Surface transport in Weyl semimetals

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Outline

- Introduction
  - Weyl Fermions in condensed matter physics
  - Topology of Weyl Fermions
  - Transport in normal metals
- Transport in bulk of Weyl semimetal (brief review)
  - Without magnetic field
  - With magnetic field
- Transport on surface of Weyl semimetal
  - Anomalous features
  - Is the surface a novel correlated liquid?
- Summary and future directions

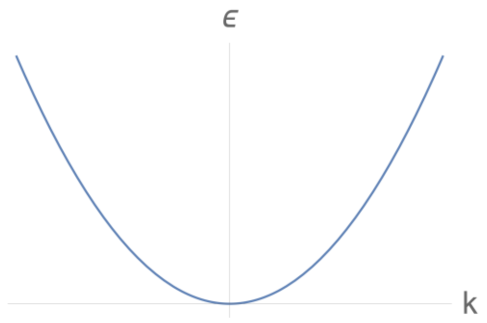
Introduction

An electron in a crystal

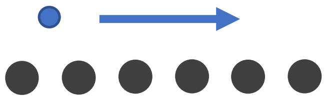
- Single particle in free space

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\varepsilon(\mathbf{k}) = \frac{k^2}{2m}$$

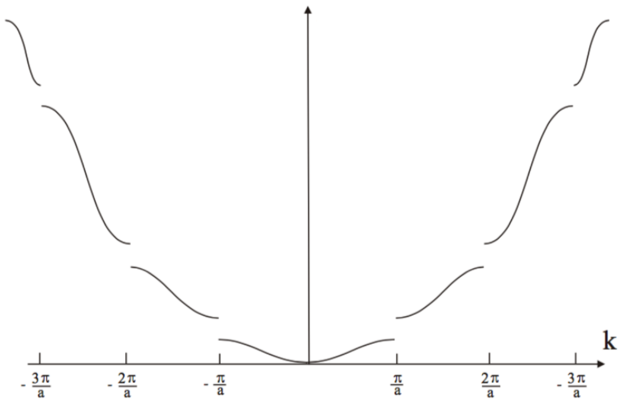


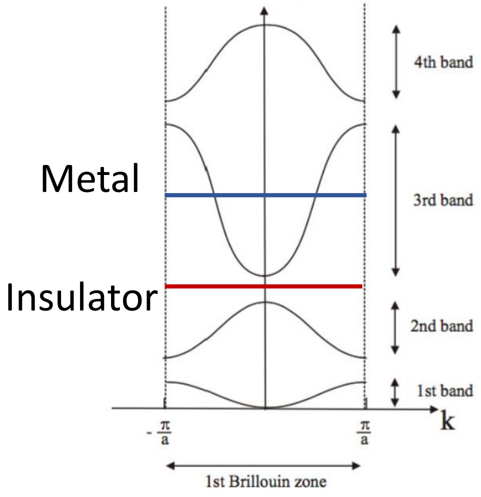
- Single particle in a crystal (periodic lattice)



$$\psi_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

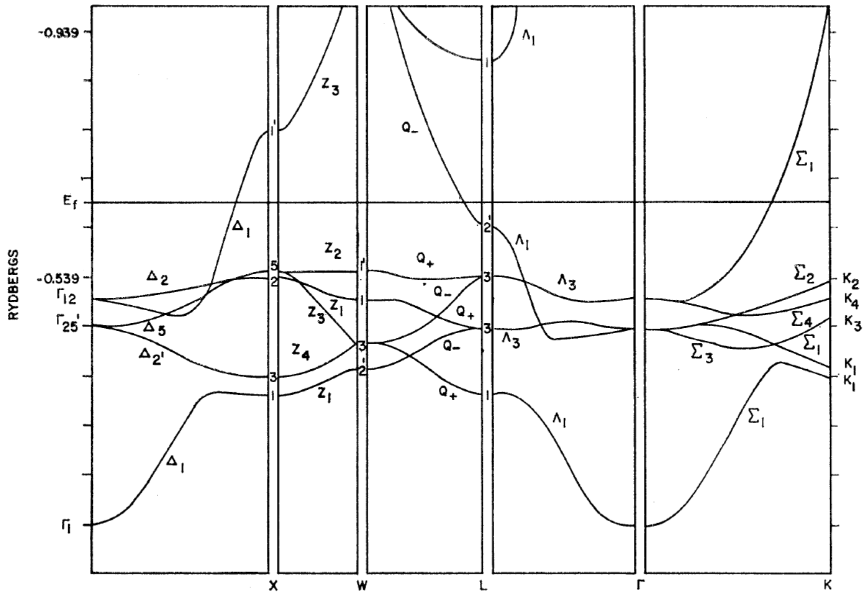
$$\varepsilon(\mathbf{k}) \implies \varepsilon_n(\mathbf{k})$$





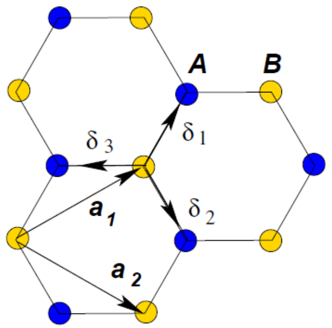
Band structures in real materials

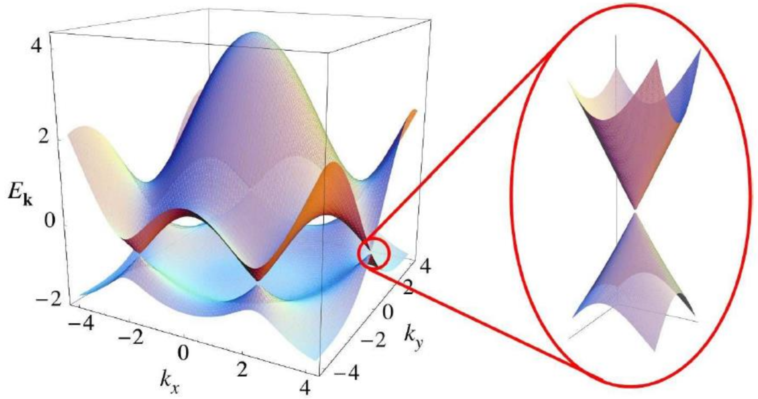
Example: Copper



Burdick, Phys. Rev. 129, 138 (1963)

Example: Graphene

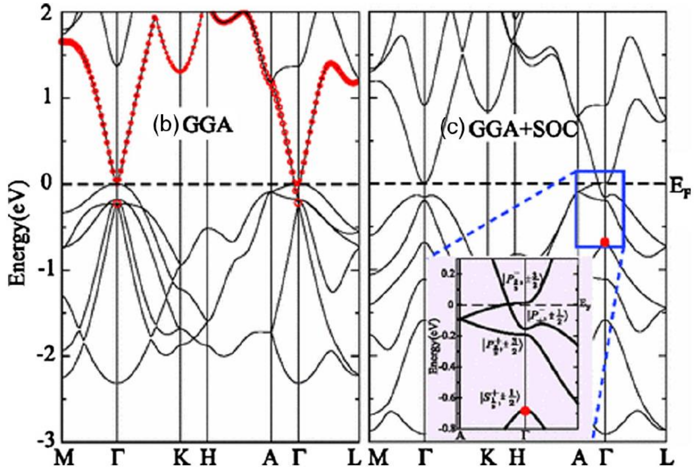




Castro Neto, et al., Rev. Mod. Phys. 81, 109 (2009)

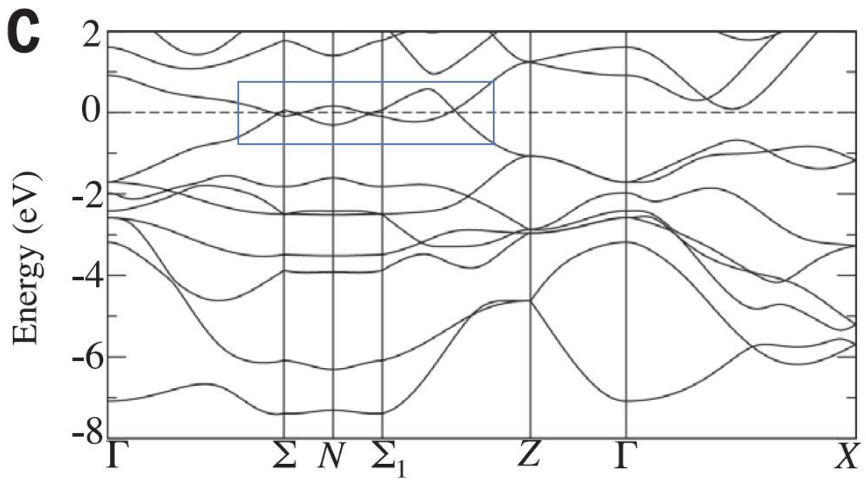
Dirac and Weyl (semi)metals

Example: Na_3Bi



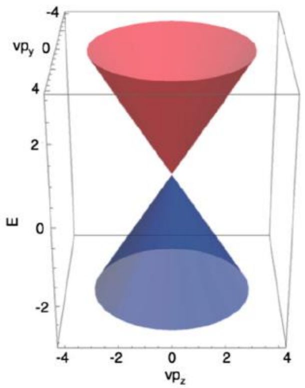
Wang, et al., Phys. Rev. B 85, 195320 (2012)

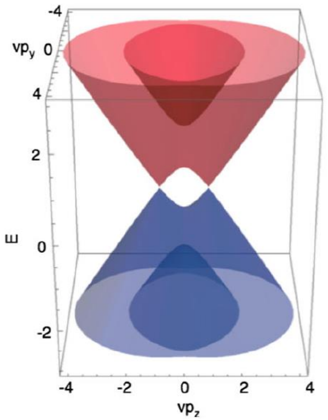
Example: TaAs



Xu, et al., Science 349, 613 (2015)

