In-medium properties of heavy quarks on the lattice

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EXTREME NONEQUILIBRIUM QCD 2020

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Outline

- Introduction
- Finite temperature lattice studies on heavy quarks
  - Spectral functions from Bayesian inference
  - Heavy quark momentum diffusion coefficients
  - Phenomenologically motivated spectral functions
- Summary

Heavy flavor

- Heavy flavor = charm & bottom

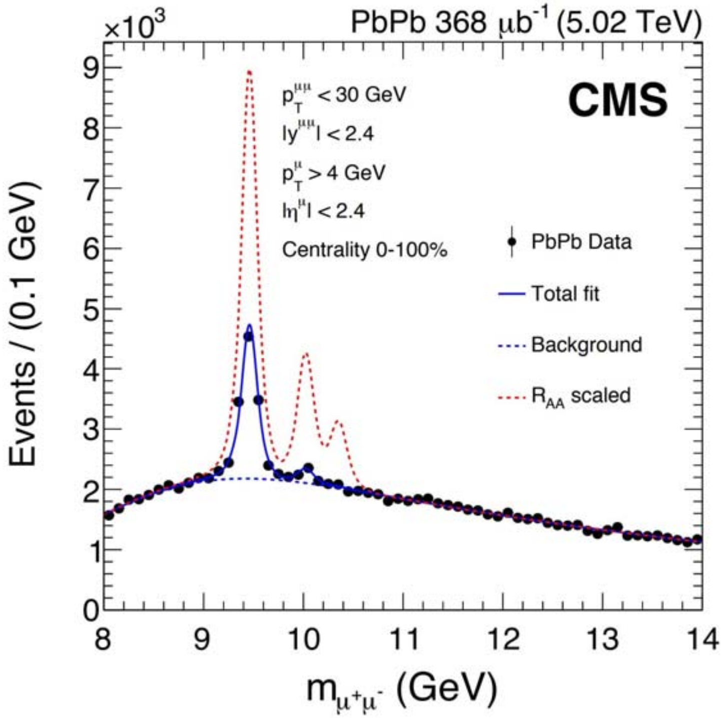
η_c, J/Ψ, χ_c, η_b, Υ, χ_b …

D, B, …

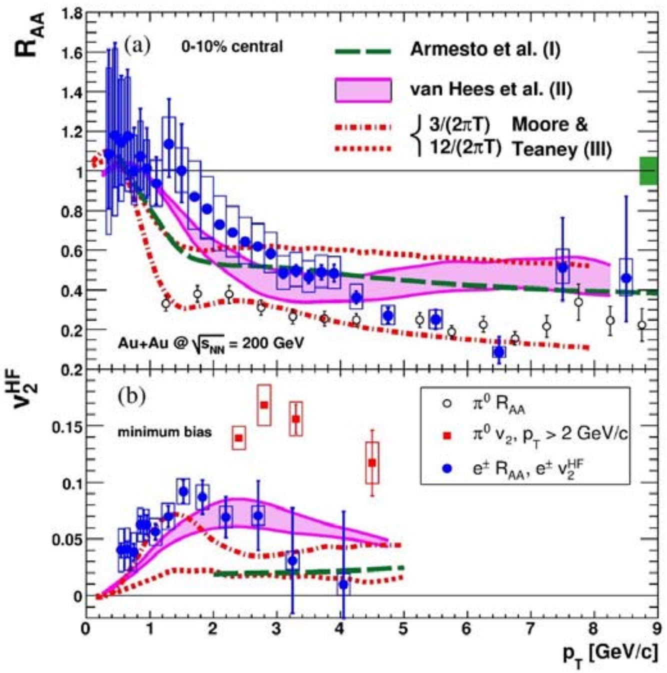
Q = c, b
q = u, d, s

- **Quarkonia ($Q\bar{Q}$)** and **open heavy flavors ($Q\bar{q}$, $q\bar{Q}$)** are important probes to investigate **Quark-Gluon Plasma (QGP)** formed in heavy ion collisions.
  - Produced in the early stage of the collisions: experiencing entire evolution of QGP
  - A signal of QGP formation: color Debye screening in QGP → suppression of quarkonium production T. Matsui and H. Satz, PLB 178 (1986) 416
  - Sequential suppression: different binding energy for different bound state → different melting temperature

What should we understand?



CMS Collaboration@QM2018



PHENIX Collaboration, PRL 98 (2007) 172301

Understanding suppression patterns

→ **dissociation temperatures**

Inputs for transport models

→ **heavy quark diffusion coefficients**

In-medium properties of open/hidden heavy flavors ← All encoded in **spectral functions**

Spectral function

Euclidian (imaginary time) mesonic correlation function

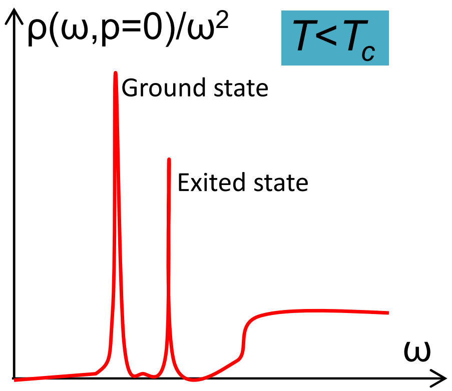
$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0})\rangle$$

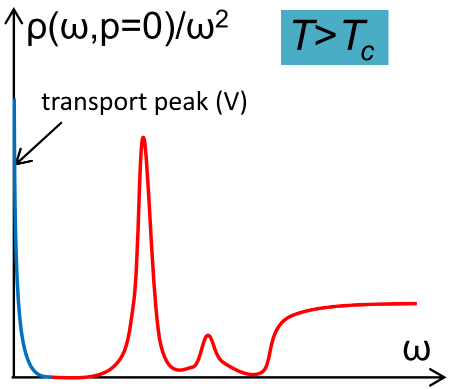
$$= \int_0^\infty \frac{d\omega}{2\pi} \boxed{\rho_H(\omega,\vec{p})} K(\omega,\tau)$$

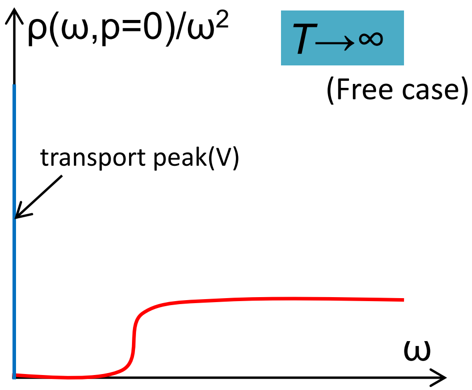
Spectral function

$$J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x})\Gamma_H \psi(\tau,\vec{x})$$

$$K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$







Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^V(\omega,\mathbf{0})}{\omega}$$

ρ^V_{ii} : Vector spectral function

χ_{00} : Quark number susceptibility

Difficulties on lattice

- **Heavy quark mass m_Q is too heavy!**
  - Basically, fine and large lattices are needed to control lattice artifacts.

- **Obtaining spectral functions: an ill-posted inverse problem!**
  - Number of correlator data points << Number of frequency bins of spectral functions
  - A simple χ^2 fit gives infinite number of possible spectra within statistical uncertainties.

Overcoming difficulties

- **Heavy quark mass m_Q is too heavy!**
  - $m_Q >> \Lambda_{QCD} \rightarrow$ separation of scales (especially for bottom)
  - Effective field theories: **Non-relativistic QCD (NRQCD), potential NRQCD (pNRQCD)**, …

Review: N. Brambilla *et al.*, Rev. Mod. Phys. 77 (2005) 1423

- **Obtaining spectral functions: an ill-posted inverse problem!**
  - Adding prior information
  - Bayesian inference: **Maximum Entropy Method (MEM), Bayesian Reconstruction (BR) Method,** …
  - **Phenomenologically motivated (perturbative) modeling of spectral functions**
  - Other approached without spectral functions

Bayesian inference

- Bayes theorem

$$P[A|\bar{G}] = \frac{P[\bar{G}|A]P[A]}{P[\bar{G}]}$$

- Prior probability: prior information

$$P[A] \propto \exp(\alpha S)$$

- Likelihood function: usual χ^2 term

$$P[\bar{G}|A] \propto \exp\left(-\chi^2/2\right)$$

- Posterior probability

$$P[A|\bar{G}] \propto e^{-F}$$

$$\boxed{\max P[A|\bar{G}] \leftrightarrow \min F \leftrightarrow \frac{\delta F}{\delta A} = 0}$$

