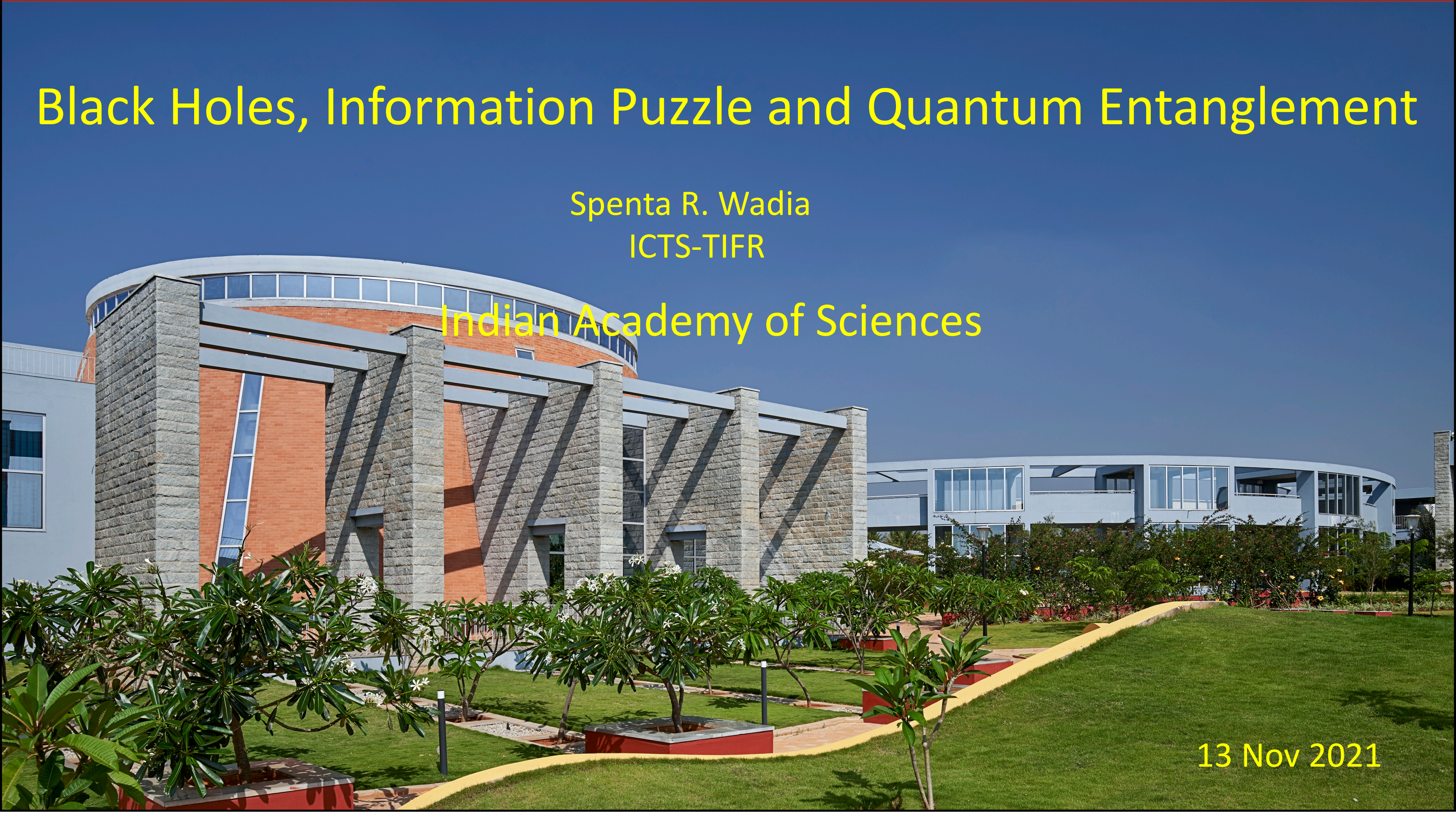
ICTS | INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Black Holes, Information Puzzle and Quantum Entanglement

Spenta R. Wadia
ICTS-TIFR

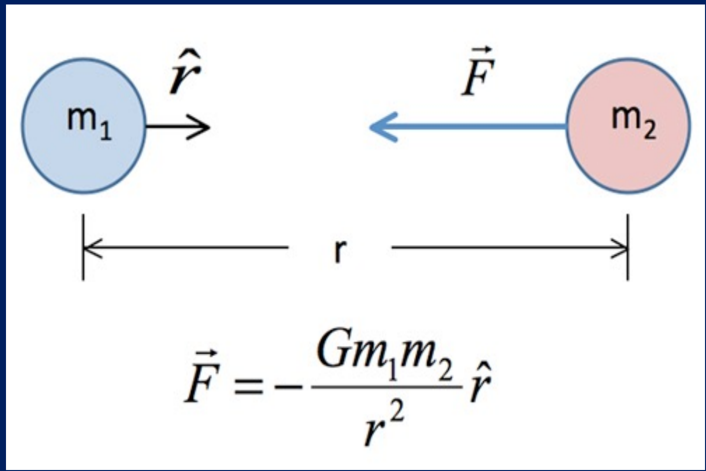
Indian Academy of Sciences

13 Nov 2021



General Relativity
and
Black Holes

Newton's law of Universal Gravitation



- Force acts instantaneously at a distance
- Always attractive
- Force (on particle 1 due to particle 2) = m_1 x Acceleration_1

In this simple case leads to the famous elliptical orbits of planets described by Kepler.

Newton (1713) "I have not yet been able to discover the cause of these properties of gravity from phenomena and I feign no hypothesis. It is enough that gravity does really exist and acts according to the laws I have explained, and that it abundantly serves to account for all the motions of celestial bodies."

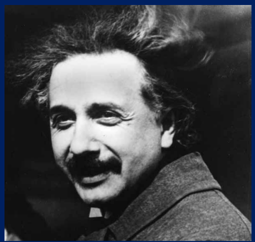
General Relativity-1

Maxwell

In 1865 James Clerk Maxwell unified electricity and magnetism and demonstrated that the interaction between changes are communicated by the `electromagnetic field' propagating with a speed $c = 3.1 \times 10^5$ kms/sec.

In 1905 Einstein used this discovery to deduce that the speed of light is the same for observers in relative motion and hence there can be no action at a distance which is simultaneous for observers in relative motion.

Hence Newton's law of gravitation needs to be modified.



