Tutorial : Flavour and QCD.

1 a) How the coupling constant + mass change with the scale.

b)
$$\frac{dg}{d\ln\mu} = \beta(g)$$

$$\beta(g) = -\beta_0\frac{g^3}{16\pi^2} - \beta_1\frac{g^5}{(16\pi^2)^2}$$

$$\alpha_s = \frac{g^2}{4\pi} \qquad d\alpha_s = \frac{2g}{4\pi}\,dg$$

$$\frac{d\alpha_s}{d\ln\mu} = \frac{2g}{4\pi}\,\beta(g)$$

$$= \frac{g}{2\pi}\left(-\beta_0\frac{g^3}{16\pi^2} - \beta_1\frac{g^5}{(16\pi^2)^2}\right)$$

$$= -\beta_0\frac{\alpha_s^2}{2\pi} - \beta_1\frac{\alpha_s^3}{8\pi^2}$$

c)
$$\frac{dm}{d\alpha_s} = \frac{dm}{d\ln\mu}\cdot\frac{d\ln\mu}{d\alpha_s}$$

$$= -\frac{\gamma_m m}{\beta}.$$

$$\frac{1}{m}\,dm = \left(-\frac{\gamma_m}{\beta}\right)d\alpha_s$$

$$\delta_{O_{4F}} = \frac{1}{\varepsilon}\lambda_1\frac{\alpha_s}{4\pi} = \frac{1}{\varepsilon}Z'_{O_{4F}}$$

2.
$$\gamma_{O_{4F}} = -2g^2\frac{d(Z'_{O_{4F}}+2Z'_2)}{dg^2}$$

See Muta QCD book or Buras notes. hep-ph/9806471

$$= (2\lambda_1 - 16/3)\frac{\alpha_s}{4\pi}$$

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)O_{4F}^R = -\gamma_{O_{4F}}O_{4F}^R$$

$$= -\left(2\lambda_1 - \frac{16}{3}\right)\frac{\alpha_s}{4\pi}O_{4F}^R$$

$$\frac{1}{O_{4F}^R}\,\partial O_{4F}^R = -\frac{\gamma_{O_{4F}}}{\beta(g)}\,dg.$$

3 QCD sum rules on the light cone

$$\Pi_+ = f_B m_B^2 \frac{f_+(q^2)}{m_B^2 - p_B^2} + \underbrace{\int_{s>m_B^2} ds \frac{\rho_{had}}{s - p_B^2}}$$

$$= \underbrace{\frac{1}{2} f_\pi m_b^2 \int_0^1 du \frac{\varphi(u,q^2)}{m_b^2 - u p_B^2 - \bar{u} q^2}} \qquad \underbrace{\int ds \frac{\rho_{T2}}{s - p_B^2}}$$

Quark hadron duality. $\Rightarrow$ Above s_0 $\int \rho_{had} + \int \rho_{T_2}$

$$\frac{f_B m_B^2 f_+(q^2)}{m_B^2 - p_B^2} = \frac{1}{2} f_\pi m_b^2 \int_0^{u_0} du \frac{\varphi(u,q^2)}{m_b^2 - u p_B^2 - \bar{u} q^2}$$

$$\left. \begin{aligned} u &= \frac{m_b^2 - q^2}{s - q^2} \\ du &= -\frac{u}{s - q^2} ds \end{aligned} \right\}$$

$$= \frac{1}{2} f_\pi m_b^2 \int_0^{s_0} ds \frac{u}{s - q^2} \cdot \frac{\varphi(u,q^2)}{(m_b^2 - q^2) - u(p_B^2 - q^2)}$$

$$= \frac{1}{2} f_\pi m_b^2 \int_0^{s_0} ds \frac{\cancel{m_b^2 - q^2}}{(s-q^2)^2} \cdot \frac{\varphi(u,q^2)}{\cancel{(m_b^2 - q^2)}\left(1 - \frac{p_B^2 - q^2}{s - q^2}\right)}$$

$$s - \cancel{q^2} - p_B^2 + \cancel{q^2}$$

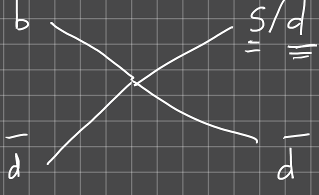
$$= \frac{1}{2} f_\pi m_b^2 \int_{m_b^2}^{s_0} ds \frac{\varphi(u,q^2)}{(s-q^2)(s-p_B^2)}$$

$$f_B m_B^2 f_+(q^2) e^{-m_B^2/M^2} = \frac{1}{2} f_\pi m_b^2 \int_{m_b^2}^{s_0} ds \frac{\varphi_2}{s - q^2} e^{-s/M^2}$$

$$f_+(q^2) = \frac{f_\pi m_b^2}{f_B m_B^2} \int_{m_b^2}^{s_0} ds \frac{\varphi(u,q^2)}{s - q^2} e^{-(s - m_B^2)/\underline{M^2}}$$

4.

$$H_{eff} = -\frac{G_F}{\sqrt{2}} \left(\lambda_t^{(d)} H_{eff}^{(t)} + \underbrace{\lambda_u^{(d)} H_{eff}^{(u)}} \right) + h.c.$$



$$\to \lambda_q^{(d)} = V_{qd}^\dagger V_{qb}$$

$V_{ud}^\dagger V_{ub}$ $\qquad$ $\underline{V_{us}^\dagger V_{ub}}$

$V_{td}^\dagger V_{tb}$ $\qquad$ $V_{ts}^\dagger V_{tb}$