ICTS-RRI Math Circle, Saturday 25 November, 2023

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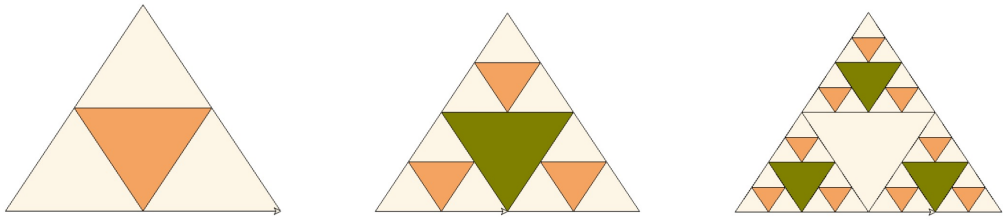
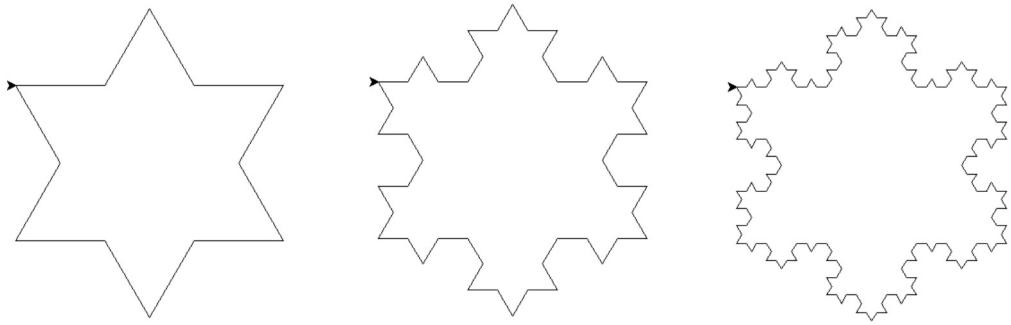
This session is going to explore a topic deeply connected to how people generate images of Fractals. As an example, there are two types of fractal images shown below generated by a Python code.





Lets discuss linguistics! How would you understand the following sentence?

"Bulldogs bulldogs bulldogs fight fight fight"

Such sentences seem confusing in the beginning but they are actually grammatically correct! And so is the following quote

"The dog the man the maid married owned died."

Such sentences use the technique of center-embedding. Lets learn how to make sense of such statements. These sentences make use of *relative pronoun* or *relative adverbs*. Lets try to understand the first sentence. To distinguish between the different uses of 'bulldogs' we color code them by imagining the first set of bulldogs to be red, the second-brown and the third-white.

Following our quote, let's add the center-embedding to the first set of bulldogs: Bulldogs [bulldogs fight] fight. Now let's use our color code and a *relative pronouns*: The (red) bulldogs [that the (brown) bulldogs fight] fight.

It's understood here that the brown bulldogs fight the red bulldogs specifically. The red bulldogs, as far as we know, don't change their character. They continue to fight generally.

Cool! Let's use the third set: Bulldogs [bulldogs bulldogs fight fight] fight. With the colors, we have this sentence: The (red) bulldogs [that the (brown) bulldogs that the (white) bulldogs fight fight] fight.

It's understood here that the white bulldogs fight the brown bulldogs specifically. Again, the description of the first two sets of bulldogs is not changed in any way.

In conclusion:

- The red bulldogs fight generally.
- The brown bulldogs fight the red bulldogs.
- The white bulldogs fight the brown bulldogs.
- The white bulldogs fight the brown bulldogs.

You may object that since the brown bulldogs fight the red bulldogs, the red bulldogs must fight them too. But this inference is not necessarily true: The red bulldogs might just keep fighting generally or among themselves, without paying any attention to the brown bulldogs, for all we know.

Do you see the pattern?

Group one fights generally, group two fights group one, group three fights group two, and so on. We can construct a recursive center-embedded sentence consisting of any number of bulldog groups and very easily determine who fights whom! A hypothetical group four would fight group three, group five would fight group four, and so on.

Another example

In a distant planet populated with only logical beings, there is a government of 100 people who meet everyday. A fraction of them are liars (who don't lie knowingly), and are thus called Pinocchios. As the name suggests, they had long noses that could be seen by everyone else, even other Pinocchios. However, this planet has no mirrors or reflective surfaces, and every Pinocchio is unaware of their own long nose, and the social customs of the planet prevented people from discussing appearances.

Due to their logical natures, any Pinocchio cannot remain in the government *(logically, an institution that runs a country cannot be composed of liars)*, and they must resign by the end of a day if they figure out they are a Pinocchio.

One fine day, the head of the government, who is considered above all laws and social customs, looks at his government and says,

"At least one of you folks is a Pinocchio."

Many days later, almost as if a ripple effect, all the Pinocchios resign simultaneously on the same day.

Question: Why would this have happened?
*(**Hint:** Think about what happens if you're one of the government people. How would you react on hearing the statement?)*