

How were the LIGO black holes created?



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Less travelled paths DM, ICTS, 11 November 2020

Special credit to my PhD students + Alex



Andrew Gow (current)



Pippa Cole
Just started in GRAPPA, Amsterdam



Sam Young
Humboldt Fellow at the MPA, Munich



Alex Hall
PDRA at Edinburgh Uni

Contents

- The optimistic take
Cosmic coincidence: The LIGO events, QCD transition, NANOGrav frequency/amplitude and DM density are all connected
- The pessimistic take
Model comparison disfavours primordial over stellar BHs
- A definitive PBH detection would be transformational

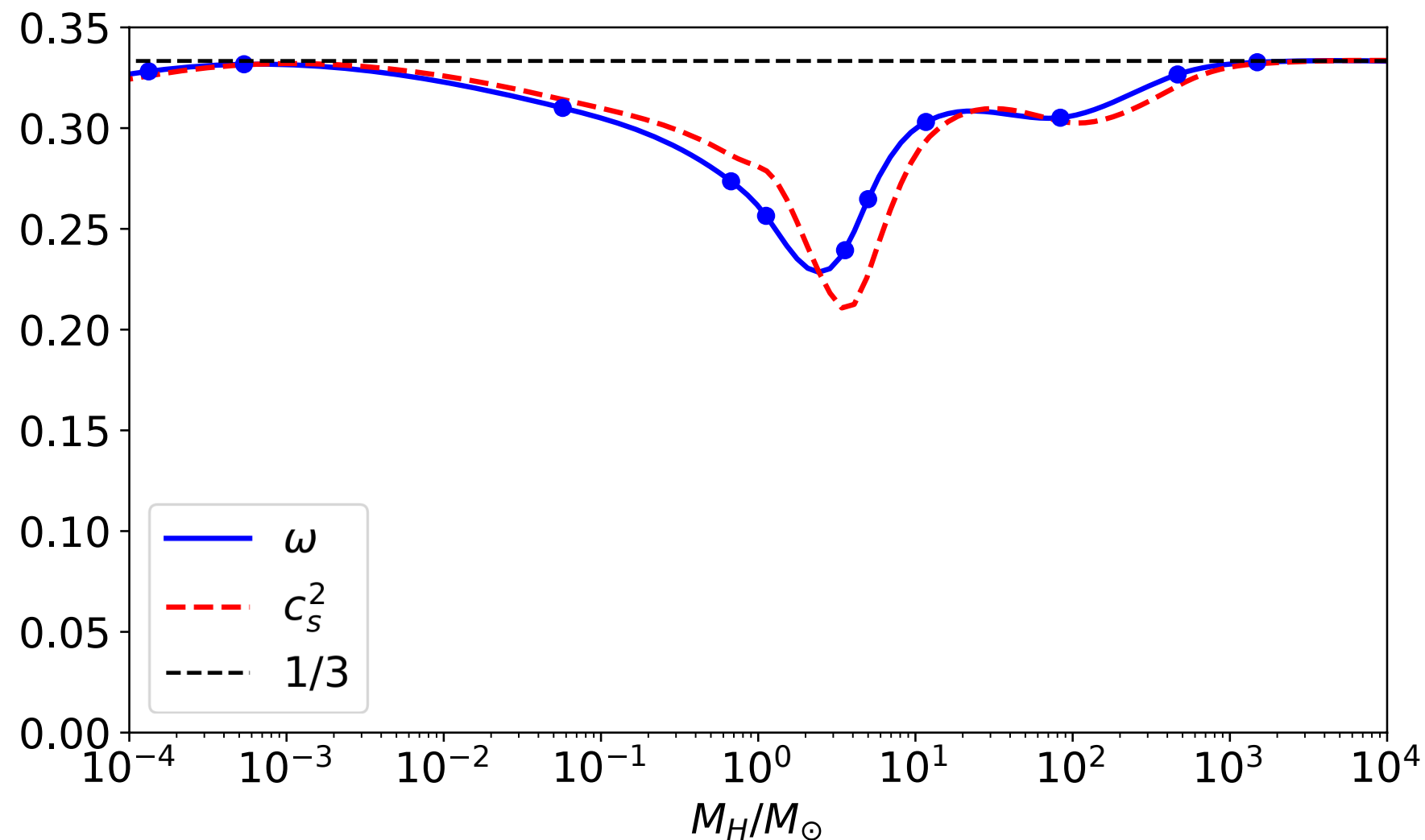
The LIGO events

- It appears unlikely that more than 1% of the dark matter can be made out of LIGO mass PBHs
- But all of the LIGO BHs could be primordial
- Black holes have no hair, so how can we know?
Total mass
Mass ratio
(Spin, redshift distribution and location)

The QCD transition

Strong interactions confine quarks into hadrons and the equation-of-state parameter w decreases. *Crawford & Schramm '82, Jedamzik '98*

QCD transition: $t \sim 10^{-6}$ s, $T \sim 200$ MeV, $M \sim 1 M_\odot$, $k \sim 10^7$ Mpc $^{-1}$

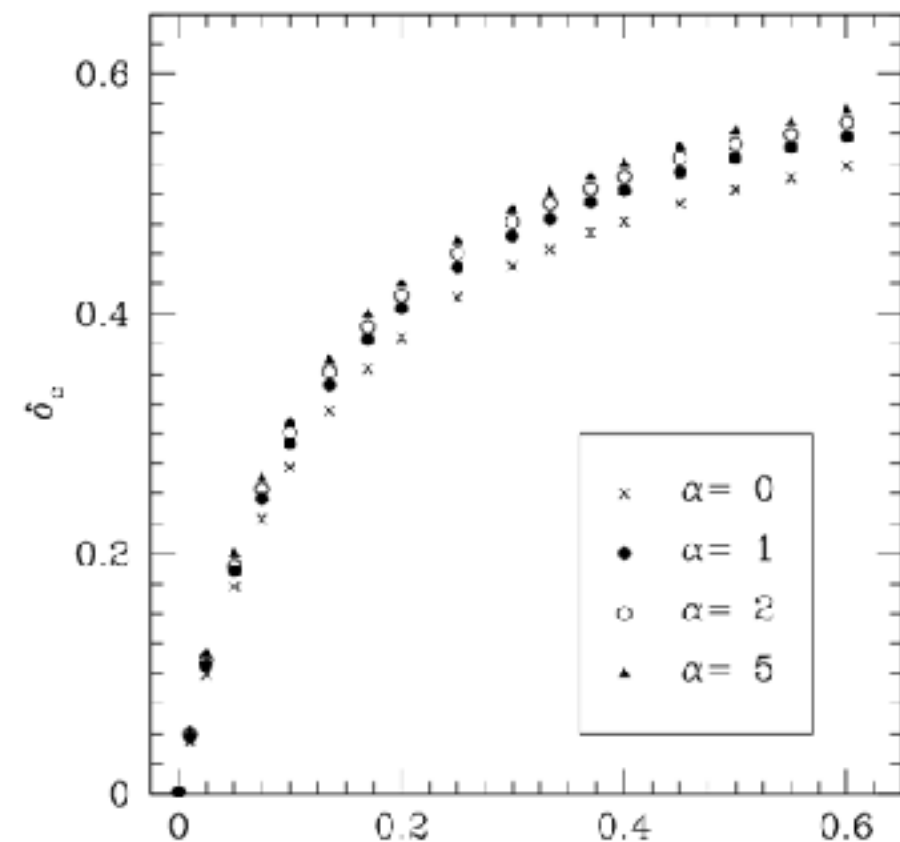


CB, Hindmarsh, Young & Hawkins 2018 using Borsanyi et al 2016

PBH formation

1. They could form from large amplitude density perturbations shortly after horizon entry
2. Causality prevents collapse before horizon entry
3. Approximate 1-to-1 relation between horizon entry time, horizon length and PBH mass

Collapse threshold

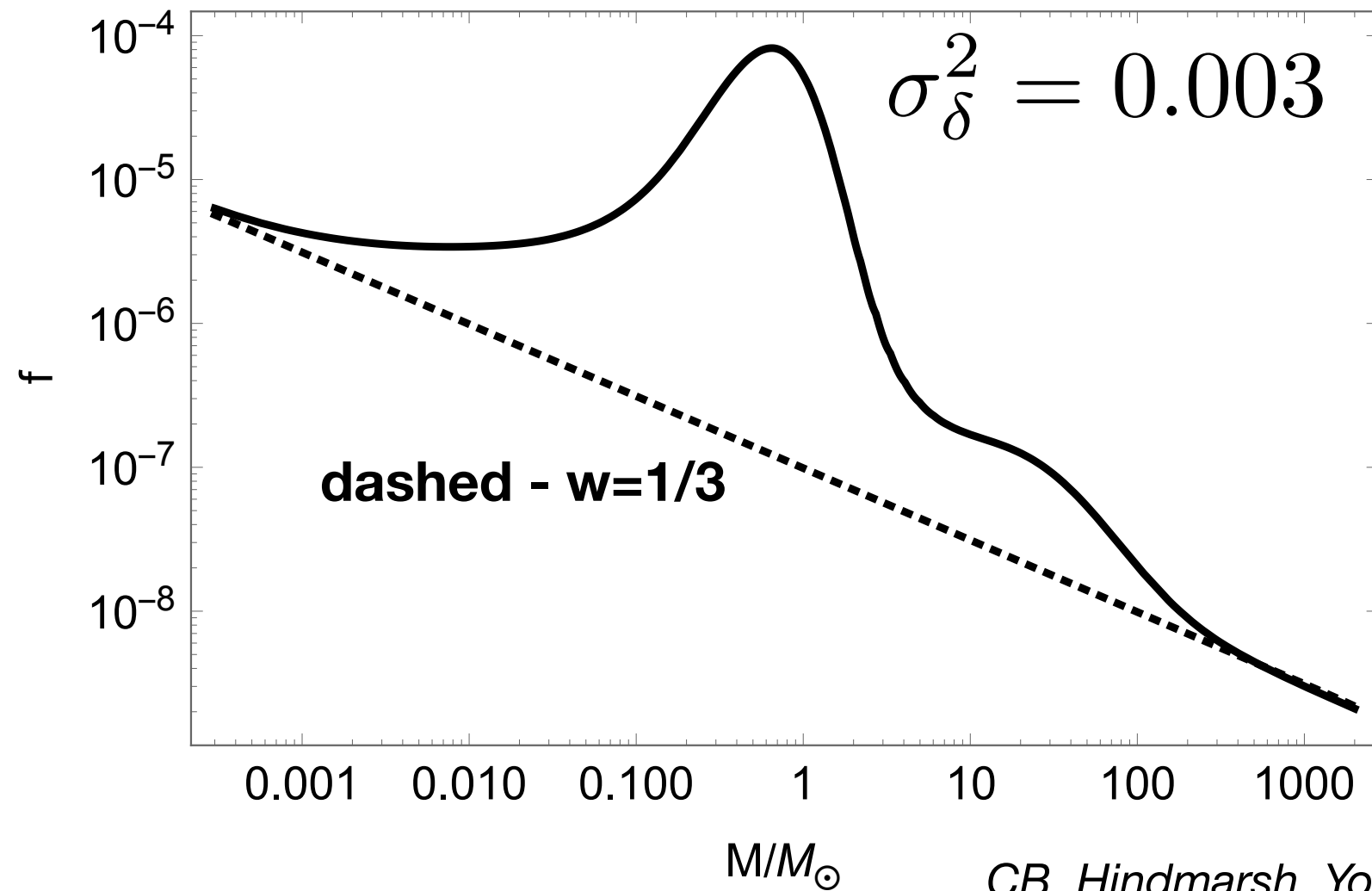


equation of state

Musco and Miller 2013

The resultant PBH-QCD mass function

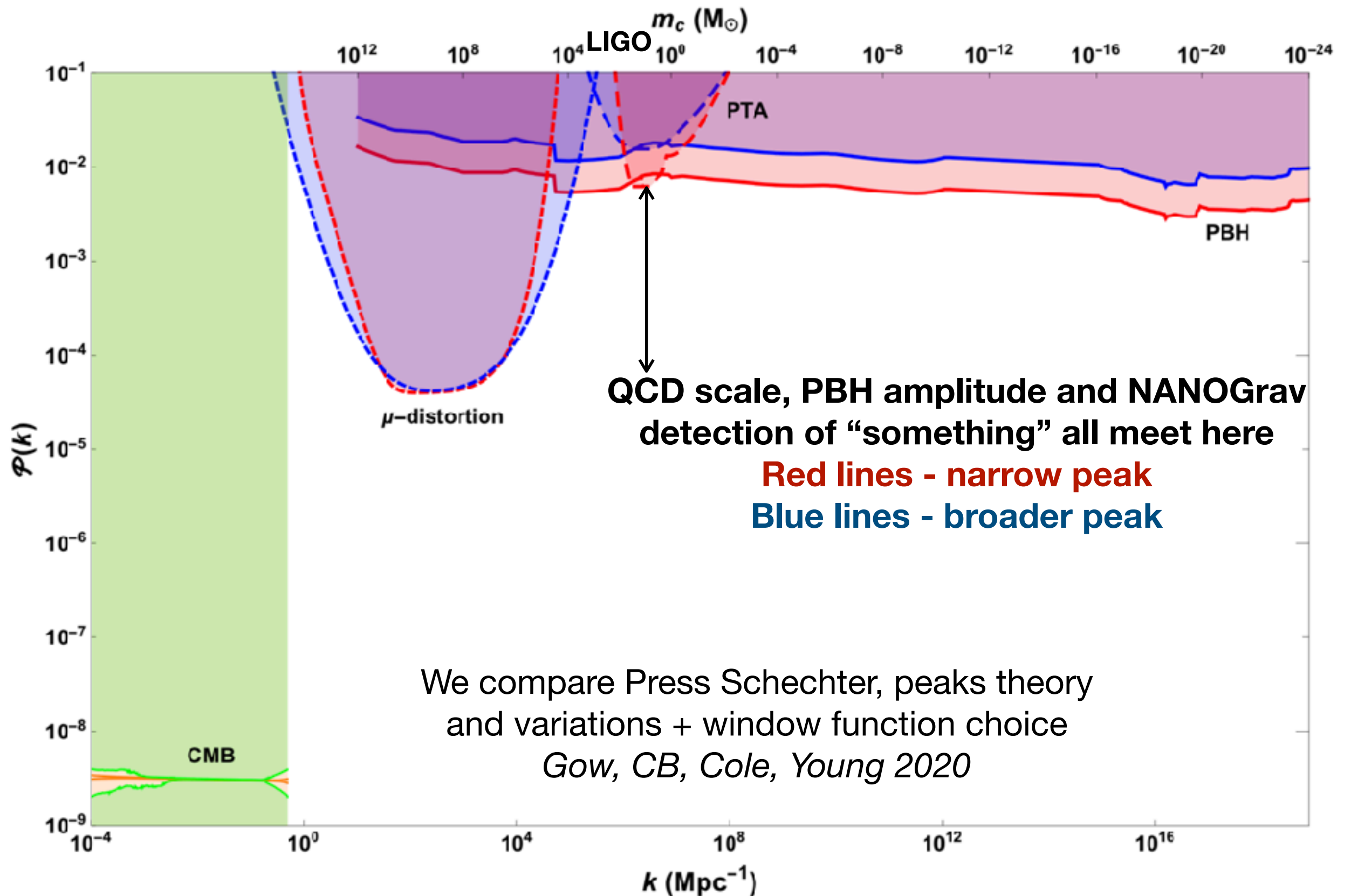
$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$



CB, Hindmarsh, Young & Hawkins 2018

The QCD phase transition took place during the time when LIGO mass PBHs would have formed. **It boosts the formation rate of solar mass PBHs by 2 orders of magnitude**
Extend to further dips in equation of state - explain microlensing, etc Carr et al '19
No detection: LIGO & Virgo collaboration 2019, Magee et al 2019

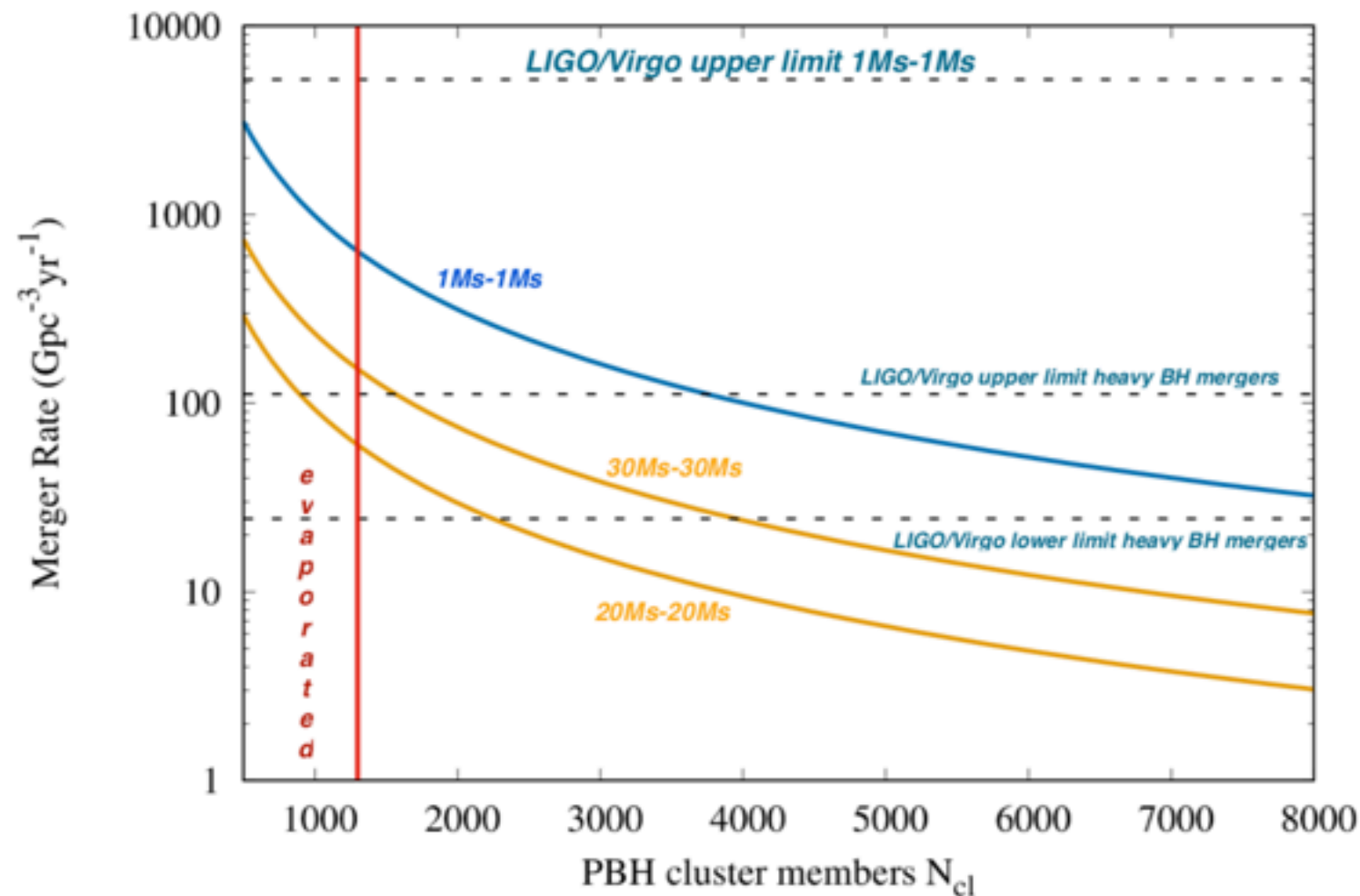
NANOGrav



The DM density as well?

