ICTS Bengaluru, November 29-December 2, 2022

Cavity Optomechanical Sensing and Manipulation of an Atomic Persistent Current*

Postdocs | Undergraduates | Faculty


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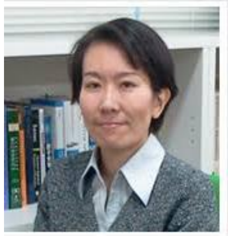
T. Biswas


K. Feliz

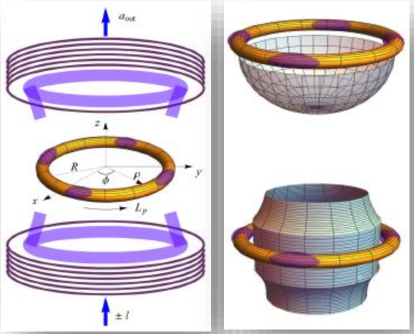

S. Kalita


R. Kanamoto


A. K. Jha


M. Chang


A. Sarma



Mishkat Bhattacharya

School of Physics and Astronomy
&
Future Photon Initiative
Rochester Institute of Technology

15,000 students
1100 faculty

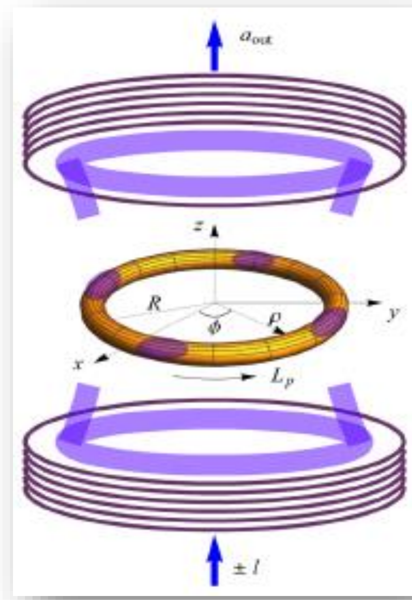
NSF CAREER

*P. Kumar, T. Biswas, K. Feliz, R. Kanamoto, M. S. Chang, A. K. Jha and M. Bhattacharya
Physical Review Letters **127**, 113601 (2021)

Office of Naval Research — Science & Technology

Talk Outline

- Atomic Superfluid Rotation
- Cavity Optomechanics
- Rotation Sensing
- Conclusion



*L. Amico et al., Roadmap on Atomtronics,
AVS Quantum Sci. **3**, 039201 (2021)

*M. Aspelmeyer et al. Cavity Optomechanics
Reviews of Modern Physics **82**, 1155 (2014)

Superfluidity

- Superconductivity

  1911: Kammerlingh Onnes (1913 Nobel Prize)

  1957: Bardeen, Cooper and Schrieffer (1972 Nobel Prize)

  1986: Bednorz and Muller (1987 Nobel prize)

  ? : Theory of High Temperature Superconductivity

- Liquid Helium

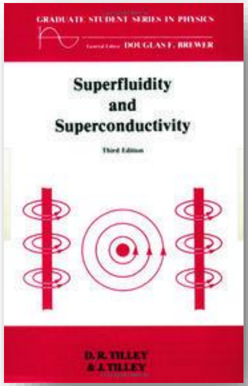
  1938: Kapitsa (1978 Nobel Prize); Allen and Misener - ^{4}He

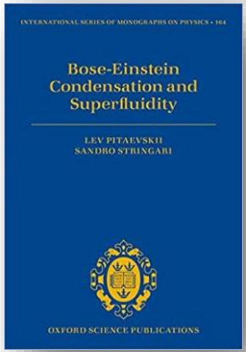
  1972: Osheroff, Lee and Richardson (1996 Nobel Prize) - ^{3}He

- Atomic (gaseous) Bose-Einstein condensates (BEC)

  1995: Cornell, Ketterle and Wieman (Nobel Prize 2001)

- Also: Polaritonic condensates, light; neutron stars, quarks, cosmic strings

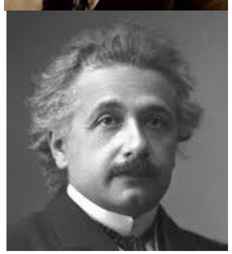


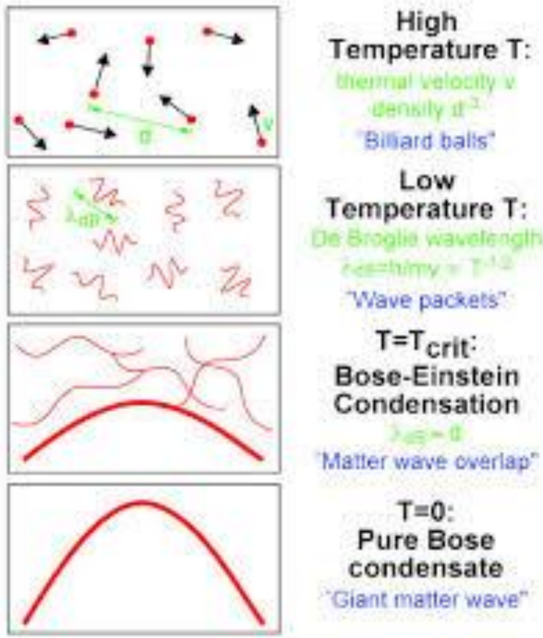


N. Cooper, *Advances in Physics* **57**, 539 (2008)
A. Fetter, *Reviews of Modern Physics* **81**, 647 (2009)
A. D. Kashkanova, *Nature Physics* **13**, 74 (2017)
Y. Sachkou et al., *Science* **366**, 6472 (2019)

Bose-Einstein condensation (BEC)

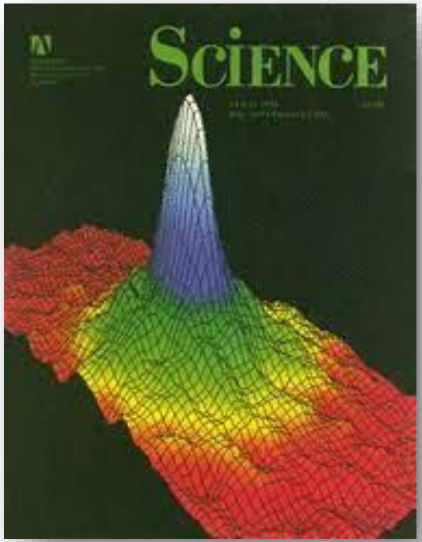
$$\lambda_{dB} = \frac{h}{\sqrt{2mk_B T}}$$



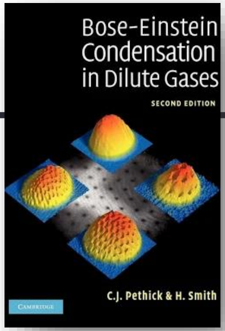


*In this talk: atoms weakly repel

First Observation of an Atomic BEC



M. H. Andersen et al, Science **269**, 198 (1995)

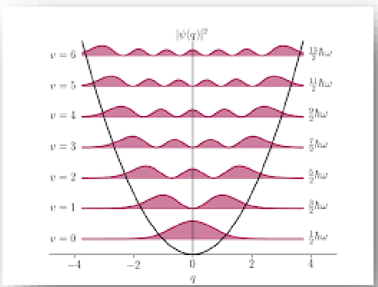


*A. J. Leggett, Bose-Einstein Condensation In alkali gases: Some fundamental concepts, *Reviews of Modern Physics* **73**, 307 (2003)

