Exercise Sheet | Flavor Physics BSM | ICTS 2022 | J. Virto

Future Flavours: Prospects for Beauty, Charm and Tau Physics

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1. Flavor symmetry:

   (a) Check explicitly that the flavor group $\mathcal{F} = U(3)^5$ is a global symmetry of the Standard Model with no Yukawa couplings.

   (b) Check explicitly that the flavor subgroup

   $$\mathcal{G} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_Y$$

   is a global symmetry of the full Standard Model.

   (c) Starting with a completely generic form for the SM Yukawa couplings, use the Flavor symmetry $\mathcal{F}$ to redefine the fermion fields such that the only change in the full SM Lagrangian is that the Yukawa couplings can be written as:

   $$\begin{aligned} y_e^{ij} &= \mathrm{diag}(y_e, y_\mu, y_\tau) \\ y_u^{ij} &= \mathrm{diag}(y_u, y_c, y_t) \\ y_d^{ij} &= V_{\mathrm{CKM}} \cdot \mathrm{diag}(y_d, y_s, y_b) \end{aligned}$$

2. Determination of the CKM Matrix:

   Consider the CKM Matrix in the Wolfenstein parametrization:

   $$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,.$$

   (a) Determine the Wolfenstein parameters $\{\lambda, A, \rho, \eta\}$ from:

   $$\begin{aligned} \Gamma(K^+ \to \mu^+\nu_\mu)^{\mathrm{exp}} &= 3.3793(79)\cdot 10^{-8}\,\mathrm{eV} \\ \Gamma(B^+ \to \tau^+\nu_\tau)^{\mathrm{exp}} &= 4.38(96)\cdot 10^{-8}\,\mathrm{eV} \\ \Delta M_{B_d}^{\mathrm{exp}} &= 3.333(13)\cdot 10^{-10}\,\mathrm{MeV} \\ \Delta M_{B_s}^{\mathrm{exp}} &= 1.1688(14)\cdot 10^{-8}\,\mathrm{MeV} \end{aligned}$$

using:

$$\Gamma(P \to \ell\nu) = |V_{uq}|^2 \frac{f_P^2 m_P m_\ell^2}{16\pi v^4}\Big(1 - \frac{m_\ell^2}{m_P^2}\Big)^2 (1+\delta_P)$$

$$\Delta M_{B_q} = |V_{tb}V_{tq}|^2 \frac{m_{B_q} f_{B_q}^2 m_W^2}{12\pi^2 v^4} B_1^q S_1(m_b)$$

(with $P = \{K, B\}$ and $q = \{d, s, b\}$ depending on the case), and the numerical parameters given in the table at the end of the exercise sheet.

Advise: Determine first the quantities $\{|V_{us}|, |V_{ub}|, |V_{tb}V_{td}|, |V_{tb}V_{ts}|\}$, and then use the Wolfesntein parametrization to order $\mathcal{O}(\lambda^4)$ to determine $\{\lambda, A, \rho, \eta\}$. You will need a computer.

(b) Consider now a BSM contribution in the Low-Energy EFT given by:

$$\mathcal{L}_{\rm BSM} = c\,(\bar{b}\gamma^\mu P_R u)(\bar{\nu}_\tau \gamma_\mu \tau) + \text{h.c.}$$

How is the determination of the CKM matrix modified as a function of the Wilson coefficient c?

3. Consider the following effective Lagrangian relevant for $b \to s\ell\ell$ processes:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD+QED} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{\alpha}{4\pi}\big(C_9 \mathcal{O}_9 + C_{10}\mathcal{O}_{10}\big)$$

with the Low-Energy EFT effective operators

$$\mathcal{O}_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)\ ; \quad \mathcal{O}_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5 \ell)\ .$$

Consider the process $\bar{B}_s(p) \to \ell^+(p_+)\ell^-(p_-)$, to leading order in G_F and α.

(a) Show that the contribution from the operator $\mathcal{O}_9$ to $\mathcal{A}(\bar{B}_s \to \ell^+\ell^-)$ vanishes.

(b) Calculate the contribution from the operator $\mathcal{O}_{10}$ to $\mathcal{A}(\bar{B}_s \to \ell^+\ell^-)$. Use the following expression for the relevant hadronic matrix element:

$$\langle 0|\bar{s}\gamma_\mu P_L b|\bar{B}_s(p)\rangle = -\frac{1}{2} f_{B_s} p^\mu\ .$$

(c) Calculate the unpolarized decay rate, using the formula (*e.g. see Peskin*)

$$\Gamma(P \to \bar{f}f) = \frac{\sqrt{1 - 4m_f^2/m_P^2}}{16\pi m_P}\ \overline{|\mathcal{A}|^2}$$

(d) Use the numerical values given in the table below to give a SM prediction to the branching fraction: $\mathcal{B}(\bar{B}_s \to \ell^+\ell^-)^{\rm SM} = \tau_{B_s}\Gamma(\bar{B}_s \to \ell^+\ell^-)^{\rm SM}$.

(e) Compare your SM prediction with

$$\mathcal{B}(\bar{B}_s \to \ell^+\ell^-)^{\text{exp}} = (2.93 \pm 0.35) \cdot 10^9$$

and use this to determine/constrain possible NP contributions to C_{10} (write $C_{10} = C_{10}^{\text{SM}} + C_{10}^{\text{NP}}$).

(f) Calculate C_{10}^{NP} in two models:

- Z' model with

$$\mathcal{L}_{Z'} = (\lambda_{sb}\, \bar{s}\not{Z}' P_L b + h.c.) + \lambda_\ell^V\, \bar{\ell}\not{Z}'\ell + \lambda_\ell^A\, \bar{\ell}\not{Z}'\gamma_5\ell + \frac{1}{2}M_{Z'}^2 Z'^\mu Z'_\mu\ .$$

- LQ model with

$$\mathcal{L}_\phi = \lambda_b \phi \bar{b} P_L \ell + \lambda_s \phi \bar{s} P_L \ell + h.c. + M_\phi^2 \phi^\dagger \phi$$

and constrain $M_{Z'}$ and M_ϕ if all $\lambda_i = 0.1$.

$v = 246.21965(6)$ GeV	$G_F = 1.166 \cdot 10^{-5}$ GeV^{-2}
$\alpha(m_b) = 1/132$	$m_W = 80.379(12)$ GeV
$m_\mu = 105.6583745(24)$ MeV	$m_\tau = 1.77686(12)$ GeV
$\delta_K = 0.0107(21)$	$\delta_B \simeq 0$
$m_{K^\pm} = 493.677(16)$ MeV	$m_{B^\pm} = 5.27932(14)$ GeV
$m_{B_d} = 5.27963(15)$ GeV	$m_{B_s} = 5.36689(19)$ GeV
$f_{K^\pm} = 155.7(0.3)$ MeV	$f_{B^\pm} = 190.0(1.3)$ MeV
$f_{B_d}^2 B_1^d = 0.0297(17)$ GeV^2	$f_{B_s}^2 B_1^s = 0.0432(22)$ GeV^2
$S_1(m_b) \simeq 1.9848$	$f_{B_s} = 230.3(1.3)$ MeV
$\lambda = 0.22537(46)$	$A = 0.828(21)$
$\rho = 0.194(24)$	$\eta = 0.391(48)$
$\tau_{B_s} = 1.527(11) \cdot 10^{-12} s$	$1s = 1.5 \cdot 10^{24}$ GeV^{-1}
$C_{10}^{\rm SM}(m_b) = -4.309$	

Table 1: Numerical values for the parameters of interest.