- Perhaps there is a 4th coincidence - that $f_{\text{PBH}}=1$ does give the correct merger rate!
- In tightly bound clusters, PBH pairs can be frequently disrupted. Numerical evidence that this could lead to 5 orders of magnitude suppression... which is (maybe) what you need to have $f_{\text{PBH}}\sim 1$ and the correct LIGO merger rate - *Jedamzik 2020 (2 papers) - Also older work by Bird et al, Clesse and Garcia-Bellido, etc*
- The lensing constraints are also less reliable if PBHs cluster tightly
- Even if the results for f_{PBH} are “wrong”, the results others made for $f_{\text{PBH}}\ll 1$ should be reliable. Perhaps $f_{\text{PBH}}\sim 5\cdot 10^{-3}$ and ~ 1 give the same merger rate
- *See the talks by Karsten, Juan, Bernard, Surhud, etc*

In Karsten's own words



PBHs formed during the QCD epoch can (pre-) post-dict, the mass scale of $\sim 30 M_{\odot}$ for PBHs observed by LIGO/Virgo, the current merger rate of such PBHs observed by LIGO/Virgo, the current merger rate of light PBHs with heavy ones observed by LIGO/Virgo, and the current non-observation of mergers on the fundamental $\sim 1 M_{\odot}$ PBH scale. It may be that nature has not chosen this pathway, but if not, has confronted us with an astonishing coincidence.

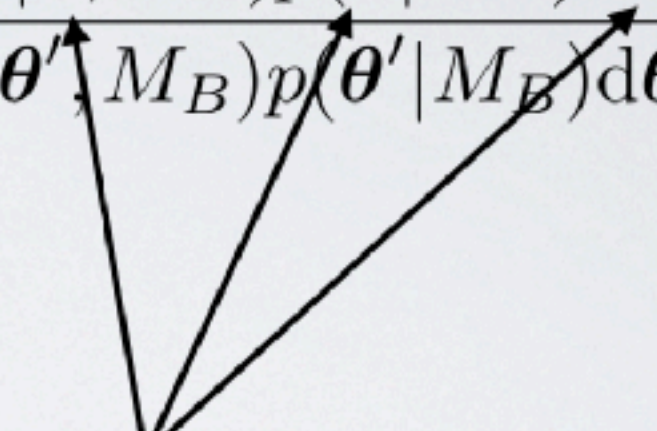
And now for something completely different

Advocates of the primordial black hole hypothesis still have a lot of convincing to do. Most physicists still believe that dark matter is made of some kind of elementary particle, one that's devilishly hard to detect. Moreover, the LIGO black holes aren't too different from what we would expect if they came from ordinary stars. "It sort of fills a hole in the theory that isn't actually there," said Carl Rodriguez, an astrophysicist at Carnegie Mellon University. "There are things that are weird about some of the LIGO sources, but we can explain everything that we've seen so far through normal stellar evolutionary process."

<https://www.quantamagazine.org/black-holes-from-the-big-bang-could-be-the-dark-matter-20200923/?fbclid=IwAR2GgveIVyYAEkvZTfitIVjAEgwnvTwpgp6TNHfNrtS1SHin6hX6Gy7L5BY>

Bayesian model comparison

The Bayesian evidence ratio

$$\frac{Z_A}{Z_B} \equiv \frac{p(M_A|\mathbf{d})}{p(M_B|\mathbf{d})} = \frac{p(M_A)}{p(M_B)} \frac{\int p(\mathbf{d}|\boldsymbol{\theta}, M_A) p(\boldsymbol{\theta}|M_A) d\boldsymbol{\theta}}{\int p(\mathbf{d}|\boldsymbol{\theta}', M_B) p(\boldsymbol{\theta}'|M_B) d\boldsymbol{\theta}'}$$


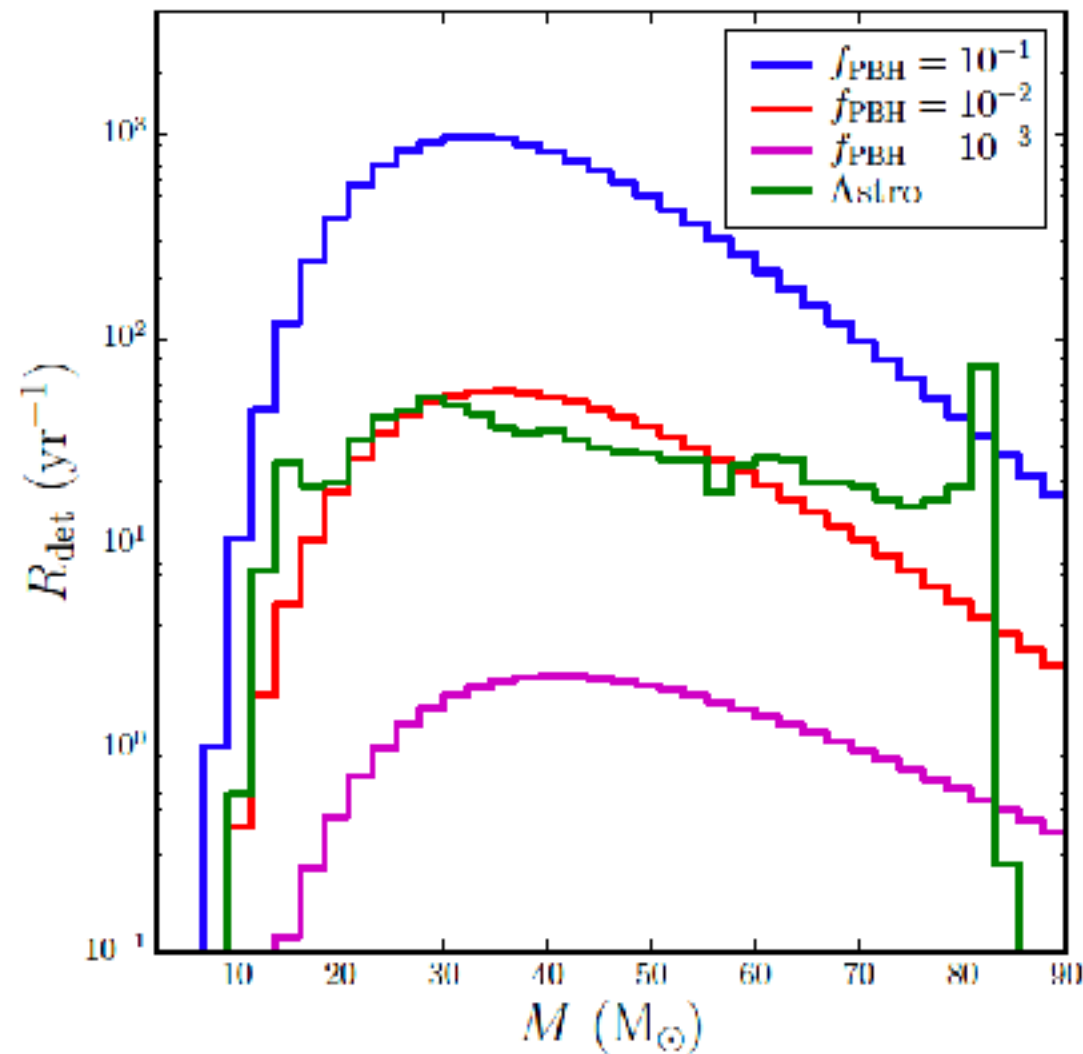
Population parameters, i.e. mass function parameters, PBH abundance etc.

Hall, Gow, CB, 2020: Bayesian comparison

Key ingredients

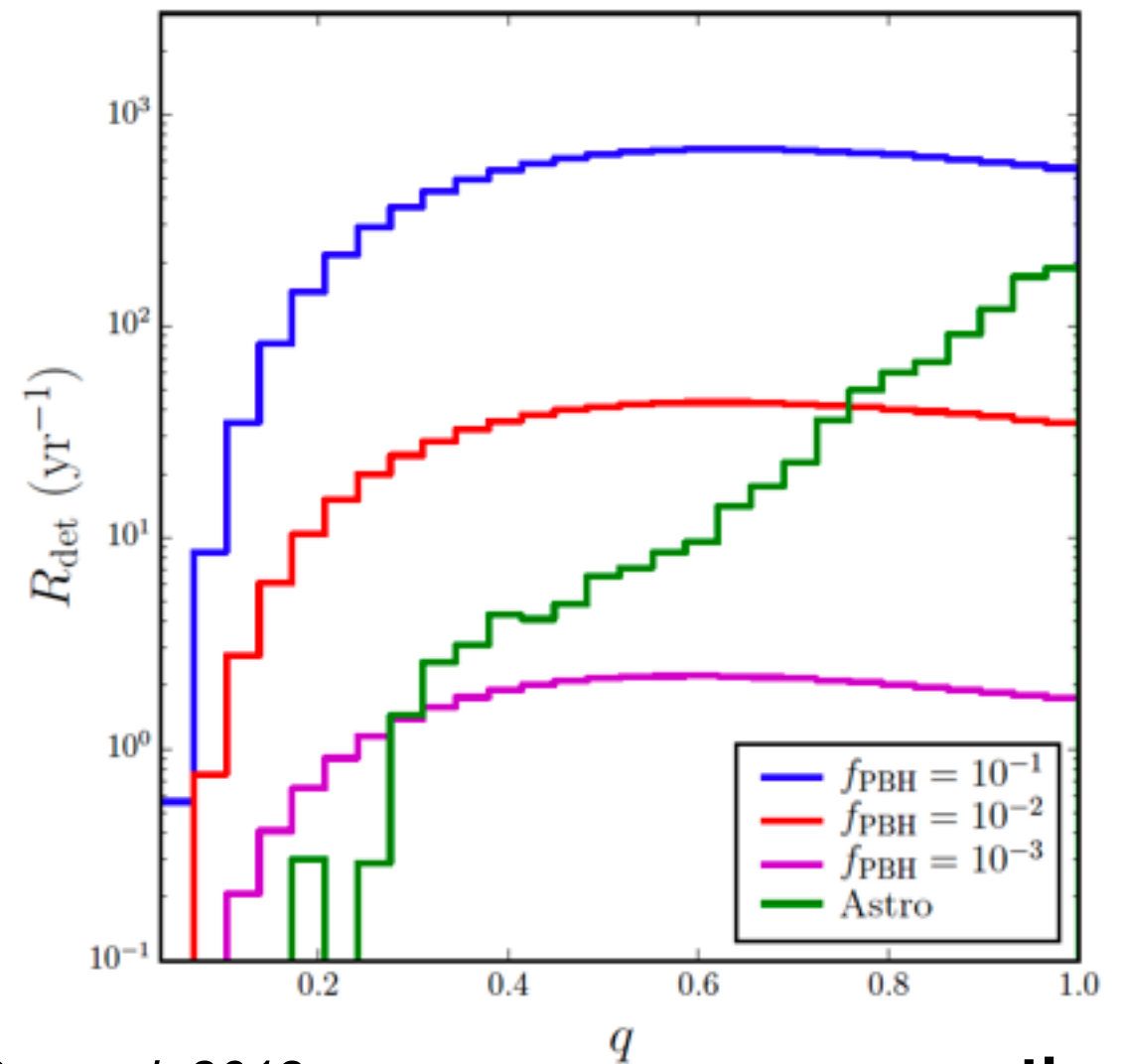
- We marginalise over the PBH merger rate, by assuming a functional form of the PBH mass function and then fit the free parameters to the data
- This typically means fitting a peak mass, a width, and amplitude (f_{PBH})
- We take broad priors on all parameters
- For the astrophysical model we take the empirical models A and B from LIGO-Virgo, which captures some key physics such as the lower and upper mass cut offs and power law dependence on the upper mass and the mass ratio

The total mass and mass ratio



total mass

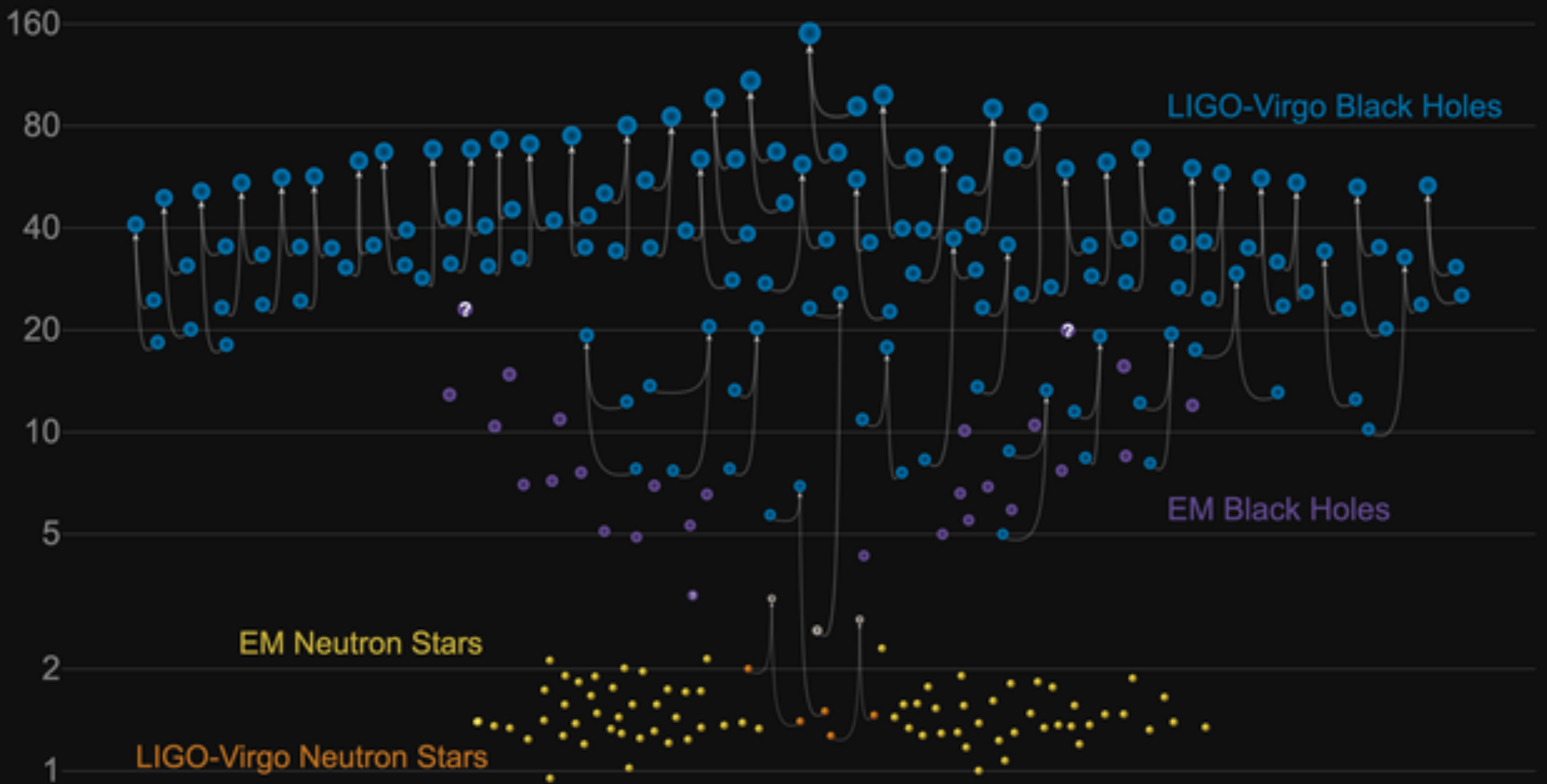
Gow, CB, Hall, Peacock 2019



mass ratio

Notice how astrophysical black holes have an expected maximum and minimum mass
The mass ratio (q) looks like a promising discriminant between the two scenarios

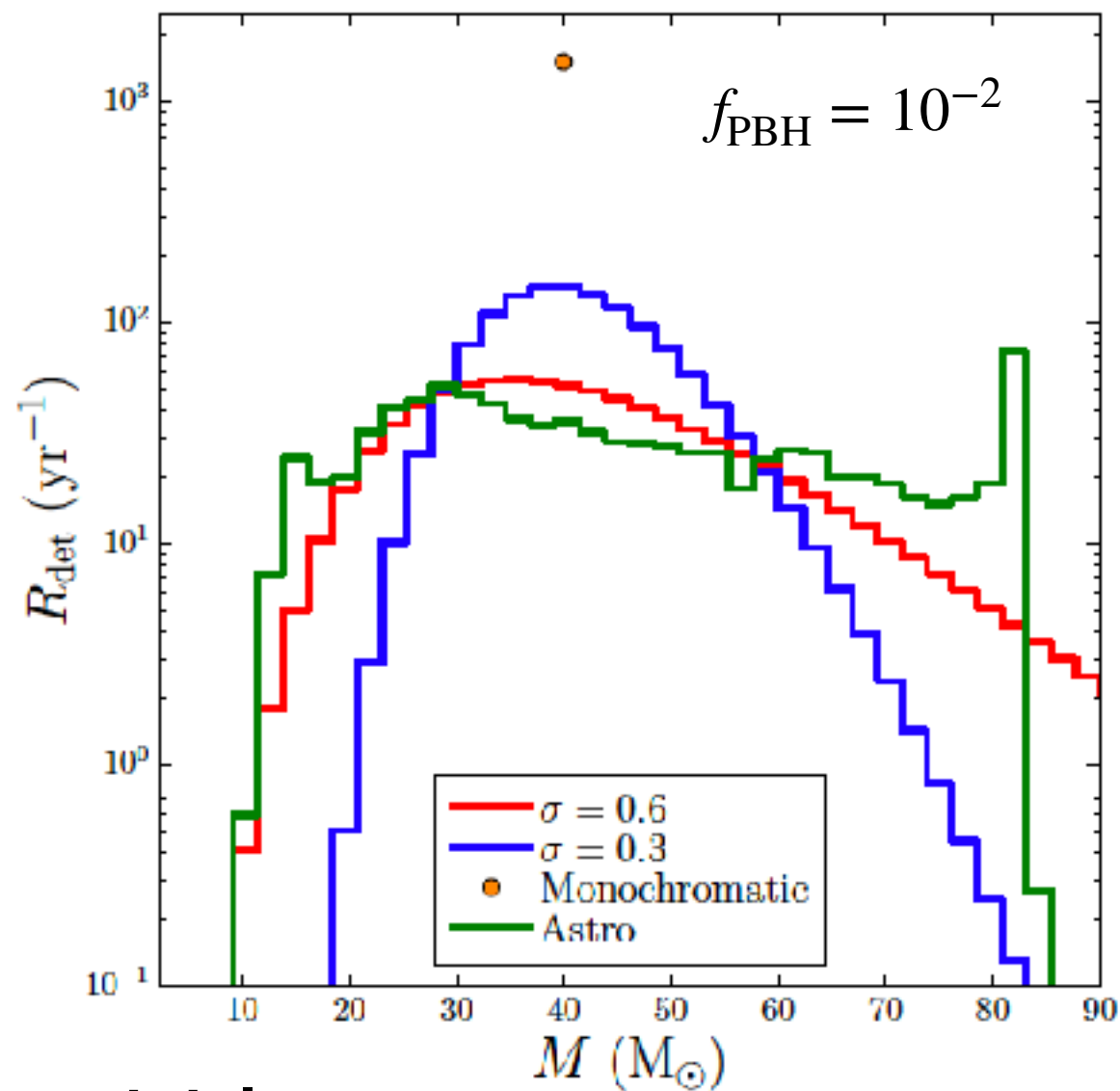
In Solar Masses



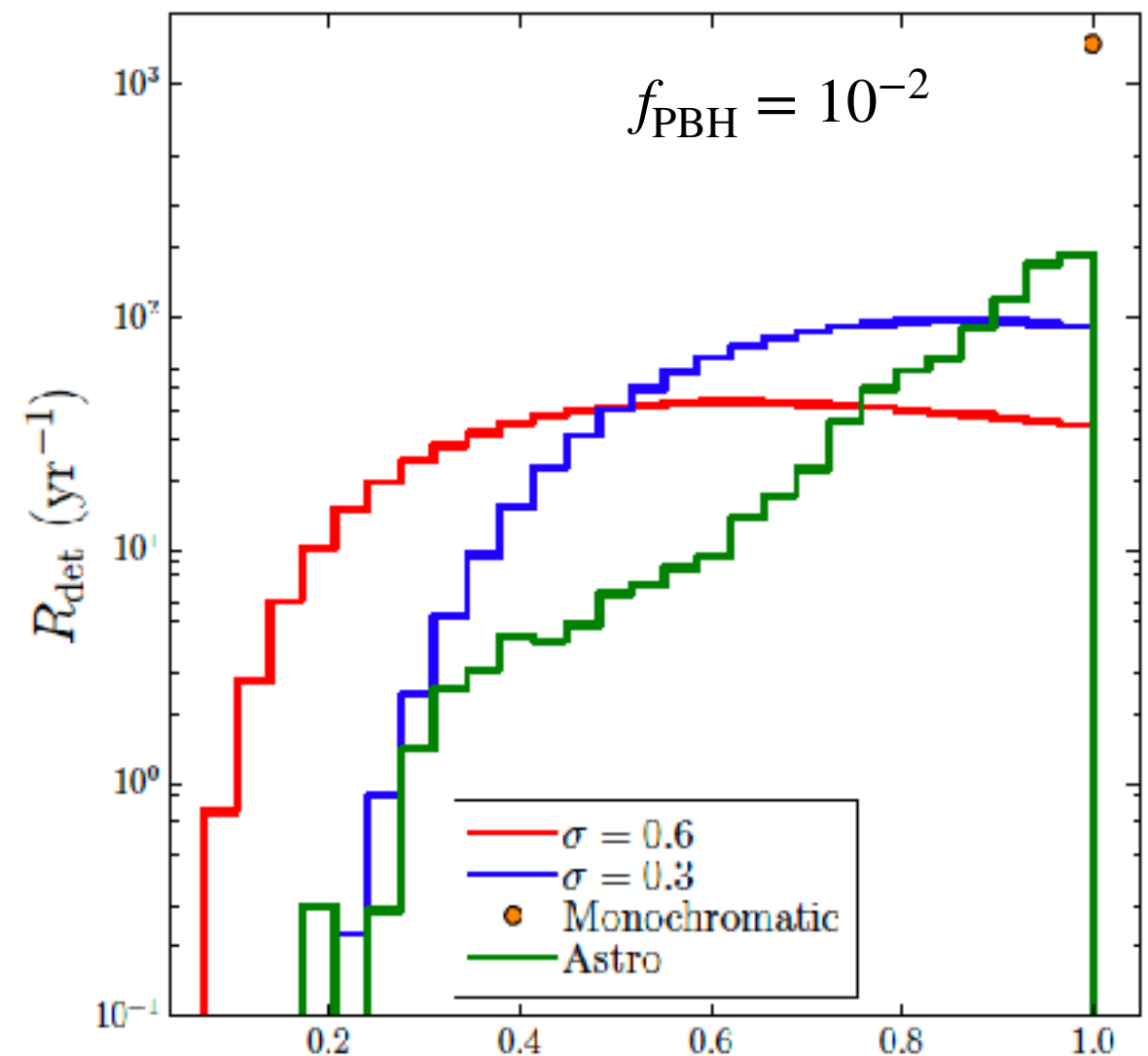
LIGO & Virgo collaboration

Varying the PBH mass function width

Wide enough to fit the masses, yet not so wide to stop $q \sim 1$



total mass



mass ratio

The “astro” distribution covers a broader range of total masses than $\sigma=0.3$, but it still prefers the mass ratio $q \sim 1$. A monochromatic mass function is ruled out.

