The TMD shape function and its applications at the EIC

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Probing Hadron Structure at the EIC

ICTS INTERNATIONAL CENTRE for THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

In collaboration with:
J. Bor, D. Boer, C.Pisano & F. Yuan

Outline

- **Part I: TMD shape function** in **TMD factorization** for Quarkonium
- **Part II: Matching procedure** to access the **TMDShF perturbative tail**
  - Relevance of the hard amplitude pole structure
- **Part III:** the TMDShF depends on Q? **Process dependence**?
- **Part IV**: Opportunities at the **EIC** to investigate the **TMDShF**

Quarkonia & gluon TMDs

Processes involving Quarkonia are **sensitive to gluons**

hadron collisions

- $p + p \to \eta_Q + X$
- $p + p \to \chi_Q + X$
- $p + p \to J/\psi + J/\psi + X$
- $p + p \to J/\psi + X$ **?**

***ep* collisions**

- $e + p \to e' + J/\psi + X$
- $e + p \to e' + J/\psi + \gamma + X$
- $e + p \to e' + J/\psi + \text{jet} + X$

and more…

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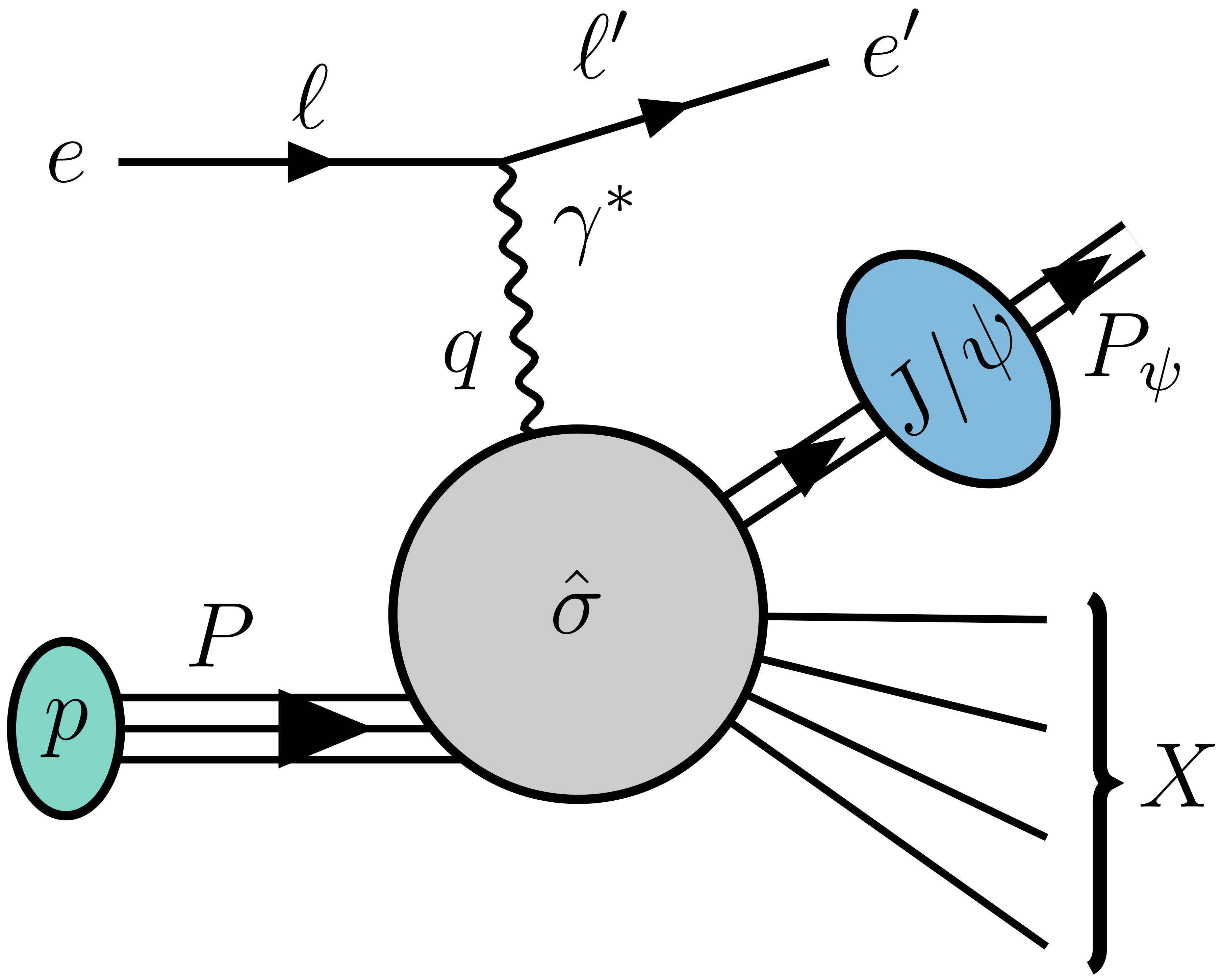
and more…

Theoretical framework

$$e\,(\ell) + p\,(P) \to e'\,(\ell') + \gamma^*\,(q) + p\,(P) \to e'\,(\ell') + J/\psi\,(P_\psi) + X$$



SIDIS variables

$$q^2 = -Q^2,\ S \approx 2P\cdot\ell$$

$$x_B = \frac{Q^2}{2\cdot q},\ y = \frac{P\cdot q}{P\cdot\ell},\ z = \frac{P\cdot P_\psi}{P\cdot q}$$

Phase spaces

$$\frac{\mathrm{d}^3\ell'}{2E'} = 2\pi\, yS\,\mathrm{d}x_B\mathrm{d}y$$

$$\frac{\mathrm{d}^3P_\psi}{2E_\psi} = \frac{\mathrm{d}z}{z}\mathrm{d}^2P_{\psi\perp}\mathrm{d}\phi_\psi$$

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02 (2007)

Sun, Zhang, EJPC 77 (2017)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\mathbf{P}^2_{\psi\perp}\,\mathrm{d}\phi_\psi} = \frac{\alpha}{yQ^2}\Big\{\left[1+(1-y)^2\right]F_{UUT} + 4(1-y)\,F_{UUL} + (2-y)\sqrt{1-y}\cos\phi_\psi\,F_{UU}^{\cos\phi_\psi} + 4(1-y)\,\cos 2\phi_\psi\,F_{UU}^{\cos 2\phi_\psi}\Big\}$$

(Some) Models for Quarkonium formation

- **Colour Singlet Model**

Baier, Rückl, Z.Phys.C 19 (1983)

$$d\sigma[\mathcal{Q}] = \int d\xi_i\, d\xi_j\, f_i(\xi_i) f_j(\xi_j)\, d\hat{\sigma}_{i+j\to\mathcal{Q}+X}\, |R(0)|^2$$

(S-wave production)

- **Non Relativistic QCD** (CS + CO mechanism)

Bodwin, Braaten, Lepage, PRD 51 (1997)

$$d\sigma[\mathcal{Q}] = \sum_n \int d\xi_i\, d\xi_j\, f_i(\xi_i) f_j(\xi_j)\, d\hat{\sigma}_{i+j\to QQ[n]+X} \langle \mathcal{O}_{\mathcal{Q}}[n] \rangle$$

Long-Distance Matrix Elements (**universal** in principle)

- **(Improved) Colour Evaporation Model**

Ma, Vogt, PRD 94 (2016)

$$\frac{d\sigma[\mathcal{Q}]}{dP_{\mathcal{Q}}} = F_{\mathcal{Q}} \int_{M_{\mathcal{Q}}}^{2M_H} dM \frac{d\sigma_{QQ}(M, P'_{\mathcal{Q}})}{dM dP_{\mathcal{Q}}}$$

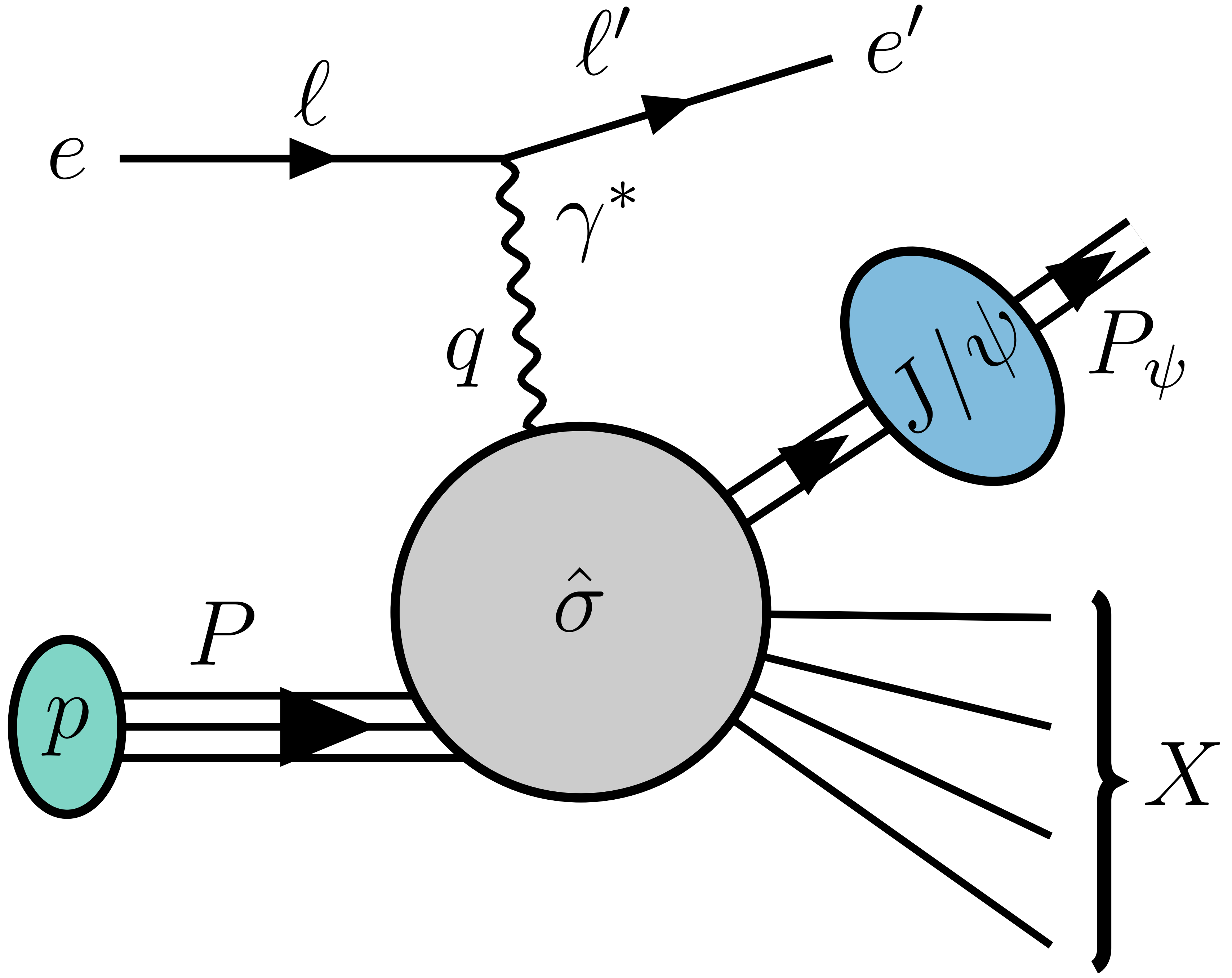
- **Fragmentation Functions**

Kang, Ma, Qiu, Sterman, PRD 90 (2014)

$$d\sigma[\mathcal{Q}] = \int d\xi_i\, d\xi_j\, dz\; f_i f_j\, d\hat{\sigma}_{i+j\to f+X}\, D_{f\to\mathcal{Q}}(z) + \int d\xi_i\, d\xi_j\, dz\; f_i f_j\, d\hat{\sigma}_{i+j\to QQ+X}\, D_{QQ\to\mathcal{Q}}(z)$$