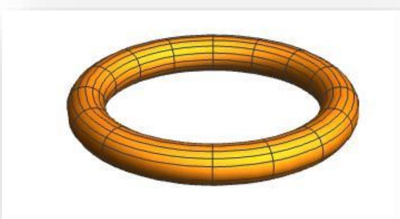
BEC and Electromagnetic fields

Magnetic fields

$$E = -\mu . B$$



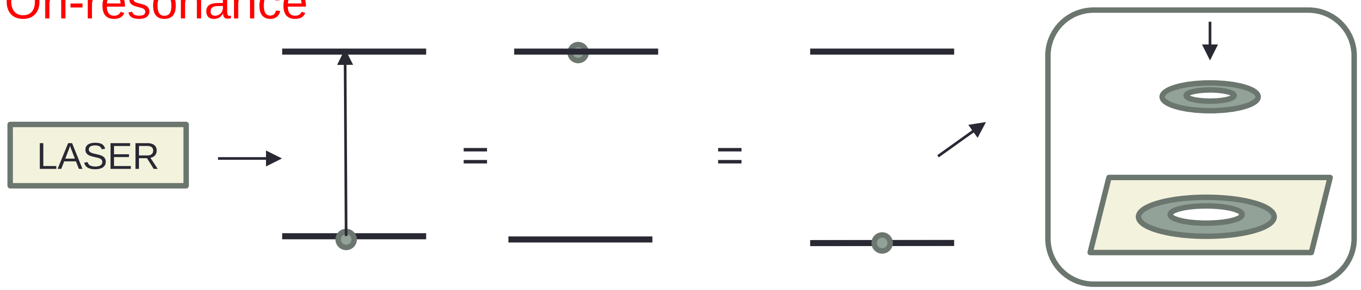
Harmonic trap



Toroidal trap

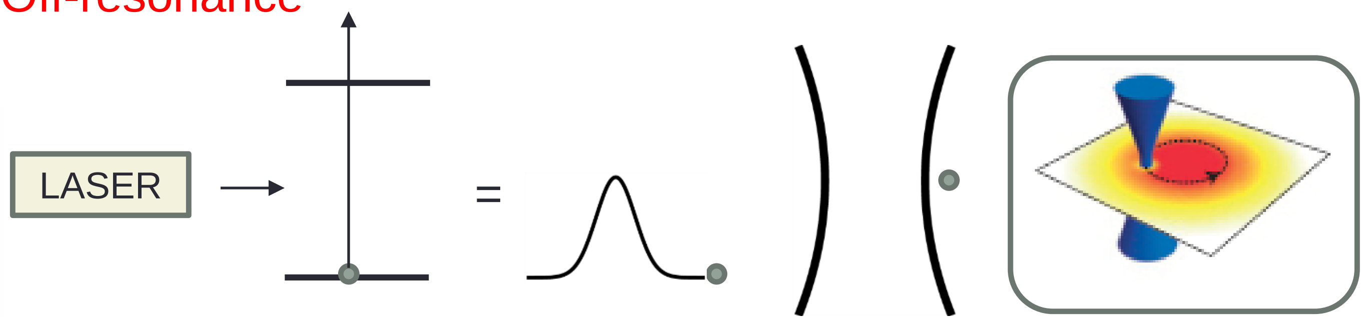
Light fields

On-resonance



Absorption Imaging

Off-resonance

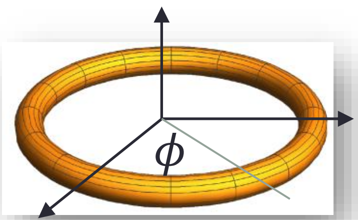


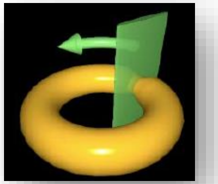
Optical stirring

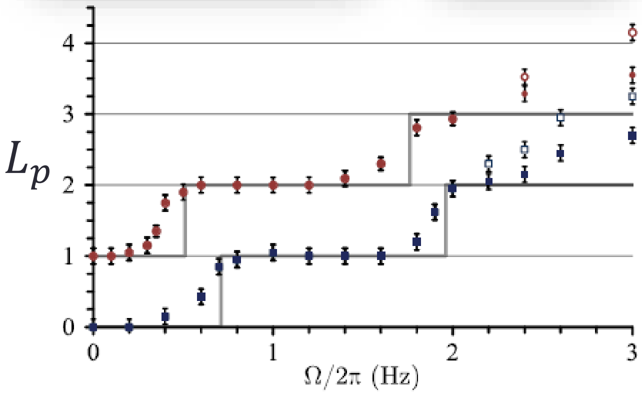
J. Fortagh and C. Zimmerman,
Reviews of Modern Physics **79**, 235 (2007)

Quantized Persistent Currents in a Ring BEC

Persistent currents







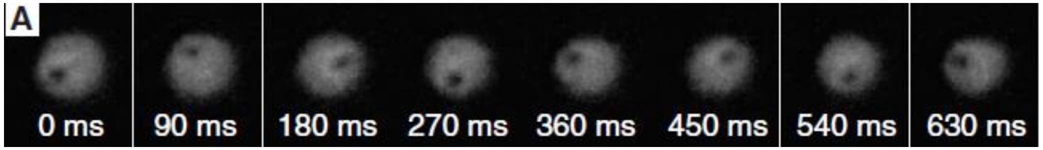
Wright et al., PRL **110**, 025302 (2013)

$$\psi = \sqrt{\frac{n}{2\pi}}\, e^{iL_p\phi},\ L_p = 0, \pm 1, \pm 2, \ldots$$

Why the ring BEC is important

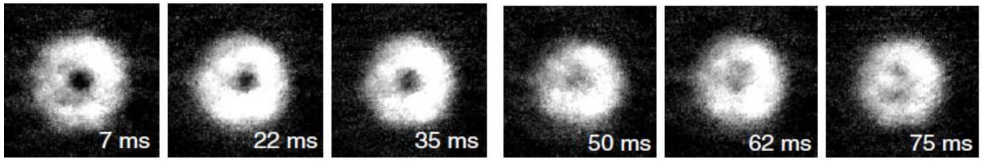
PRL **99**, 260402 (2007)

1. The ring trap pins the vortex down.



$L_p = 1$ in a harmonically trapped BEC [Science **329**, 1182 (2010)]

2. It topologically stabilizes high circulation.

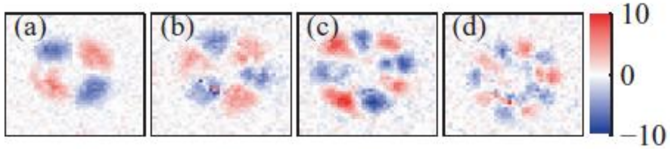


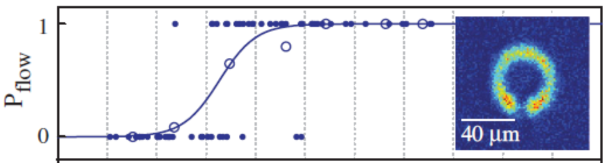
$L_p = 2$ in a harmonically trapped BEC [PRL **93**, 160406 (2004)]

