Topological aspects of strong correlations and gauge theories,

TIFR, India, September 6-10, 2021

Chiral Matter

from quarks to quantum computers

Lecture 2

Dmitri Kharzeev

Center for Nuclear Theory
Stony Brook University

C2QA Co-design Center for Quantum Advantage

Brookhaven National Laboratory

Chiral fermions

Fermions:
E. Fermi, Florence, 1925

Dirac equation:
P. Dirac, 1928

$$(i\partial\!\!\!/ - m)\psi = 0$$

Weyl fermions:
H. Weyl, 1929

$$\sigma^\mu \partial_\mu \psi = 0$$

Majorana fermions:
1937
E.Majorana, 1906-38?

$$-i\partial\!\!\!/\psi + m\psi_c = 0$$

$$\psi_c := i\psi^*$$

Currents in a magnetic field

$$\vec{J} \sim \vec{B} \quad ?$$

vector — pseudo-vector

An electric current parallel to B requires a parity breaking

Currents in a magnetic field

Consider a gas of massless charged Weyl fermions of a certain chirality, say left-handed (cf weak interactions)

Put this gas in an external magnetic field B; the interaction of spin with B, and the locking of momentum to spin

$$\langle \vec{\sigma} \cdot \vec{p} \rangle = -1$$

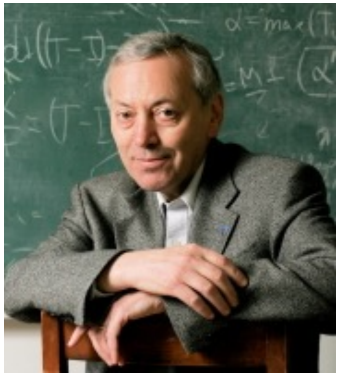
induce the current $\vec{J} \sim \vec{B}$

PHYSICAL REVIEW D VOLUME 22, NUMBER 12 15 DECEMBER 1980

Equilibrium parity-violating current in a magnetic field

Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155
(Received 1 August 1980)

It is argued that if the Hamiltonian of a system of charged fermions does not conserve parity, then an equilibrium electric current parallel to $\vec{B}$ can develop in such a system in an external magnetic field $\vec{B}$. The equilibrium current is calculated (i) for noninteracting left-handed massless fermions and (ii) for a system of massive particles with a Fermi-type parity-violating interaction. In the first case a nonzero current is found, while in the second case the current vanishes in the lowest order of perturbation theory. The physical reason for the cancellation of the current in the second case is not clear and one cannot rule out the possibility that a nonzero current appears in other models.



But: no current in equilibrium

Bloch theorem, …

C.N. Yang

PHYSICAL REVIEW D VOLUME 22, NUMBER 12 15 DECEMBER 1980

Cancellation of equilibrium parity-violating currents

Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155

Early work on currents in magnetic field due to P violation

(see DK, Prog.Part.Nucl.Phys. 75 (2014) 133 for a complete (?) list of references)

A.Vilenkin (1980) “Equilibrium parity-violating current in a magnetic field”;
(1980) “Cancellation of equilibrium parity-violating currents”

G. Eliashberg (1983) JETP 38, 188
L. Levitov, Yu.Nazarov, G. Eliashberg (1985) JETP 88, 229

M. Joyce and M. Shaposhnikov (1997) PRL 79, 1193;
M. Giovannini and M. Shaposhnikov (1998) PRL 80, 22

A. Alekseev, V. Cheianov, J. Frohlich (1998) PRL 81, 3503

The way out: chiral anomaly

For massless fermions, the axial current

$$J_\mu^A = \bar{\Psi}\gamma_\mu\gamma_5\Psi = J_\mu^R - J_\mu^L$$

is conserved classically due to the global $U_A(1)$ symmetry:

$$\partial^\mu J_\mu^A = 0$$

This is because left- and right-handed fields decouple in the massless limit:

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \rightarrow 0$$

However, this conservation law is destroyed by quantum effects

Chiral anomaly

The axial current is not conserved:

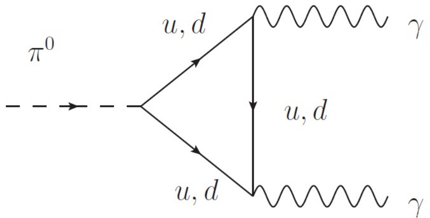
$$\partial_\mu J^\mu_A = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

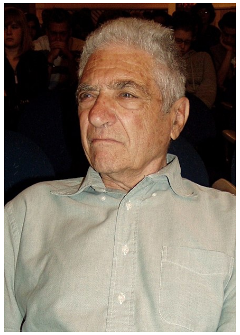
S. Adler '69
J. Bell, R. Jackiw '69

This is a consequence of UV regularization of QFT.