The TMD shape function

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)



“light-hadron” SIDIS

$$\sigma^{ep\to e'hX} = \hat{\sigma}^{[a]}(\mu_H) \otimes f_a(\hat{x};\mu_H) \otimes D_{a\to h}(\hat{z};\mu_H)$$

Bodwin, Braaten, Lepage, PRD 51 (1997)

“Quarkonium” SIDIS (adopting NRQCD)

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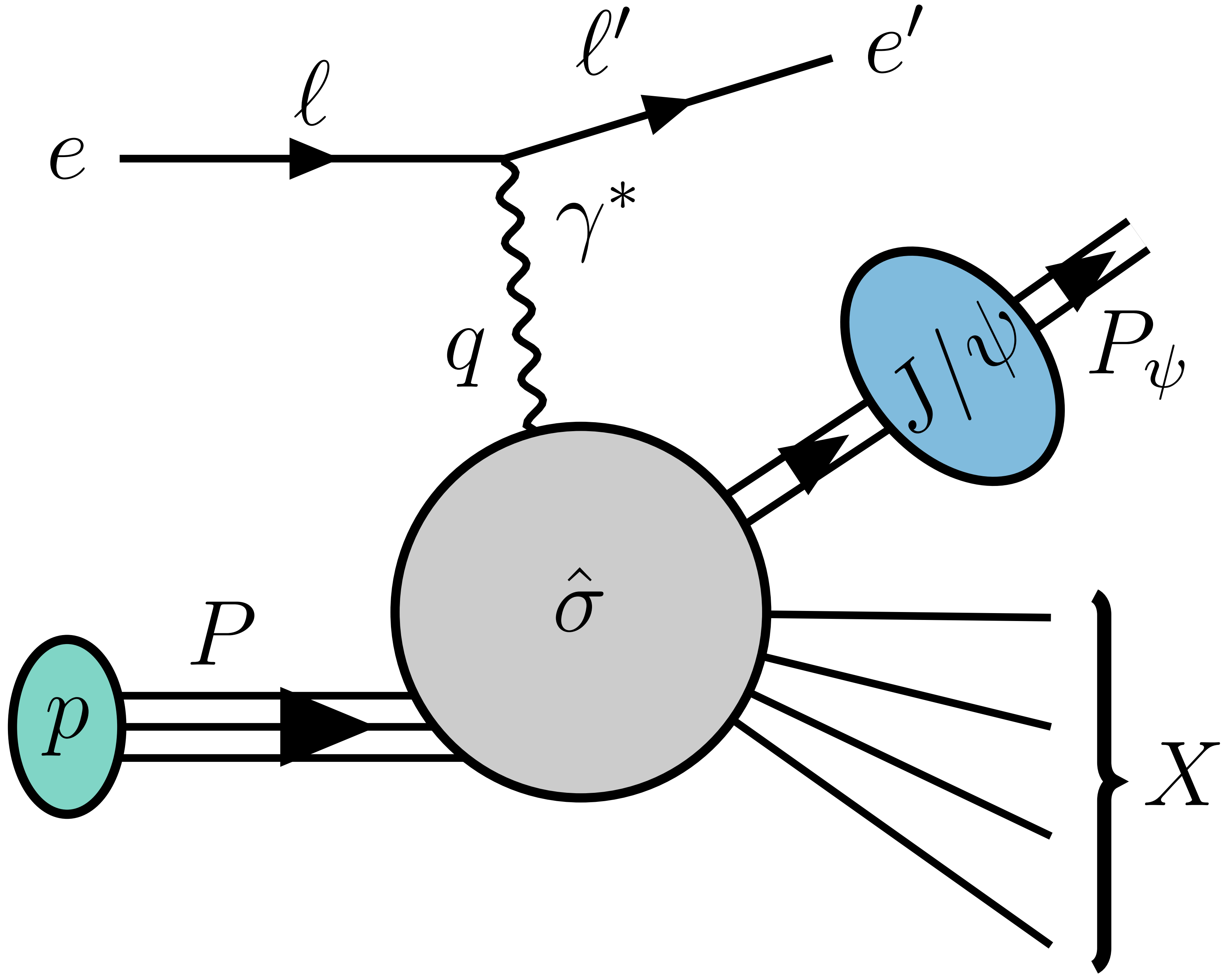

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As for $D_{a\to h}(\hat{z}) \to D_{a\to h}(\hat{z}, k_T)$, we have $\langle \mathcal{O}_\psi[n]\rangle \delta(\hat{z}-z) \to \Delta^{[n]}(\hat{z}, k_T)$

[Echevarría, JHEP 144 (2019)](#)

[Fleming, Markis, Mehen, JHEP 112 (2020)](#)

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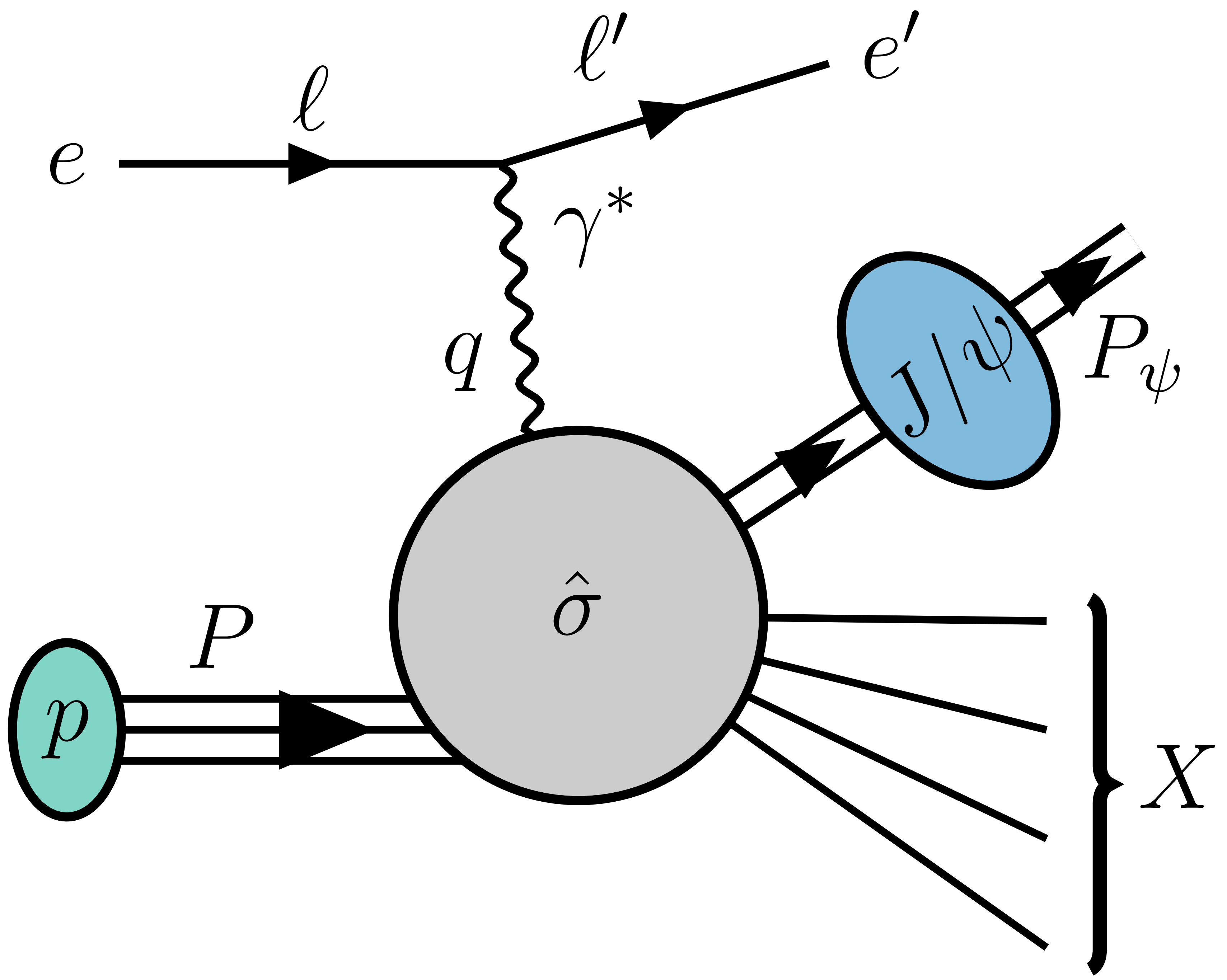

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$\boldsymbol{\Delta^{[n]}}$ encodes hadronization *plus* exchange of soft gluons

Echevarría, JHEP 144 (2019)

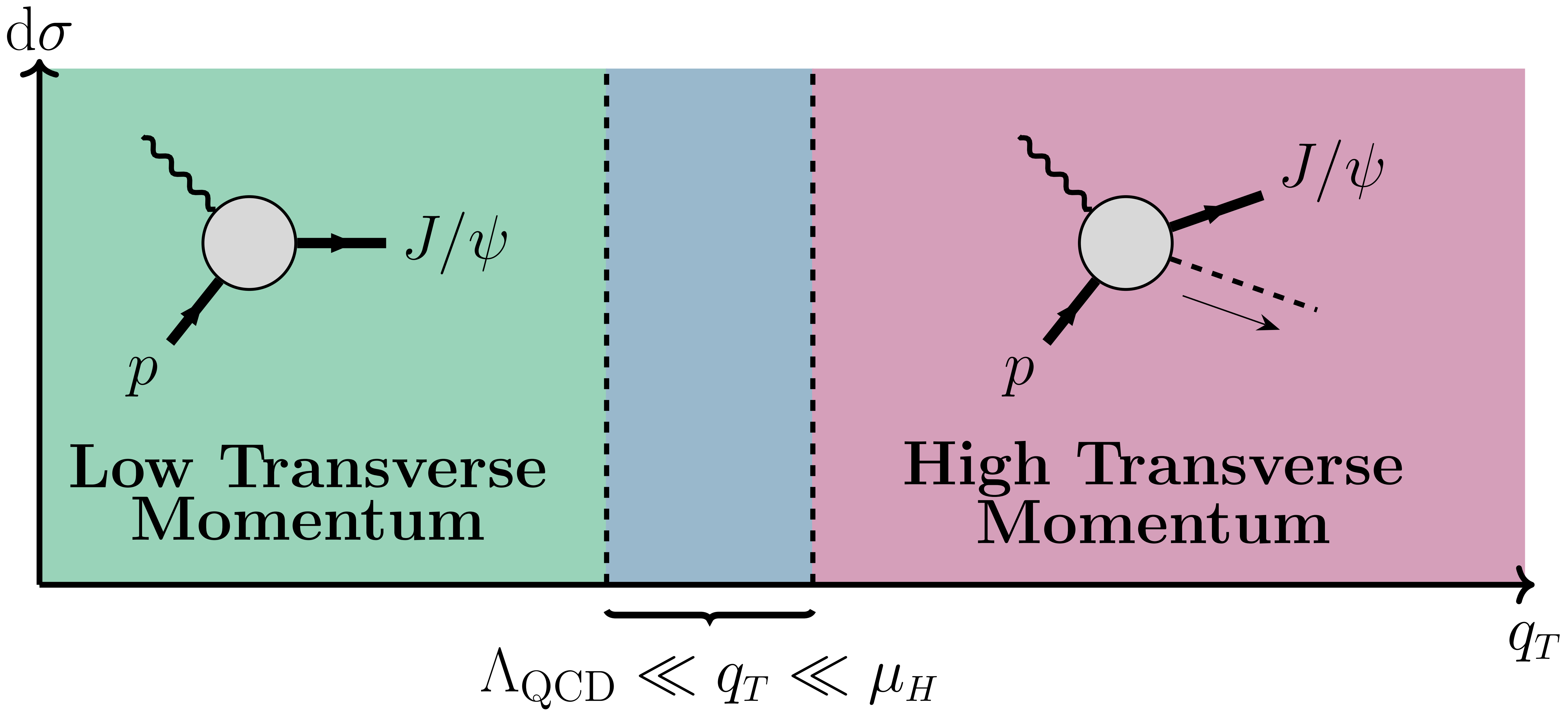
Fleming, Markis, Mehen, JHEP 112 (2020)