3. Metastable currents persist for minutes !
   - Beattie et al., PRL **110**, 025301 (2013)

Supersonic: $L_p \simeq 350$, PRL **124**, 025301 (2020)
Hypersonic: $L_p \simeq 40{,}000$, Nature **570**, 205 (2019)

Ring BECs: A Paradigm of Quantum Rotation





Superfluid hydrodynamics

Z. Mehdi et al., SciPost Phys. **11**, 080 (2021)
J. Polo et al., PRL **123**, 195301 (2019)
K. C. Wright et al., PRL **110**, 025302 (2013)

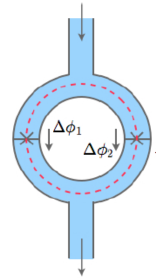
Matter wave interferometry

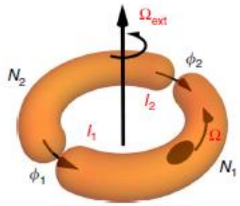
S. Pandey et al., PRL **126**, 170402 (2021)
C. Ryu et al., Nat. Comm. **11**, 3338 (2020)

Atomtronic circuits

J. Polo et al., PRL **121**, 0904040 (2018)
S. Eckel et. al, Nature **506**, 200 (2014)

SQUID



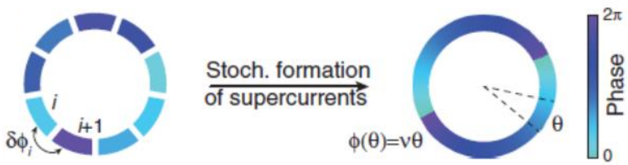


Topological defect formation

Aidelsburger et. al, PRL **119**, 190403 (2017)

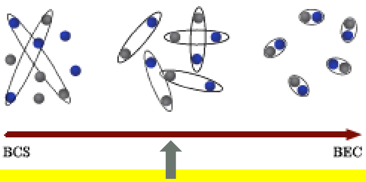
Cosmological simulations

S. Eckel et. al, PRX **8**, 021021 (2018)



Fermionic supercurrents (BCS-BEC crossover)

Y. Cai et al. PRL **128**, 150401 (2022)
G. Del Pace et al., arxiv:2204.06542 (2022)
G. Pecci et al., PRR **3**, L032064 (2021)
J. W. Park et al, PRL **121**, 225301 (2018)

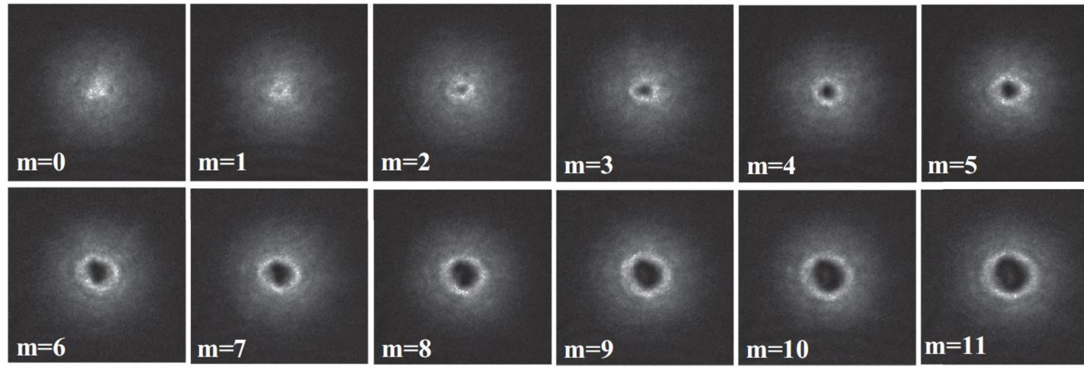


High T_c, QGP, Graphene

Measuring the Ring BEC Rotation

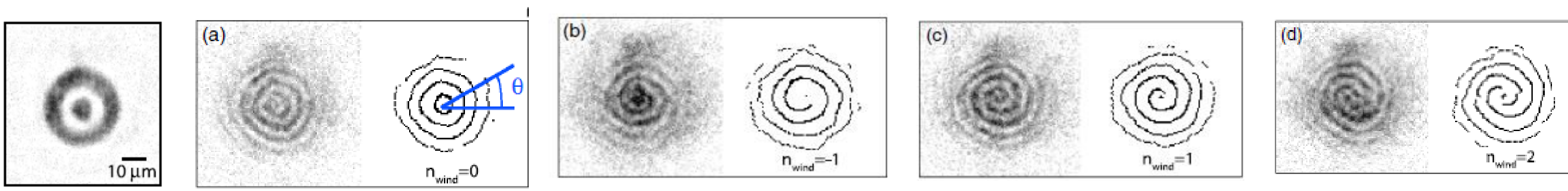
Measure the radius after time of flight expansion

N. Murray et al.,
Phys. Rev. A **88**, 053615 (2013)
K. C. Wright et. al,
PRL **110**, 025302 (2013)



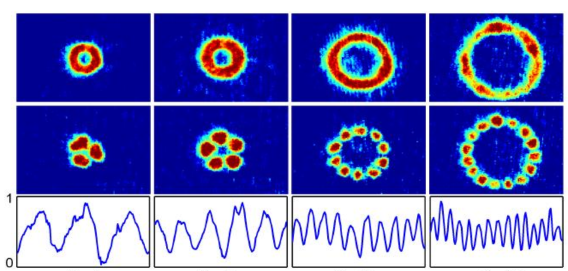
Interfere with central non-rotating BEC

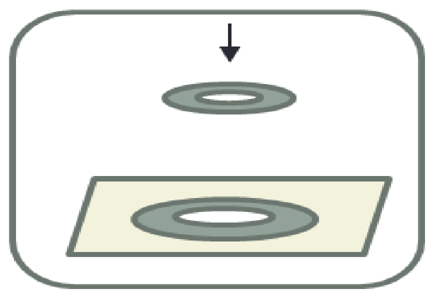
Corman et. al, PRL **113**, 135302 (2014)



More complicated schemes

Moulder et. al, PRA **86**, 013629 (2012)





Absorption imaging

← Destructive!!

Measuring the Ring BEC rotation

Required: A measurement technique that works

1. Non-destructively

   +

2. *In situ*

   +

3. In real time

→

Aims:

- Observing system dynamics in real time + *in situ* - the ideal aim of experiments
- Following historical precedent
- Coherent and reversible control - undoing phase slips
- Storing and retrieve information - quantum memory
- Detecting fermionic superfluidity directly without projecting to the BEC regime…

Superconductors

- Nat. Comm. **8**, 85 (2017)
- Nature Physics **5**, 651 (2009)
- Nature **397**, 308 (1999)
- Nature **360**, 51 (1992)

Superfluid He

- Nat. Comm. **12**, 2645 (2021)
- Science **366**, 1480 (2019)
- Nature **441**, 588 (2006)
- PNAS **102**, 14952 (2005)

Simply connected BECs

- *In situ* detection of vortices in BEC, PRA **91**, 023631 (2015)
- Real-time dynamics of single vortex lines in BECs, Science **329**, 1182 (2010)
- Vortex precession in BECs, PRL **85**, 2857 (2000)

