

# Dark Matter Caustics

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ICTS e-Conference

Less Travelled Path of Dark Matter:  
Axions and Primordial Black Holes

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The dark matter (WIMPs, axions, sterile neutrinos, ...) is thought to be cold and collisionless

If this is so,

- its velocity distribution at any physical location is a sum of discrete flows
- dark matter caustics, surfaces where the physical space density quasi-diverges, exist in galactic halos.



James R. Ipser

# Galactic halos live in phase-space

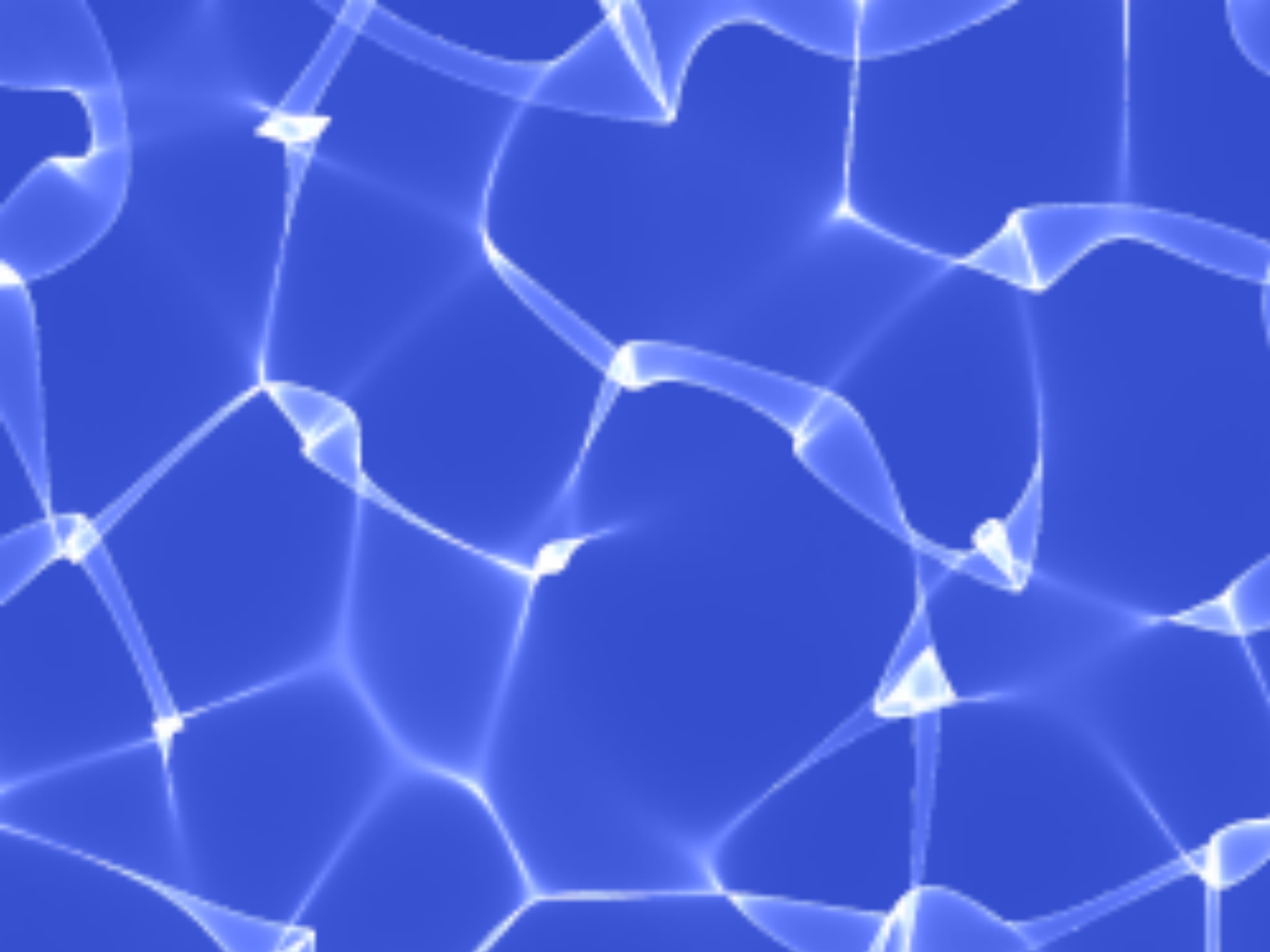
ordinary fluid

$$d(\vec{r}; t) \qquad \vec{v}(\vec{r}; t)$$

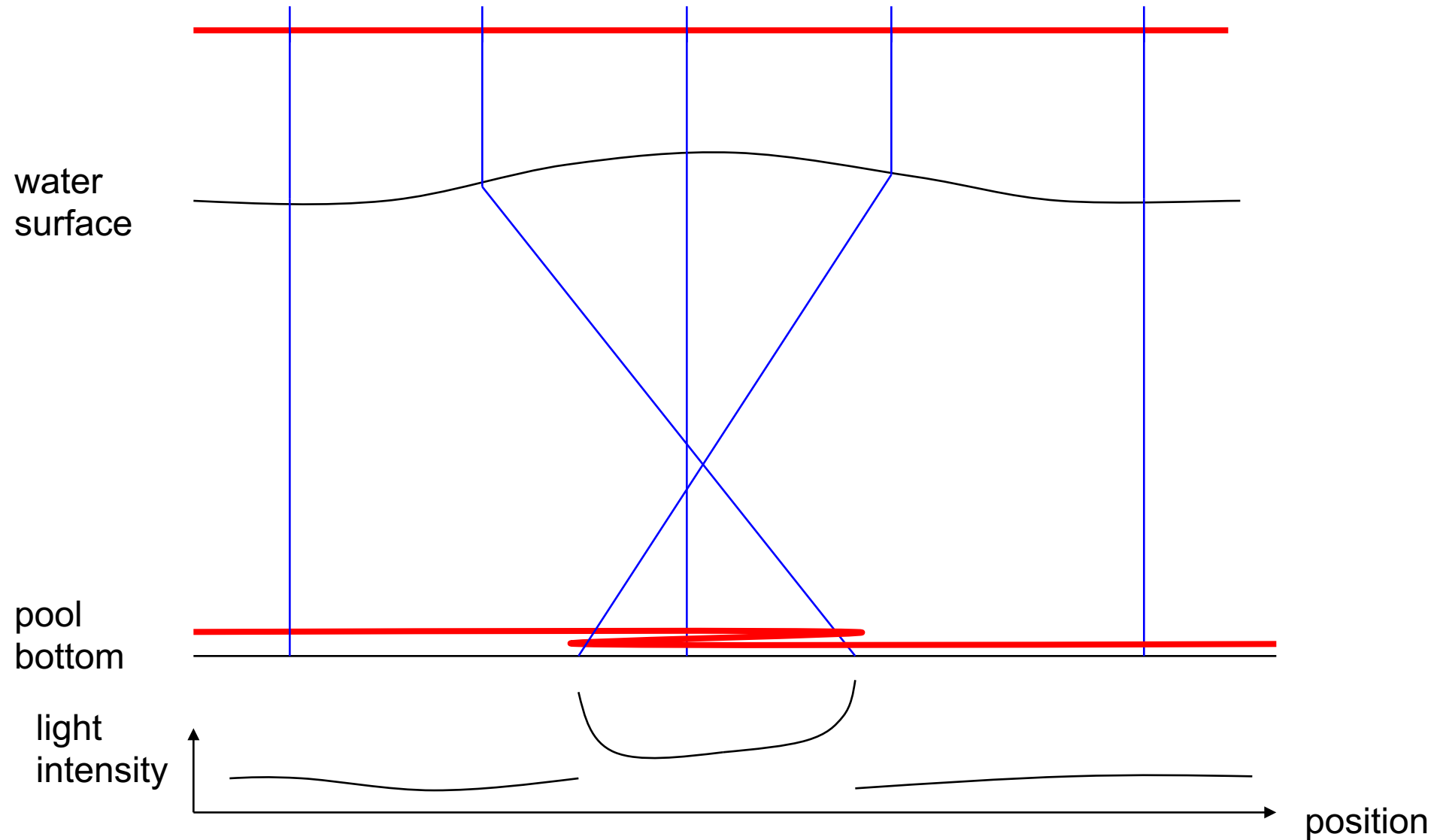
dark matter (collisionless) fluid

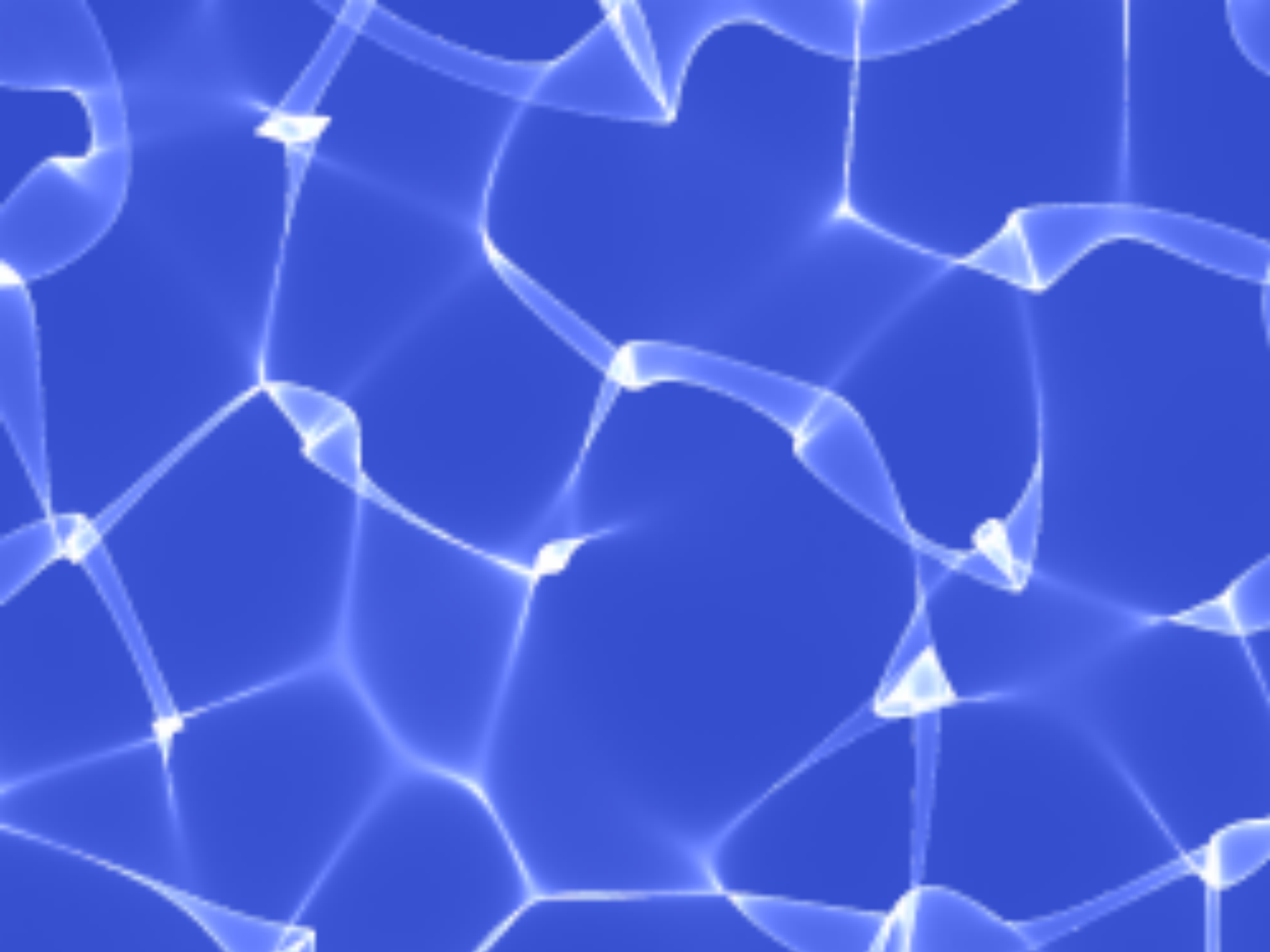
$$f(\vec{r}, \vec{v}; t)$$





# Caustics of light at the bottom of a swimming pool on a sunny breezy day





Caustics are common in the propagation of light  
(rainbows, the twinkling of stars, shimmering  
of the sea, gravitational lenses ... )

because

1. light is collisionless
2. in many cases, the flow of light  
has small velocity dispersion

Caustics are rare in the propagation of ordinary matter because of collisions.

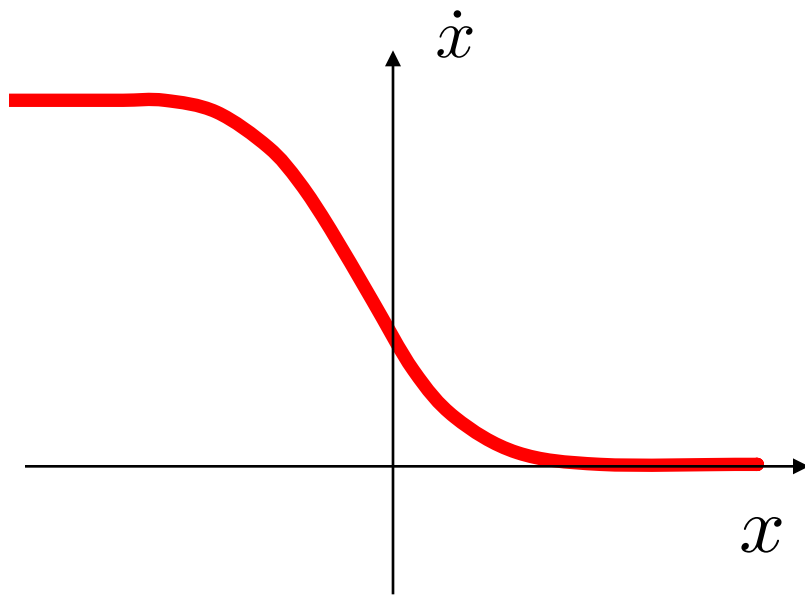
On the other hand, cold dark matter

1. is collisionless

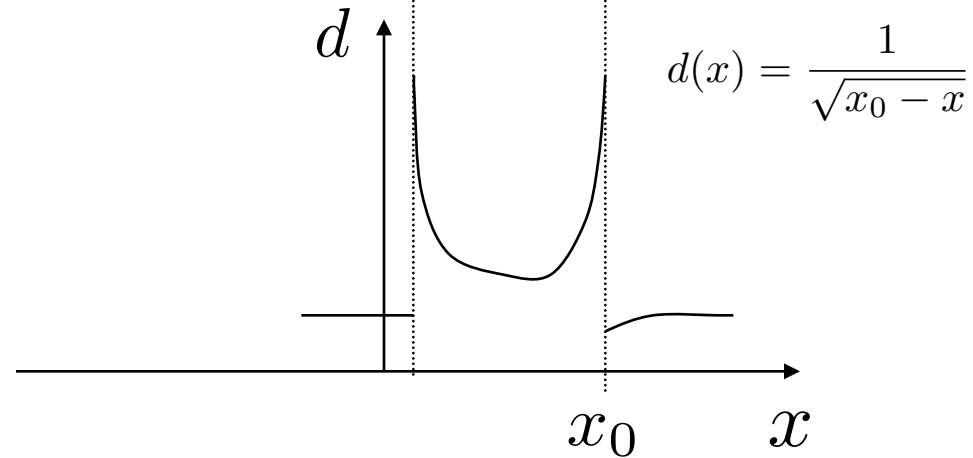
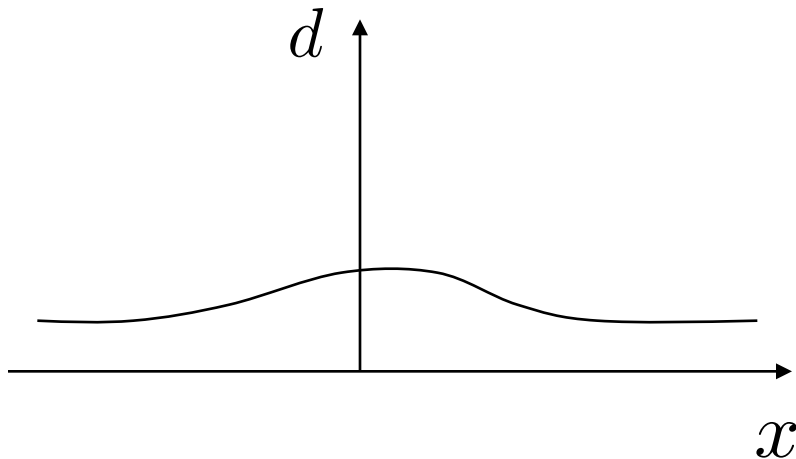
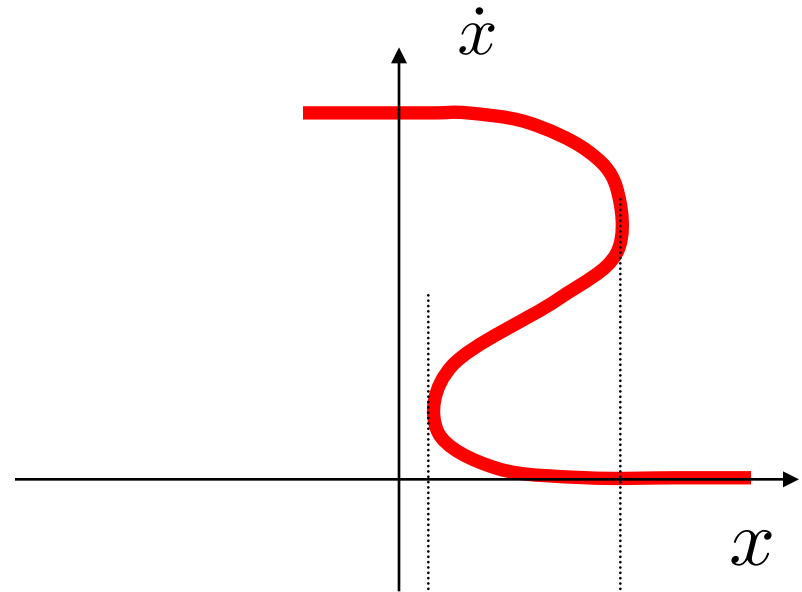
2. has negligible primordial velocity dispersion

hence will form caustics.