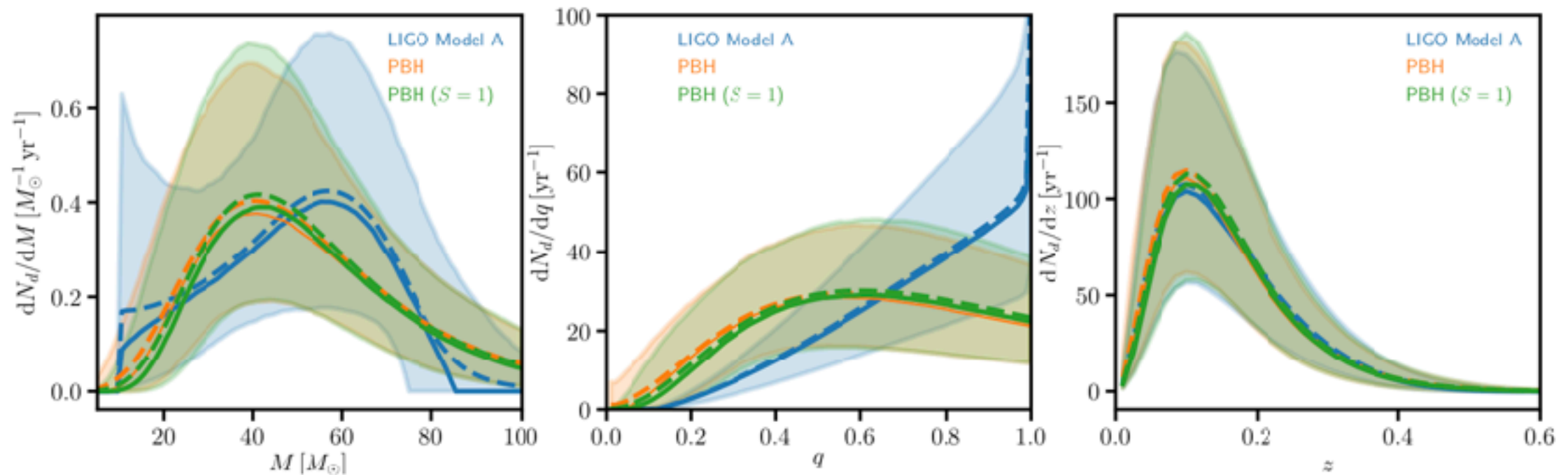
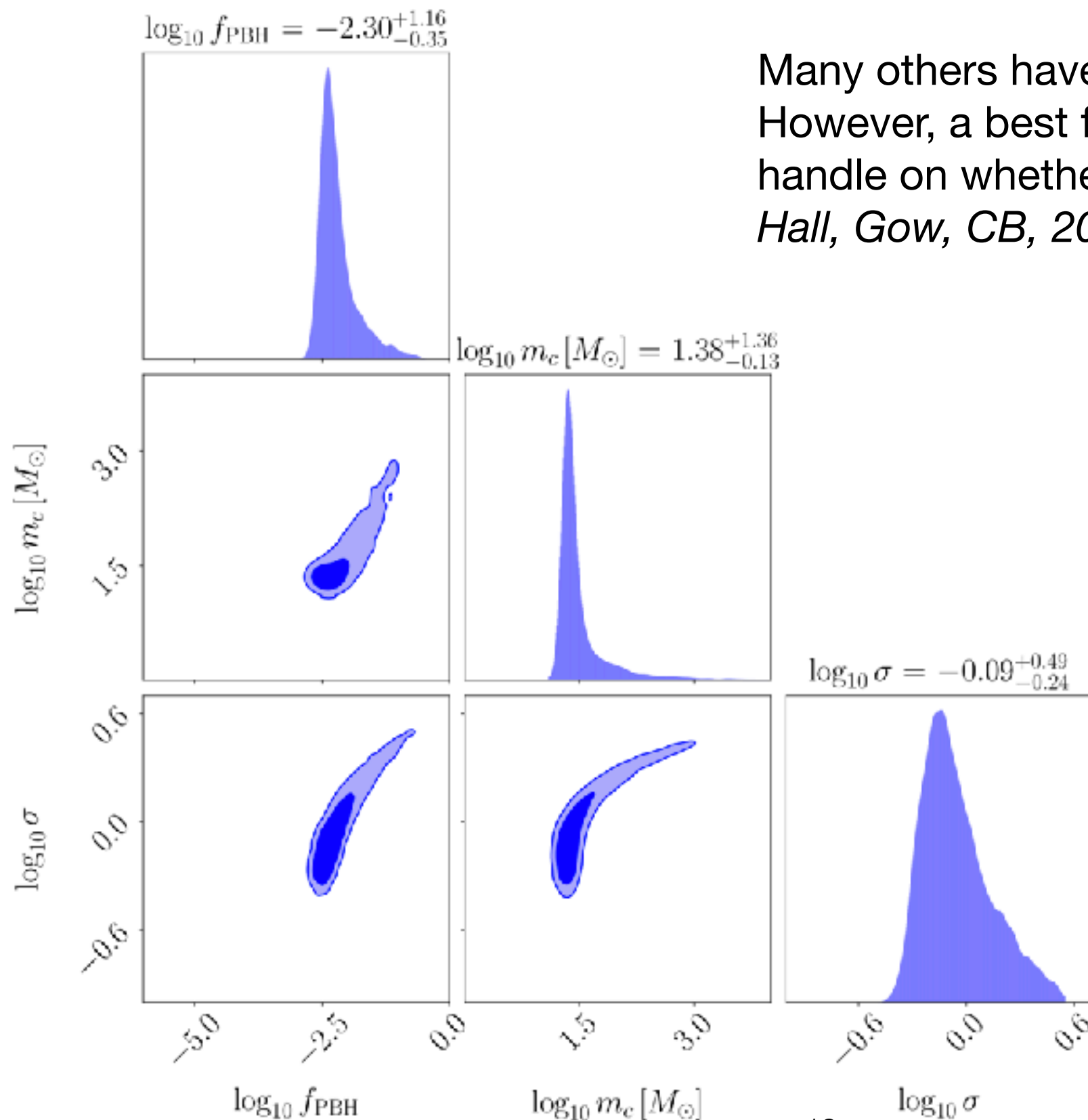


FIG. 11: Differential detector-frame merger rates with respect to total mass (left panel), mass ratio (middle panel) and redshift (right panel) for LIGO Model A (blue), the lognormal PBH model (orange) and the lognormal PBH model with suppression factor set to unity (green). In each case we plot the median and 90% quantiles over the posterior samples for each model given the GWTC-1 data (solid lines and shaded bands), and the (weighted) mean over the samples (dashed lines).

Hall, Gow, CB, 2020: Bayesian comparison

PBHs are better at explaining events with a small mass ratio, but don't naturally explain the upper and lower mass gaps predicted by stellar models
 PBHs are more flexible at explaining individual events

Fitting a lognormal mass function



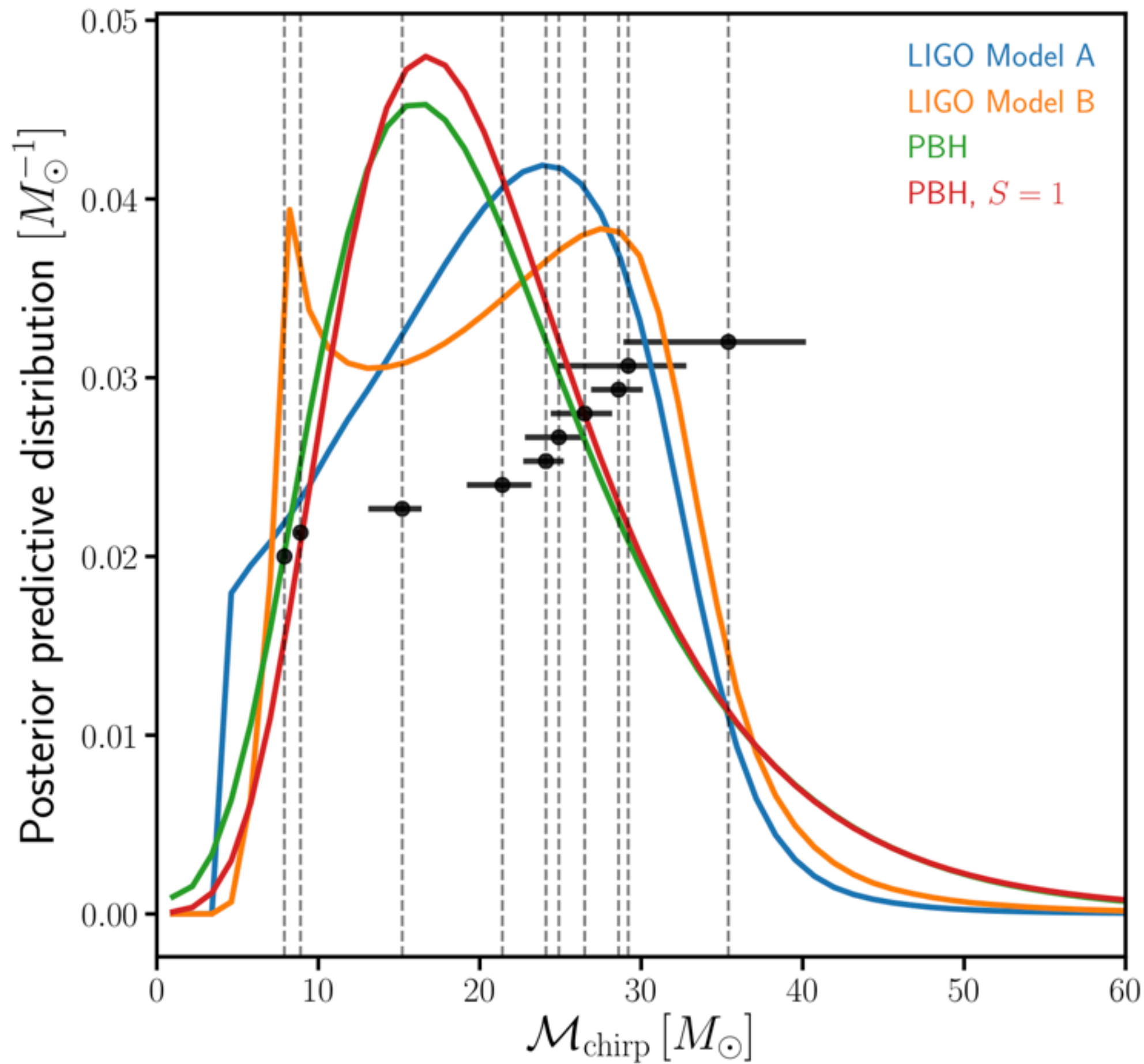
Many others have made similar fits
However, a best fit analysis does not give any
handle on whether the best fit is also a good fit
Hall, Gow, CB, 2020

Bayesian results

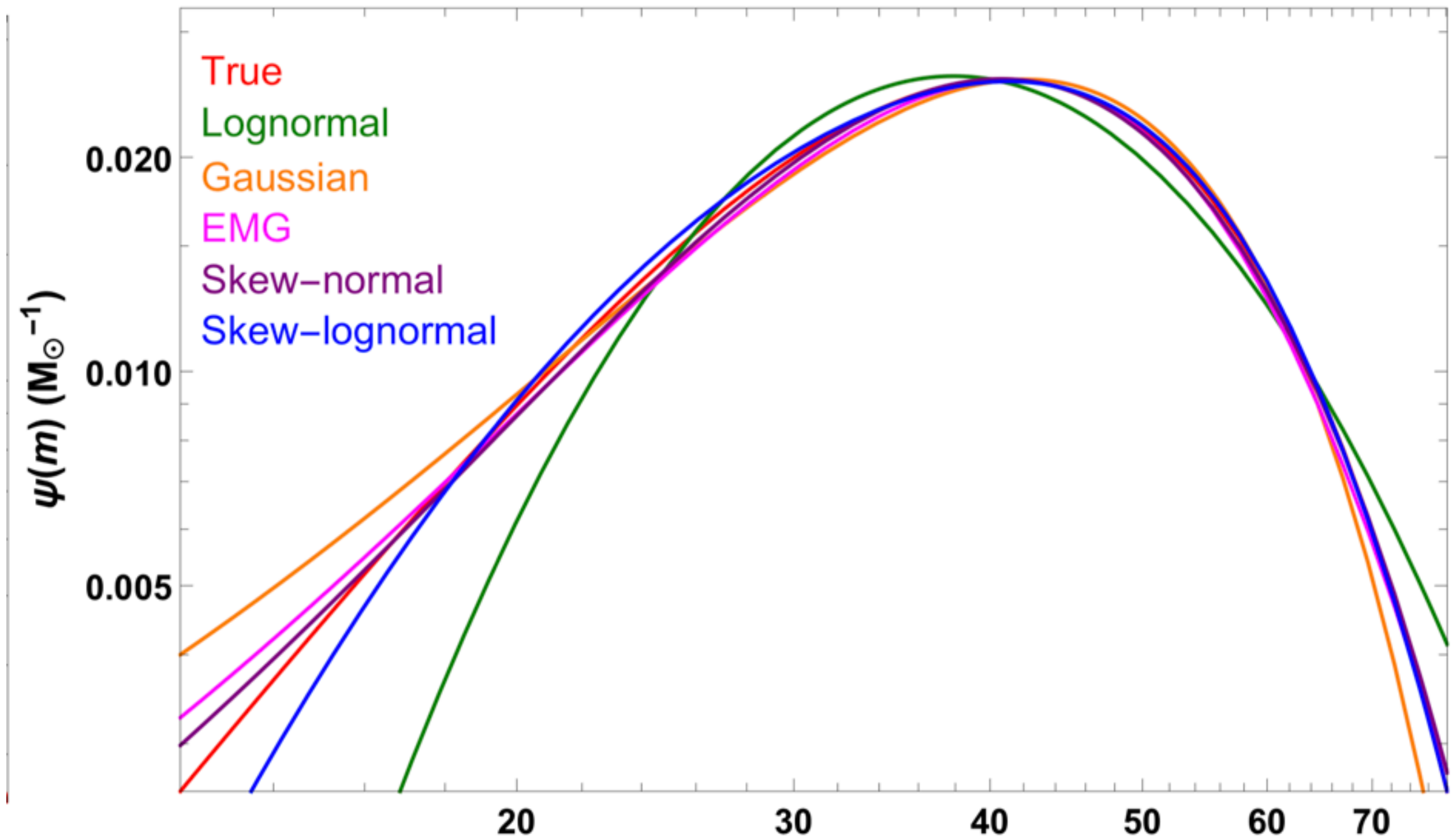
- Our models are: All mergers are due to PBHs vs all due to stellar BHs
- We use 01/02 data only and carefully use the LIGO sensitivity curve.
- The Bayesian evidence can be approximated as the likelihood of the best fit model * the Occam factor
- Both are important but the Occam factor is prior dependent and more controversial
- PBHs are disfavoured by both terms - assuming the “normal” lognormal mass function

PBH models are disfavoured decisively

$$\ln Z_{\text{PBH}}/Z_{\text{stellar}} = -7.35 \pm 0.23$$



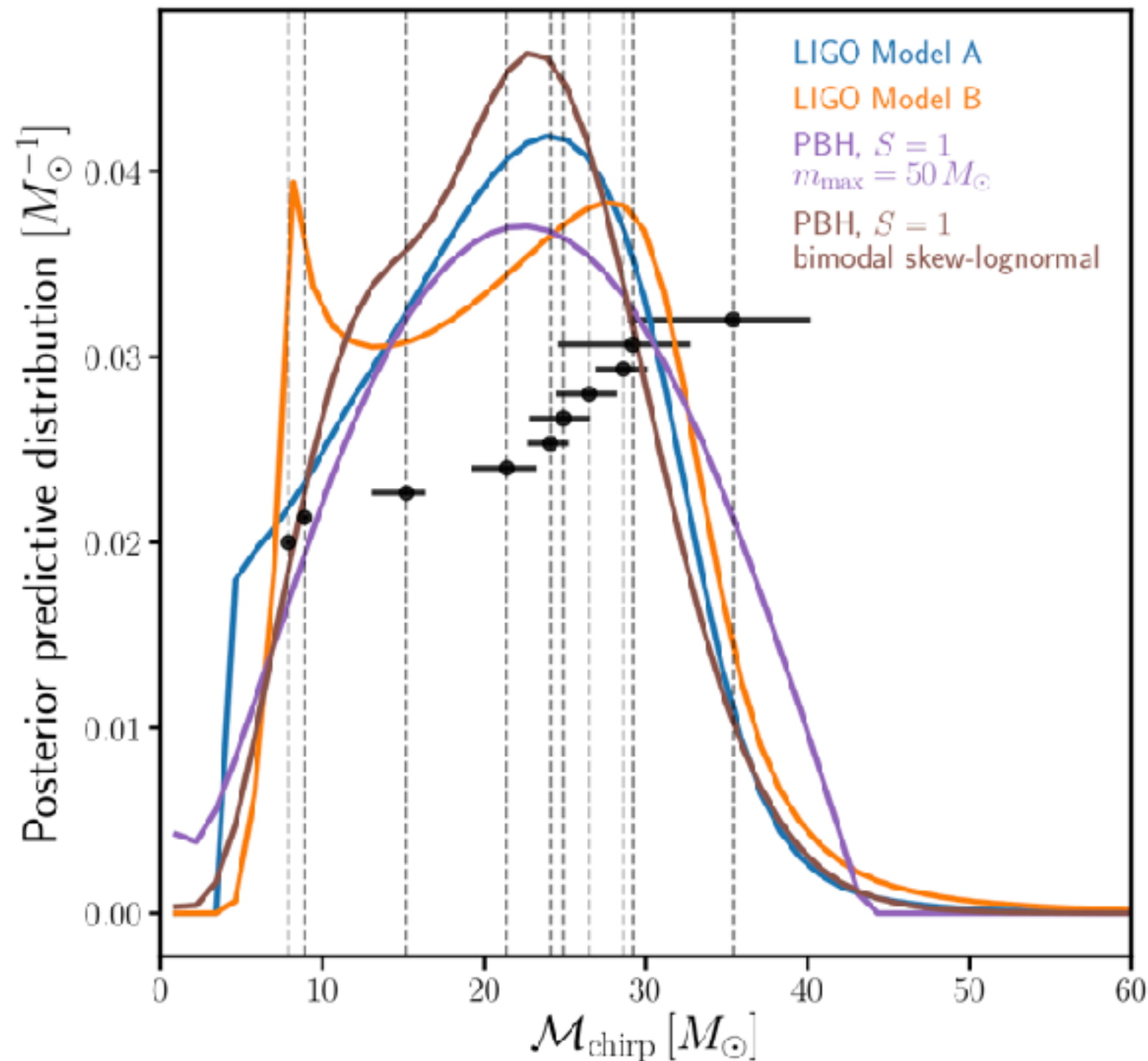
Is the lognormal mass function correct?