Einstein

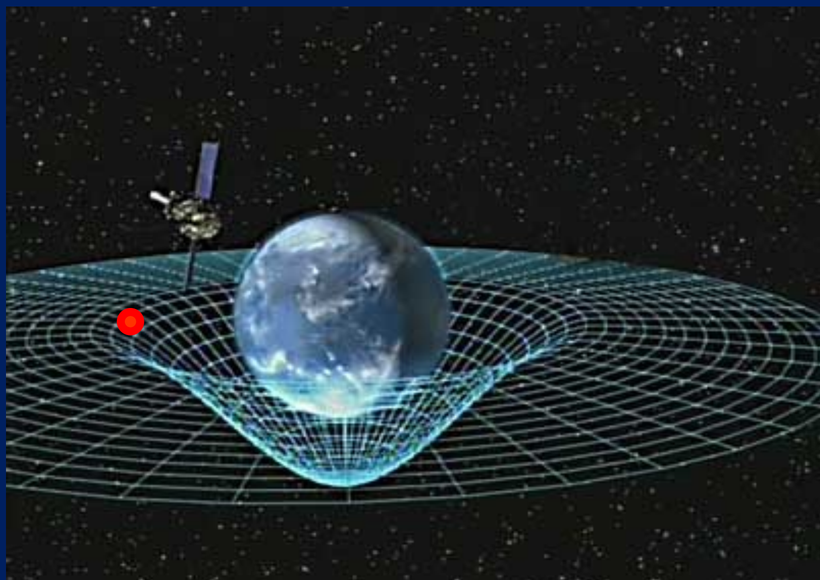
Waging one of the great struggles in modern scientific history in 1915 Einstein completed the General Theory of Relativity, which is a theory of gravity in which the force of gravity is not instantaneous. It is communicated at a finite speed c (the same as the speed of light) by changes in the geometry of space-time. Here the role of the electromagnetic field is played by the `metric' of spacetime.

General Relativity-2

GR describes the changes of the geometry of space-time caused by massive objects to which other objects respond.

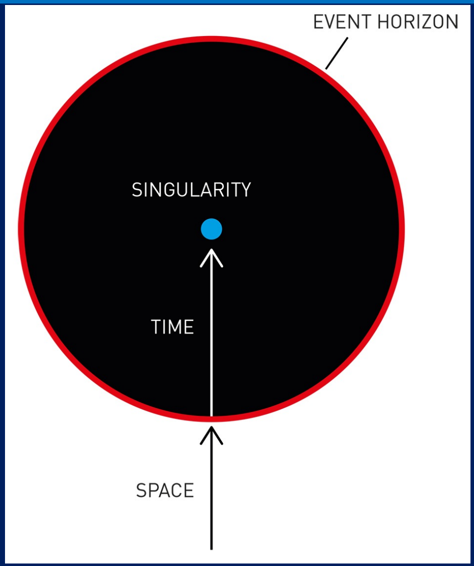
Space-time is like an 'elastic' grid, communicative and causal…but very very stiff…10^{20} times stiffer than steel!

In a curved space-time an object follows a path that maximizes the proper time in the frame of the object.



Black Holes-1

Karl Schwarzschild
1916



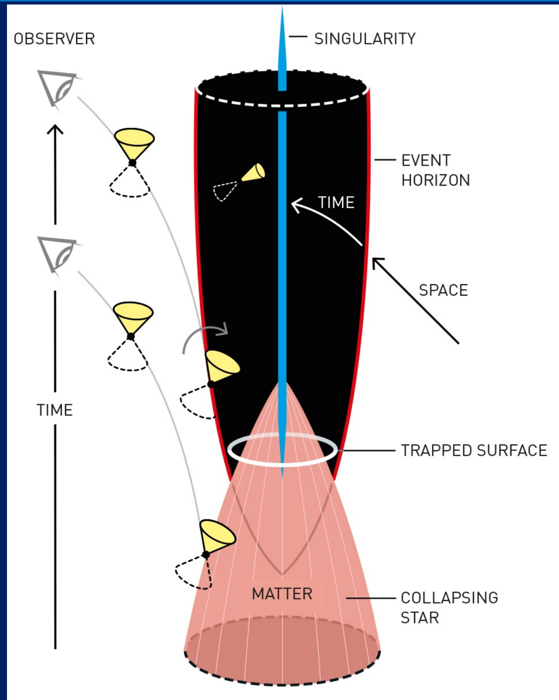
nobelprize.org

They are solutions of Einstein's equations which divide space-time into 2 parts. Interior and exterior separated by a surface called the horizon, which is a one-way gate. Even light entering it cannot get out. Hence the name BH.

The BH space-time has a curvature singularity in the interior. Inside the horizon the radial direction is time-like, and time ends at the singularity.

Black Holes-2

Roger Penrose



nobelprize.org

- Penrose (1973) developed topological arguments to show that BH formation is a robust prediction of the General Theory of Relativity.

- The horizon is a null surface of radius $r_s=2GM/c^2$ (Schwarzschild radius), G is Newton's constant. $r>r_s$ describes the exterior and $r<r_s$ the interior.

The horizon has area: $A_h = 4\pi r_s^2 = 16\pi G^2M^2/c^4$

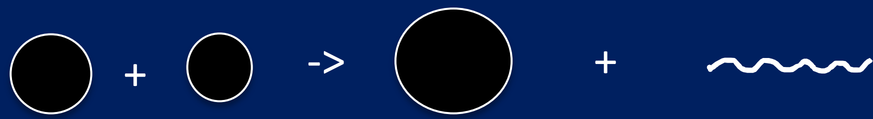
Black Holes exist in Nature
They are an observational reality!

The first image of a supermassive black hole and its shadow in the Messier 87 galaxy in the Virgo cluster. (Event Horizon Telescope collaboration - 2019). The key point is that no light was observed coming along the line of sight.

BHs have been indirectly observed in the galactic center (Ghez and Genzel) and also by the observation of gravitational waves by LIGO.

Black Hole Entropy -1

Area theorems of GR (Hawking, 1971)



Bh_1 + Bh_2 → Bh_{12} + radiation

Horizon area always increases $A_{12} > A_1 + A_2$

A BH cannot be split apart and hence the reverse process is impossible.

Bekenstein (1973):

- Take a hot object and throw it into a black hole, it disappears decreasing the entropy of the universe, and that violates the 2nd law of thermodynamics.
- A contradiction is avoided if one attributes a thermodynamic entropy to a black hole proportional to the area of the horizon.