A textbook example: neutral pion decay

$$\pi^0 \to \gamma\gamma$$





J. Steinberger
(1921- Dec 2020;
Nobel prize 1988)

computed the decay
rate in 1949!

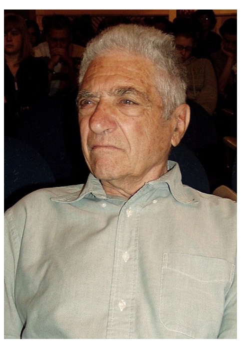
PHYSICAL REVIEW VOLUME 76, NUMBER 8 OCTOBER 15, 1949

On the Use of Subtraction Fields and the Lifetimes of Some Types of Meson Decay

J. STEINBERGER*
The Institute for Advanced Study, Princeton, New Jersey

(Received June 13, 1949)

The method of subtraction fields in current meson perturbation theory is described, and it is shown that it leads to finite results in all processes. The method is, however, not without ambiguities, and these are stated. It is then applied to the following problems in meson decay: Decay of a neutral meson into two and three γ-rays, into a positron-electron pair, and into another neutral meson and photon; decay of a charged meson into another charged meson and a photon, and into an electron (or μ-meson) and neutrino. The lifetimes are tabulated in Tables I, II and III. The results are quite different from those of previous calculations, in all those cases in which divergent and conditionally convergent integrals occur before subtraction, but identical whenever divergences are absent. The results are discussed in the light of recent experimental evidence.



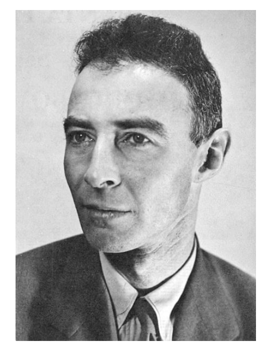
J. Steinberger
(1921- Dec 2020;
Nobel prize 1988)

(A) Decay of a Neutral Scalar Meson into 2 Photons[10]

(1) Scalar meson with scalar coupling.

$$M=\frac{ge^2}{(2\kappa)^{\frac{1}{2}}\pi^4}A_\mu(k_1)A_\nu(k_2)[I_{\mu\nu}+J_{\mu\nu}],$$

[7] S. Tomonaga, Prog. Theor. Phys. 1, 27 (1946). Koba, Tati, and Tomonaga, Prog. Theor. Phys. 2, 101 (1947); 2, 198 (1947). S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. 3, 1 (1948).
[8] R. P. Feynman, Phys. Rev. 76, 748 (1949).
[9] J. Schwinger, Phys. Rev. 74, 1439 (1948); 75, 651 (1949).
[10] J. R. Oppenheimer was the first to point out that present theory requires the γ-instability of neutral mesons coupled to nucleons. The calculations were first made by R. Finkelstein, Phys. Rev. 72, 415 (1949).

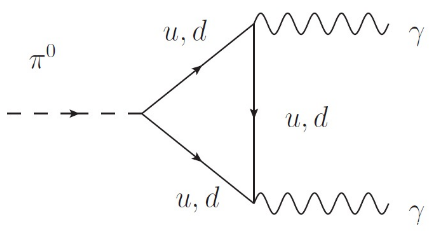


J. R. Oppenheimer
(1904 - 1967)

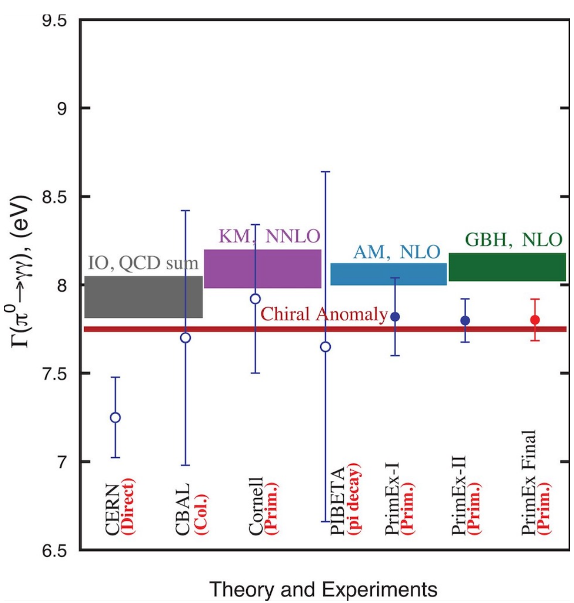
Science

Science 01 May 2020:
Vol. 368, Issue 6490, pp. 506-509
DOI: 10.1126/science.aay6641