For a Dirac-delta function power spectrum $m (M_{\odot})$

Gow, CB, Hall 2020

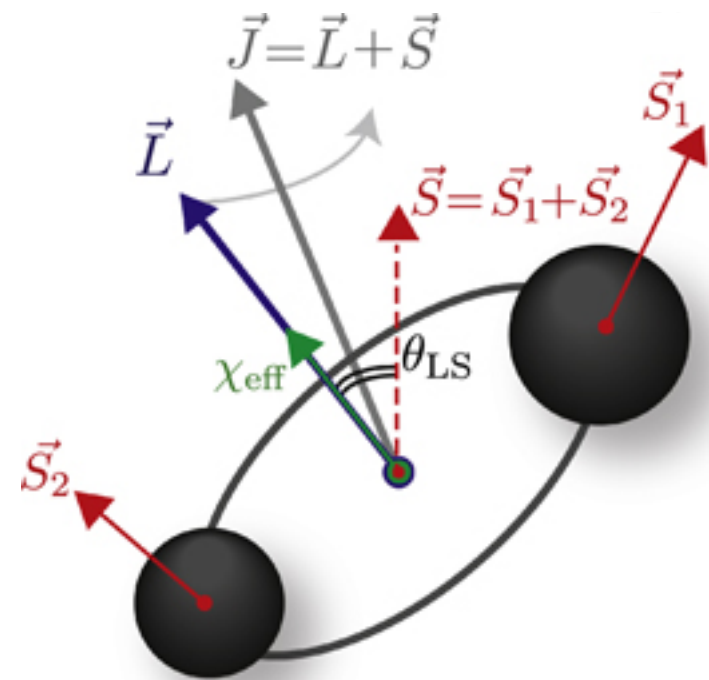
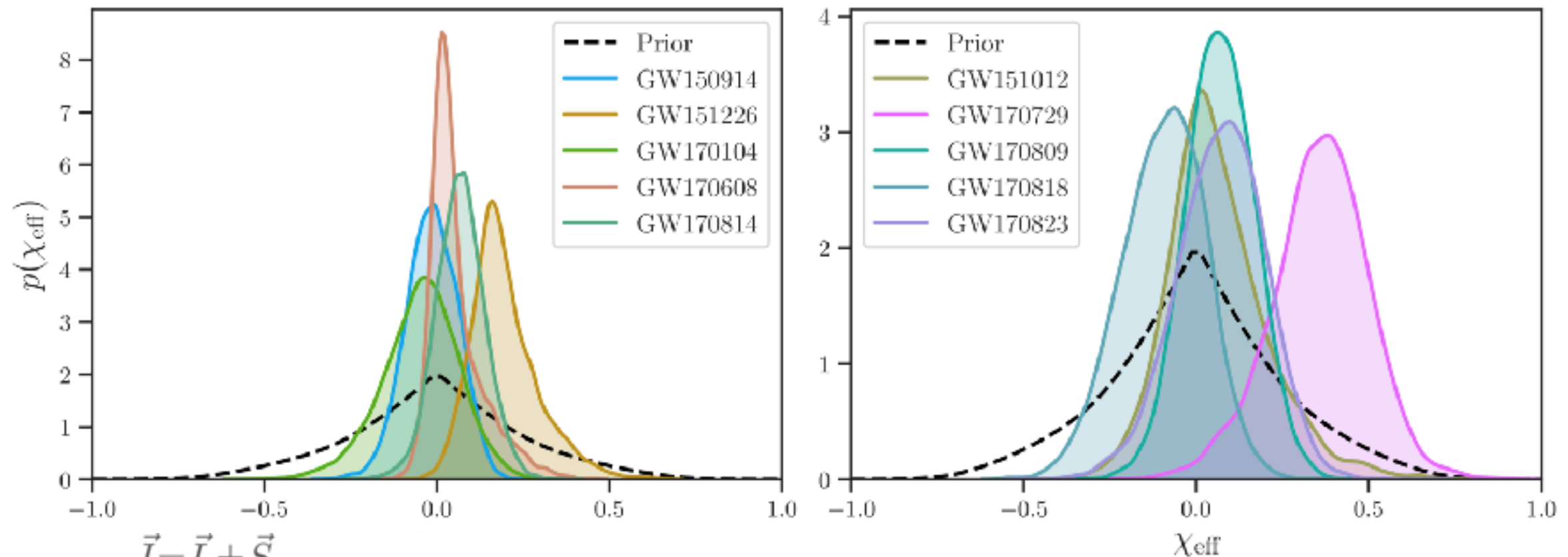
Trying hard to fit the data - cutoff or bimodal mass function



Hall, Gow, CB, 2020

These alternatives are a better fit, but still not a good fit compared to the stellar models
The late time PBH capture and merger rate is also a bad fit
Accretion broadens the mass function at large masses (*de Luca et al '20*): => worse fit

Black hole spin - in isolation favours PBHs



$$\chi_{\text{eff}} = \frac{c}{G(m_1 + m_2)} \left(\frac{\vec{S}_1}{m_1} + \frac{\vec{S}_2}{m_2} \right) \cdot \vec{L} \quad a_* = \frac{c |\vec{S}|}{Gm^2} \leq 1$$

Model comparison based on spins

Fernandez & Profumo '19, Garcia-Bellido et al '20, Wong et al 2020 (uses O3a data + not only spins)

However, we find that by far the strongest constraining power comes from the masses

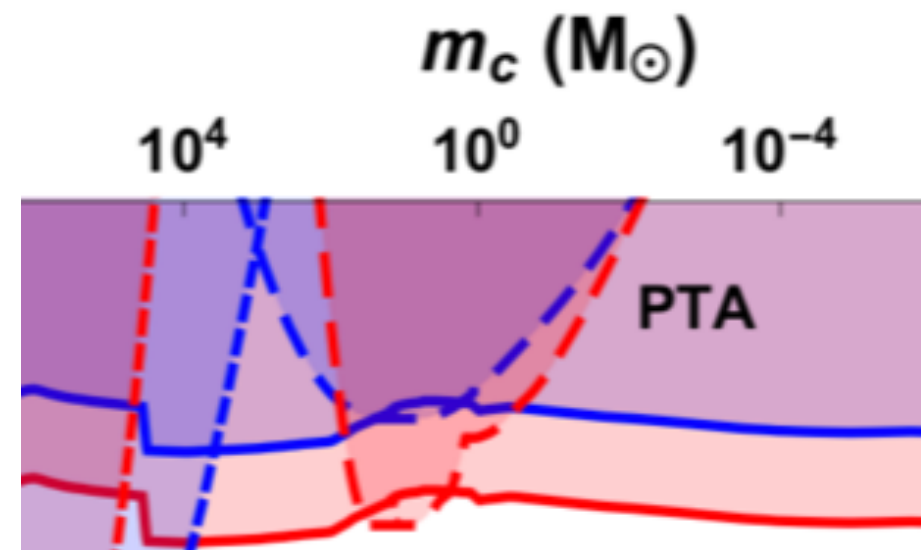
See talks by Juan and Toni

PBHs do not undergo much collapse before formation, small spin expected

Belczynski et al. '17; Mirbabayi et al '19; De Luca et al '19, Harada et al '20 + many more

Optimistic vs pessimistic take

- There is no contradiction
- No reason why the merging black holes should be ALL stellar or ALL primordial
- The PBBH merger rates are still very uncertain
- However, the late time PBH mergers and accretion won't improve things much
- The required power spectrum amplitude hardly varies when f_{PBH} changes by 10 orders of magnitude



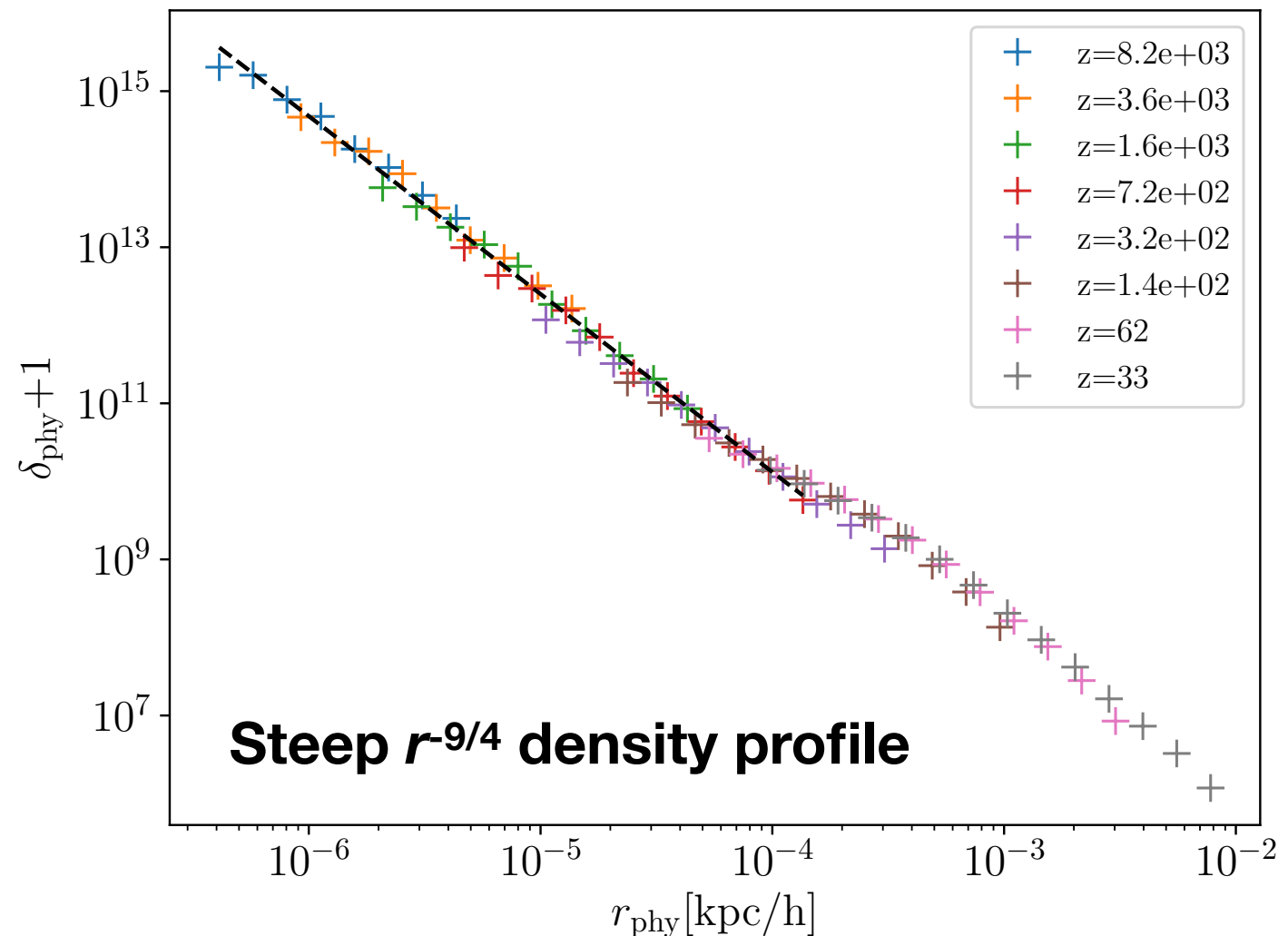
What if one PBH was detected?

- We would know what some of the DM was (and what it is not)
- It would be the oldest relic detected, predating the primordial element abundance generated by BBN
- Other potential relics include: gravitational waves, topological defects such as cosmic strings, CMB spectral distortions, ultracompact minihalos
- Requires non-trivial inflationary dynamics, perhaps with an early-matter-dominated phase and/or topological defects.
PBH review article: Green 2015

WIMPs and PBHs are incompatible

- Assuming WIMPs have the standard, velocity independent cross section which gets the right abundance, and $M_{\text{PBH}} > 10^{-6} M_{\text{sun}}$.
- If $f_{\text{PBH}} < 1$, then another DM component is inevitable
- Steep and high density profiles form around PBHs (density $\sim r^{-9/4}$). WIMPs would rapidly annihilate to gamma rays.
- In contrast to ultracompact minihalos without a PBH seed.
Gosenca et al '17, Delos et al '17

- A detection of WIMPs or PBHs may effectively rule out the existence of the other



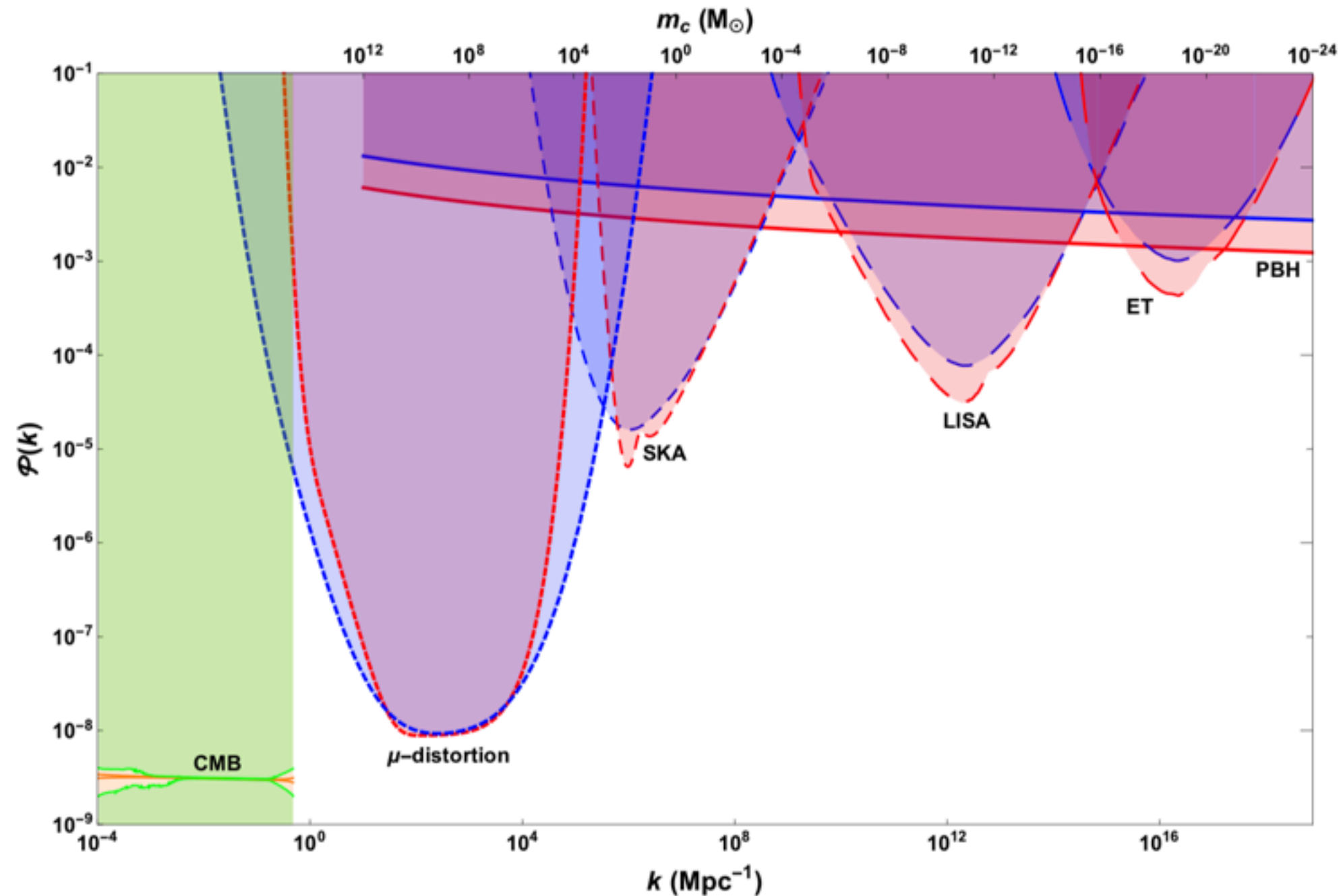
Adamek, CB, Gosenca & Hotchkiss 2019;

Lacki & Beacom 2010; Eroshenko 2016;
Boucenna, Kühnel, Ohlsson & Visinelli 2017
The 3 papers above all find different profiles.
We made the first simulations of this scenario

Looking forwards

- LIGO/Virgo have many new events not included in our analysis, and many many more to come
- The PTA collaborations will firm up the “detection” or reach a sensitivity which rules out PBH generation from large amplitude scalar perturbations on LIGO related mass scales
- LISA and future ground based detectors will be sensitive to very high redshift mergers + better probe of the spins, mass ratio, etc
- Theoretical and numerical work, especially on the merger rate and accretion, is required
- The detection of a sub Chandrasekhar mass object is what I want for Christmas

Future constraints



The PBH lines
correspond to
zero PBHs
Cole & CB '17

Gow, CB, Cole, Young 2020

Summary

- PBHs could still form the DM, if lighter than ??
- It remains possible that some of the LIGO black holes were primordial, but not all of them
- There are some mass gap candidates in O3 data
- We can constrain the PBH mass function and compare it to astrophysical models. Ideally we would incorporate spin as well
- The detection of a single small black hole would be transformative - please keep searching LIGO-Virgo

Finding just one black hole of sub-solar mass — which should be common, according to the primordial black hole scenario, and which can't form from stars — would transform this entire debate. And with

Quanta magazine

Backup slides

Parameter	Prior
$\log_{10} f_{\text{PBH}}$	$[-6, 0]$
$\log_{10} m_c [M_\odot]$	$[0, 4]$
$\log_{10} \sigma$	$[-1, 0.7]$
$\log_{10} R_0$	$[-1, 3]$
$m_{\text{max}} [M_\odot]$	$[30, 100]$
$m_{\text{min}} [M_\odot]$	$[5, 10]$
α	$[-4, 12]$
β_q	$[-4, 12]$
λ	$[0, 0.5]$
$\log_{10} m_{c,1} [M_\odot]$	$[-1, 3]$
$\log_{10} m_{c,2} [M_\odot]$	$[-1, 3]$