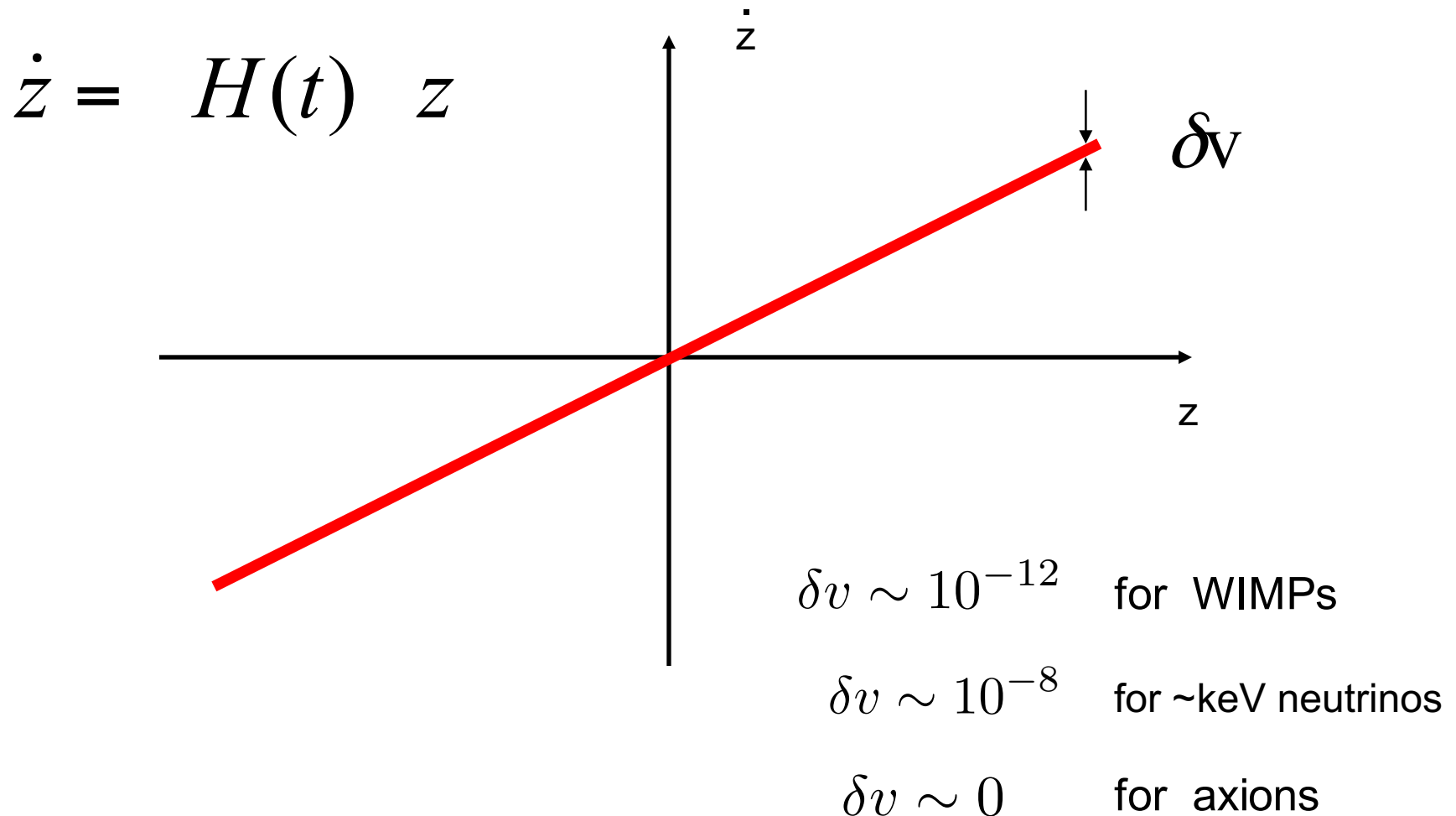
# Caustics in 2-dimensional phase-space



Dark matter particles are red here

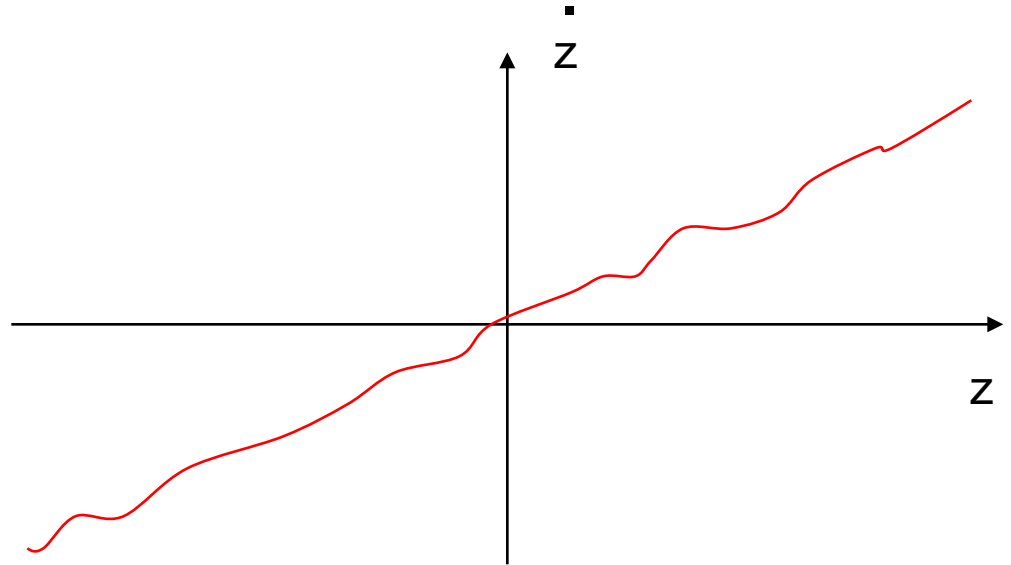


# Phase space distribution of CDM in a homogeneous universe



The cold dark matter particles lie on a  
3-dimensional sheet in  
6-dimensional phase-space

the physical density is  
the projection of the  
phase-space sheet onto  
position space

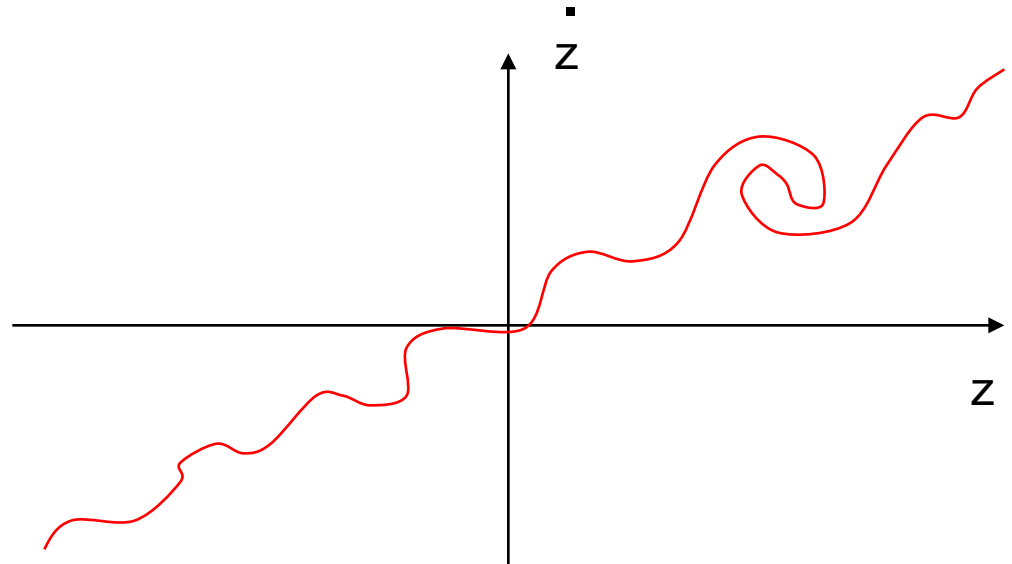


$$\vec{v}(\vec{r}, t) = H(t)\vec{r} + \Delta\vec{v}(\vec{r}, t)$$

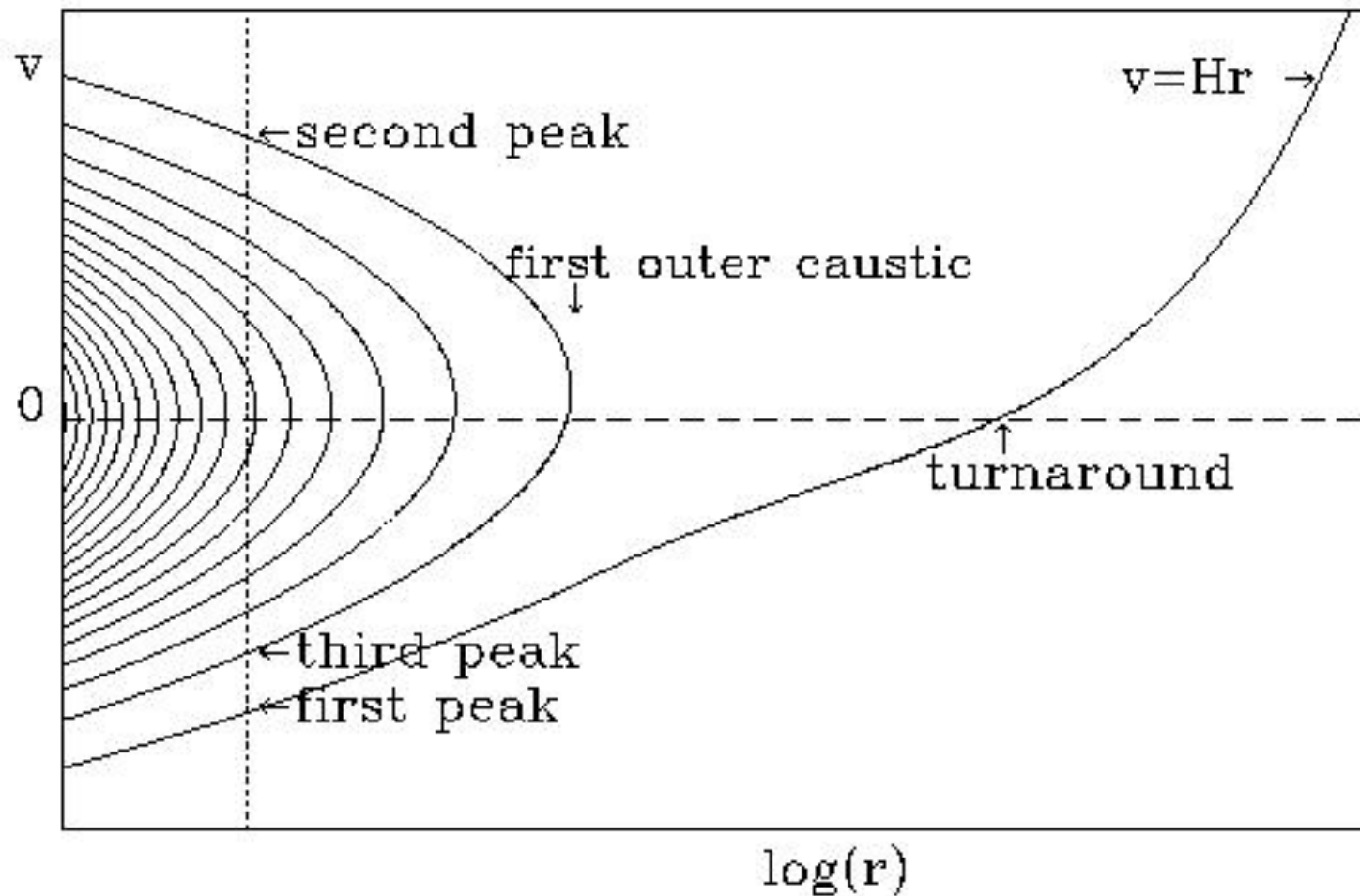


# The cold dark matter particles lie on a 3-dimensional sheet in 6-dimensional phase space

the physical density is  
the projection of the  
phase space sheet onto  
position space



# Phase space structure of spherically symmetric halos



# Implications:

1. At every point in physical space, the distribution of velocities is discrete, each velocity corresponding to a particular flow at that location.
2. At some locations in physical space, where the number of flows changes, there is a caustic, i.e. a surface where the density of dark matter is very high.

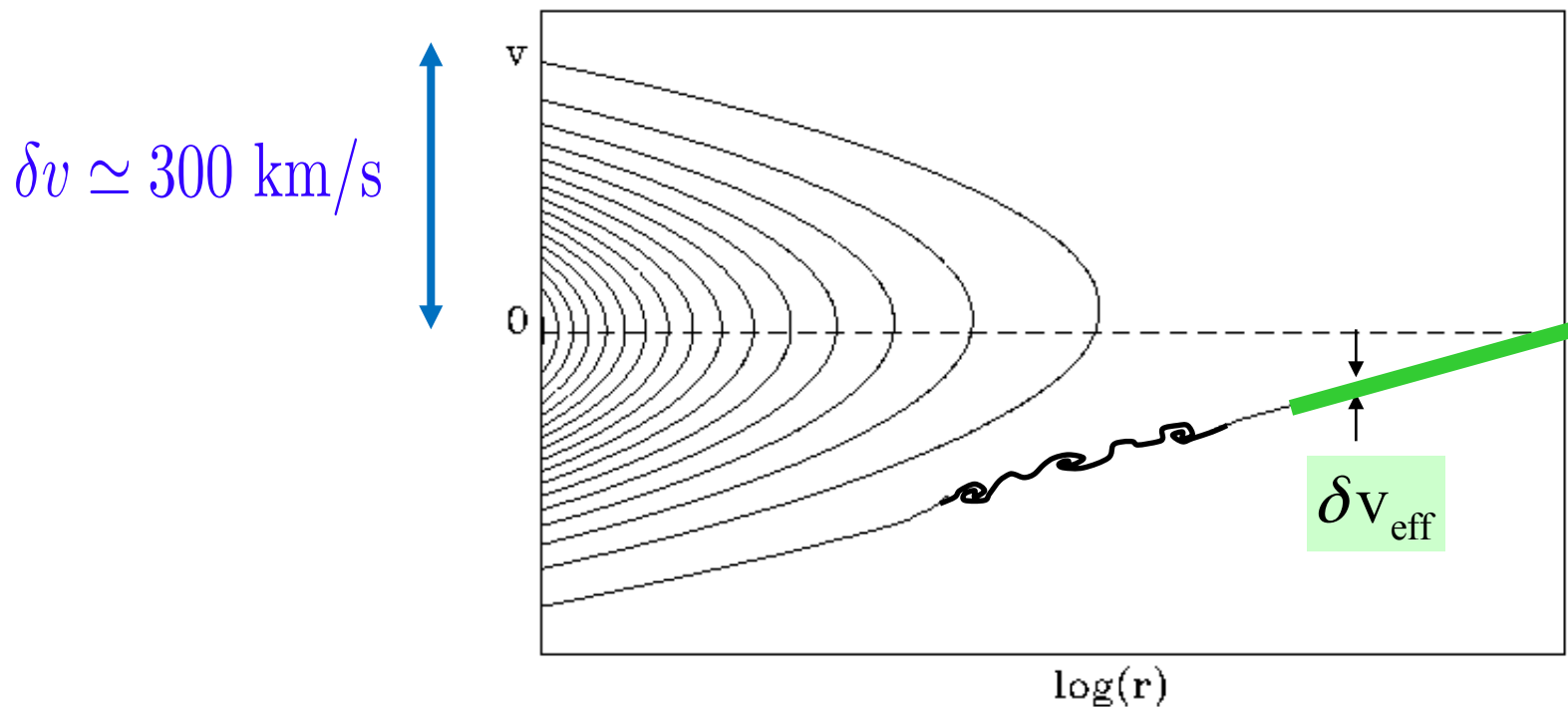
- the number of flows at our location in the Milky Way halo is of order 100
- small subhalos from hierarchical structure formation produce an effective velocity dispersion

$$\delta v_{\text{eff}} \lesssim 10 \text{ km/s}$$

but do not destroy the sheet structure in phase space

- the known inhomogeneities in the distribution of matter are insufficient to diffuse the flows by gravitational scattering
- present N-body simulations do not have enough particles to resolve the expected flows and caustics

# Hierarchical clustering introduces effective velocity dispersion



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Gravitational scattering by a class of objects of mass  $M$  and density  $n$  results in diffusion over a cone of opening angle  $\Delta\theta$  :

J. Ipser and PS, 1992

$$(\Delta\theta)^2 = 7 \times 10^{-7} \left( \frac{10^{-3}c}{v} \right)^3 \left( \frac{M}{M_{\odot}} \right)^2 \left( \frac{n}{\text{pc}^{-3}} \right) \left( \frac{t}{10^{10} \text{ year}} \right)$$

