Topological aspects of strong correlations and gauge theories,

TIFR, India, September 6-10, 2021

Chiral Matter

from quarks to quantum computers

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Stony Brook University

C^2QA
Co-design Center for Quantum Advantage

Brookhaven National Laboratory

Contents

Chirality: the definition

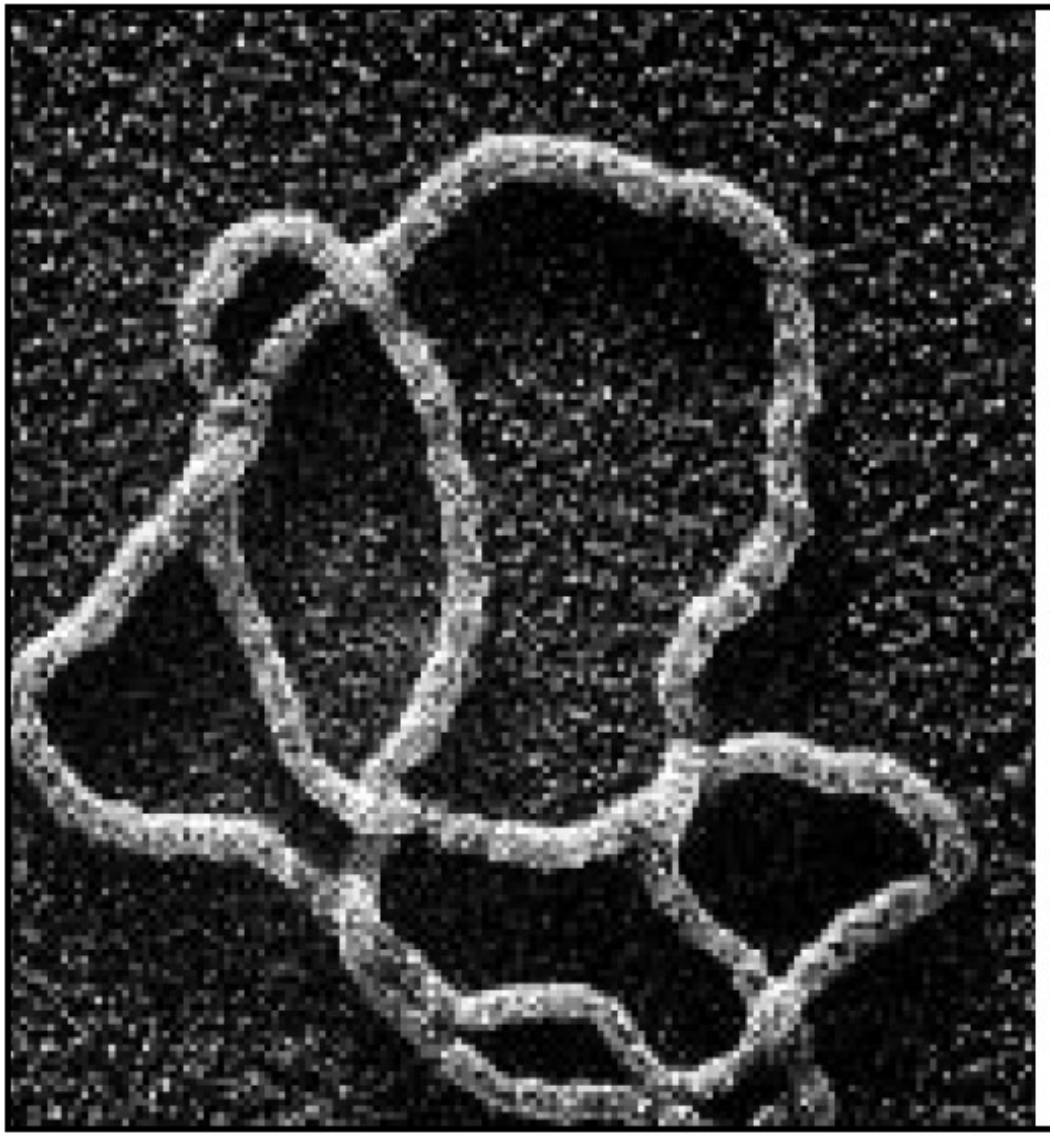
Greek word: χειρ (cheir) - hand

Lord Kelvin (1893):

"I call any geometrical figure, or groups of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself."

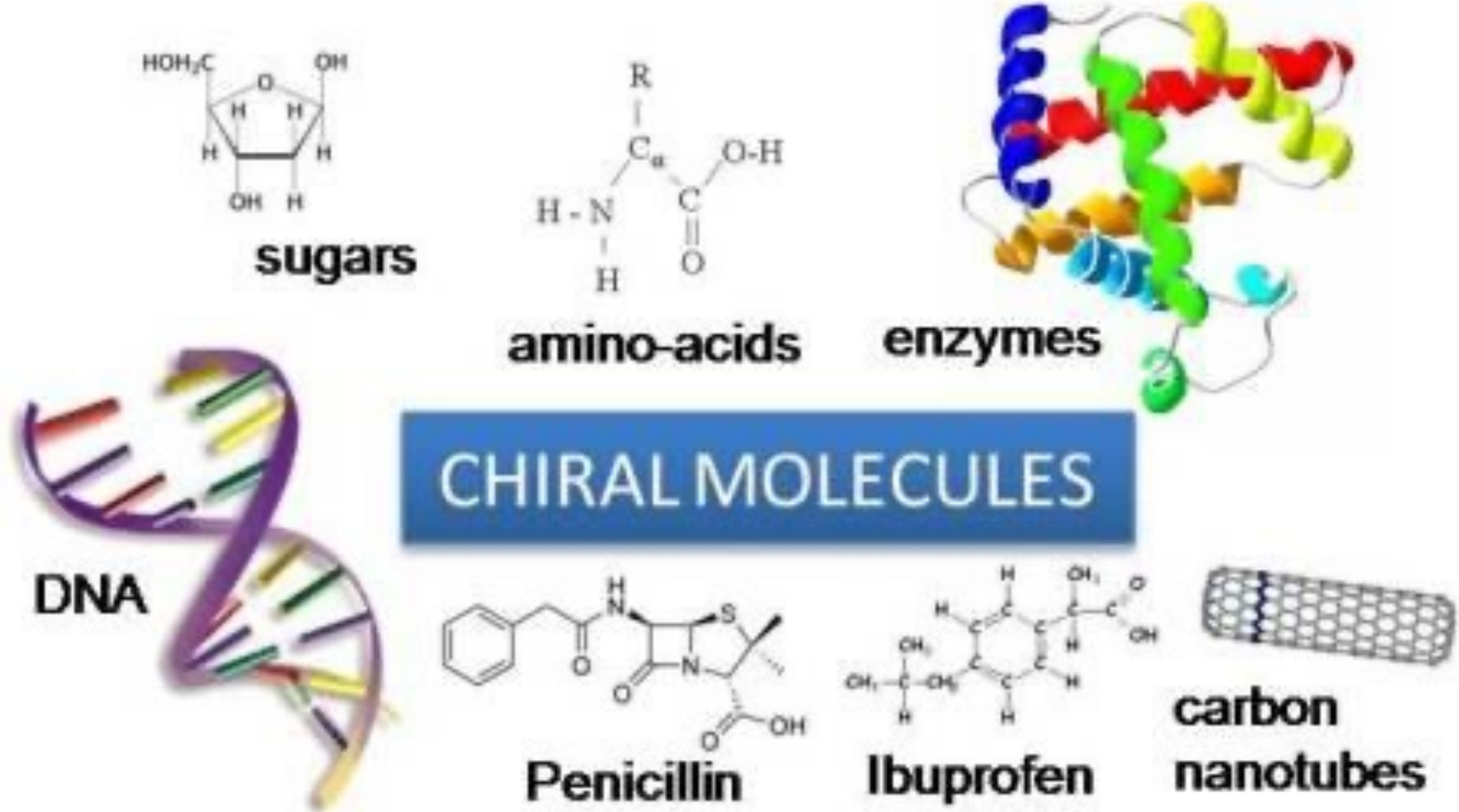
Chirality: DNA

Chirality: DNA



De Witt Sumners, Notices of the AMS, 1995

Living organisms contain almost only left-handed (LH) amino-acids and right-handed (RH) sugars – you would starve on LH sugar! (artificial sweeteners)