Matching procedure

q_T → photon transverse momentum

$P_{\psi\perp}$ → J/ψ transverse momentum

$$q_T = \frac{P_{\psi\perp}}{\hat{z}}$$



TMD factorization $q_T \ll \mu_H$

↓

Overlapping at $\Lambda_{\text{QCD}} \ll q_T \ll \mu_H$

↑

Collinear factorization $q_T \gg \Lambda_{\text{QCD}}$

Matching procedure

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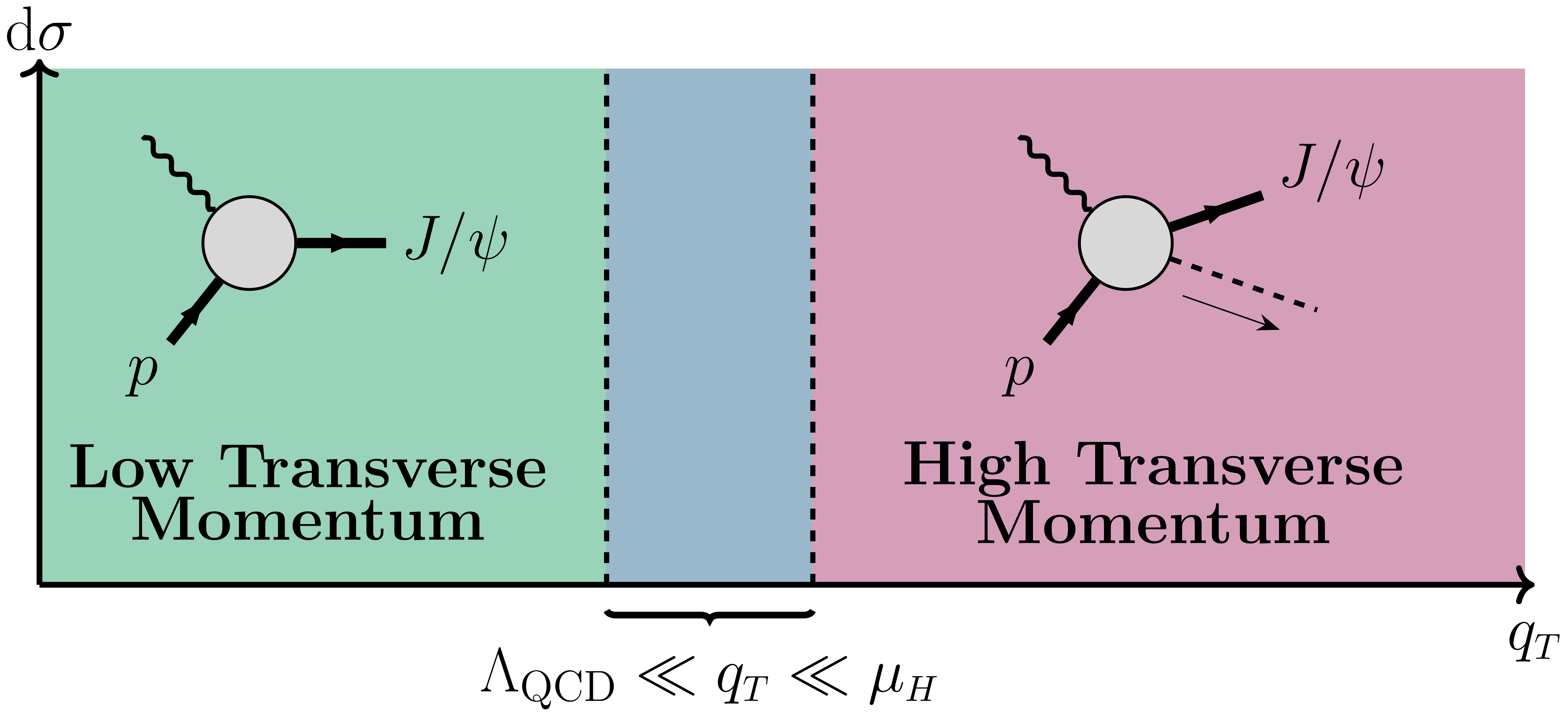
→ $q_T = \frac{P_{\psi\perp}}{\hat{z}}$



Description of the same dynamics?

Thus they should match!

TMD factorization $q_T \ll \mu_H$

Overlapping at $\Lambda_{\rm QCD} \ll q_T \ll \mu_H$

Collinear factorization $q_T \gg \Lambda_{\rm QCD}$

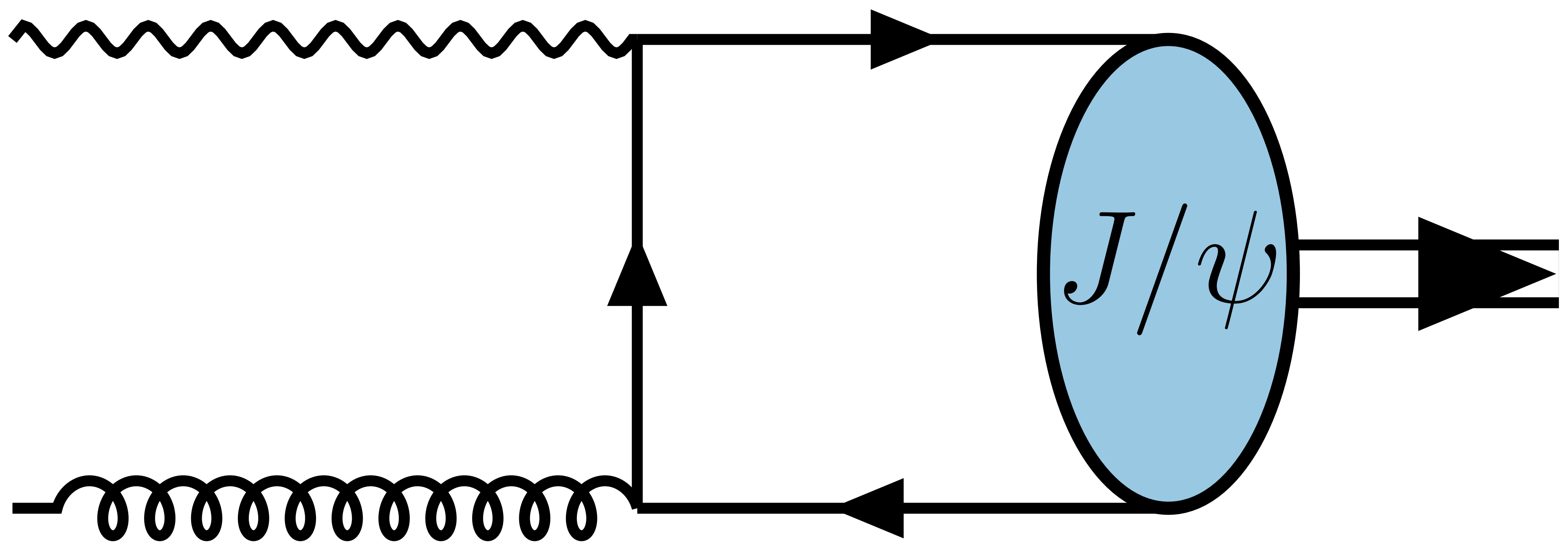
Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)

Boer, D'Alesio, Murgia, Pisano, Taels, JHEP 09 (2020)

Structure function at small-q_T (TMD region)

J/ψ production at the lowest α_s-order: $\gamma^* + g \to c\bar{c}[n]$

Bacchetta, Boer, Pisano, Taels, EPJC 80 (2020)



Kinematic fixes most of the variables:

- $\hat{x} = x$ $\left(\text{where } x = x_B \dfrac{M_\psi^2 + Q^2}{Q^2}\right)$
- $\hat{z} = 1$
- $p_{aT} = q_T$

$$\mathrm{d}\sigma|_{\mathrm{TMD}} = \frac{\alpha}{yQ^2}\left\{ \left[1 + (1-y)^2\right] \mathcal{F}_{UUT} + 4(1-y)\left(\mathcal{F}_{UUL} + \cos 2\phi\, \mathcal{F}_{UU}^{\cos 2\phi}\right) \right\}$$

Involves the convolutions:
$$\begin{cases} \mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, q_T) \\ \mathcal{C}\left[w\, h_1^{\perp g} \Delta_h^{[n]}\right](x, q_T) \end{cases}$$

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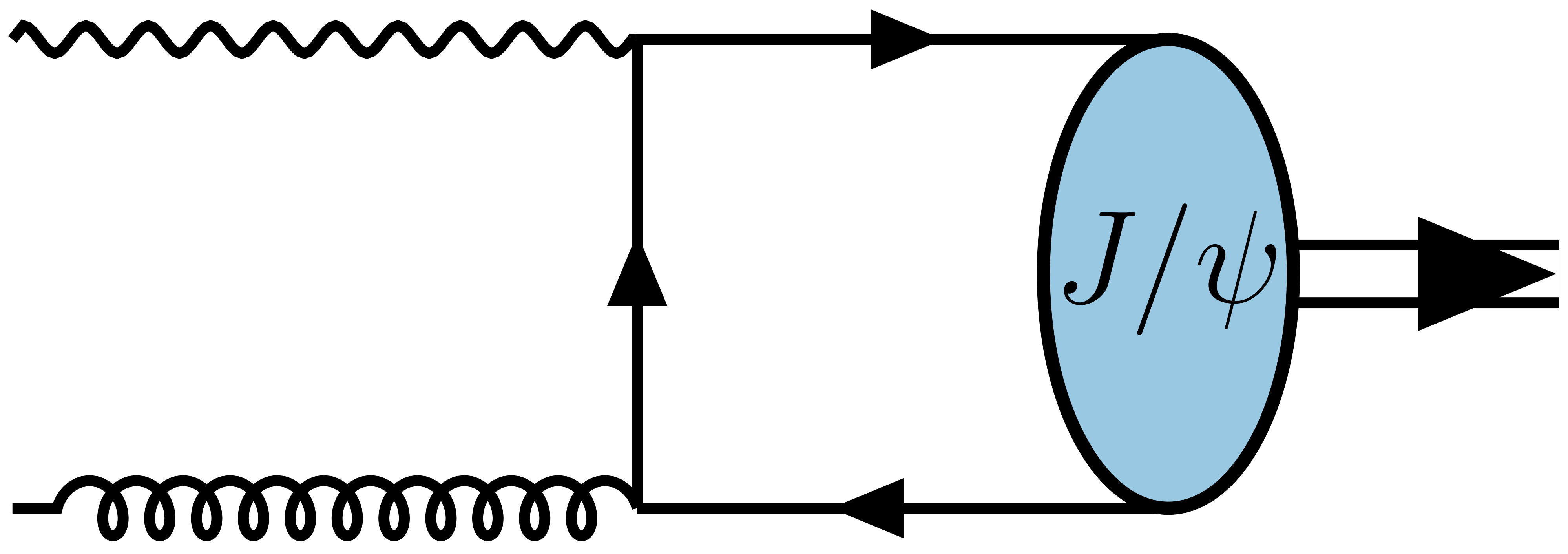

