Gluon distributions in the proton

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Dipankar Chakrabarti, IIT Kanpur

Refs: DC, P. Choudhary, B. Gurjar, R Kishore, T. Maji, C. Mondal, A. Mukherjee,
PRD 108, 014009(2023);
manuscript in preparation.

Introduction

- One of the main goals of EIC is to understand the three dimensional structure of nucleons in terms of quarks and gluons as well as their spin and angular momentum distributions.
- Form factors, PDFs, GPDs, TMDs, Wigner distributions.... Encode different informations.
- Gluon distributions are not yet well understood.
- Gluon PDFs are mainly small-x dominated.
- Large uncertainty in small-x, specially for polarized pdf.
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Lattice talks

TMDs

$$f(x, p_\perp^2)$$

- TMD factorization: $\text{SIDIS} = \int TMD * Hard * FF$
- TMDs: 3D spatial structure of proton
- —> Transverse motion of partons, spin-transverse momentum correlations
- SIDIS and Drell-Yan processes are sensitive to TMDs.
- TMDS: → spin asymmetries
- Azimuthal asymmetry of unpolarised quarks in transversely polarised proton: Sivers effect. T-odd!
- Final State Interaction (FSI) in SIDIS (ISI in DY): gluon exchange between the struck quark and the remnant produces nonzero Sivers effect.

Brodsky, Hwang, Schmidt, PLB530, 99

Non-perturbative

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Gluon TMDs

- EIC can probe the gluon distributions for both unpolarised and polarized proton.
- To study gluon TMDs, high energy and/or small x are required.
- Gluon Sivers TMD: $e\ p^{\uparrow} \rightarrow eQ\bar{Q}X$, $ep \rightarrow e'D\bar{D}X$ [back to back D meson pair production], $ep^{\uparrow} \rightarrow e'J/\Psi\ X$
- TMDs are not universal [due to FSI/ISI dependence]- - Sivers TMDs for quarks in SIDIS and DY differ by an overall negative sign.
- At small x , two unpolarised gluon distributions[Weizsacker-Williams(WW) and dipole].

C.Pisano 1912.13020
D. Boer, 1601.01813

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For more discussions on gluon TMDs: see the talks by Christine Aidala And Cristian Pisano

C.Pisano 1912.13020
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Model with active gluon

- Spectator model studies provide good insight into the different partonic distributions.
- Simplified, but insightful, help to understand the proton structure.
- consider the proton as a composite state of spin 1/2 spectator+ gluon(active parton).
- $$|P;\uparrow(\downarrow)\rangle = \int \frac{\mathrm{d}^2\mathbf{p}_\perp \mathrm{d}x}{16\pi^3\sqrt{x(1-x)}} \times \Bigg[\psi^{\uparrow(\downarrow)}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp)\Bigg|+1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\Bigg\rangle + \psi^{\uparrow(\downarrow)}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp)\Bigg|+1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\Bigg\rangle$$
$$+\psi^{\uparrow(\downarrow)}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp)\Bigg|-1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\Bigg\rangle + \psi^{\uparrow(\downarrow)}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp)\Bigg|-1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\Bigg\rangle\Bigg],$$
- $\psi^{\uparrow(\downarrow)}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp)$ = LFWF corresponding to the two particle state $|\lambda_g,\lambda_X;xP^+,\mathbf{p}_\perp\rangle$

Bacchetta, Celiberto, Radici, Taels, EPJC80, 733

DC et al, PRD 108, 014009

$$\psi^{\uparrow}_{+1+\frac{1}{2}}(x, \mathbf{p}_\perp) = -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x, \mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{+1-\frac{1}{2}}(x, \mathbf{p}_\perp) = -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x, \mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{-1+\frac{1}{2}}(x, \mathbf{p}_\perp) = -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x}\varphi(x, \mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{-1-\frac{1}{2}}(x, \mathbf{p}_\perp) = 0,$$

With

$$\varphi(x, \mathbf{p}^2_\perp) = N_g \frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^b(1-x)^a \exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2 x^2}\mathbf{p}^2_\perp\right].$$

The model parameters are fitted to the unpolarised gluon PDF(NNPDF3.0 data) at $Q_0 = 2$ GeV

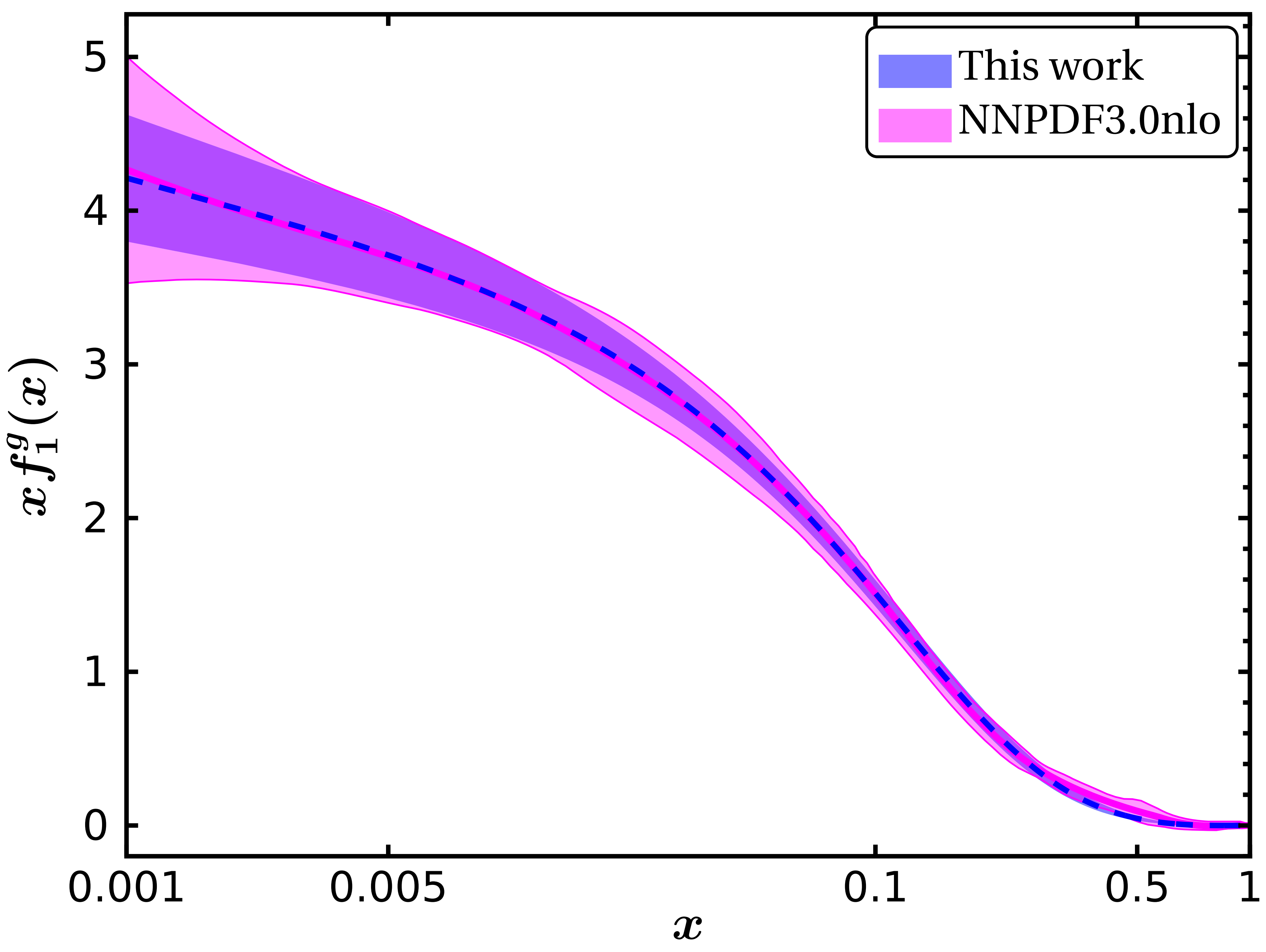
DC eta. PRD108

Fixing the parameters

- 4 parameters in the model: a, b, N_g, M_X
- N_g : fixed by normalization condition,
- Spectator mass $M_X > M$ (proton mass)
- Behaviour of the distribution is determined by a and b
- The parameters in the model are fixed by fitting the unpolarised gluon pdf $f_1^g(x)$ with NNPDF3.0 NLO data at $Q_0 = 2$ GeV.
- $f_1^g(x) = 2N_g^2 x^{2b+1}(1-x)^{2a-2}\left[\kappa^2 \frac{(1+(1-x)^2)}{\log[1/(1-x)]} + (M(1-x) - M_X)^2\right].$

Fitting the parameters

- We take 300 NNPDF3.0NLO data points in the interval $0.001 < x < 1$



Large uncertainty in small-x region
Excluded in our model.

-

(Considered 100 replica of the NNPDF data)

- Except the unpolarized gluon PDF, everything else is our model prediction.
- Average longitudinal momentum
- $\langle x \rangle_g = \int_{0.001} dx\ x\ f_1^g(x) = 0.416^{+0.048}_{-0.041}$ [lattice result: $\langle x \rangle_g = 0.427(92)$]