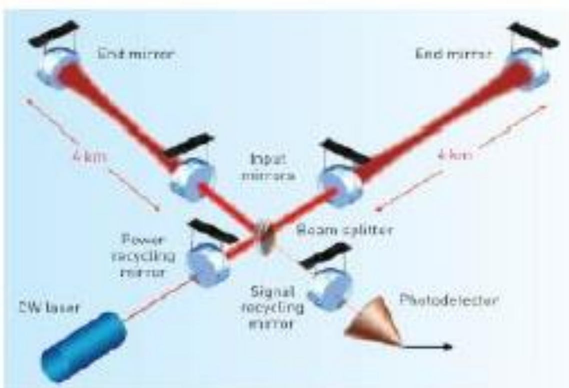
Minimally destructive, Doppler measurement of a Quantized flow in a ring-shaped BEC,
A. Kumar et al., NJP **18**, 025001 (2016)

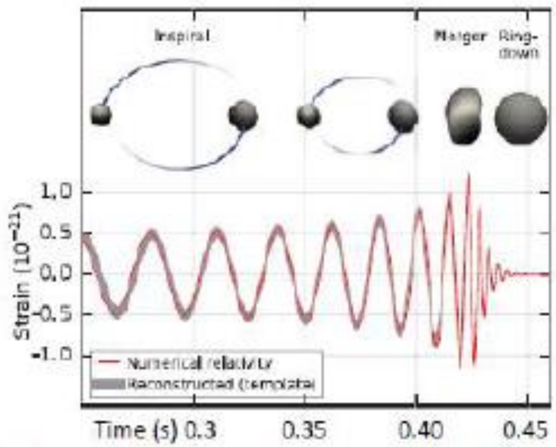
Cavity Optomechanics

PRL 116, 061102 (2016) PHYSICAL REVIEW LETTERS 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)





Also...

Accelerometry
A. G. Krause, M. Winger, T. D. Blasius, Q. Lin and O. Painter, *Nat. Photon.* 6 768 (2012)

Magnetometry
S. Forstner et. al, *PRL* **108** 120801 (2012)

Thermometry
J. Millen et. al, *Nat. Nano.* 9 425(2012)

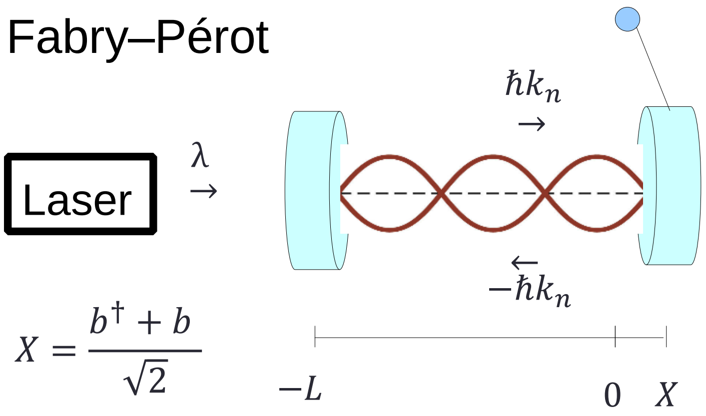
→ LIGO is a cavity optomechanical device !!

Displacement = Strain x Length of int. arm $\approx 10^{-18} m$

*M. Aspelmeyer et. al
Reviews of Modern Physics **82**, 1155 (2014)

The Law interaction

Fabry–Pérot



C. K. Law, PRA **51**, 2537(1995)

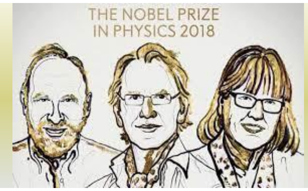
$$H = \hbar\omega_c a^\dagger a + \hbar\omega_m b^\dagger b + \hbar G a^\dagger a X + H_E$$

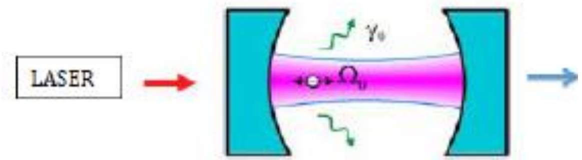
- $\hbar\omega_c a^\dagger a$: optical mode
- $\hbar\omega_m b^\dagger b$: mechanical oscillator
- $\hbar G a^\dagger a X$: radiation pressure interaction
- H_E: environment

TOOLBOX FOR MEASURING AND MANIPULATING MECHANICAL MOTION

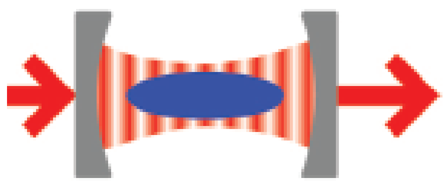
1. Sub-quantum limit detection of mechanical motion..Nature Physics **15**, 745 (2019)
2. Detection of quantum back-action……………………………. Science **339**,801(2013)
3. Back-action-evading measurement of motion………..Nat. Comm. **10**, 2086 (2019)
4. Squeezing of mechanical quadrature of motion…………..Science **349**, 952 (2015)
5. Preparation of quantum mechanical ground state…..MB…PRL **99**, 073601 (2007)
6. Storing and retrieving optical information from mechanical elements……………………………..……….PRL **107**, 133601 (2011)
7. Phonon lasing……………………….…..PRL **113**, 030802 (2014)
8. Entangling multiple mechanical oscillators…….Nature **556**, 478 (2018)

Levitated optomechanics





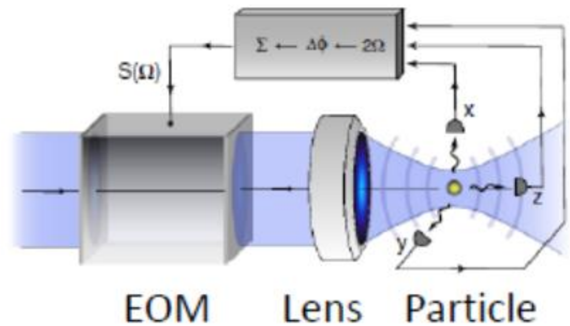
U. Delic et al., Science **367**, 892 (2020)
N. Keisel et. al, PNAS **110**, 14180 (2013)

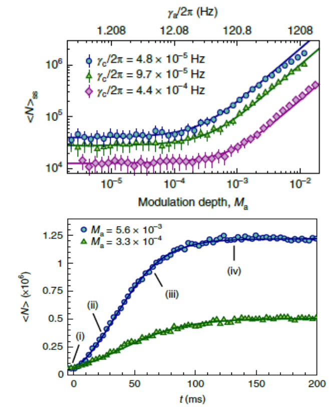


F. Brennecke et al.,
Cavity Optomechanics with a Bose-Einstein Condensate
Science **322**, 235 (2007)

Cavity QED with Quantum Gases
Mivehvar et al.,
Advances in Physics **70**, 1 (2021)

Beyond the Law interaction





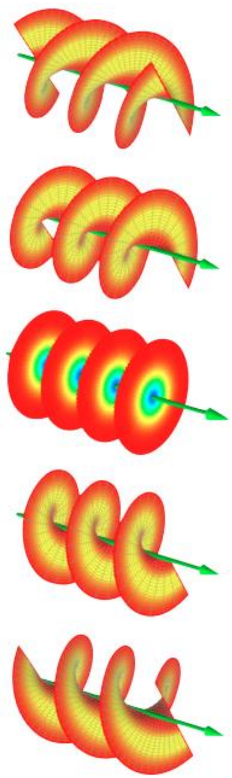
Collaboration with A. N. Vamivakas @ UofR