Main contributors: molecular clouds, globular clusters

$$(\Delta\theta)^2 \simeq 10^{-6} \times (\text{number of throughfalls})$$

- the number of flows at our location in the Milky Way halo is of order 100
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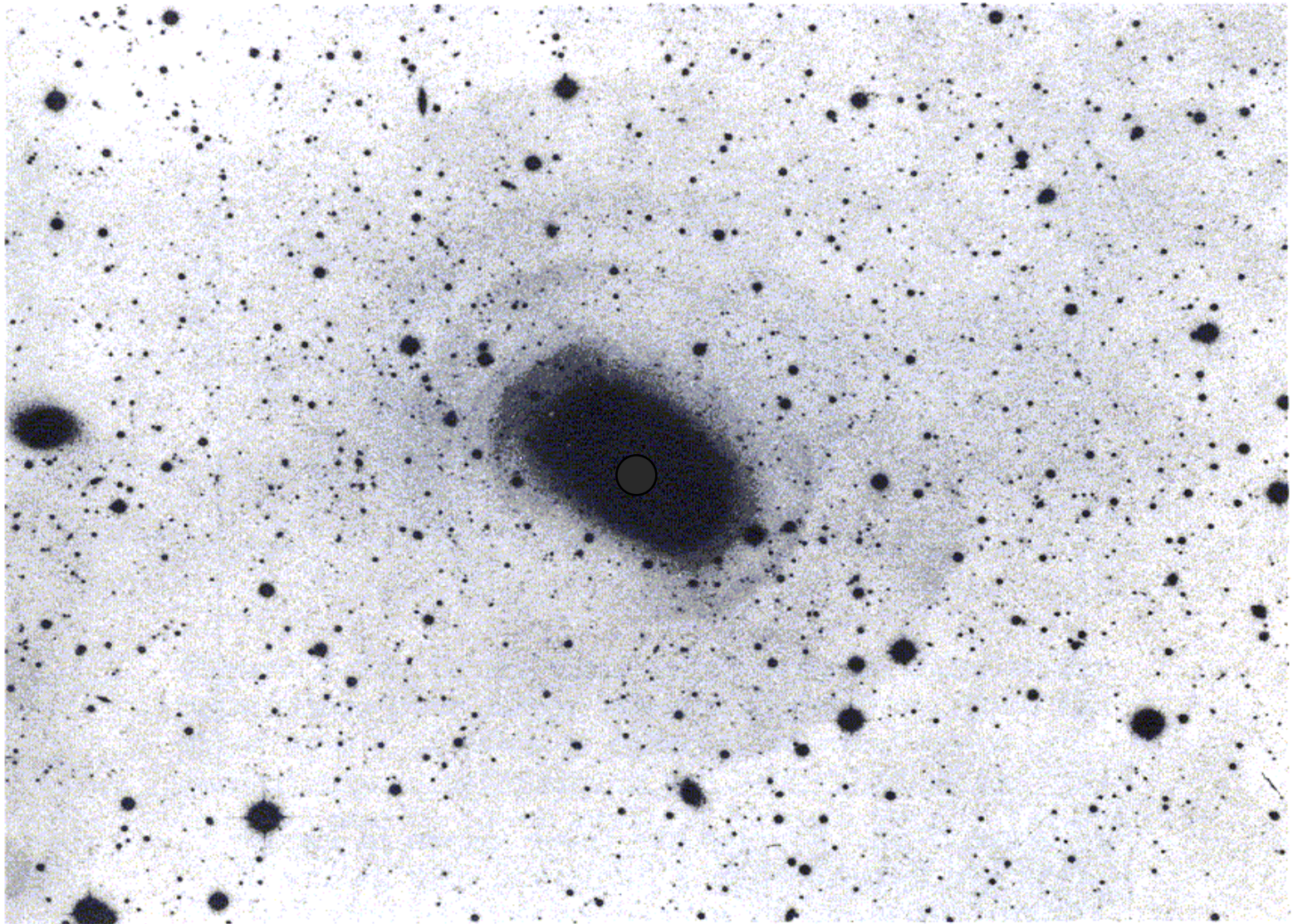
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$$N \sim 10^8$$

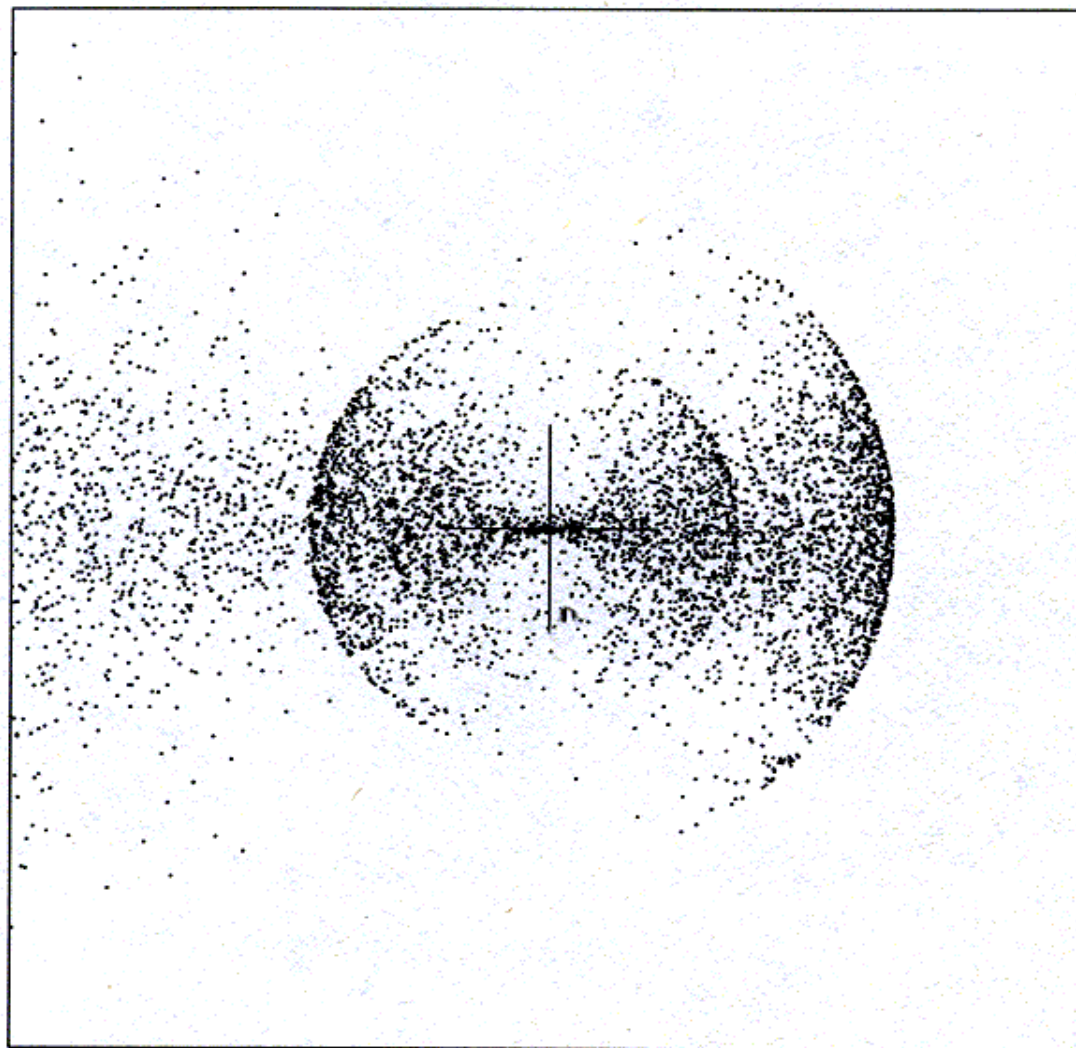
$$N^{\frac{1}{6}} \sim 21$$

$$M \sim 10^5 M_{\odot}$$





**Figure 7-22.** The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.  
(from Binney and Tremaine's book)



**Figure 7-23.** Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)



The flow of cold collisionless particles from all directions in and out of a region necessarily forms an inner caustic (Arvind Natarajan and PS, 2005).

Galactic halos have inner caustics as well as outer caustics.

If the initial velocity field is dominated by net overall rotation, the inner caustic is a 'tricusp ring'.

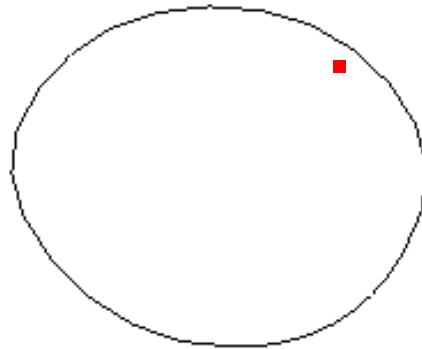
If the initial velocity field is irrotational, the inner caustic has a 'tent-like' structure.

A shell of particles, part of a continuous flow.

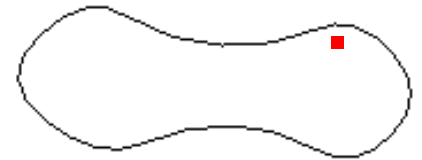
The shell has net angular momentum.

As the shell falls in and out of the galaxy, it turns itself inside out.

a)



b)



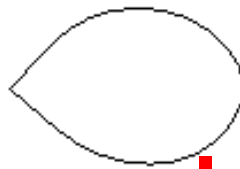
c)



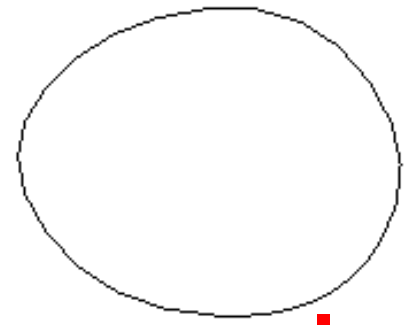
d)



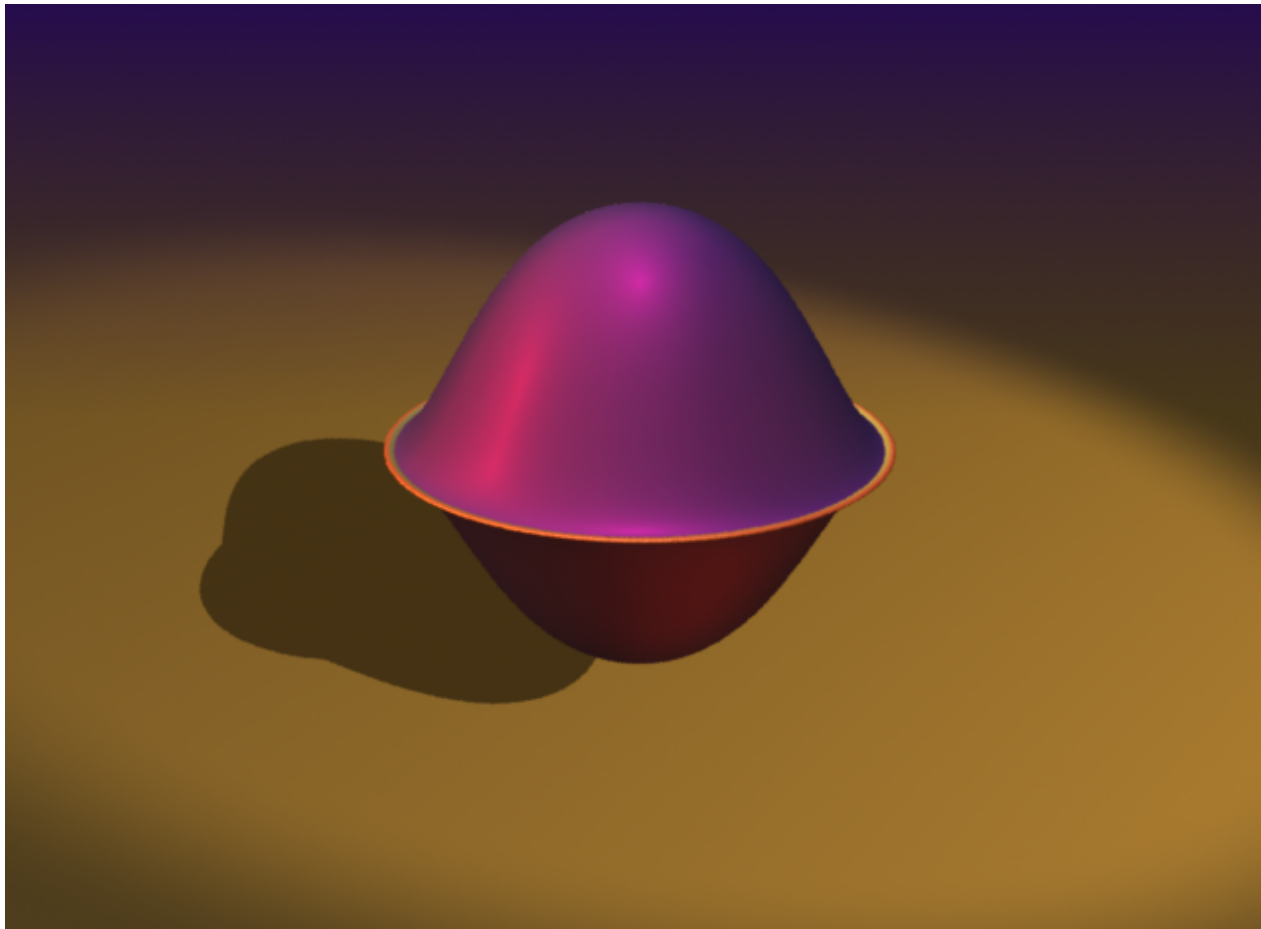
e)



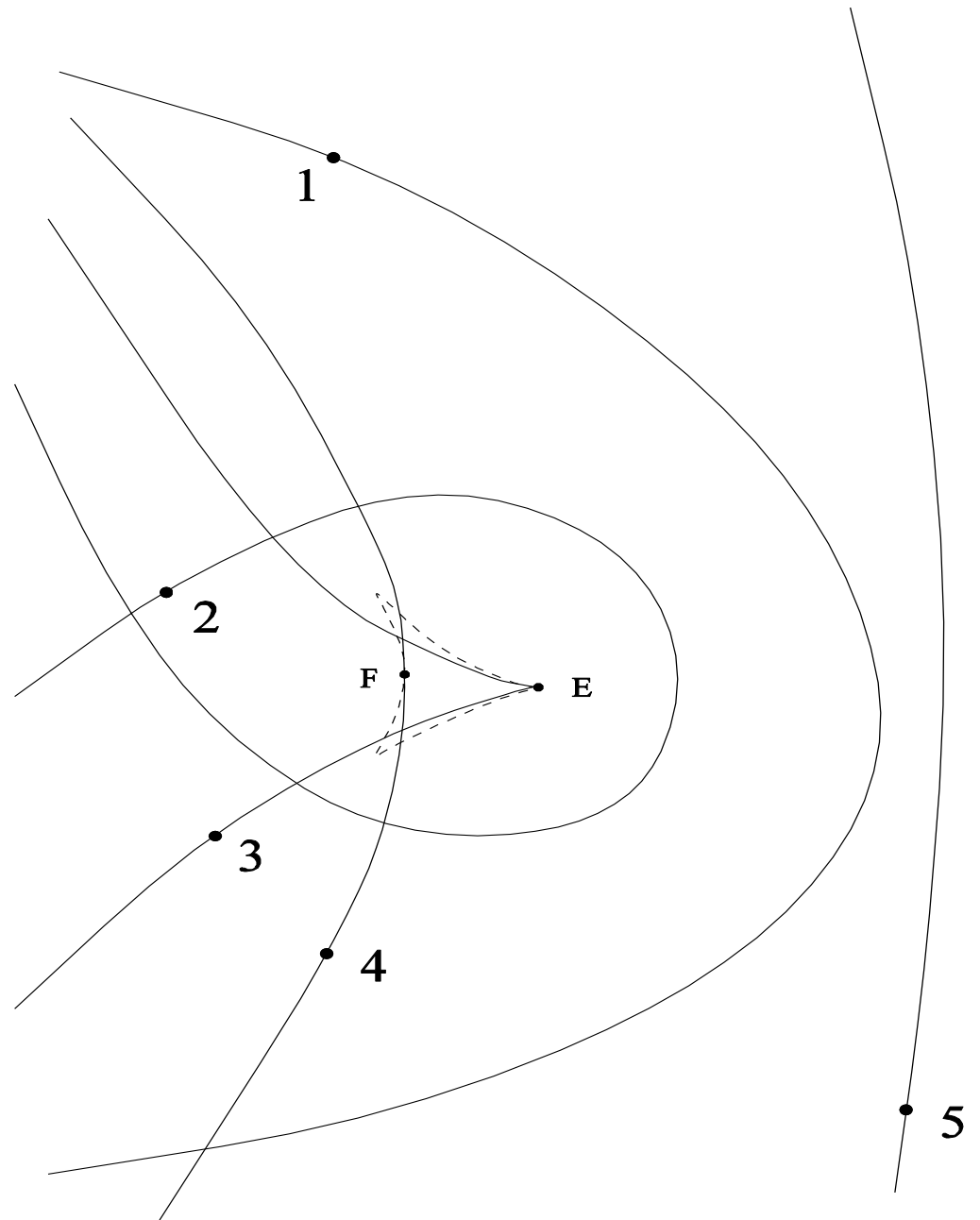
f)



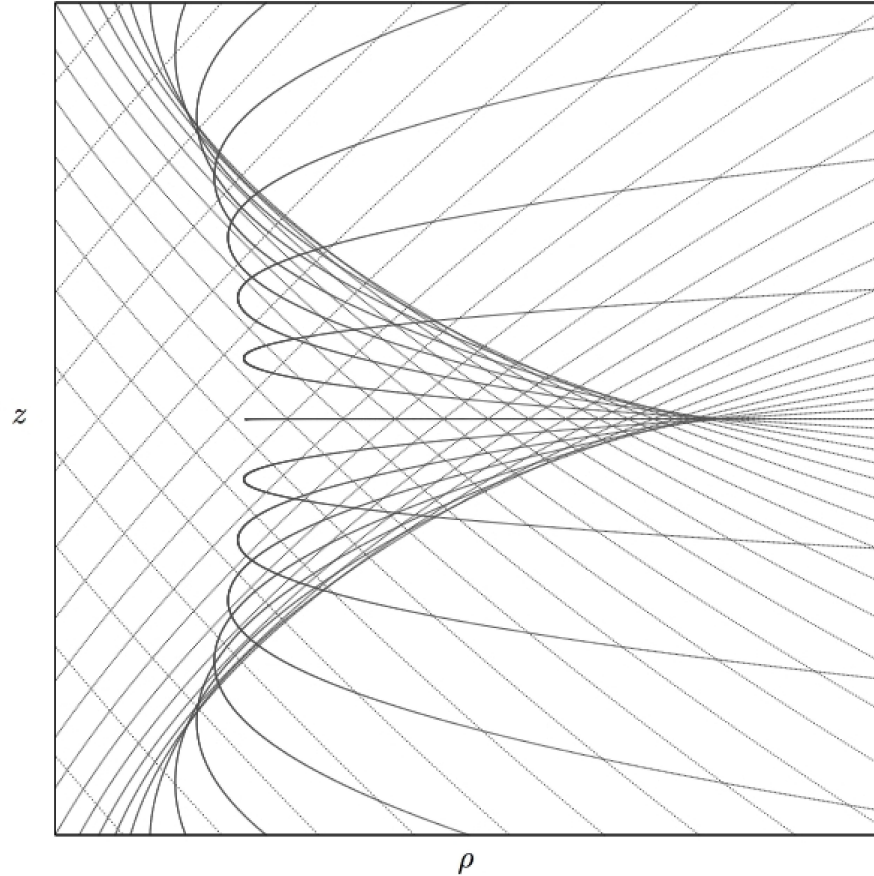
# Sphere turning inside out



A caustic forms  
where the particles  
with the most angular  
momentum are at their  
closest approach to  
the galactic center.