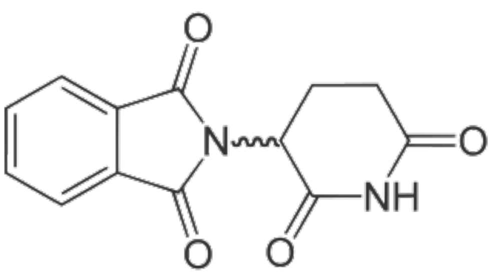
2500 chiral drugs! (most of the new)

www.rowland.harvard.edu/rjf/fischer

Thalidomide

W. White, “**Breaking Bad**”, Season 1, episode 2

1962: FDA pharmacologist Frances Oldham Kelsey receives the President's Award for Distinguished Federal Civilian Service from President John F. Kennedy for blocking sale of thalidomide in the United States.



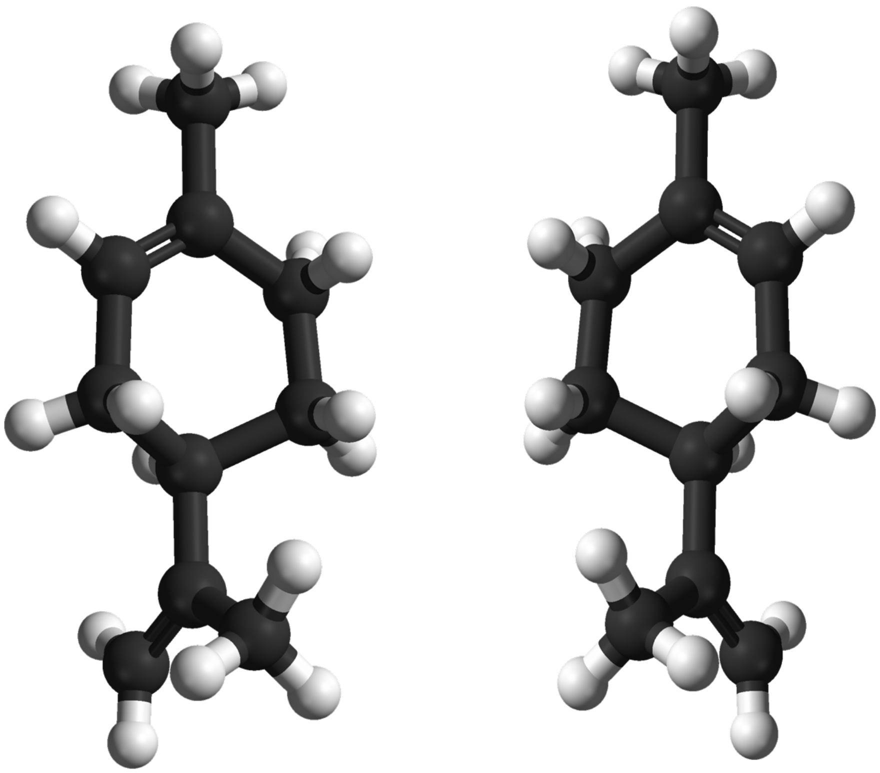
Left-handed molecule is a drug;
right-handed is a poison:
birth defects, 50% death rate.

Synthetic: equal mixture

Can you smell chirality?

Limonene

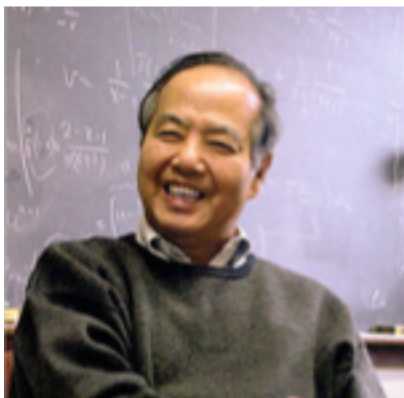
Left: Right:



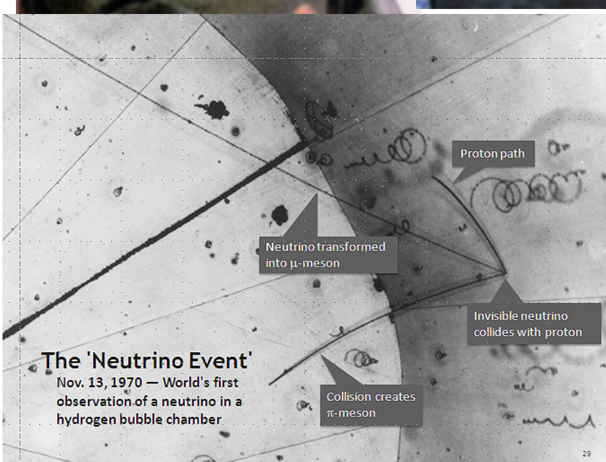
hydrocarbon

Chirality in subatomic world as the origin of left-right asymmetry of life?

Weak interactions violate parity (T.D. Lee and C.N. Yang)
(symmetry between left and right)




Weak interactions thus induce tiny differences between the binding energies of left- and right-handed molecules



A.Salam: this could be at the origin of left-right asymmetry of life

⚠ highly controversial

The chirality of life

Force of nature gave life its asymmetry

'Left-handed' electrons destroy certain organic molecules faster than their mirror versions.

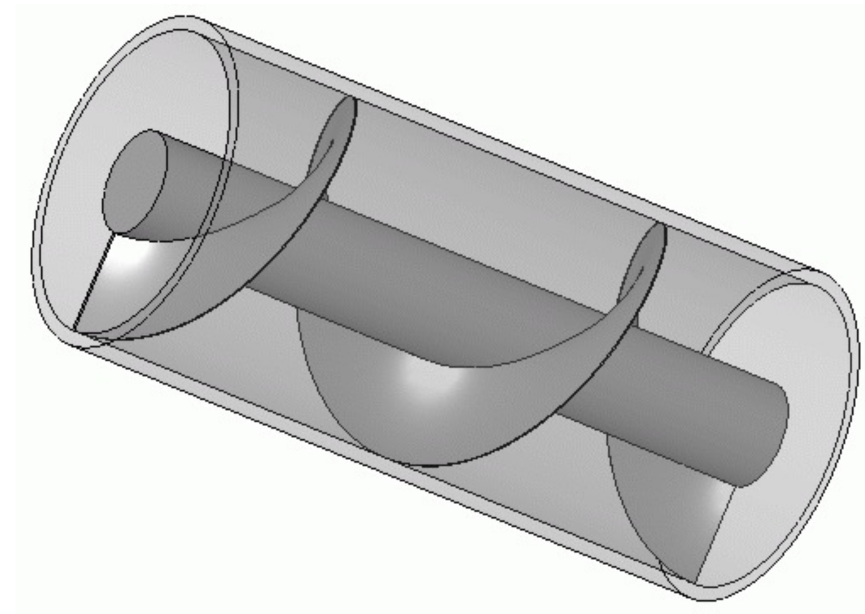
E. Gibney, Nature, 25 September 2014

Chirally Sensitive Electron-Induced Molecular Breakup and the Vester-Ulbricht Hypothesis

Phys. Rev. Lett. 113, 118103 (2014)

J. M. Dreiling and T. J. Gay

Chirality and hydrodynamics 240 B.C.