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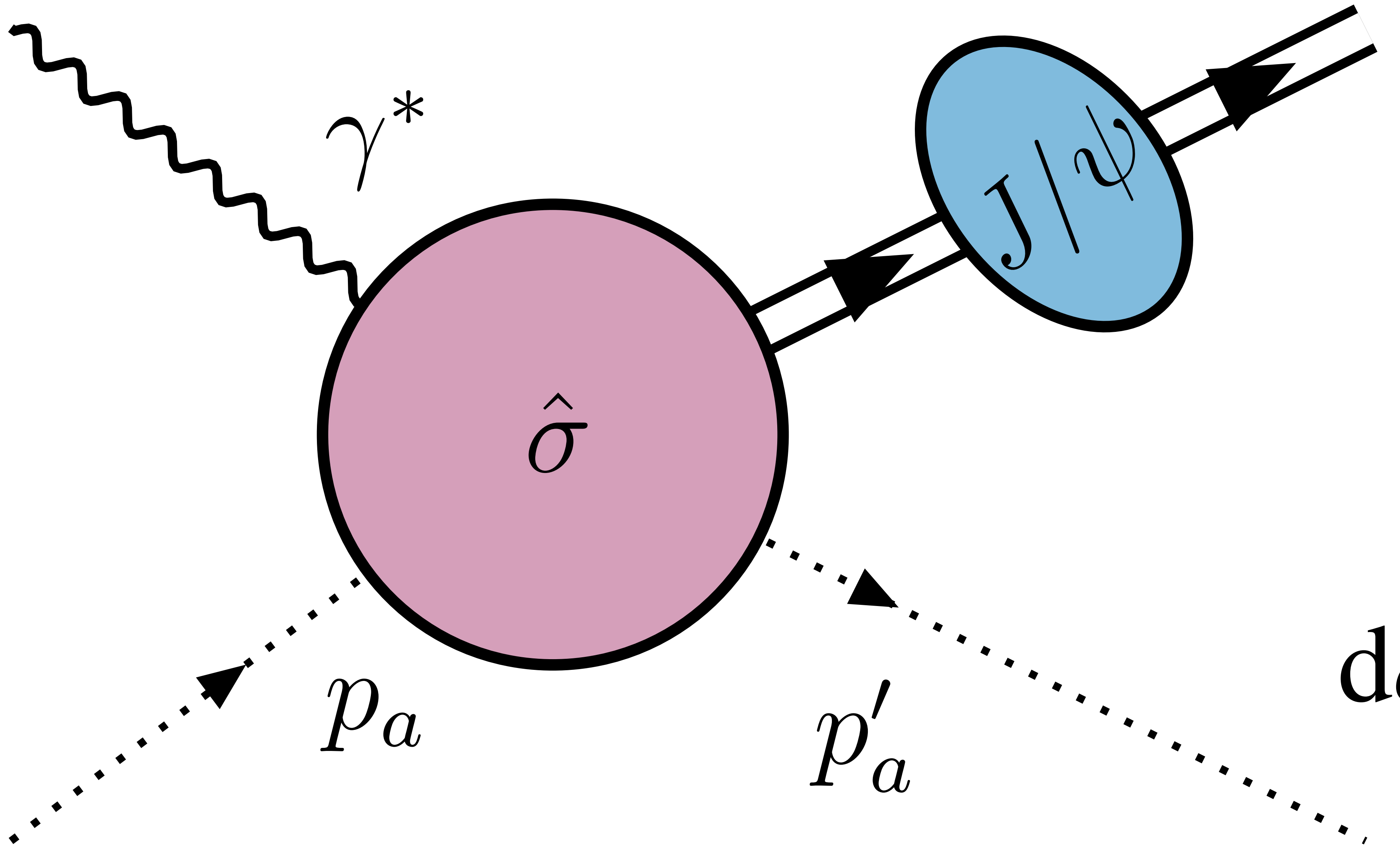
Structure function at high-q_T (collinear region)

J/ψ production at the lowest α_s-order: $\gamma^* + a \to c\bar{c}[n] + a$ $(a = q,\, \bar{q},\, g)$



$$\mathrm{d}\sigma^{ep\to e'J/\psi X} = \mathrm{d}\hat{\sigma}^{a\,[n]}(\mu_H) \otimes f_p^a(\hat{x};\mu_H) \otimes \langle \mathcal{O}_\psi[n]\rangle\, \delta(\hat{z}-z)$$

Lepton tensor from

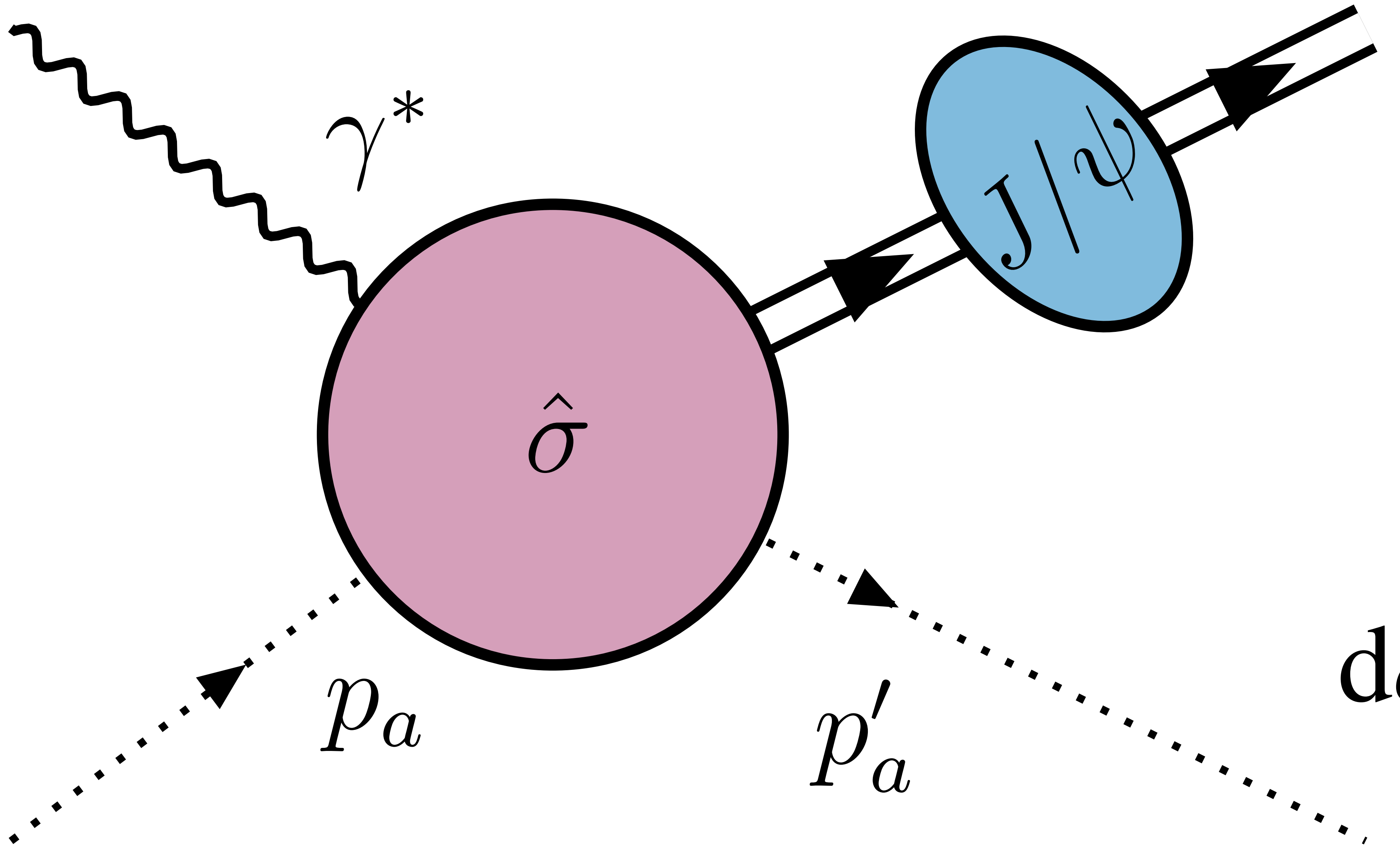
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02 (2007)

$$\mathrm{d}\hat{\sigma}^{a\,[n]} \propto \int \frac{\mathrm{d}\hat{x}}{\hat{x}} \frac{\mathrm{d}\hat{z}}{\hat{z}} \frac{L^{\mu\nu}}{Q^4} H_\mu^{a\,[n]} H_\nu^{*\,a\,[n]}\ \delta(\hat{x}', \hat{z})$$

$$\hat{x}' = \frac{x_B}{\hat{x}} \frac{M_\psi^2 + Q^2}{Q^2}$$

Structure function at high-q_T (collinear region)

J/ψ production at the lowest α_s-order: $\gamma^* + a \to c\bar{c}[n] + a$ $(a = q,\ \bar{q},\ g)$



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$$\delta(\hat{x}', \hat{z}) = \delta\left(\frac{(1-\hat{x}')(1-\hat{z})}{\hat{x}'\hat{z}} + \frac{1-\hat{z}}{\hat{z}}\frac{\hat{z}-\hat{x}'}{\hat{x}'\hat{z}}\frac{M_\psi^2}{Q^2} + \frac{q_T^2}{Q^2}\right)$$

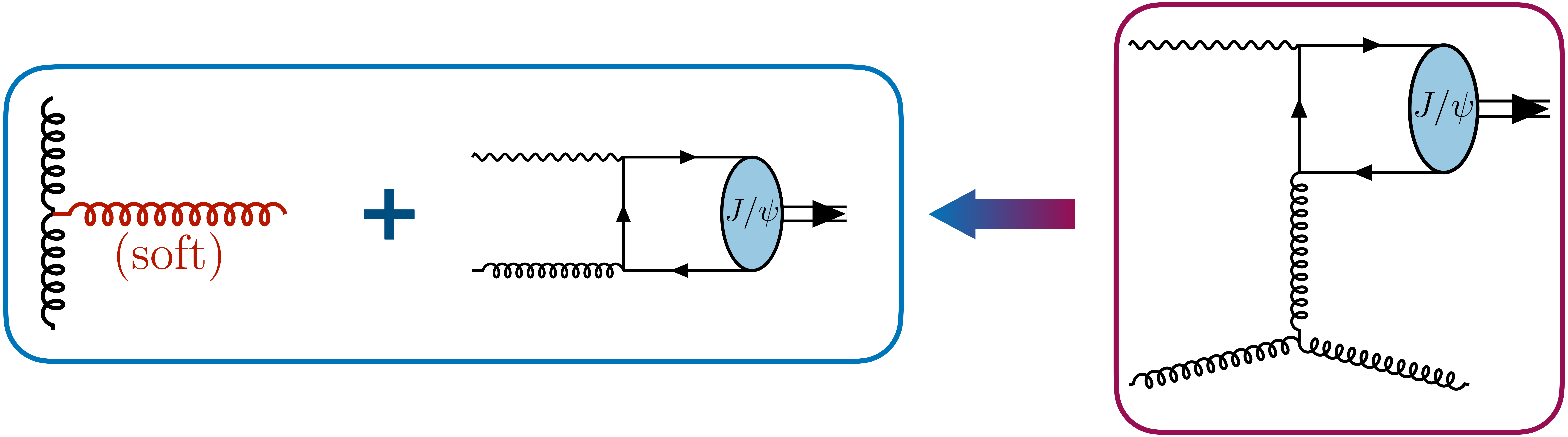
for $M_\psi \ll Q$ in agreement with

[Meng, Olness, Soper JHEP 11 (2019)]

