$

Gravitational perturbation (e.g. galaxy-galaxy interaction) $\Rightarrow$ Size of the condensate would oscillate about R_{h0}.

Characteristic frequency ω:

$$\omega = \sqrt{\frac{GM}{R_{h0}^3}} \ \Rightarrow \ \tau \cong 2\times 10^6\,(n^2 + 1/4)^{3/2}\left(\frac{m}{10^{-22}\ \text{eV}}\right)^{-3}\left(\frac{M}{10^9\ M_\odot}\right)^{-2} \text{yrs}$$

• Gravitational potential due to a rotating BEC would not be spherically symmetric

• Undulations in the ALP-BEC triggered by galactic encounters $\Rightarrow$ Gravitational potential can vary on time scales of $\sim 10^6$ yrs in a region of size $\lesssim 100$ pc $\Rightarrow$ Collisions of Giant Molecular Clouds $\Rightarrow$

• Very rapid creation of stars, on a time scale of $\sim 10^7$ yrs, confined to nuclear regions of size $\lesssim 100$ pc

• $\Rightarrow$ Star burst galaxies

ALP-BEC will implode to form a BH if its size R_{h0} is less than the Kerr EH radius:

$$R_{BH} = \frac{R_s}{2} + \sqrt{\left(\frac{R_s}{2}\right)^2 - \left(\frac{L}{Mc}\right)^2}\ .$$

Schwarschild Radius:

$$R_s = \frac{2GM}{c^2}$$

Angular Momentum:

$$L = Nn\hbar = \left(\frac{M}{m}\right) n\hbar$$

$$R_{h0} = \left(n^2 + \frac{1}{4}\right)\frac{\hbar^2}{GMm^2}$$

$$R_{h0} < R_{BH} \;\Rightarrow$$

$$m\; M_{SMBH} \gtrsim \frac{n^2 + 1/4}{\sqrt{n^2 + 1/2}}\; m_{Pl}^2$$

For n=1, the above result implies,

$$m\; M_{SMBH} \gtrsim 1.02\; m_{Pl}^2$$

$$m\; M_{SMBH} \gtrsim 1.02\; m_{Pl}^2 \;\Rightarrow\; m \gtrsim 1.3 \times 10^{-20} \left(\frac{M_{SMBH}}{10^{10}\; M_{\odot}} \right) \text{eV}$$

• We proceed to a more accurate description by studying the evolution of ALPs in the BEC phase using the framework of Gross-Pitaevskii equation.

• Bose-Einstein condensates:

For N weakly interacting, identical bosons at temperature $T \approx 0^\circ K \rightarrow$ Many body wavefunction $\Psi(\vec{r_1}, \vec{r_2}, .., \vec{r_N}, t)$ describing the condensate can be expressed as,

$$\Psi(\vec{r_1}, \vec{r_2}, .., \vec{r_N}, t) \cong \prod_{j=1}^{N} \psi(\vec{r_j}, t)$$

$\psi(\vec{r}, t)$ is the normalized ground state wavefunction for a single boson $\rightarrow$ $\psi(\vec{r}, t)$ acts as a macroscopic wavefunction for the condensate

• BEC critical temperatures for bosonic dark matter particles with rest mass less than ~ 1 eV always exceed the temperature of the Universe, and therefore such particles can undergo Bose condensation

(Fukuyama & Morikawa 2007, Das & Bhaduri 2015)

The dynamical evolution of the condensate wavefunction $\psi(\vec{r}, t)$ (normalized to unity) is governed by the Gross-Pitaevskii equation

- Gross-Pitaevskii equation (GPE):

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + N\int V(\vec{r}-\vec{u})|\psi(\vec{u},t)|^2 d^3u \right]\psi(\vec{r},t)$$

where,

$$V(\vec{r}-\vec{u}) = \frac{4\pi\hbar^2 a}{m}\delta^3(\vec{r}-\vec{u}) + V_g(|\vec{r}-\vec{u}|)$$

so that,

$$i\hbar\frac{\partial\psi}{\partial t} = \Bigg[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + Ng|\psi(\vec{r},t)|^2 + \\ +N\int V_g(|\vec{r}-\vec{u}|)|\psi(\vec{u},t)|^2 d^3u \Bigg]\psi(\vec{r},t)$$

where m is the dark matter boson mass and $g \equiv \frac{4\pi\hbar^2 a}{m}$.