The Archimedes screw

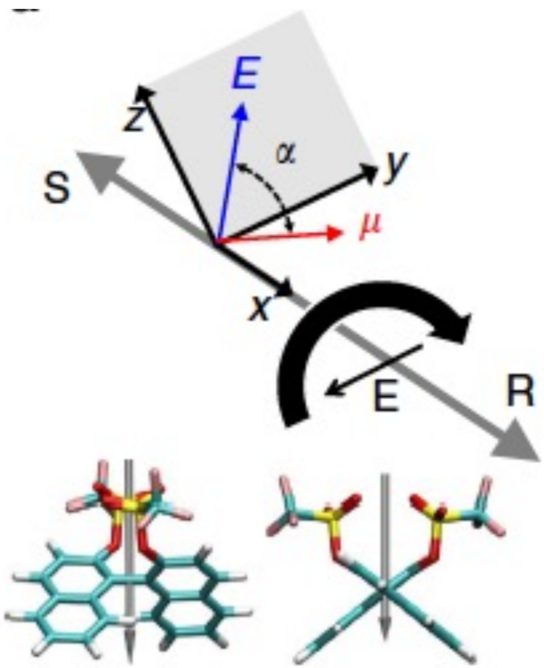
Chiral propulsion

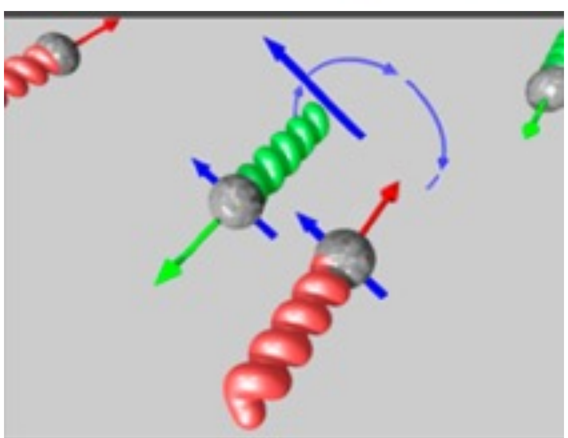
$$\vec{V} \sim \vec{\Omega} \quad ?$$

vector — pseudo-vector

Velocity parallel to angular velocity requires the breaking of parity

Propeller effect in a fluid



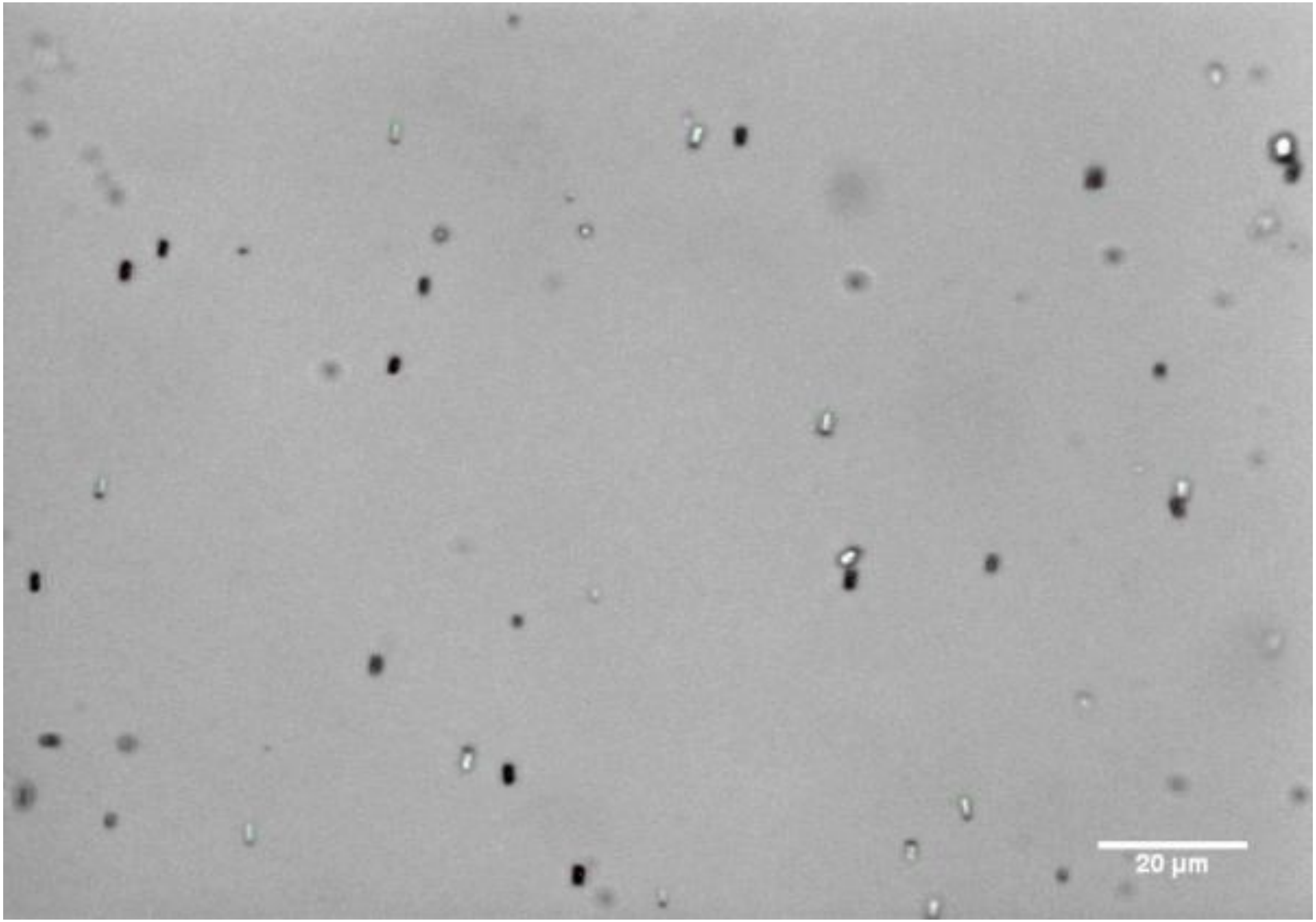


How to rotate the chiral molecule in a fluid?

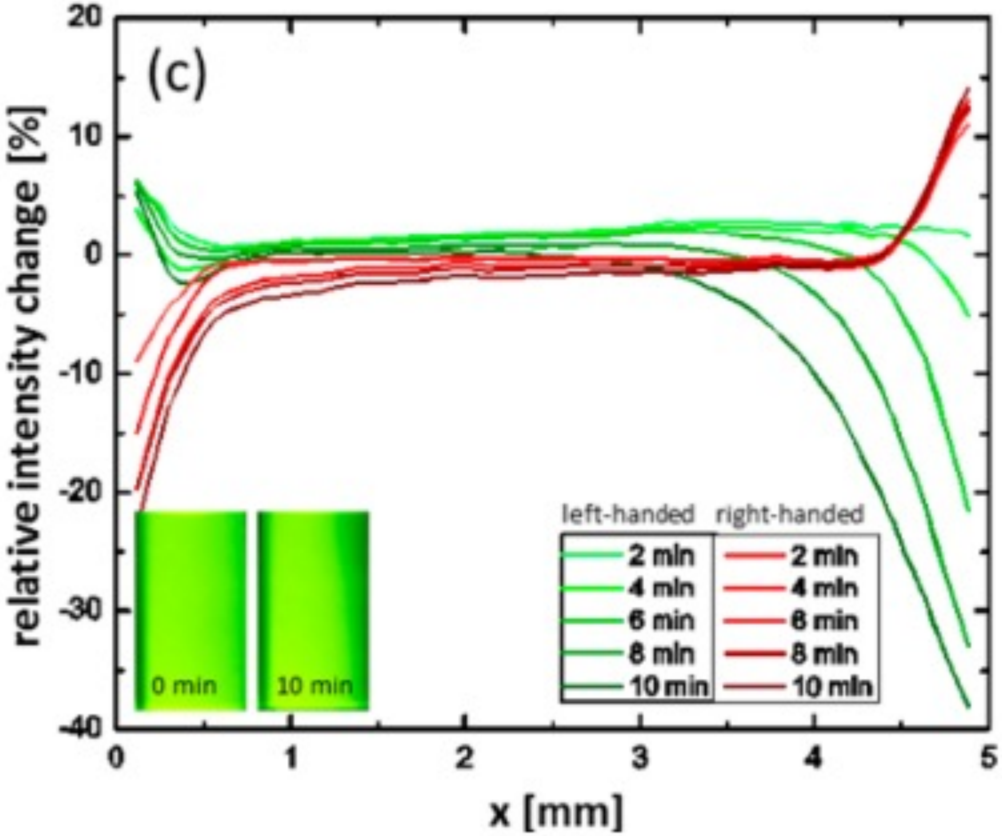
Use the coupling of an external electric field to the molecule's electric dipole moment!