$$\partial_\mu J^\mu_A = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$



$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = \frac{m^3_{\pi^0}\alpha^2 N_c^2}{576\pi^3 F^2_{\pi^0}} = 7.750 \pm 0.016 \ \text{eV}$$



REPORT

Precision measurement of the neutral pion lifetime

I. Larin[1,2], Y. Zhang[3,4], A. Gasparian[5,*], L. Gan[6], R. Miskimen[2], M. Khandaker[7], D. Dale[8], S. Danagoulian[5], E
Pasyuk[9], H. Gao[3,4], A. Ahmidouch[5], P. Ambrozewicz[5], V. Baturin[9], V. Burkert[9], E. Clinton[2], A. Deur[9], A.
Dolgolenko[1], D. Dutta[10], G. Fedotov[11,12], J. Feng[6], S. Gevorkyan[13], A. Glamazdin[14], L. Guo[15], E. Isupov[11], M. M.
Ito[9], F. Klein[16], S. Kowalski[17], A. Kubarovsky[9], V. Kubarovsky[9], D. Lawrence[9], H. Lu[18], L. Ma[19], V. Matveev[1], B.
Morrison[20], A. Micherdzinska[21], I. Nakagawa[22], K. Park[9], R. Pedroni[5], W. Phelps[23], D. Protopopescu[24], D. Rimal[15], D
Romanov[25], C. Salgado[7], A. Shahinyan[26], D. Sober[16], S. Stepanyan[9], V. V. Tarasov[1], S. Taylor[9], A. Vasiliev[27], M. Wood[2
L. Ye[10], B. Zihlmann[9], PrimEx-II Collaboration[†]

[1]Alikhanov Institute for Theoretical and Experimental Physics, National Research Center (NRC) "Kurchatov Institute," Moscow, 11721
Russia.

[2]Department of Physics, University of Massachusetts, Amherst, MA 01003, USA.

[3]Department of Physics, Duke University, Durham, NC 27708, USA.

[4]Triangle Universities Nuclear Laboratory, Durham, NC 27708, USA.

[5]Department of Physics, North Carolina A&T State University, Greensboro, NC 27411, USA.

[6]Department of Physics and Physical Oceanography, University of North Carolina Wilmington, Wilmington, NC 28403, USA.

[7]Department of Physics, Norfolk State University, Norfolk, VA 23504, USA.

[8]Department of Physics and Nuclear Engineering, Idaho State University, Pocatello, ID 83209, USA.

[9]Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA.

[10]Department of Physics and Astronomy, Mississippi State University, Mississippi State, MS 39762, USA.

[11]Department of Physics, Moscow State University, Moscow 119991, Russia.

[12]B. P. Konstantinov Petersburg Nuclear Physics Institute, NRC "Kurchatov Institute," Gatchina, St. Petersburg, 188300, Russia.

[13]Joint Institute for Nuclear Research, Dubna, 141980, Russia.

[14]Kharkov Institute of Physics and Technology, Kharkov, 310108, Ukraine.

[15]Department of Physics, Florida International University, Miami, FL 33199, USA.

[16]Department of Physics, The Catholic University of America, Washington, DC 20064, USA.

[17]Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

[18]Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA.

[19]School of Nuclear Science and Technology, Lanzhou University, Lanzhou, 730000, China.

[20]Department of Physics, Arizona State University, Tempe, AZ 85281, USA.

[21]Department of Physics, George Washington University, Washington, DC 20064, USA.

[22]RIKEN Nishina Center for Accelerator-Based Science, Wako, Saitama 351-0198, Japan.

[23]Department of Physics, Computer Science and Engineering, Christopher Newport University, Newport News, VA 23606, USA.

[24]School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK.