Black Hole Entropy - 2

$$S = bArea_h = b4\pi r_s^2 = b4\pi(2GM)^2$$

where b is an undetermined constant

The area theorems imply the 2nd law for the generalized coarse-grained entropy:

$$S_{gen} = S_{BH} + S_{matter}$$

S_{matter} is the entropy of matter/radiation outside the BH horizon

(Bekenstein, A. Wall)

When the hot water bucket falls into the BH, S_{matter} decreases but S_{BH} increases by more.

Quantum Mechanics, Hawking Radiation and Information Puzzle

Hawking Temperature and Entropy

In one of the celebrated calculations of the second half of the 20th century Hawking (1974) discovered that BHs radiate due to quantum effects. He calculated the temperature of the BH and found his famous formula which is on his tombstone:

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_{\mathrm{B}} M}$$

$T_{sun} = 3.6 \times 10^{-7}$ K

$T_{earth} = 0.1$ K

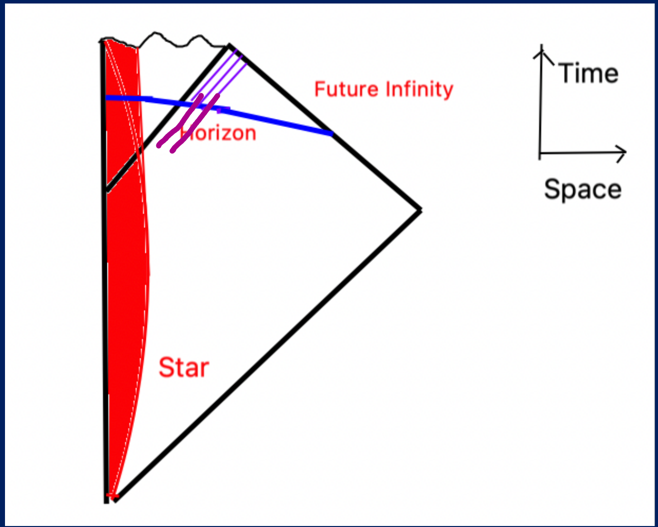
$T_{M=10^{18}\,kg} = 7000$ K

Using the first law of thermodynamics dM=TdS the BH entropy is (note b=1/4)

$$S_{BH} = \frac{k_B c^3}{4\,\hbar\, G} A$$

$$\frac{c^3}{\hbar G} = \frac{1}{l_p^2}$$

Universal Black Hole Temperature & Entropy



Penrose diagram of a BH formed by collapsing matter.

The formula for BH temperature and entropy is universal and independent of how the BH was formed.

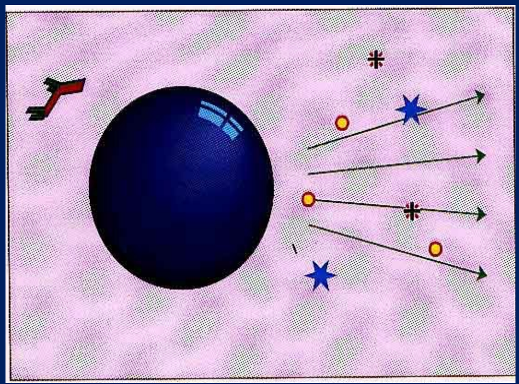
This is because the answer at late times depends only on very high energy Hawking quanta (or radiation) that emanate from near the horizon of the BH, after the collapsing matter has crossed the horizon.

The radiation appears to be thermal because we are NOT keeping track of the part that is on the other side of the horizon and is entangled with the radiation we are observing.

Information Loss is in conflict with Quantum Mechanics!

Like all hot bodies black holes radiate.

A black hole forms in various ways, but it always evaporates in the same way into thermal radiation leading to information loss as there is no memory of its initial state.



This violates a fundamental principle of unitarity in quantum mechanics:

A state cannot evolve to a mixed state in a closed system.

Hence, we have a serious problem

GR leads to information loss and QM cannot allow it!

Some people called it a crisis, but it presented an opportunity for progress

Resolving the Information Puzzle - Black Hole Microstates

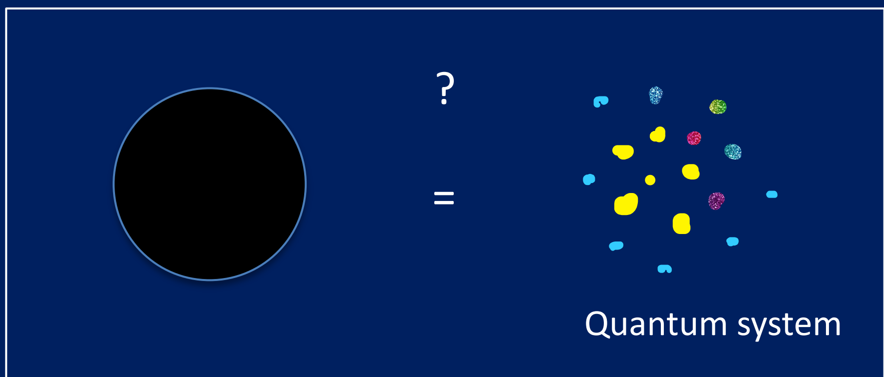
Attempts to resolve the information puzzle 1990 onwards.....

Since a BH is a thermodynamic system, can information loss for black holes be understood as due to an averaging process of many internal states, like in usual treatments of quantum statistical mechanics? Like the burning of a piece of wood?

Is there a theory of quantum gravity in which black hole entropy is given by Boltzmann's formula ?

$$S_{BH} = k \log(N)$$

What are the internal states of the BH, that would account for BH entropy?



Black hole micro-states

In 1996 Strominger and Vafa provided the first concrete evidence in a calculable supersymmetric model that the black hole space-time is a sort of a hydrodynamic description of more basic underlying quantum system of D-branes in string theory.

D-branes which are domain walls of string theory were discovered by Polchinski.

Hawking radiation and BH thermodynamics can be calculated in the framework of statistical mechanics in this constituent model of the black hole!

Microscopic model of Hawking radiation was developed post 1996 over many years:

J. David, G. Mandal, S. R. Wadia - Physics Reports (2002)

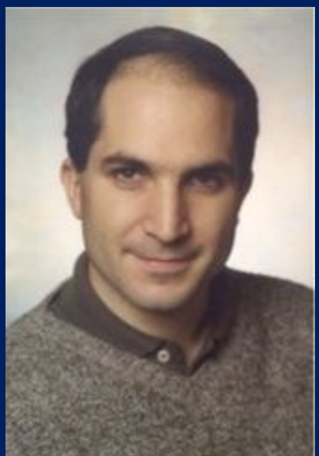
Higher order corrections to BH entropy were computed by A. Dabholkar, A. Sen and others in the framework of string theory.