- C. Alexandrou et al, PRD 101, 094512

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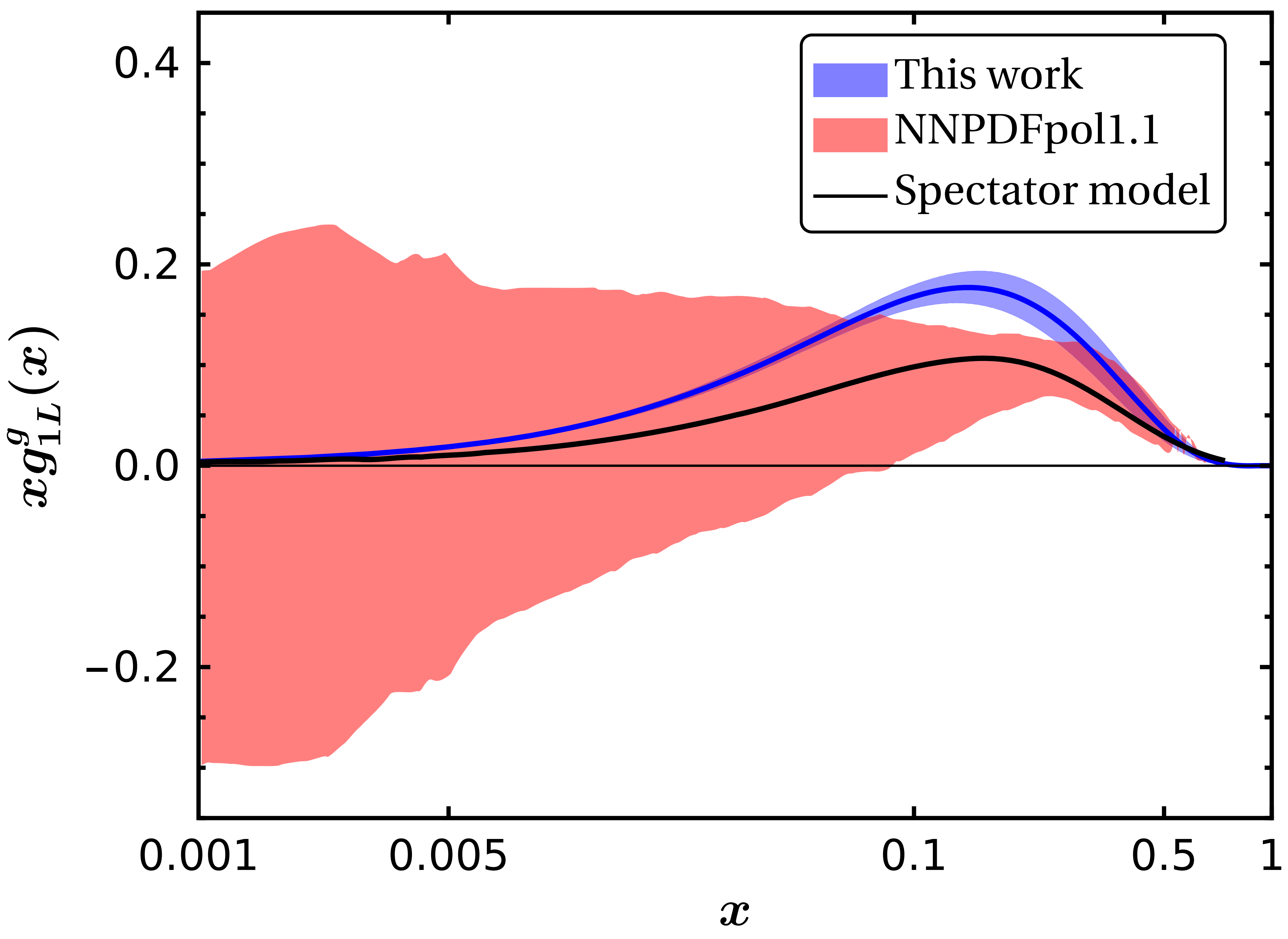
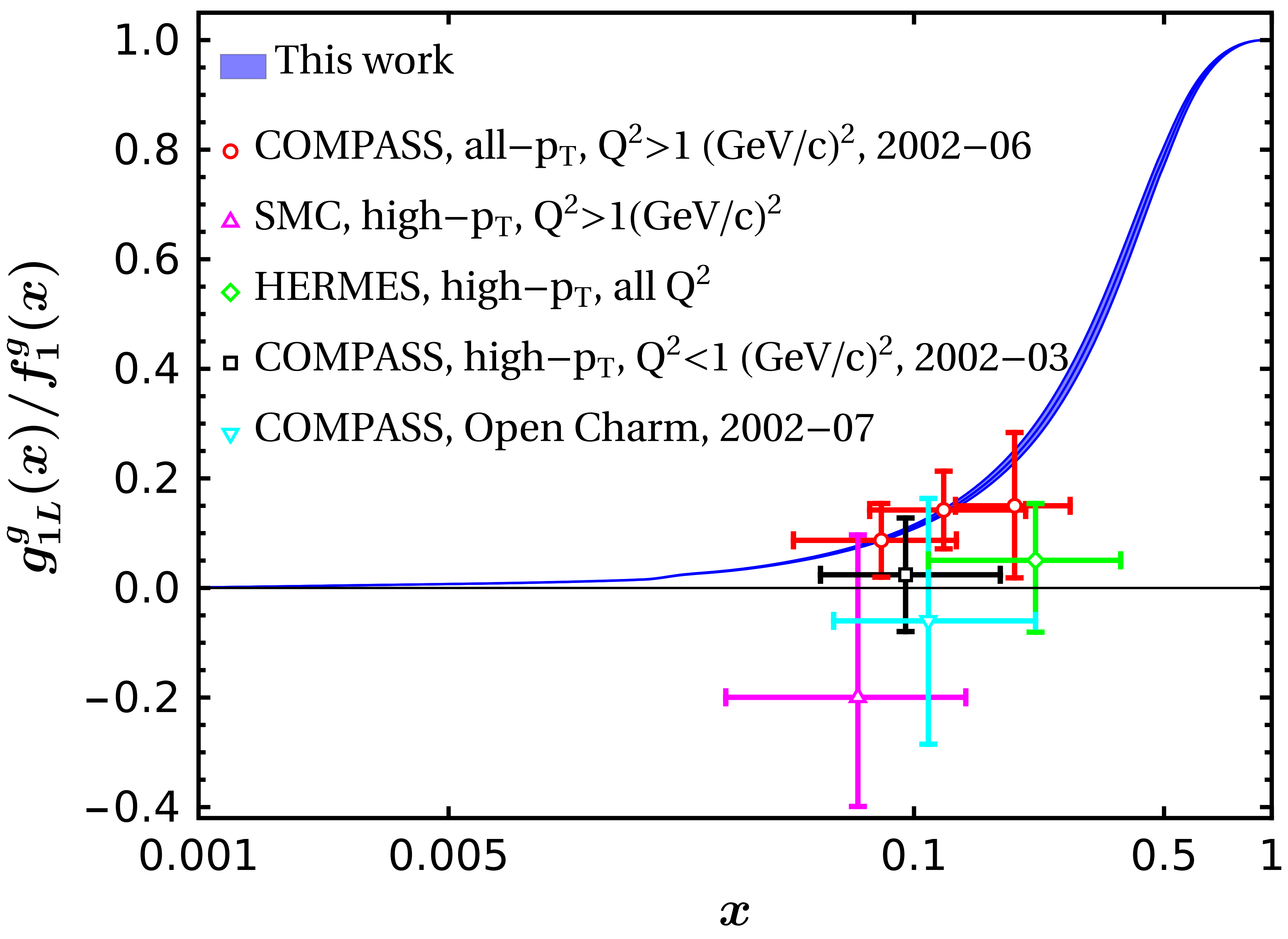
Gluon helicity pdf

Gluon helicity pdf



Spectator model: Bacchetta et al, EPJC 80, 733

Gluon helicity asymmetry

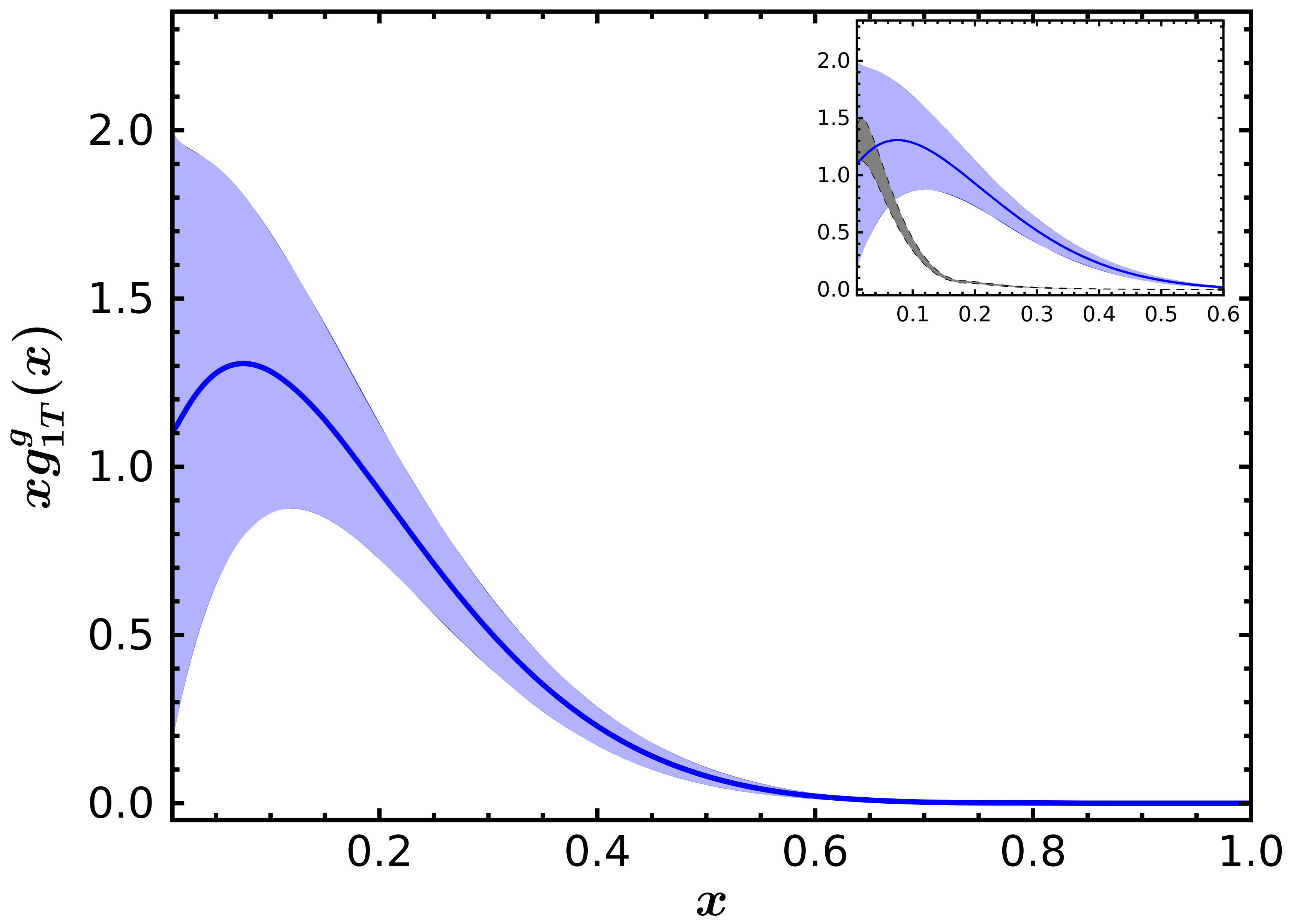


- $$\lim_{x\to 0}\frac{g_{1L}^g(x)}{f_1^g(x)} = 0, \quad \text{and} \quad \lim_{x\to 1}\frac{g_{1L}^g(x)}{f_1^g(x)} = 1.$$

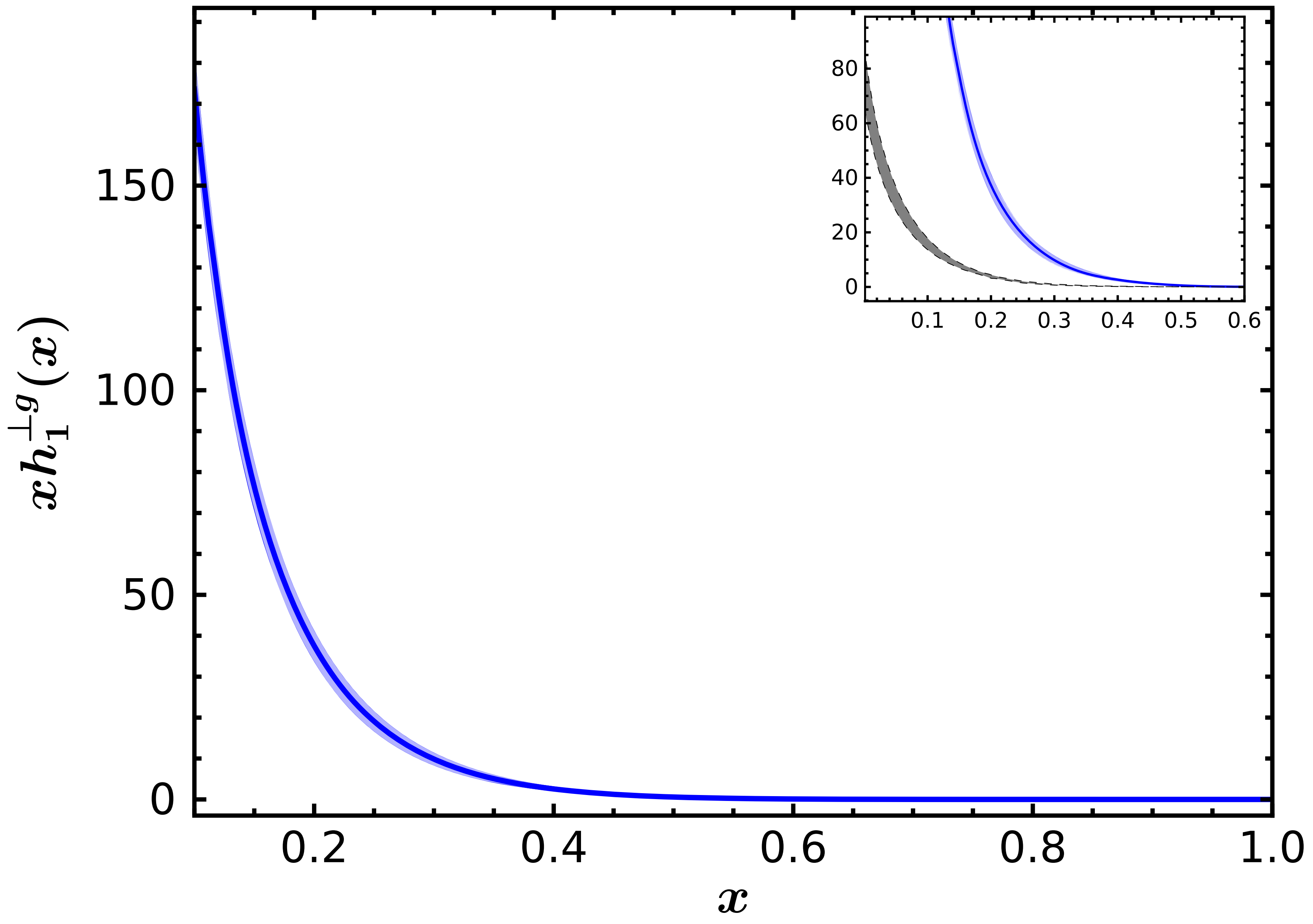
Gluon spin contribution:

	Our prediction	Comparison
$\Delta G = \int_{0.05}^{0.3} dx \Delta g(x)$	0.28	0.20 [A.Adare et al[Phenix] PRD90]
$\Delta G = \int_{0.05}^{0.2} dx \Delta g(x)$	0.22	0.23 [Nocera et al. [NNPDF] NPB886]
$\Delta G = \int_{0.05}^{1} dx \Delta g(x)$	0.32	0.19 [Florian et al, PRL113]

Gluon worm gear pdf



Gluon Boer-Mulders pdf



Inset: compared with the predictions of V.E. Lyubovitskij and I. Schmidt, PRD 103, 094017

Gluon TMDs

- The correlator for gluon TMDs in SIDIS:

$$\Phi^{g[ij]}(x, \mathbf{p}_\perp; S) = \frac{1}{xP^+} \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ik\cdot\xi} \langle P; S | F_a^{+j}(0) \mathcal{W}_{+\infty,ab}(0;\xi) F_b^{+i}(\xi) | P; S \rangle \Big|_{\xi^+=0^+},$$

- At leading twist 8 gluon TMDs: 4 are T-even and 4 are T-odd.