Doubly degenerate



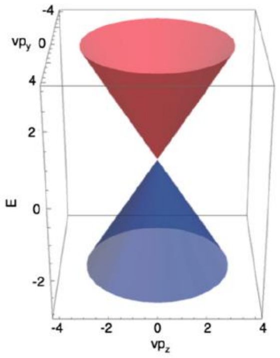


Not degenerate

Fig.: Rev. Mod. Phys. 90 (2018)

Effective Dirac and Weyl descriptions

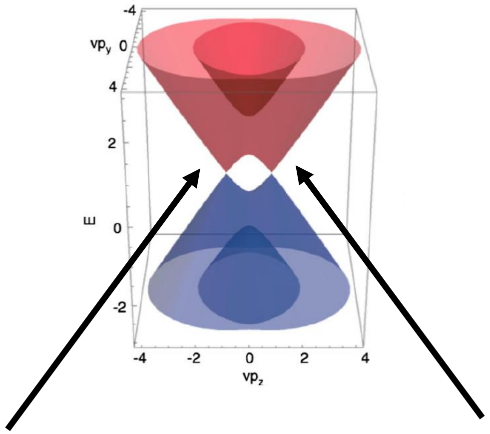
Dirac



$$k \to -i\,\nabla$$

$$c \to 1$$

Weyl



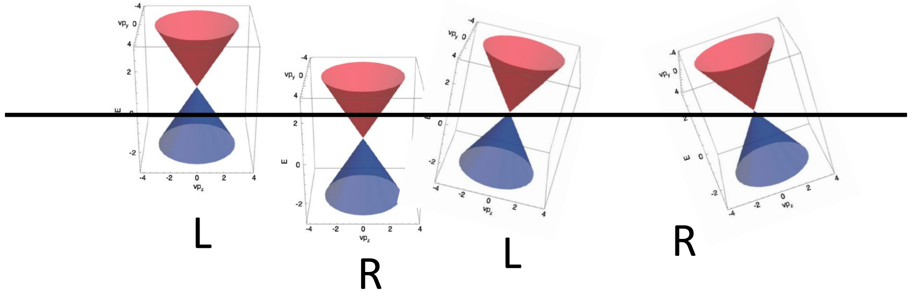
$$i\gamma^\mu \partial_\mu \psi(x) = 0$$

$$i(\partial_0 - \sigma \cdot \nabla)\psi_L = 0 \qquad i(\partial_0 + \sigma \cdot \nabla)\psi_R = 0$$

- In condensed matter Weyl systems

$$H_{\mathbf{k}} = \sum_j \sum_{lm} \left[\mu^j I + (-1)^j v_{lm}\sigma_l(\mathbf{k} - \mathbf{K}^j)_m\right]$$

σ : Pseudospin



Topology of Weyl Fermions

Berry connection $\mathbf{A}(\mathbf{k}) = i\langle u(\mathbf{k})|\nabla_{\mathbf{k}}|u(\mathbf{k})\rangle$

Berry curvature $\Omega(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$

Monopole charge $\chi = \frac{1}{2\pi}\int_S \Omega(\mathbf{k}) \cdot d\mathbf{s}_{\mathbf{k}}$

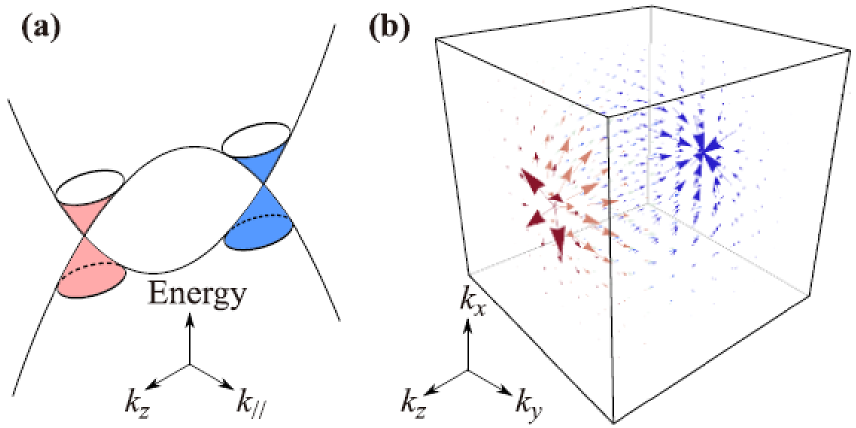


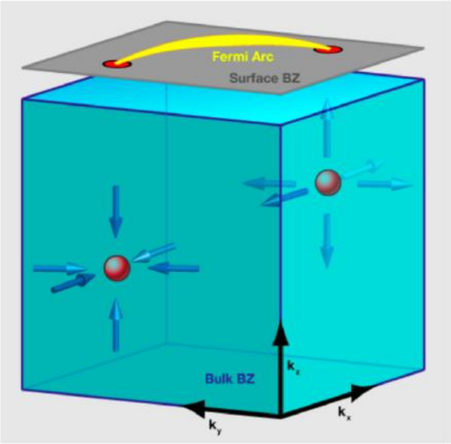
Fig. from Front. Phys. 12(3), 127201 (2017)

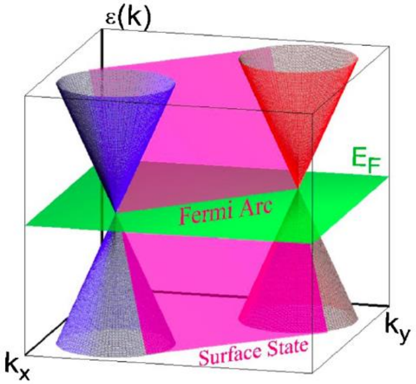
$\chi = \pm 1$ $\equiv$ Chirality (left or right)

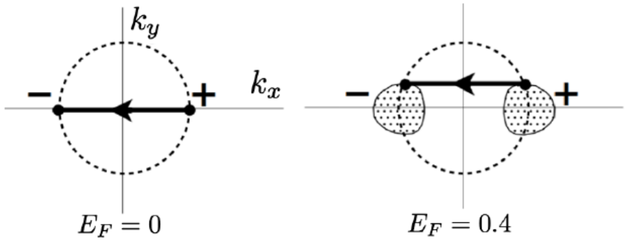
$\sum_j \chi_j = 0$ Nielsen-Ninomiya theorem

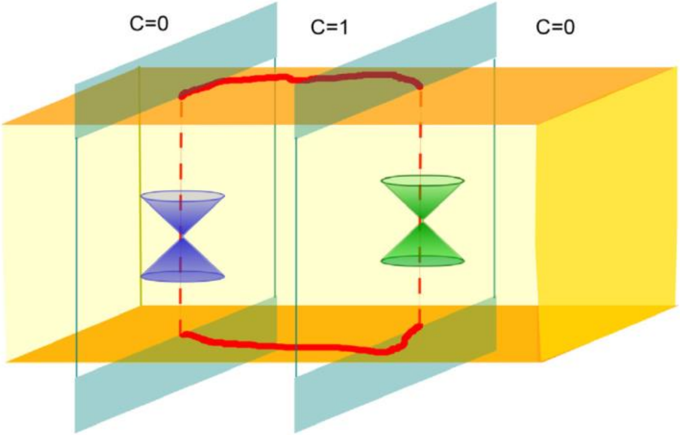
Surface states: Fermi arcs

- Surface states form open segments of isoenergy contours
- Guaranteed by topology









