Geometry Explorations: What Changes? What Stays the Same?

Instructions for Students

In this session, you will draw figures, take measurements, and compare results. Your goal is not to prove anything formally, but to observe carefully and look for patterns.

For every activity: - Change positions, dimensions, angles and shapes deliberately. - Record measurements clearly. - Compare results across different trials. - Ask yourself: *What changes? What stays the same?*

Exploration 1: Distances Inside a Triangle

Construction 1. Draw an equilateral triangle of side 10 cm. 2. Mark 6 different points inside the triangle. 3. From each point, draw perpendiculars to all three sides. 4. Measure the three perpendicular distances.

Record - Write down the three distances for each point. - Compute their sum.

Explore - How does the sum change as the point moves? - Compare results for different interior points. - What happens when the point is close to a side or a vertex? - What features of the triangle might matter here?

Extension - Test points outside the triangle

Exploration 2: Squares Built on a Quadrilateral

Construction 1. Draw any quadrilateral. 2. Construct a square externally on each side. 3. Mark the center of each square. 4. Join the centers of opposite squares.

Record - Measure the lengths of the two segments. - Measure the angle between them.

Explore - Compare the two segment lengths. - How does the angle behave? - Try this with at least three very different quadrilaterals. - Which changes seem to matter? Which do not?

Extension - Replace squares with other regular polygons.

Exploration 3: Chords Through a Point in a Circle

Construction 1. Draw a circle and a chord AB. 2. Mark the midpoint M of AB. 3. Through M, draw two other chords at different angles. 4. Mark all intersection points clearly.

Record - Measure the two segments formed on chord AB.

Explore - How do the two segments compare? - Does changing the angles affect the result? - What role does the midpoint appear to play?

Extension - Will the result hold true if M is not the midpoint.

Exploration 4: Distances from a Point Inside a Rectangle

Construction 1. Draw a rectangle ABCD. 2. Choose any point O inside the rectangle. 3. Measure the distances OA, OB, OC, and OD.

Record - Compute $OA^2 + OC^2$. - Compute $OB^2 + OD^2$.

Explore - Compare the two values. - Move point O and repeat. - Does the shape of the rectangle affect the result? - What seems stable across trials?

Extension - Explore whether the result holds for other quadrilaterals.

Exploration 5: Building New Triangles from a Triangle

Construction 1. Draw any triangle. 2. Construct equilateral triangles externally on each side. 3. Find the centroid of each equilateral triangle. 4. Join the three centroids.

Record - Measure the sides and angles of the new triangle.

Explore - What kind of triangle is formed? - Does the result depend on the original triangle? - Try very different shapes of triangles.

Extension - Construct triangles internally instead of externally.

Reflection
- Which quantities stayed the same even when the figure changed?
- Which changes affected the outcome?
- What kinds of constructions seem to create stability?