$(f_1^g,\ g_{1L}^g,\ g_{1T}^g,\ \text{and}\ h_1^{\perp g})$

$(f_{1T}^{\perp g}, h_{1L}^{\perp g}, h_{1T}^g, h_{1T}^{\perp g})$

Unpolarised TMD $f_1^g(x, p_\perp^2)$

- overlap representation of light front wave functions:

$$f_1^g(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big[|\psi^{\uparrow}_{+1+1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{+1-1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{-1+1/2}(x, \mathbf{p}_\perp^2)|^2\Big].$$

-

- With our wave functions we get:

$$f_1^g(x, p_\perp^2) = N^2 \frac{2}{\pi\kappa^2} \frac{ln[1/(1-x)]}{x} x^{2b}(1-x)^{2a}\Big[(M - \frac{M_X}{1-x})^2 + p_\perp^2 \frac{1+(1-x)^2}{x^2(1-x)^2}\Big] Exp[-\frac{ln[1/(1-x)]}{\kappa^2 x^2} p_\perp^2]$$

- When integrated over the transverse momentum, it reduces to the unpolarised pdf $f_1^g(x)$.

- Helicity TMD: Circularly polarized gluon in longitudinally polarized proton

$$g^g_{1L}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big[|\psi^{\uparrow}_{+1+1/2}(x,\mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{+1-1/2}(x,\mathbf{p}_\perp^2)|^2 - |\psi^{\uparrow}_{-1+1/2}(x,\mathbf{p}_\perp^2)|^2\Big]$$

$$= N_g^2\frac{2ln[1/(1-x)]}{\pi\kappa^2 x}x^{2b}(1-x)^{2a}\Big[\Big(M-\frac{M_X}{1-x}\Big)^2 + p_\perp^2\frac{1-(1-x)^2}{x^2(1-x)^2}\Big]exp[-C(x)p_\perp^2]$$

- Gluon helicity pdf. $g^g_{1L}(x) = \int d^2p_\perp g^g_{1L}(x,p_\perp^2)$

$$C(x) = \frac{\log[1/(1-x)]}{\kappa^2 x^2}.$$

- Worm-gear TMD: circularly polarized gluon in transversely polarized proton

$$\frac{\mathbf{p}_\perp.\mathbf{S}_\perp}{M}g^g_{1T}(x,\mathbf{p}_\perp^2) = -\frac{1}{16\pi^3}i\epsilon_T^{\mu\nu}\sum_{\lambda_g\lambda'_g\lambda_X}\epsilon_\mu^{\lambda'_g *}\epsilon_\nu^{\lambda_g}\psi^{\uparrow *}_{\lambda'_g\lambda_X}(x,\mathbf{p}_\perp^2)\psi^{\downarrow}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp^2)$$

$$= \frac{1}{16\pi^3}\frac{i}{2}\sum_{\lambda_g\lambda'_g\lambda_X}(\epsilon_1^{\lambda'_g *}\epsilon_2^{\lambda_g} - \epsilon_2^{\lambda'_g *}\epsilon_1^{\lambda_g})[\psi^{\uparrow *}_{\lambda'_g\lambda_X}(x,\mathbf{p}_\perp^2)\psi^{\downarrow}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp^2) + \psi^{\downarrow *}_{\lambda'_g\lambda_X}(x,\mathbf{p}_\perp^2)\psi^{\uparrow}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp^2)]$$

- In our model:

$$g^g_{1T}(x,\mathbf{p}_\perp^2) = -\frac{4M}{\pi\kappa^2}N_g^2(M(1-x)-M_X)\log[1/(1-x)]x^{2b-2}(1-x)^{2a-1}\exp[-C(x)\mathbf{p}_\perp^2].$$

- Boer-Mulders TMD: linearly polarized gluon inside unpolarised proton [interference between ±1 gluon helicities]

- $$\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x, \mathbf{p}_\perp^2) = \frac{1}{2}\eta_T^{\mu\nu} \sum_{\lambda_N \lambda_g \neq \lambda'_g \lambda_X} \frac{1}{16\pi^3} \left[\epsilon_\mu^{\lambda'_g *} \epsilon_\nu^{\lambda_g} \psi^{*\lambda_N}_{\lambda'_g \lambda_X}(x, \mathbf{p}_\perp) \psi^{\lambda_N}_{\lambda_g \lambda_X}(x, \mathbf{p}_\perp) \right],$$

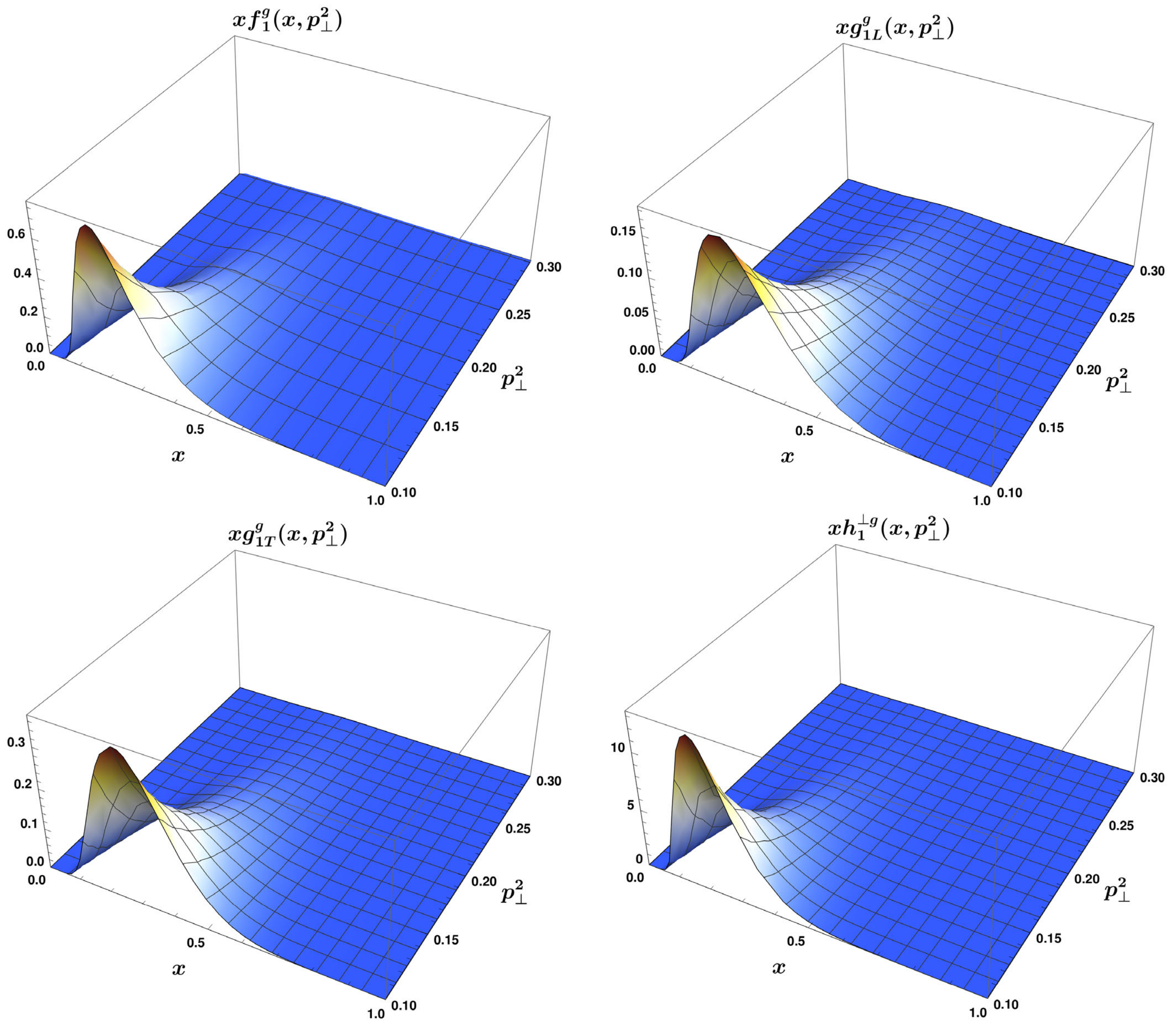
- Analytic form in our model:

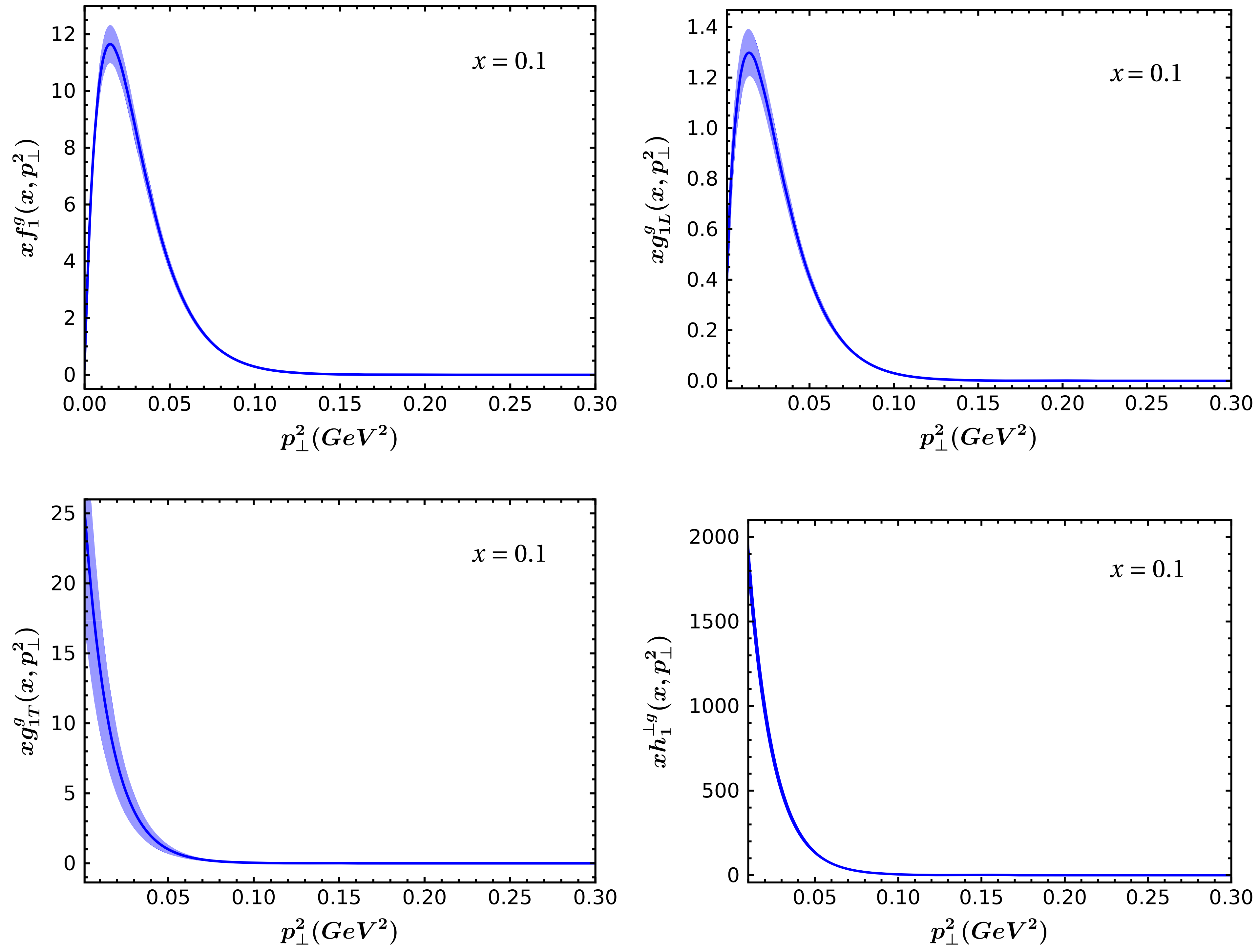
$$h_1^{\perp g}(x, \mathbf{p}_\perp^2) = \frac{8M^2}{\pi\kappa^2} N_g^2 \log[1/(1-x)] x^{2b-3} (1-x)^{2a-1} \exp[-C(x)\mathbf{p}_\perp^2].$$

- Corresponding PDFs :

- $$g_{1T}^g(x) = \int d^2\mathbf{p}_\perp g_{1T}^g(x, \mathbf{p}_\perp^2) \qquad h_1^{\perp g}(x) = \int d^2\mathbf{p}_\perp h_1^{\perp g}(x, \mathbf{p}_\perp^2)$$