These development gave enormous evidence that string theory is the correct theory of quantum gravity and led to the holographic description of quantum gravity.

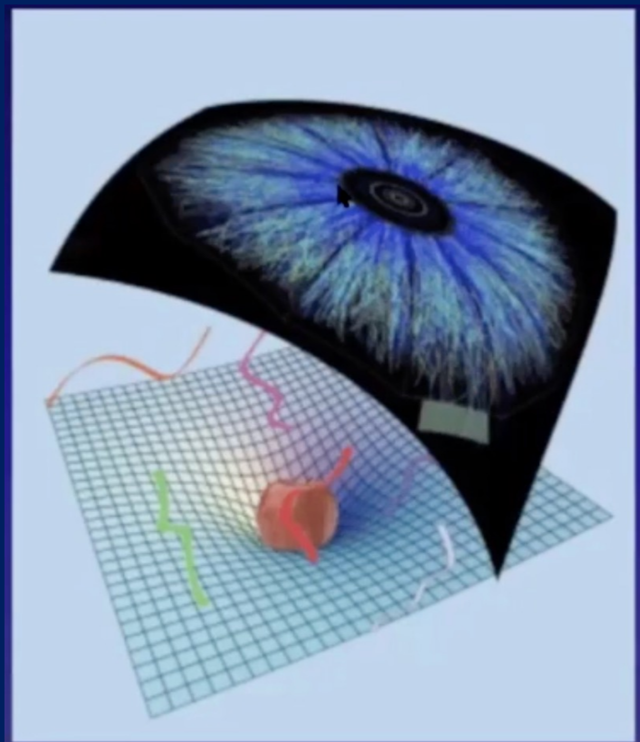
Holography and non-perturbative Quantum Gravity

Another approach to resolve the information puzzle is the AdS/CFT holography proposed by Maldacena (1997). It says that gravity and string theory in an asymptotically AdS (hyperbolic) space-time are holographically coded in a unitary QFT on its boundary.

Hence in principle one can track BH formation and evaporation in the QFT and that by the holographic correspondence would be a unitary process.



Juan Maldacena



Gravity in the interior →
described by interacting particles on the boundary.

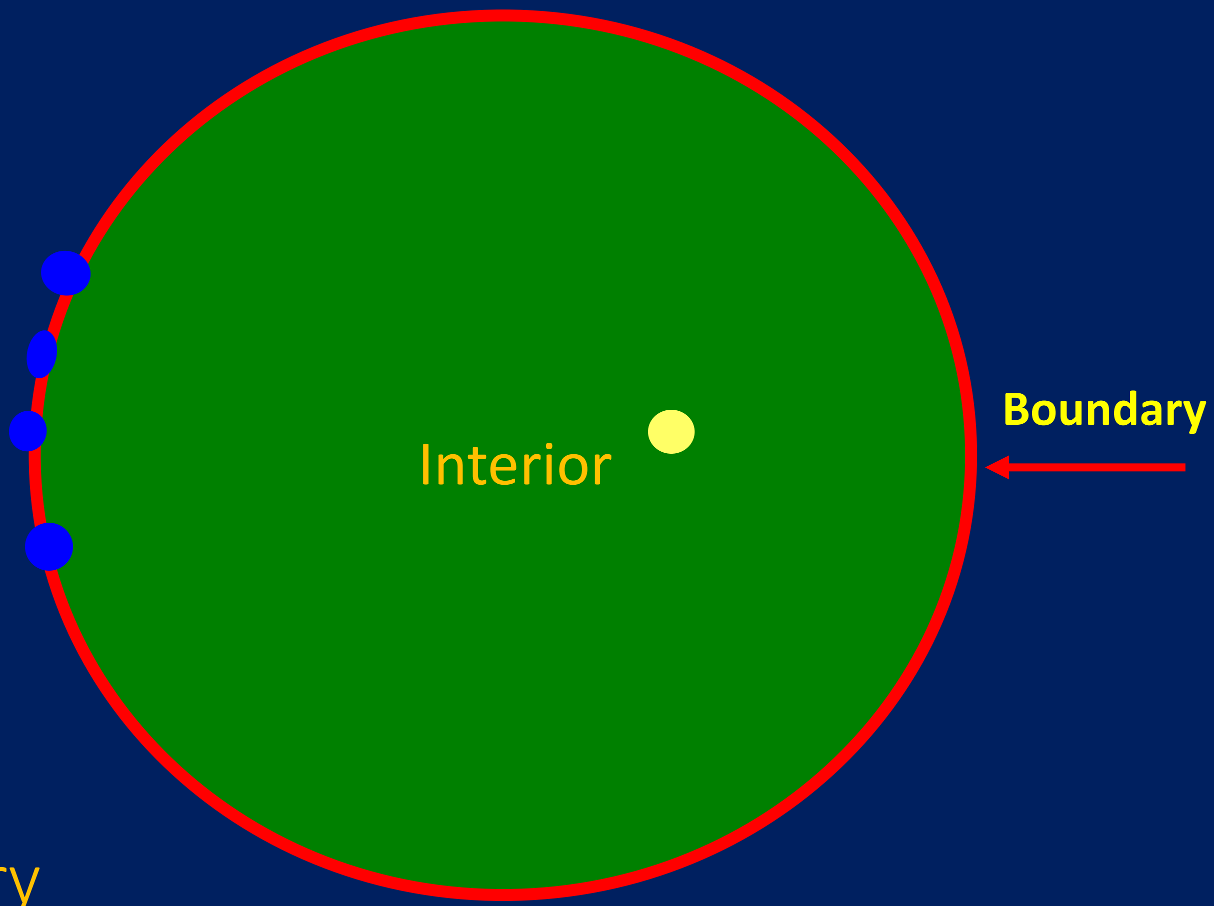
In the interior

is a string theory

(gravity) in

anti-de Sitter

space-time.