MEM: Shannon-Jaynes Entropy

M. Asakawa, T. Hatsuda and Y. Nakahara,
Prog.Part.Nucl.Phys. 46 (2001) 459-508

$$S = \int d\omega \left(A - D - A \log\left(\frac{A}{D}\right)\right)$$

BR: new regulator

Y. Burnier and A. Rothkopf, PRL 111 (2013) 18, 182003

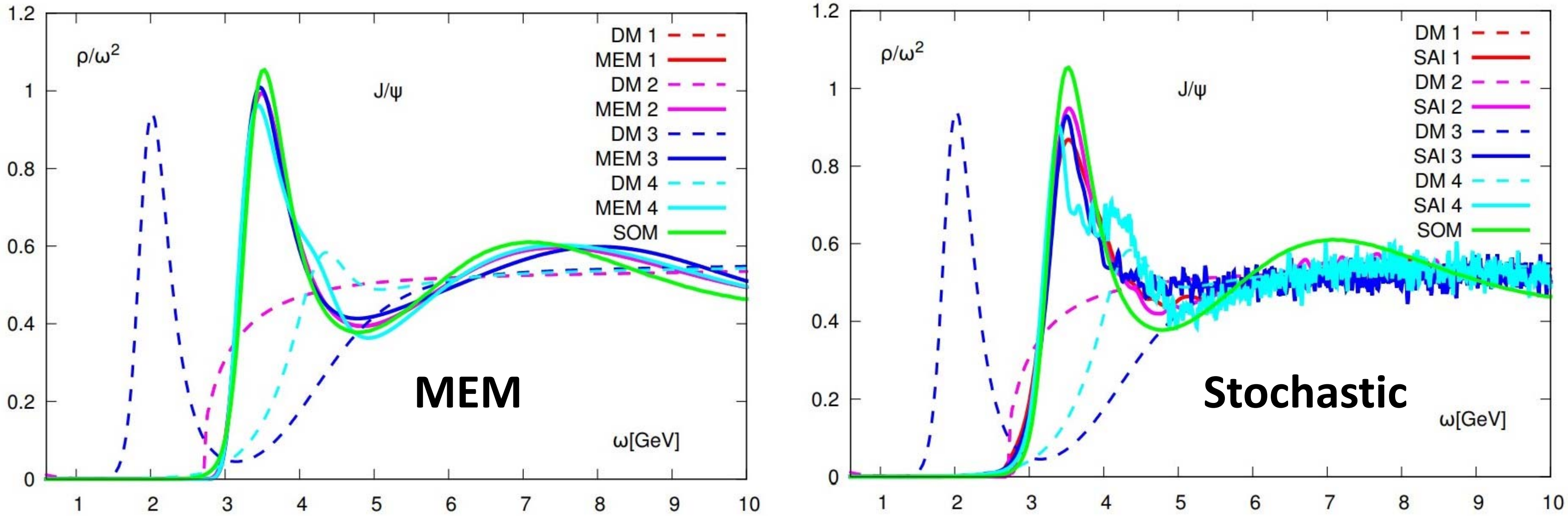
$$S = \int d\omega \left(1 - \frac{A}{D} + \log\left(\frac{A}{D}\right)\right)$$

D: Default model = prior knowledge

Stochastic method

- Stochastically finding a spectral image
- Formally equivalent to MEM in a certain limit K.S.D. Beach, arXiv:cond-mat/0403055

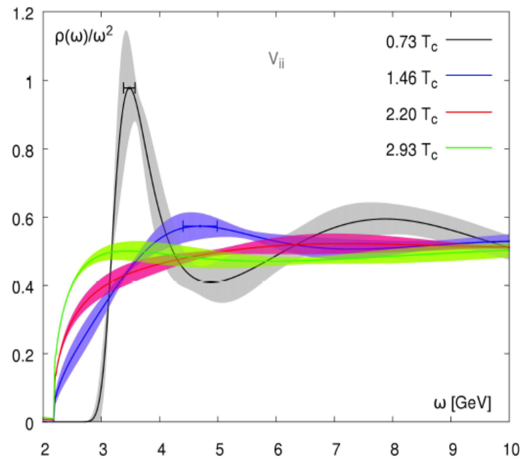
J/Ψ@0.75T_c, 128^3x96, Quenched H.-T. Ding, O. Kaczmarek, Swagato Mukherjee, HO and H.-T. Shu, PRD 97 (2018) 9, 094503



Qualitatively consistent results

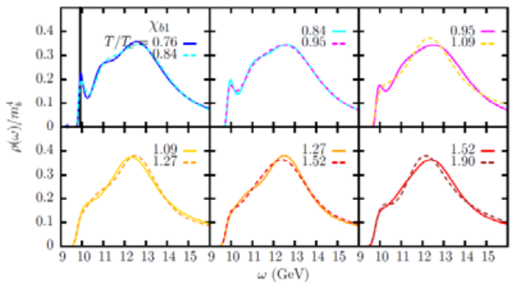
Lattice studies on quarkonium spectral functions so far

- Charmonia
  - Several studies both in quenched QCD and with dynamical quarks
  - **Dissociation temperatures are still not conclusive**
    - **J/Ψ seems to survive up to 1.5T_c**
    - **P-wave states may melt just above T_c**

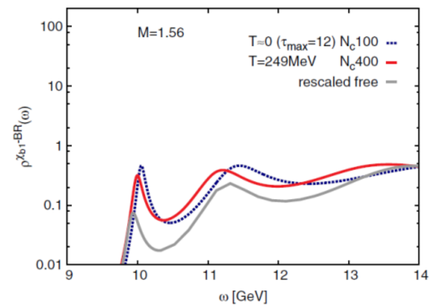


H.-T. Ding *et al.*, PRD 86 (2012) 014509

- Bottomonia
  - NRQCD
  - **Υ(1S) seems to survive at least up to 2T_c**
  - **Different conclusions on melting of a P-wave state with different methods.**
  - It is good to crosscheck with other methods and without NRQCD