- PRA, submitted (2022)
- Optics Express **28**, 4234 (2020)
- *Nature Photonics **13**, 372 (2019)
- Optics Express **25**, 19568 (2017)
- Optica **3**, 318 (2016)
- …

Included in **Breakthroughs of 2019** (*Optics and Photonics News*)

*US Patent App. PCT/US2021/032444

Beams with Orbital Angular Momentum



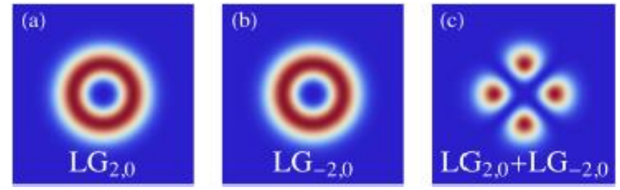
$l = +2$

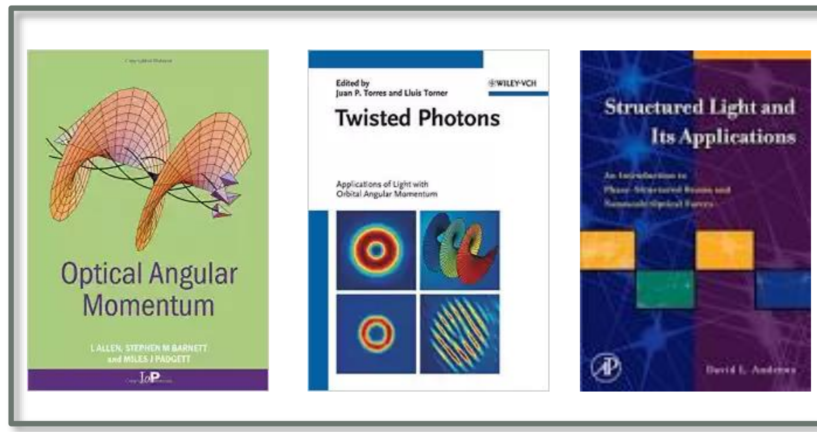
$l = +1$

$l = 0$

$l = -1$

$l = -2$

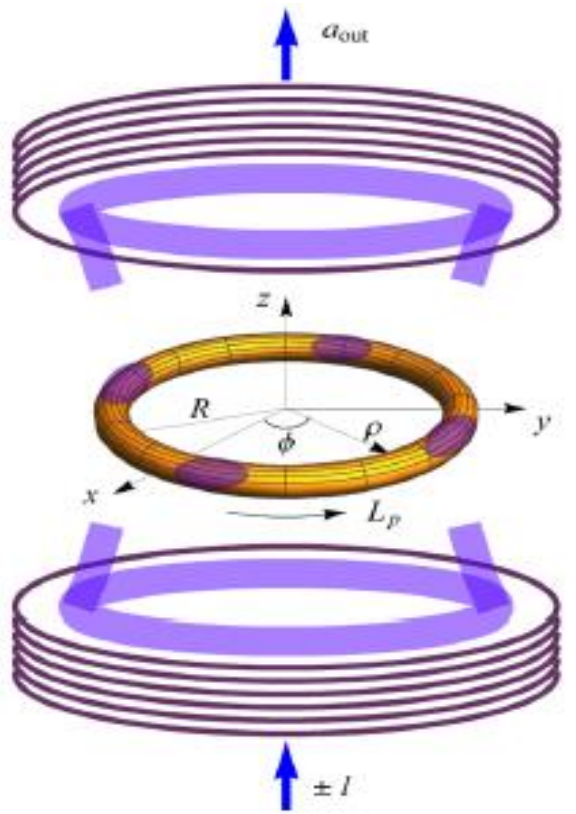




Mirhosseini et al., NJP **17**, 033033 (2015)
Leach et al., Science **329**, 662 (2010)

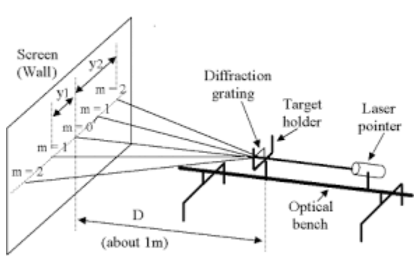
M. Padgett, Optics Express **25**, 11265 (2017)
MB and P. Meystre, PRL **99,** 153603 (2007)
H. Shi and MB, J. Phys. B **49**, 153001 (2016)

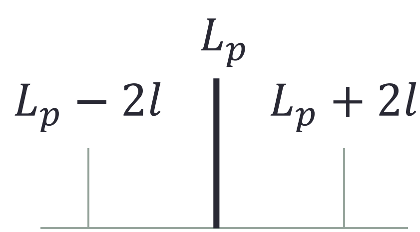
Ring BEC in a cavity: Detecting L_p



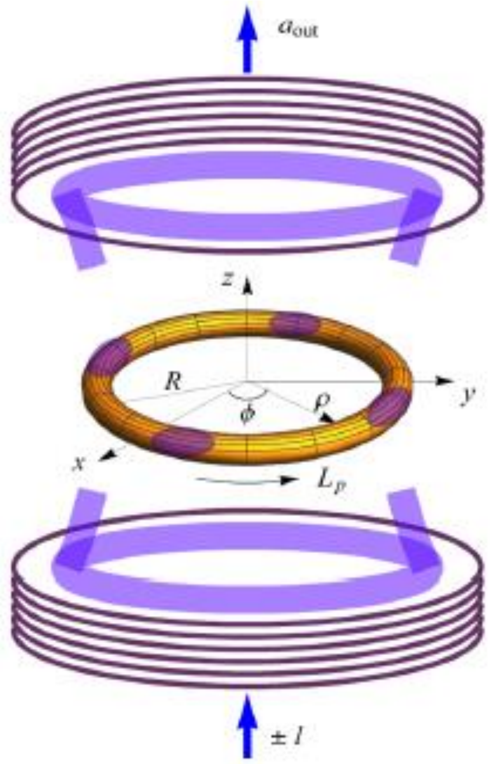
Measurement Scheme

1. We start with a ring BEC in a cavity.
2. The BEC has winding number L_p.
3. We drive the cavity with beams carrying OAM $\pm l\hbar$ far detuned from any atomic transition
4. The BEC matter wave Bragg diffracts from the angular lattice: $L_p \rightarrow L_p \pm 2l$
5. These matter waves modulate the BEC density.
6. These modulations are picked up by the light transmitted by the cavity (Rotational Doppler shift)
7. Cavity transmission yields L_p.