On the boundary

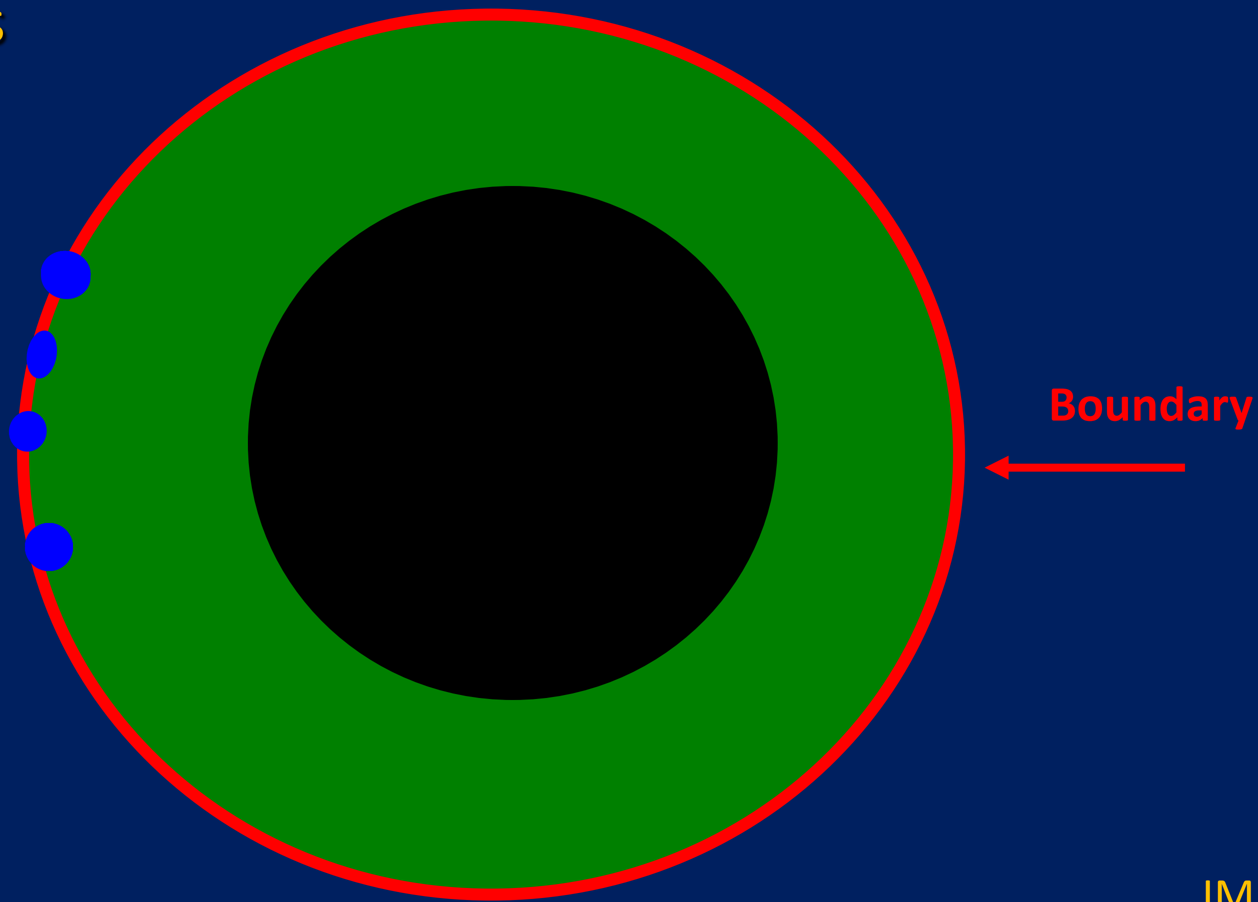
of AdS lives a unitary

QFT

JM

Black holes in AdS

Temperature and entropy
→ motion of particles
on the boundary
(Witten)



JM

In principle
solves information
puzzle because
BH formation is thermalization of the theory on the boundary.
Hawking conceded that there is in principle no information loss.

How does gravity solve the Information Puzzle?

There is recent progress in answering this question. It involves understanding `Information loss' as `quantum information loss' and uses concepts form quantum information theory. In particular, the concept of Quantum Entanglement and its measure by the fine grained or entanglement entropy (Von Neumann, 1932) and its geometrization.

Work along these lines was initiated by S. Ryu and T. Takayanagi (2006), Hayden and Preskill (2007), Maldacena and Susskind, S. Mathur, A. Almheiri, D. Marolf, J. Polchinski, J. Sully (AMPS), S. Raju, K. Papadodimas, Dong, D. Harlow, N. Engelhardt, A. Wall and many others.

A good diagnostic is the Page curve (1993) of the time evolution of the entanglement entropy of an entangled composite quantum system. It is a direct consequence of the unitarity of quantum mechanics.

In what follows I will give a rough sketch of recent developments.

Entanglement Entropy in Gravity
and
Information Puzzle

Thermodynamic entropy and Entanglement entropy

Repeat:

- The resolution of the information puzzle is enabled by thinking about the BH and radiation in the conceptual framework of quantum information theory and understanding the entanglement (Von Neumann) entropy (EE) of quantum systems as a diagnostic of `quantum information loss'.
- The EE is defined for ANY quantum state of a system and not restricted to dealing with a large number of degrees of freedom like in the thermodynamic course grained entropy (TE).
- Unlike TE the EE does NOT satisfy the 2nd law and is not necessarily an increasing function of time.
- For systems 1 and 2, $(TE)_{12} > (TE)_1 + (TE)_2$ but $(EE)_{12} \leq (EE)_1 + (EE)_2$
- and $EE \leq TE$
- EE of a given quantum state is zero and time independent under unitary time evolution.
- If one divides a system into 2 parts and observes either one of them, the subsystem EEs are positive and equal $(EE)_1 = (EE)_2$ and can increase or decrease in time.
- $(EE)_1$ and $(EE)_2$ time evolve according to the Page curve (see later).

Entangled states and density matrix

- Entanglement is a property of QM that distinguishes it from classical mechanics and accounts for its very non-intuitive consequences. Here we illustrate this very simple point in the case of two entangles two state systems.

- A and B are 2 state systems A: $\{|0>_A, |1>_A\}$, B: $\{|0>_B, |1>_B\}$, and C = AxB is the composite system

- We can consider separable product states like $|0>_A \times |1>_B$ in which both systems A and B are in a definite state.

- However, the state $|AB> = \frac{1}{\sqrt{2}}(|0>_A \times |1>_B - |1>_A \times |0>_B)$ is not a product of two states. It is an `entangled' state of the system C. It does not belong to A or B.

- A state of a system in QM can be equivalently described by a `density matrix' e.g. $\rho(AxB) = |AB><AB|$. Since $|AB>$ is normalized we have $\rho(AxB)^2 = \rho(AxB)$, a projection operator.