# The caustic ring cross-section



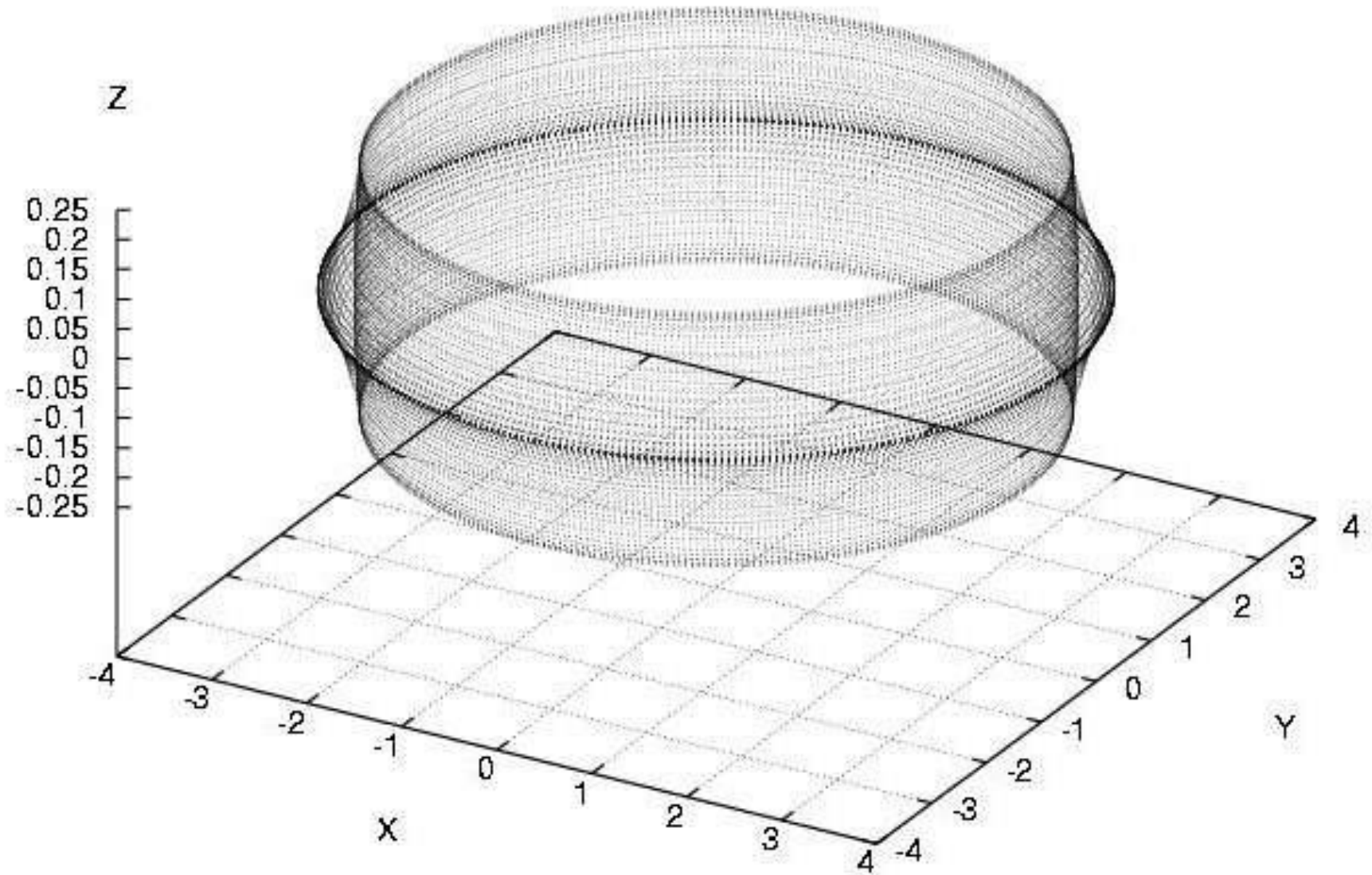
$D_{-4}$

an elliptic umbilic catastrophe

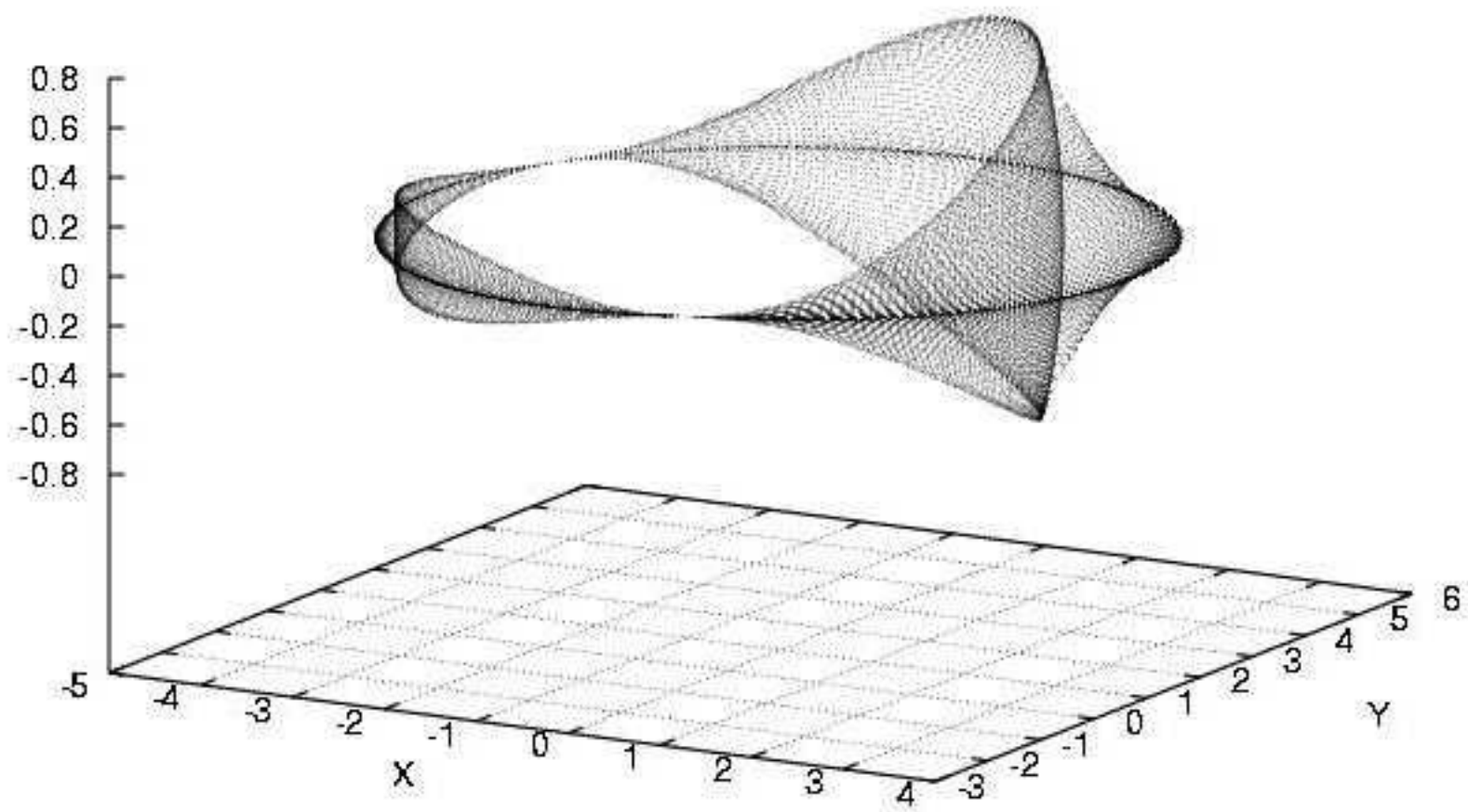


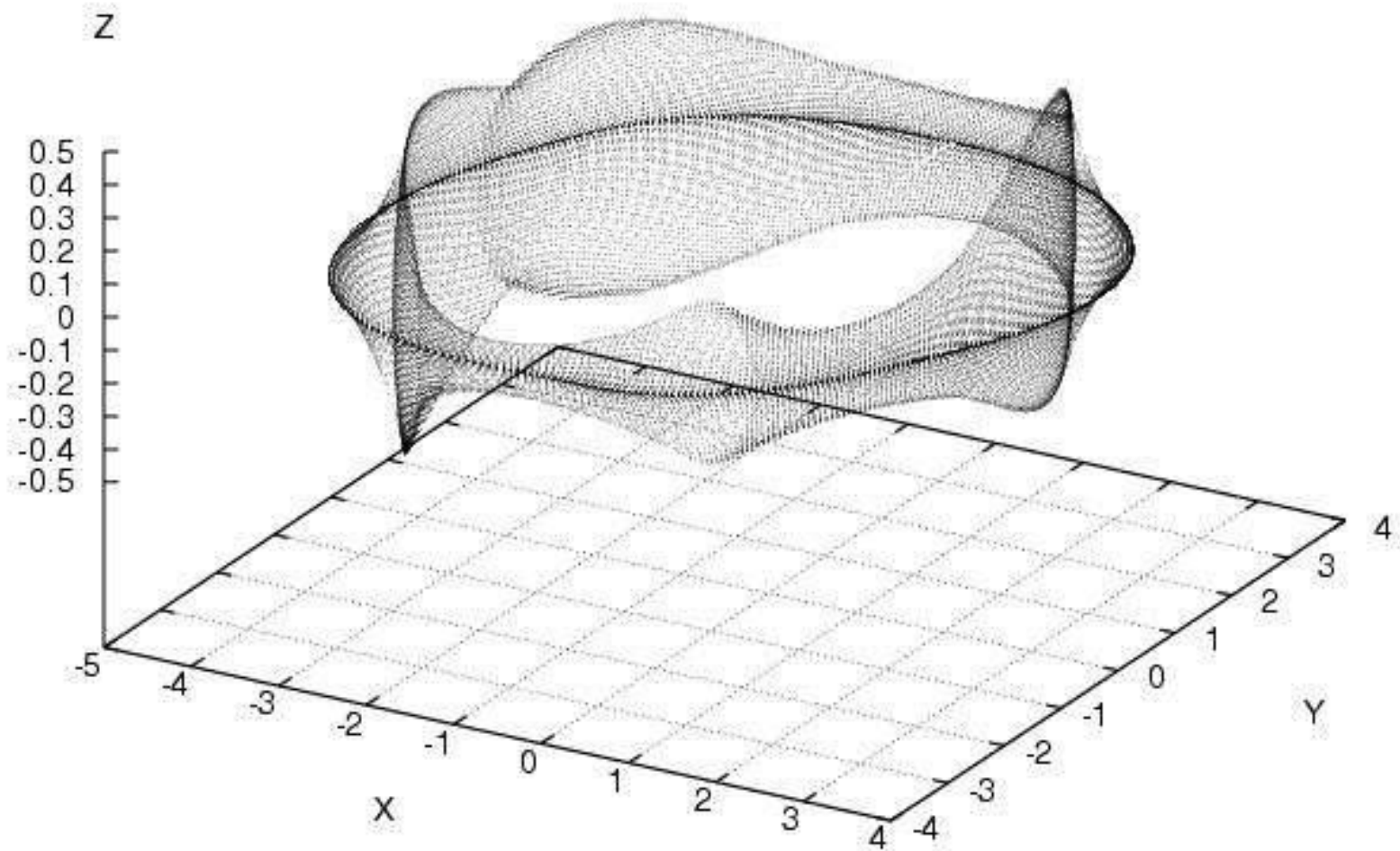


simulations by Arvind Natarajan



$z$





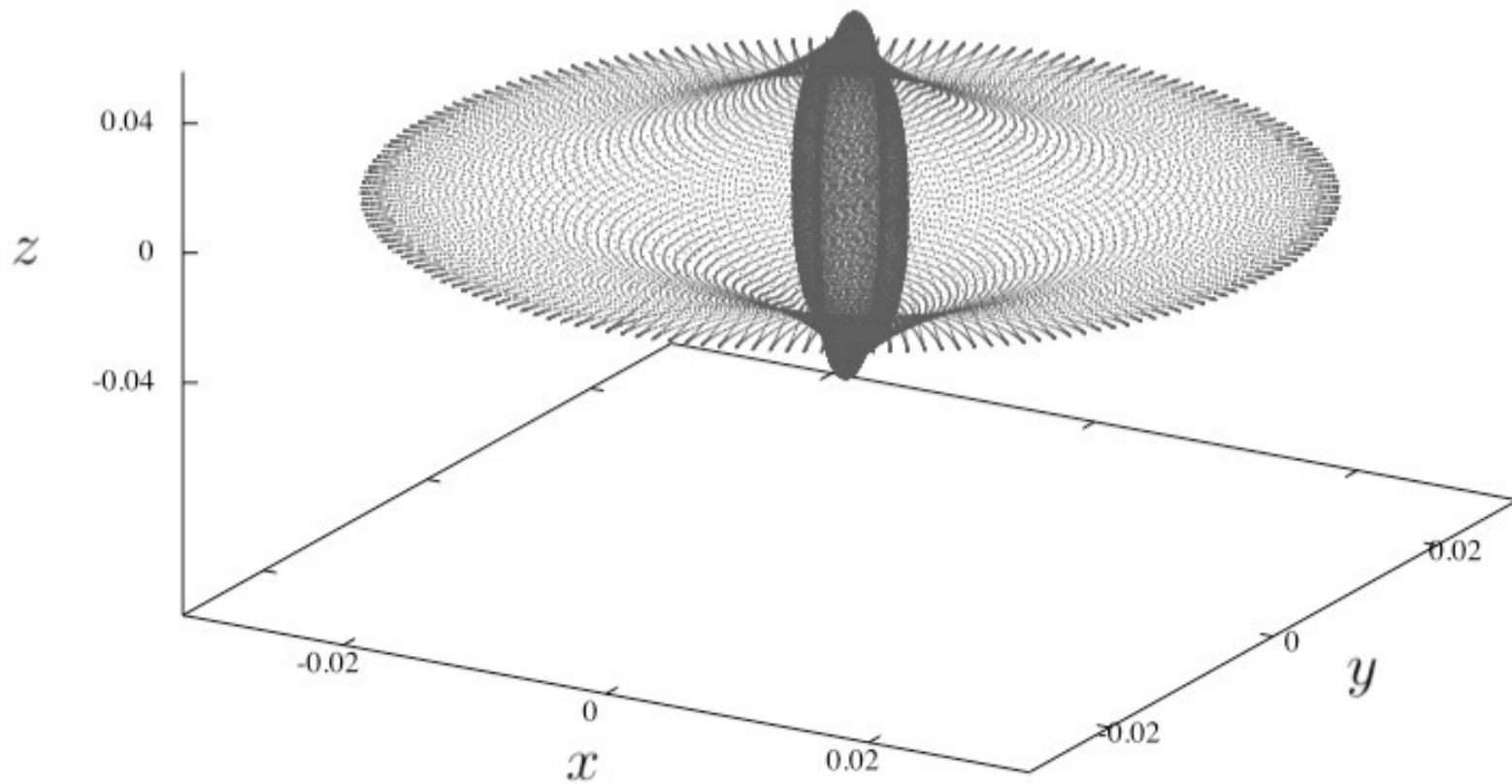
The flow of cold collisionless particles from all directions in and out of a region necessarily forms an inner caustic (Arvind Natarajan and PS, 2005).

Galactic halos have inner caustics as well as outer caustics.