Ring BEC in a cavity: Detecting L_p



Hamiltonian: $$H = \int_0^{2\pi} \Psi^\dagger(\phi)\left[-\frac{\hbar^2}{2I}\frac{d^2}{d\phi^2} + \hbar U_o \cos^2(l\phi) a^\dagger a\right]\Psi(\phi)d\phi + \frac{g}{2}\int_0^{2\pi}\Psi^\dagger(\phi)\Psi^\dagger(\phi)\Psi(\phi)\Psi(\phi)d\phi - \hbar\Delta_o a^\dagger a - i\hbar\eta(a - a^\dagger),$$

$$I = mR^2;\ U_0 = \frac{g_a^2}{\Delta_a};\ g_a = E_0\langle e|d|g\rangle;\ \eta = \sqrt{P_{in}\gamma_o/\hbar\omega_o},$$
$$g = 2\hbar\omega_\rho a_{Na}/R$$

Ansatz: $$\Psi(\phi) = \frac{e^{iL_p\phi}}{\sqrt{2\pi}}c_p + \frac{e^{i(L_p+2l)\phi}}{\sqrt{2\pi}}c_+ + \frac{e^{i(L_p-2l)\phi}}{\sqrt{2\pi}}c_-,$$

Persistent current — Sidemodes

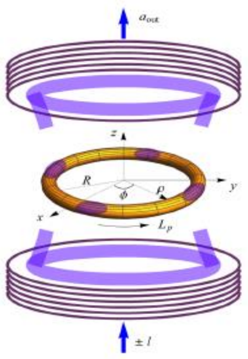
Normalization: $$c_p^\dagger c_p + c_+^\dagger c_+ + c_-^\dagger c_- = N$$

Recognize: $c_p \sim \sqrt{N}$ is a macroscopically occupied mode.

$$H = \hbar\omega_c c^\dagger c + \hbar\omega_d d^\dagger d + \hbar\left[G(X_c + X_d) - \tilde{\Delta}\right]a^\dagger a - i\hbar\eta\left(a - a^\dagger\right) + \hbar g\tilde{F}$$

$$\omega_{c,d} = \frac{\hbar(L_p \pm 2l)^2}{2I} \quad \left(c = \frac{c_p^\dagger c_+}{\sqrt{N}}, d = \frac{c_p^\dagger c_-}{\sqrt{N}}, G = \frac{U_0\sqrt{N}}{2\sqrt{2}}, \tilde{\Delta} = \Delta_0 - \frac{U_0 N}{2}\right)$$

Detecting L_p: Equations of Motion



Collisionless Hamiltonian ($g = 0$)

$$H = \hbar\omega_c c^\dagger c + \hbar\omega_d d^\dagger d + \hbar\left[G(X_c + X_d) - \tilde{\Delta}\right]a^\dagger a - i\hbar\eta(a - a^\dagger)$$

Equations of motion

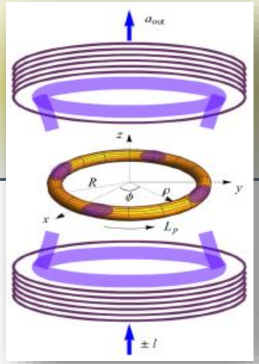
$$\ddot{X}_c + \gamma_m \dot{X}_c + \omega_c^2 X_c = -\omega_c G a^\dagger a + \omega_c \epsilon_c$$
$$\ddot{X}_d + \gamma_m \dot{X}_d + \omega_d^2 X_d = -\omega_d G a^\dagger a + \omega_d \epsilon_d$$
$$\dot{a} = i\left[\tilde{\Delta} - G(X_c + X_d)\right]a - \frac{\gamma_o}{2}a + \sqrt{\gamma_o}\, a_{in}$$

Heuristic argument (neglecting noise and damping)

For weak optical driving, $X_{c,d}$ will be modulated at $\omega_{c,d}$.
And therefore a will also.
→ So the cavity output will show two peaks at $\omega_{c,d}$.

W. Chen, D. S. Goldbaum, MB and P. Meystre,
PRA **81**, 053833 (2010)

Detecting L_p: Experimental Parameters

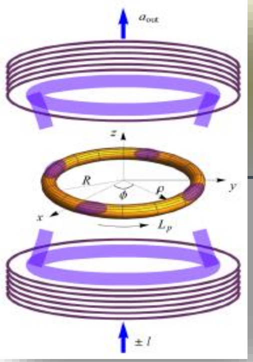


1. Mass of Sodium atom	$m = 23$ amu
2. Number of atoms	$N \sim 1.2 \times 10^4$
3. BEC radius	$R = 12$ μm
4. Trap frequencies	$\omega_{\rho,z} = 2\pi \times 4 \times 10^{1-2}$ Hz
5. Temperature	$T = 10 - 20$ nK
6. Chemical potential	$\frac{\mu}{\hbar} = 200 - 500$ Hz
7. Winding number of BEC	$L_p = 1 - 5$
8. OAM of light	$l = 1 - 15$
9. Atom velocity	$v_a \sim 0.3 mm/s$
10. Sound velocity	$v_c \sim 1.5 mm/s$
11. Healing length	$\xi = 4\mu$m
12. Atomic detuning	$\Delta_a = 2\pi \times 4.7$ GHz
13. Mechanical damping	$\gamma_m \sim 2\pi \times 1$ Hz
14. Cavity linewidth	$\gamma_0 = 2\pi \times 1.3$ MHz
15. Cavity detuning	$\Delta' = 0 - 500$ Hz
16. Cavity finesse	$\sim 10^4$
17. Optomechanical coupling	$G = 2\pi \times 260$ kHz
18. Input power	$P_{in} \sim 10$ fW
19. Angular lattice recoil	≤ 1KHz

P. Kumar et al., PRL **127**, 113601 (2021)

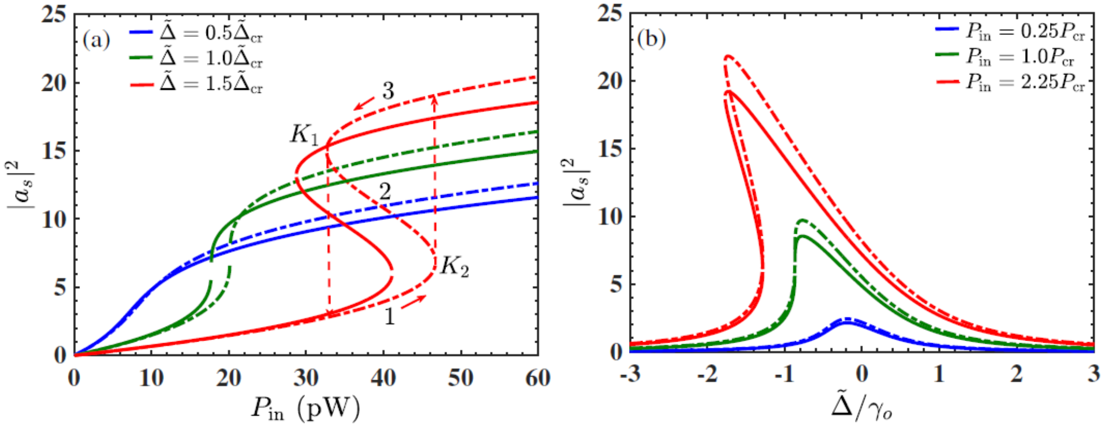
S. Kalita et al., PRA (2022), under review.

Steady state: Bistability



Photon Number vs Input Power

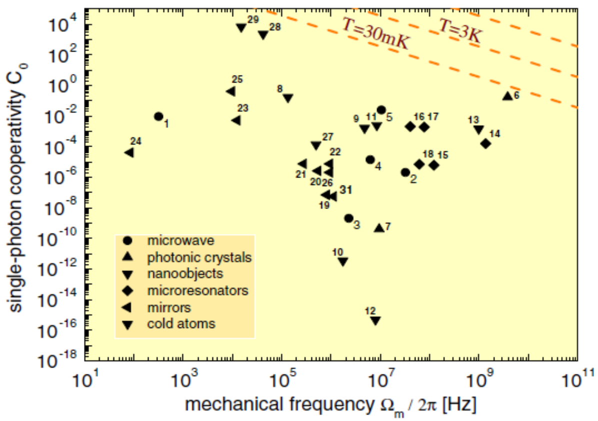
Photon Number vs Detuning



No collisions

Collisions

$$\tilde{\Delta} = \Delta_o - U_o N/2,$$



→ Bistability at very low photon numbers!