Rotating electric field – rotating molecule

Baranova, Zel'dovich '78



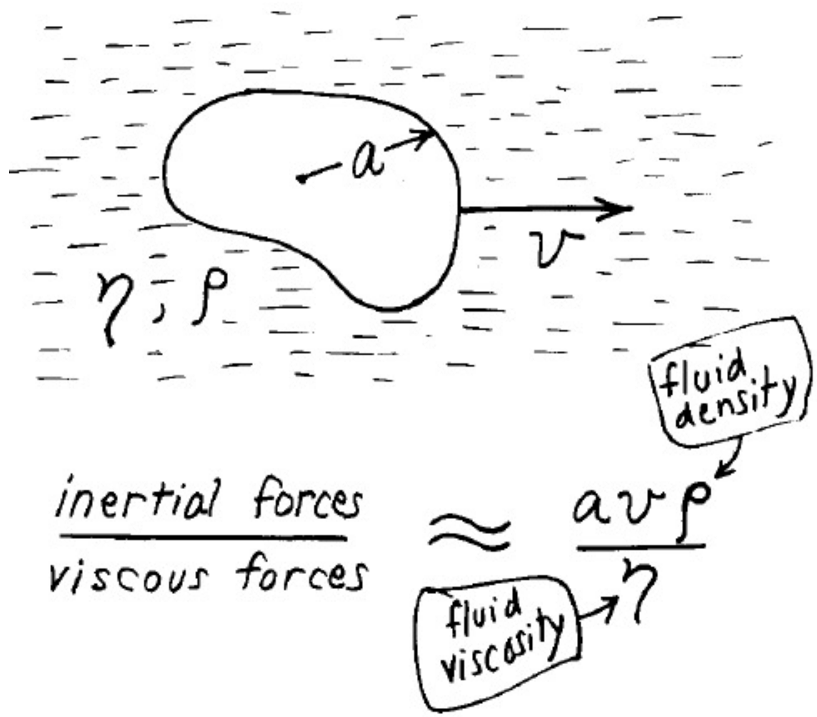
D.Schamel et al, JACS 135, 12353 (2013)



D.Schamel et al, JACS 135, 12353 (2013)

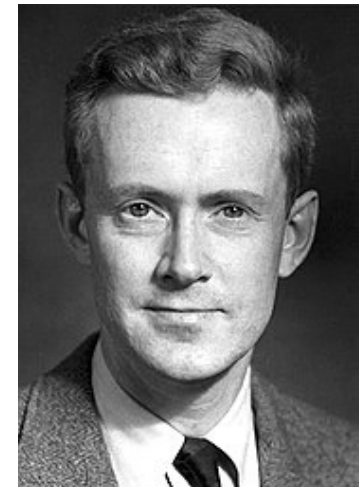
Life at low Reynolds number

E. M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 12 June 1976)



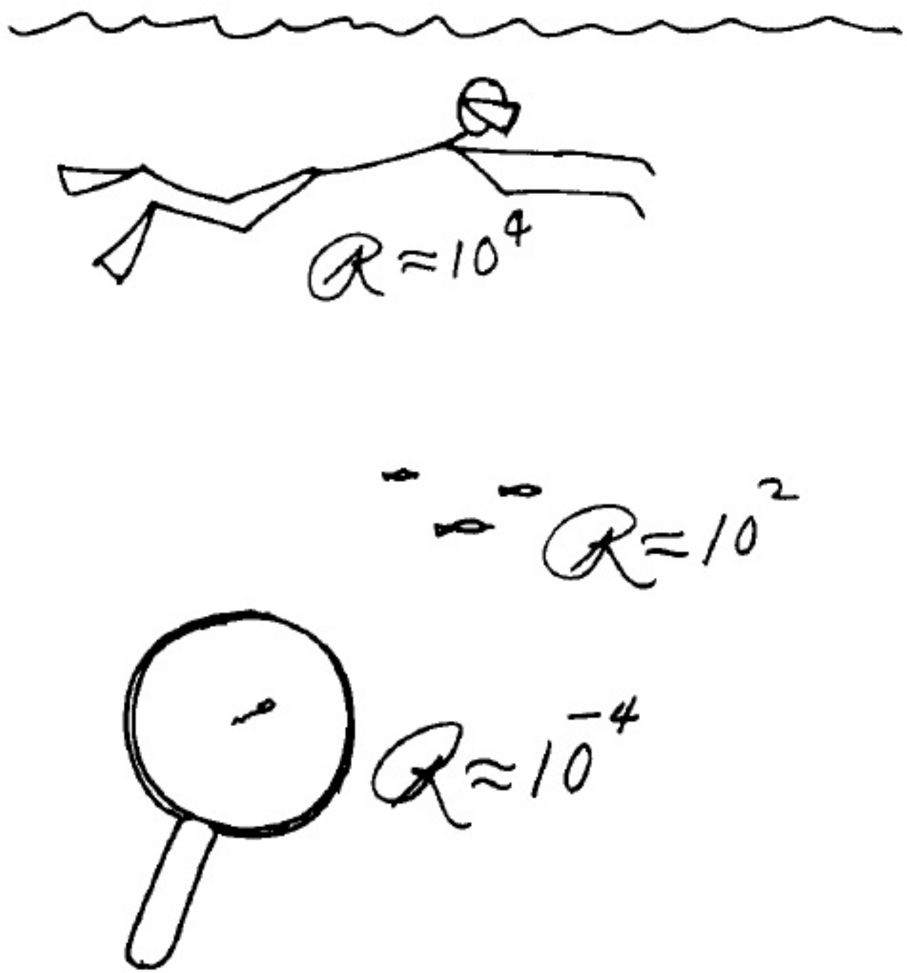
$$\mathcal{R} \equiv \frac{av\rho}{\eta} = \frac{av}{\nu}$$

$\nu = 10^{-2}\ \frac{cm^2}{sec}$ for water



E.M.Purcell
(1912-1997)

Nobel prize, 1952
(Nuclear Magnetic
Resonance)


$\mathcal{R} \approx 10^4$
$\mathcal{R} \approx 10^2$
$\mathcal{R} \approx 10^{-4}$

Navier - Stokes:

$$-\nabla p + \eta \nabla^2 \vec{v} = \cancel{\rho \frac{\partial \vec{v}}{\partial t}} + \cancel{\rho (\vec{v} \cdot \nabla) \vec{v}}$$

Stokes equation ("creeping flow")

T-invariance!

If $\mathcal{R} \ll 1$:

Time doesn't matter. The pattern of motion is the same, whether slow or fast, whether forward or backward in time.