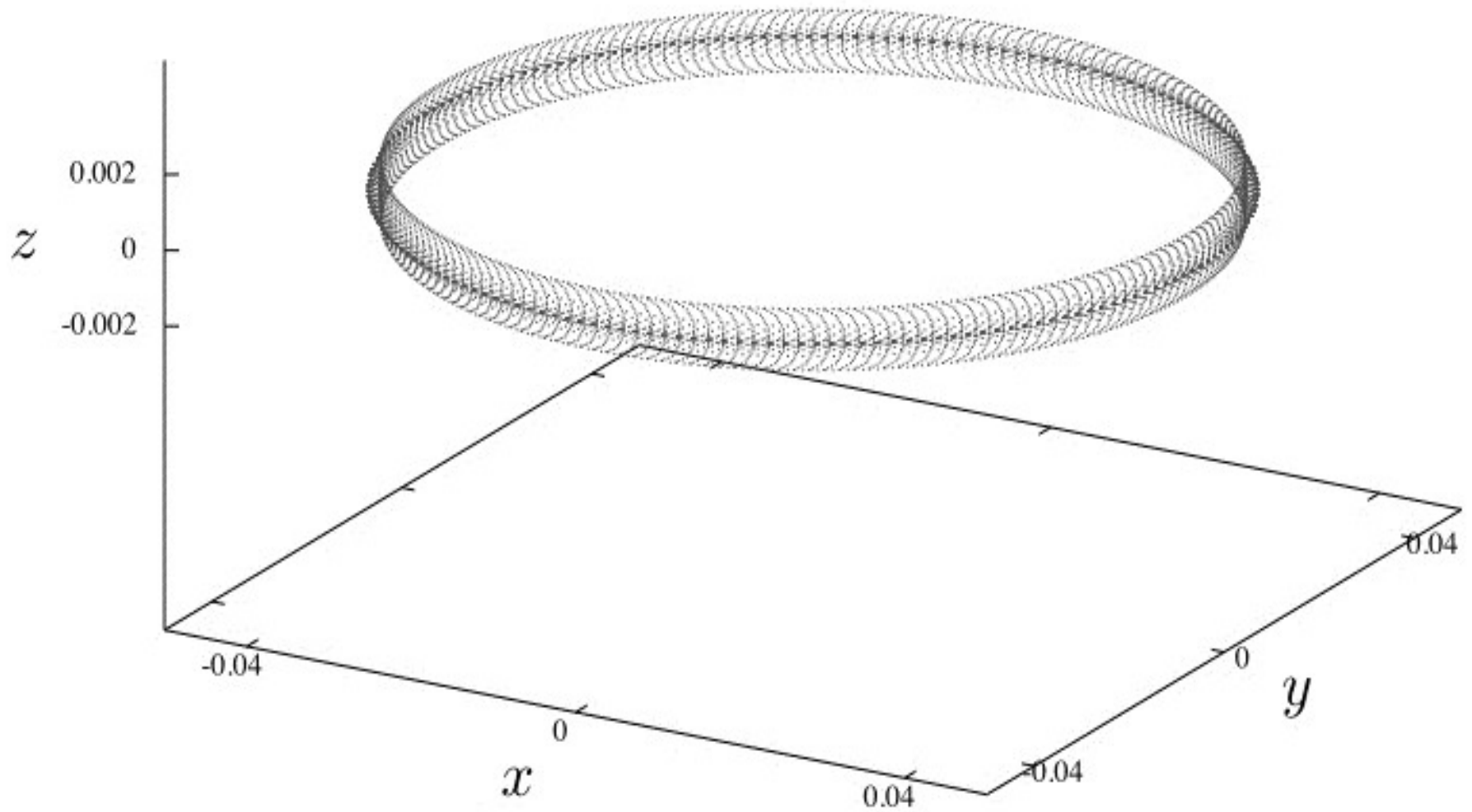
If the initial velocity field is dominated by net overall rotation, the inner caustic is a 'tricusp ring'.

If the initial velocity field is irrotational, the inner caustic has a 'tent-like' structure.

in case of irrotational flow



in case of rotational flow



On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

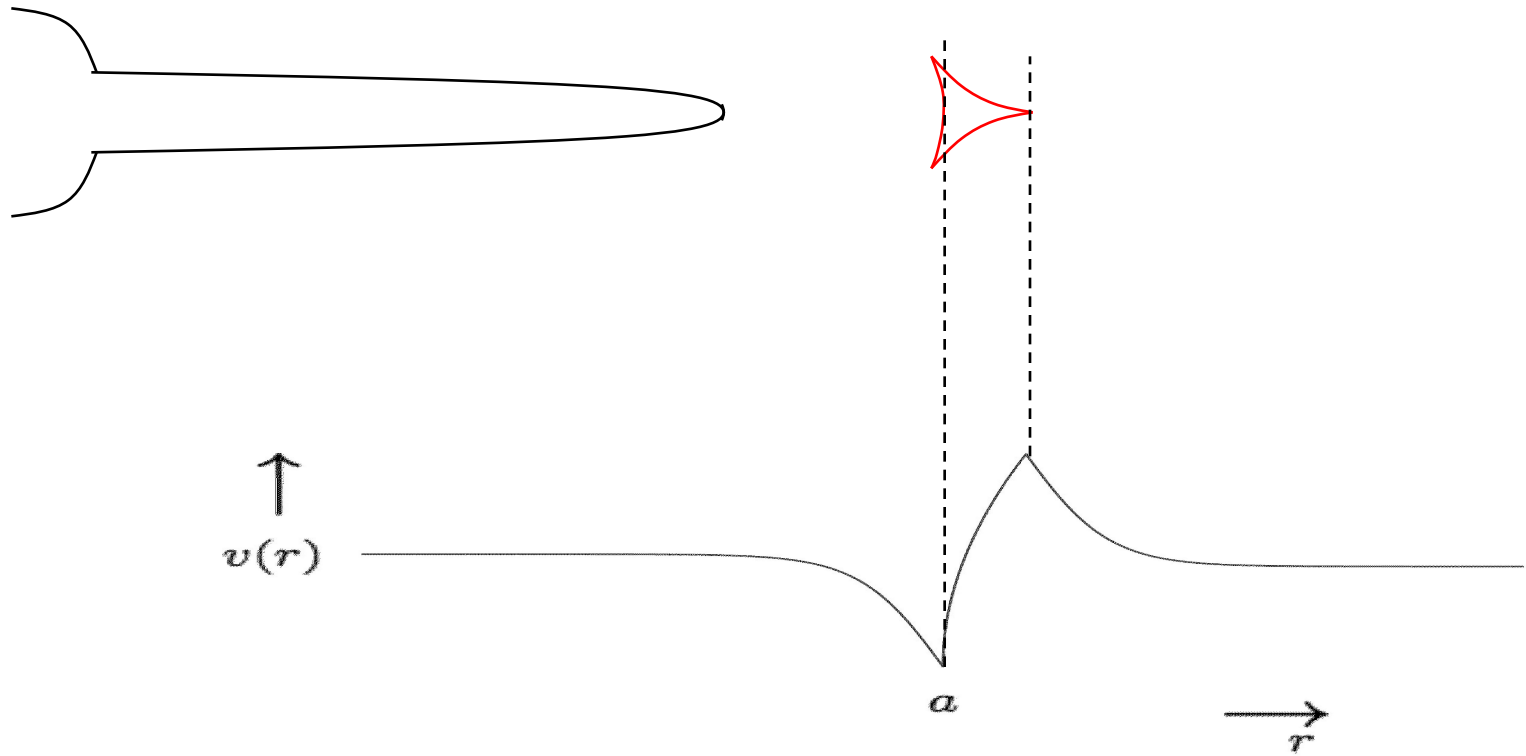
in the galactic plane

with radii  $(n = 1, 2, 3 \dots)$

$$a_n = \frac{40 \text{kpc}}{n} \left( \frac{V_{\text{rot}}}{220 \text{km/s}} \right) \left( \frac{j_{\text{max}}}{0.26} \right)$$

$j_{\text{max}} \cong 0.26$  was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

# Effect of a caustic ring of dark matter upon the galactic rotation curve





# Rotation curve of Andromeda Galaxy

from L. Chemin, C. Carignan & T. Foster, arXiv: 0909.3846

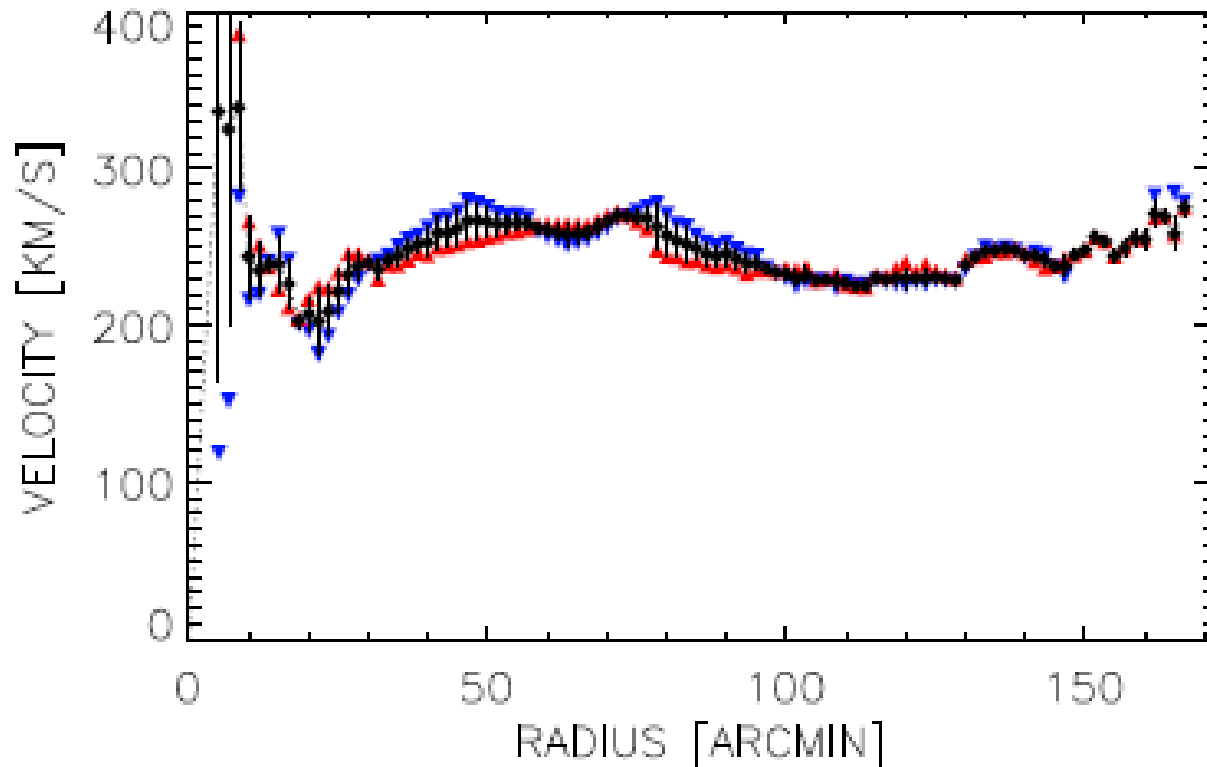
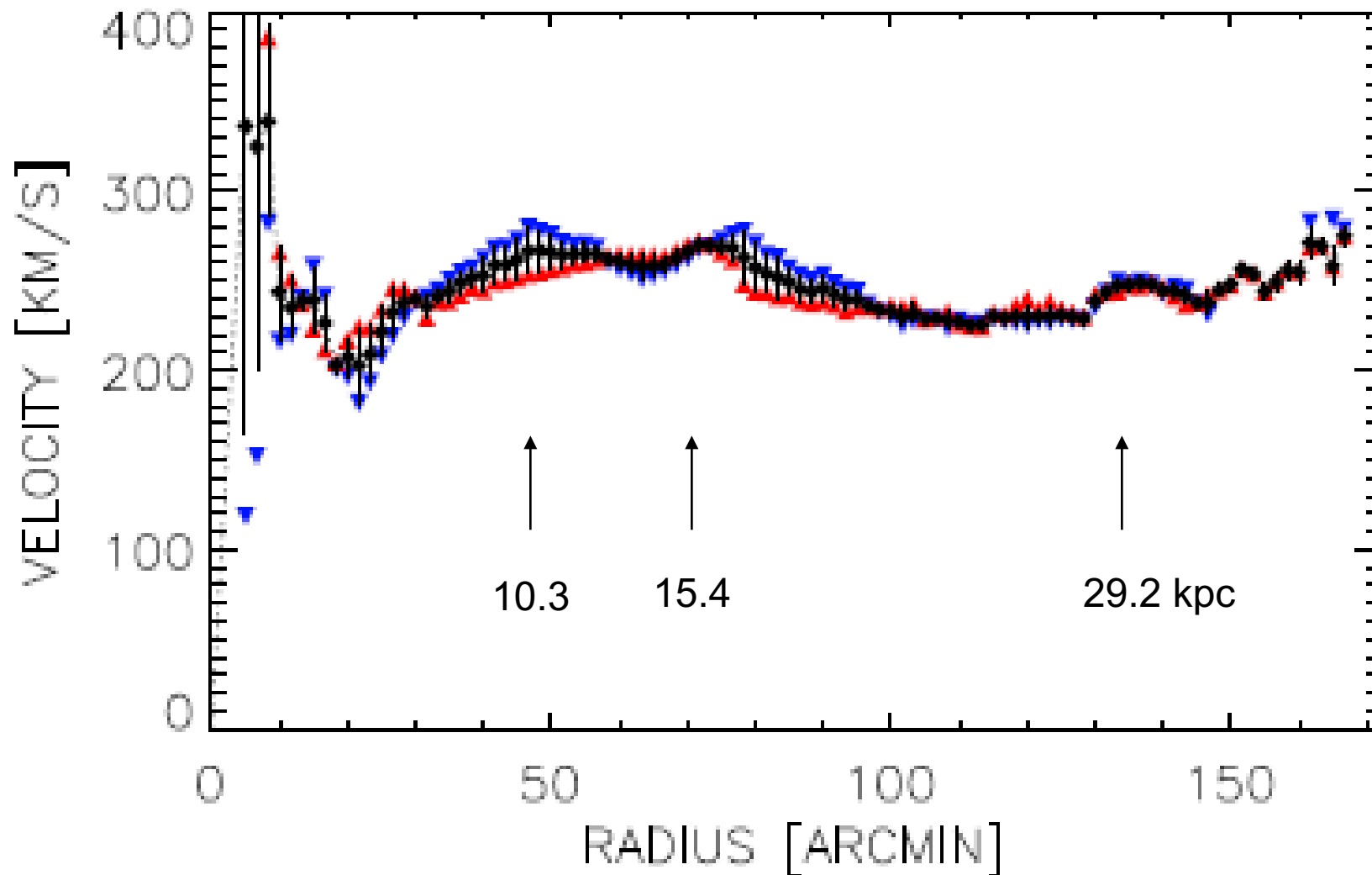


FIG. 10.— H I rotation curve of Messier 31. Filled diamonds are for both halves of the disc fitted simultaneously while blue downward/red upward triangles are for the approaching/receding sides fitted separately (respectively).