Schematic small-q_T limit valid at $\Lambda_{\rm QCD} \ll q_T \ll \mu_H$

$\Lambda_{\rm QCD} \ll q_{_T} \ll \mu_H$

$q_{_T} \gg \Lambda_{\rm QCD}$



Limit is obtained by expanding $\delta(\hat{x}', \hat{z})$ at small-$q_{_T}$

$$\delta(\hat{x}', \hat{z}) \sim \frac{M_\psi^2 + Q^2}{M_\psi^2/\hat{z} + Q^2} \frac{\hat{z}}{(1-\hat{z})_+} \delta(1-\hat{x}') + \log \frac{M_\psi^2 + Q^2}{\boldsymbol{q}_T^2} \delta(1-\hat{x}')\, \delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x})_+} \delta(1-\hat{z})$$

Delta expansion

Boer, D'Alesio, Murgia, Pisano, Taels, JHEP 09 (2020)

$$I = \int_0^1 \mathrm{d}\hat{z}\, g(\hat{z}) \int_0^{\hat{x}_{\max}} \mathrm{d}\hat{x}\, f(\hat{x})\, \delta(\hat{x}, \hat{z}) = \hat{x}_{\max} \int_0^1 \mathrm{d}\hat{z}\, \hat{z}^2\, g(\hat{z}) \int_0^1 \mathrm{d}\hat{x}'\, \hat{x}' f(\hat{x}')\ \delta\Big((1-\hat{x}')(1-\hat{z}) + (1-\hat{z})(\hat{z}-\hat{x}')\frac{M_\psi^2}{Q^2} + \frac{q_T^2}{Q^2}\Big)$$

continuous test functions

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continuous test functions

$$\hat{x}_0' = (1 + \frac{M_\psi^2}{Q^2}) \left[1 + \frac{M_\psi^2}{\hat{z}\, Q^2} + \frac{\hat{z}}{1-\hat{z}} \frac{\boldsymbol{q}_T^2}{Q^2} \right]^{-1}$$

$$I = \int_0^1 \mathrm{d}\hat{z}\, \frac{(1-\hat{z})\,(\hat{z}\, Q^2 + M_\psi^2)}{(1-\hat{z})\,(\hat{z}\, Q^2 + M_\psi^2) + \hat{z}^2\, \boldsymbol{q}_T^2}\, \tilde{g}(\hat{z})\, \tilde{f}(\hat{x}_0') = I_1 + I_2 + I_3$$

Obtained from:

$$\tilde{g}(\hat{z})\, \tilde{f}(\hat{x}_0') = \big(\tilde{g}(\hat{z}) - \tilde{g}(1)\big)\, \tilde{f}(1) + \tilde{g}(1)\, \tilde{f}(1) + \tilde{g}(\hat{z})\, \big(\tilde{f}(\hat{x}_0') - \tilde{f}(1)\big)$$

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$$\hat{x}'_0 = (1 + \frac{M_\psi^2}{Q^2}) \left[1 + \frac{M_\psi^2}{\hat{z}\, Q^2} + \frac{\hat{z}}{1-\hat{z}} \frac{\boldsymbol{q}_T^2}{Q^2} \right]^{-1}$$

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Poles in the structure functions

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

$$F_{UU}\left(\hat{x}', \hat{z}\right) = F_{UU}^{(0)}\left(\hat{x}', \hat{z}\right) + \sum_{k=1}\left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^{k} F_{UU}^{(k)}\left(\hat{x}', \hat{z}\right)$$

(general notation)

Continuous functions of $\hat{x}'$ and $\hat{z}$

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Continuous functions of $\hat{x}'$ and $\hat{z}$

✓ Delta expansion is applicable

✗ Delta expansion is not applicable

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Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

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Relevant for $\gamma^* g$ in $F_{UUT}^{(k)}$ and $F_{UUL}^{(k)}$ with $k = 1,\, 2$

Impact on the double delta

$$\delta(\hat{x}', \hat{z}) \sim \frac{\hat{x}'}{(1-\hat{x})_+} \delta(1-\hat{z}) + \log \frac{M_\psi^2 + Q^2}{q_T^2} \delta(1-\hat{x}')\, \delta(1-\hat{z})$$

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$$\log\frac{M_\psi^2 + Q^2}{q_T^2} \to \frac{1}{2}\left(\log\frac{M_\psi^2 + Q^2}{q_T^2} - 1 - \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right)$$

Eikonal method

Same term is found by considering the soft gluon emission

$$x_g = \frac{p_a \cdot p_g}{p_a \cdot q}$$

$$\mathrm{d}\sigma_1 \propto \int_{\frac{-p_{g\perp}^2}{M_\psi^2 + Q^2}}^{1} \frac{\mathrm{d}x_g}{x_g} \left[2\, S_g(p_a, P_\psi) + S_g(P_\psi, P_\psi) \right]$$

from momentum conservation and $2p_g^+ p_g^- = -p_{g\perp}^2 = P_{\psi\perp}^2$

$$S_g(v_1, v_2) = \frac{v_1 \cdot v_2}{(v_1 \cdot p_g)(v_2 \cdot p_g)}$$

Eikonal method

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Relation to quark-pair Fragmentation Function?

Kang, Ma, Qiu, Sterman, PRD 90 (2014) & PRD 91 (2015)

Ma, Qiu, Sterman, Zhang, PRL 113 (2014)

$$\mathrm{d}\sigma[\mathcal{Q}] = \int \mathrm{d}\xi_i\,\mathrm{d}\xi_j\,\mathrm{d}z\; f_i\, f_j\,\mathrm{d}\hat{\sigma}_{i+j\to f+X}\, D_{f\to\mathcal{Q}}(z)$$
$$+\int \mathrm{d}\xi_i\,\mathrm{d}\xi_j\,\mathrm{d}z\; f_i\, f_j\,\mathrm{d}\hat{\sigma}_{i+j\to Q\bar{Q}+X}\, \boldsymbol{D_{Q\bar{Q}\to\mathcal{Q}}}(z)$$

TMD shape function perturbative tail

Comparison at $\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

TMD-PDFs evolved according to

Echevarria, Kasemets, Mulders, Pisano, JHEP 07 (2015)

Sun, Xiao, Yuan, PRD 84 (2011)

$$\mathcal{F}_{UU}^{\cos 2\phi}\big|_{\mathrm{TMD}} = F_{UU}^{\cos 2\phi}\big|_{\mathrm{coll}} \quad \Rightarrow \quad \Delta_{h,\psi}^{[n]} = \delta^{(2)}(k_T^2)\,\langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

$$\left.\begin{array}{l} \mathcal{F}_{UUT}\big|_{\mathrm{TMD}} \neq F_{UUT}\big|_{\mathrm{coll}} \\ \mathcal{F}_{UUL}\big|_{\mathrm{TMD}} \neq F_{UUL}\big|_{\mathrm{coll}} \end{array}\right\} \quad \Rightarrow \quad \Delta_{\psi}^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Up to the precision considered, bulk of the expression driven by **CO waves**

$${}^1S_0^{(8)} \qquad {}^3P_J^{(8)}$$

Scale dependence of the TMD shape function

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

Previous equation is obtained for $\mu_H \equiv \sqrt{M_\psi^2 + Q^2}$

(in b_T-space)

$$\tilde{\Delta}_\psi^{[n]}\left(z, b_T; \sqrt{M_\psi^2 + Q^2}\right) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2}{\left(M_\psi^2 + Q^2\right)}\right) \log \frac{M_\psi^2 + Q^2}{\mu_b^2}\right] \langle \mathcal{O}[n] \rangle \, \delta(1-z)$$

$$\mu_b = \frac{2\, e^{-\gamma_E}}{b_T}$$

Scale dependence of the TMD shape function

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$$\mu_b = \frac{2\,e^{-\gamma_E}}{b_T}$$

for a general (hard) scale μ_H

$$\tilde{\Delta}_{ep}^{[n]}(z, b_T; \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2\,\mu_H^2}{\left(M_\psi^2 + Q^2\right)^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle \mathcal{O}[n]\rangle\,\delta(1-z)$$

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! It is process dependent !

Note that a Q^2-dependent soft factor is present in the open-quark production too

Zhu, Sun, Yuan, Phys. Lett. B 727 (2013)

Scale dependence of TMD-PDFs

TMD-PDF matching coefficients taken from Echevarria, Kasemets, Mulders, Pisano, JHEP 07 (2015)

$$\tilde{f}_1^{g/A}(x, b_T; \zeta, \mu) = \int_x^1 \frac{\mathrm{d}\bar{x}}{\bar{x}}\, \tilde{C}_{g/j}\, f_{j/A}(\bar{x}/x; \mu)$$

(Only terms relevant in region $\Lambda_{\mathrm{QCD}} \ll \mu$)

gluon-gluon splitting function

$$\tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{2}\log^2\frac{Q^2}{\mu_b^2}\delta(1-x) - \log\frac{Q^2}{\mu_b^2}\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right]$$

$$\beta_0 = \frac{11}{3}C_F - \frac{4}{3}T_F n_f$$

$$\mu_b = \frac{2\,\mathrm{e}^{-\gamma_E}}{b_T}$$

$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{Q^2}{\mu_b^2}P_{g/q}\right]$$

$$\mu_b = \frac{2\,\mathrm{e}^{-\gamma_E}}{b_T}$$

quark-gluon splitting function

Scale dependence of TMD-PDFs

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$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{Q^2}{\mu_b^2}P_{g/q}\right]$$

Q comes from the scale choice ($\mu = Q$)

Scale dependence of TMD-PDFs

TMD-PDF matching coefficients taken from Echevarria, Kasemets, Mulders, Pisano, JHEP 07 (2015)

$$\tilde{f}_1^{g/A}(x, b_T; \zeta, \mu) = \int_x^1 \frac{\mathrm{d}\bar{x}}{\bar{x}}\, \tilde{C}_{g/j}\, f_{j/A}(\bar{x}/x; \mu)$$

(Only terms relevant in region $\Lambda_{\mathrm{QCD}} \ll \mu$ and for general μ)

$$\tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[C_A\left(-\frac{1}{2}\log^2\frac{\mu^2}{\mu_b^2} + \log\frac{\mu^2}{Q^2}\log\frac{\mu^2}{\mu_b^2}\right)\delta(1-x) - \log\frac{\mu^2}{\mu_b^2}\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right]$$

$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{\mu^2}{\mu_b^2}P_{g/q}\right]$$

Scale dependence of TMD-PDFs

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$$\tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[C_A\left(-\frac{1}{2}\log^2\frac{\mu^2}{\mu_b^2} + \log\frac{\mu^2}{Q^2}\log\frac{\mu^2}{\mu_b^2}\right)\delta(1-x) - \log\frac{\mu^2}{\mu_b^2}\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right]$$