Sir G. Stokes (1819-1903)

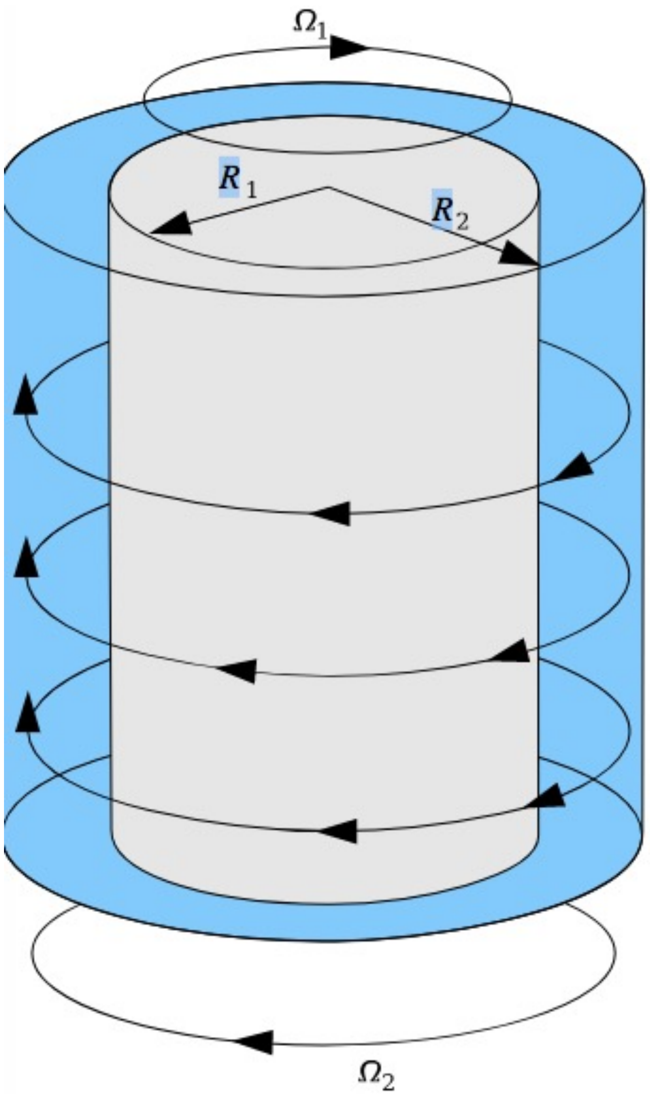
The Scallop Theorem

Geometry of the gauge field on the space of shapes

A.Shapere, F.Wilczek '88

Time reversability in hydrodynamics

Couette flow



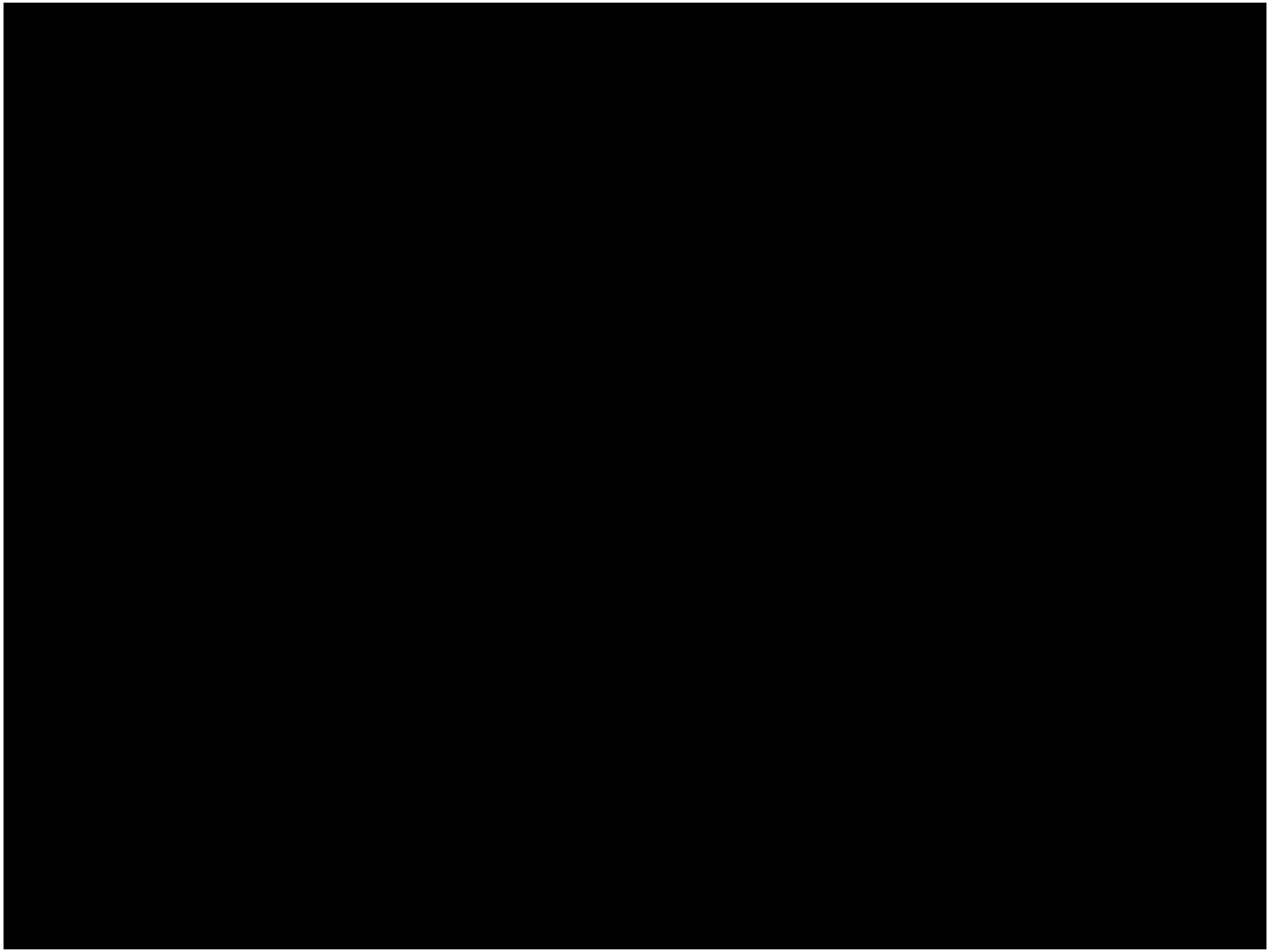
M.Couette
1858-1943

Born: Tours, France

Time reversability in hydrodynamics

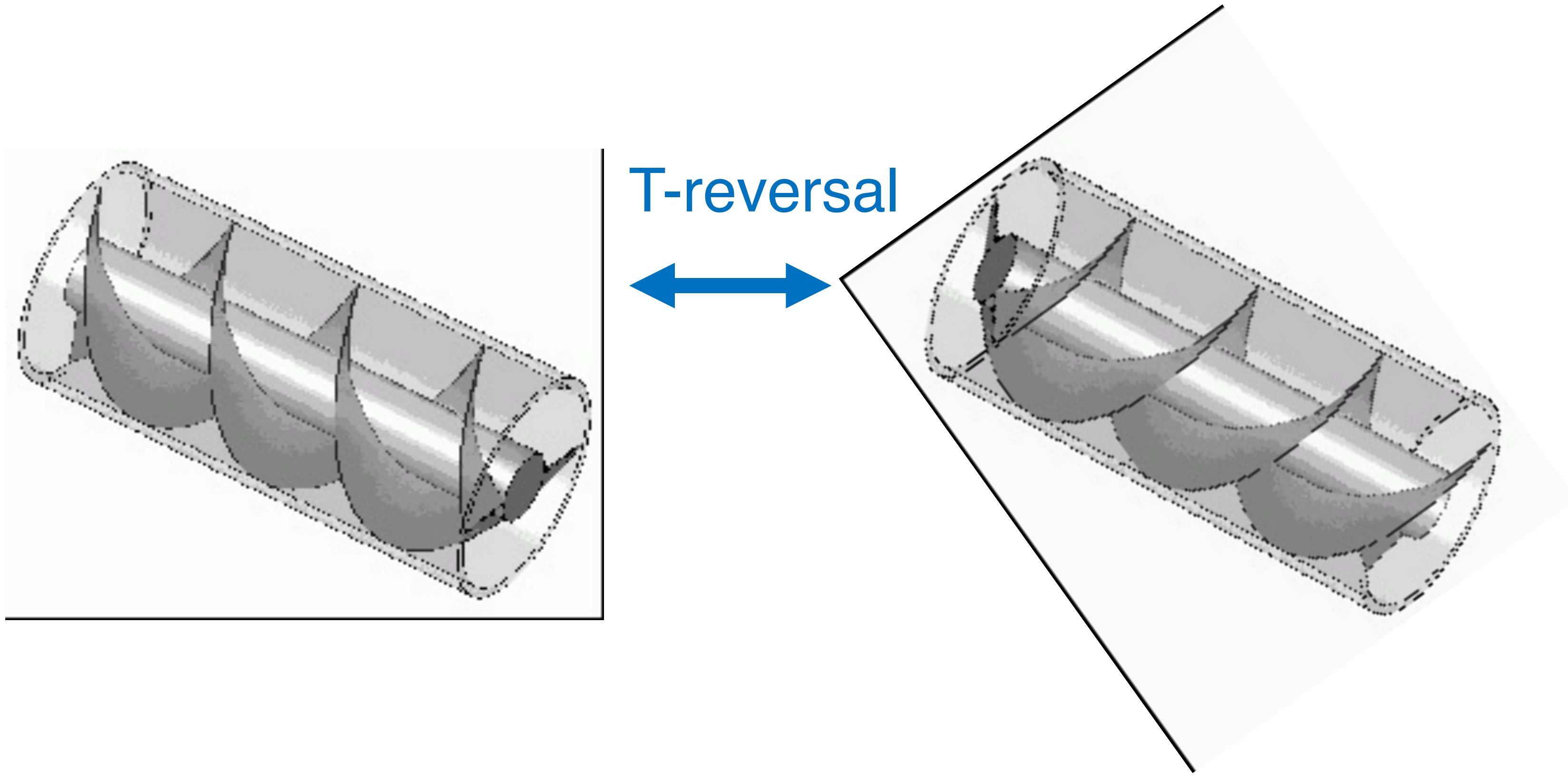
Couette flow

Video credit: UNM

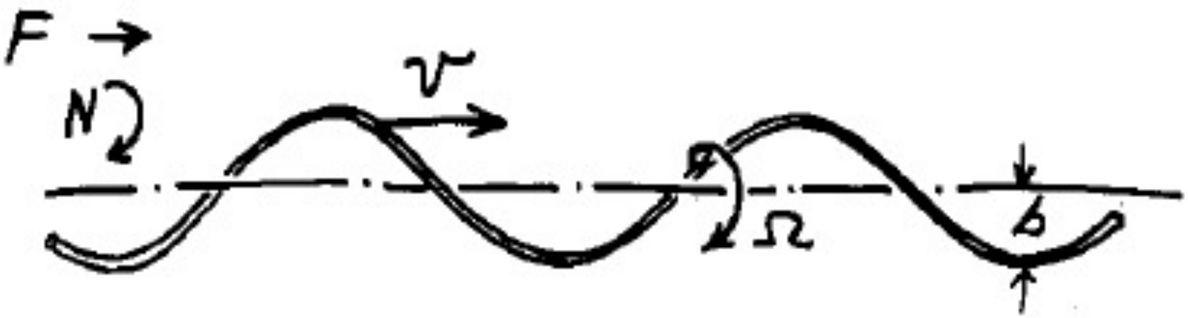


Need to break T-invariance to move – chirality!

Left-handed screw



The propulsion matrix



(force) $F = Av + B\Omega$

(torque) $N = Cv + D\Omega$

$$|P| \equiv \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

Chirality imbalance = transport

Chiral propulsion:

the Green's function approach

The Stokes equation with a point force:

$$-\nabla P + \nu \nabla^2 \mathbf{v} = -\mathbf{F}\ \delta(\mathbf{r})$$

The Green's function ("Stokeslet"):

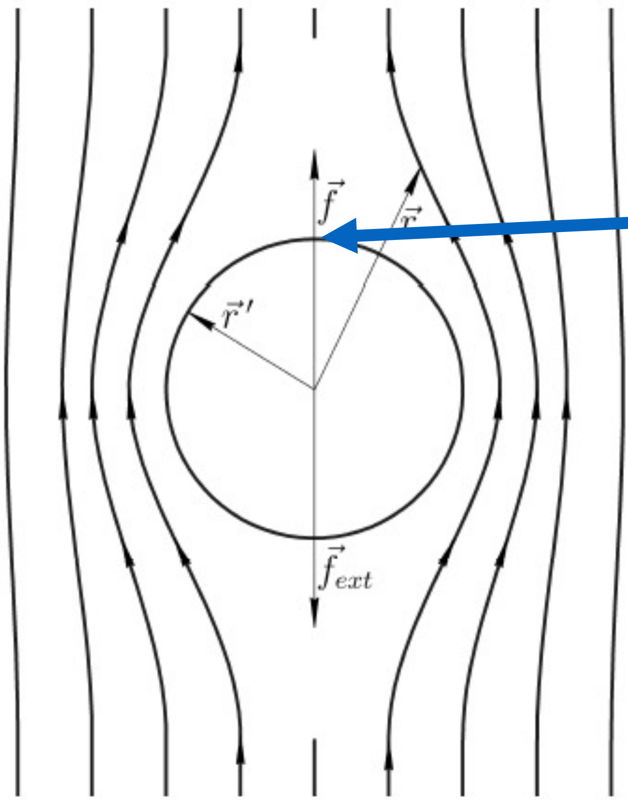
$$G_{ij}(\mathbf{r}) = \frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3}$$

The solution for an arbitrary moving body:

$$v_i(\mathbf{r}) = \frac{1}{8\pi\nu} \int_S G_{ij}(\mathbf{r} - \mathbf{r}')f_j(r')d\sigma'$$

Sphere in a viscous fluid: the Stokes' law

$$v_i(\mathbf{r}) = \frac{1}{8\pi\nu}\int_S G_{ij}(\mathbf{r}-\mathbf{r}')f_j(r')d\sigma'$$



Let us evaluate the fluid velocity here; due to the "non-slip" boundary condition, this will be the velocity of the sphere.