G. Aarts *et al.*, JHEP1407(2014)097

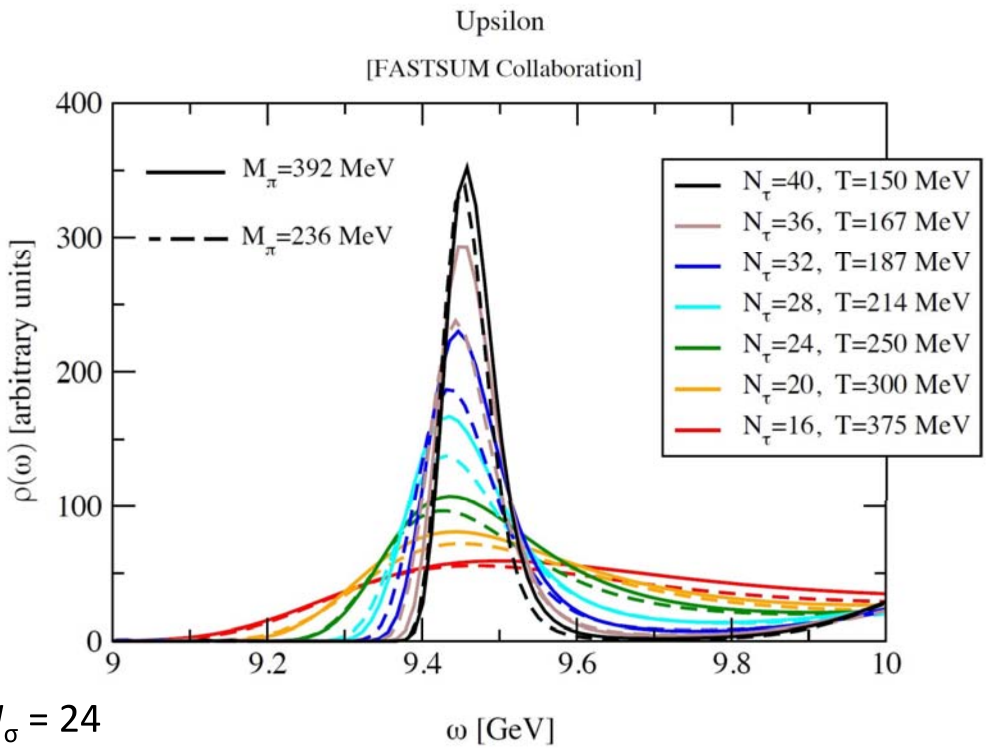


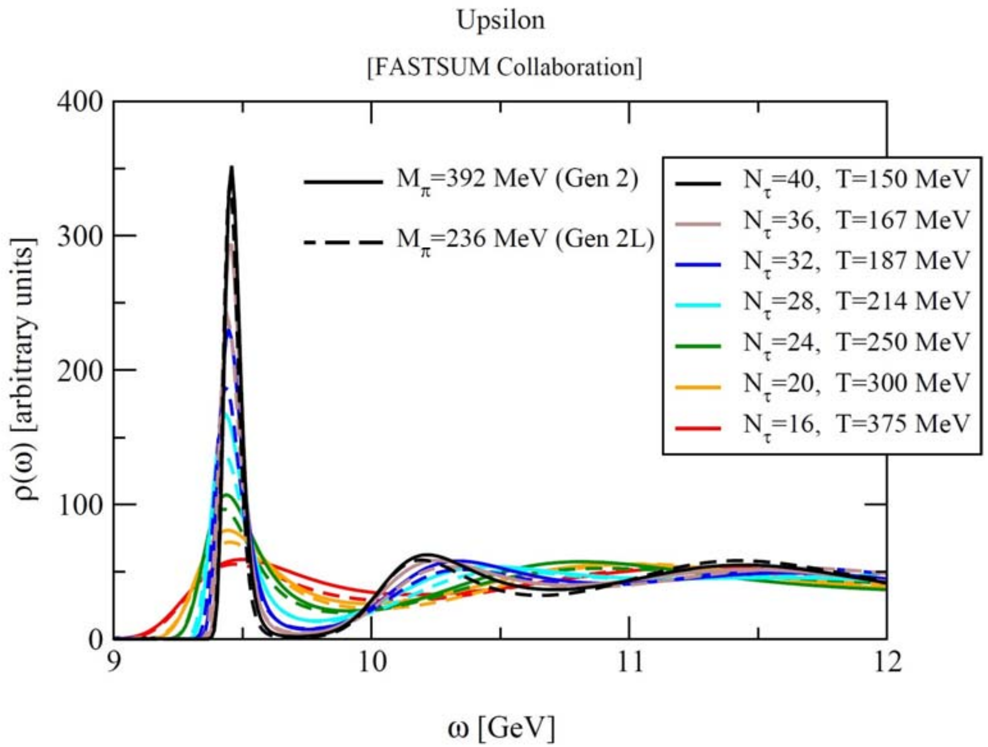
S. Kim *et al.*, PhysRevD.91.054511

Recent results: bottomonium spectral functions

- FASTSUM Collaboration, 2+1 flavor NRQCD, anisotropic a_σ/a_τ = 3.5, MEM

Samuel Offler's talk@Lattice 2019





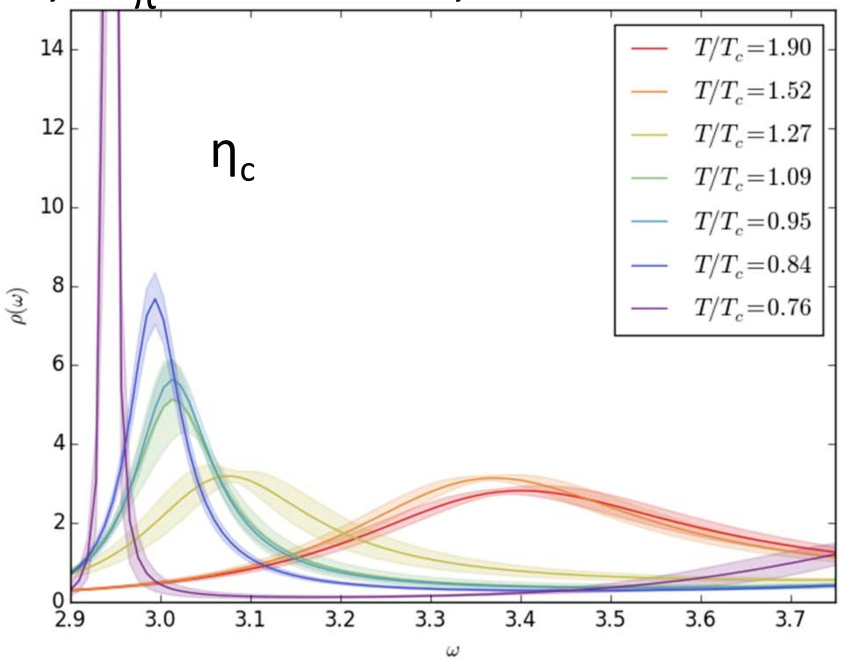
N_σ = 24
a^{-1}_τ = 5.63(4) GeV
$T_c \simeq$ 185 MeV

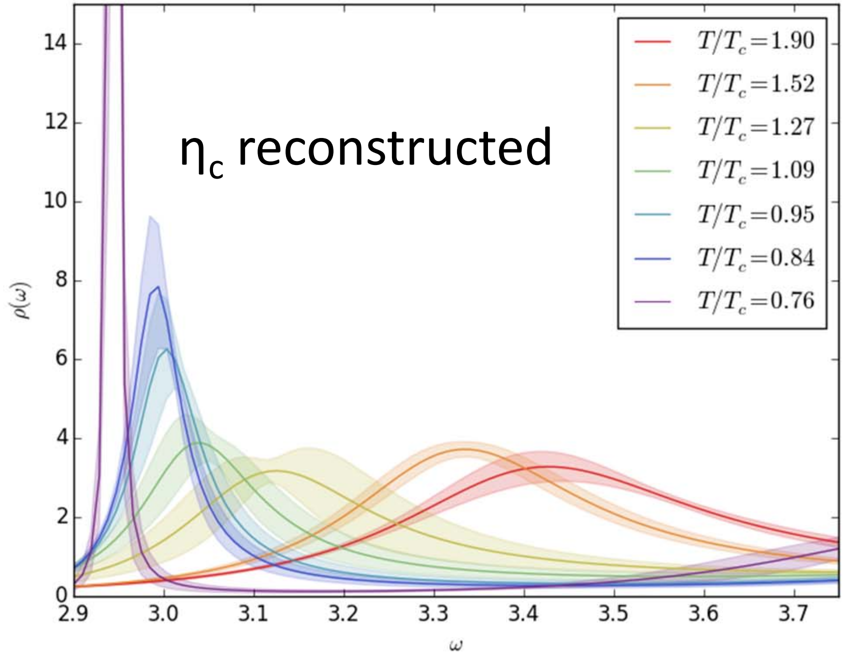
Virtually no effect of pion mass in the Υ spectral function
Survival of Υ(1S) up to the highest temperature (1.9T_c)

Recent results: open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic a_σ/a_τ = 3.5, m_π = 390 MeV, BR method

R. Quinn *et al.*, arXiv:1903.11006 [hep-lat]





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Shifting and broadening of the peak in spectral functions from the thermal and reconstructed correlators are consistent with each other up to 1.9T_c.
→ An effect of the limited number of temporal points available

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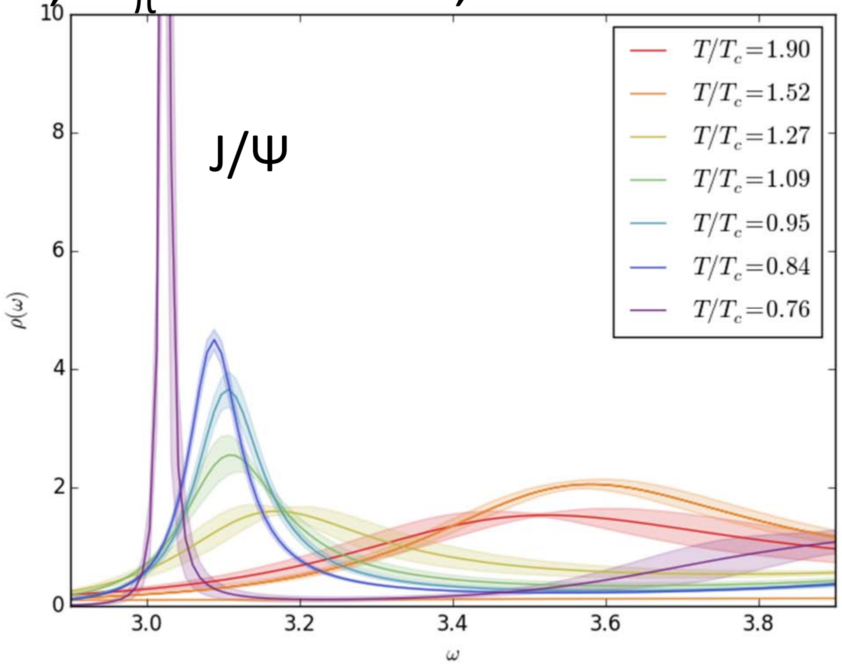