- In case we do not want to observe system B we trace over all the states of system B. $tr_B\, \rho(AxB) = \rho(A)$ is the reduced density matrix of A and it turns out that $\rho(A)^2 \neq \rho(A)$

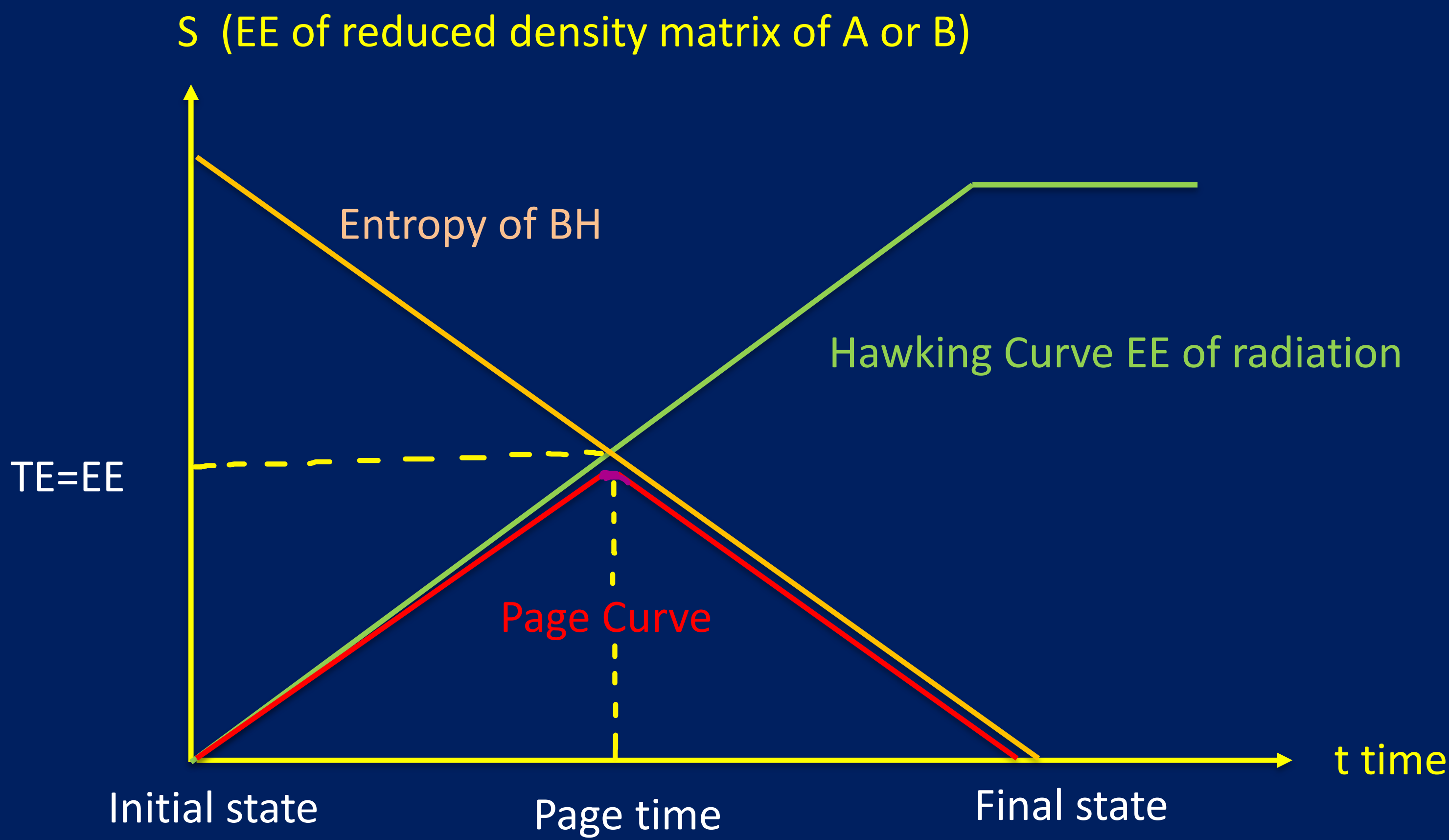
Entanglement Entropy (Von Neumann, 1932)

- Von Neumann entropy: $S = -\mathrm{tr}\rho \ln \rho$ and $\mathrm{tr}\rho = 1$
- $S = -\mathrm{tr}\rho\ln\rho = 0$, iff ρ is a `pure state'. A non-zero S is a measure of deviation from a maximally entangled pure state. $S(\rho(AxB)) = 0$ but $S(\rho(A)) = \ln 2$
- $S(\rho(A)) = S(\rho(B))$ EE of the reduced density matrices are equal.
- $S(\rho) = S(U^{+}\rho U)$, where U is a unitary matrix. Hence S(ρ) is constant under unitary time evolution.
- $S(\rho) \leqslant \log N$, where Nis the dim of the Hilbert space.

  $\log N = S_{th}$ is the thermodynamic entropy, which has no information
- $S_{th} - S_{vn}(\rho) \geqslant 0$, is a measure of information loss.
- For a bipartite system AxB, $S(A \cup B) \leqslant S(A) + S(B)$, which is very different from the thermodynamic entropy, where the entropy of the combined system is always $\geqslant$ the entropy of the parts.

Hawking and Page curves for the BH-Radiation entangled system

Imagine evaporating BH is system A and Hawking radiation is system B



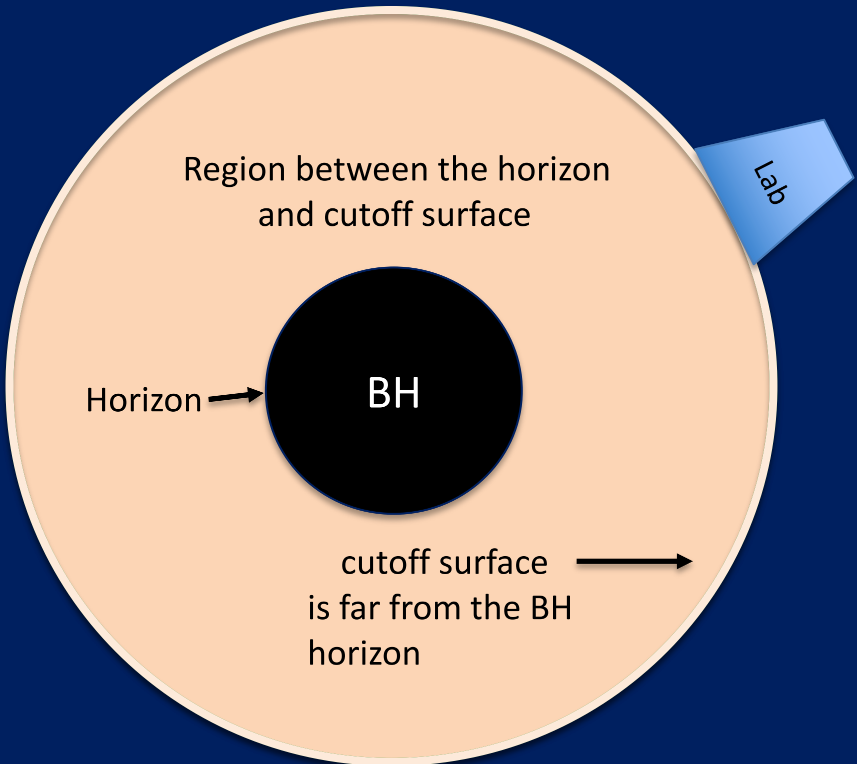
Hawking curve violates unitarity Page curve is consistent with Unitarity