Even though the Green's function is singular, parity symmetry of the sphere leads to the cancellation of the divergent terms:

S.Ayf, I.Kuk, DK, arXiv:1804.08664

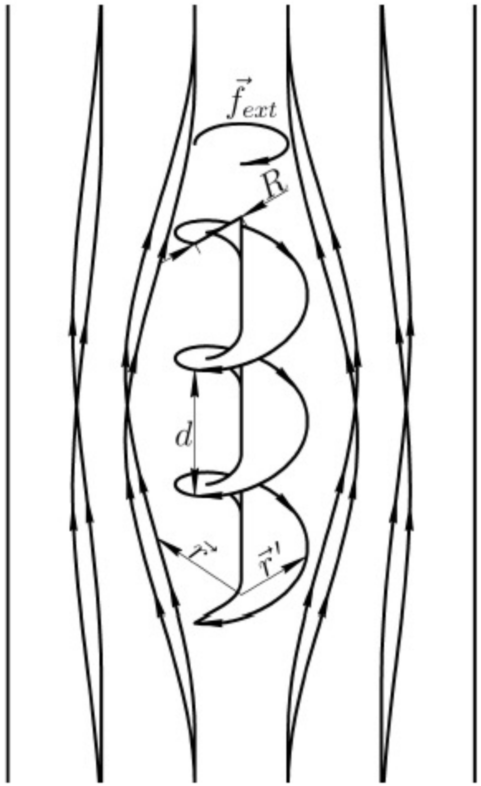
$$v_z^{sphere} = \lim_{z\to R} v_z = \lim_{z\to R}\frac{F_z(R^2-3z^2)}{12\pi\eta z^3} = -\frac{F_z}{6\pi\eta R} = \frac{F_z^{ext}}{6\pi\eta R}$$

Sphere in a viscous fluid: the Stokes' law

$$v_z^{sphere} = \frac{F_z^{ext}}{6\pi\eta R}$$

For the sphere (an achiral object),
we have encountered no divergences,
but the final result contains the shear viscosity –
an UV cutoff of hydrodynamics

Propeller in a viscous fluid



This time, no parity symmetry,
and thus no cancelation of divergence:

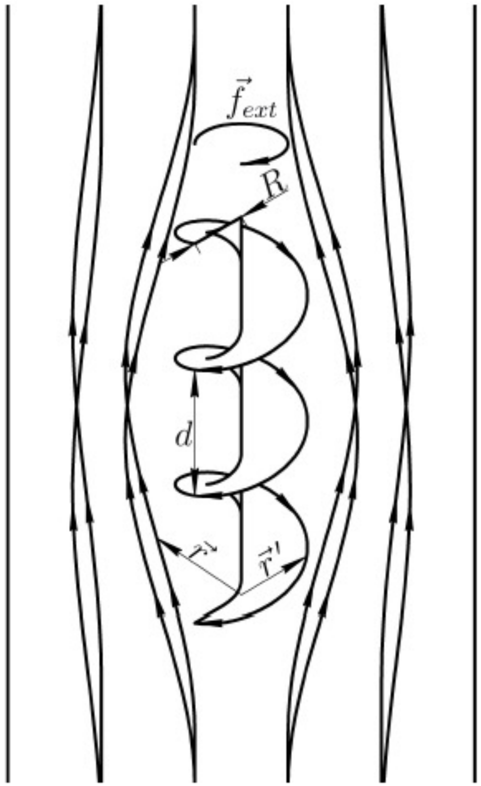
$$v_z(z) = \frac{1}{8\pi\eta}\int\int \frac{(z-z')(r_j - r'_j) f_j(\mathbf{r})'}{(\rho^2+\epsilon^2)^{\frac{3}{2}}} d\sigma'$$

However, the same divergence appears for other components (torque):

$$v_y(x) = \frac{1}{8\pi\eta}\iint\left(\frac{f_y(\mathbf{r}')(\rho^2+2\epsilon^2)}{\rho^2+\epsilon^2} + \right.$$
$$\left. + \frac{(x'y' - xy')f_x(\mathbf{r}')}{(\rho^2+\epsilon^2)^{\frac{3}{2}}} + \frac{y'y' f_y(\mathbf{r}')}{(\rho^2+\epsilon^2)^{\frac{3}{2}}}\right) d\sigma',$$

S.Ayf, I.Kuk, DK,
arXiv:1804.08664

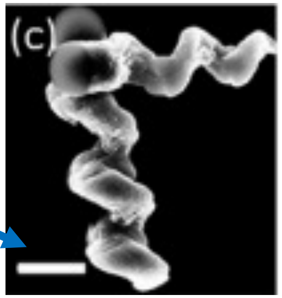
Propeller in a viscous fluid



Therefore, the final expression for the propulsion velocity expressed in terms of the angular velocity

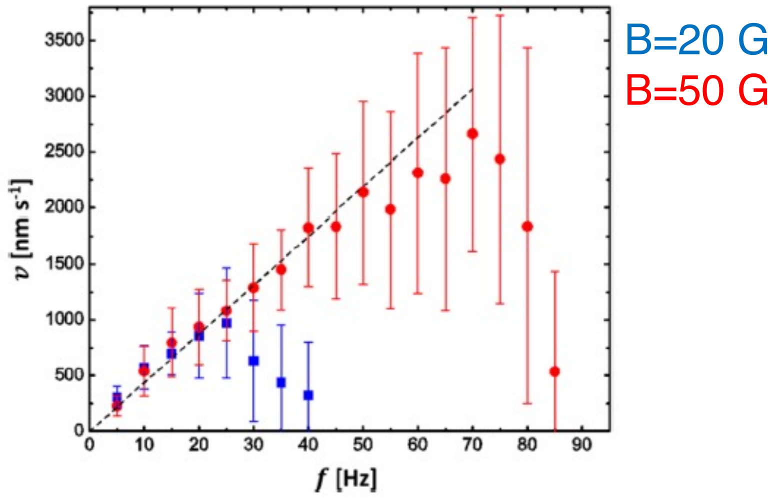
$$v_z = \alpha\ \omega$$

contains no divergences and does not depend on viscosity of the fluid!

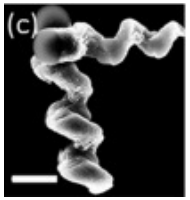


0.5 µm

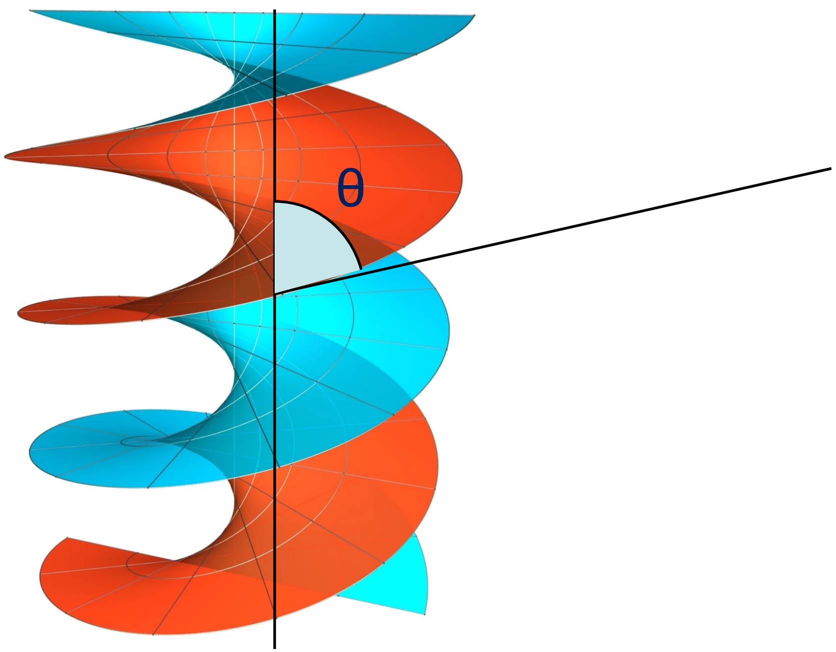
S.Ayf, I.Kuk, DK, arXiv:1804.08664



Chiral propulsion



Chiral propulsion in the Stokes regime
is determined entirely by geometry.
What is the value of the pitch angle θ that is optimal for chiral propulsion?