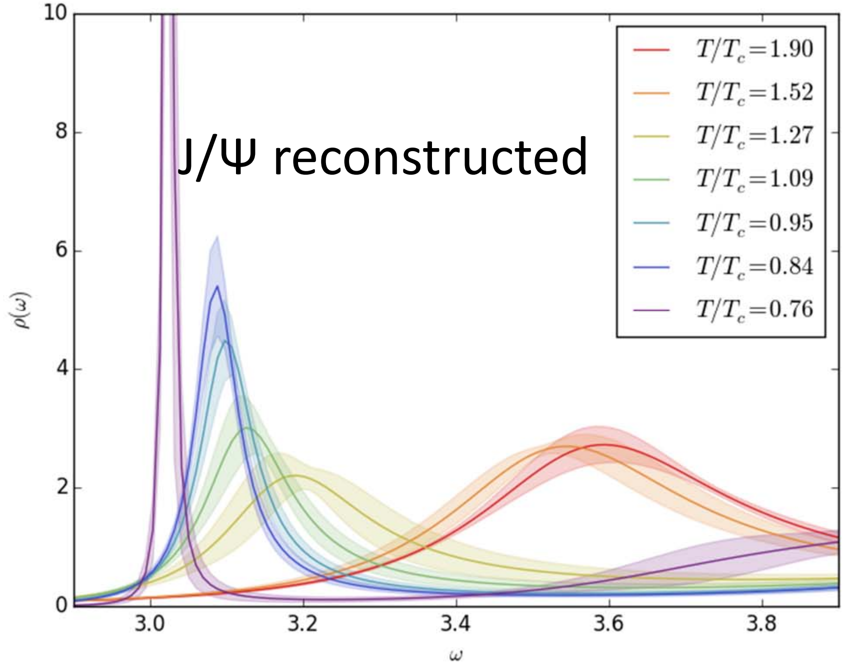

N_σ = 24
a^{-1}_τ = 5.63(4) GeV
$T_c \simeq$ 185 MeV

Shifting and broadening of the peak in spectral functions from the thermal and reconstructed correlators are consistent with each other up to 1.5T_c.
→ An effect of the limited number of temporal points available

Recent results: open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic $a_\sigma/a_\tau = 3.5$, m_π = 390 MeV, BR method

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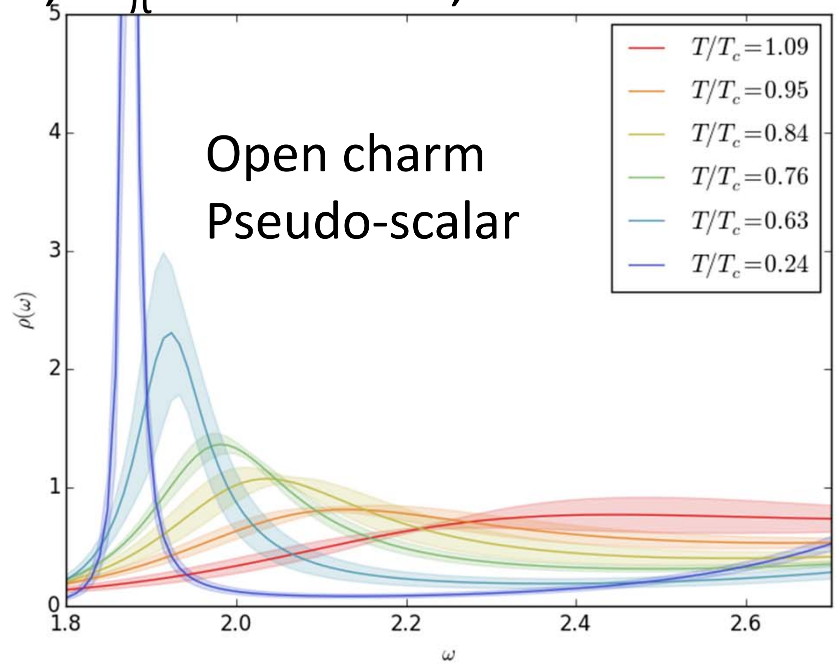
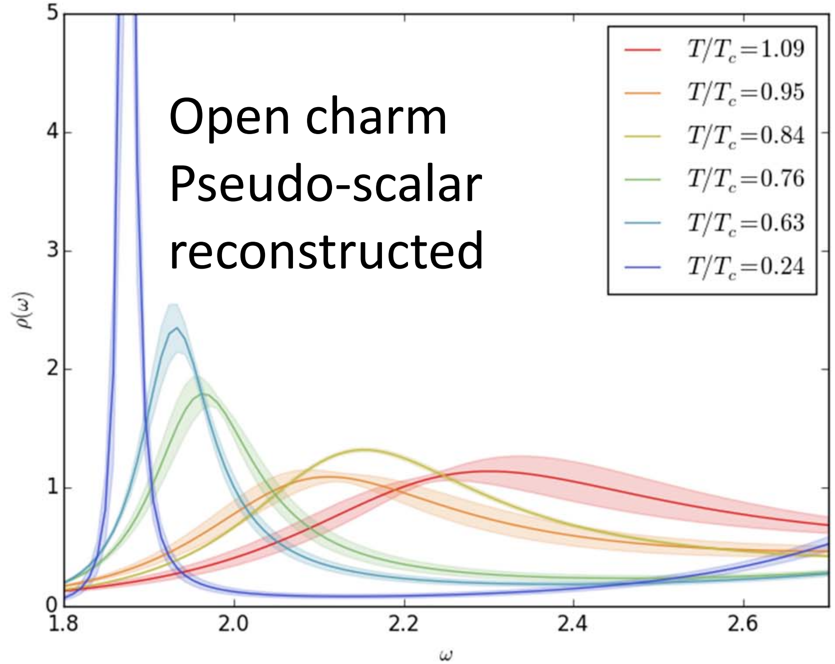




$N_\sigma = 24$
$a^{-1}_\tau = 5.63(4)$ GeV
$T_c \simeq 185$ MeV

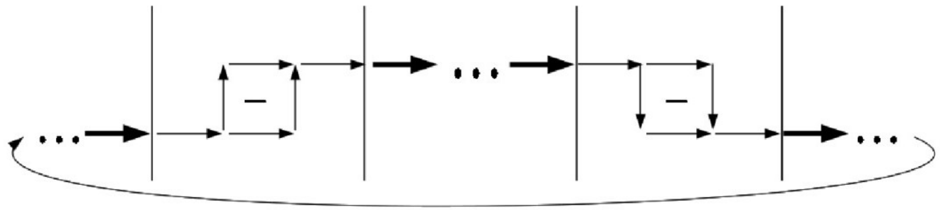
Spectral functions from the thermal and reconstructed correlators are different already below T_c.
→ No signal of survival of open charm

Heavy quark momentum diffusion coefficient

- Quenched QCD, Clover Wilson, continuum extrapolated
- Heavy quark effective theory → color electric correlator

S. Caron-Huot, M. Laine and G.D. Moore, JHEP 04 (2009) 053

$$G_{\rm E}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re\,Tr}\left[U(\beta;\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}\left[U(\beta;0)\right]\right\rangle}\,, \quad \beta \equiv \frac{1}{T}$$



$$G_{\rm E}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\rm E}(\omega)\,\frac{\cosh[\omega(\frac{\beta}{2}-\tau)]}{\sinh[\frac{\omega\beta}{2}]}\,.$$

- Heavy quark momentum diffusion coefficient

$$\kappa \equiv \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega}$$

Heavy quark momentum diffusion coefficient

A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus and HO, PRD 92 (2015) no.11, 116003

- Modeling the spectral function
  - IR part $(\omega \ll T)$

$$\kappa \equiv \lim_{\omega \to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \quad \Rightarrow \quad \boxed{\phi_{\mathrm{IR}}(\omega) \equiv \frac{\kappa\omega}{2T}}$$

  - UV part $(\omega \gg T)$

  : computed from perturbation theory, two different ansatzes

$$\boxed{\phi_{\mathrm{UV}}^{(a)}(\omega) \qquad \phi_{\mathrm{UV}}^{(b)}(\omega)}$$

  - Interpolations

  : two different interpolations, three different models

$$\rho_{\mathrm{E}}^{(1\mu i)}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{\max}} c_n e_n^{(\mu)}(y)\right]\left[\phi_{\mathrm{IR}}(\omega) + \phi_{\mathrm{UV}}^{(i)}(\omega)\right] \qquad \rho_{\mathrm{E}}^{(2\mu i)}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{\max}} c_n e_n^{(\mu)}(y)\right]\sqrt{\left[\phi_{\mathrm{IR}}(\omega)\right]^2 + \left[\phi_{\mathrm{UV}}^{(i)}(\omega)\right]^2}$$