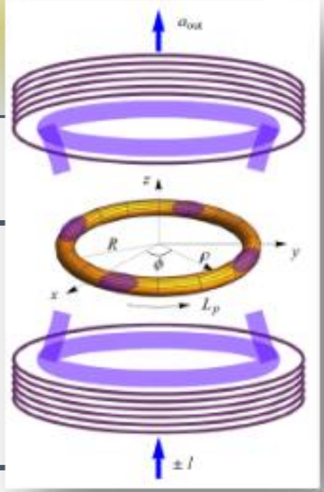
*Brennecke et al., Science **322**, 235 (2007)

- Bistability at $|a_s|^2 \cong 0.2$.

Purdy et al., PRL **105**, 133602 (2010)

RMP **82**, 1155 (2014)

Detecting L_p: Noise Spectrum



Linear response $\mathcal{M} \to \mathcal{M}_s + \delta\mathcal{M}$ for $\mathcal{M} = X_c, X_d, a$

Phase quadrature

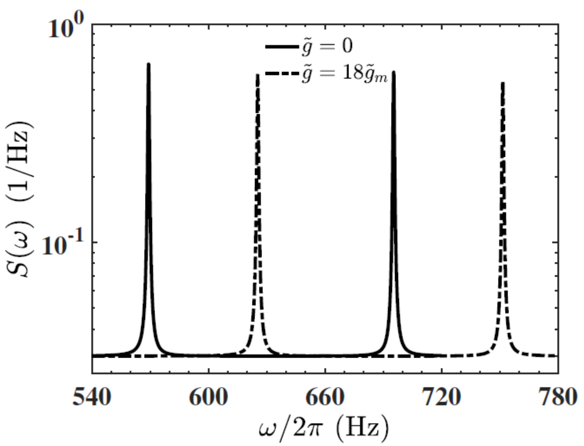
$$P = \frac{i(a_{out} - a_{out}^{\dagger})}{\sqrt{2}}$$

Input-output relation

$$a_{out} = -a_{in} + \sqrt{\gamma_o}\, a$$

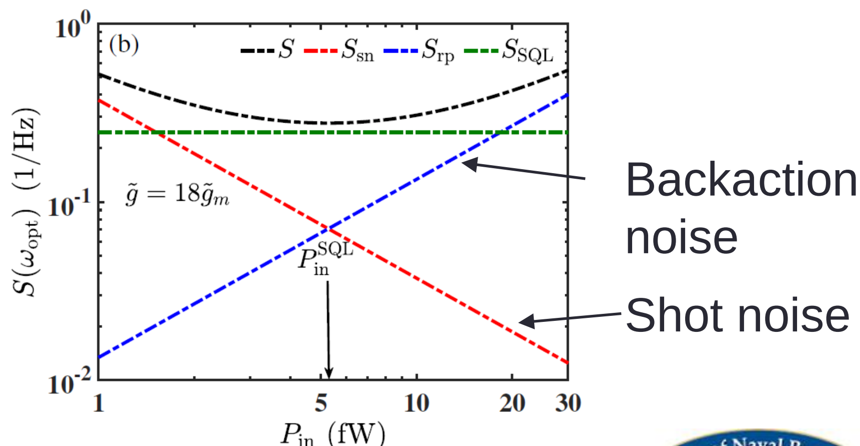
Output Noise Spectrum

$$S(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \delta P(t) \delta P(t) \rangle$$

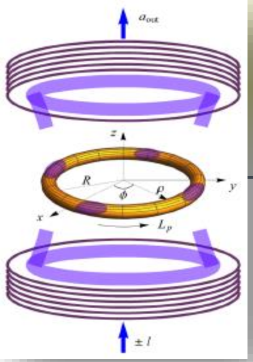


$$\omega_c - \omega_d \cong \frac{4lL_p\hbar}{I}$$

$$S(\omega) = S_{sn}(\omega) + S_{rp}(\omega) + S_{th}(\omega)$$



Detecting L_p: Sensitivity



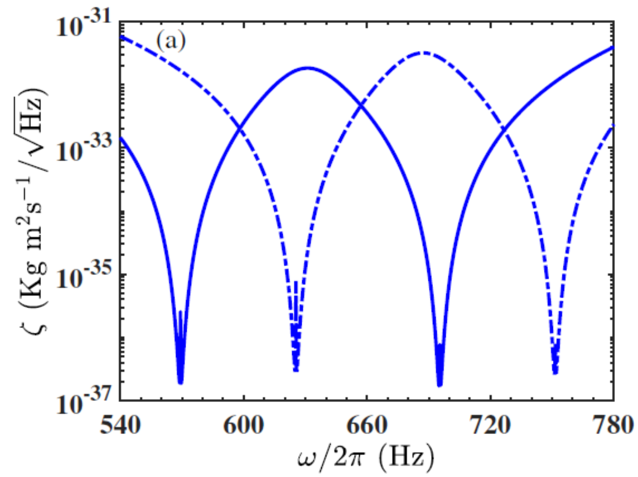
$$\zeta = \frac{S(\omega)}{\partial S(\omega)/\partial \Lambda}\sqrt{t_{meas}}$$

$$\Lambda = \hbar L_p$$

$$t_{meas} = \frac{\gamma_0}{8(a_s G)^2} \sim 60 \text{ ms} \ll 1 \text{ min}$$

→ *in situ*

→ ~ Real time



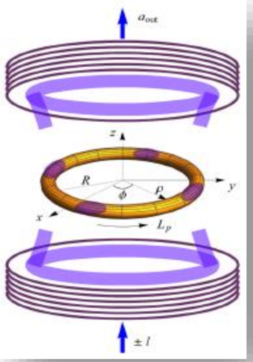
Number of atoms in the sidemodes ~ 0.1! → Minimally destructive

$10^{-3}\text{Hz}/\sqrt{\text{Hz}}$: → Thousand times more sensitive than demonstrated.

*Kumar et al., New Journal of Physics **18**, 025001 (2016)

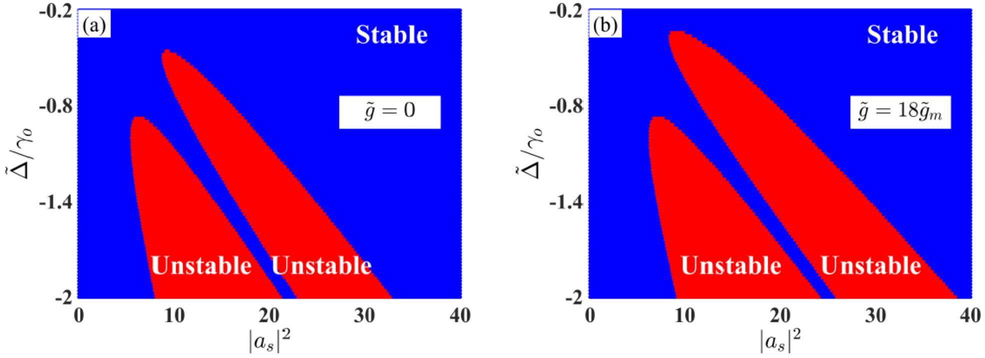
Ragole and Taylor, PRL **117**, 203002 (2016)