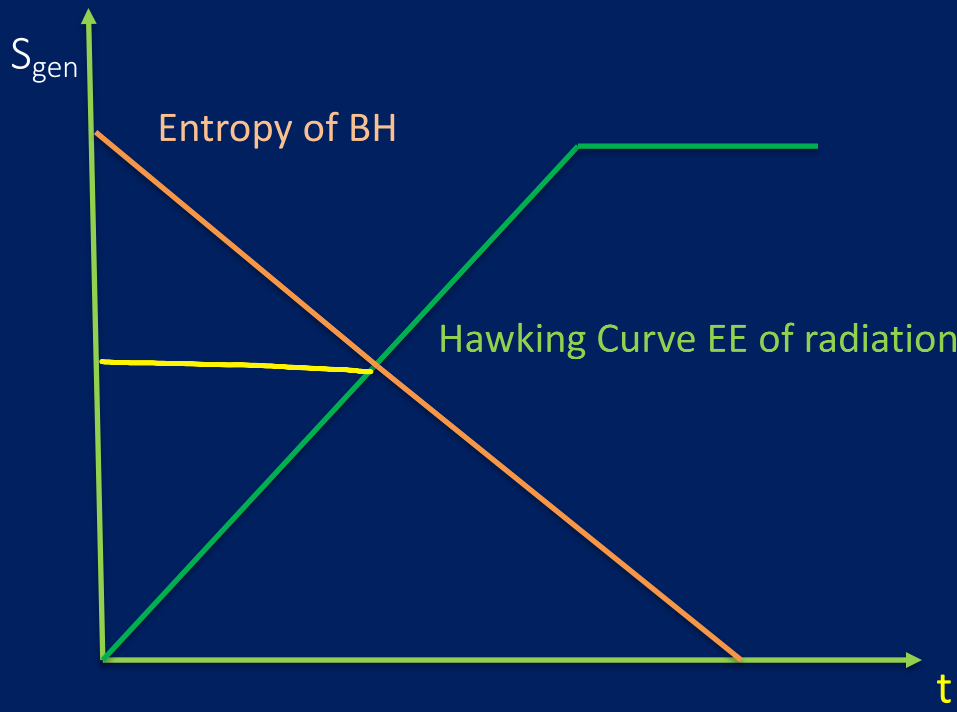
Entanglement Entropy formula that computes the Hawking curve of an evaporating BH

Recall the Bekenstein-Hawking formula for the generalized entropy. It can be shown that it is a formula for EE. The area term is the EE of the Hawking quanta near the inside and outside the horizon (Sorkin 1982) and S_{matter} is the EE of matter/radiation outside the horizon.

$$S_{gen} = \frac{A}{4G} + S_{matter}$$ (units $c=\hbar=1$) (N. Engelhardt and A. Wall, 2015)

(The UV divergences between the two terms cancel, and the Newton constant is finite).





The EE of radiation at the Page time exceeds the the TE of the BH and that is a violation of unitarity

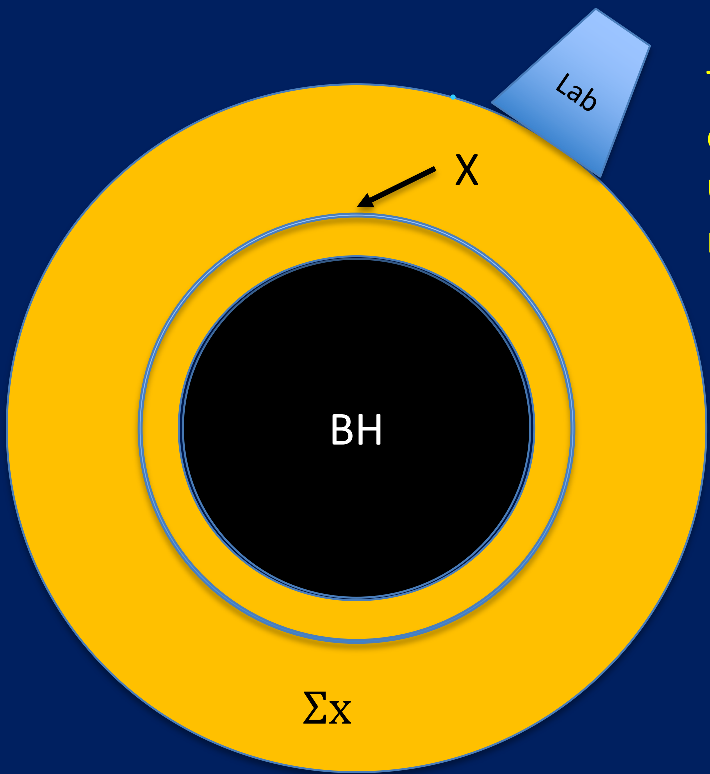
What is the correct EE formula formula in gravity?