$$C(k_z) = \frac{1}{2\pi} \int dk_x dk_y \, \Omega(\mathbf{k}) \cdot \hat{k_z}$$

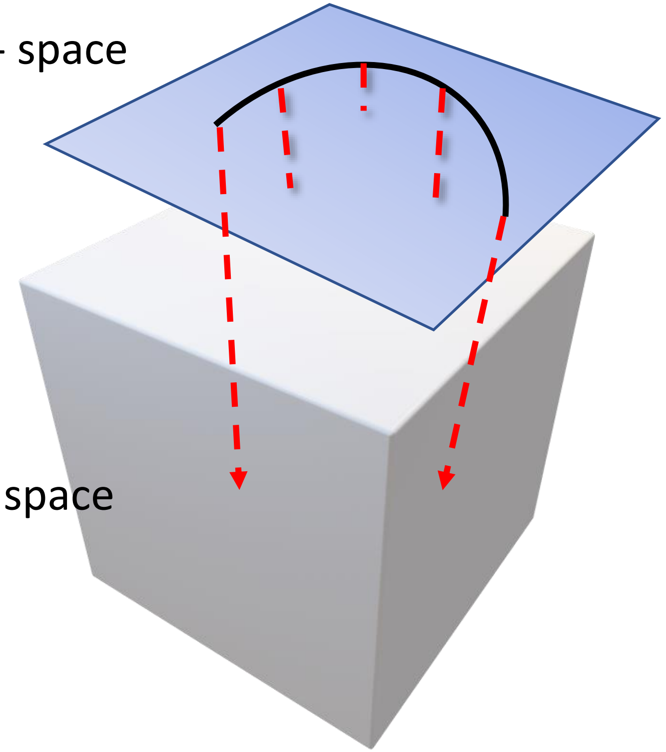
: Chern number

Figs.: Rev. Mod. Phys. 90 (2018)
& C. R. Physique 14 (2013) 857–870

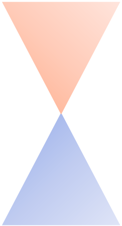
Surface leaking into bulk

Weyl semimetal

2D k- space



3D r- space

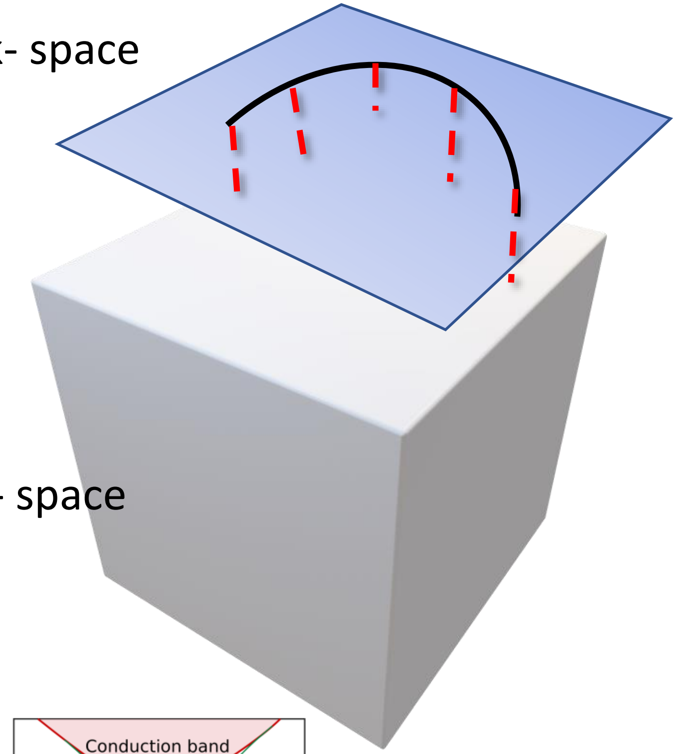


3D k- space

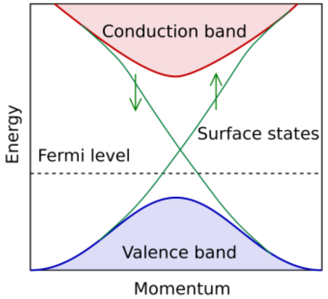
$$\psi_s \sim e^{-z/\lambda}$$

Topological insulator

2D k- space



3D r- space



2D k- space

Transport: Basics

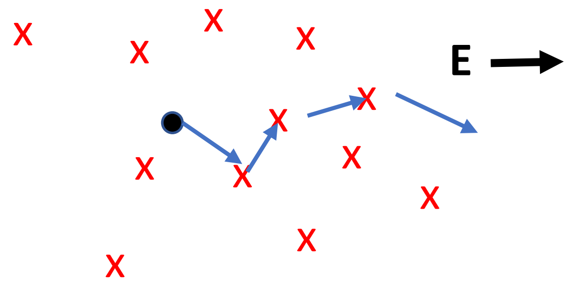
- Response to external electric field

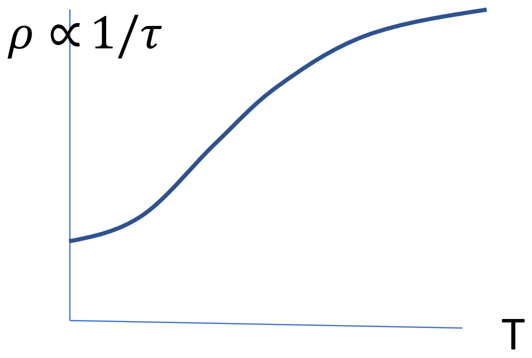
$$\mathbf{j} = ne\mathbf{v}$$

$$\frac{d\mathbf{k}}{dt} = e\mathbf{E} - \frac{\mathbf{k}}{\tau} = 0 \quad \Rightarrow \quad \mathbf{v}_{avg} = \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{j} = \left(\frac{ne^2\tau}{m}\right)\mathbf{E} \quad \Rightarrow \quad \sigma = \frac{ne^2\tau}{m}$$

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2}\frac{1}{\tau}$$ usually increases with temperature in a metal

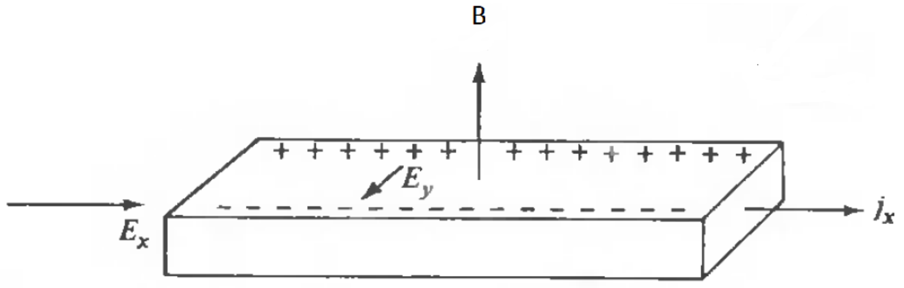




Transport: Basics (Cont'd.)

- Response to external electric and magnetic fields

$$\frac{d\mathbf{k}}{dt} = e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) - \frac{\mathbf{k}}{\tau} = 0$$



$$\sigma_{xx} = \frac{\sigma_0}{1 + \omega_c^2 \tau^2} \quad , \quad \sigma_{xy} = \frac{\sigma_0 \omega_c \tau}{1 + \omega_c^2 \tau^2} \quad , \quad \sigma_{zz} = 0 \quad \text{where} \quad \omega_c = \frac{eB}{m}$$

- ➢ Transverse conductivity decreases with **B**
- ➢ Hall conductivity depends on the direction of **B**
- ➢ Longitudinal conductivity is zero*

* Can be nonzero if the dispersion is of a certain kind, see Pal and Maslov, Phys. Rev. B 81, 214438; Pal, arXIv: 2107.04222

Transport: Basics (Cont'd.)

- Quantum calculation: Kubo formula

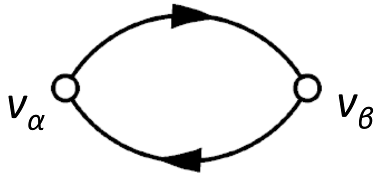
$$\mathrm{Re}\,\sigma_{\alpha\beta} = -e^2 \lim_{\omega\to 0}\lim_{q\to 0}\frac{1}{\omega}\,\mathrm{Im}\,\Pi^R_{\alpha\beta}(\mathbf{q},\omega)$$

$$\Pi_{xx}(\mathbf{q}, iq_n) = -\frac{1}{\mathcal{V}\beta}\langle J_x(\mathbf{q}, iq_n) J_x(-\mathbf{q}, -iq_n)\rangle$$

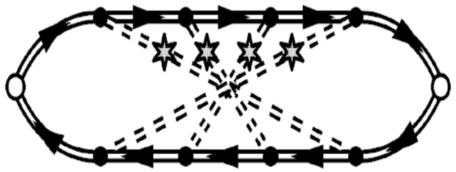
$$G(\mathbf{k},\omega) = \frac{1}{\omega - \xi_{\mathbf{k}} - \Sigma} \qquad \text{(B=0)}$$

$$G(k_z, k_y, x, x', \omega) = \sum_n \frac{\phi_n^*(x - k_y l_B^2)\phi_n(x' - k_y l_B^2)}{\omega - \xi_n(k_z) - \Sigma} \qquad \text{(B≠0)}$$

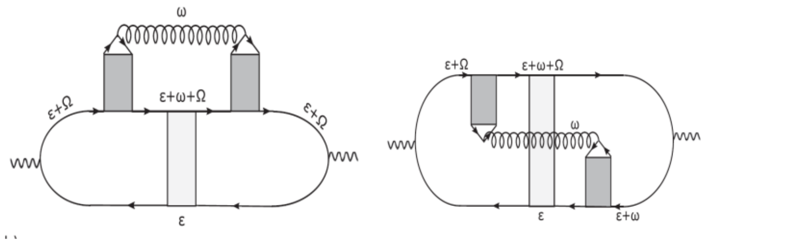
- Drude



- Quantum corrections



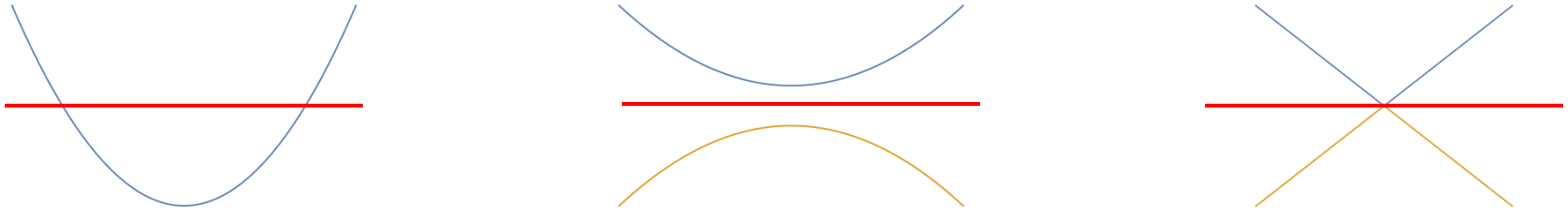
Weak localization (disorder)



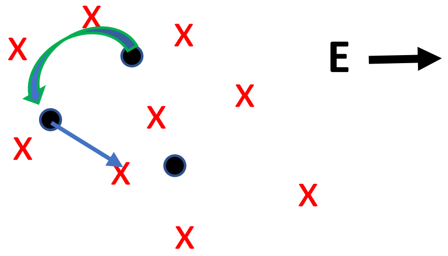
Altshuler-Aronov (disorder+e-e interaction)

Transport in bulk of Weyl semimetal (brief review)

Without magnetic field (Nontopological)



metal | Insulator | Weyl semimetal



Metal: τ

- Disorder: Relaxes momentum, gives rise to residual resistivity at T=0
- Electron-electron: cannot relax momentum (normally), operates at T ≠0

Insulator: n

- Conductivity only arises at T ≠0

$$\sigma = \frac{ne^2\tau}{m}$$

Without magnetic field (cont'd.)