Gluon TMDs:





For x=0.1

TMD relations:

- Positivity bound:

$$f_1^g(x,\mathbf{p}_\perp^2) > 0, \quad f_1^g(x,\mathbf{p}_\perp^2) \geq |g_{1L}^g(x,\mathbf{p}_\perp^2)|.$$

$$f_1^g(x,\mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|}{M}|g_{1T}^g(x,\mathbf{p}_\perp^2)|,$$

$$f_1^g(x,\mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|^2}{2M^2}|h_1^{\perp g}(x,\mathbf{p}_\perp^2)|.$$

-

- Mulders-Rodrigues relation put more stringent conditions on TMDS:

$$\sqrt{[g_{1L}^g(x,\mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M} g_{1T}^g(x,\mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{p}_\perp^2),$$

$$\sqrt{[g_{1L}^g(x,\mathbf{p}_\perp^2)]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x,\mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{p}_\perp^2),$$

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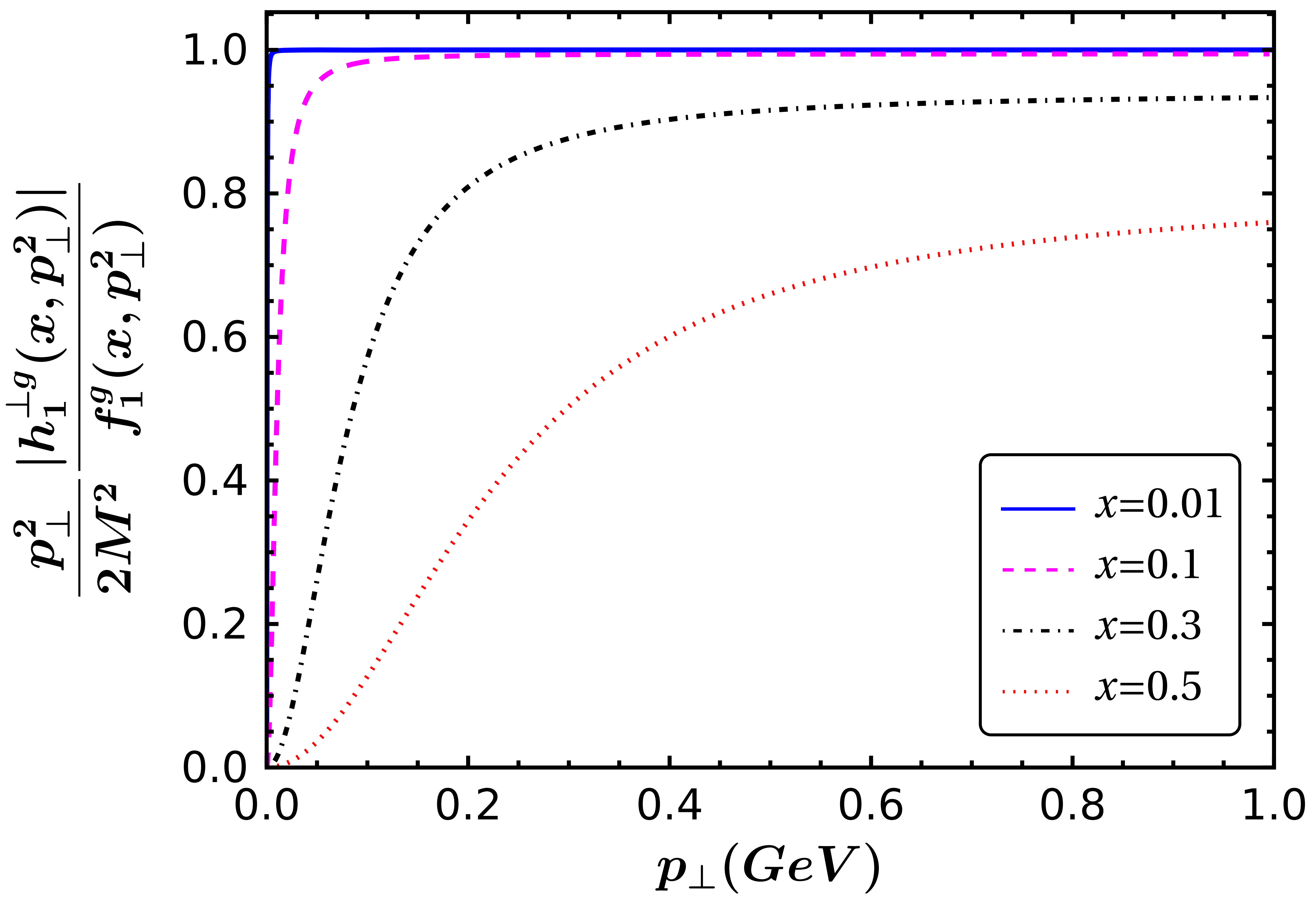
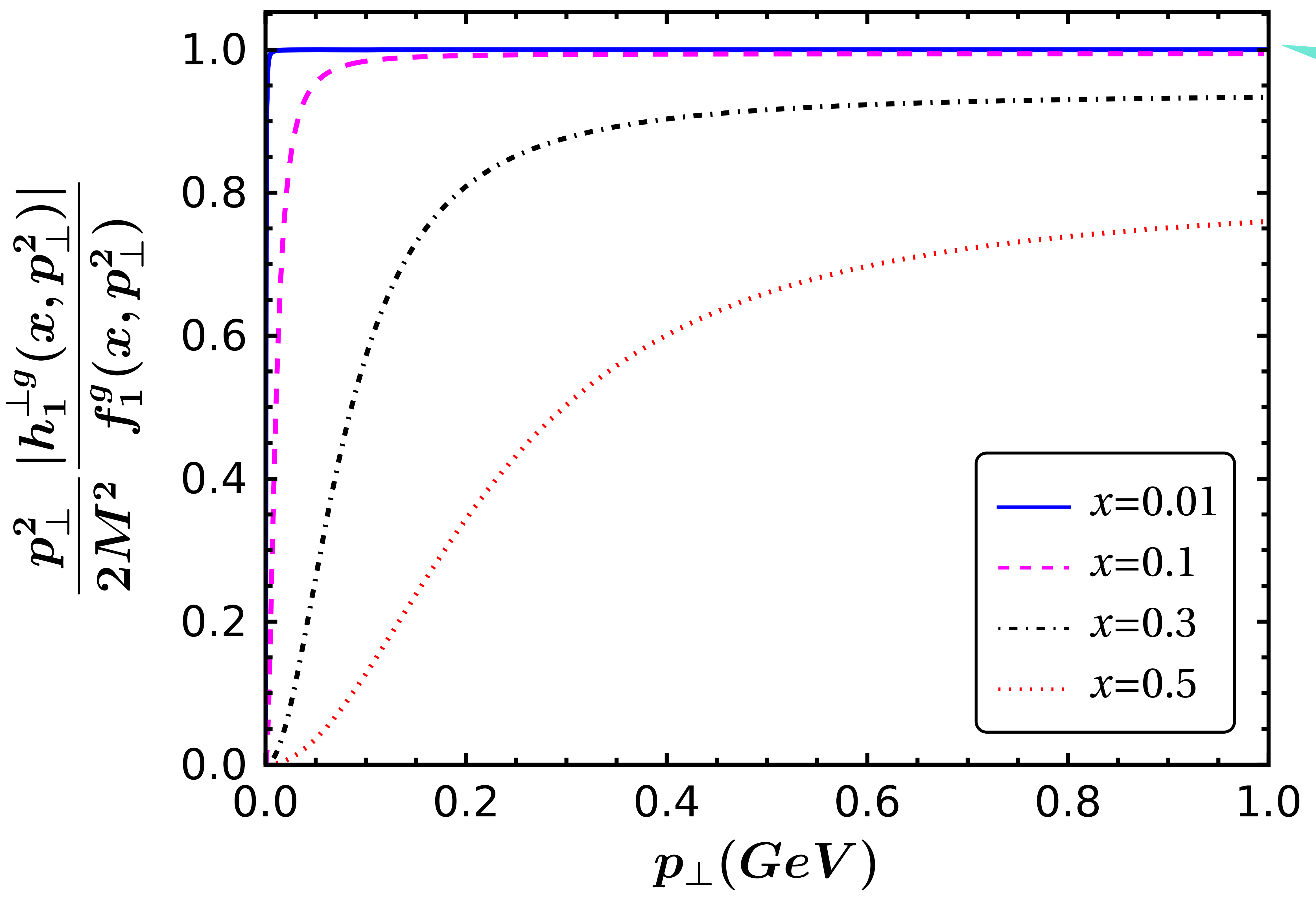
Positivity bound



Positivity bound



Saturation occurs for small x

Gluon densities

- Unpolarised gluon density in an unpolarised proton

$$x\rho_g(x, p_x, p_y) = xf_1^g(x, \mathbf{p}_\perp^2),$$

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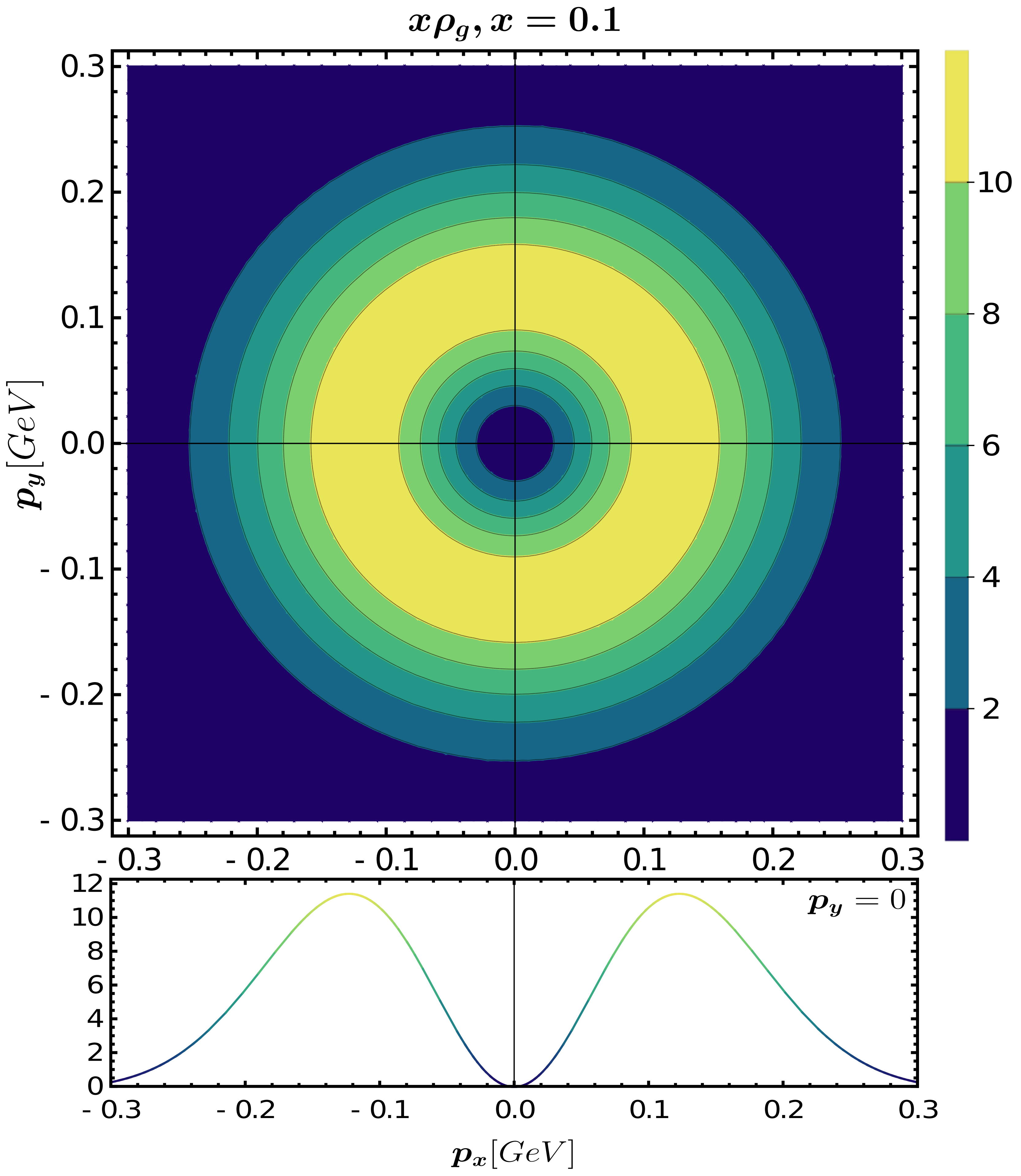

- Boer-Mulders density: longitudinally polarized gluon density
- $$x\rho_g^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\left[xf_1^g(x, \mathbf{p}_\perp^2) + \frac{p_x^2 - p_y^2}{2M^2} xh_1^{\perp g}(x, \mathbf{p}_\perp^2)\right]$$
- Spherical symmetry gets distorted due to the second term...shows dipolar structure in momentum space.