[25]Department of Physics, Moscow Engineering Physics Institute, Moscow, Russia.

[26]Yerevan Physics Institute, Yerevan 0036, Armenia.

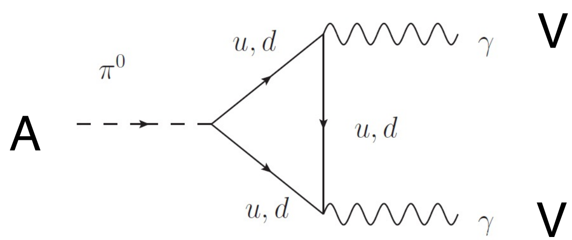
[27]Institute for High Energy Physics, NRC "Kurchatov Institute," Protvino, 142281, Russia.

Chiral anomaly

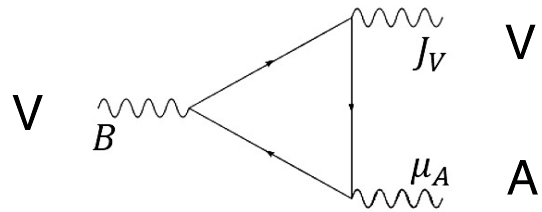
The axial current is not conserved:

S. Adler '69
J. Bell, R. Jackiw ' 69

$$\partial_\mu J^\mu_A = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$



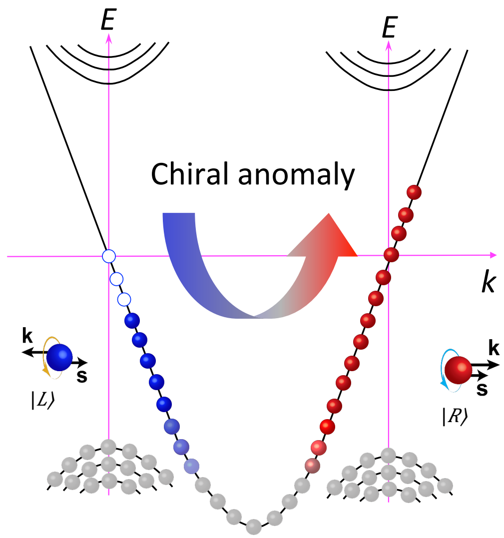
The chiral charge is not conserved;
a chirally imbalanced state of chiral fermions is not
a true ground state of the system!



Chiral anomaly

$$J_A \equiv -J_L + J_R$$

LEFT **RIGHT**



In classical background fields (E and B), chiral anomaly induces an imbalance between left- and right-handed fermions;

$$\partial_\mu J^\mu_A = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

chiral chemical potential:

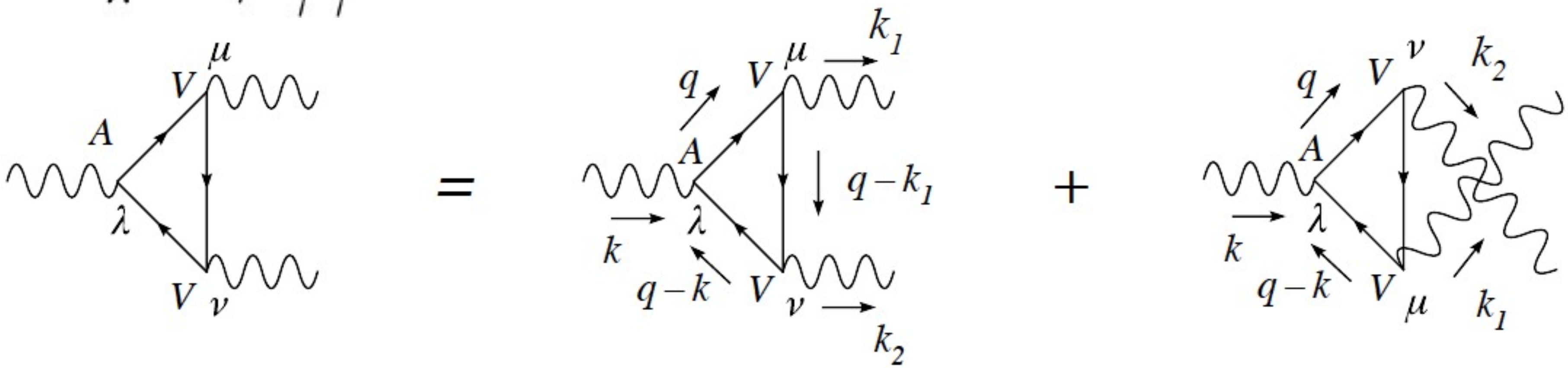
$$\mu_5 = \frac{1}{2}(\mu_R - \mu_L)$$

Adler; Bell, Jackiw (1969); Nielsen, Ninomiya (1983)