- Electron-electron interaction

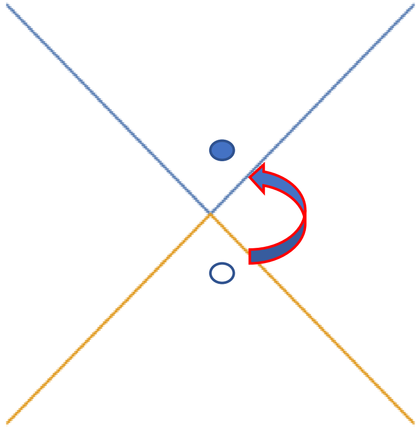
Net momentum=0

$$\sigma = \frac{ne^2\tau}{m}$$

$$\sigma_{\rm dc}^{(N)}(T) = \frac{e^2}{h}\,\frac{k_B T}{\hbar v_F(T)}\,\frac{1.8}{\alpha_T^2 \ln\alpha_T^{-1}}.$$

$$v_F(T) = v_F(\alpha_0/\alpha_T)^{2/N+2}$$

$$\alpha_T = \alpha_0[1 + \tfrac{(N+2)\alpha_0}{3\pi}\ln(\tfrac{\hbar\Lambda}{k_B T})]^{-1}$$



- Disorder

$$\sigma_{\rm dc}(\tau_d) = \frac{2e^2 v_F^2}{3h\gamma}, \quad \gamma \sim \frac{1}{\tau_d}$$

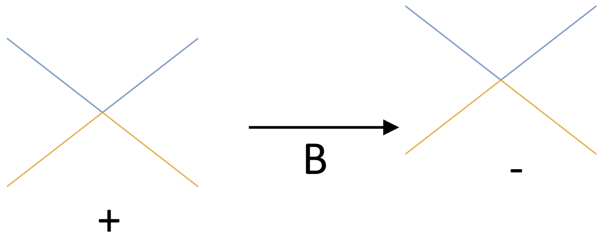
Hosur, et al., Phys.Rev.Lett.108, 046602 (2012)
Goswami and Chakravarty, Phys.Rev.Lett.107, 196803 (2011)
Burkov and Balents, Phys.Rev.Lett.107, 127205 (2011)

With magnetic field (Topological)

- Chiral anomaly: From Landau levels

$$\epsilon_n = v_{\rm F}\,{\rm sign}(n)\sqrt{2\hbar|n|eB + (\hbar\boldsymbol{k}\cdot\hat{\boldsymbol{B}})^2}, \quad n = \pm1, \pm2, \ldots$$

$$\epsilon_0 = -\chi\hbar v_{\rm F}\boldsymbol{k}\cdot\hat{\boldsymbol{B}}$$

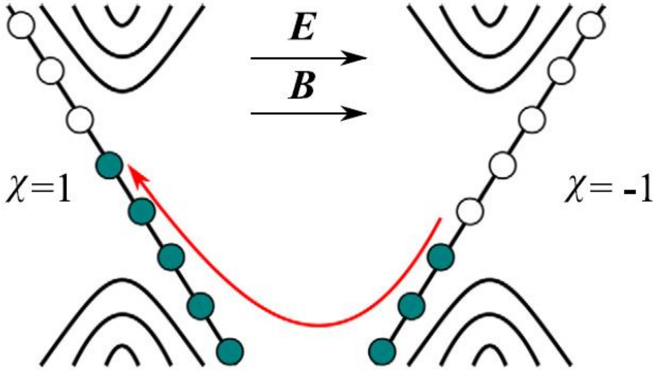


$$\frac{dQ^z_\chi}{dt} = (-e)\chi\frac{\frac{\Delta k_z}{2\pi/L_z}}{\Delta t} = -e\chi\frac{\dot{k}_z}{2\pi/L_z} \quad ; \quad \dot{k}_z = -eE_z$$

$$\Rightarrow \quad \frac{dQ^z_\chi}{dt} = \frac{e^2}{2\pi}\chi E_z L_z$$

Landau level degeneracy $\frac{D}{A_{xy}} = \frac{eB_z}{2\pi}$; Volume $V = A_{xy}L_z$

$$\Rightarrow \quad \frac{dQ_\chi}{dt} = D\frac{dQ^z_\chi}{dt} = \chi V\frac{e^3}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$



Hosur and Qi, C. R. Physique 14 (2013) 857–870

- Total particles conserved but not in each valley

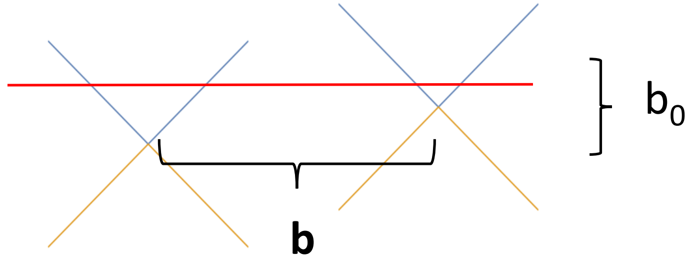
With magnetic field (cont'd.)

- Chiral anomaly: From Field theory

Fuzikawa, Phys. Rev. Lett. 42, 1195 (1979)
Zyuzin and Burkov, Phys. Rev. B 86, 115133 (2012)

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{[-S(\bar{\psi},\psi)]}$$

$$S = \int d^4x \bar{\psi} i\gamma^\mu (\partial_\mu + ieA_\mu + ib_\mu\gamma^5)\psi \qquad \mathcal{D}\bar{\psi}\mathcal{D}\psi$$



$$\psi \rightarrow e^{-i\theta(x)\gamma^5/2}\psi \ , \ \bar{\psi} \rightarrow \bar{\psi}e^{-i\theta(x)\gamma^5/2}$$

$$\theta(x) = 2b_\mu x_\mu = 2\mathbf{b}\cdot\mathbf{r} - 2b_0 t$$

$$S = \int d^4x \bar{\psi} i\gamma^\mu (\partial_\mu + ieA_\mu)\psi \qquad \mathcal{J}^{-2}\mathcal{D}\bar{\psi}\mathcal{D}\psi \ , \ \mathcal{J} = e^{-i\int d^4x\theta(x)\left(\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}\right)}$$

⇨ Extra term in the action: $S_\theta = \frac{ie^2}{16\pi^2}\int d^4x\theta(x)\epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}$

⇨ Non conservation of axial charge: $\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}$

With magnetic field (cont'd.)

- Chiral anomaly: From semiclassics

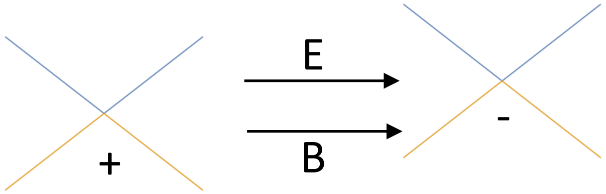
$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{r}} \cdot \boldsymbol{\nabla}_{\mathbf{r}} + \dot{\mathbf{k}} \cdot \boldsymbol{\nabla}_{\mathbf{k}}\right) f_{\mathbf{k},\mathbf{r},t} = I_{\text{coll}}\{f_{\mathbf{k},\mathbf{r},t}\}$$

$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v}_{\mathbf{k}} + \dot{\mathbf{k}} \times \boldsymbol{\Omega}^j_{\mathbf{k}} \\ \dot{\mathbf{k}} = e\mathbf{E} + e\dot{\mathbf{r}} \times \mathbf{B} \\ D^j(\varepsilon) = \int \frac{d^3k}{(2\pi)^3} \left[1 + e(\mathbf{B} \cdot \boldsymbol{\Omega}^j_{\mathbf{k}})\right]^{-1} \delta(\varepsilon_{\mathbf{k}} - \varepsilon) \end{cases}$$

Sundaram and Niu, Phys. Rev. B 59, 14915 (1999)

- Suppose $\nabla_{\mathbf{r}} \to 0, \quad I_{\text{coll}} \to 0$

➢ Then $\frac{\partial N^j}{\partial t} + \nabla \cdot \mathbf{j}^j = \chi^j \frac{e^2}{4\pi^2}(\mathbf{E} \cdot \mathbf{B})$; $\chi = \frac{1}{2\pi} \int_S \Omega(\mathbf{k}) \cdot d\mathbf{s}_{\mathbf{k}}$



➢ Particles in each valley not conserved, but total is conserved.

Son and Spivak, Phys. Rev. B 88, 104412 (2013)

Anomalous Transport due to chiral anomaly

=0

Solve $\left(\frac{\partial}{\partial t} + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}}\right) f_{\mathbf{k},\mathbf{r},t} = I_{\text{coll}}\{f_{\mathbf{k},\mathbf{r},t}\}$ with $I_{\text{coll}} = -\frac{f - f_{eq}}{\tau}$

Drude

NLMR

$$f_{\mathbf{k}} = f_{eq} - eD\tau \mathbf{E} \cdot \left[\mathbf{v} + e\mathbf{B}(\mathbf{\Omega}^j_{\mathbf{k}} \cdot \mathbf{v})\right] \frac{\partial f_{eq}}{\partial \varepsilon} \; ; \quad \mathbf{j}^j = \int \frac{d^3k}{(2\pi)^3} \left[\mathbf{v} + e\mathbf{E} \times \mathbf{\Omega}^j_{\mathbf{k}} + e(\mathbf{\Omega}^j_{\mathbf{k}} \cdot \mathbf{v})\mathbf{B}\right] f_{\mathbf{k}}$$

Anomalous Hall effect

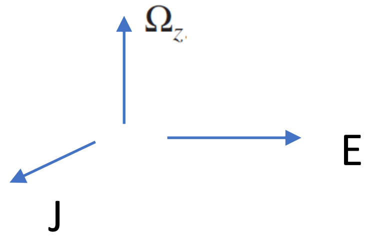
Chiral magnetic effect

- These anomalous features can be derived from the other two methods as well

Anomalous transport (cont'd.)