<https://arxiv.org/pdf/2008.13704.pdf>

Parameter	Model					
	PBH	PBH, $S = 1$	Model A	Model B	PBH, $S=1$, $m_{\max} = 50 M_{\odot}$	PBH, $S=1$, skew- bimodal
$\log_{10} f_{\text{PBH}}$	$-2.30^{+1.16}_{-0.35}$	$-2.76^{+0.25}_{-0.24}$	—	—	$-2.72^{+0.25}_{-0.25}$	$-2.74^{+0.23}_{-0.23}$
$\log_{10} m_c [M_{\odot}]$	$1.38^{+1.36}_{-0.13}$	$1.26^{+0.12}_{-0.22}$	—	—	$1.91^{+1.91}_{-0.76}$	—
$\log_{10} \sigma$	$-0.09^{+0.49}_{-0.24}$	$-0.21^{+0.24}_{-0.16}$	—	—	$0.27^{+0.23}_{-0.47}$	—
$m_c [M_{\odot}]$	$24.23^{+528.62}_{-6.31}$	$18.06^{+5.72}_{-7.10}$	—	—	$81.28^{+6525.7}_{-67.15}$	—
σ	$0.82^{+1.71}_{-0.35}$	$0.61^{+0.45}_{-0.19}$	—	—	$1.86^{+1.30}_{-1.23}$	—
$\log_{10} R_0$	—	—	$1.63^{+0.50}_{-0.45}$	$1.55^{+0.41}_{-0.43}$	—	—
$m_{\max} [M_{\odot}]$	—	—	$42.65^{+18.96}_{-5.99}$	$42.73^{+35.11}_{-6.31}$	50.0	—
$m_{\min} [M_{\odot}]$	—	—	5.00	$7.88^{+1.30}_{-2.64}$	—	—
α	—	—	$0.94^{+1.59}_{-2.38}$	$1.93^{+1.70}_{-1.96}$	—	—
β_q	—	—	0.00	$6.62^{+5.04}_{-6.62}$	—	—
λ	—	—	—	—	—	$0.35^{+0.14}_{-0.27}$
$\log_{10} m_{c,1} [M_{\odot}]$	—	—	—	—	—	$1.08^{+0.57}_{-0.38}$
$\log_{10} m_{c,2} [M_{\odot}]$	—	—	—	—	—	$1.57^{+0.08}_{-0.62}$
$m_{c,1} [M_{\odot}]$	—	—	—	—	—	$12.02^{+32.65}_{-10.82}$
$m_{c,2} [M_{\odot}]$	—	—	—	—	—	$37.15^{+7.52}_{-28.24}$
$\ln L^*/L_{\text{B}}^*$	-6.99	-7.14	-2.51	0.00	-5.44	-3.53
$\ln \text{Occam}$	-6.13	-8.21	-5.71	-6.74	-5.46	-7.73
$\ln Z_{\text{Lap}}/Z_{\text{NS}}$	$1.60^{+0.16}_{-0.16}$	$0.26^{+0.17}_{-0.17}$	$0.77^{+0.15}_{-0.15}$	$0.63^{+0.16}_{-0.16}$	$0.54^{+0.13}_{-0.13}$	$1.92^{+0.18}_{-0.18}$
$\ln Z_{\text{NS}}/Z_{\text{NS,B}}$	$-7.35^{+0.23}_{-0.23}$	$-8.25^{+0.23}_{-0.23}$	$-1.62^{+0.22}_{-0.22}$	0.00	$-4.01^{+0.21}_{-0.21}$	$-5.79^{+0.24}_{-0.24}$

TABLE II: Median and 95% credible intervals for the parameters of each model considered. The bottom four rows display difference in best-fit log-likelihood between each model and LIGO Model B, the log of the Occam factor defined in the text, the difference in log-evidence between the DYNESTY nested sampling estimate and the Laplace approximation defined in the text, and the Bayesian evidence ratios computed from nested sampling along with uncertainties.

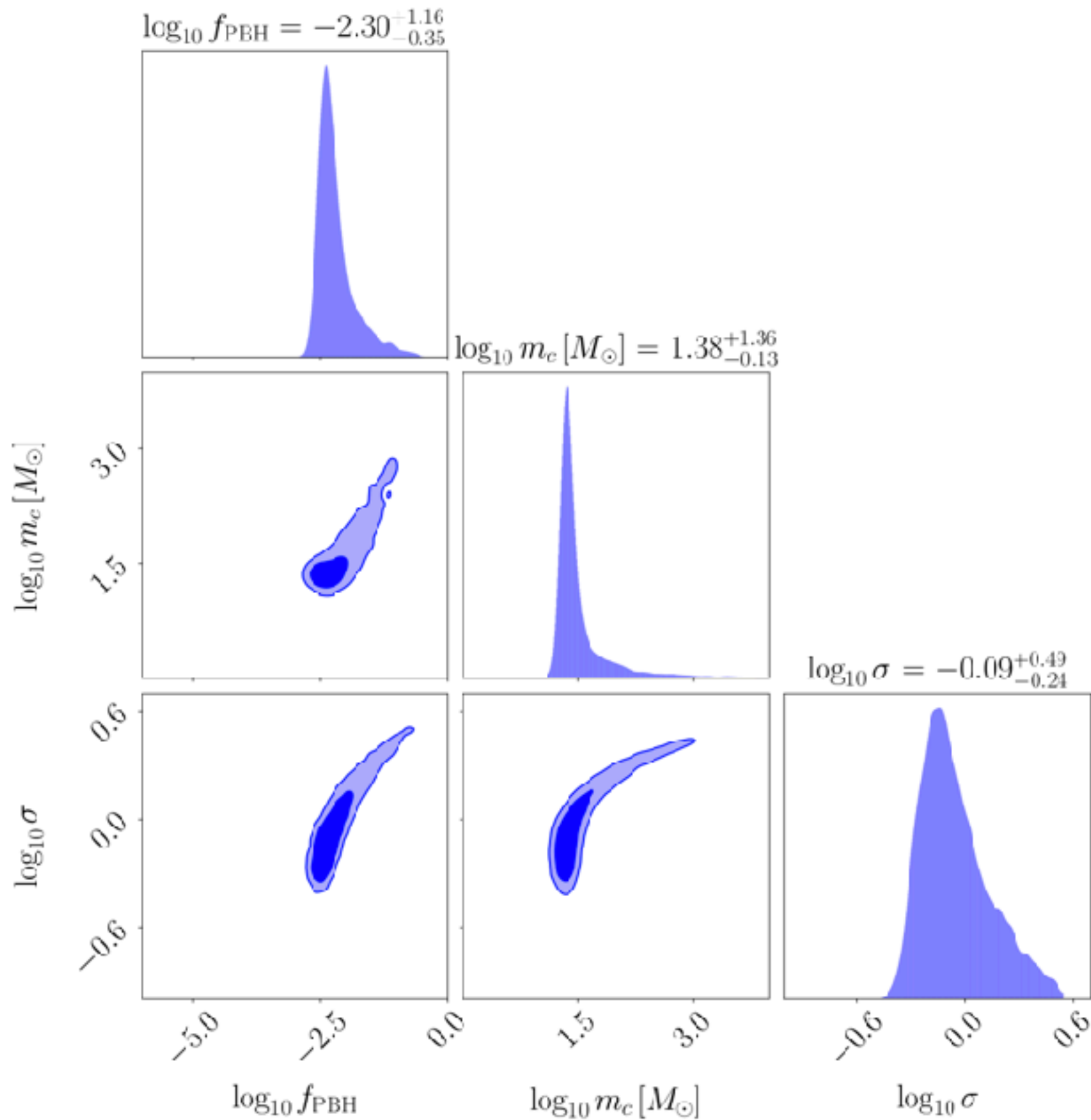


FIG. 8: *Off-diagonal panels:* Two-dimensional 68% and 95% marginal posterior quantiles for the parameters of the lognormal PBH model including the 3-body suppression factor, given the GWTC-1 data. The plot boundaries correspond to the extent of the (uniform) priors on the parameters shown. *Diagonal panels:* One-dimensional marginal posterior densities for the parameters. Above each panel are the marginalised posterior median and 95% posterior quantiles for each parameter.

The merger rate

The merger time of a binary $\tau = \frac{3}{85} \frac{r_a^4}{\eta M^3} j^7$

$$e = \sqrt{1 - j^2} \quad \eta \equiv \mu/M \quad M \equiv m_1 + m_2 \text{ and } \mu \equiv m_1 m_2 / M$$

The differential merger rate

$$dR = S \times dR_0$$

S is a suppression factor

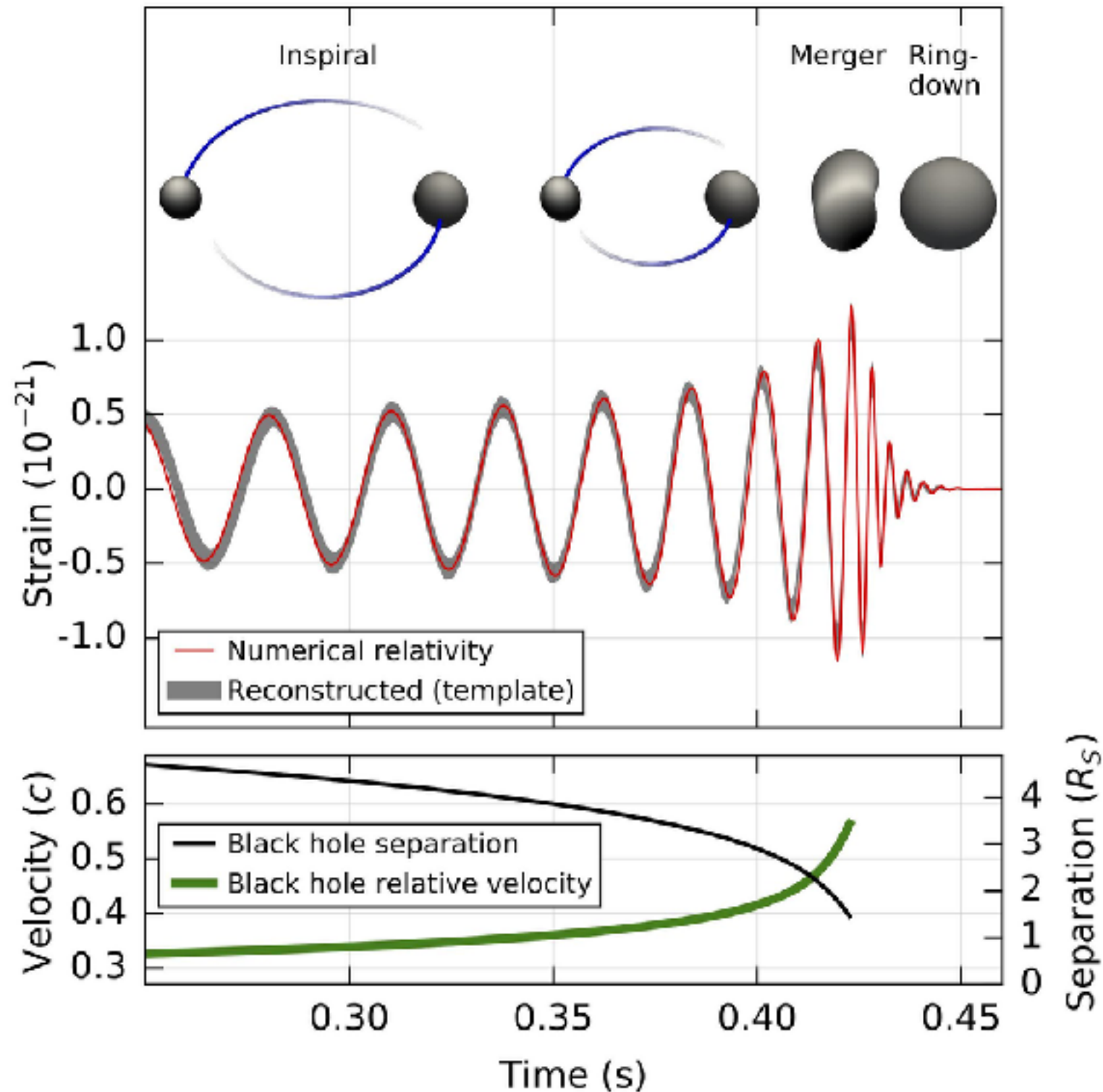
$$\begin{aligned} dR_0 &= \frac{0.65}{\tau} \left(\frac{\tau \eta M^{14}}{f_{\text{PBH}}^7 c_j^7 c_a^4 \rho_M^{11}} \right)^{\frac{3}{37}} dn(m_1) dn(m_2) \\ &\approx \frac{1.6 \times 10^6}{\text{Gpc}^3 \text{yr}} f_{\text{PBH}}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{M}{M_\odot} \right)^{-\frac{32}{37}} \left(\frac{\tau}{t_0} \right)^{-\frac{34}{37}} \psi(m_1) \psi(m_2) dm_1 dm_2 \end{aligned}$$

Raidal et al <https://arxiv.org/pdf/1812.01930.pdf>

PBH formation comments

- The formation rate is exponentially sensitive to the amplitude of the power spectrum, and the collapse threshold
- Inflationary models posit an inflection point (ultra-slow-roll inflation) or other feature
- The power spectrum can't grow faster than about k^4 (in canonical single-field inflation), impacts the constraints. *Byrnes, Cole & Patil '18; Carrilho, Malik & Mulryne '19*
- PBHs are very rare - very sensitive to non-Gaussianity
- The formation criteria depends on the density profile. Many spherically symmetric simulations exist, e.g. *Niemeyer & Jedamjik, Musco & Miller, Harada ++, Nakama ++...*
- Extensive recent analytic work has been done to relate the power spectrum to PBH formation rate at, but (at least) an order unity uncertainty remains (= tens of orders of magnitude in terms of the formation rate). *Germani & Musco '17, Yoo et al '17, Kawasaki & Nakatsuka '19, de Luca et al '19, Young et al '19, Young '19, Kalaja et al '19*

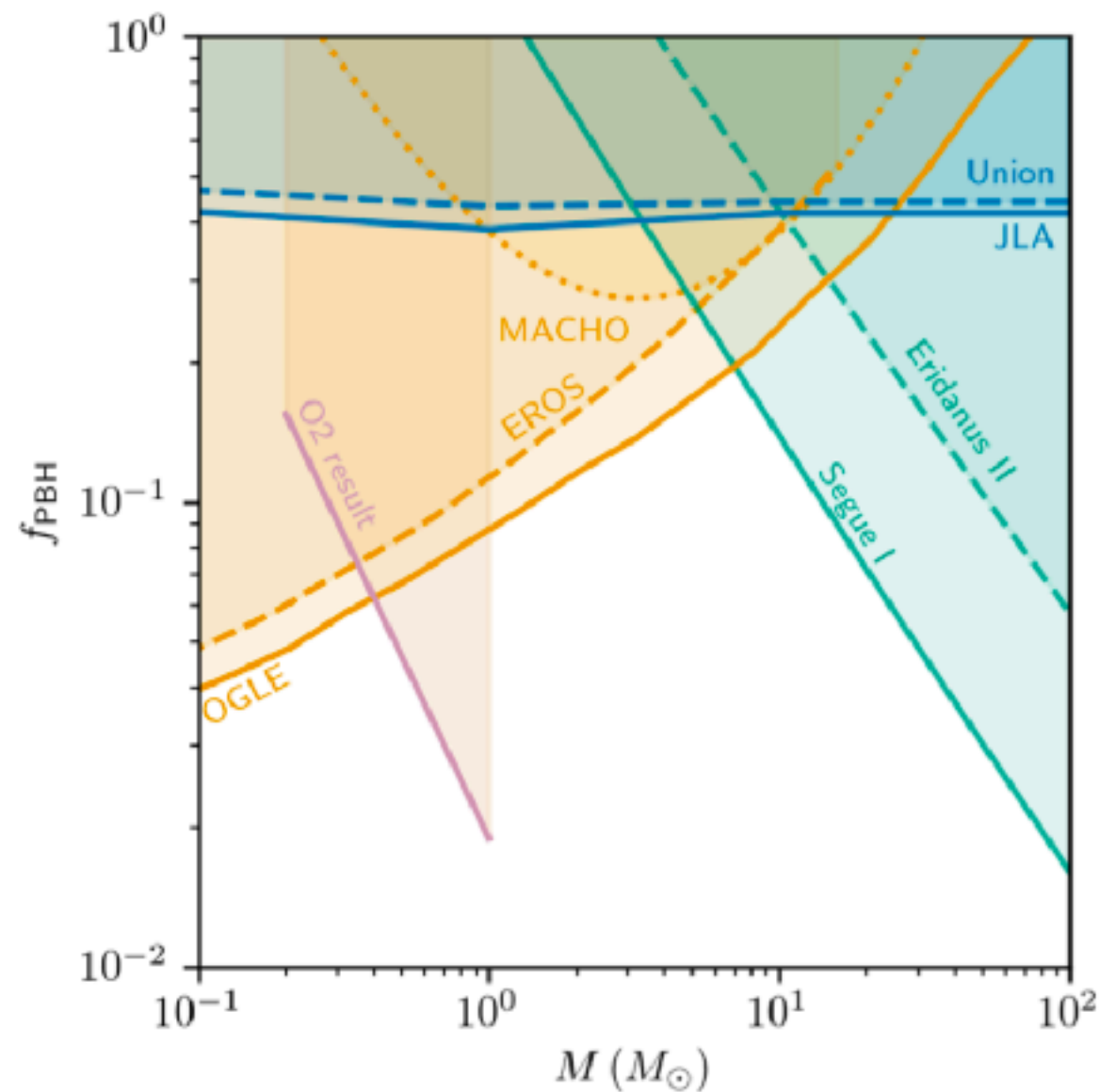
The three merger phases



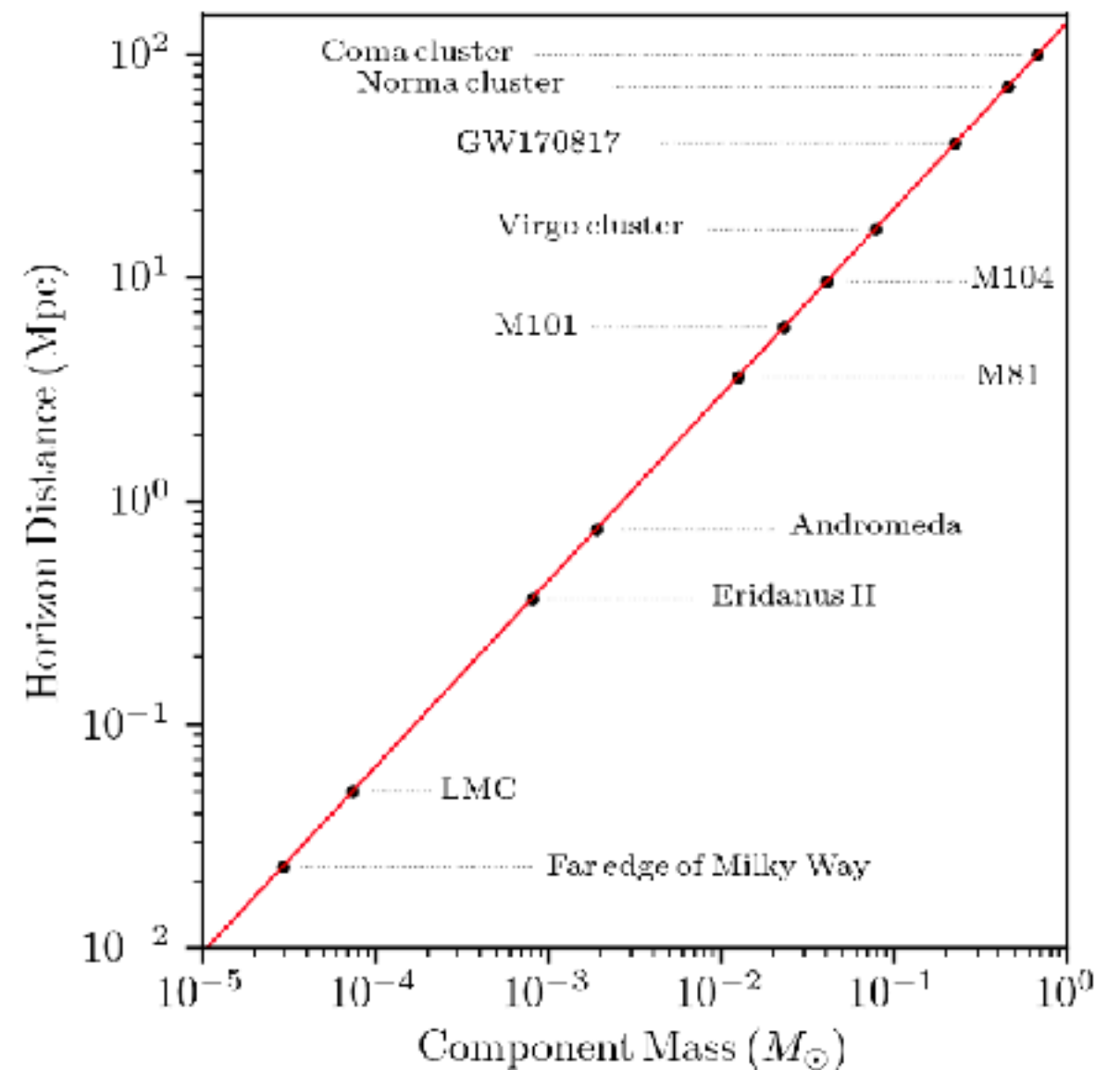
Sub-solar mass GW searches

GW searches have been made, with no detections so far

These are below the Chandrasekhar mass, hence potential proof of a primordial origin