Chiral anomaly

S. Adler '69
J. Bell, R. Jackiw ' 69

$\pi^0 \to \gamma\gamma$



$$k_\lambda \Delta^{\lambda\mu\nu}(k_1, k_2) = a_n \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \qquad a_n = -i/2\pi^2$$

$$\Delta^{\lambda\mu\nu} = a_n \frac{k^\lambda}{k^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}$$

The chiral anomaly does not vanish at finite mass, and mass corrections have been evaluated, see e.g.

A.D. Dolgov, V.I. Zakharov, Nucl. Phys. B27 (1971) 525
R. Armillis et al, JHEP 0912 (2009) 029

Possibility of anomalous transport in systems with a finite gap (strange quarks, semiconductors)?

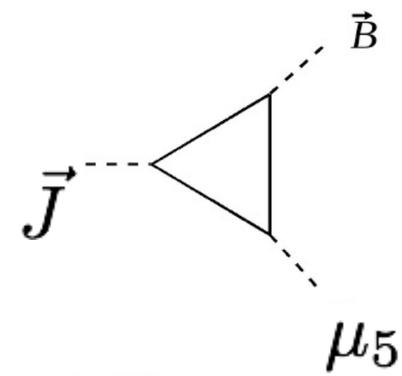
$$w_L = -\frac{4\,i}{s} - \frac{4\,i\,m^2}{s^2} \log\left(-\frac{s}{m^2}\right) + O(m^3) \quad s \equiv k^2$$

Chiral Magnetic Effect

DK'04; DK, A. Zhitnitsky '07; DK, L.McLerran, H.Warringa '07; K.Fukushima, DK, H.Warringa, "Chiral magnetic effect" PRD'08; Review and list of refs: DK, arXiv:1312.3348 [Prog.Part.Nucl.Phys]

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of B, vector e.m. current is generated:



$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$

Compute the current through $$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

Absent in Maxwell theory!

$$\vec{J} = \frac{e^2}{2\pi^2} \, \mu_5 \, \vec{B}$$

Coefficient is fixed by the chiral anomaly, no corrections

Chirally imbalanced system is a non-equilibrium, steady state

Chiral Magnetic Effect

Alternative derivation:

K.Fukushima, DK, H.Warringa,
"Chiral magnetic effect" PRD'08;

Consider the thermodynamical potential at finite : $\mu_5 = A_5^0$

$$\Omega = \frac{|eB|}{2\pi} \sum_{s=\pm} \sum_{n=0}^{\infty} \alpha_{n,s} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \left[\omega_{p,s} + T \sum_{\pm} \log(1 + e^{-\beta(\omega_{p,s} \pm \mu)}) \right]$$