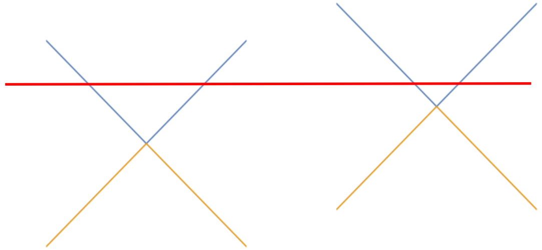
Anomalous Hall effect

$$\sigma_{yx} = -\frac{e^2}{\hbar}\int \frac{d^3k}{(2\pi)^3}\Omega_z f_{eq}$$



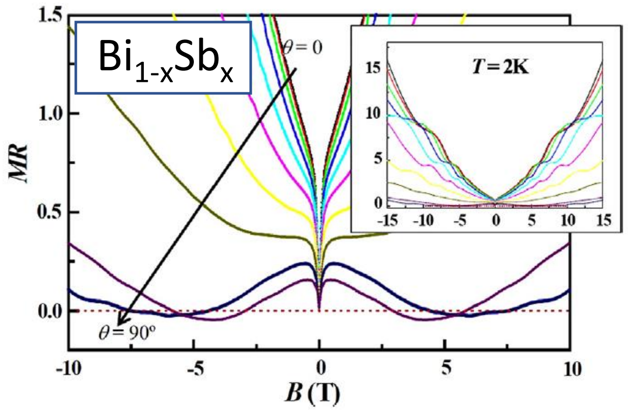
Chiral magnetic effect

$$\mathbf{j} = e\sum_j \mathbf{j}^j = \frac{e^2}{4\pi^2}\mathbf{B}\sum_j \chi^j \mu^j$$



Negative Longitudinal Magnetoresistance

$$\sigma_{zz} = \frac{e^2}{4\pi^2}\frac{v^3 e^2 B^2 \tau}{\mu^2}$$



Kim et al., Phys. Rev. Lett. 111, 246603 (2013)

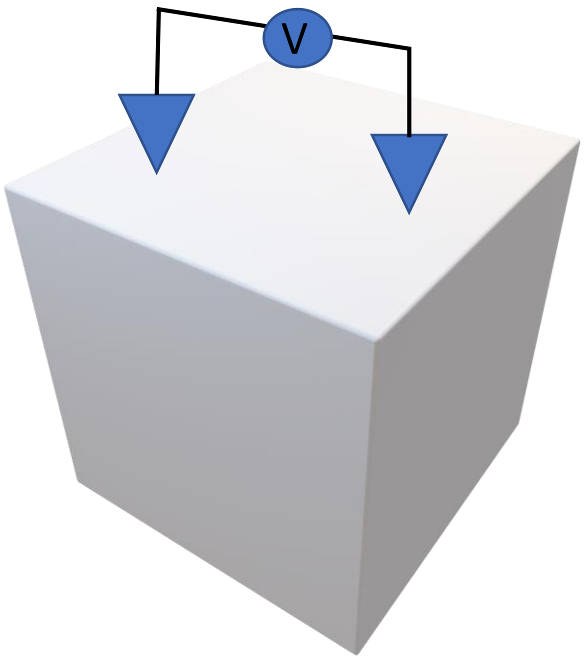
Other transport features

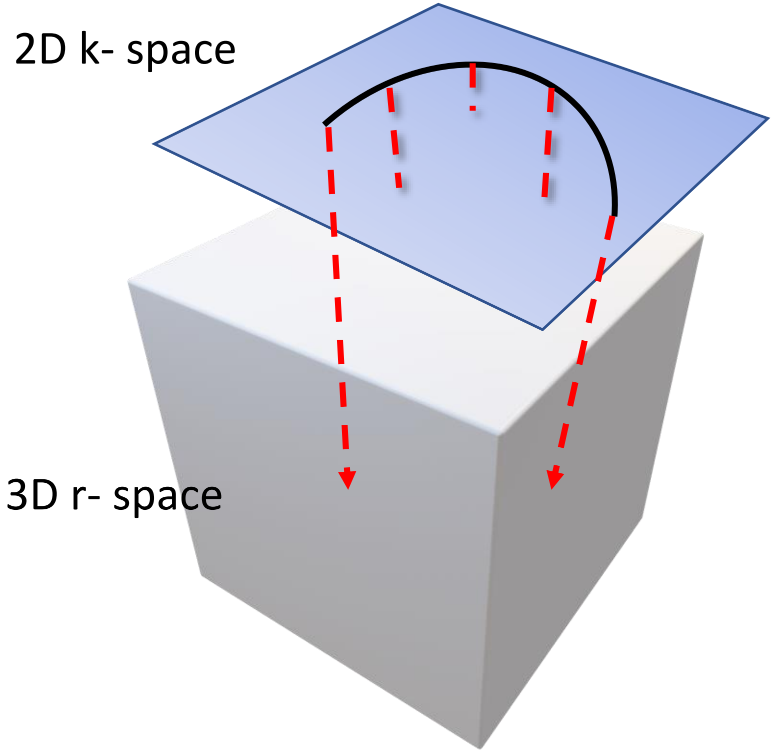
- Various other features exist. But most of them share the same common orign.
- Reviews on transport in Weyl semimetals

  - Hosur and Qi, C. R. Physique 14 857 (2013)
  - Lu and Shen, Front. Phys. 12(3), 127201 (2017)
  - Gorbar, et al., Low Temp. Phys. 44, 487 (2018)
  - Burkov, Ann. Rev. Con. Matt. Phys. 9, 359 (2018)
  - Armitage, Mele, Vishwanath, Rev. Mod. Phys. (2018)
  - Ong and Liang, Nat. Rev. Phys. 3**,** 394–404 (2021)

Transport on surface of Weyl semimetal

Transport on surface: Issues





- Not possible to write down a surface Hamiltonian as a starting point.
- Any response on surface intrinsically connected to the bulk.

Gorbar, et al., Phys. Rev. B 93, 235127 (2016)

Layered model for a Weyl semimetal

$$H_{\mathbf{k}} = \sum_{z=1}^{L} \psi^{\dagger}_{z,\mathbf{k}} (-1)^z \xi_{\mathbf{k}} \psi_{z,\mathbf{k}} + \sum_{z=1}^{L-1} \psi^{\dagger}_{z,\mathbf{k}} h_{z\ z+1,\mathbf{k}} \psi_{z+1,\mathbf{k}} + \mathrm{H.c.}$$

$$\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \varepsilon^{2D}$$

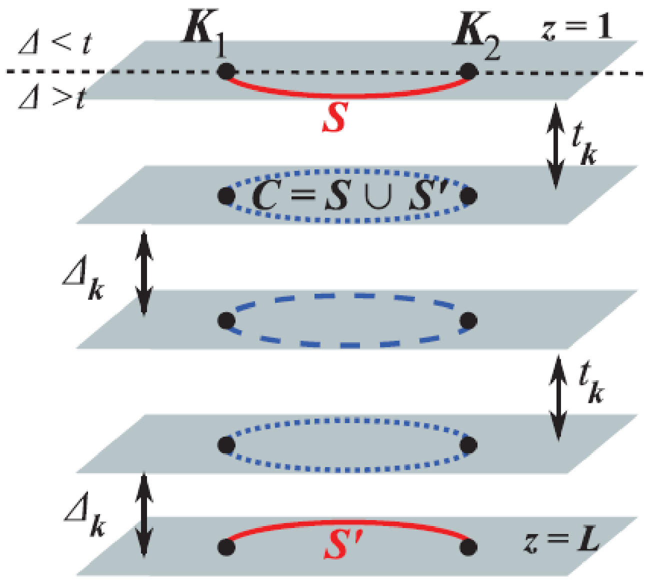
$\xi_{\mathbf{k}} = 0$: electron

$-\xi_{\mathbf{k}} = 0$: hole

$$h_{z\ z+1,\mathbf{k}} = -t_{\mathbf{k}} \quad z\text{: odd}$$

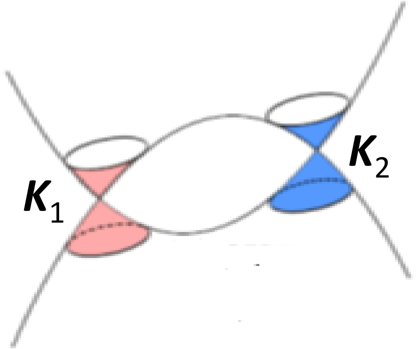
$$h_{z\ z+1,\mathbf{k}} = -\Delta_{\mathbf{k}} \quad z\text{: even}$$

$t_{\mathbf{k}} - \Delta_{\mathbf{k}}$: Changes sign at discrete points $\boldsymbol{K}_j$