• The above GPE can be derived by extremizing the following action: $S = \int dt \int d^3r \; \mathcal{L}$, where:

$$\mathcal{L} = \frac{i\hbar}{2}\left\{\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right\} + \frac{\hbar^2}{2m}\nabla\psi^*.\nabla\psi + V_{ext}\;|\psi|^2 +$$

$$+\frac{gN}{2}|\psi|^4 + \frac{N}{2}|\psi|^2\int V_g(|\vec{r}-\vec{u}|)|\psi(\vec{u},t)|^2 d^3u$$

where,

$$V_{ext}(r) = -\frac{GM_0 m}{r}$$

• Gravitational interaction between ultra-light dark matter particles:

$$V_g(r) = -\frac{Gm^2}{r} \quad .$$

The above action can also be expressed as.

$$S = \int dt \int d^3r \ \mathcal{L} = \int dt \int d^3r \ \psi^* \Big\{ - i\hbar \frac{\partial \psi}{\partial t} - \frac{\hbar^2}{2m} \nabla^2 \psi + V_{ext} \ \psi +$$

$$+ \frac{gN}{2} |\psi|^2 + \frac{N}{2} \int V_g(|\vec{r} - \vec{u}|) |\psi(\vec{u}, t)|^2 d^3u \Big\} \psi$$

• Time dependent Variational Method with trial wavefunction

(Perez-Garcia et al. 1996, 1997, PDG 2015)

• We obtain approximate but analytical solution of GPE with $g = 0$ using variational method:

$$\psi(\vec{r}, t) = A(t) \ r \ \exp\left(-r/\sigma(t)\right) \exp\left(-iB(t) \ r\right) \ Y_{lm}(\theta, \phi)$$

• From the normalization condition, $A(t)$ and $\sigma(t)$ are related by,

$$|A(t)|^2 = \frac{4}{3} (\sigma(t))^{-5} \Rightarrow A(t) = \frac{2}{\sqrt{3}} (\sigma(t))^{-5/2}$$

- The Lagrangian:

$$L = \int d^3r\ \mathcal{L} = -\ \frac{5}{2}\hbar\sigma\dot{B} + \frac{\hbar^2}{2m}B^2 + \frac{\hbar^2}{2m\sigma^2} + L_{int}\ -\ \frac{GM_0 m}{2\sigma}$$

The self-gravity term L_{int} is given by,

$$L_{int} \equiv \frac{N}{2}\int d^3r|\psi(\vec{r},t)|^2 \int V_g(|\vec{r}-\vec{u}|)|\psi(\vec{u},t)|^2 d^3u$$

If we choose $l = 1$ and $m = 1$ so that.

$$\psi(\vec{r},t) = A(t)\ r\ \ \exp\left(-r/\sigma(t)\right)\exp\left(-iB(t)\ r\right)\ Y_{11}(\theta,\phi)$$

then,

$$L_{int} = -\ \frac{0.37NGm^2}{2\sigma}$$

Euler-Lagrange equations:

$\frac{d}{dt}(\partial L/\partial \dot{q}_j) - (\partial L/\partial q_j) = 0$, for j=1 and 2, with $q_1 \equiv B$ and $q_2 \equiv \sigma$:

$$B(t) = -\ \frac{5m\ \dot{\sigma}}{2\hbar}$$

and,

$$\frac{5}{2}\hbar\dot{B} + \frac{\hbar^2}{m\sigma^3} - \frac{G[0.37Nm + M_0]m}{2\sigma^2} = 0$$

The above two equations can be combined to give,

$$m\ddot{\sigma} = -\frac{dV_{eff}}{d\sigma}$$

where,

$$V_{eff} \equiv \frac{2}{25}\left[\frac{\hbar^2}{m\sigma^2} - \frac{G(0.37Nm + M_0)m}{\sigma}\right]$$

$$\Rightarrow \frac{1}{2}m\dot{\sigma}^2 + V_{eff} = \text{Constant} \equiv K_0$$

$\Rightarrow$

$$\dot{\sigma} = \pm \frac{2}{5} \sqrt{\frac{G \bar{M}}{a} - \frac{\hbar^2}{m^2 \sigma^2} + \frac{25}{2} K_0}$$

where,

$$\bar{M} \equiv 0.37 Nm + M_0$$

If we choose $l = 1$ and $m = 1$ so that.

$$\psi(\vec{r}, t) = A(t)\ r\ \exp\left(-r/\sigma(t)\right) \exp\left(-iB(t)\ r\right)\ Y_{11}(\theta, \phi)$$

then,

$$L_{int} = -\ \frac{0.37 N G m^2}{2\sigma}$$

The time evolution of the trial wavefunction is determined by specifying the initial data $\sigma(t_i)$ and $\dot{\sigma}(t_i)$ at time t_i.