$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{\mu^2}{\mu_b^2}P_{g/q}\right]$$

Comes from the rapidity regulator choice ($\zeta = Q^2$)

No rapidity divergences for the TMD shape function $\tilde{\Delta}^{[n]}_{ep}(z, b_T; \mu_H)$

in agreement with

[del Castillo, Echevarria, Makris, Scimemi, JHEP 03 (2022)]

[Bor, Boer, Phys.Rev.D 106 (2022)]

Process dependence of the TMD shape function

[Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)](#)

$$\tilde{\Delta}^{[n]}_{ep}(z, b_T; Q, \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2\,\mu_H^2}{(M_\psi^2 + Q^2)^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle \mathcal{O}[n]\rangle\,\delta(1-z)$$

Reasons to split-up this term:

1. A purely quarkonium quantity should depend on M_ψ solely

2. In open-quark production the soft-factor may produce azimuthal dependeces

[Catani, Grazzini, Torre, Nucl.Phys. B 890 (2014)](#)

$$\frac{d\sigma^{(res)}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c}}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}$$
$$\times\; S_c(M,b)\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\,[(\mathbf{H}\boldsymbol{\Delta})C_1C_2]_{c\bar{c};a_1a_2}$$
$$\times\; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)\,,$$

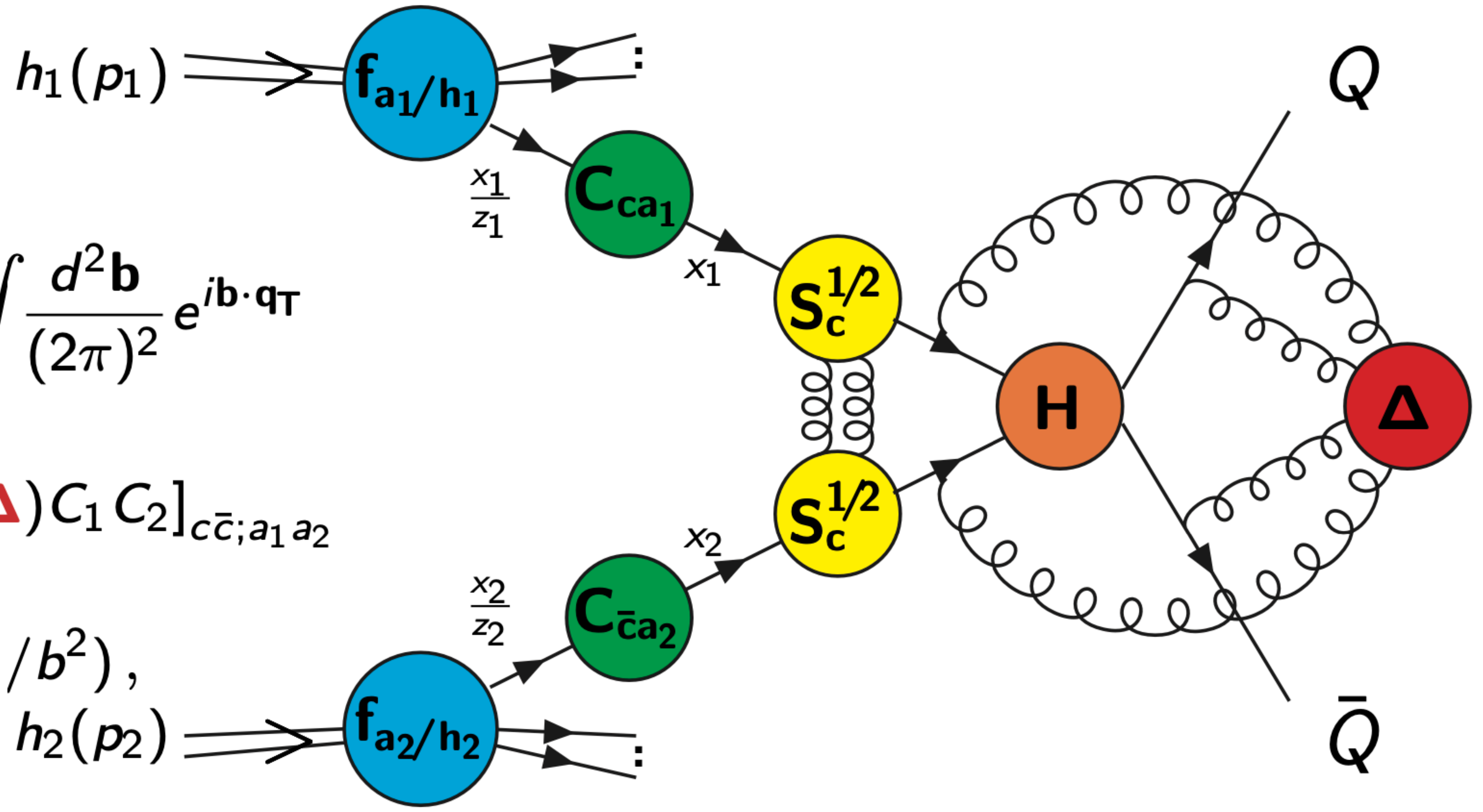

Figure taken from Ferrera's talk
@ Heavy-Quark Hadroproduction from
Collider to Astroparticle Physics (2019)

Process dependence of the TMD shape function

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

$$\tilde{\Delta}_{ep}^{[n]}(z, b_T; Q, \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2\,\mu_H^2}{\left(M_\psi^2 + Q^2\right)^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\,\delta(1-z)$$

split up: $\boldsymbol{\Delta_{ep}^{[n]} = \Delta_\psi^{[n]} \times S_{ep}}$

$$\Delta_\psi^{[n]}(z, b_T; \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2}{\mu_H^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\,\delta(1-z)$$

➡ **Universal**

$$S_{ep}(b_T; Q, \mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2\log\frac{\mu_H^2}{M_\psi^2 + Q^2}\right)\log\frac{\mu_H^2}{\mu_b^2}$$

➡ **Process dependent**

Single vs double scales processes

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

J/ψ production in:

SIDIS (2 hard scales)

$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

hadron collisions (1 hard scale)

$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

Single vs double scales processes

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

J/ψ production in:

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$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

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$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$S_{pp}(M_\psi) \approx 0$$

Easier extraction of $\Delta_\psi^{[n]}(M_\psi)$

Single vs double scales processes

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

J/ψ production in:

SIDIS (2 hard scales)

$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$\Delta_\psi^{[n]}\left(\sqrt{M_\psi^2 + Q^2}\right)$$

hadron collisions (1 hard scale)

$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$S_{pp}(M_\psi) \approx 0$$

Easier extraction of $\Delta_\psi^{[n]}(M_\psi)$

Evolution to

Single vs double scales processes

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

J/ψ production in:

SIDIS (2 hard scales)

$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$\Delta_\psi^{[n]}\left(\sqrt{M_\psi^2 + Q^2}\right)$$

Tested at other scales, e.g. Υ production

hadron collisions (1 hard scale)

$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$S_{pp}(M_\psi) \approx 0$$

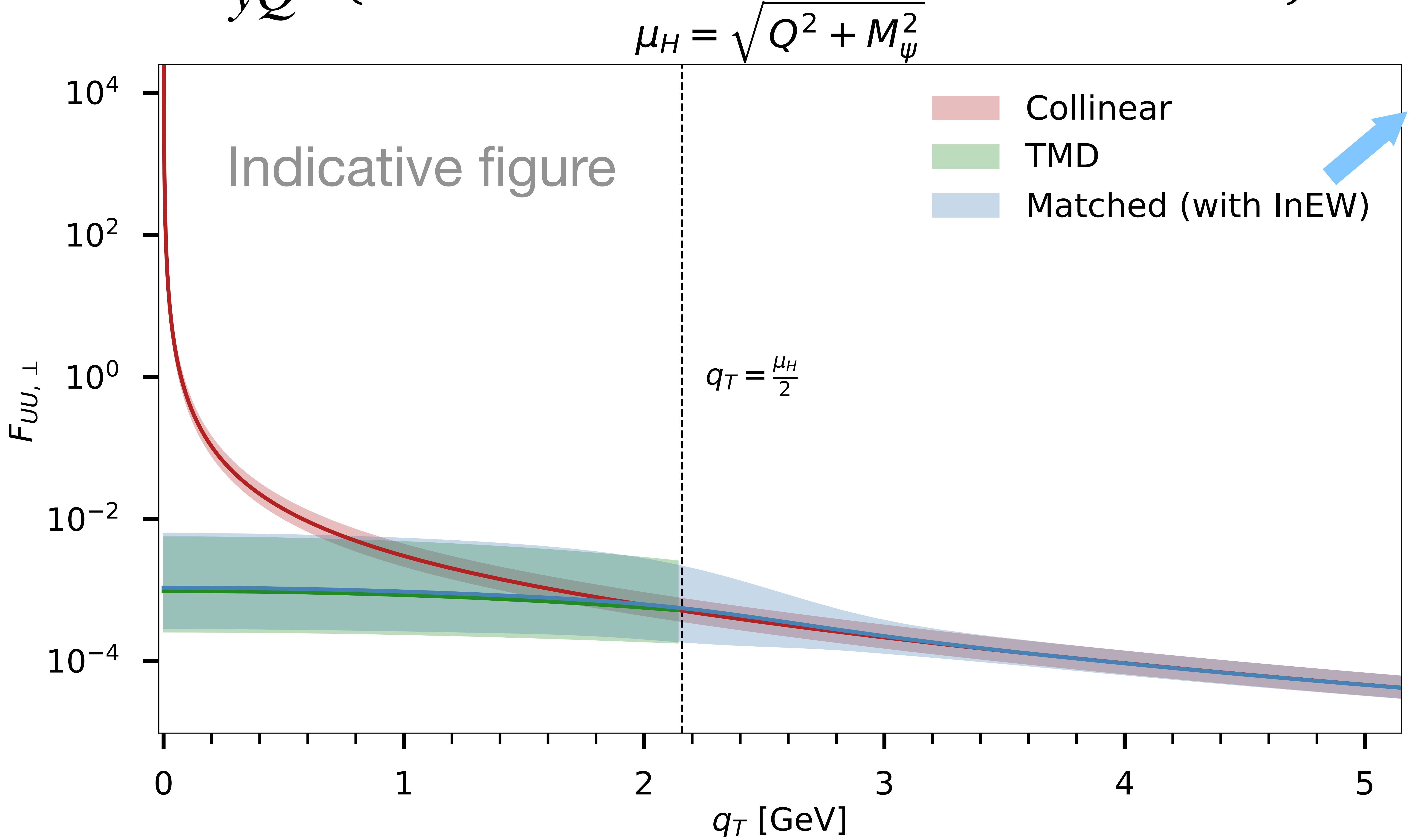
Easier extraction of $\Delta_\psi^{[n]}(M_\psi)$

Evolution to

Gluon TMDs at the EIC (unpolarized)

EIC has the opportunity to access the **unpolarized gluon TMDs**

$$d\sigma|_{\mathrm{TMD}} = \frac{\alpha}{yQ^2}\left\{ \left[1+(1-y)^2\right] \mathcal{F}_{UUT} + 4(1-y)\, \mathcal{F}_{UUL} \right\}$$