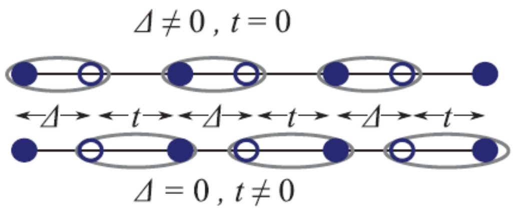


P. Hosur, Phys. Rev. B 86, 195102 (2012).

Bulk →



Surface →



Bulk Hamiltonian

$$H^{\text{bulk}}_{\boldsymbol{k},k_z} = \mathcal{E}_{\boldsymbol{k}}\sigma_z + (\Delta_{\boldsymbol{k}} - t_{\boldsymbol{k}}\cos k_z)\sigma_x + t_{\boldsymbol{k}}\sin k_z \sigma_y$$

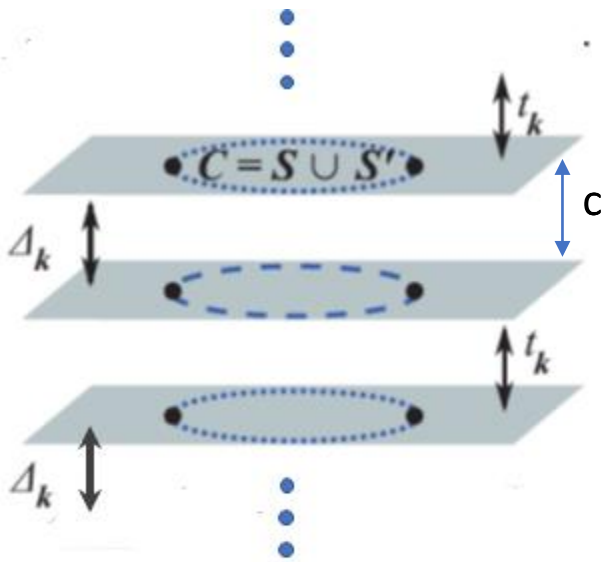
Weyl nodes at $(\boldsymbol{K}_i, 0)$ satisfying $\xi_{\mathbf{k}} = t_{\mathbf{k}} - \Delta_{\mathbf{k}} = 0$

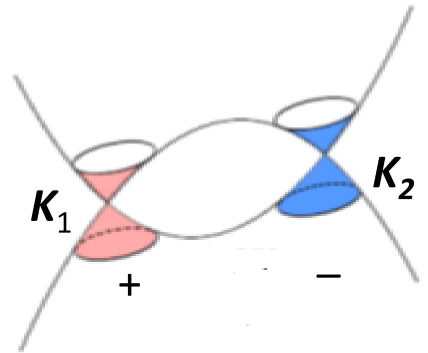
$$H_{\text{Weyl},i} = \boldsymbol{k}_{3D} \cdot (\boldsymbol{v}_i\sigma_z + \boldsymbol{u}_i\sigma_x + \boldsymbol{w}_i\sigma_y)$$

$\boldsymbol{k}_{3D} = (\boldsymbol{k}, k_z)$, measured from Weyl node

$$\boldsymbol{v}_i = \boldsymbol{\nabla}_{\boldsymbol{k}}\mathcal{E}_{\boldsymbol{K}_i} \quad \boldsymbol{u}_i = \boldsymbol{\nabla}_{\boldsymbol{k}}(\Delta_{\boldsymbol{K}_i} - t_{\boldsymbol{K}_i}) \quad \boldsymbol{w}_i = 2t_{\boldsymbol{K}_i}c\hat{\mathbf{z}}$$

$$\chi_i = \text{sign}[\mathbf{u}_i.\hat{\mathbf{e}}_t(\mathbf{K}_i)]$$



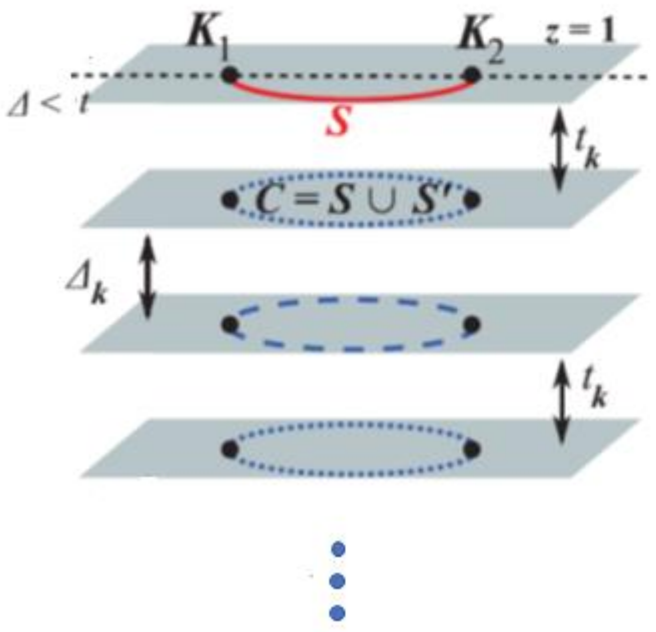


Surface Green's function

$$G_s(\mathbf{k}, E) = \frac{g_1(\mathbf{k}, E) + g_2(\mathbf{k}, E)}{2t_{\mathbf{k}}^2(E - \xi_{\mathbf{k}})}$$

$$g_1(\mathbf{k}, E) = E^2 - \xi_{\mathbf{k}}^2 + t_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2$$

$$g_2(\mathbf{k}, E) = \sqrt{(E^2 - \xi_{\mathbf{k}}^2 + t_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2)^2 - 4t_{\mathbf{k}}^2\,(E^2 - \xi_{\mathbf{k}}^2)}$$

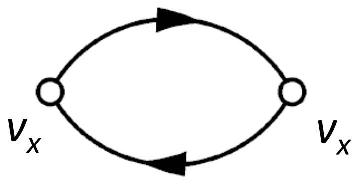


$$G_s(\mathbf{k}, E) = \frac{t_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2 + |t_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2|}{2t_{\mathbf{k}}^2(E - \xi_{\mathbf{k}})}$$

- Nonzero only on the Fermi arc

Surface conductivity

$$\sigma_s = \frac{e^2}{2\pi A}\sum_{\mathbf{k}} v_x^2(\mathbf{k}) \int_{-\infty}^{\infty} (-n_F')\, G_s^R(\mathbf{k},E) G_s^A(\mathbf{k},E)\, dE$$



$$G_s(\mathbf{k},E) = \frac{g_1(\mathbf{k},E) + g_2(\mathbf{k},E)}{2t_{\mathbf{k}}^2(E-\xi_{\mathbf{k}})}$$

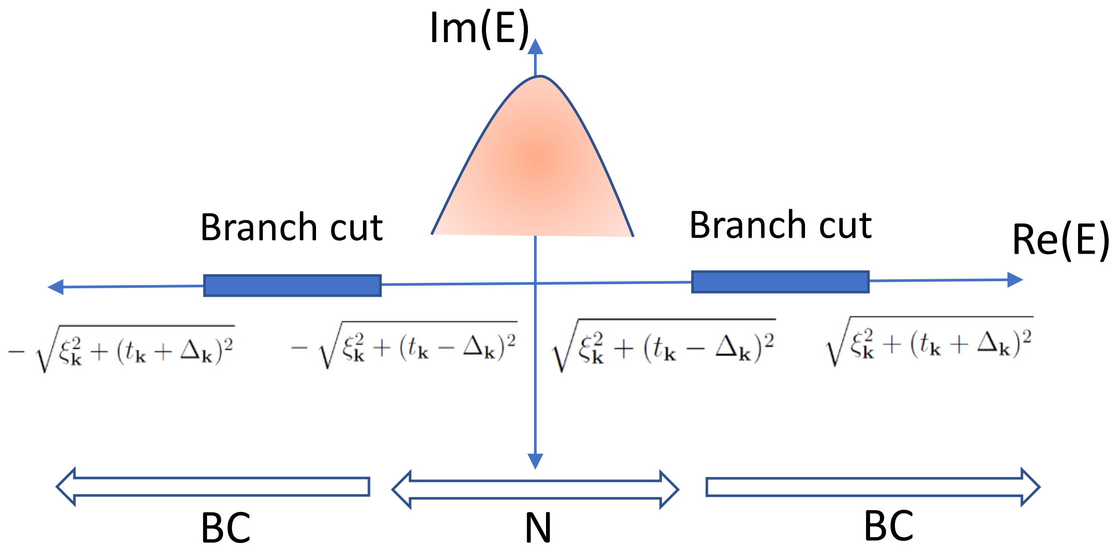
$$g_1(\mathbf{k},E) = E^2 - \xi_{\mathbf{k}}^2 + t_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2$$

$$g_2(\mathbf{k},E) = \sqrt{[E^2 - \xi_{\mathbf{k}}^2 - (t_{\mathbf{k}} - \Delta_{\mathbf{k}})^2]\,[E^2 - \xi_{\mathbf{k}}^2 - (t_{\mathbf{k}} + \Delta_{\mathbf{k}})^2]}$$

$$G_s^{R,A} = \frac{g_1 + g_2}{2t_{\mathbf{k}}^2(E - \xi_{\mathbf{k}} \pm \frac{i}{2\tau})} \quad (\mathrm{N-region}),$$