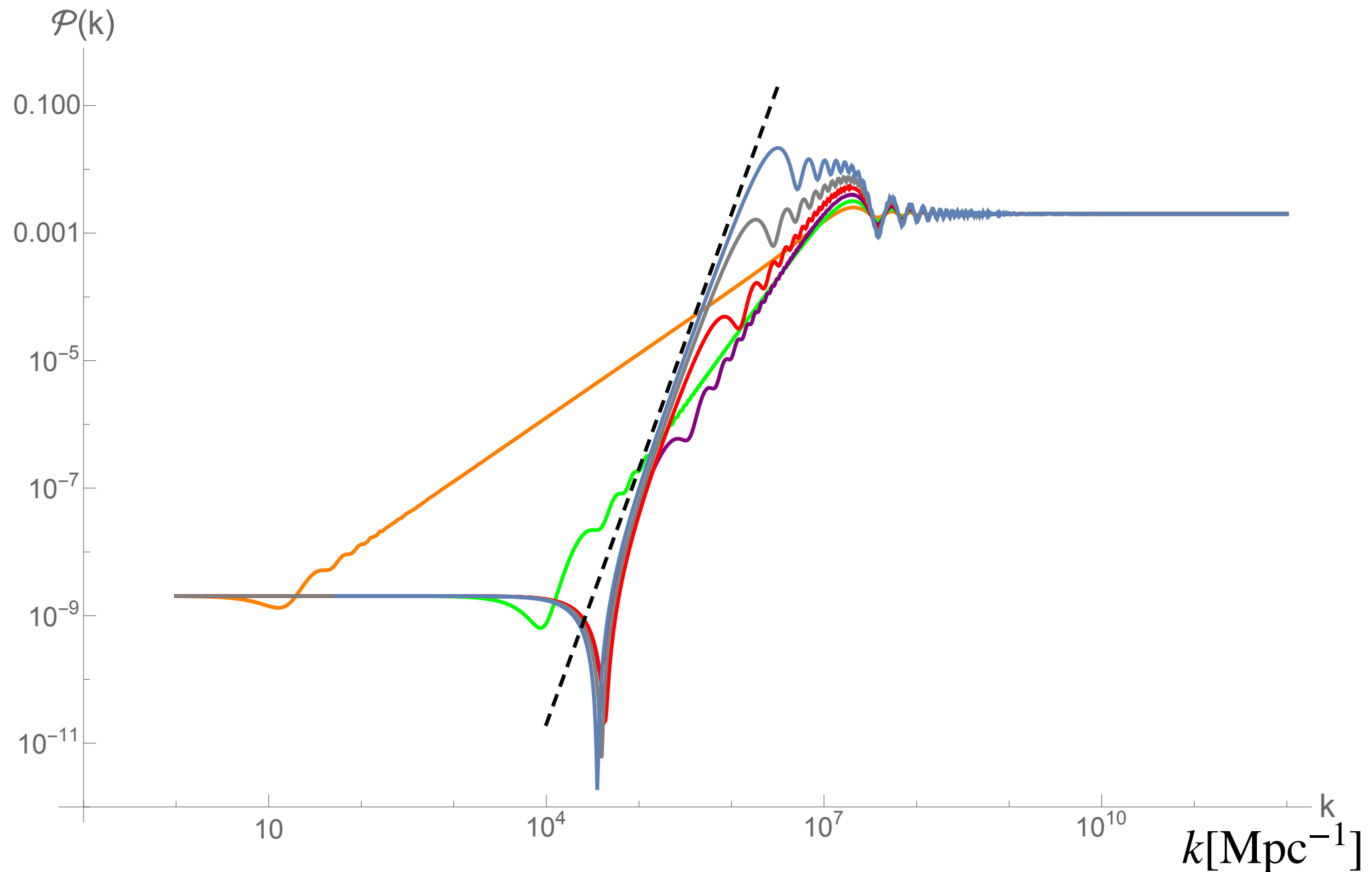


LIGO & Virgo collaboration 2019



Magee et al 2018

Steepest possible power spectra



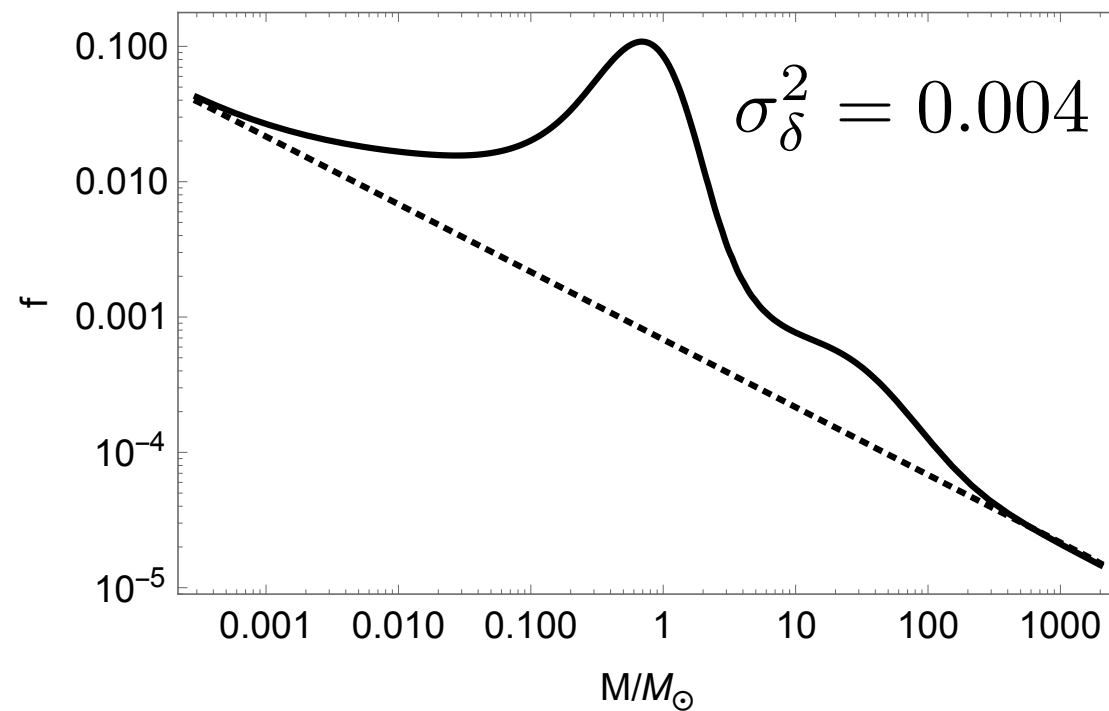
From single-field inflation with an inflection point (ultra slow-roll inflation).
The power spectrum cannot be steeper than k^4

CB, Cole & Patil 2018

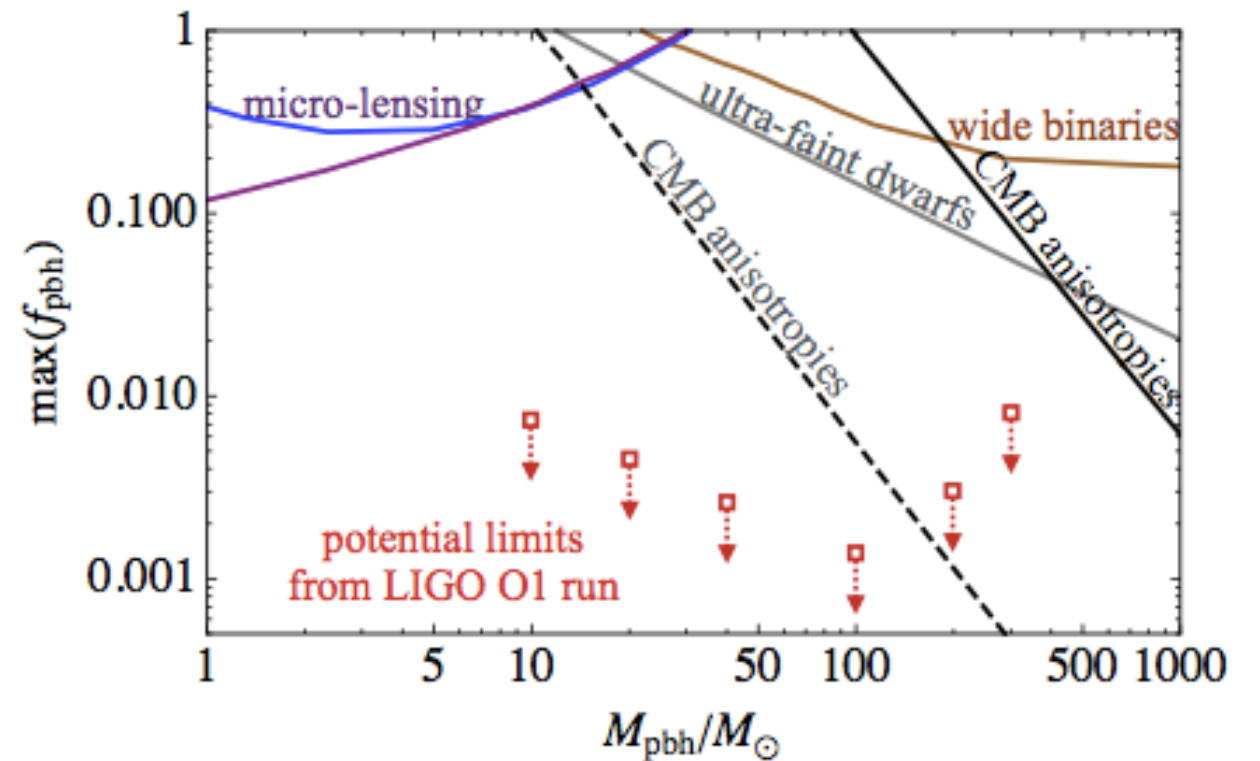
The resultant PBH mass function

$$f(M) = \frac{1}{\Omega_{\text{CDM}}} \frac{d\Omega_{\text{PBH}}}{d \ln M_{\text{H}}}$$

$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$



CB, Hindmarsh, Young & Hawkins 2018



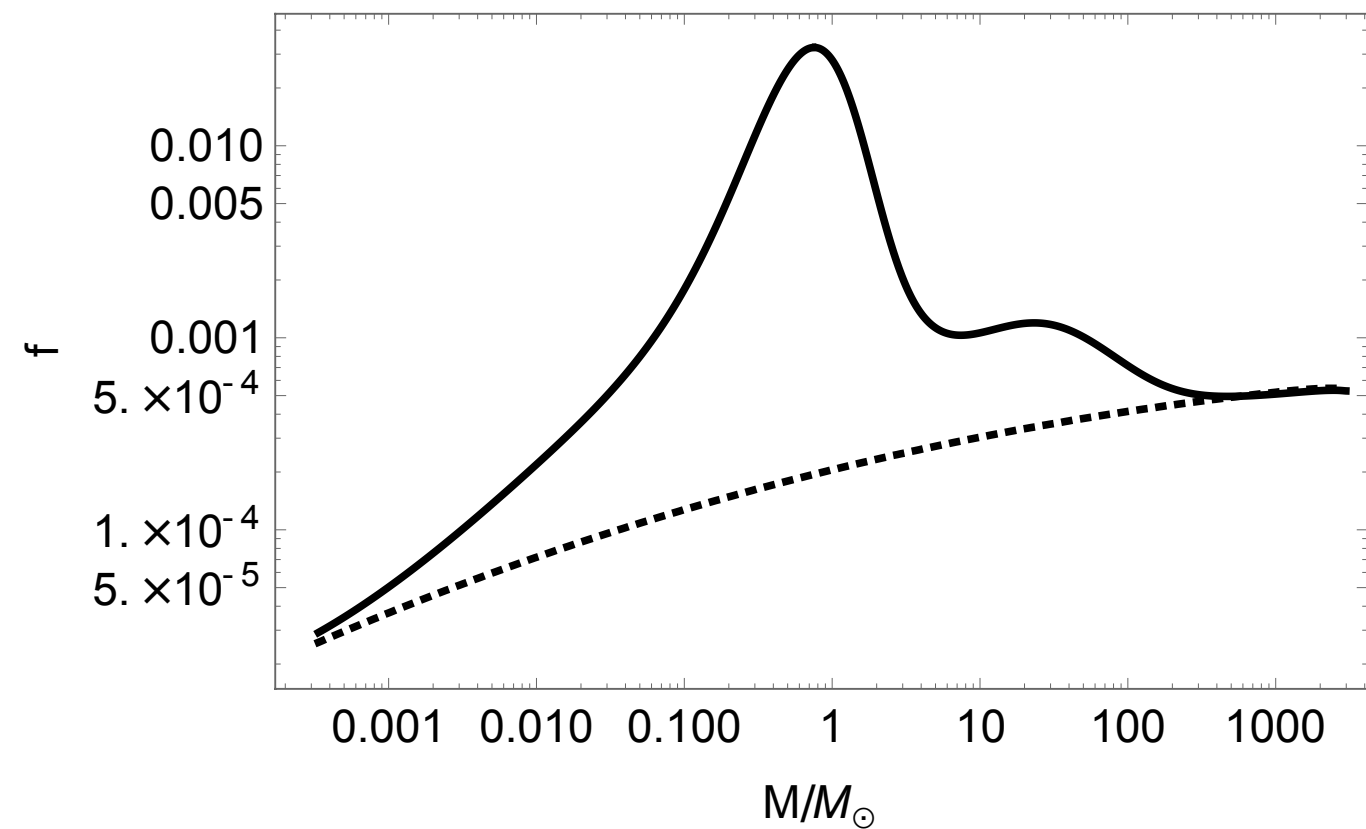
Haimoud et al 2017

For the left plot, approx 10% of DM is made up of \sim solar mass PBHs and 0.1% lies in the LIGO mass range - enough to get the merger rate LIGO detects
Sasaki et al + Haimoud et al + Chen & Huang + Raidal et al + many more

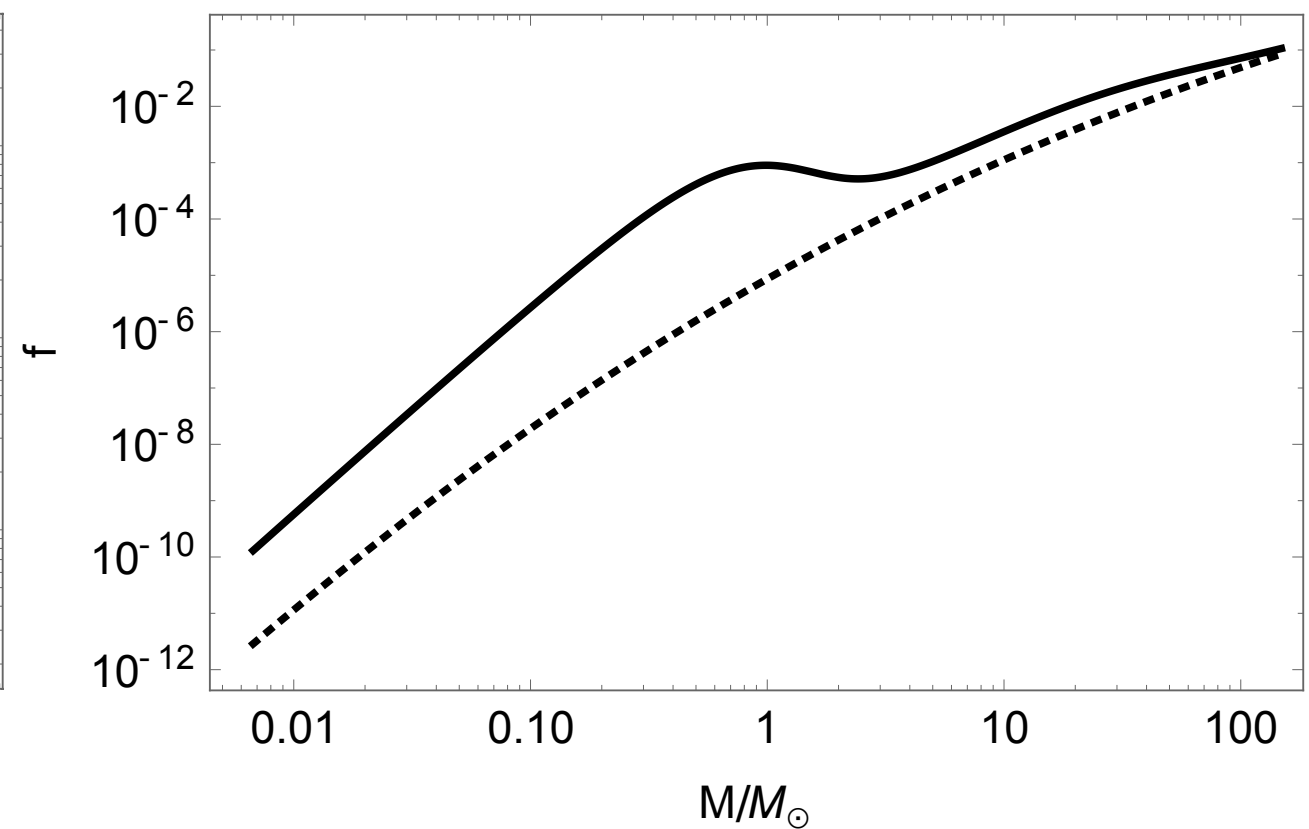
Varying the primordial perturbations

If the primordial power spectrum is not scale invariant on the relevant scales then the mass function changes, but a peak remains