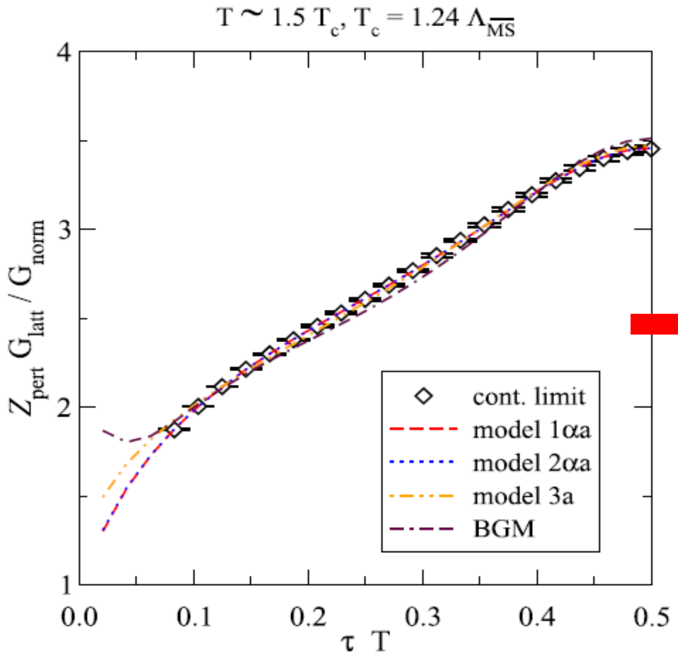
$$\rho_{\mathrm{E}}^{(3i)}(\omega) \equiv \max\left[\phi_{\mathrm{IR}}(\omega), c\,\phi_{\mathrm{UV}}^{(i)}(\omega)\right]$$

Heavy quark momentum diffusion coefficient

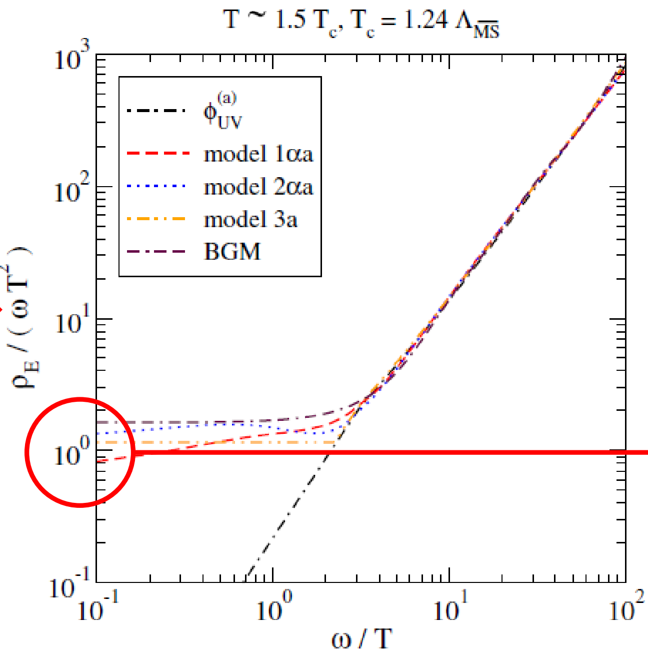
$1.5T_c$

A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus and HO, PRD 92 (2015) no.11, 116003

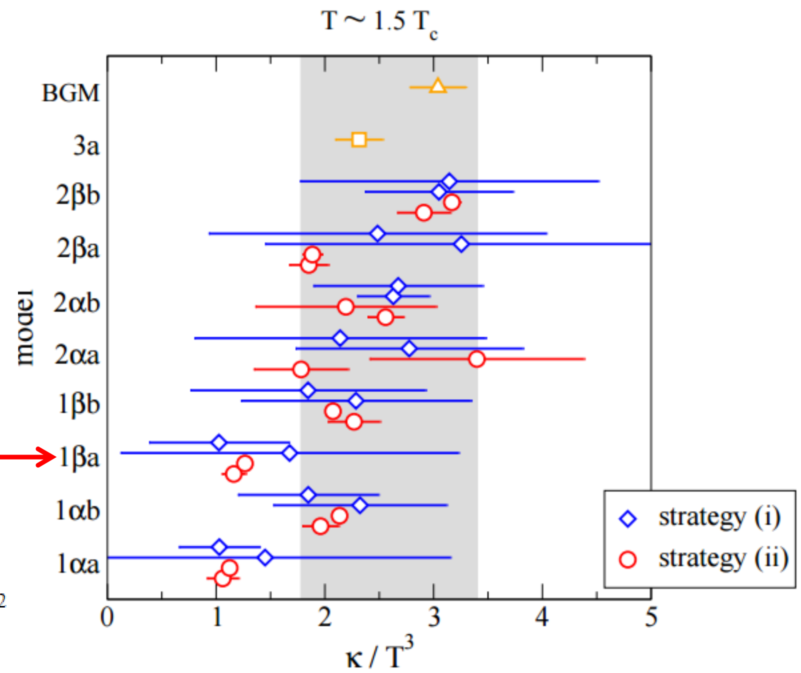
Correlator in the cont. limit



Perturbatively constrained model spectral functions



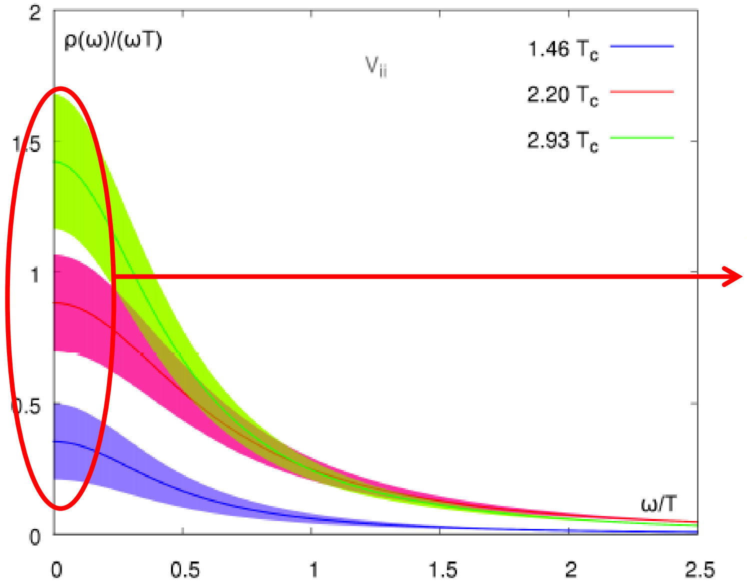
Heavy quark momentum diffusion coeff.

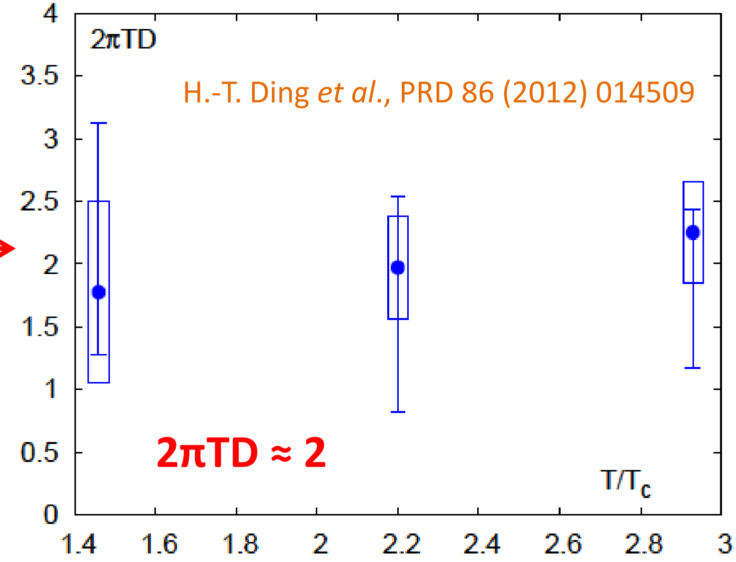


$$D = \frac{2T^2}{\kappa} \rightarrow \mathbf{2\pi TD \approx 3.7 - 7.0}$$

Heavy quark momentum diffusion coefficient

- Perturbative estimate
  $2\pi DT \approx 71.2$ in LO
  Moore and Teaney, PRD 71 (2005) 064904
  $2\pi DT \approx 8.4$ in NLO
  Caron-Hout and Moore, PRL 100 (2008) 052301
- Strong coupling limit
  $2\pi DT \approx 1$
  Kovtun, Son and Starinets, JHEP 0310 (2004) 064