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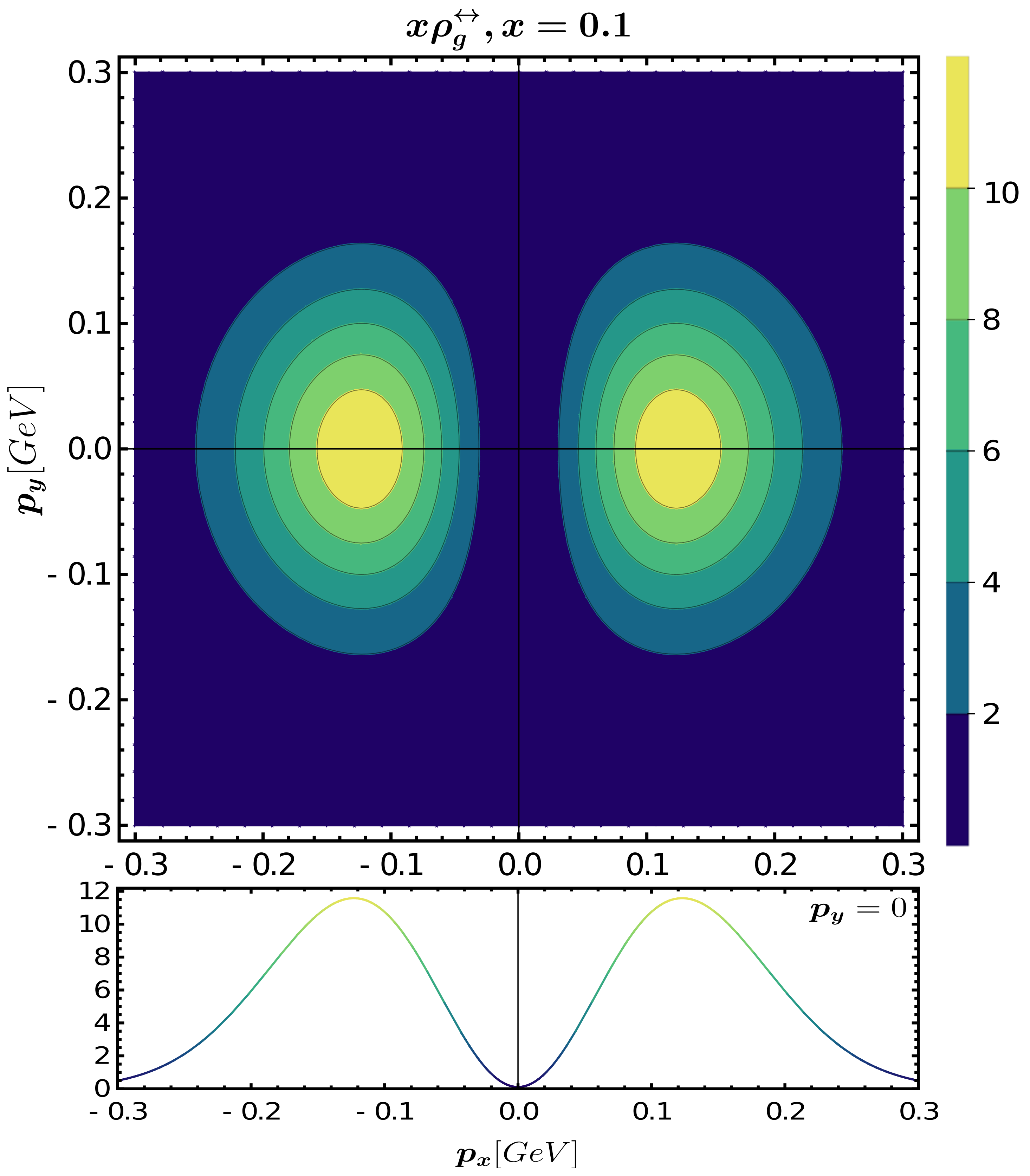

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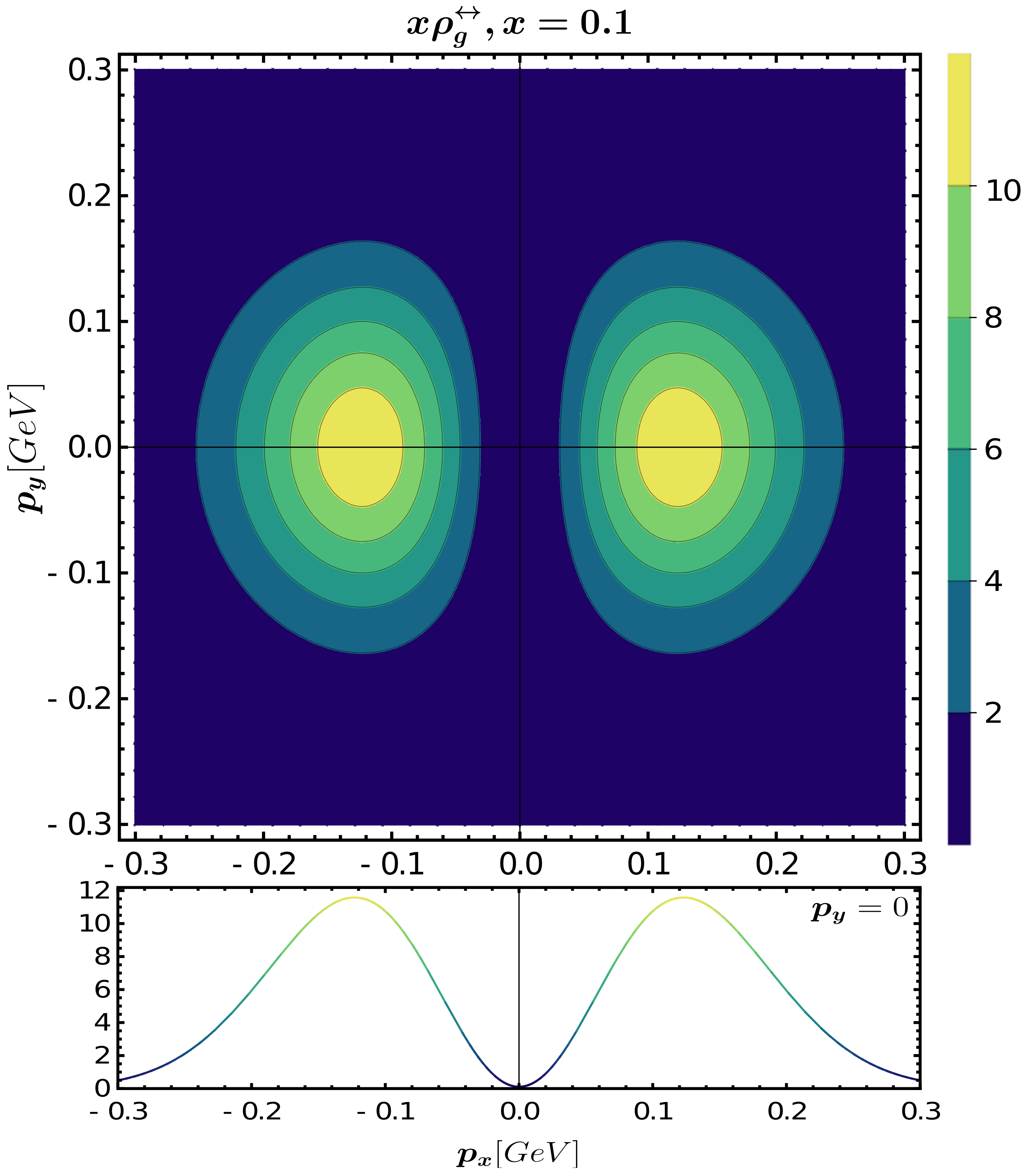

Results are similar to Bacchetta et al EPJC80

- Helicity density: circularly polarized gluon density in a longitudinally polarized proton.

- $$x\rho_g^{\circlearrowleft/+}(x,p_x,p_y)=\frac{1}{2}[xf_1^g(x,\mathbf{p}_\perp^2)+xg_{1L}^g(x,\mathbf{p}_\perp^2)]$$

- Worm-gear density: circularly polarized gluon density in a transversely polarized proton

- $$x\rho_g^{\circlearrowleft/\leftrightarrow}(x,p_x,p_y)=\frac{1}{2}[xf_1^g(x,\mathbf{p}_\perp^2)-\frac{p_x}{M}xg_{1T}^g(x,\mathbf{p}_\perp^2)]$$

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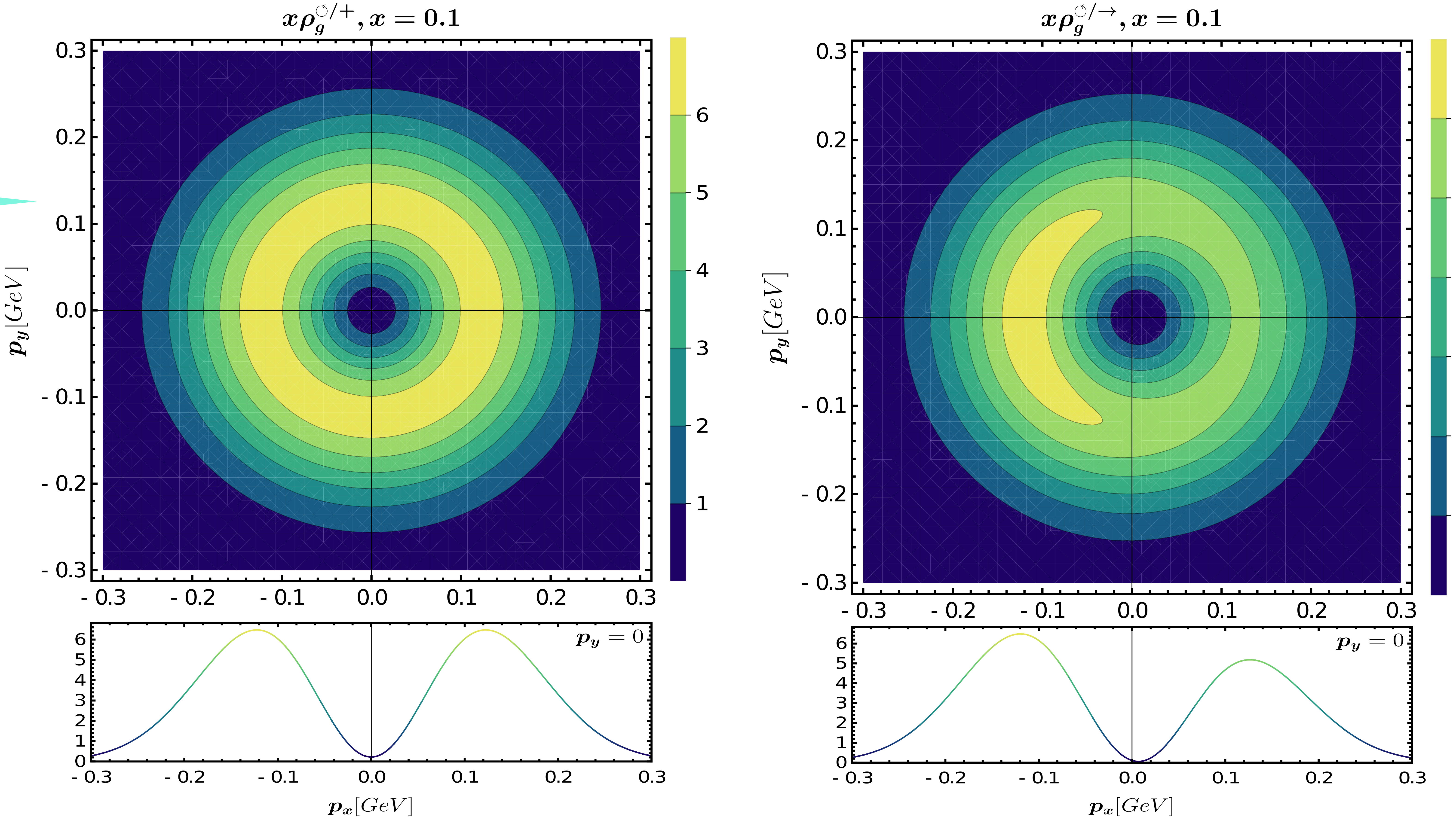
Cylindrically symmetric



Cylindrical symmetry is broken

GPDs

- GPDs appear in exclusive processes e.g., DVCS/ vector meson production
- are off-forward matrix elements of the bilocal operator and functions of (x, ξ, t).
- encode spatial as well as spin structure of the nucleon.
- don't have probabilistic interpretation.
- for skewness $\xi = 0$, in impact parameter space can have probabilistic interpretation.
- In the forward limit GPDs —> PDFs.