• Scenario: A large number N of dark bosons with $\sim$ zero momentum and initially spread over a very large scale $\sim$ 20 - 30 kpc evolve quantum mechanically

If initially at time $t = t_i$ when $\sigma_i \equiv \sigma(t_i) \approx 25$ kpc and $\dot{\sigma}(t_i) = \ -\epsilon$ where $\epsilon \approx 0$ then,

$$K_0 \approx 0$$

$$\Rightarrow t - t_i = \frac{5}{3}\sqrt{\frac{\sigma_i^3}{G\bar{M}}}[(1 - \frac{\hbar^2}{G\bar{M}m^2\sigma_i})^{3/2} - (\frac{\sigma(t)}{\sigma_i} - \frac{\hbar^2}{G\bar{M}m^2\sigma_i})^{3/2}] -$$

$$-\frac{2\hbar^2}{G\bar{M}m^2\sigma_i}[(1 - \frac{\hbar^2}{G\bar{M}m^2\sigma_i})^{1/2} - (\frac{\sigma(t)}{\sigma_i} - \frac{\hbar^2}{G\bar{M}m^2\sigma_i})^{1/2}]$$

The turning point occurs at σ_{min} when $\dot{\sigma}$ =0 so that,

$$\sigma_{min} = \frac{\hbar^2}{G\bar{M}m^2}$$

After the bounce at the turning point, in general, $\sigma(t)$ starts increasing again.

- Total mass enclosed within a sphere of radius R at time t:

$$M_0 + M_{bec}(< 2\sigma(t), t) = Nm\int_0^R |\psi(r,t)|^2 d^3r$$

$$= \frac{4Nm}{3(\sigma(t))^5}\int_0^R r^4 \exp\left(-2r/\sigma(t)\right)dr \cong 0.37Nm + M_0 = \bar{M}$$

The corresponding angular momentum of the condensate:

$$\bar{L} = N\int_0^{2\sigma(t)}\int_0^{2\pi}\int_0^{\pi} |\psi(r,t)|^2 d^3r = 0.37N\hbar \quad . \tag{1}$$

Then, the gravitational radius :

$$R_{BH} = \frac{G\bar{M}}{c^2} + \sqrt{\frac{G\bar{M}}{c^2} - (\frac{L}{\bar{M}c})^2} = \frac{G\bar{M}}{c^2}[1 + \sqrt{1 - (\frac{Lc}{G\bar{M}^2})^2}]$$

with,

$$L = 0.37N\hbar$$

so that,

$$R_{BH} = \frac{G\bar{M}}{c^2}[1 + \sqrt{1 - \frac{m_{pl}^4}{\bar{M}^2 m^2}(1 - M_0/\bar{M})^2}]$$

• As long as black hole is not formed, $\sigma(t)$ decreases steadily with time till it reaches the turning point,

• Criteria for the formation of black hole:

ALP condensate collapses to form a black hole when,

$$\sigma_{min} = \frac{\hbar^2}{G\bar{M}m^2} < R_{BH}$$

$$\Rightarrow m\ M_{SMBH} > \frac{m_{pl}^2}{\sqrt{1 - \frac{1}{4}(1 - M_0/M_{SMBH})^2}} \cong 1.15 m_{pl}^2$$

$$m\ M_{SMBH} \gtrsim 1.15\ m_{Pl}^2\ \ \Rightarrow\ \ m \gtrsim 1.5 \times 10^{-20} \left(\frac{M_{SMBH}}{10^{10}\ M_{\odot}}\right) \text{eV}$$

Very Light ($m \sim 10^{-20}$ eV) ALPs can lead to SMBHs of mass $\sim 10^{10}\ M_\odot$

Collapse Time Scale,

$$t - t_i \cong \frac{5}{3}\sqrt{\frac{\sigma_i^3}{G\bar{M}}} \approx 10^8\ -\ 10^9\ \text{yrs}$$

The lower bound of m can be smaller if the mass $\bar{M}$ of the SMBH formed is bigger

• Possibility of very low frequency gravitational radiation during the collapse of the ALP-BEC:

Energy released in the collapse process,

$$\Delta E \approx \frac{GN^2 m^2}{\sigma_0}$$

Energy carried by gravitational radiation,

$$E_{GW} \approx \epsilon \, \frac{GN^2 m^2}{\sigma_0} \cong \epsilon \; 10^{64} \text{ erg}$$

over a time scale of,

$$\tau_{dyn} \cong 4 \times 10^8 \text{ yrs}$$

Average gravitational wave luminosity,

$$L_{GW} \approx \frac{E_{GW}}{\tau_{dyn}} = \epsilon \; 10^{48} \text{ erg/s}$$

This is comparable with the power radiated by strong quasars in the electromagnetic regime if ϵ is of the order unity.

SUMMARY:

- Ultra-light ALP-BEC can lead to:
- Star burst when the condensate is perturbed
- Formation of SMBHs
- Very low frequency gravitational waves
- But more detailed analysis is needed

Thank You