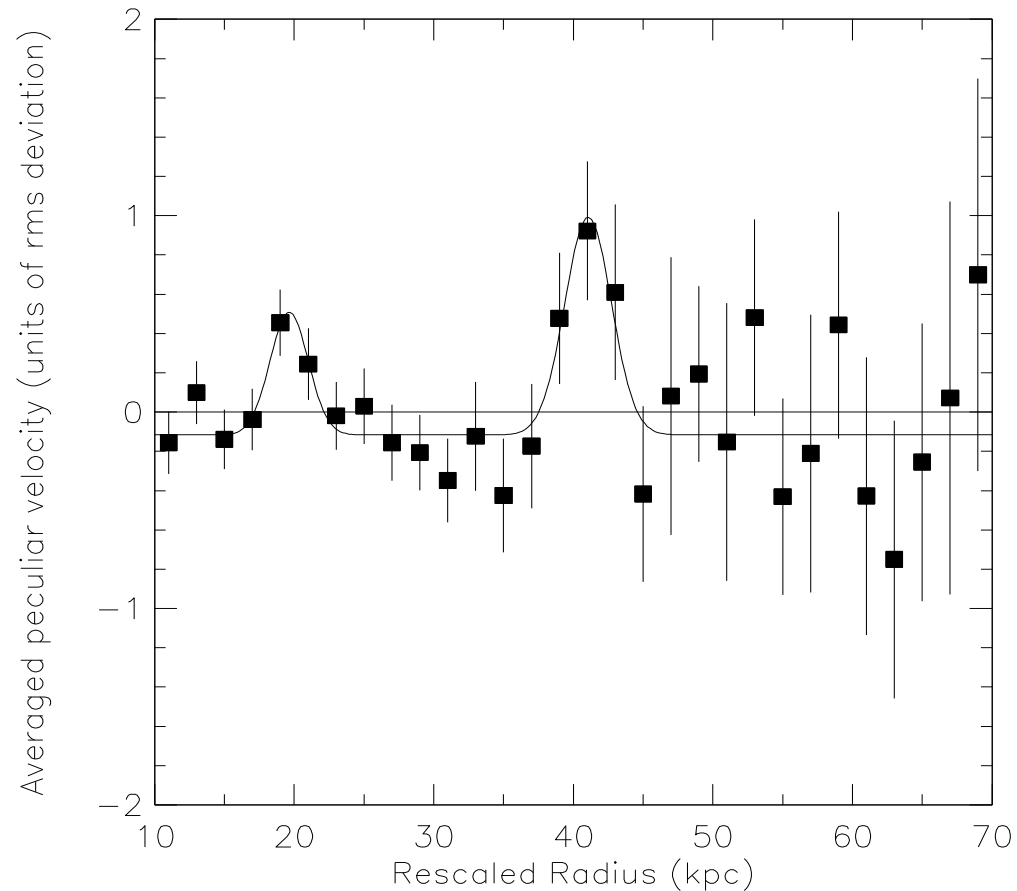
10 arcmin = 2.2 kpc

S. Chakrabarty, Y. Han, A. Gonzalez  
& PS, arXiv:2007.10509

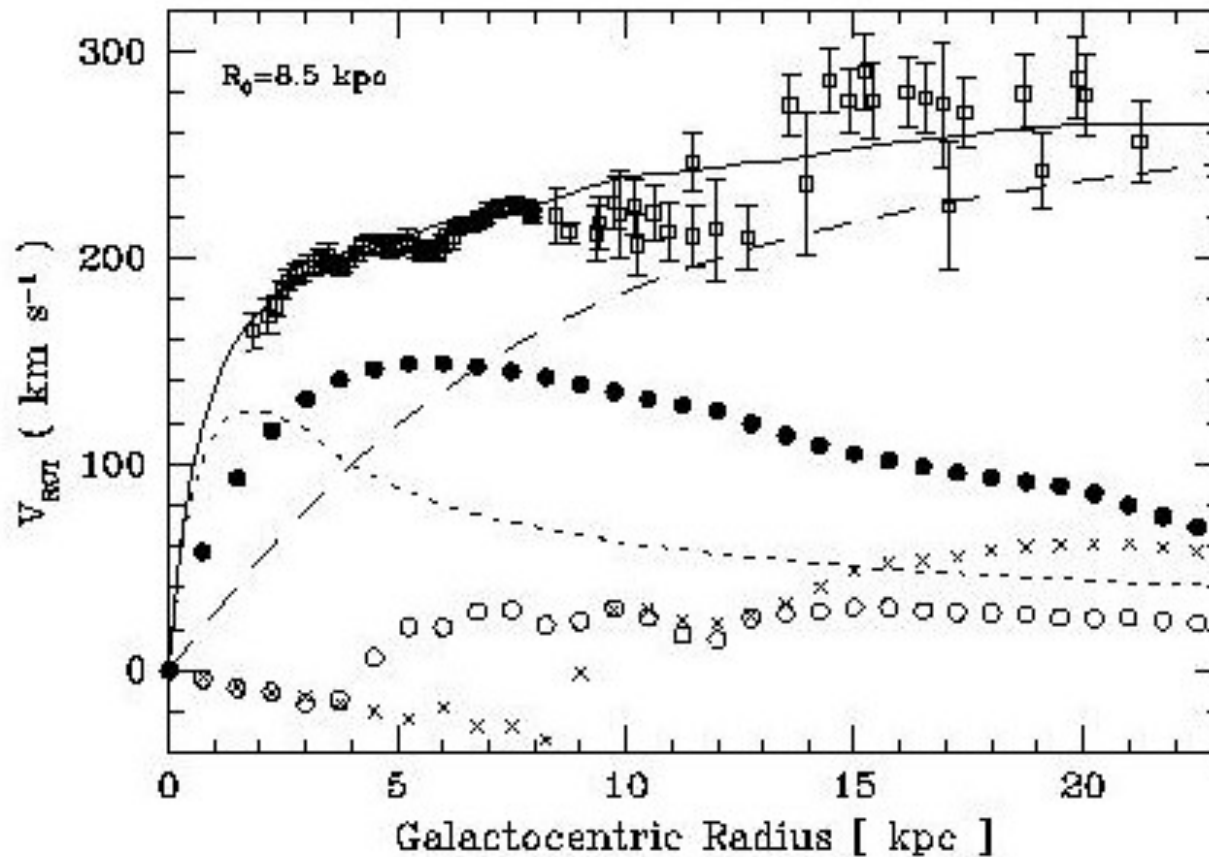
# Composite rotation curve

(W. Kinney and PS, astro-ph/9906049)

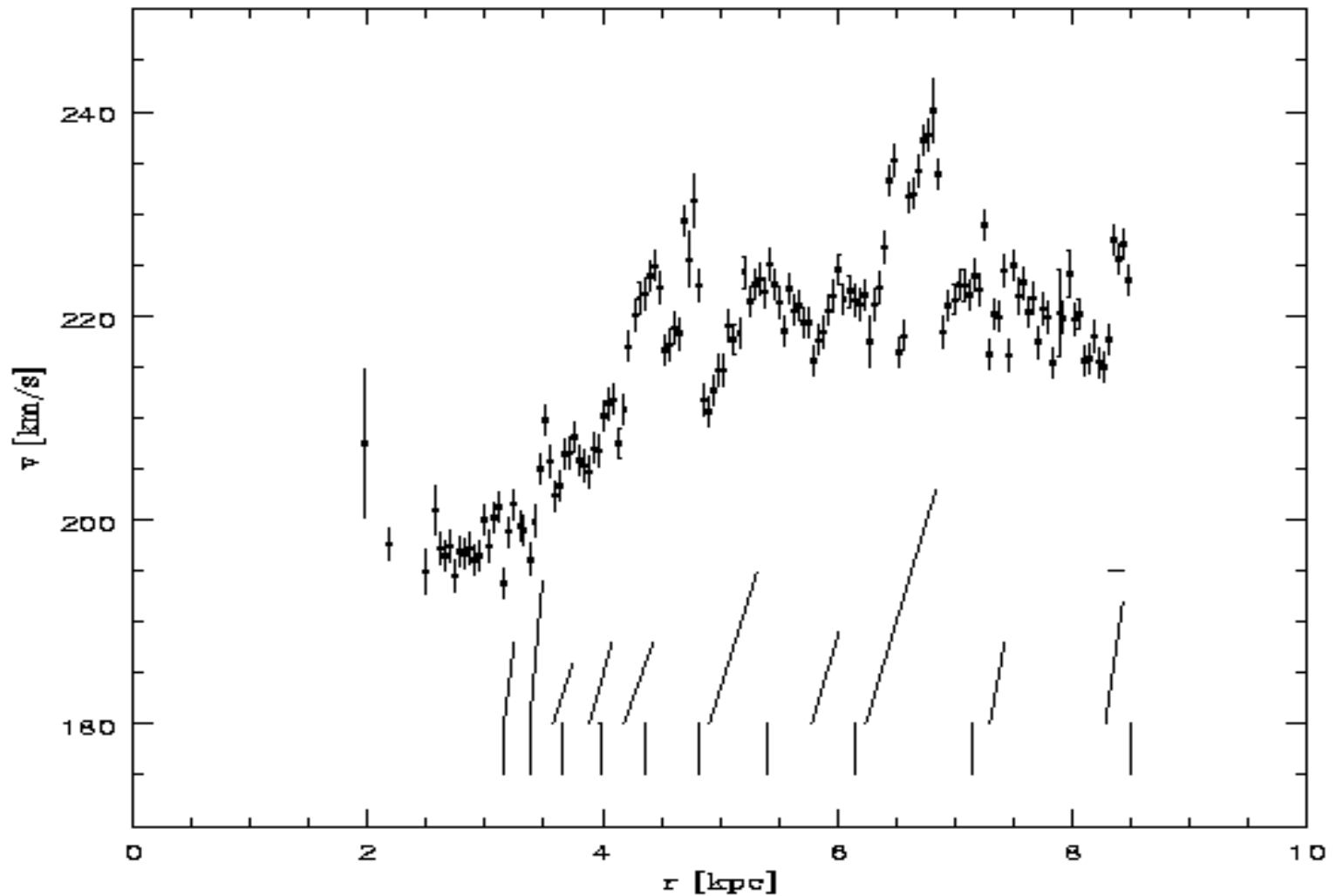
- combining data on 32 well measured extended external rotation curves
- scaled to our own galaxy



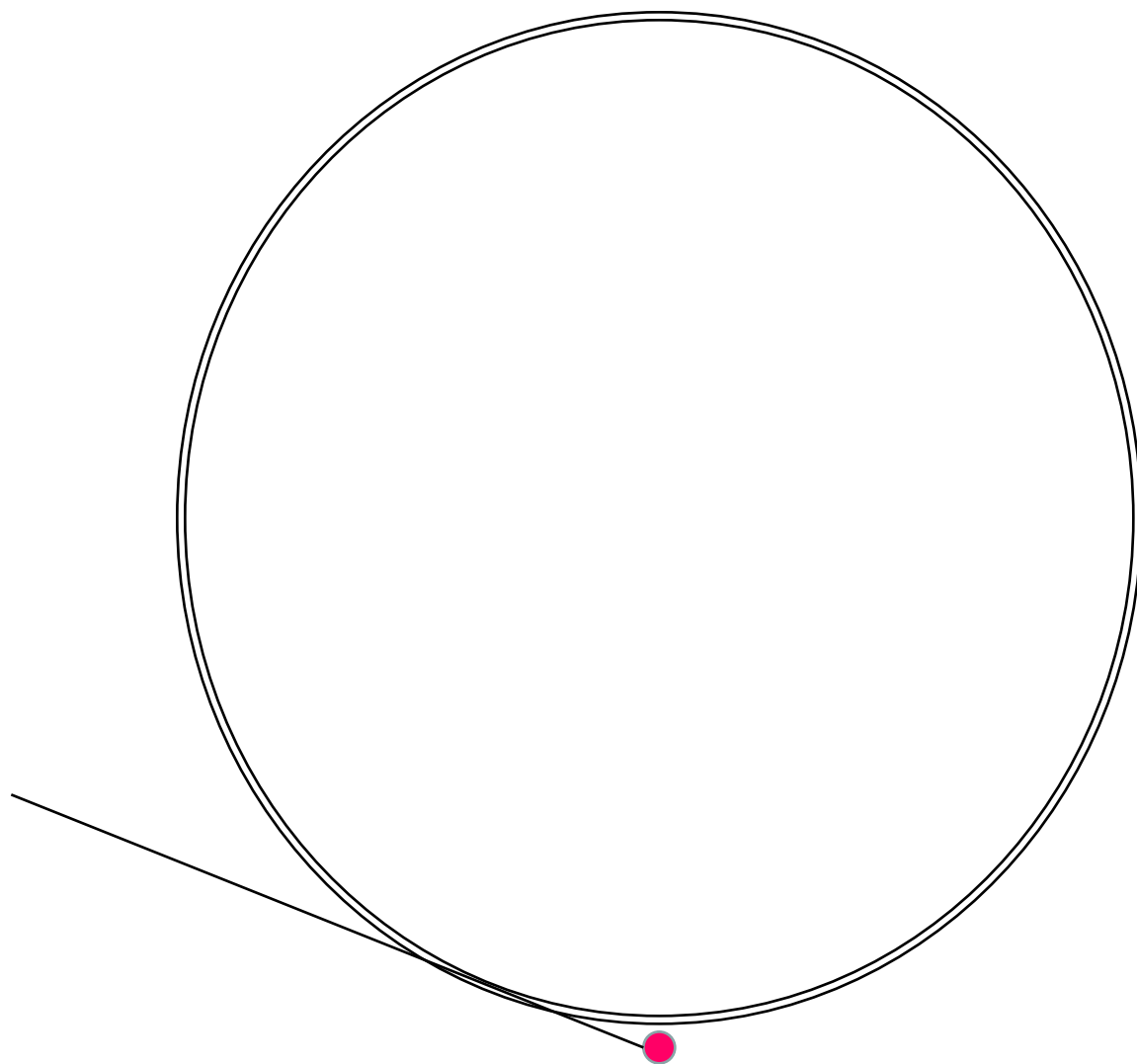
# Outer Galactic rotation curve



# Inner Galactic rotation curve



from Massachusetts-Stony Brook North Galactic Plane CO Survey (Clemens, 1985)



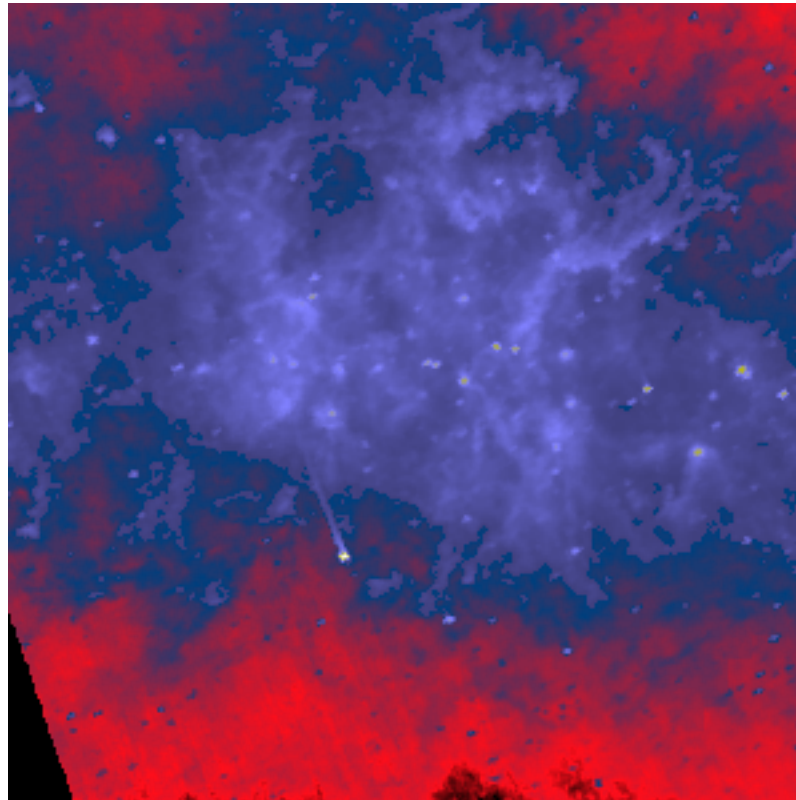
Sun

IRAS

$12\ \mu\text{m}$

$(l, b) = (80^\circ, 0^\circ)$

$10^\circ \times 10^\circ$

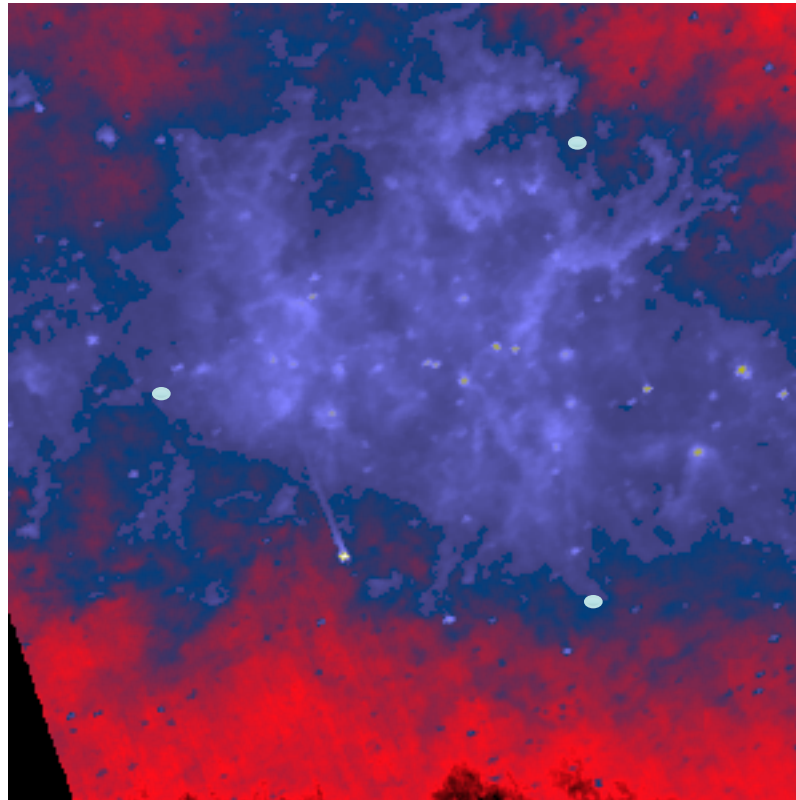


IRAS

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$(l, b) = (80^\circ, 0^\circ)$