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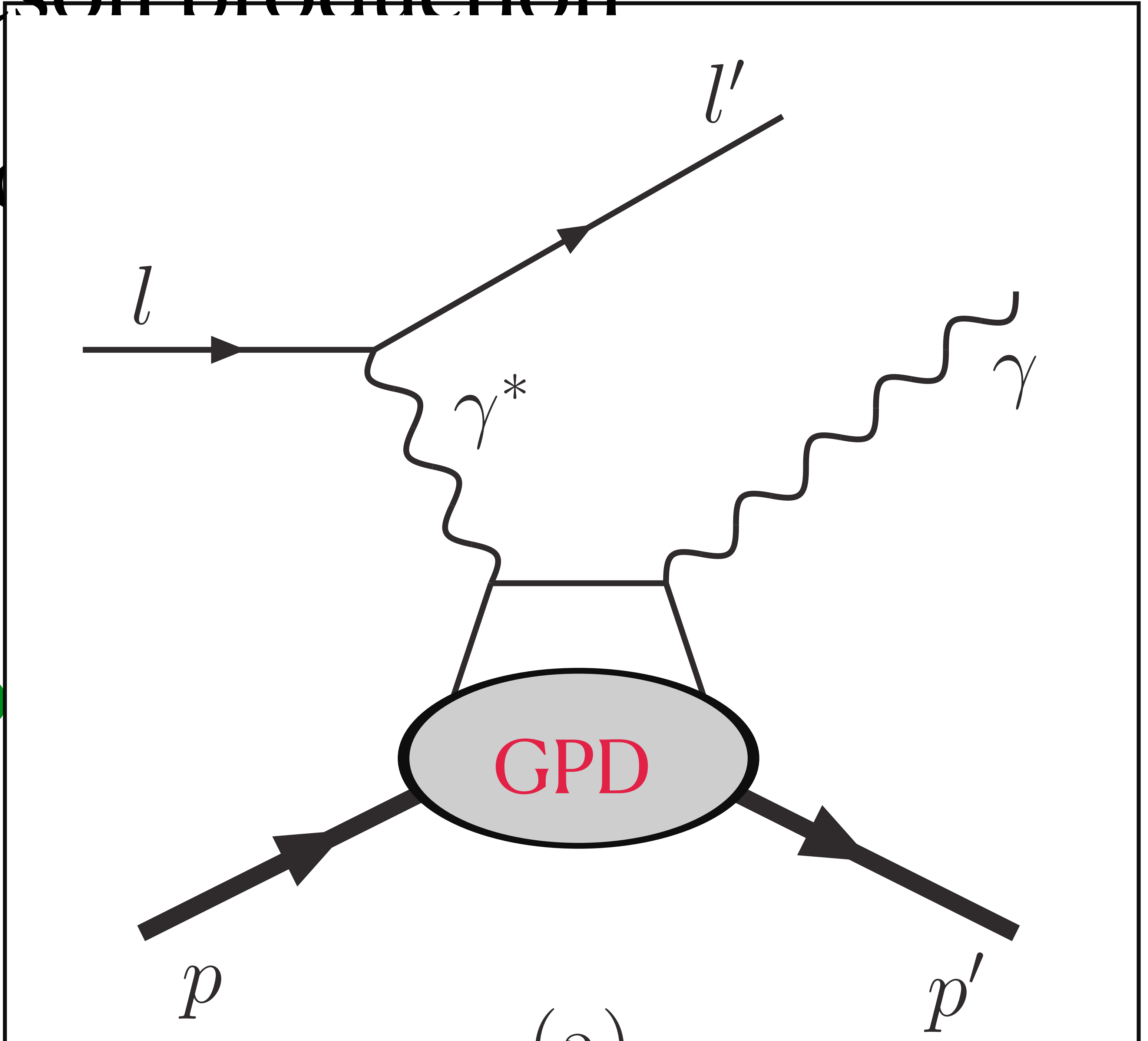

(a)

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Gluon GPDs

- Since nonperturbative QCD evaluations are not yet feasible, it is important to constraints the GPDs by using different model predictions.

- We analyze the gluon GPDs in our model for both $\xi = 0$ and $\xi \neq 0$ (in experiments $\xi \neq 0$).

- In the light cone gauge $(A^+ = 0)$, 4 helicity conserving gluon GPDs:

$$\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, F^{+i}(-\tfrac{z}{2})\, F^{+i}(\tfrac{z}{2})\, |p, \lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p', \lambda') \left[H^g\, \gamma^+ + E^g\, \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} \right] u(p, \lambda),$$

$$-\frac{i}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, F^{+i}(-\tfrac{z}{2})\, \tilde{F}^{+i}(\tfrac{z}{2})\, |p, \lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p', \lambda') \left[\tilde{H}^g\, \gamma^+\gamma_5 + \tilde{E}^g\, \frac{\gamma_5 \Delta^+}{2M} \right] u(p, \lambda),$$

- 4 gluon helicity flip GPDs:

$$-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, \mathbf{S} F^{+i}(-\tfrac{z}{2})\, F^{+j}(\tfrac{z}{2})\, |p, \lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0} = \mathbf{S}\, \frac{1}{2P^+}\, \frac{P^+\Delta^j - \Delta^+ P^j}{2MP^+}$$

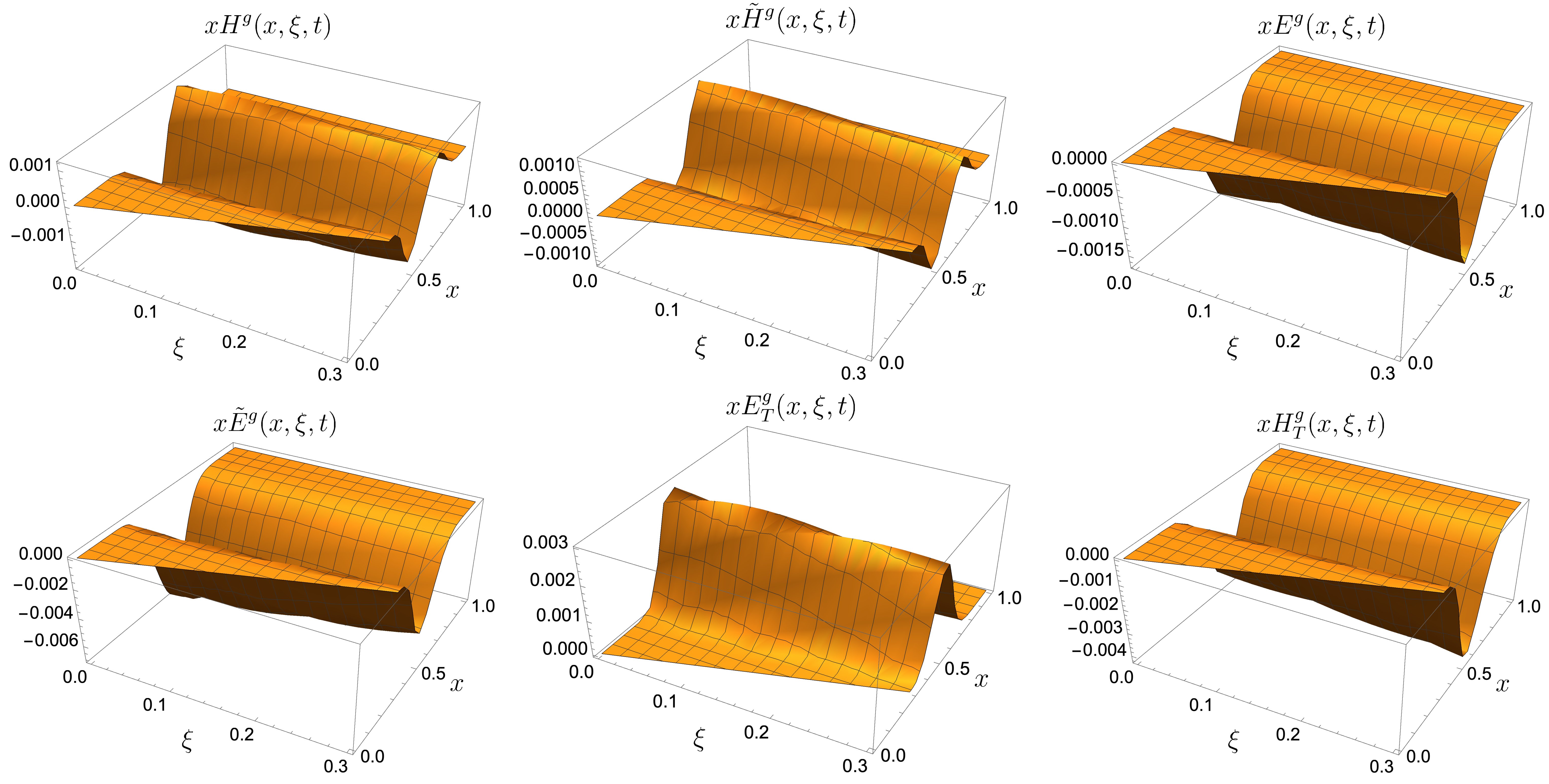
$$\times\, \bar{u}(p', \lambda') \left[H_T^g\, i\sigma^{+i} + \tilde{H}_T^g\, \frac{P^+\Delta^i - \Delta^+ P^i}{M^2} + E_T^g\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M} + \tilde{E}_T^g\, \frac{\gamma^+ P^i - P^+\gamma^i}{M} \right] u(p, \lambda)$$

- We consider $x \geq \xi$ only: a quark with longitudinal momentum fraction $(x + \xi)$ gets hit by the photon and comes back to the proton with $(x - \xi)$

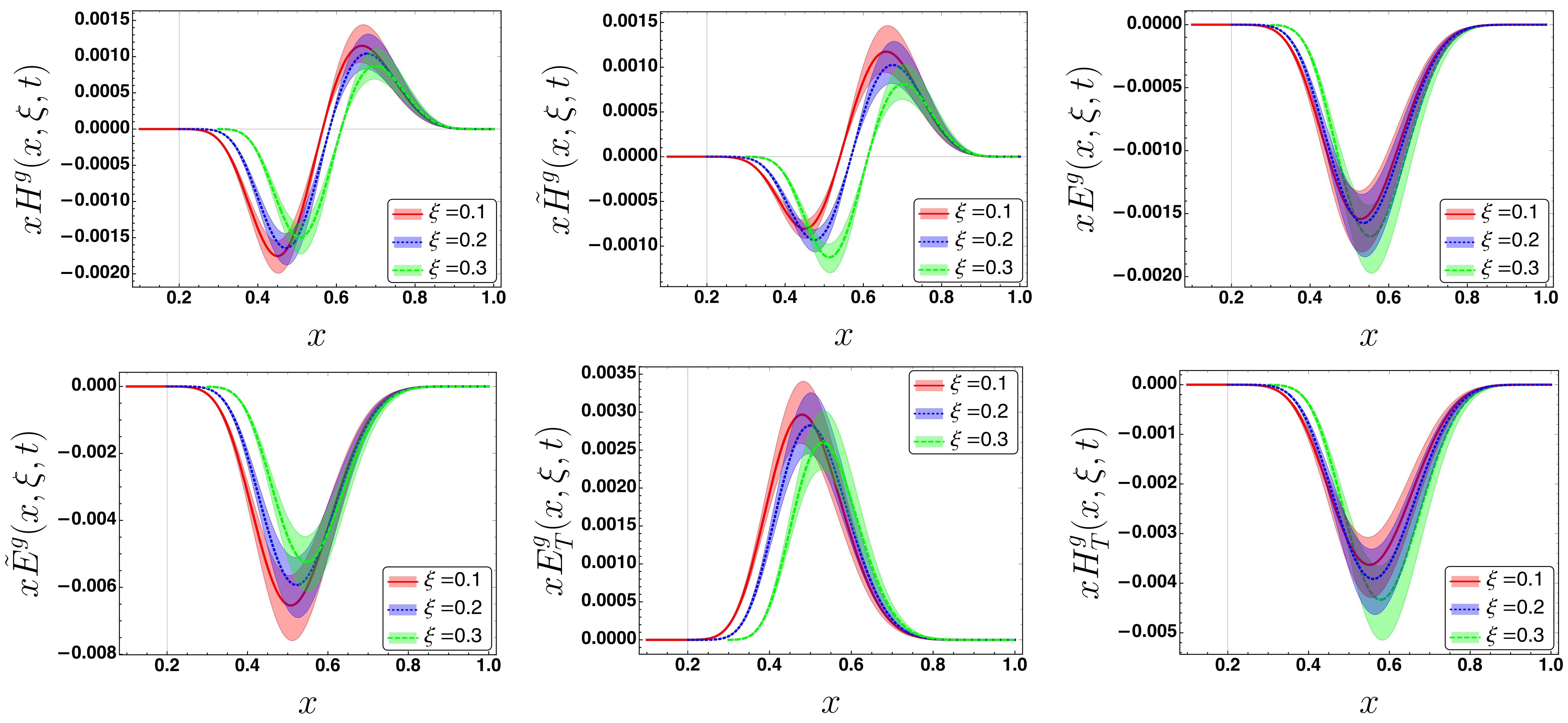
Particle number conserving process

- In the forward limit GPDs $\longrightarrow$ PDFs
- Unpolarised gluon GPD $H^g(x, \xi = 0, t = 0) = f^g(x)$ = unpolarised gluon pdf
- Helicity dependent GPD $\tilde{H}^g(x, \xi = 0, t = 0) = g_{1L}^g(x)$ = helicity pdf