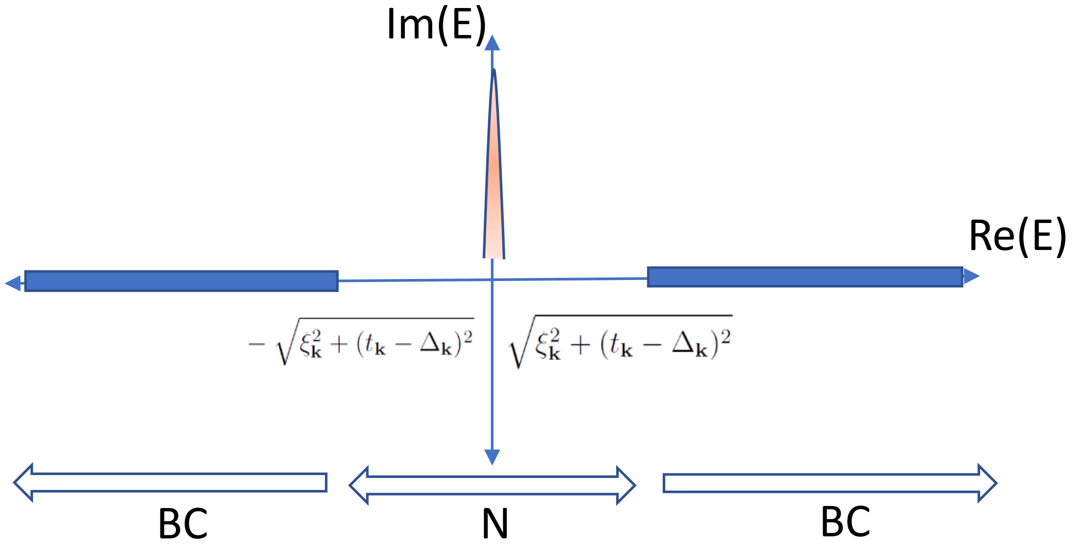
$$G_s^{R,A} = \frac{g_1 \pm i|g_2|}{2t_{\mathbf{k}}^2(E - \xi_{\mathbf{k}} \pm \frac{i}{2\tau})} \quad (\mathrm{BC-region}).$$

$1/\tau \to 0$



N-Region: Drude part

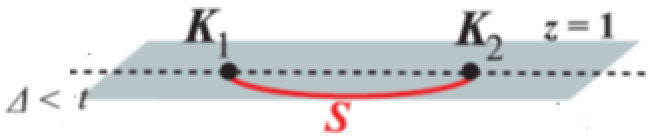
$$\sigma_s = \frac{e^2}{2\pi A}\sum_{\mathbf{k}} v_x^2(\mathbf{k})\int_{-\infty}^{\infty}(-n'_F)\,G_s^R(\mathbf{k},E)G_s^A(\mathbf{k},E)dE$$



$$\sigma_s^N = e^2\nu_F\langle v_x^2\rangle\tau$$

$$\langle v_x^2\rangle = \frac{1}{4\pi^2\nu_F}\int d\mathbf{k}\delta(\varepsilon_{\mathbf{k}}-\varepsilon^{2D})v_x^2(\mathbf{k})\left(1-\frac{\Delta_{\mathbf{k}}^2}{t_{\mathbf{k}}^2}\right)^2\Theta(t_{\mathbf{k}}^2-\Delta_{\mathbf{k}}^2)$$

$$\nu_F = \frac{1}{4\pi^2}\int d\mathbf{k}\delta(\varepsilon_{\mathbf{k}}-\varepsilon^{2D})$$



BC-region: Anomalous part

- Nonzero T

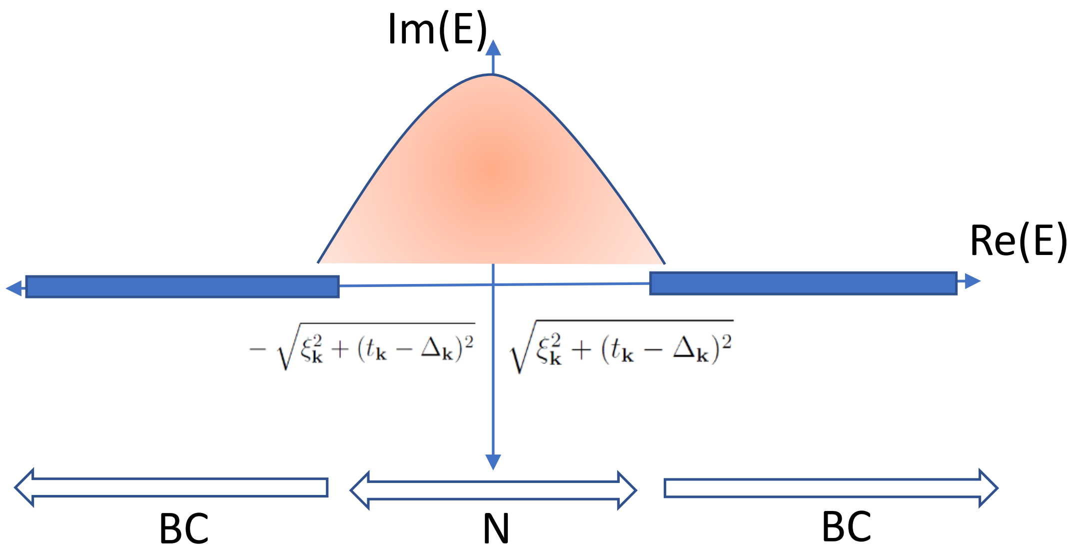
$$\sigma_s = \frac{e^2}{2\pi A}\sum_{\mathbf{k}} v_x^2(\mathbf{k}) \int_{-\infty}^{\infty} (-n'_F)\, G_s^R(\mathbf{k},E) G_s^A(\mathbf{k},E) dE$$

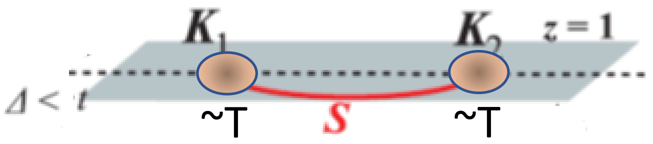
Main contribution near $\xi_{\mathbf{k}} = t_{\mathbf{k}} - \Delta_{\mathbf{k}} = 0$

➢ Near the Weyl nodes

$$\sigma_z^{BC} = \frac{T^2}{12} e^2 d \sum_j \frac{v_{x,j}^2}{|(\mathbf{u}_j \times \mathbf{v}_j)\cdot \mathbf{w}_j|} \frac{1}{t_{\perp 0j}}$$

➢ Independent of the scattering time!



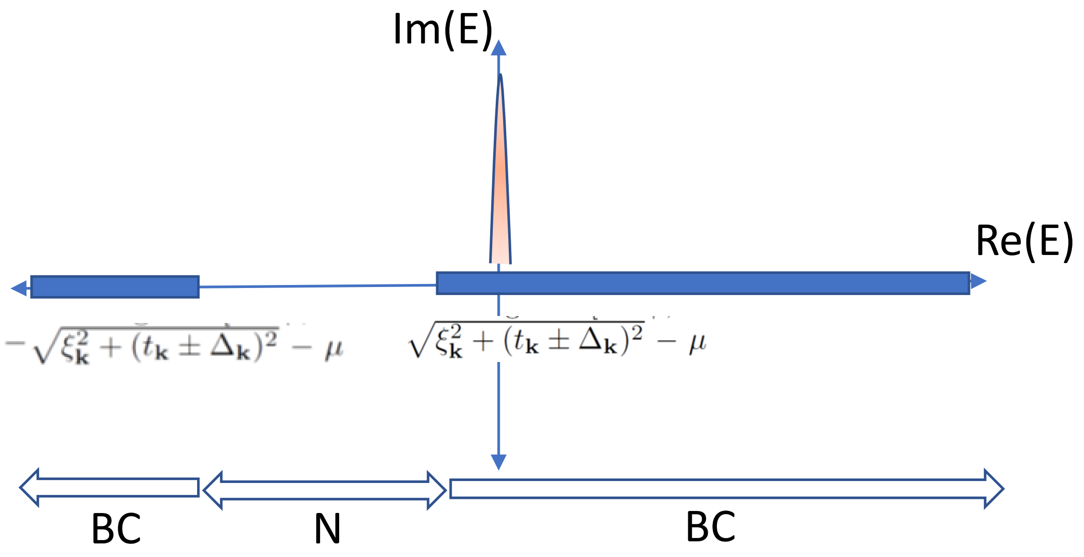


BC-region: Anomalous part (cont'd.)