$10^\circ \times 10^\circ$



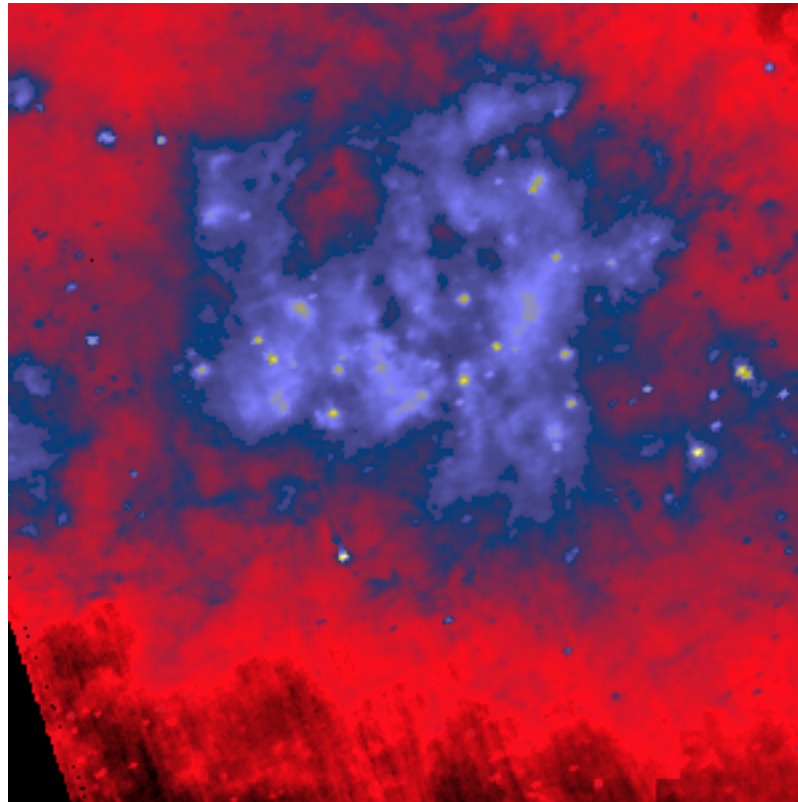


IRAS

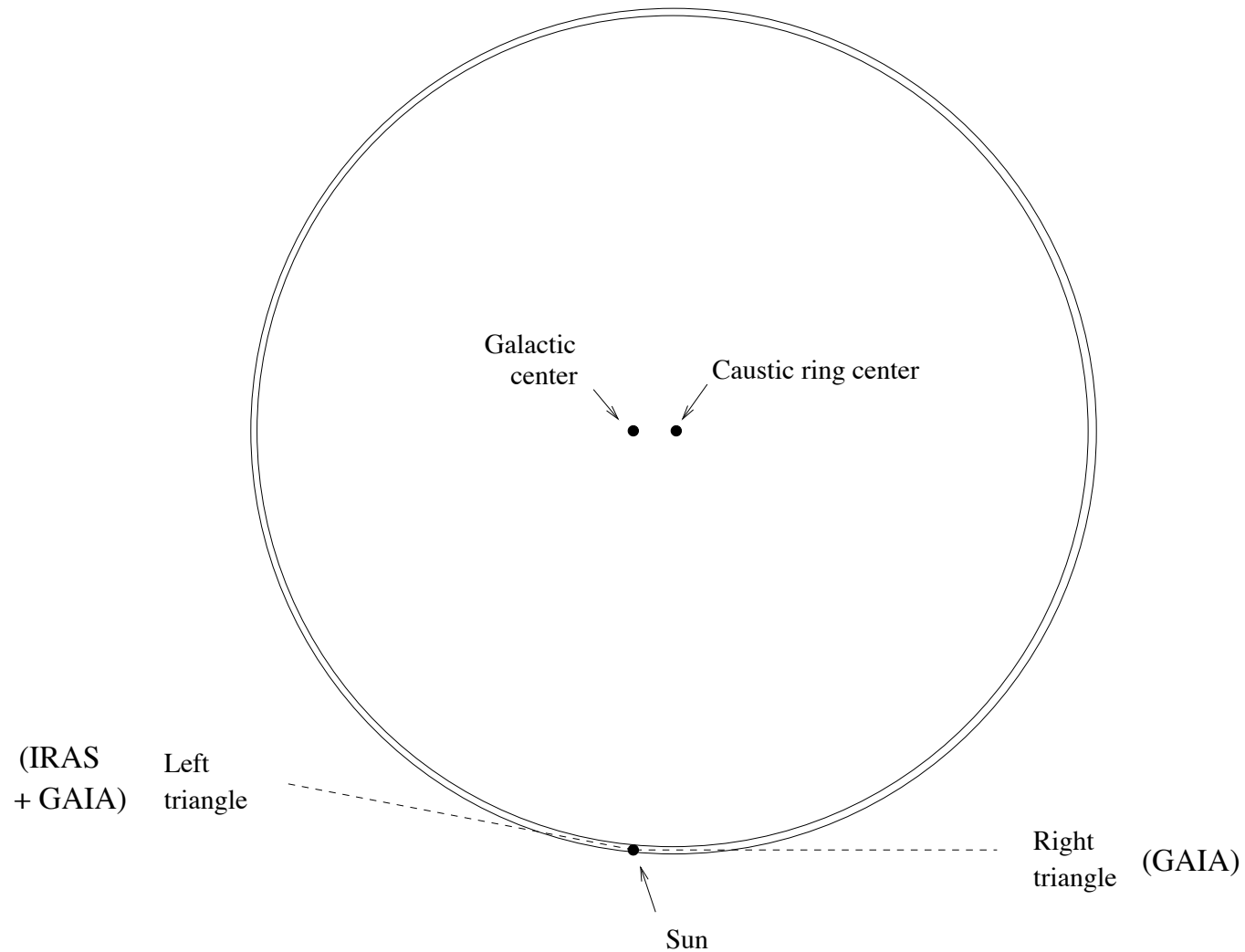
$25\ \mu\text{m}$

$(l, b) = (80^\circ, 0^\circ)$

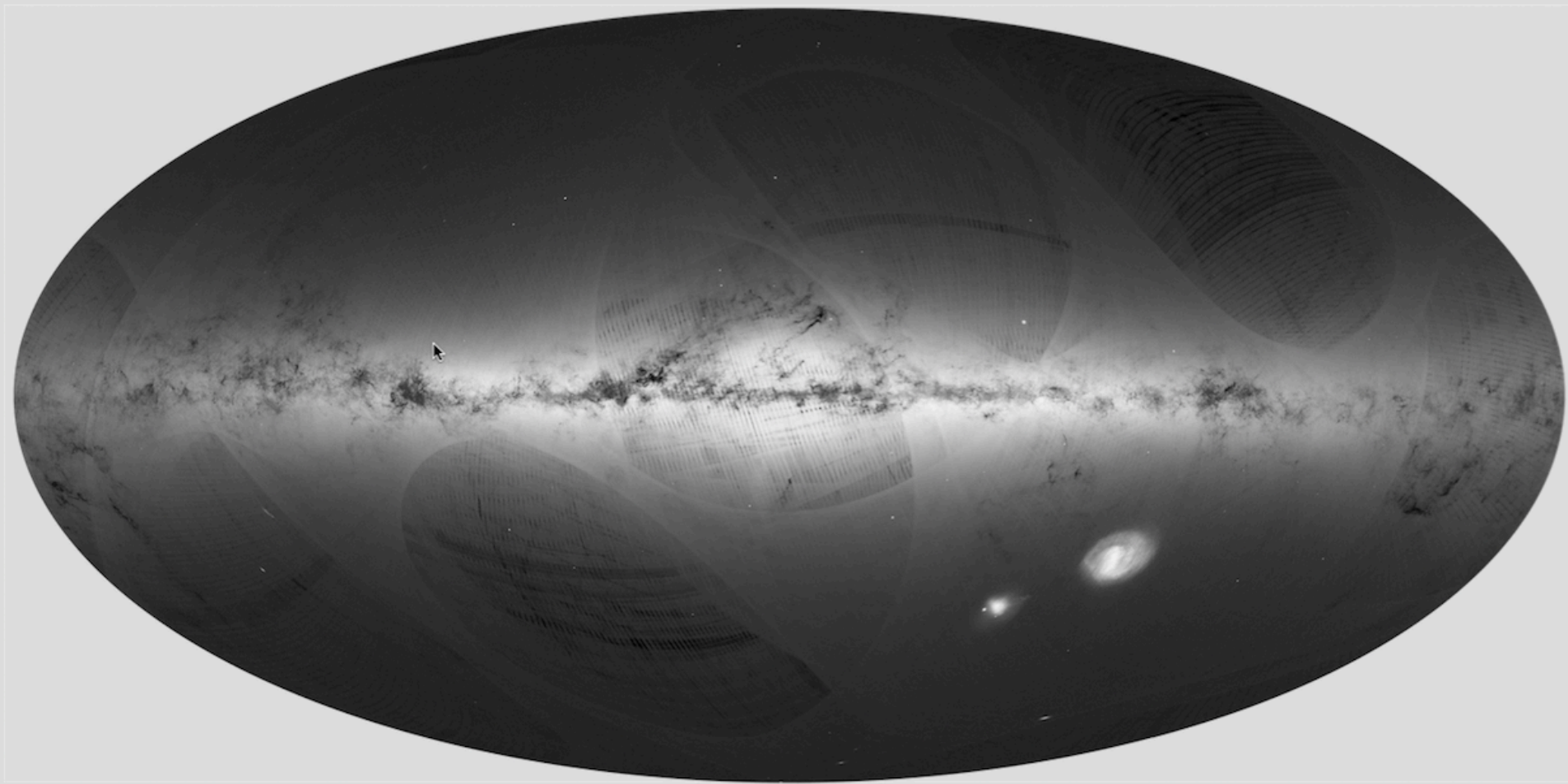
$10^\circ \times 10^\circ$

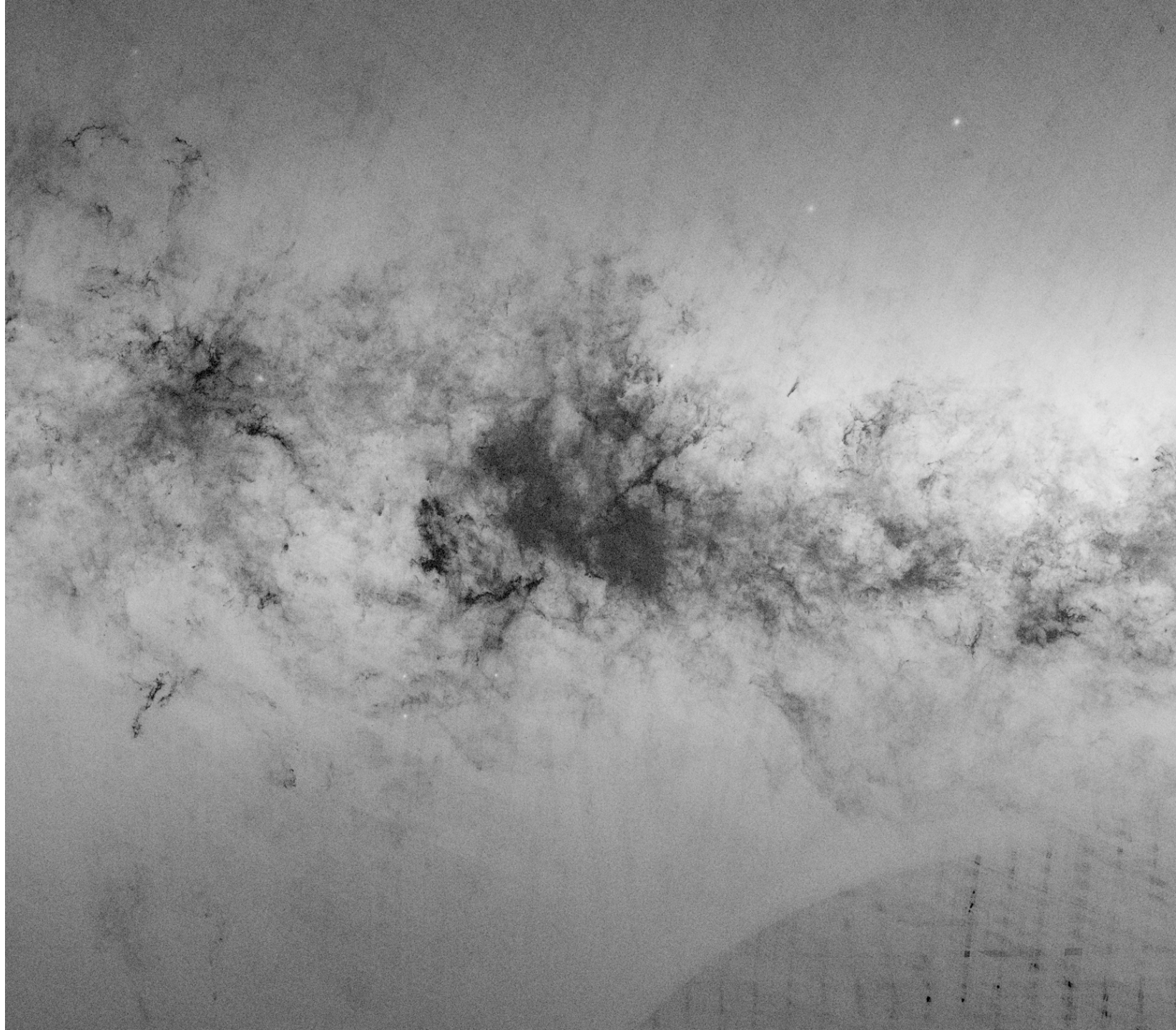


S. Chakrabarty, Y. Han, A. Gonzalez and PS, arXiv: 2007.10509

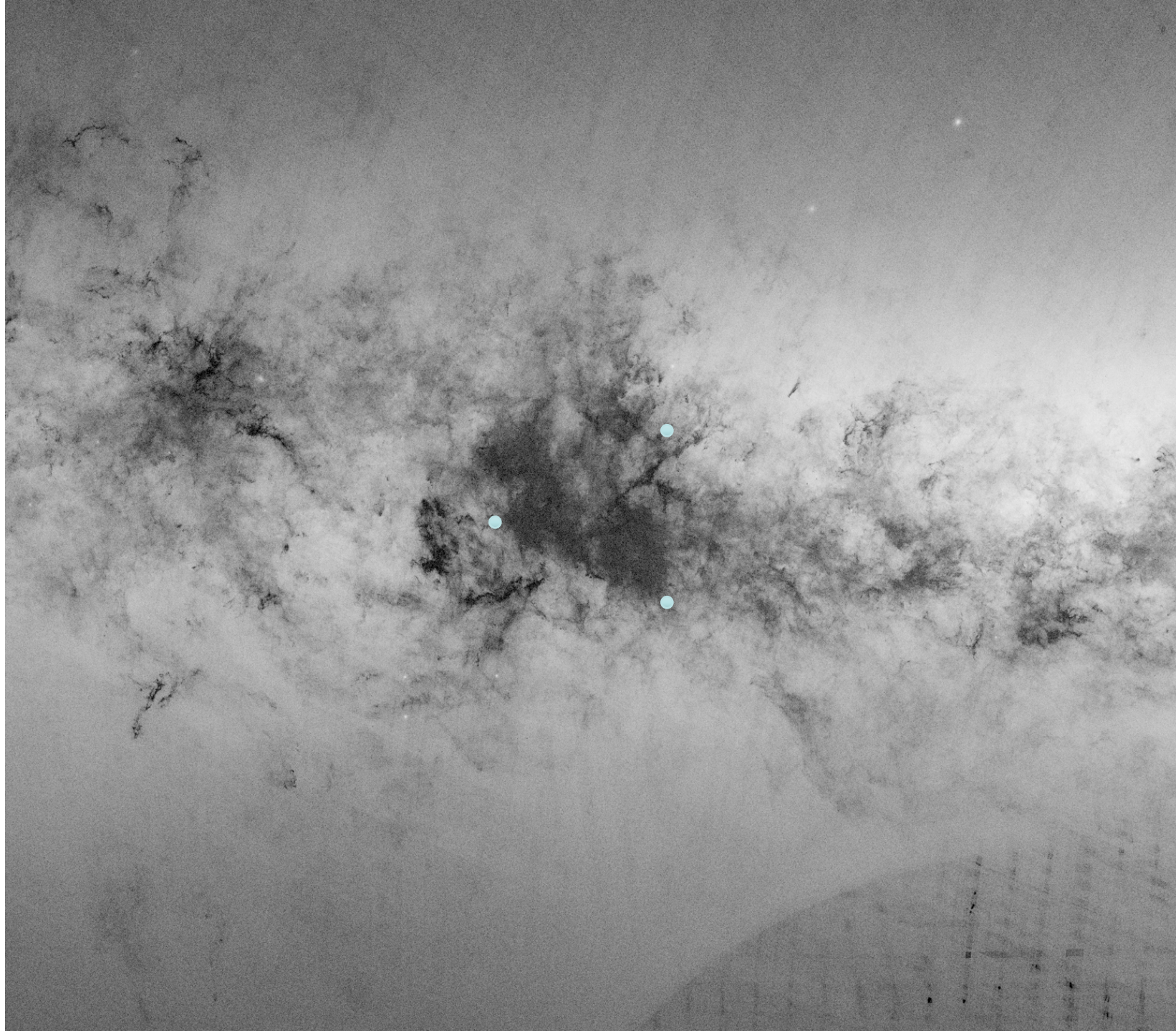


# GAIA sky map

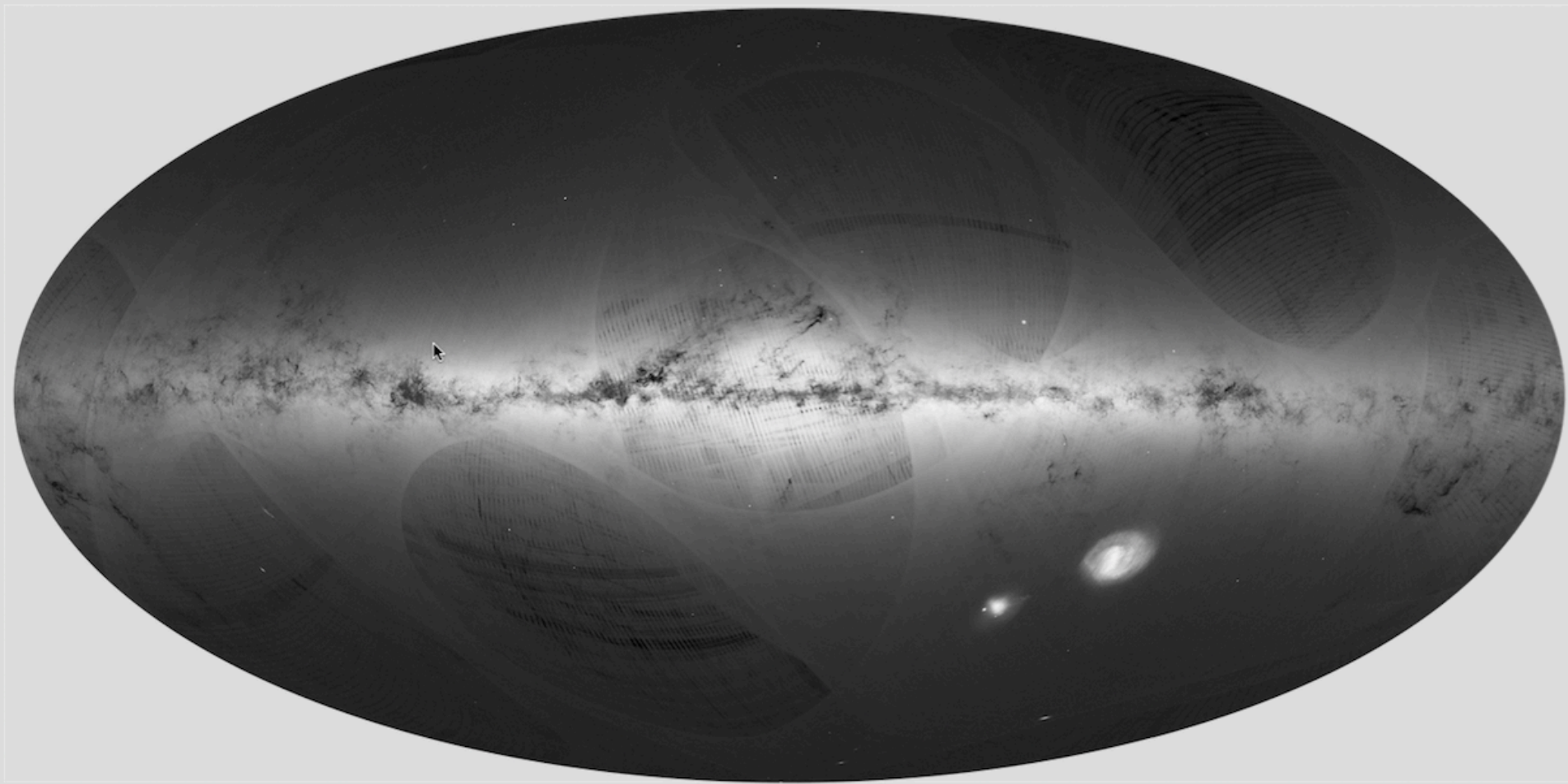