Gluon GPDs at $-t = 3\ GeV^2$ as functions of (x, ξ)



Out of 8 GPDs, 6 are nonzero in our model



$xH^g(x,\xi,t)$
$\tilde{H}^g(x,\xi,t)$
$xE^g(x,\xi,t)$
$x\tilde{E}^g(x,\xi,t)$
$xE_T^g(x,\xi,t)$
$xH_T^g(x,\xi,t)$
x
ξ =0.1
ξ =0.2
ξ =0.3

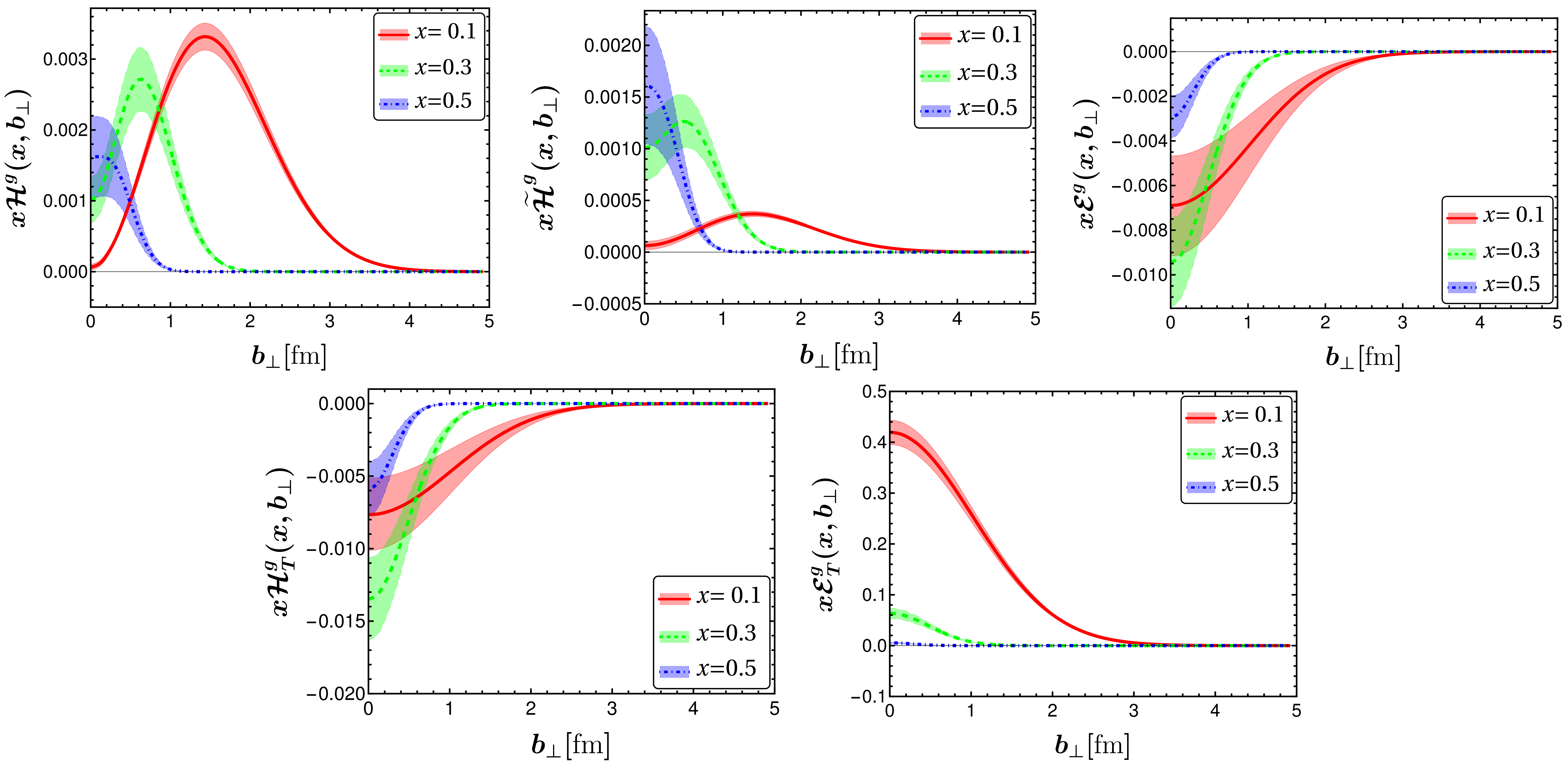
GPDs in impact parameter space

- 2D Fourier transform with respect to the transverse momentum transferred at $\xi = 0$ gives the GPD in impact parameter space.
- $$\mathcal{F}(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} F^g(x, \xi = 0, t = -\Delta_\perp^2)$$

  Transverse distance of the struck quark from the CoM

- GPDs in impact parameter have probabilistic interpretation. M. Burkardt, IJMPA18, 187(2003)

- $\langle b_\perp^2 \rangle$: transverse size of the nucleon. At small x, gluons show larger transverse radius than quarks. At larger x, the radius decreases, becomes point-like at $x = 1$.



$x\mathcal{H}^g(x,b_\perp)$
$x\widetilde{\mathcal{H}}^g(x,b_\perp)$
$x\mathcal{E}^g(x,b_\perp)$
$x\mathcal{H}_T^g(x,b_\perp)$
$x\mathcal{E}_T^g(x,b_\perp)$
$b_\perp$[fm]
x= 0.1
x=0.3
x=0.5

•

Orbital angular momentum

- According to Ji's sum rule: $$J_z^g = \frac{1}{2}\int dx x\left[H^g(x,0,0)+E^g(x,0,0)\right]$$
- Our estimate $J_z^g = 0.058$ consistent with BLFQ result of $J_z^g = 0.066$

B. Lin et al. 2308. 08275

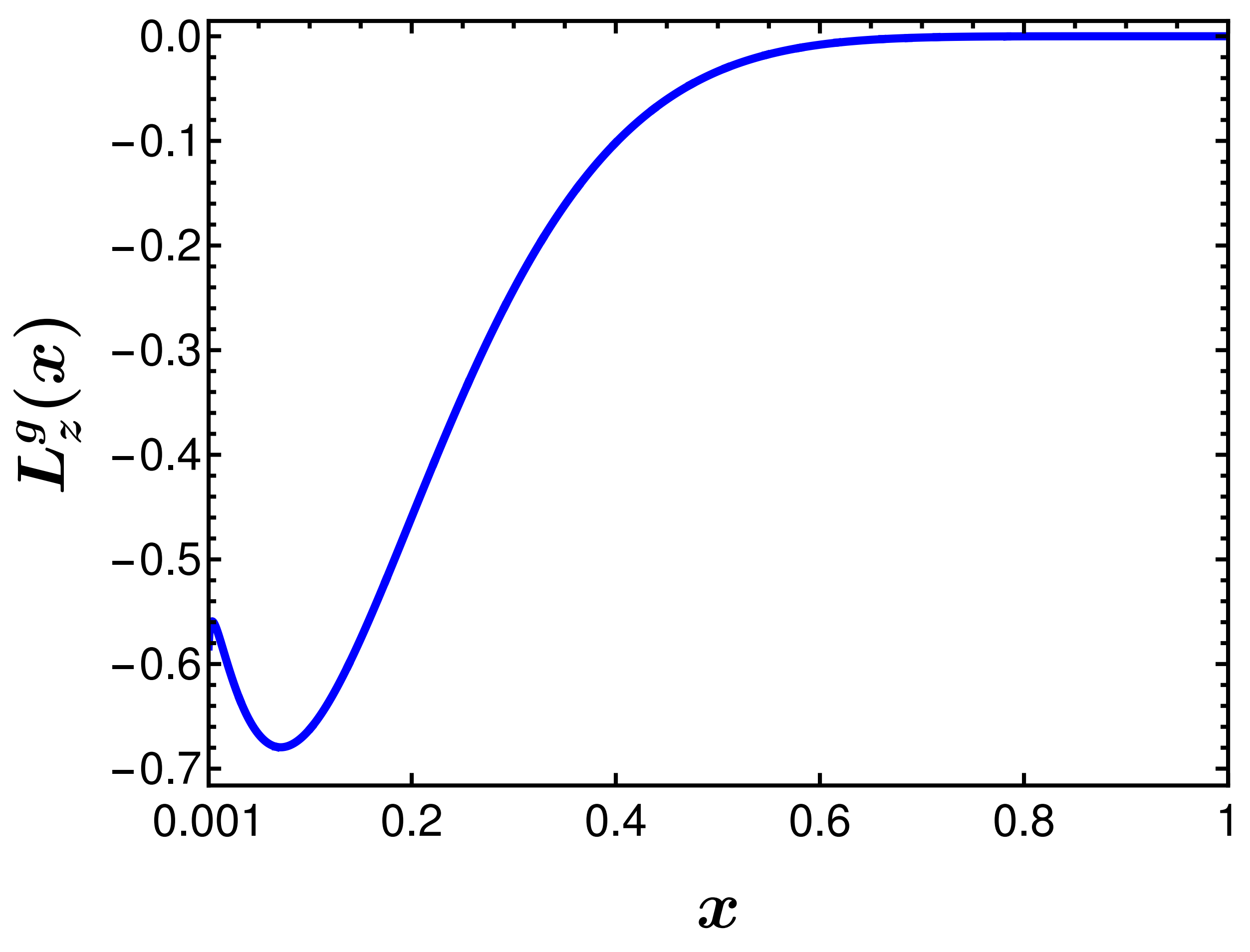
- Helicity GPD gives the spin contribution of gluon:

$$\Delta G = \int dx\ g_{1L}(x) = \int dx\ \tilde{H}^g(x,0,0)$$

- (Kinetic) OAM:

$$L_z^g = \frac{1}{2}\int \left\{x\left[H^g(x,0,0)+E^g(x,0,0)\right]-\tilde{H}^g(x,0,0)\right\}$$

-

- 

- Our result: $L_z^g = -0.18$ [Comparable to $L_z^g = -0.123$ (Tan & Lu, PRD108, 054038)]
- Canonical OAM can be defined by GTMDs
- $$\ell_z^g(x) = -\int d^2\mathbf{p}_\perp \frac{\mathbf{p}_\perp^2}{M^2} F_{1,4}^g(x,0,\mathbf{p}_\perp,0,0).$$

GTMDs

- GTMDs : higher dimensional distributions ⟷ Wigner distributions.