It is the same formula as before except the `surface' under consideration is not the horizon!
The `surface' is obtained by looking at the extrema of

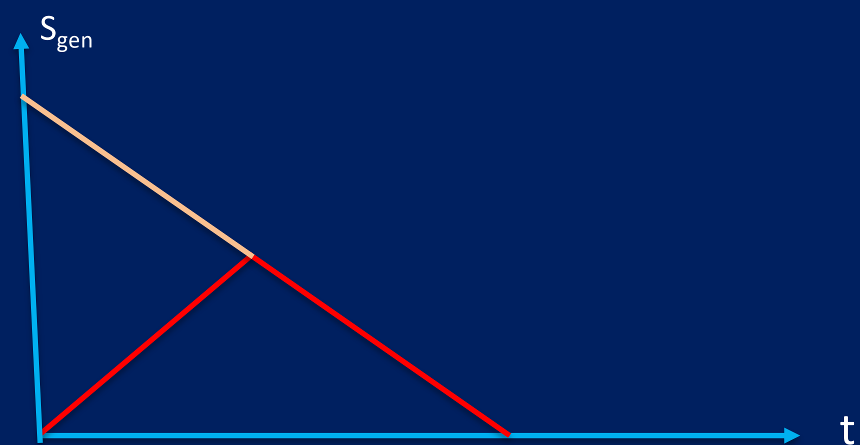
$$S(X(t))_{gen} = \left[\frac{A(X(t))}{4G} + S(\Sigma x(t))_{matter} \right]$$

Pennington (2019)
Almheiri, Engelhardt, Marolf, Maxfield (2019)

where X(t) is a surface or a union of surfaces, $\Sigma x(t)$ is the region bounded by X(t) and the cut-off surface. (If X is the horizon, then we get Hawking's result).



The key point is that X(t) can also be partly or fully inside the horizon and can be a union of surfaces…around islands, as required by the variational principle.



Summary

We have reviewed a particular promising development in theoretical physics which brings us closer to a reconciliation of Quantum Mechanics and General Relativity within semi-classical gravity. This has been enabled by a new gravity formula for the entanglement entropy.

Explicit verification of the Page curve has been done in low dimensional systems.

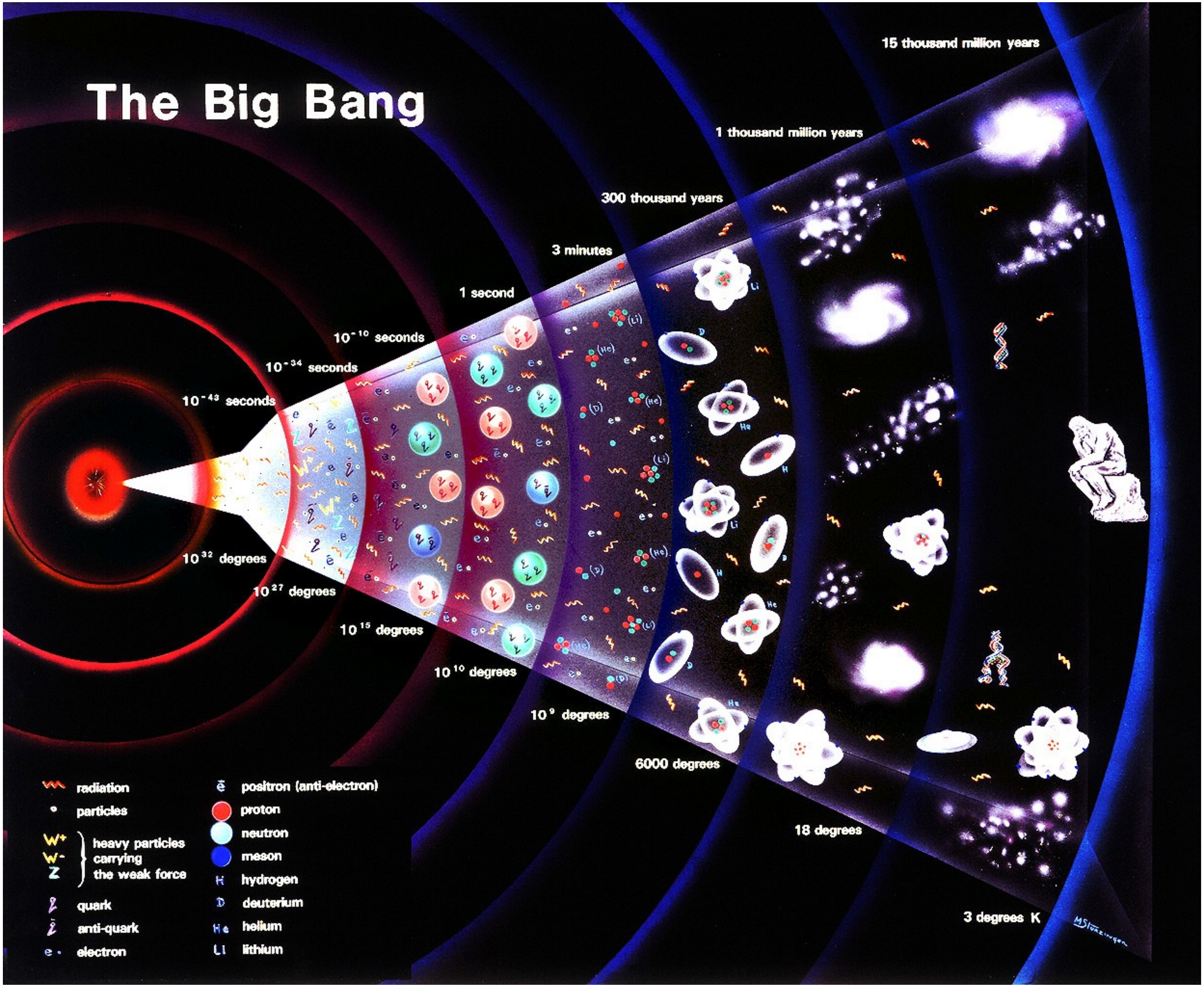
In this talk I have not included the following topics:

1. Discussion of the interior of the BH and its imprint on the Hawking radiation.

2. Wormhole geometries and entanglement entropy.

3. Explicit dynamical evolution models of BH formation and evaporation in lower dimensional systems like the SYK model of random spins which is holographic to 2-dim gravity.
(A. Dhar, A. Gaikwad, L. Joshi, A. Kaushal, G. Mandal, SRW; Maldacena, Milekhin)

One of the reasons to study BHs is to understand the principles and degrees of freedom of quantum gravity and develop a framework to discuss the quantum nature of the cosmological event horizon which also has a characteristic temperature and Hawking radiation.



The Big Bang
15 thousand million years
1 thousand million years
300 thousand years
3 minutes
1 second
10⁻¹⁰ seconds
10⁻³⁴ seconds
10⁻⁴³ seconds
10³² degrees
10²⁷ degrees
10¹⁵ degrees
10¹⁰ degrees
10⁹ degrees
6000 degrees
18 degrees
3 degrees K
radiation
particles
heavy particles carrying the weak force
quark
anti-quark
electron
positron (anti-electron)
proton
neutron
meson
hydrogen
deuterium
helium
lithium

Thank you!