$$D = \frac{2T^2}{\kappa}$$ → **2πTD ≈ 3.7—7.0**

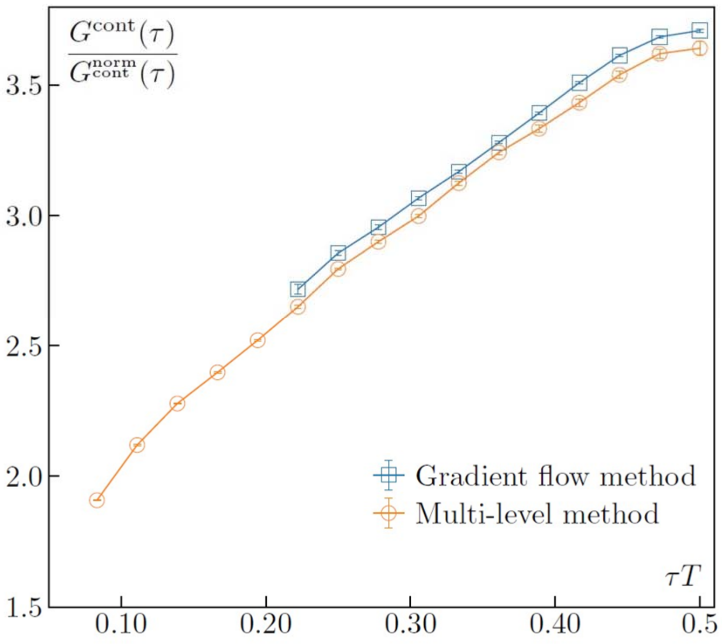
A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus and HO, PRD 92 (2015) no.11, 116003

Heavy quark momentum diffusion coefficient: update

- Improvement with gradient flow

L. Altenkort, A. M. Eller, O. Kaczmarek, L. Mazur, G. D. Moore, H.-T. Shu, arXiv:2009.13553 [hep-lat]

1.5T$_c$ Correlator in the cont. limit



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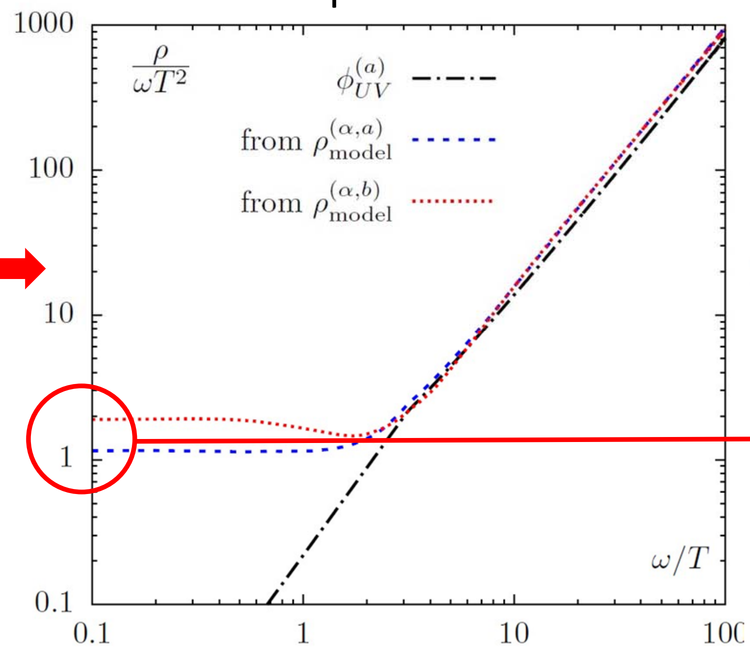
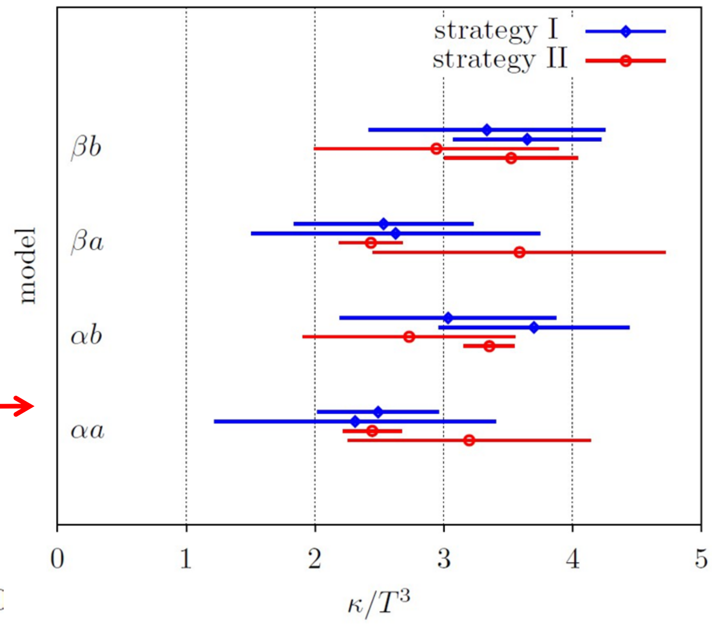


Heavy quark momentum diffusion coeff.



$$D = \frac{2T^2}{\kappa}$$ → **2πTD ≈ 1.7－2.7**

Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated, pseudo-scalar

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

$$\rho^{pert}(\omega) = \underbrace{\rho^{vac}(\omega)}_{\text{Vacuum asymptotics}} \theta(\omega - \omega^{match}) + A^{match} \underbrace{\rho^{NRQCD}(\omega)}_{\text{pNRQCD}} \theta(\omega^{match} - \omega) \underbrace{\Phi(\omega)}_{\text{Suppression}}$$

- High energy ρ^{vac}: Vacuum asymptotics Burnier, Laine, Eur.Phys.J.C 72 (2012) 1902
- Threshold region ρ^{NRQCD}: pNRQCD Laine, JHEP 0705:028,2007
- Suppressed at low energy

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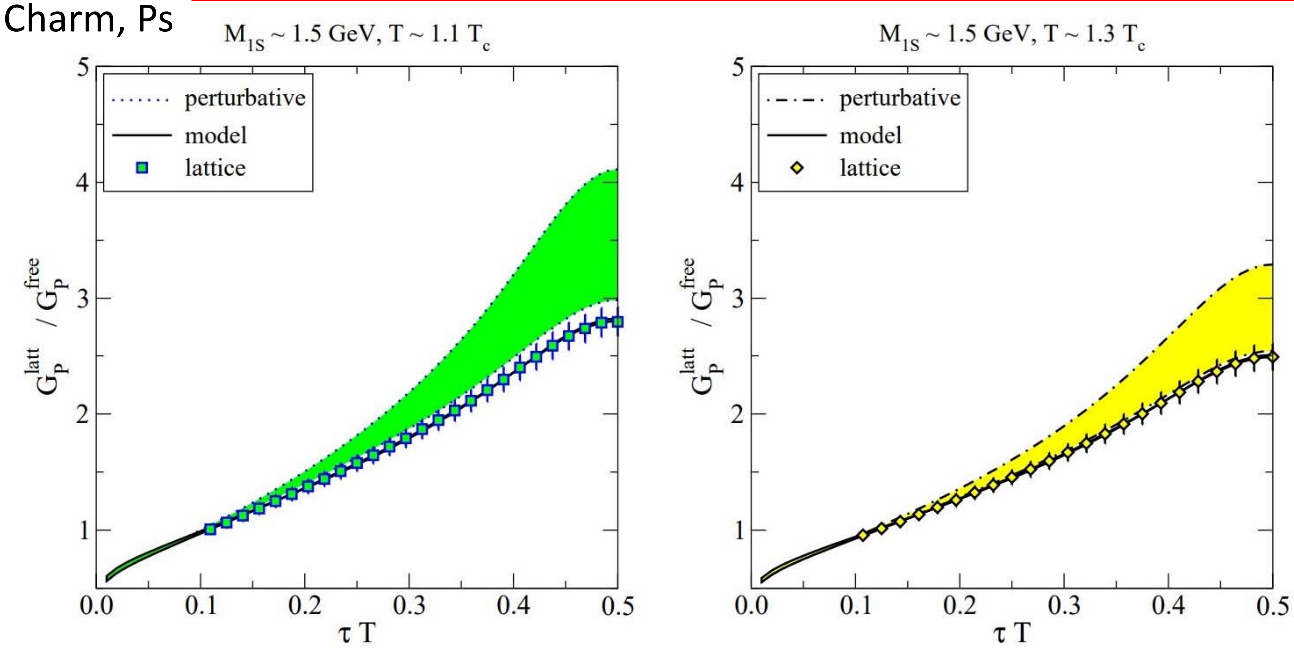
Charm, Ps



$$\rho^{\mathbf{mod}}(\omega) = \mathbf{A}\rho^{\mathbf{pert}}(\omega - \mathbf{B})$$