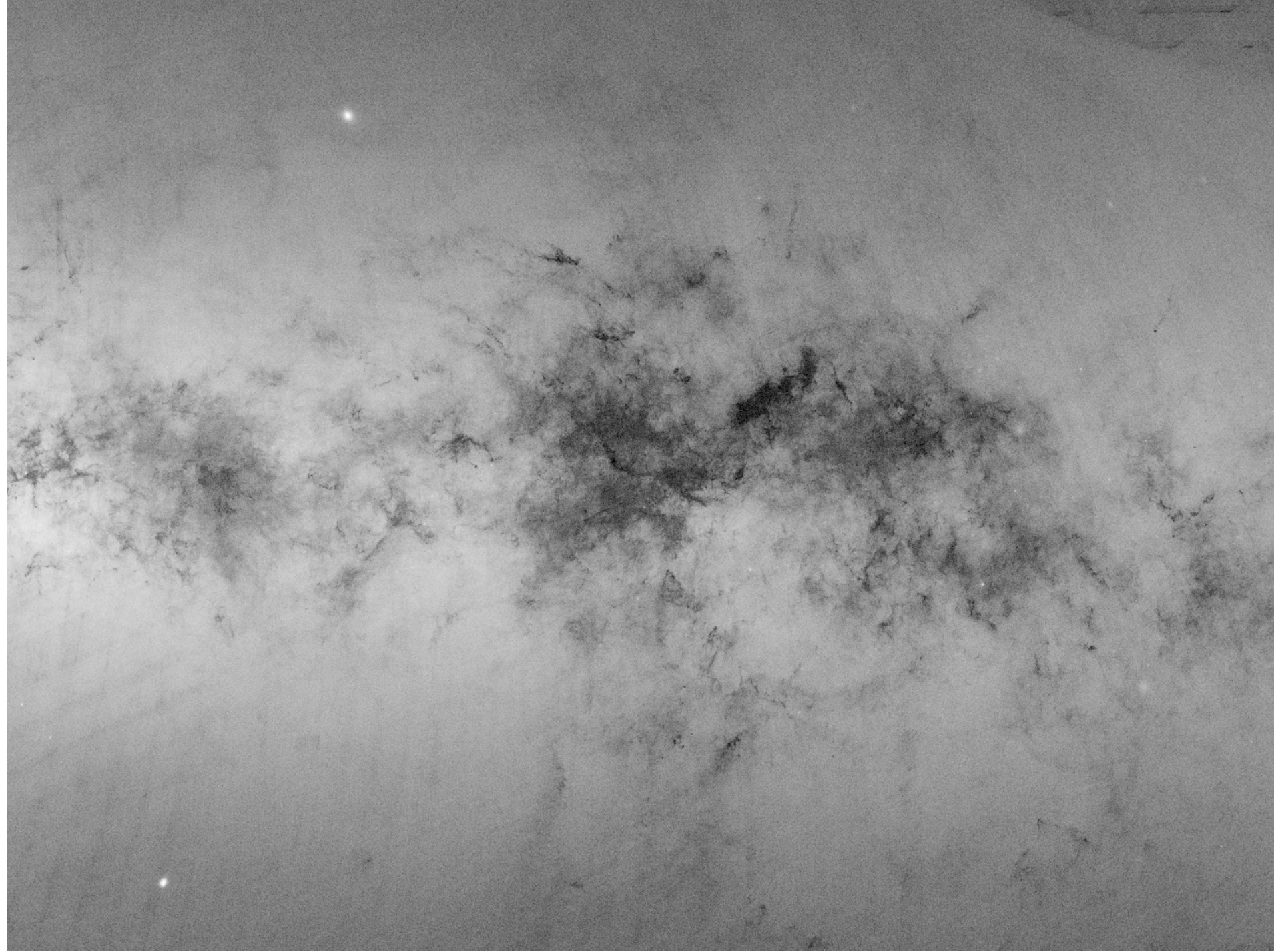




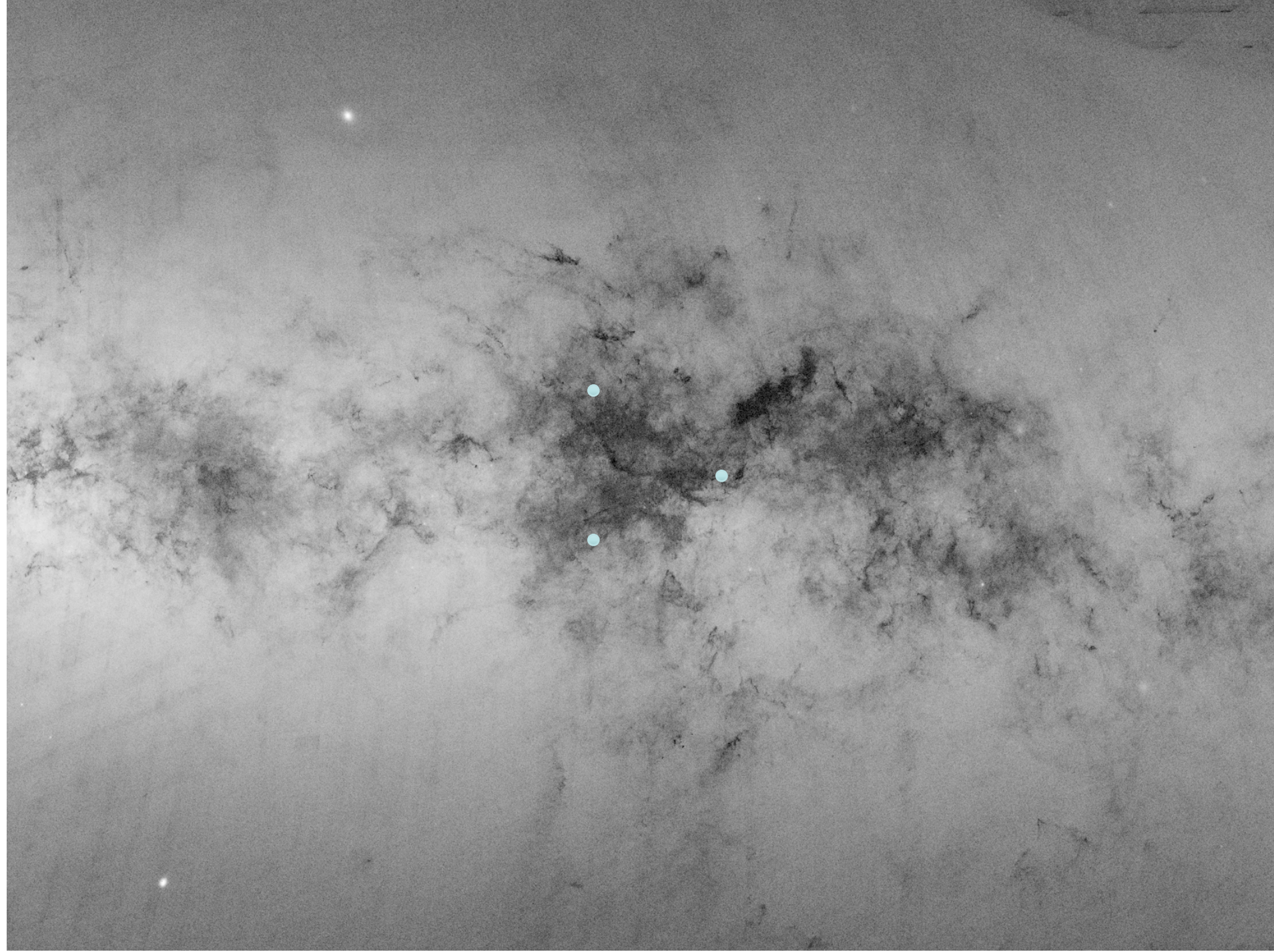
# GAIA sky map



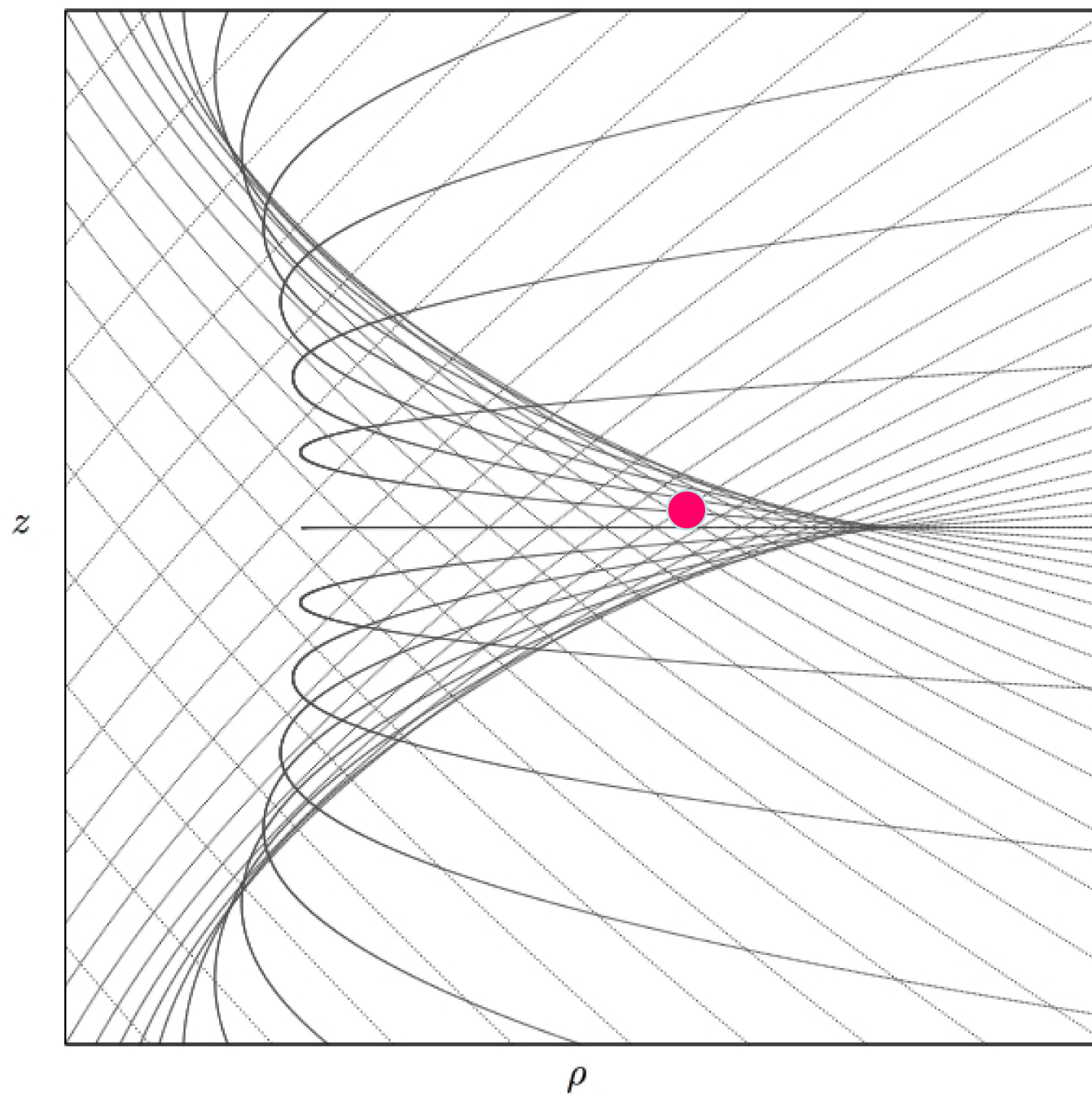


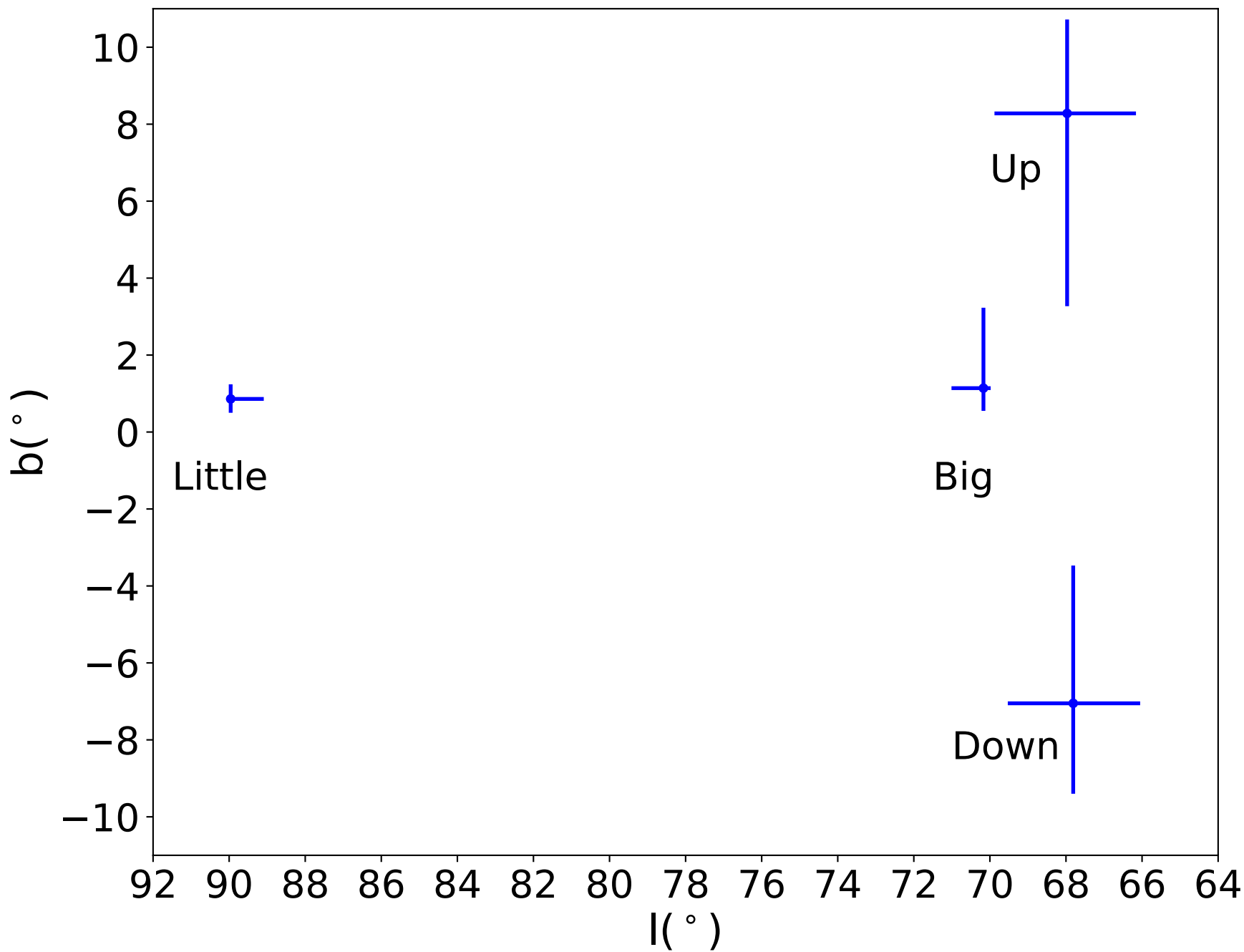












# Conclusions

Discrete flows and caustics are a robust prediction of cold dark matter cosmology.

There is observational evidence for caustic rings in galactic halos. It implies that the dark matter falls in with a rotational velocity field.