Detecting L_p: Stability and Nonlinear response



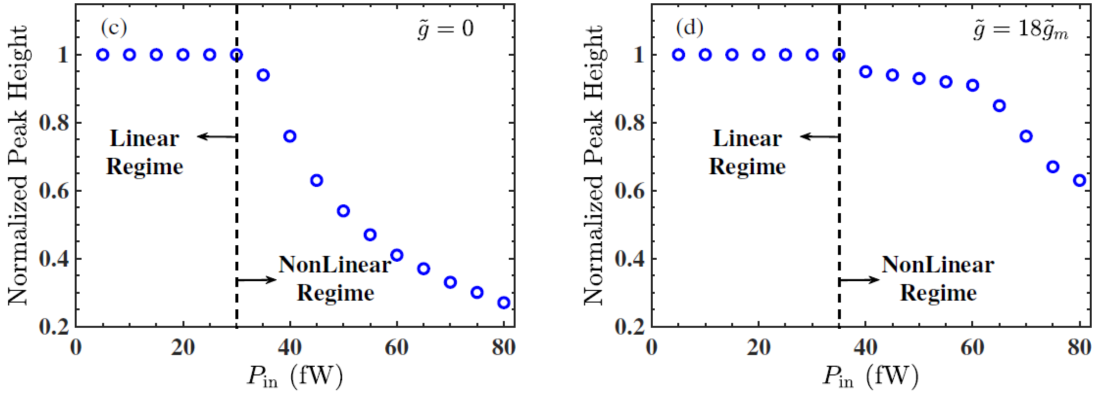
No collisions | Collisions

Routh-Hurwitz criterion



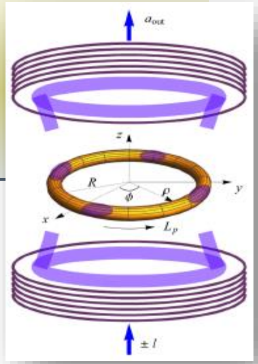
$$\tilde{g}_m = \frac{\hbar}{4IN}$$

Linear and nonlinear response regimes



P. Kumar…MB, PRL **127**, 113601 (2021)

Matter wave (bipartite) entanglement



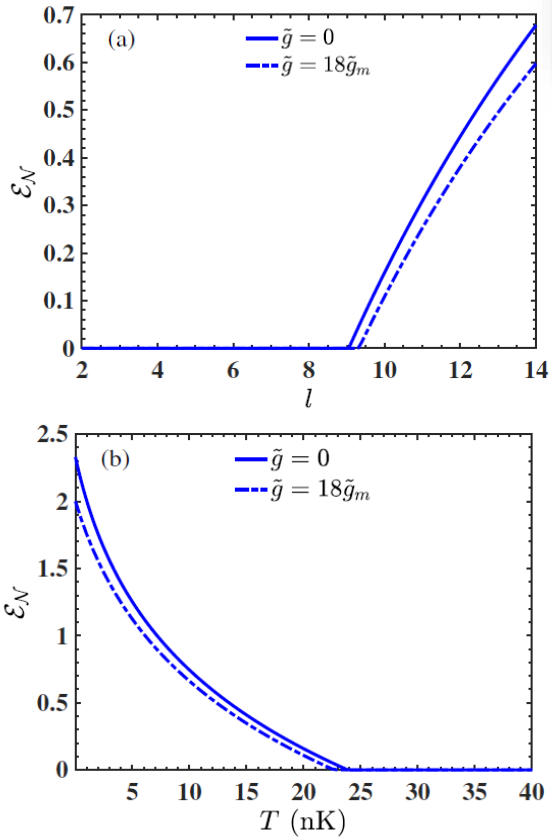
Logarithmic negativity

$$\mathcal{E}_N = \max[0, -\ln(2\sigma_-)]$$

$$\sigma_- = 2^{-1/2}\left[\Sigma - \sqrt{\Sigma^2 - 4\det(V_{sub})}\right]^{1/2}$$

$$\Sigma = \det A + \det B - 2\det C$$

$$V_{\rm sub} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$



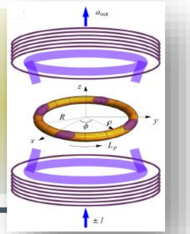
$$V_{X_c X_c} = \langle \delta X_c \delta X_c \rangle$$

$$FV + VF^T = -D$$

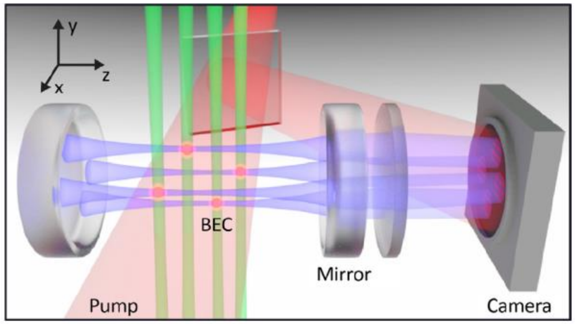
$$F = \begin{pmatrix} 0 & \Omega_c & 0 & 0 & 0 & 0 \\ -\Omega_c & -\gamma_m & -\frac{\mathcal{A}}{\Omega_c} & 0 & -\frac{\tilde{\omega}_c G_r}{\Omega_c} & -\frac{\tilde{\omega}_c G_i}{\Omega_c} \\ 0 & 0 & 0 & \Omega_d & 0 & 0 \\ \frac{\mathcal{A}}{\Omega_d} & 0 & -\Omega_d & -\gamma_m & -\frac{\tilde{\omega}_d G_r}{\Omega_d} & -\frac{\tilde{\omega}_d G_i}{\Omega_d} \\ G_i & 0 & G_i & 0 & -\frac{\gamma_o}{2} & -\Delta' \\ -G_r & 0 & -G_r & 0 & \Delta' & -\frac{\gamma_o}{2} \end{pmatrix}$$

$$D = \mathrm{diag}\left(0, \gamma_m(2n_c+1), 0, \gamma_m(2n_d+1), \frac{\gamma_o}{2}, \frac{\gamma_o}{2}\right)$$

Conclusions: Creating a Stir



* We have theoretically shown cavity optomechanics can be used to measure the superfluid rotation of a ring BEC in a minimally destructive manner, *in situ,* in real time and with three orders of magnitude better sensitivity than demonstrated.



Marsh et al., PRX **11**, 021048(2021)

Future work

Applying the optomechanical toolbox
to problems in atomic superfluid rotation

- Supercurrents in the BCS-BEC crossover
- Solitonic interferometry
- Andreev-Bashkin effect (superfluid drag)
- Entangling, squeezing, synchronizing rotating matter waves
- Atomic memory for photons carrying OAM (5 orders of magnitude better storage time)
- Directional amplifier (Nunnenkamp et al.) ↘
- Entanglement distiller (S. Bose et al.)
- Wavelength converter (Lin Tian et al.)
- …

Coherent and reversible interface between optics and atomic superfluid rotation.

**Pump-probe cavity optomechanics with a rotating atomic superfluid in a ring*
S. Kalita, P. Kumar, R. Kanamoto, M. Bhattacharya and A. K. Sarma,
Phys. Rev. A (submitted).

CAREER

THANKS !!!