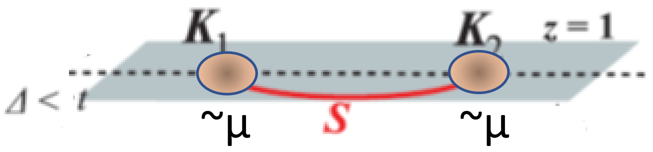
- Nonzero μ

$$\sigma_s = \frac{e^2}{2\pi A}\sum_{\mathbf{k}} v_x^2(\mathbf{k}) \int_{-\infty}^{\infty} (-n_F')\, G_s^R(\mathbf{k},E) G_s^A(\mathbf{k},E)\, dE$$

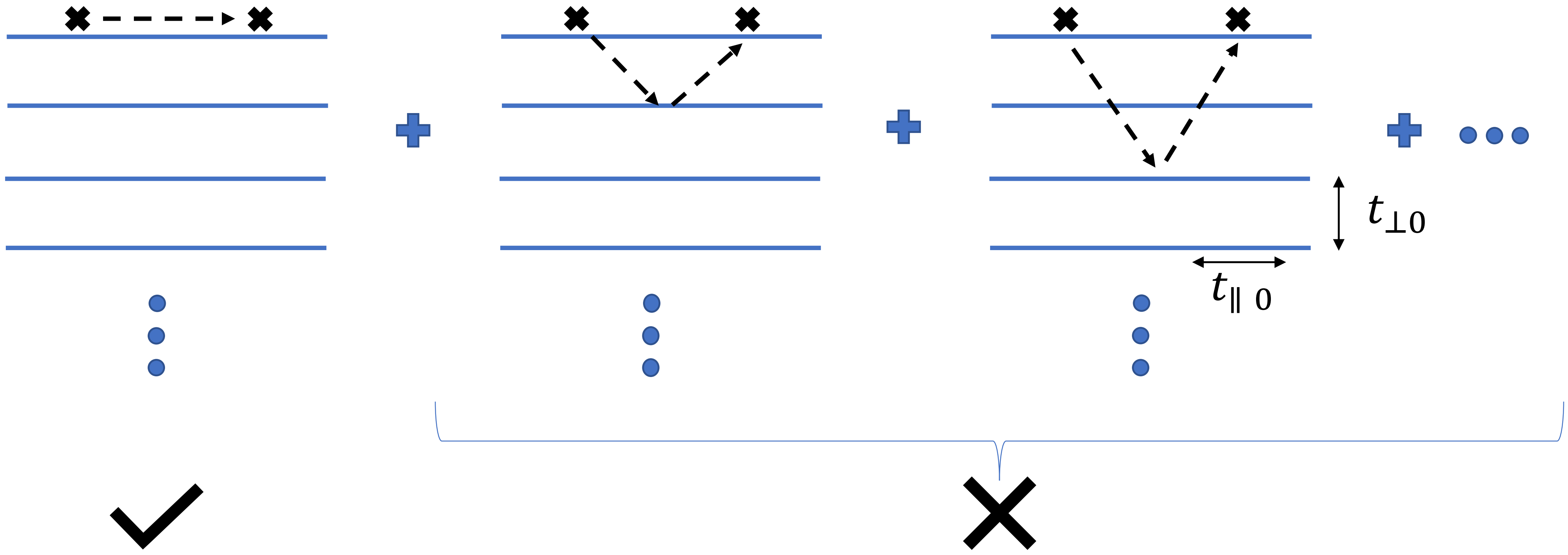
$$\sigma_z^{BC} = \frac{\mu^2}{4\pi^2} e^2 d \sum_j \frac{v_{x,j}^2}{|(\mathbf{u}_j \times \mathbf{v}_j)\cdot \mathbf{w}_j|} \frac{1}{t_{\perp 0j}}$$



➢ Also, independent of the scattering time!



Layered Weyl semimetal approximation



Assume interlayer coupling small so that these can be neglected.

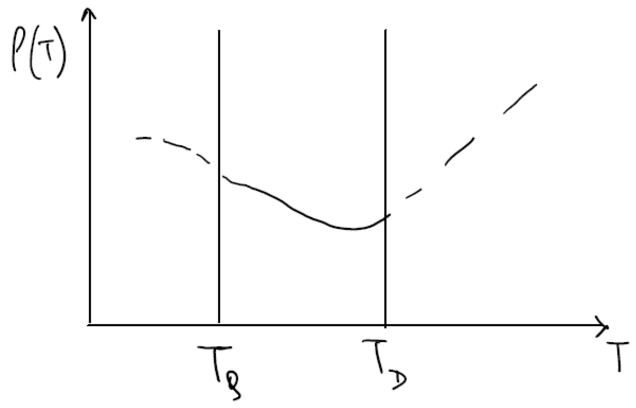
Final result

❑ Layered Weyl semimetal surface

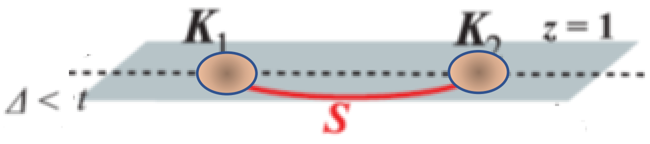
$$\sigma_s(\mu, T) = e^2 \nu_F \langle v_x^2 \rangle \tau + \frac{\mu^2 + T^2\pi^2/3}{4\pi^2} e^2 d \sum_j \frac{v_{x,j}^2}{|(\mathbf{u}_j \times \mathbf{v}_j)\cdot \mathbf{w}_j|} \frac{1}{t_{\perp 0 j}}$$

Drude
(Fermi arc)

Anomalous
(Weyl node)



Pal, Obakpolor, and Hosur, in preparation

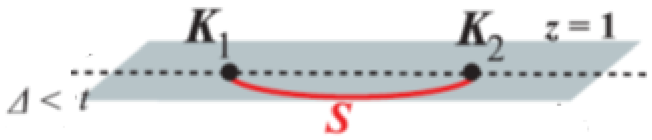


- T_Q: Temperature below which quantum corrections become important
- T_D: Above which temperature dependence of τ becomes important

Origin of anomalous part

- Total surface density

$$n_s = -2\mathrm{Im}\int_{\mathbf{k},E} G_s^R(\mathbf{k},E) = n_s^N + n_s^{BC}$$



$$n_{s,j}^{BC}(T) = \frac{\mu\left(\mu^2+\pi^2T^2\right)}{6\pi^2}\frac{c}{|(\mathbf{u}_j\times\mathbf{v}_j)\cdot\mathbf{w}_j|}$$

$$\nu_j^{BC}(T) = dn_{s,j}^{BC}(T)/d\mu = \frac{\mu^2+T^2\pi^2/3}{2\pi^2}\frac{c}{|(\mathbf{u}_j\times\mathbf{v}_j)\cdot\mathbf{w}_j|}$$

$$\Rightarrow \quad \sigma_s^{BC}(T) = \sum_j \frac{v_{j,x}^2\nu_j^{BC}(T,\mu)}{2t_{\perp j}}$$

- Anomalous part results from an effective bulk density on surface

Effective correlation on surface

- Why no dependence on τ

$$G_s(\mathbf{k},E) = \frac{g_1(\mathbf{k},E)+g_2(\mathbf{k},E)}{2t_{\mathbf{k}}^2(E-\xi_{\mathbf{k}})}$$

$$\begin{cases} g_1(\mathbf{k},E) = E^2-\xi_{\mathbf{k}}^2+t_{\mathbf{k}}^2-\Delta_{\mathbf{k}}^2 \\ g_2(\mathbf{k},E) = \sqrt{[E^2-\xi_{\mathbf{k}}^2-(t_{\mathbf{k}}-\Delta_{\mathbf{k}})^2]\,[E^2-\xi_{\mathbf{k}}^2-(t_{\mathbf{k}}+\Delta_{\mathbf{k}})^2]} \end{cases}$$

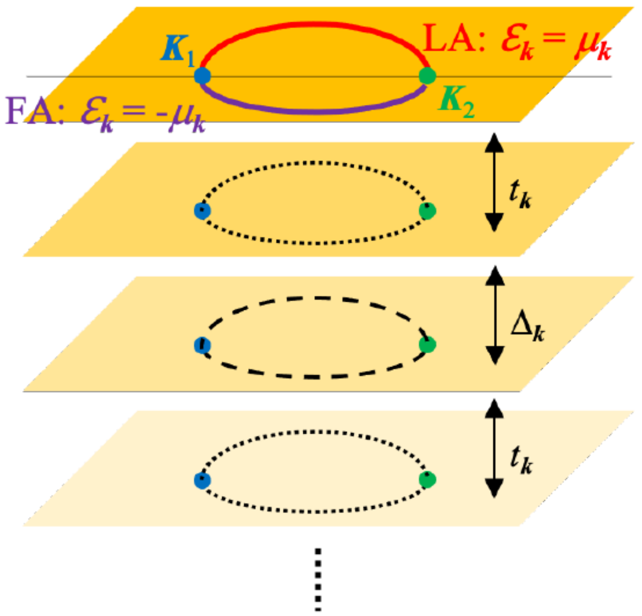
$$G^R(E,\mathbf{k}) = \frac{Z(E,\mathbf{k})}{E-\xi_{\mathbf{k}}-\Sigma^R(E,\mathbf{k})}$$ in the region $|E| > \sqrt{\xi_{\mathbf{k}}^2+(t_{\mathbf{k}}-\Delta_{\mathbf{k}})^2}.$

With $Z = (g_1^2+|g_2|^2)/2g_1t_{\mathbf{k}}^2$ and $\Sigma^R = i(E-\xi_{\mathbf{k}})|g_2|/g_1$

- ➢ 2D Correlated liquid on the surface of a 3D noninteracting Weyl semimetal
- ➢ The effect is most near the Weyl nodes

Effective correlation on surface (cont'd.)

- The portion absent on the surface is a Luttinger arc
- Definition of Luttinger arc: G=0
- Usually signature of strong correlations due to divergence of self-energy.



Obakpolor and Hosur, arXiv: 2108:05380

Summary and future directions

The gist

- The electromagnetic response of Weyl semimetals contains anomalous behaviors
- In the bulk
  - Due to semimetal band structure in an electric field (non topological)
  - Due to chiral anomaly in parallel electric and magnetic field (topological)
- On the surface
  - In addition to Drude response from Fermi arc, an anomalous part due to intrinsic surface-bulk connection which is independent of surface disorder

What lies ahead?

- Transport on surface
  - Microscopic models of scattering
  - Quantum corrections
  - In a magnetic field

- New phases on the surface
  - Phases which can reside only on the surface due to the effect from the bulk

Interplay between intrinsic correlation due to bulk and other interactions.

Acknowledgments

- Pavan Hosur (University of Houston)
- Obakpolor Eki Osakpolor (University of Houston)

➢ Financial support from IRCC, IIT Bombay

❖ Will be posted soon (Pal, Obakpolor, and Hosur, arXiv: 2109.xxxxx)

Thank you