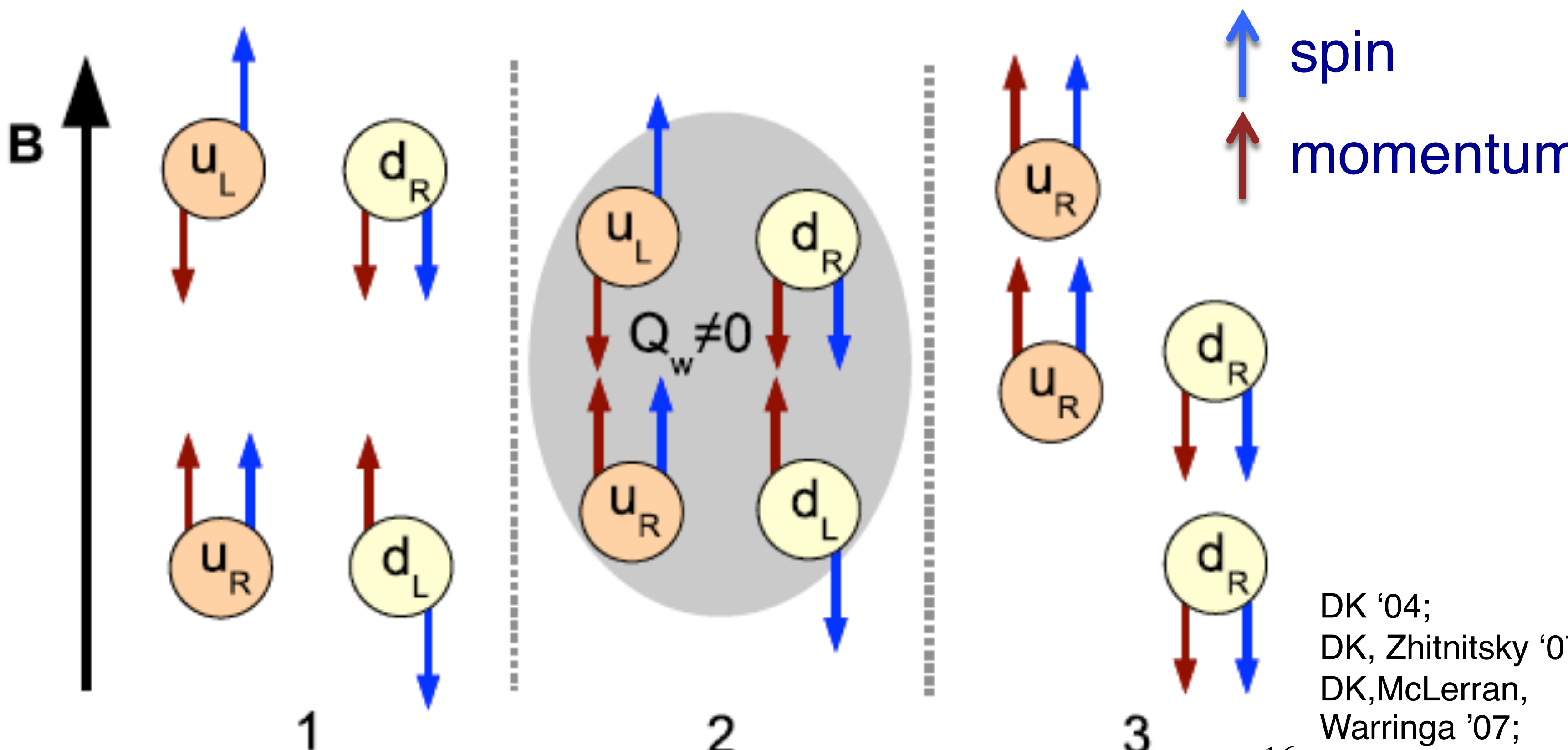
$$\omega_{p,s}^2 = \left[\mathrm{sgn}(p_3)(p_3^2 + 2|eB|n)^{1/2} + s\mu_5 \right]^2 + m^2$$

Compute the current through $j_3 = \left. \frac{\partial \Omega}{\partial A_3} \right|_{A_3=0}$ using $\partial/\partial A_3 = ed/dp_3$

$\Rightarrow$ $$\vec{J} = \frac{e^2}{2\pi^2} \, \mu_5 \, \vec{B}$$

Chirality in 3D: the Chiral Magnetic Effect

chirality + magnetic field = current



Review: DK, arxiv:1312.3348 (Prog.Part.Nucl.Phys'14)

DK '04;
DK, Zhitnitsky '07
DK,McLerran, Warringa '07;
Fukushima, DK, Warringa '08