- **Perturbative spectral function describes lattice data perfectly.**

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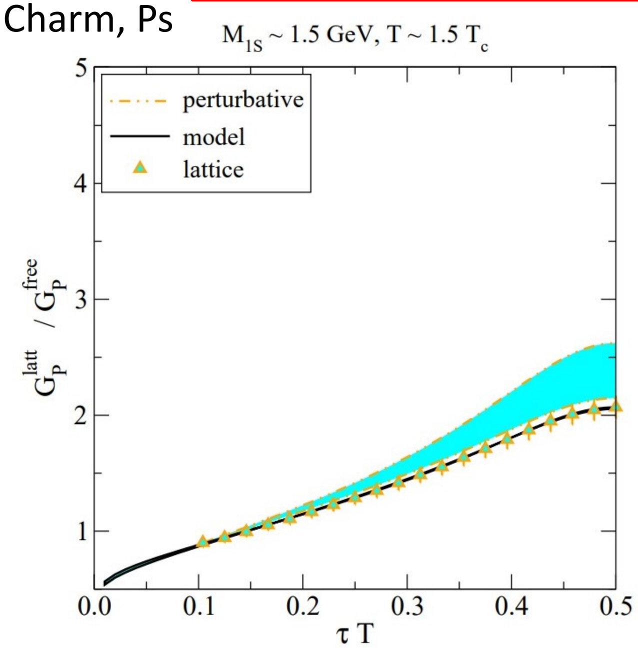
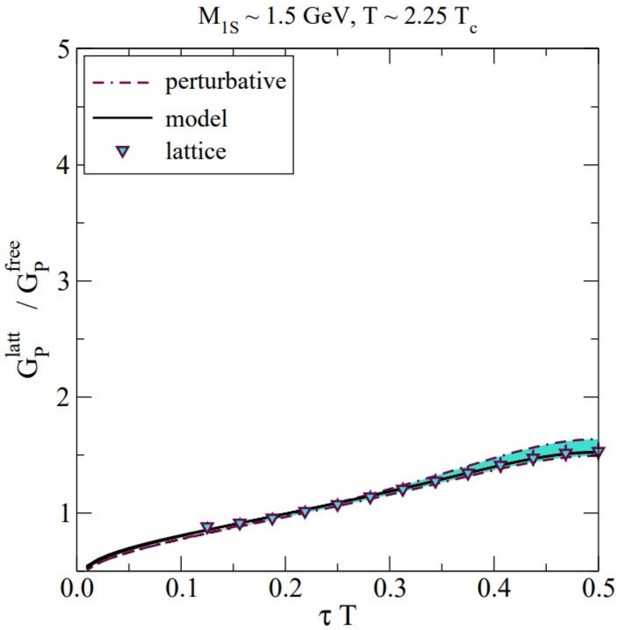
Charm, Ps





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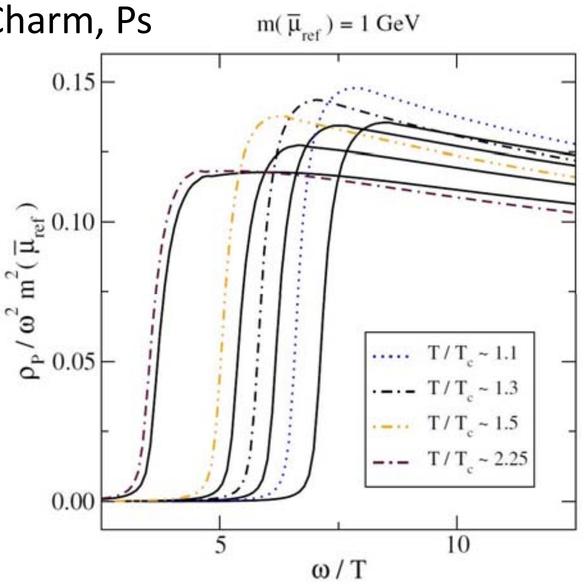
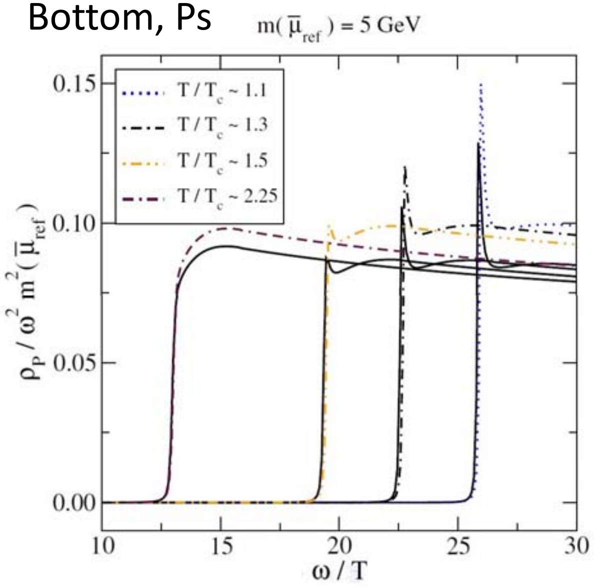
Charm, Ps



Bottom, Ps



$$\rho^{\mathbf{mod}}(\omega) = \mathbf{A}\rho^{\mathbf{pert}}(\omega - \mathbf{B})$$

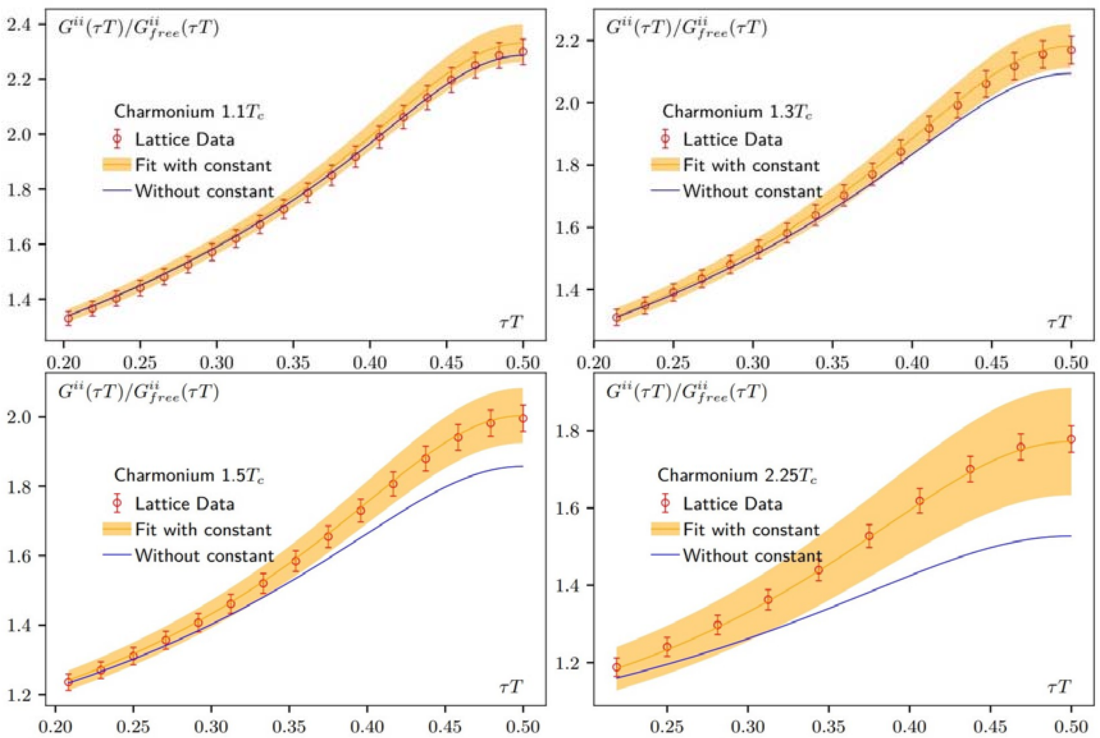
- **Perturbative spectral function describes lattice data perfectly.**
- **No resonance peak needed for charm.**
- **A resonance peak may be well present for bottom at T $\lesssim$ 1.5 Tc.**

Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H.-T. Shu *et al.* in preparation

$$\rho^{ii}(\omega) = A\rho^{pert}(\omega - B) + \rho^{trans}(\omega)$$



Charm, V

- Fit parameters: A, B, ρ^{trans}
- ρ^{trans}: constant

- **Perturbative spectral function describes lattice data perfectly.**

Fit to a perturbative spectral function

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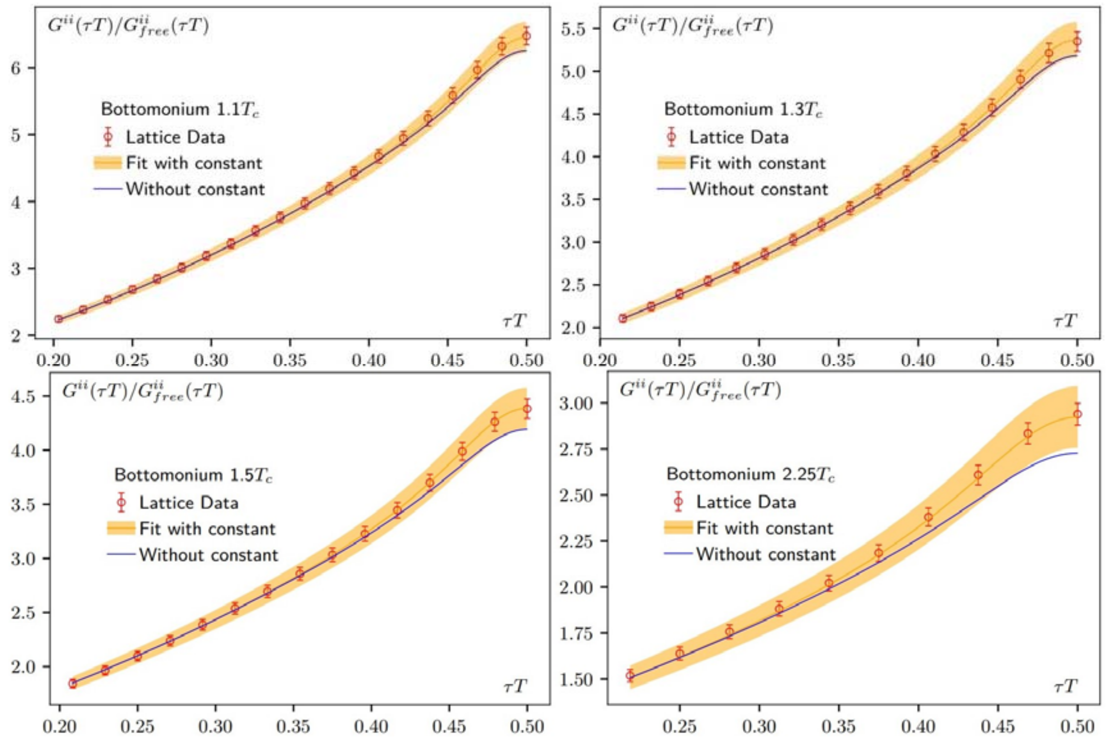

Bottom, V

- Fit parameters: A, B, ρ^{trans}
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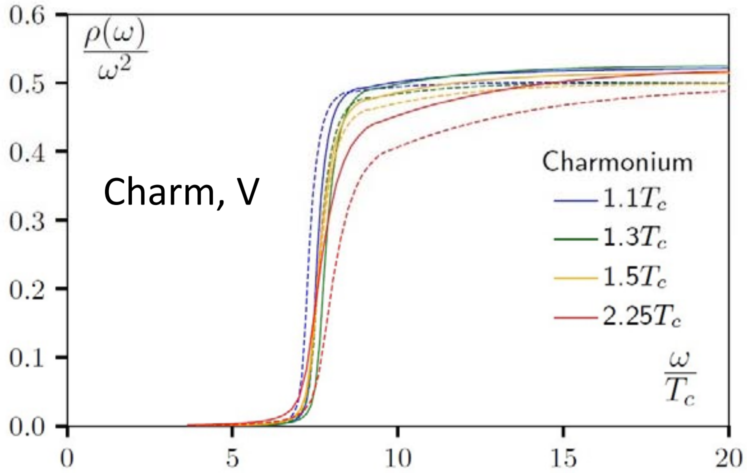
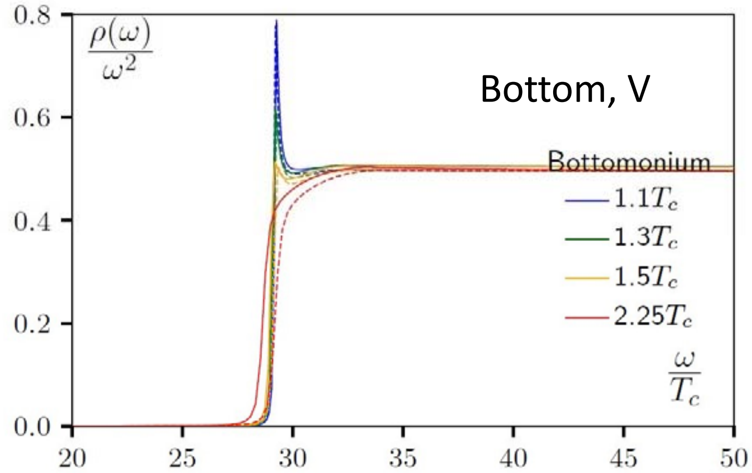




- Fit parameters: A, B, ρ^{trans}
- ρ^{trans}: constant

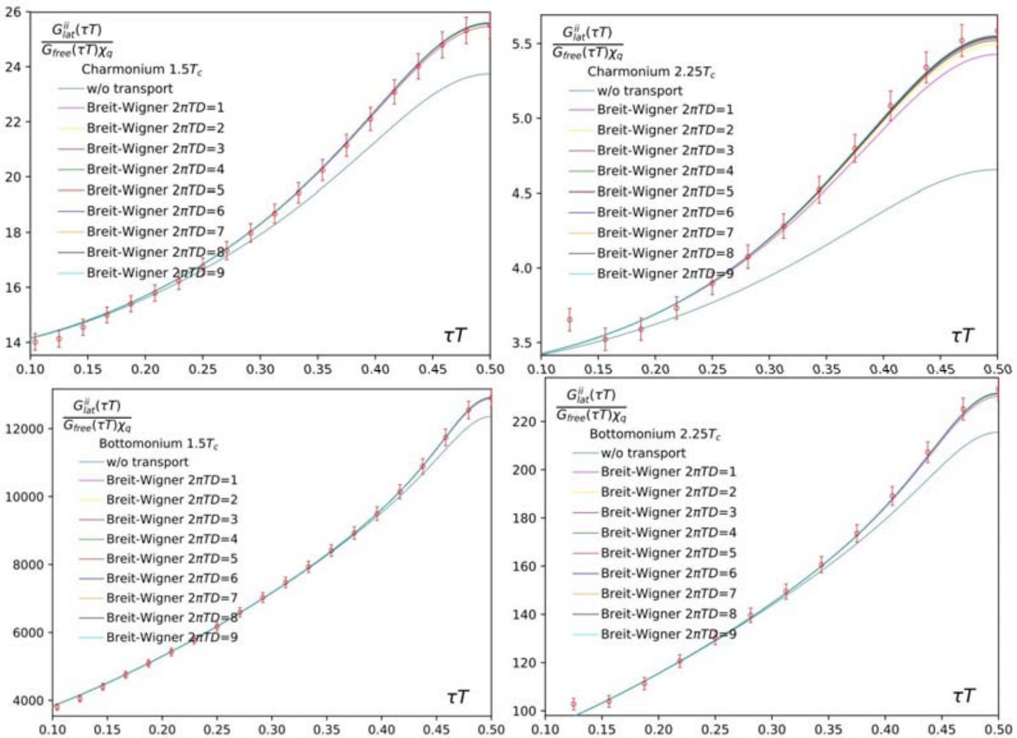
- **Perturbative spectral function describes lattice data perfectly.**
- **No resonance peak needed for charm.**
- **A resonance peak may be well present for bottom at T $\lesssim$ 1.5 Tc.**

Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H.-T. Shu *et al*. in preparation

$$\rho^{ii}(\omega) = A\rho^{pert}(\omega - B) + \rho^{trans}(\omega)$$



$$\boxed{\rho^{trans}(\omega) = 6D\chi_q \frac{\omega\eta_D^2}{\omega^2+\eta_D^2}\frac{1}{\cosh\left(\frac{\omega}{2\pi T}\right)}}$$

- A, B fixed
- $2\pi TD$ varied from 1 to 9

Possible lower bound at $2\pi TD$ = 2?

Summary

- Heavy quarks are important probes to investigate quark-gluon-plasma
- Spectral functions contain all information about in-medium properties of heavy quarks
- There are some difficulties to study the spectral functions on the lattice
- A couple of approaches to overcome the difficulties and study the spectral functions
- Further studies are needed for better understanding of in-medium properties of heavy quarks