* TMDs can be obtained from GTMDs at $\Delta_\perp = 0$ limit

** GPDs in impact parameter space are obtained by integrating GTMDs over $p_\perp$

- GTMD correlator:
- $$W(x, \xi = 0, p_\perp, \Delta_\perp) = \frac{1}{xP^+} \int \frac{dz^- d^2 z^\perp}{(2\pi)^3} e^{ip.z} \langle p + \frac{\Delta_\perp}{2} | F^{+i}(-z/2) \mathscr{W} F^{+j} | p - \frac{\Delta_\perp}{2} \rangle |_{z^+=0}$$

- $$\frac{i(\mathbf{p}_\perp \times \mathbf{\Delta}_\perp)_z}{M^2} F^g_{1,4} = \frac{1}{2(2\pi)^3} \frac{1}{2} \sum_{\Lambda,\lambda,\mu} \text{sign}(\Lambda) \left[\psi^{\Lambda *}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}'_\perp)\psi^{\Lambda}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}_\perp)\right]$$

Λ =proton helicity

λ= quark helicity

μ=gluon helicity

GTMD $F_{1,4}$ describes the distortion of unpolarised parton in a longitudinally polarised nucleon

- $$-\frac{i(\mathbf{p}_\perp \times \mathbf{\Delta}_\perp)_z}{M^2} G^g_{1,1} = \frac{1}{2(2\pi)^3} \frac{1}{2} \sum_{\Lambda,\lambda,\mu} \text{sign}(\mu) \left[\psi^{\Lambda *}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}'_\perp)\psi^{\Lambda}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}_\perp)\right]$$

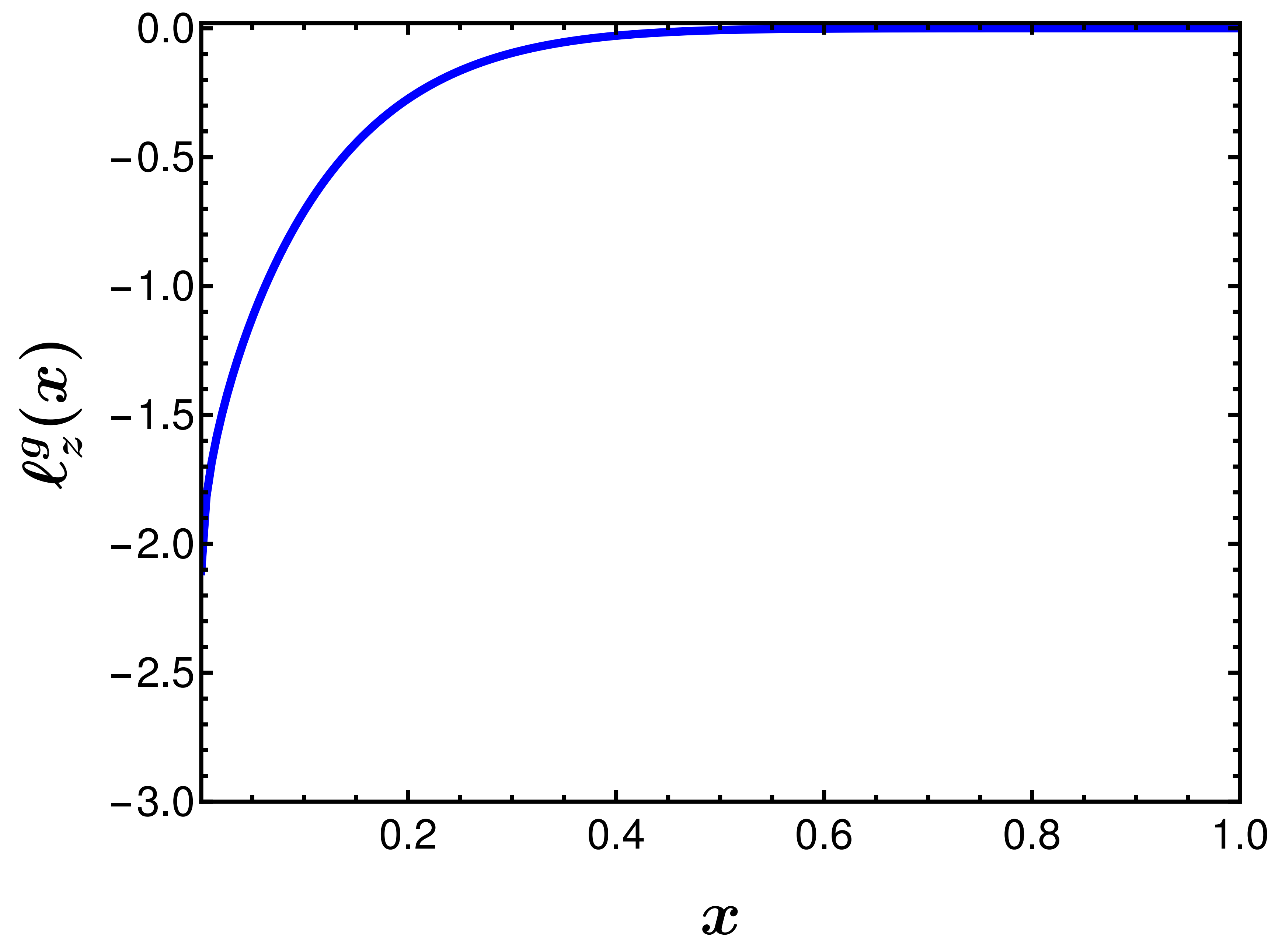
$G_{1,1}$: distortion of longitudinally polarised gluon inside a unpolarised nucleon

Canonical OAM:

$$\ell_z^g(x) = -\int d^2\mathbf{p}_\perp \frac{\mathbf{p}_\perp^2}{M^2} F_{1,4}^g(x,0,\mathbf{p}_\perp,0,0).$$

- Our model result:

$$\ell_z^g(x) = -N_g^2\kappa^2 \frac{1-(1-x)^2}{x^2(1-x)^2} x^{2b+3}(1-x)^{2a+1} \frac{1}{\log[\frac{1}{1-x}]}$$



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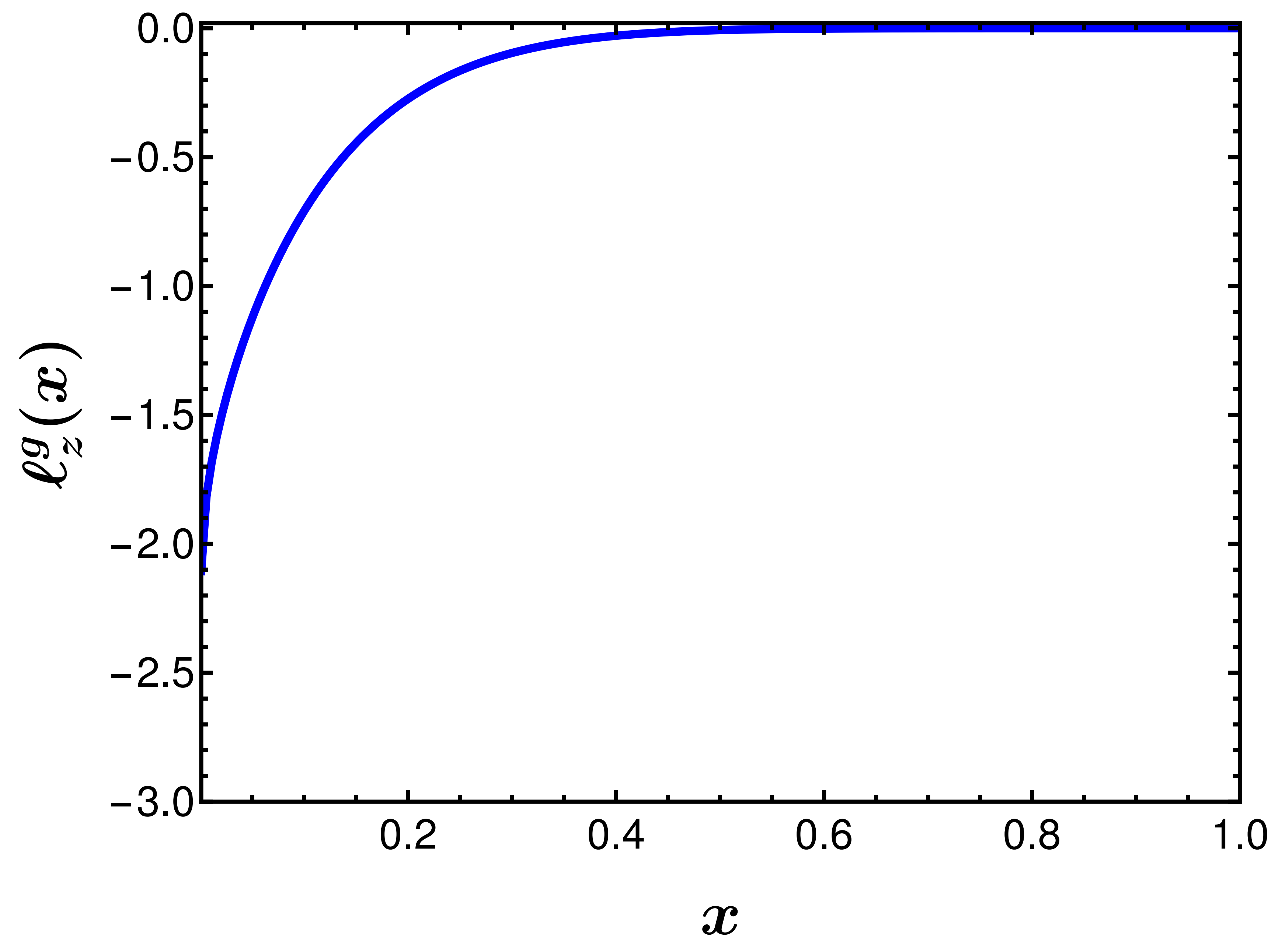

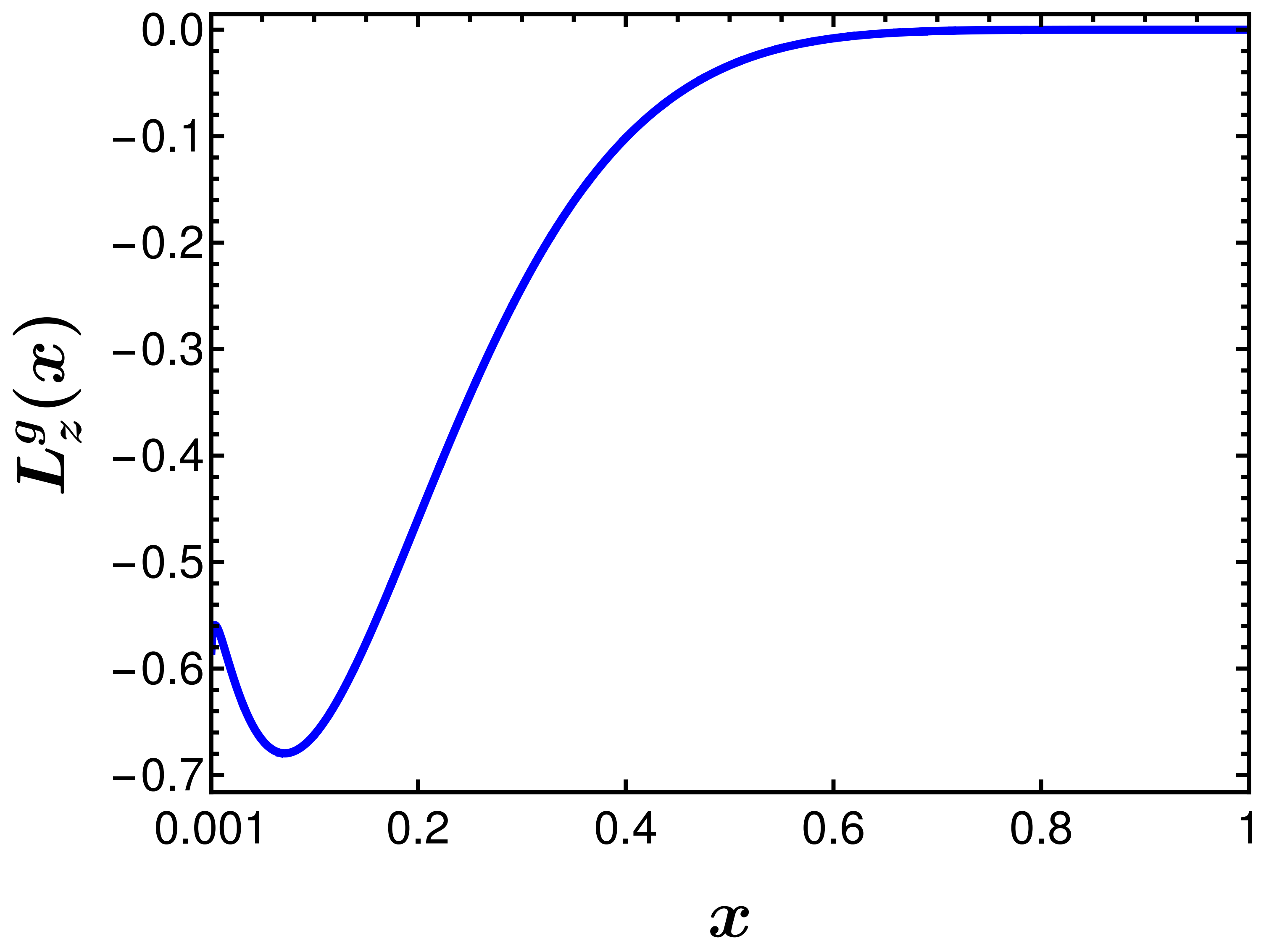

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Spin-orbit correlation

$$C_z^g(x) = \int d^2\mathbf{p}_\perp \frac{\mathbf{p}_\perp^2}{M^2} G_{1,1}^g(x, 0, \mathbf{p}_\perp, 0, 0)$$

- $G_{1,1}$ gives the spin-OAM correlation

- $C_z^g < 0$: spin and OAM are anti-aligned
- $C_z^g > 0$: spin and OAM are aligned.

Model predicts $C_z^g < 0$

Lorce, Pasquini, PRD84, 014015

Talk by Y. Hatta in the morning.

Summary and conclusions

- To understand the three dimensional structure and partonic level description of spin/OAM , we need to investigate both quark and gluons (and sea quarks too!).
- Gluon distributions are not yet well understood/studied.
- We presented the study of different gluon distributions in a simple model of proton.
- gluon contributions to spin/OAM .
- We require more experiments , lattice results, better models with gluons...

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EIC

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