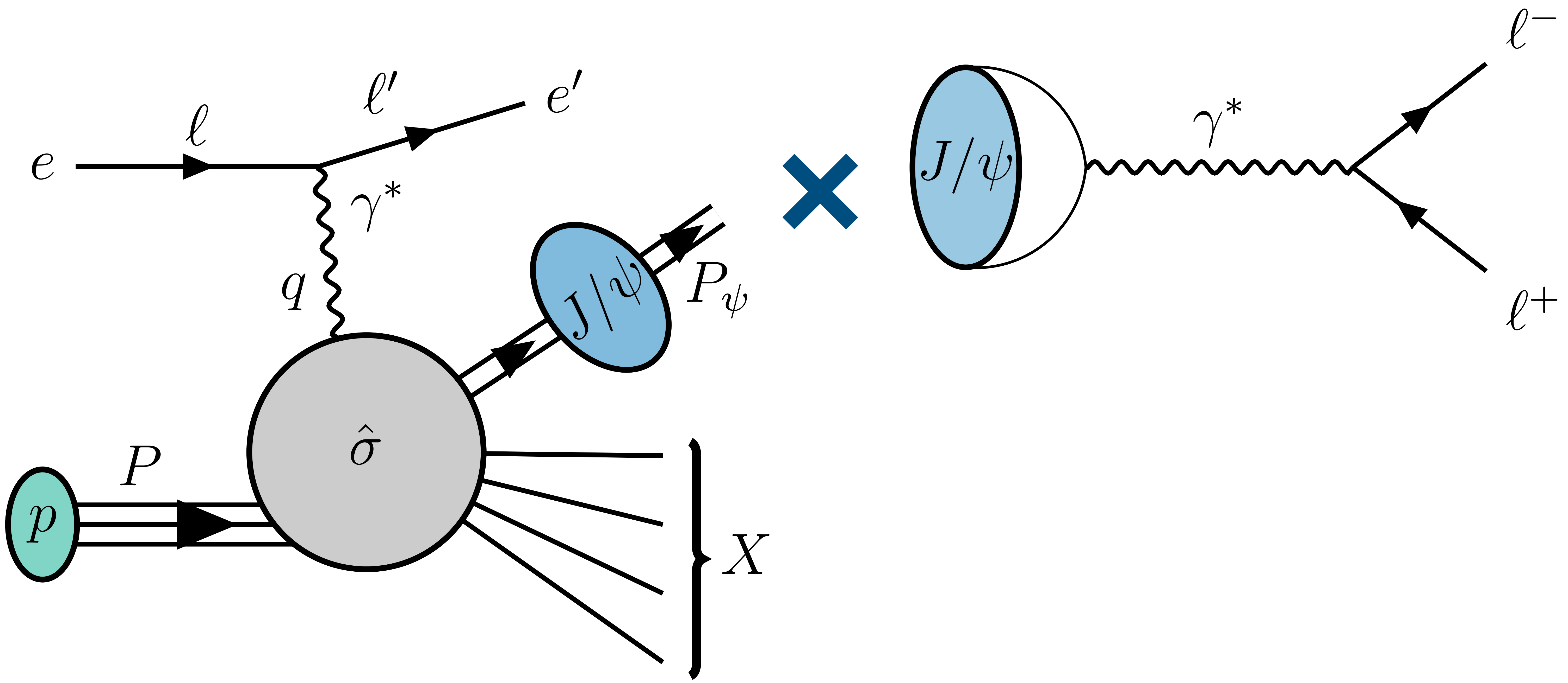
Echevarria, Kasemets, Lansberg, Pisano, Signori, PLB 781 (2018)

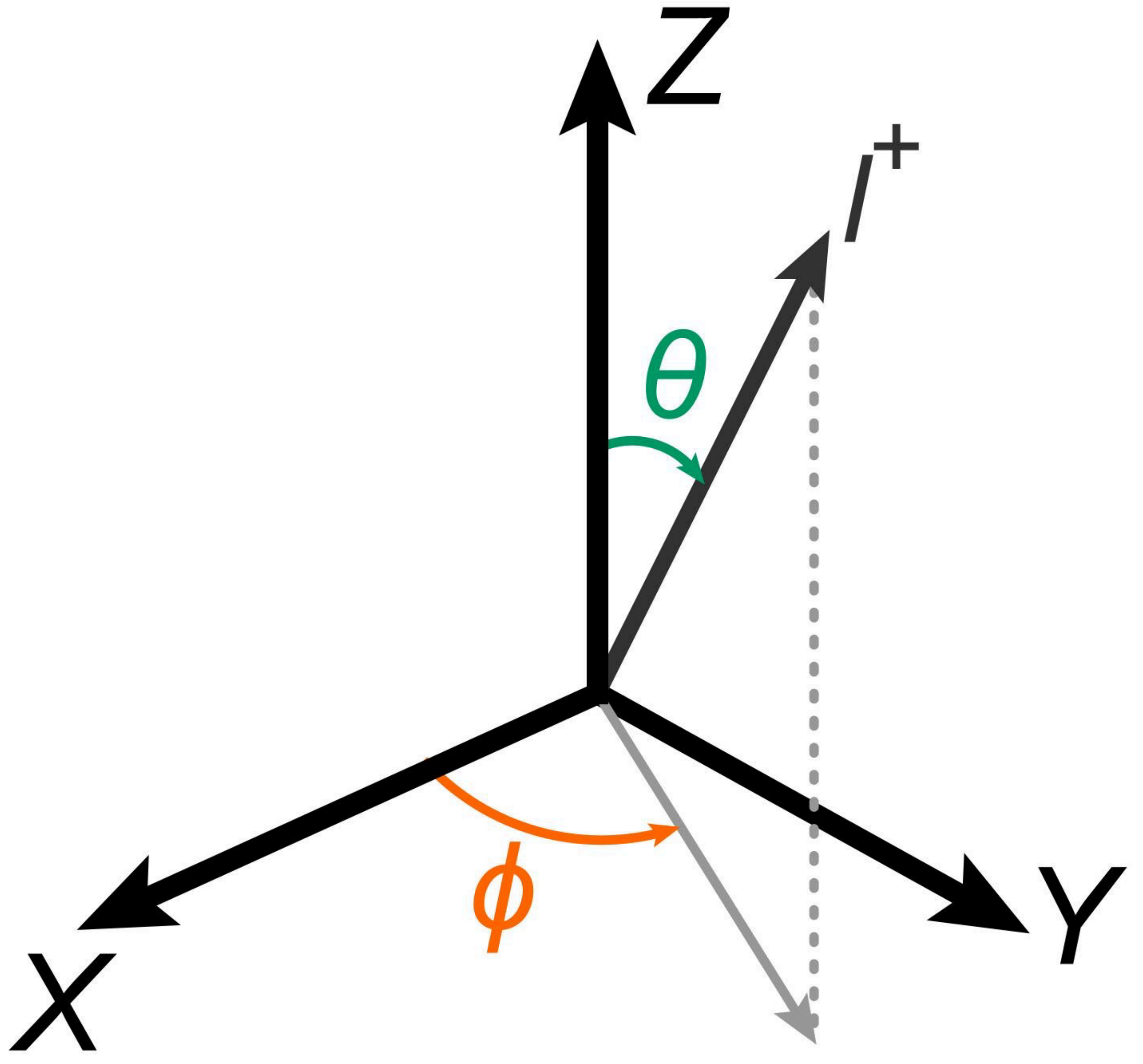
- Gluon TMDs at (relatively) low scale
  (Here $Q = 3$ GeV)
- TMDShF non-perturbative component
- Match as a function of Q?

J/ψ polarization

We can study the J/ψ polarization by considering its decay into a lepton pair

$(\mathcal{B}_{e^+e^-} \sim \mathcal{B}_{\mu^+\mu^-} \sim 6\,\%)$





$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega} \propto 1+\lambda\cos^2\theta+\mu\cos 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi$$

J/ψ polarization and NRQCD

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega} \propto 1+\lambda\cos^2\theta+\mu\cos 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi$$

D'Alesio, LM, Murgia, Pisano, Sangem, JHEP 03 (2022)

Angular parameters are connected to **helicity amplitudes** $\mathscr{W}_{\Lambda\Lambda'}$

with $\Lambda = -1, 0, +1$

Parameterization is in accordance to **model-independent** arguments!

Hermeticity **Parity** **Gauge Invariance**

Within **NRQCD** helicity amplitudes involve interferences among waves!

up to v^4 order

$$\mathscr{W}_{\Lambda\Lambda'} = \mathscr{W}_{\Lambda\Lambda'}[{}^3S_1^{(1)}] + \mathscr{W}_{\Lambda\Lambda'}[{}^1S_0^{(8)}] + \mathscr{W}_{\Lambda\Lambda'}[{}^3S_1^{(8)}] + \mathscr{W}_{\Lambda\Lambda'}[\{S=1,L=1\}^{(8)}]$$

Beneke, Krämer, Vänttinen, PRD 57 (1998)

J/ψ polarization and the Boer-Mulders

Angular parameters within **TMD** factorization

D'Alesio, LM, Murgia, Pisano, Sangem, JHEP 03 (2022)

$$\lambda = \frac{\mathscr{W}_T - \mathscr{W}_L}{\mathscr{W}_T + \mathscr{W}_L} \rightarrow \begin{cases} \mathscr{C}\left[f_1^g \Delta_T^{[n]}\right] \\ \mathscr{C}\left[f_1^g \Delta_L^{[n]}\right] \end{cases}$$

! Matching procedure works in the same way !

$(\Lambda_\psi = T,\, L)$

$$\Delta_{\Lambda_\psi}^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathscr{O}_\psi[n] \rangle\, \delta(1-z)$$

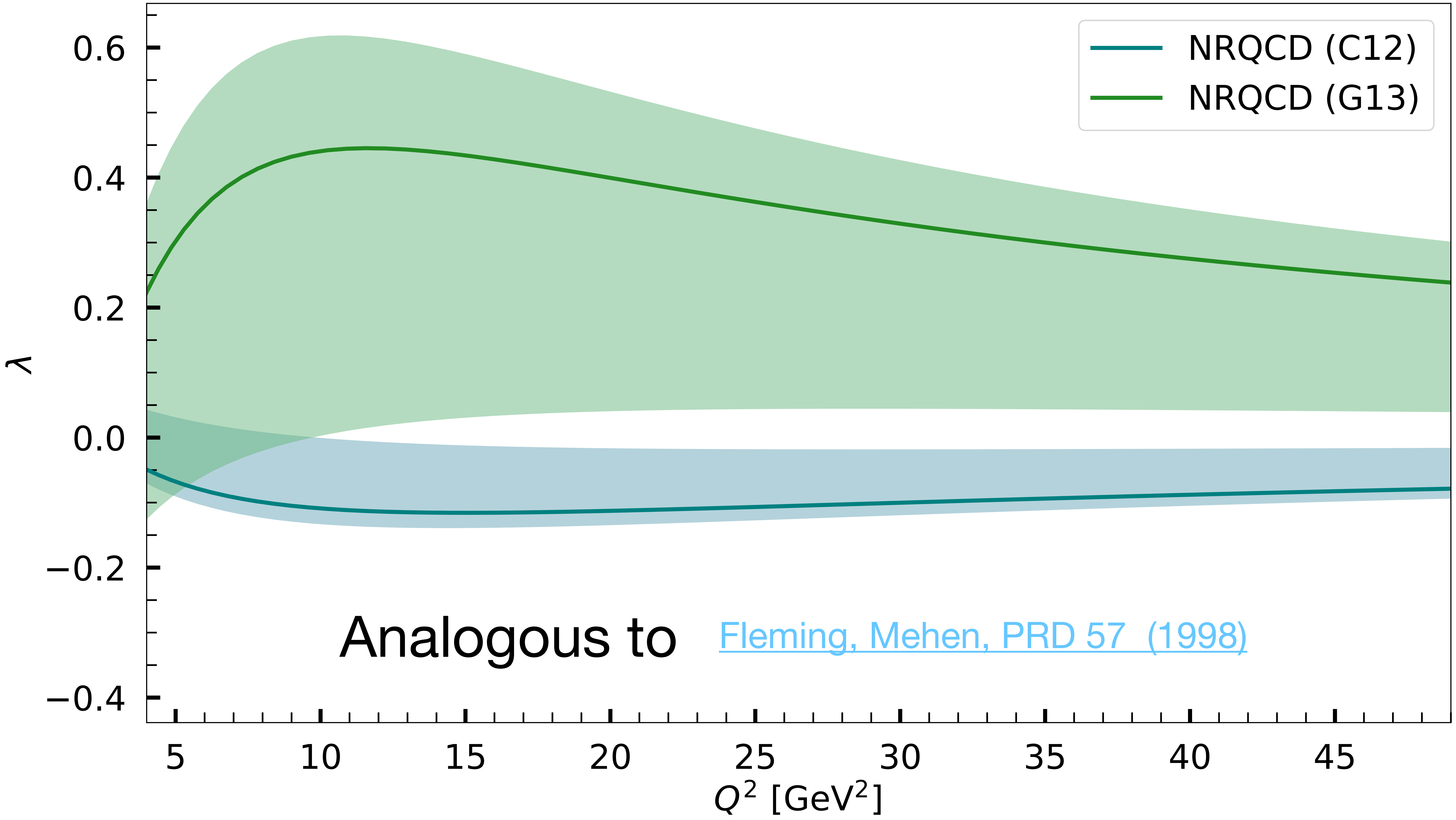
~~$$\mu = \frac{\mathscr{W}_\Delta}{\mathscr{W}_T + \mathscr{W}_L}$$~~