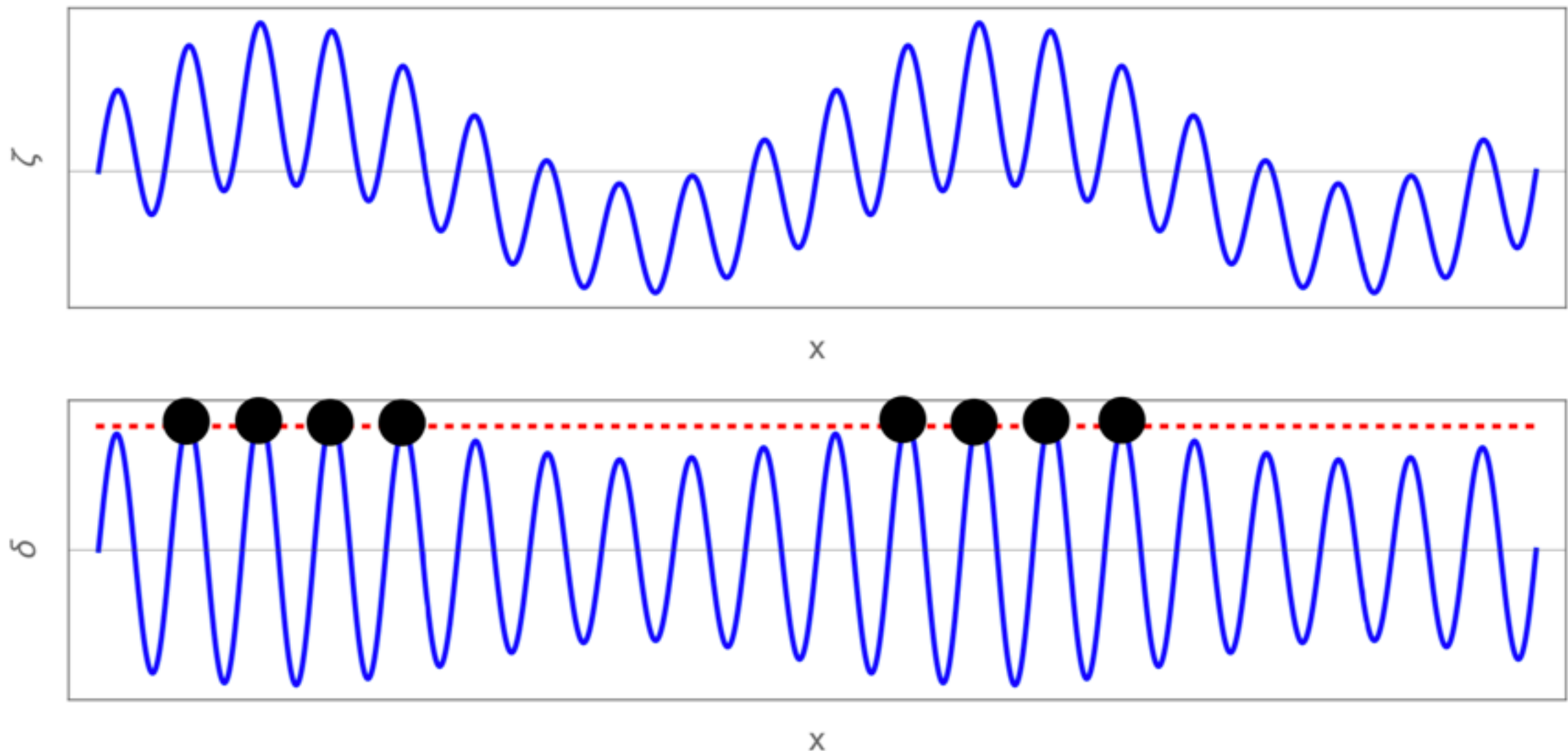
$$n_s - 1 = -0.05$$



$$n_s - 1 = -0.2$$



The impact of non-Gaussianity



Local non-Gaussianity boosts the PBH fraction and creates an initial spatial clustering
Suyama & Yokoyama 2019

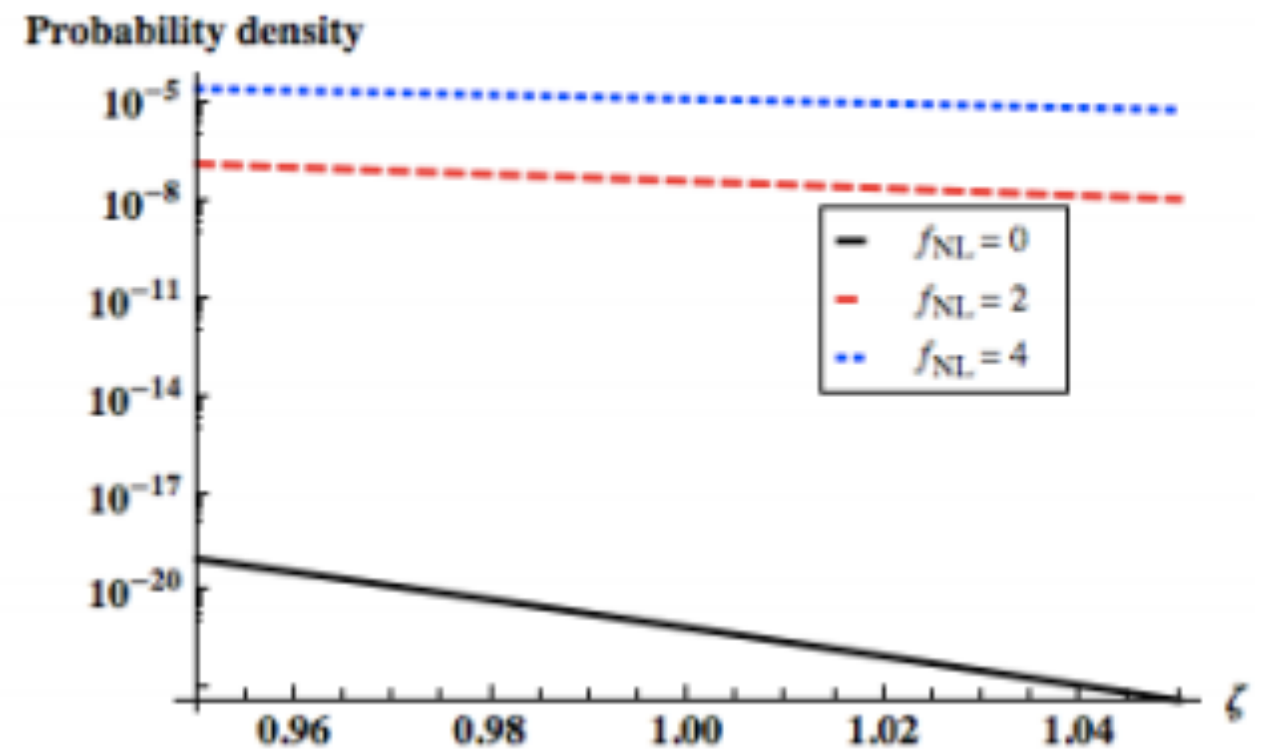
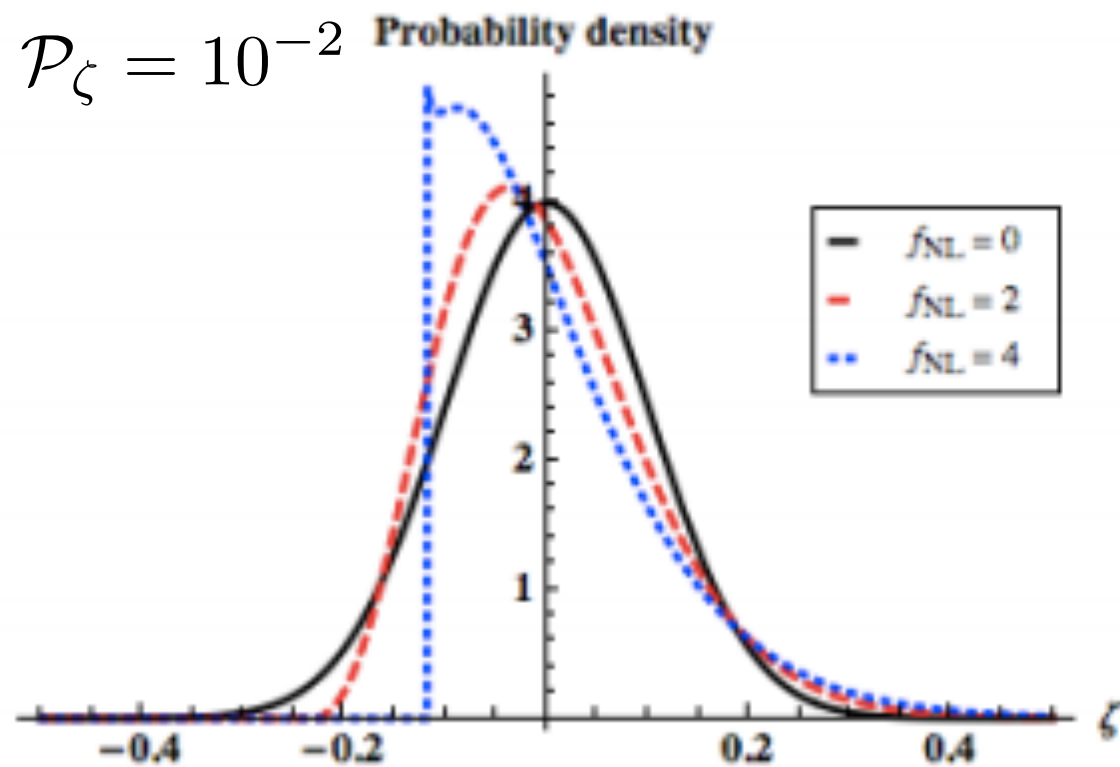
This (probably) increases the merger rate

It may also rule out the PBH scenario entirely, by generating a large **DM-photon isocurvature perturbation** - *Tada & Yokoyama 2015, Young & CB 2015*

PBH abundance is exponentially sensitive to non-Gaussianity

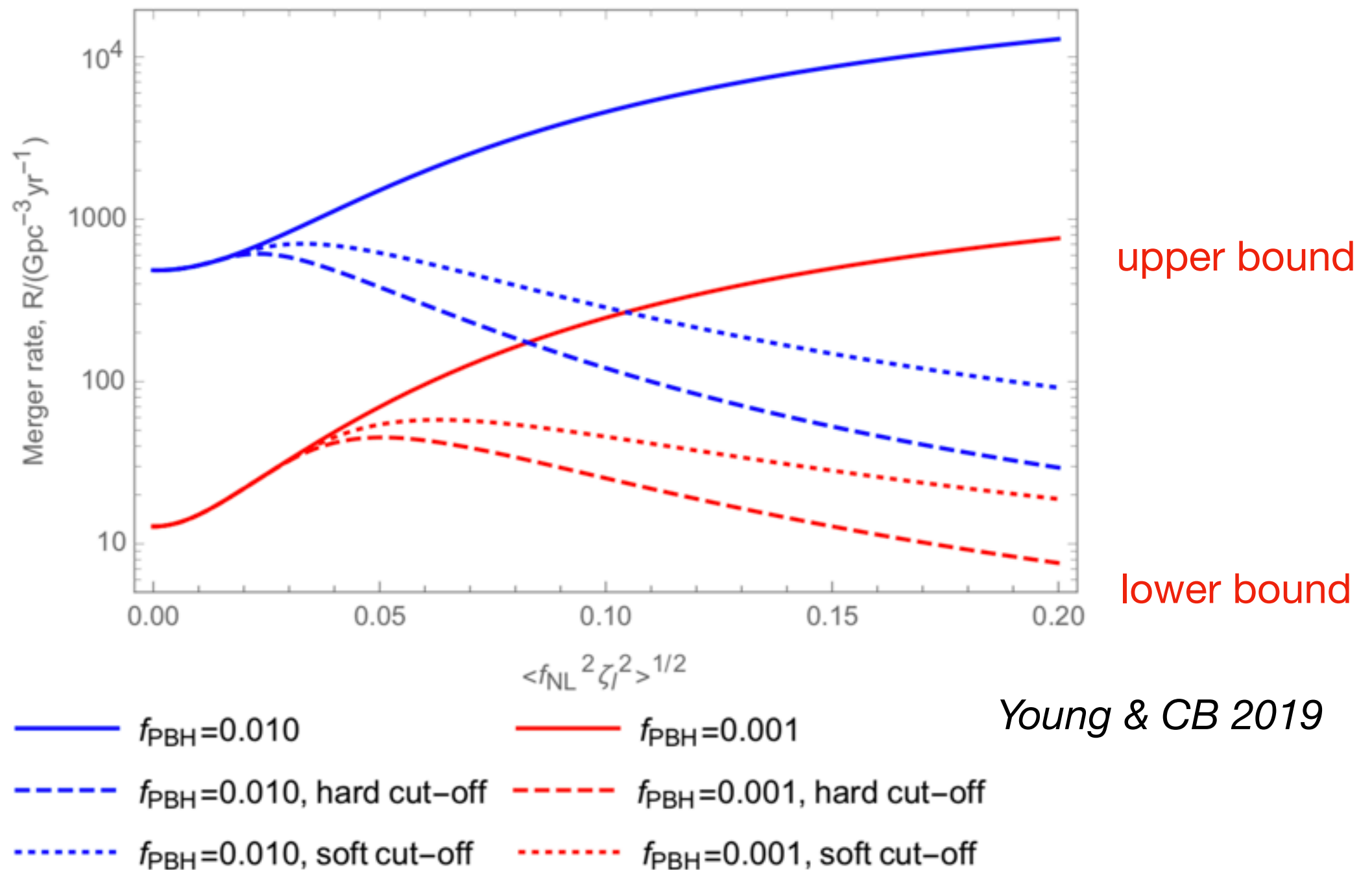
Local non-Gaussianity

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} (\zeta_g^2 - \sigma^2)$$



Young & CB 2013

Clustering and the merger rate



With non-Gaussianity => spatial clustering => large local PBH densities

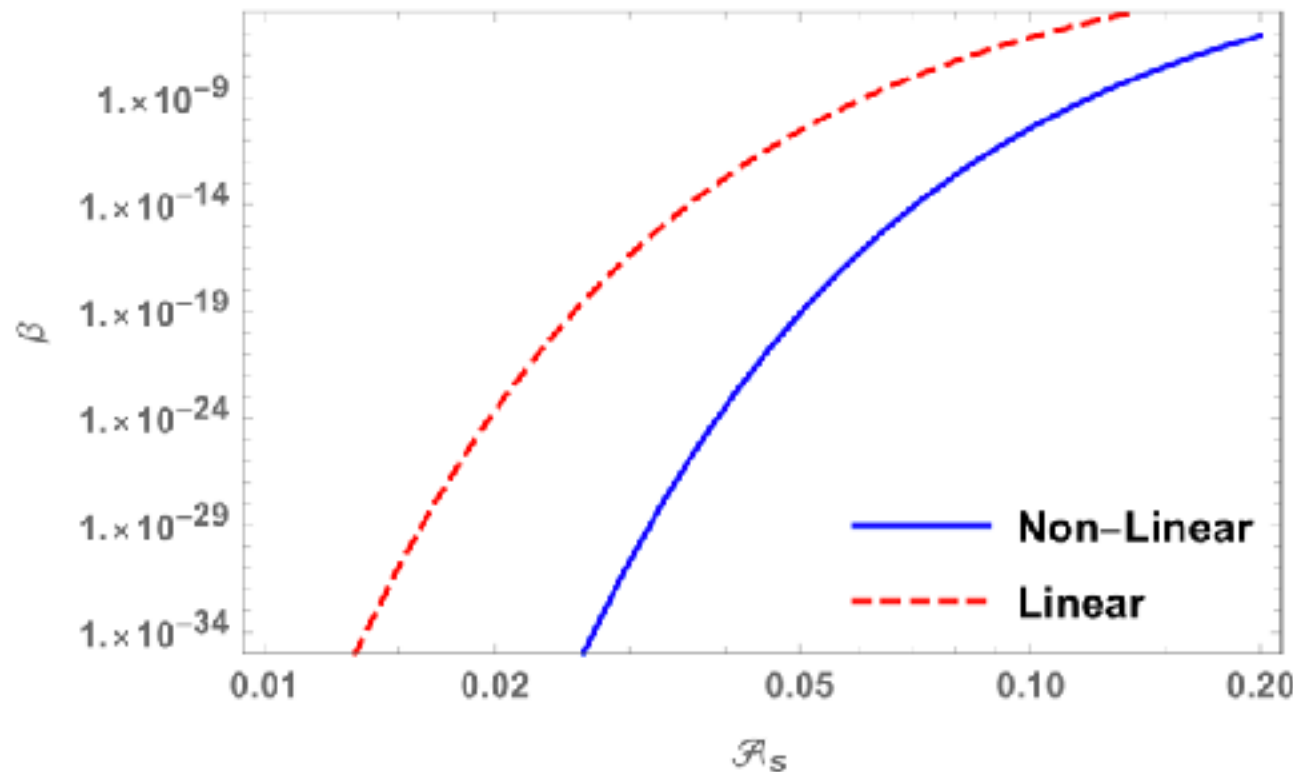
We don't know the merger rate in such cases - binaries are likely to be disrupted

One millionth of DM in PBHs may be large enough to explain the LIGO events

Non-Gaussianity take-away message

- Beware of invoking non-Gaussianity to “evade” constraints, since it introduces new challenges
- The PBH abundance, initial clustering, merger rate and isocurvature fraction are all very sensitive to non-Gaussianity
- Even slow-roll suppressed values of f_{NL} and τ_{NL} potentially rule out PBHs being the dark matter

Power spectrum constraints are weakened by a factor ~ 2



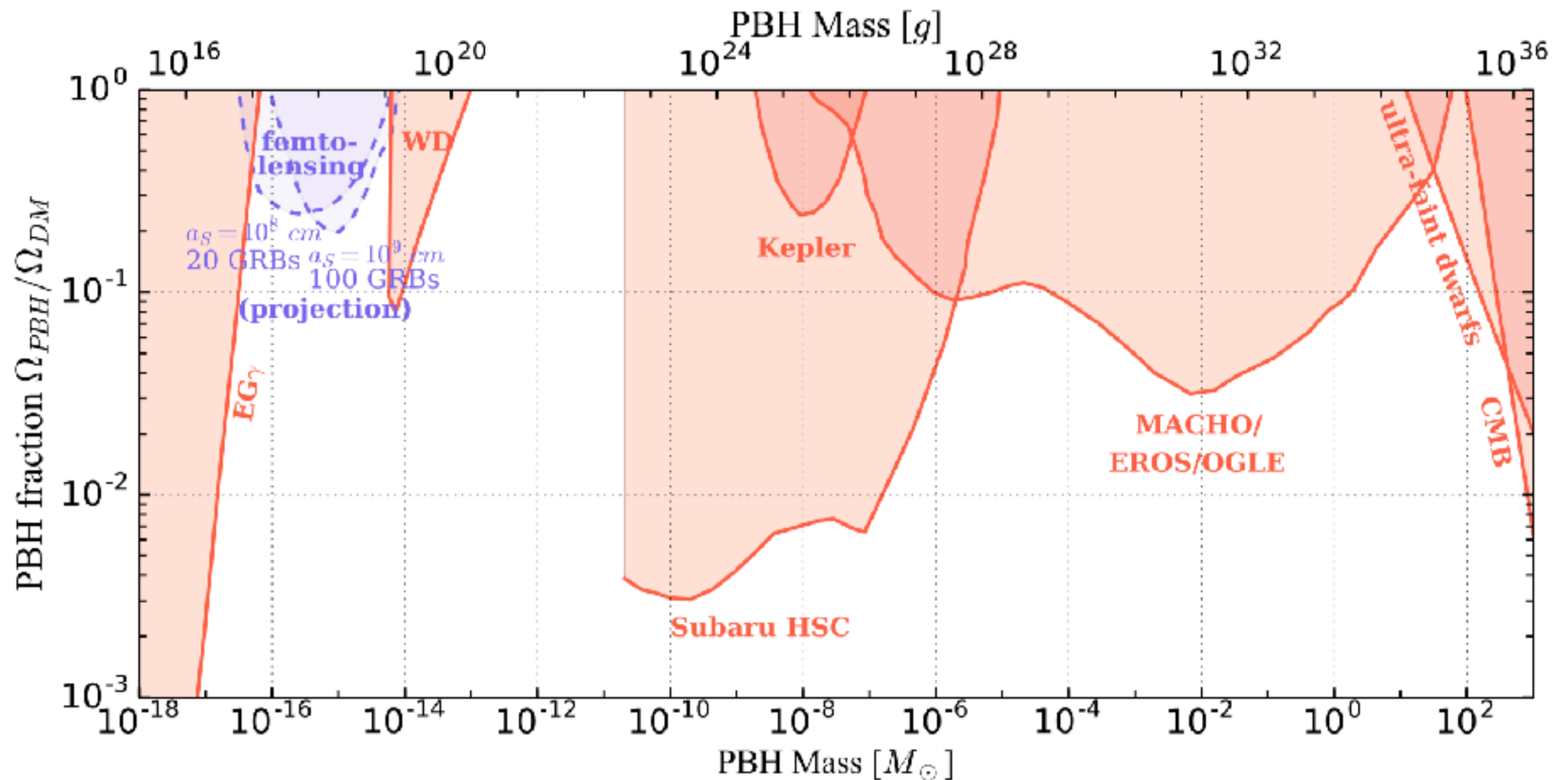
Delta function power spectrum
Young, Musco, CB '19

In order to generate the same number of PBHs when taking the non-linear (NL) relation into account, compared to the normal/wrong case that you use the linear relation, the power spectrum amplitude needs to increase by the ratio

$$1.5 \lesssim \frac{\mathcal{A}_{NL}}{\mathcal{A}_L} = \frac{16 \left(1 - \sqrt{\frac{2 - 3\delta_c}{2}} \right)^2}{9\delta_c^2} \lesssim 4$$