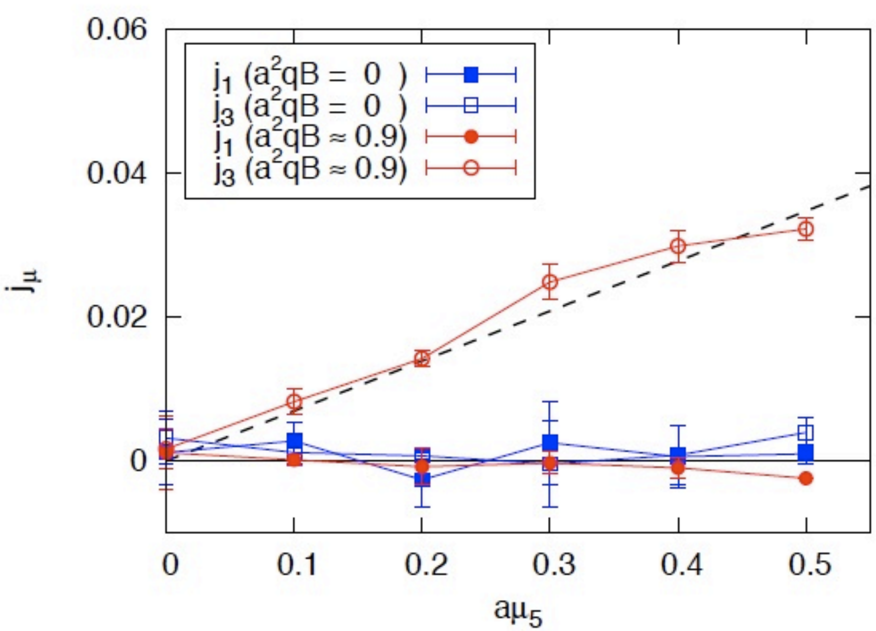
arXiv:1105.0385, PRL

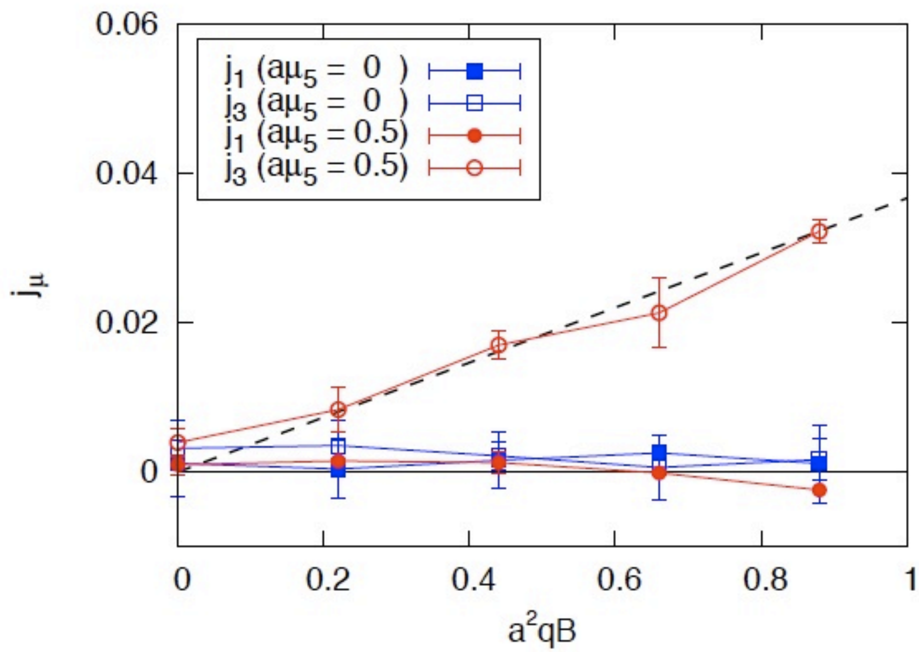
Chiral magnetic effect in lattice QCD with chiral chemical potential