$$\nu = \frac{2\mathscr{W}_{\Delta\Delta}}{\mathscr{W}_T + \mathscr{W}_L} \rightarrow \mathscr{C}\left[w\, h_1^{\perp g} \Delta_{\Delta\Delta}^{[n]}\right] \rightarrow \Delta_{\Delta\Delta}^{[n]} = ??$$

Non-perturbative components may differ with Λ

J/ψ polarization and production mechanisms

Within **TMD** factorization the **CO channel** dominates



λ polarisation is related to the ratio

$$R = \frac{\langle \mathcal{O}_8[^3P_0] \rangle}{m_c^2 \langle \mathcal{O}_8[^1S_0] \rangle}$$

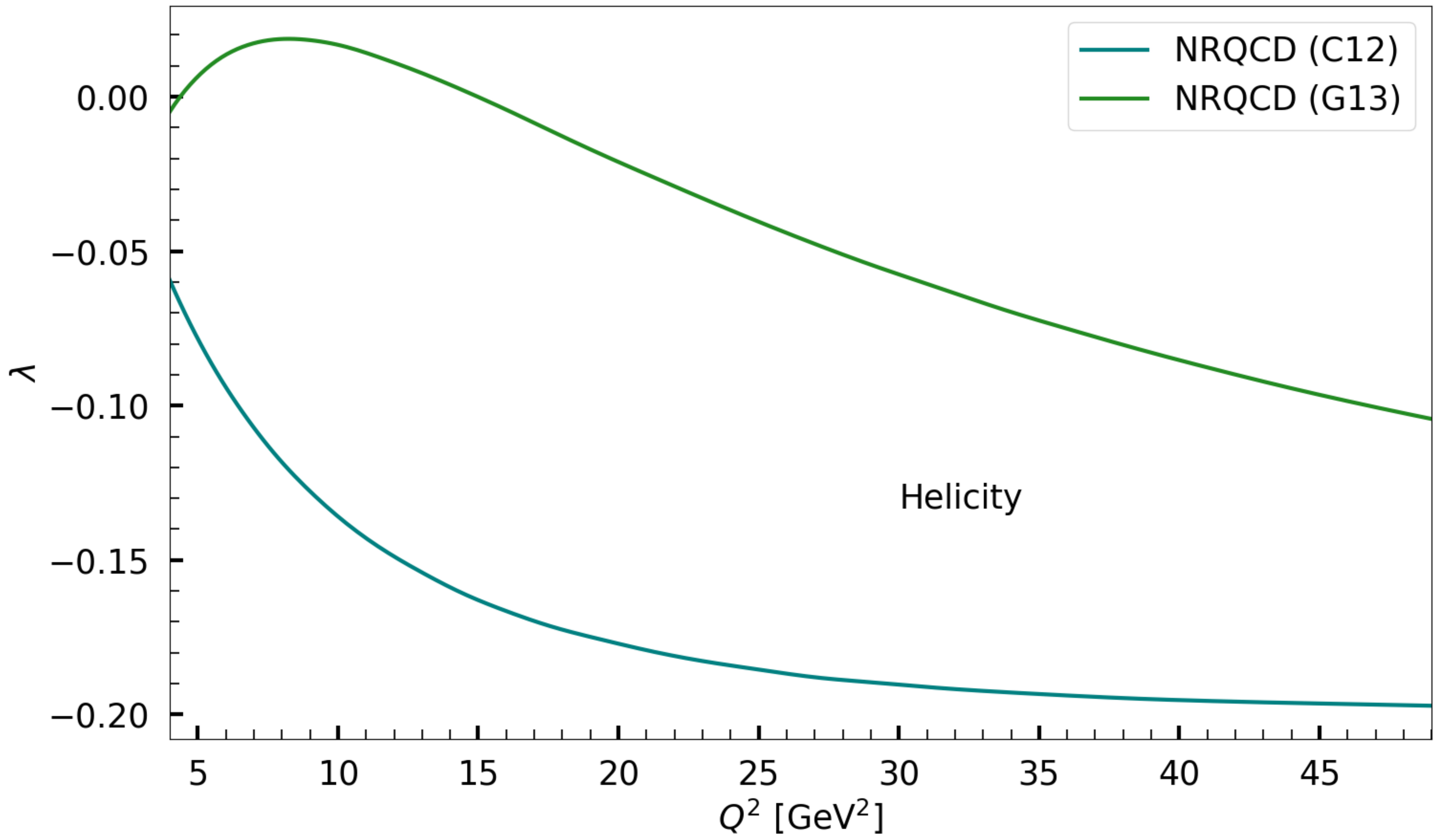
Band given by y variation

$$\left(0 \leq y = \frac{P \cdot q}{P \cdot \ell} \leq 1 \right)$$

Valid only for $z = 1$!

J/ψ polarization and production mechanisms II

At $P_T \neq 0$ **CS channel** is **very** relevant

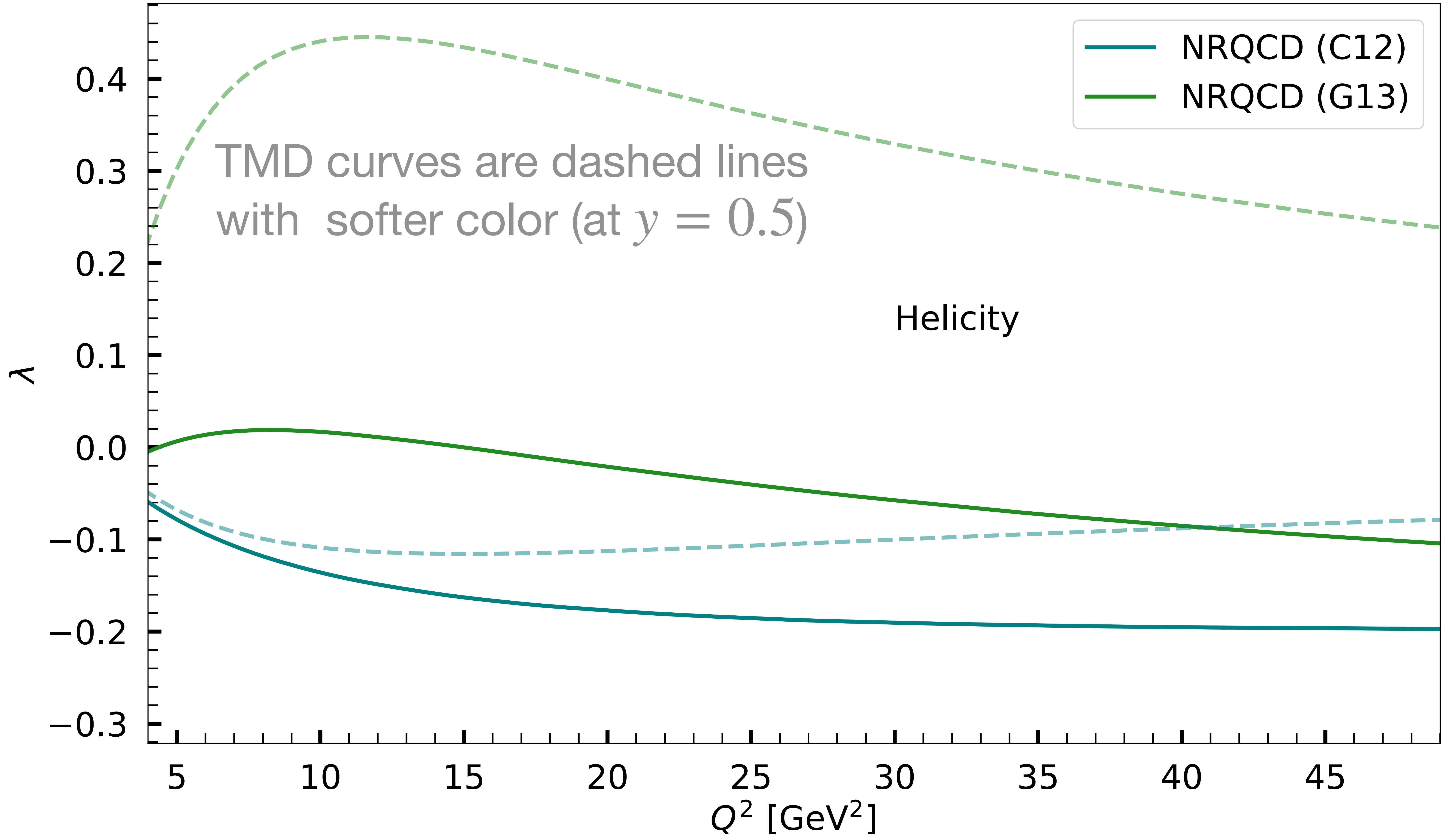


z **integrated** in a phase space accessible at the **EIC**

$\sqrt{s} = 140$ GeV
$P_T > 1$ GeV
$0.2 < z < 0.9$
$y = 0.5$

J/ψ polarization and production mechanisms II

At $P_T \neq 0$ **CS channel** is **very** relevant



z **integrated** in a phase space accessible at the **EIC**

$\sqrt{s} = 140$ GeV
$P_T > 1$ GeV
$0.2 < z < 0.9$
$y = 0.5$

Connection between **TMD** and **collinear** regions **is neeed!**

And the **TMDShF** sits in the middle of this story!

Conclusions

- Factorization involves the presence of TMD shape functions
- We present a matching procedure to extract the TMDShF perturbative tail
- TMD shape functions separated in universal and process-dependent components

- Perturbative tail at higher order → Relevant for $\Delta_h^{[n]}$
- Non-perturbative dependence
- Role of the TMD shape function in other processes
- The EIC is a promising playground to study the TMD shape function