For the typical value of $\delta_c \sim 0.55$, power spectrum constraints are weakened by a factor of 2

Constraints on PBHs as DM



Katz et al 2018 “Femtolensing revisited”; see also Sasaki et al 2017 Review

The constraints have shifted and some have disappeared over time *Niikura et al `18*

Following *Raidal et al 2019* we consider a log-normal mass function with “central mass” $m_c=20 M_\odot$ and $\sigma=0.6$

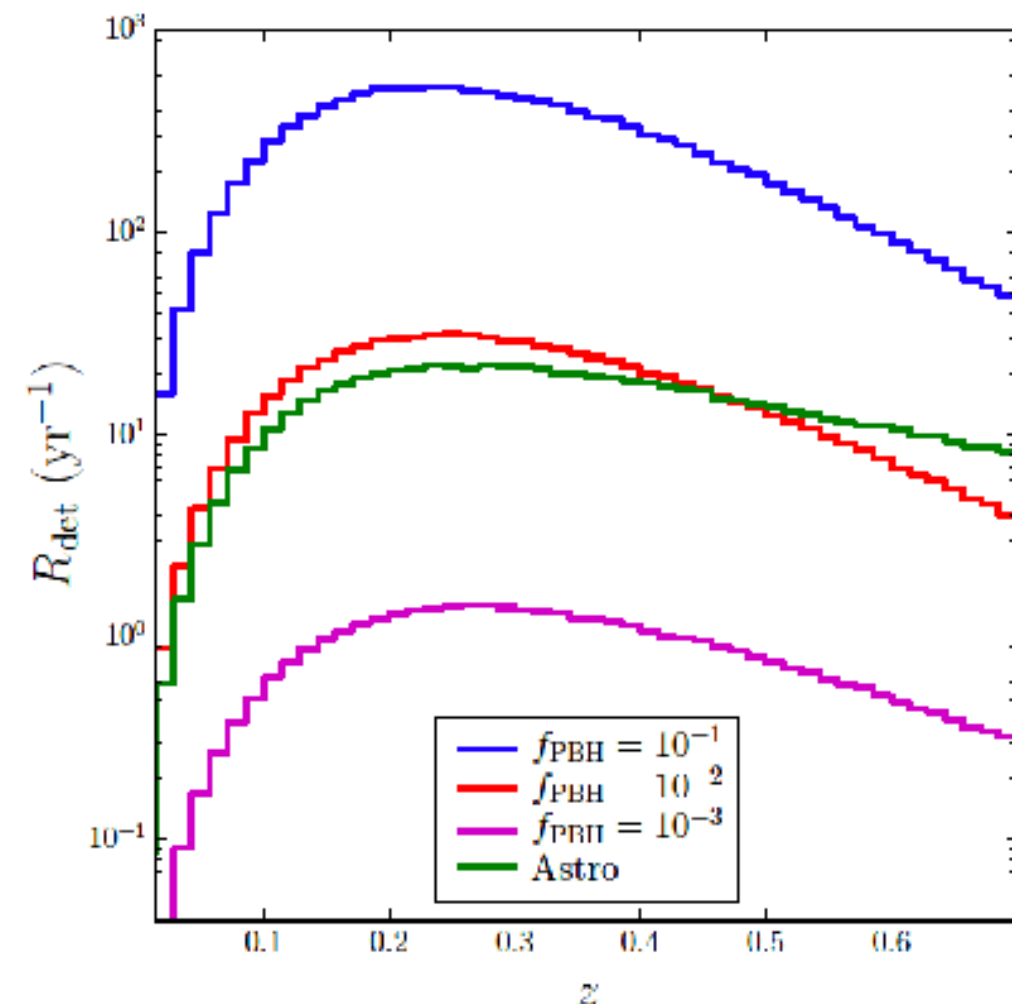
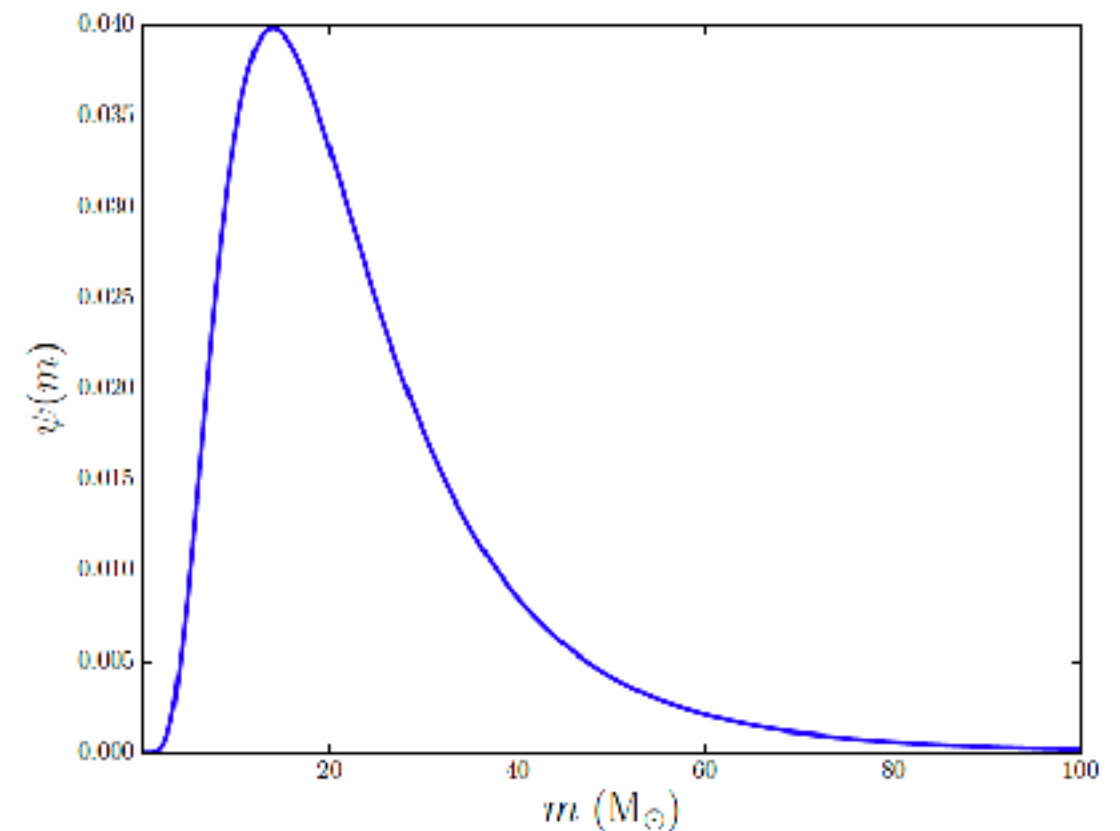
$$\psi(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left(-\frac{\ln^2(m/m_c)}{2\sigma^2}\right)$$

We match the expected astrophysical merger rate when about 1% of DM is in PBHs

The intrinsic BH merger rate estimated by LIGO assumes a mass function. We calculate the observed merger rate in order to avoid this assumption

Despite knowing the number density of binary stars, the “astro” prediction is very uncertain

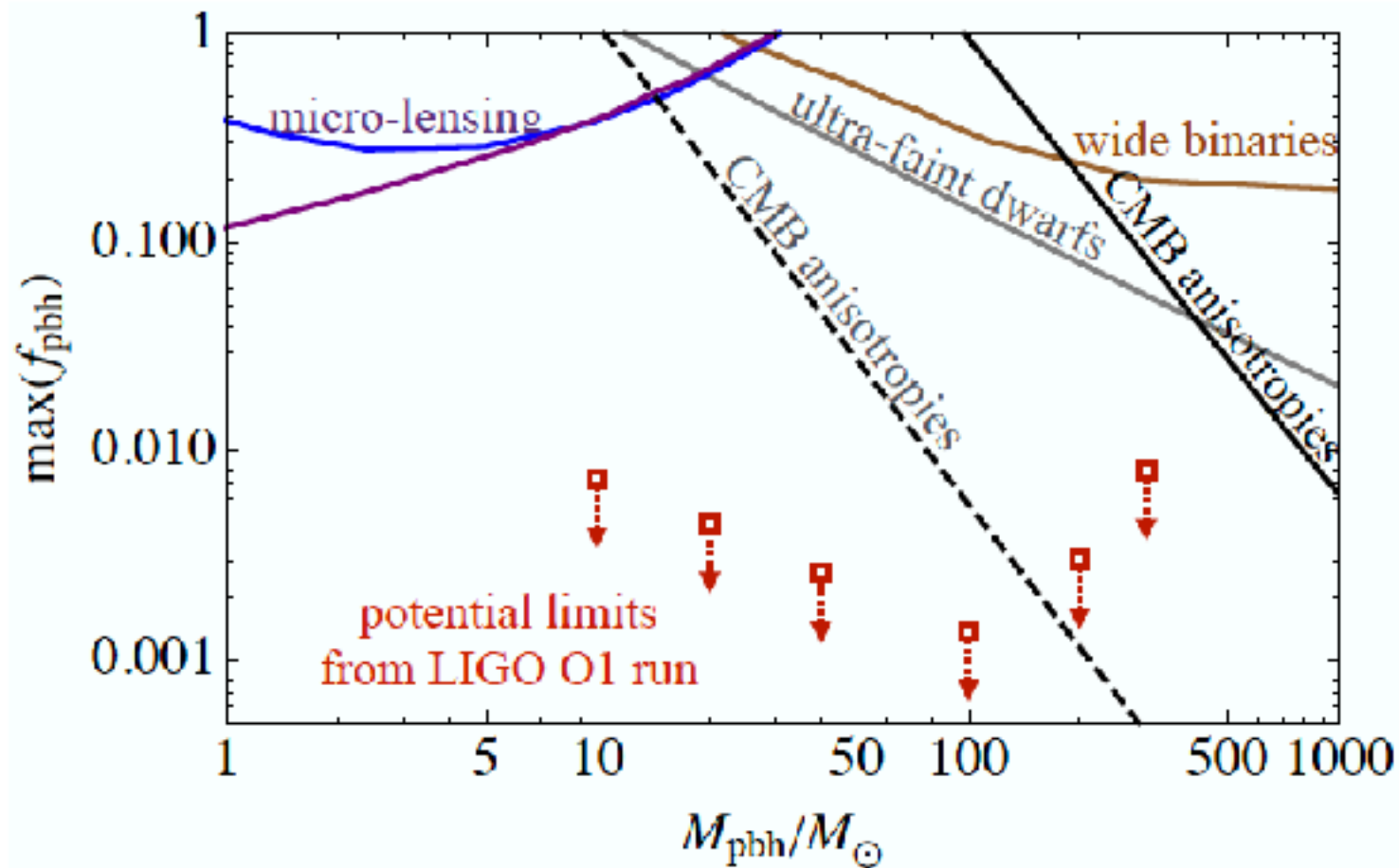
The astrophysical line in the plots is always based on *Gerosa et al 2019*



PBHs as DM

- There is (disputed) evidence that PBHs cannot form all of the dark matter unless $M \sim 10^{-15} - 10^{-11} M_{\text{sun}}$
- The heaviest DM candidate by 20-30 orders of magnitude
- Uniquely not a new particle
- Constraints normally assume an (unrealistic) monochromatic mass spectrum, but broad mass functions are not a “get out of jail free” card e.g. *Bellomo et al 2018*

The PBH merger rate places the tightest constraint

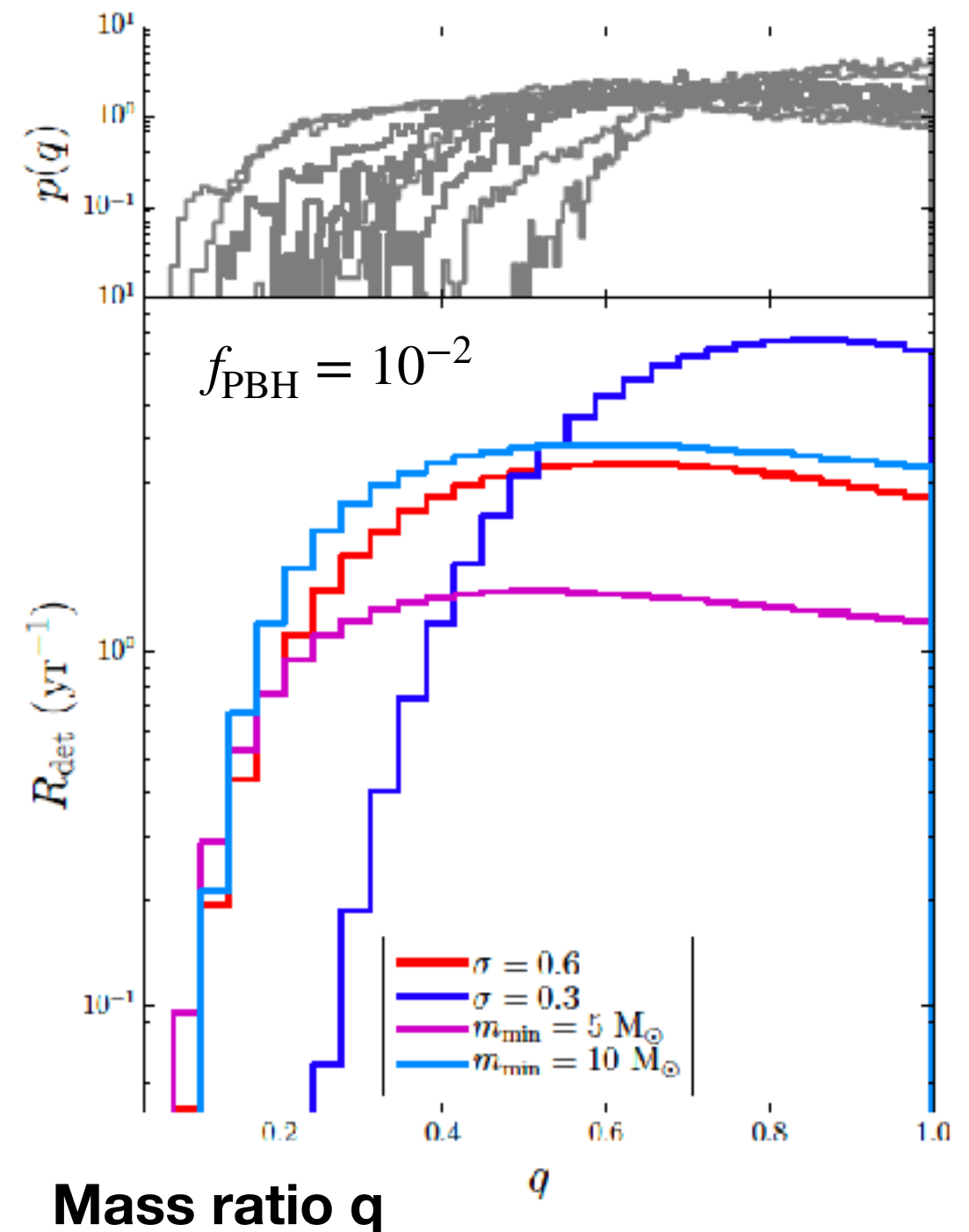
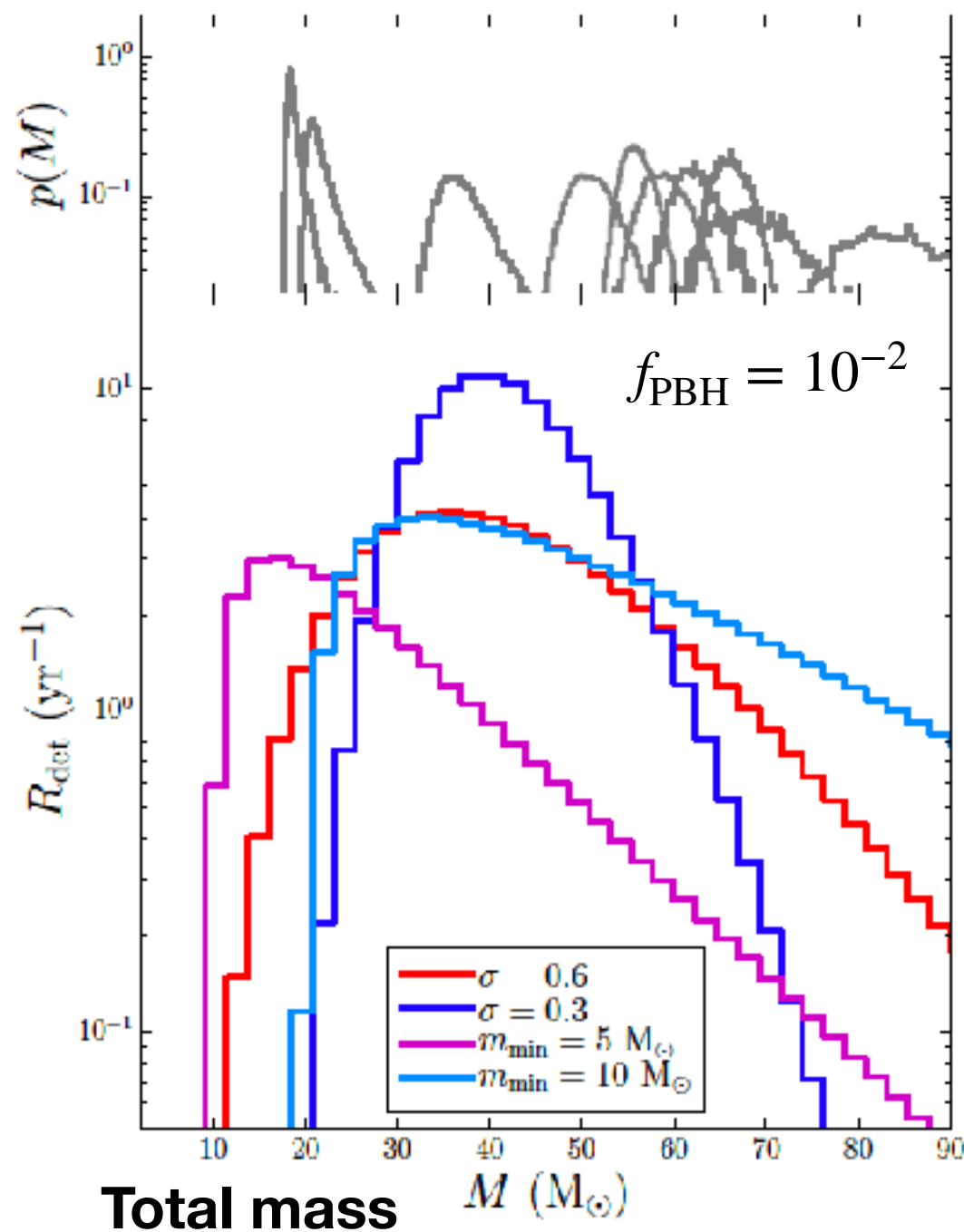


Haimoud et al 2017

- Caveats:

1. Assumes a monochromatic mass spectrum. Extended by *Chen & Huang '18, Raidal et al '18 ++*
2. Assumes PBHs are randomly placed initially, true if Gaussian initial conditions. Clustering does not help *Bringmann et al '18, Young & CB '19*
3. Assumes BH binaries are not disrupted. Recently tested to $z \sim 1000$ by simulations (*Raidal et al '18*) and even disrupted PBHs can merge *Vaskonen & Veermäe '19*
4. Neglects DM halo formation around the BHs. Not a big effect overall *Kavanagh, Gaggero & Bertone '18*

Wide enough to fit the masses, yet not so wide to stop $q \sim 1$



We use the LIGO 0102 sensitivity curves. 1/2 a year of data with 10 events.

Gow, CB, Hall, Peacock 2019

Primordial Black holes (PBHs)

- Are “black holes from the big bang”
- Naturally have the properties required of dark matter
- Lots of interesting phenomenology:
They could change early astrophysics, seed supermassive black holes, generate Hawking radiation...
- Require special initial conditions to form
- They might not exist, but are useful anyway

Zel'dovich and Novikov 1967; Hawking 1974; Carr and Hawking 1974