Arata Yamamoto
Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

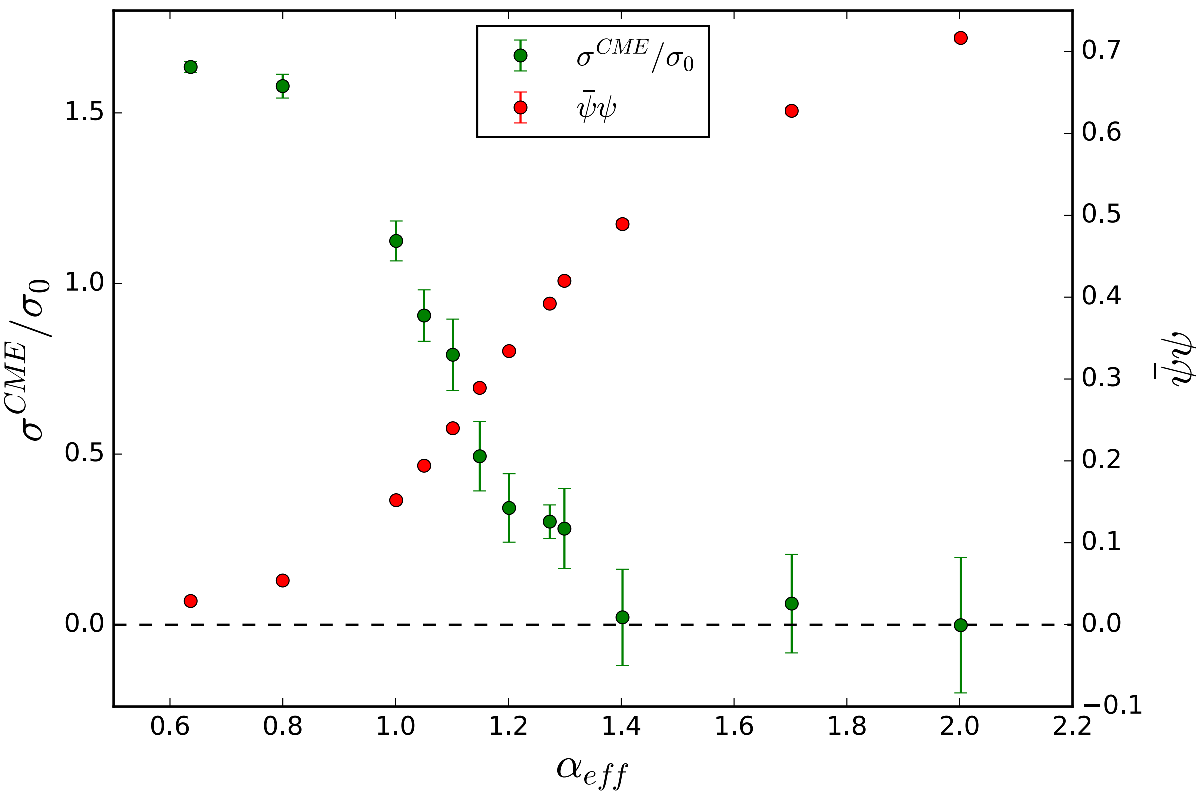
(Dated: May 3, 2011)

We perform a first lattice QCD simulation including two-flavor dynamical fermion with chiral chemical potential. Because the chiral chemical potential gives rise to no sign problem, we can exactly analyze a chirally asymmetric QCD matter by the Monte Carlo simulation. By applying an external magnetic field to this system, we obtain a finite induced current along the magnetic field, which corresponds to the chiral magnetic effect. The obtained induced current is proportional to the magnetic field and to the chiral chemical potential, which is consistent with an analytical prediction.





Chiral magnetic effect as a signature of chiral symmetry restoration



V.Braguta et al, arxiv:1704.07132, and to appear

The spontaneous breaking of chiral symmetry does not allow the chiral magnetic current to propagate

Systematics of anomalous conductivities

	Magnetic field	Vorticity
Vector current	$\frac{\mu_A}{2\pi^2}$	$\frac{\mu\mu_A}{2\pi^2}$
Axial current	$\frac{\mu}{2\pi^2}$	$\frac{\mu^2+\mu_A^2}{4\pi^2}+\frac{T^2}{12}$

Chiral magnetic conductivity: discrete symmetries

P – parity
T – time reversal

$$\vec{J} = \boxed{\frac{e^2}{2\pi^2}\,\mu_5}\,\vec{B}$$

P-odd
T-odd

P-even
T-odd

P-odd

P-odd effect!

T-even

Non-dissipative current!

(*reversible dynamics*)

Effect persists in hydrodynamics!

cf Ohmic conductivity:

$$\vec{J} = \sigma \vec{E}$$

T-odd, dissipative

CME vs superconductivity

Fritz and Heinz London

London theory of superconductors, '35:

$$\vec{\mathbf{J}} = -\lambda^{-2}\vec{\mathbf{A}} \qquad \nabla \cdot \vec{\mathbf{A}} = 0$$

$$\vec{\mathbf{E}} = -\dot{\vec{\mathbf{A}}}$$

$$\boxed{\vec{\mathbf{E}} = \lambda^2 \dot{\vec{\mathbf{J}}}}$$

Chiral anomaly:

$$\partial_\mu J^\mu_A = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

CME:

$$\vec{J} \sim \mu_5 \vec{B}$$

$$\mu_5 \sim \vec{E}\vec{B}\, t$$

for $\vec{E}||\vec{B}$

$$\boxed{\vec{\mathbf{E}} \sim \mathrm{B}^{-2}\, \dot{\vec{\mathbf{J}}}}$$

superconducting current, tunable by magnetic field!

DK, arXiv:1612.05677