A very simple answer:

$$\alpha \sim \cos\theta \;\; \sin 2\theta$$

$$\theta = \tan^{-1}(1/\sqrt{2}) \simeq 35.26^\circ$$

L.Korneev, DK, A.Abanov
arXiv:2105.12181 [physics.flu-dyn]
Phys.Fluids ‘21

Motility modes of Spiroplasma melliferum BC3: A helical, wall-less bacterium driven by a linear motor

March 2003 · Molecular Microbiology 47(3):657-69

DOI:10.1046/j.1365-2958.2003.03200.x

Source · PubMed

$$\theta = \tan^{-1}(1/\sqrt{2}) \simeq 35.26^\circ$$

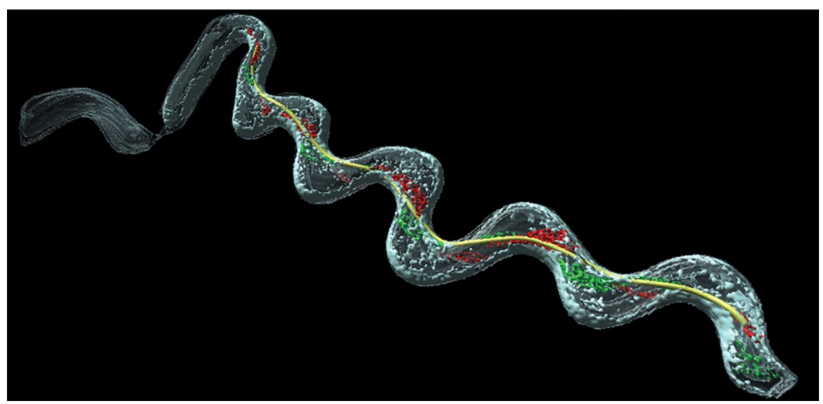
Authors:

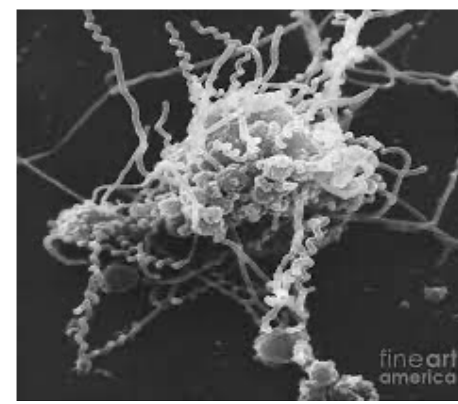
Rami Gilad

Asher Porat

Shlomo Trachtenberg

L.Korneev, DK, A.Abanov
arXiv:2105.12181 [physics.flu-dyn]





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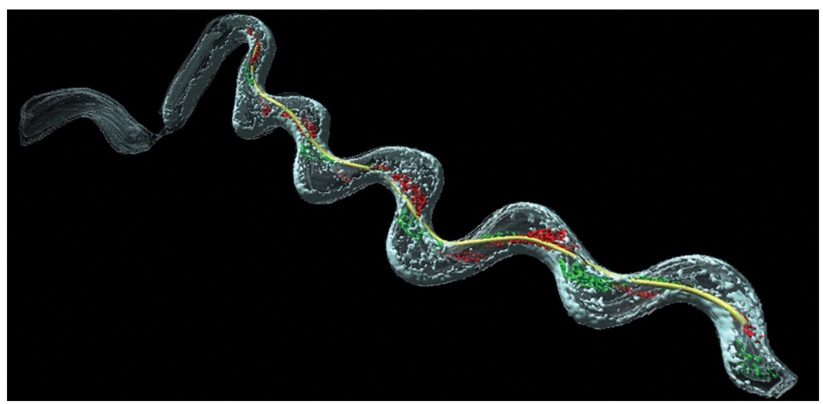
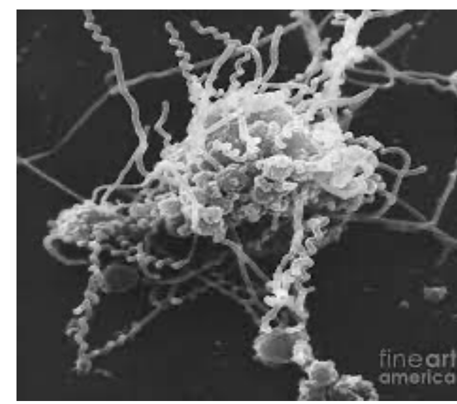
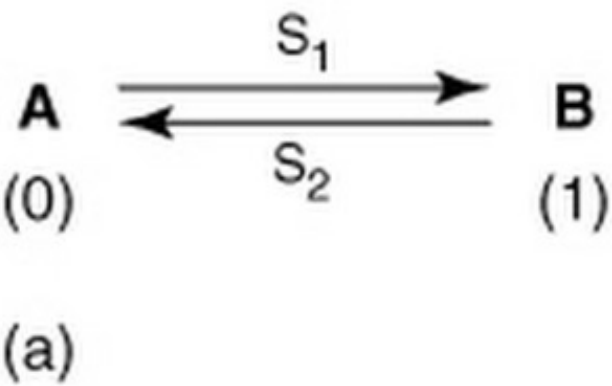


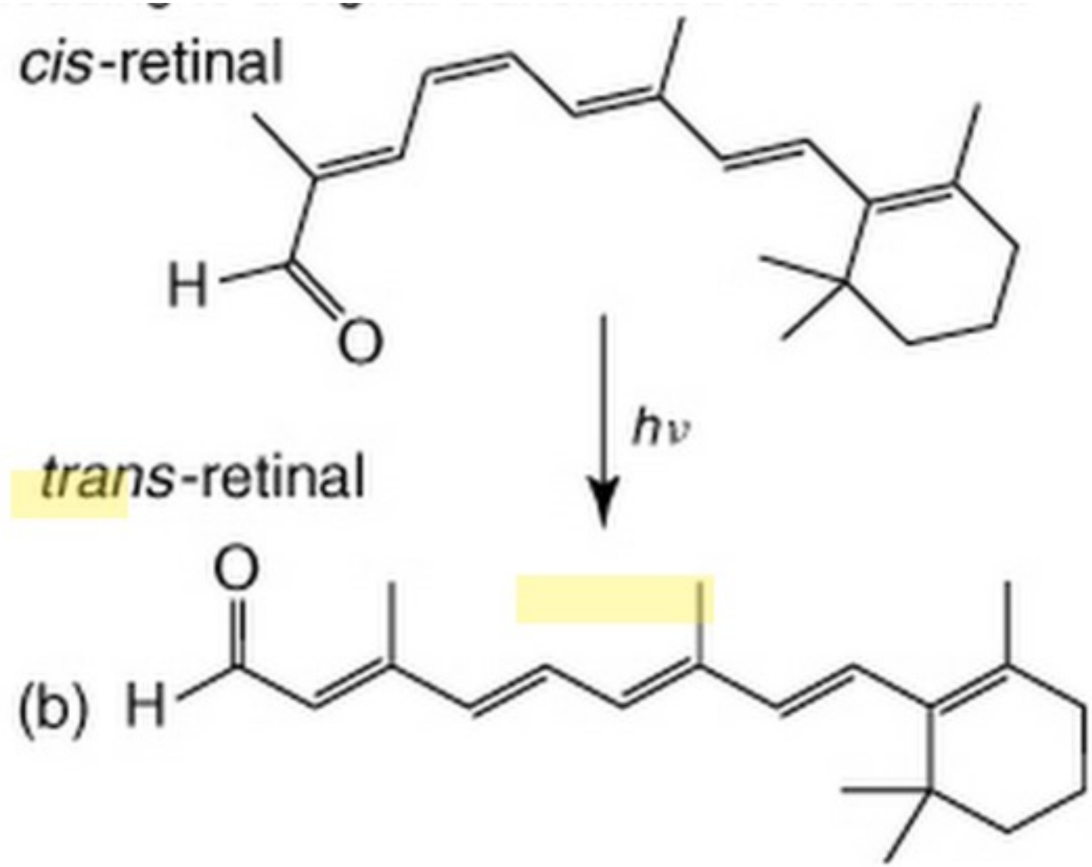


"…helical pitch angle observed for the helical Spiroplasma melliferum is 35 degrees…"

Chirality and vision



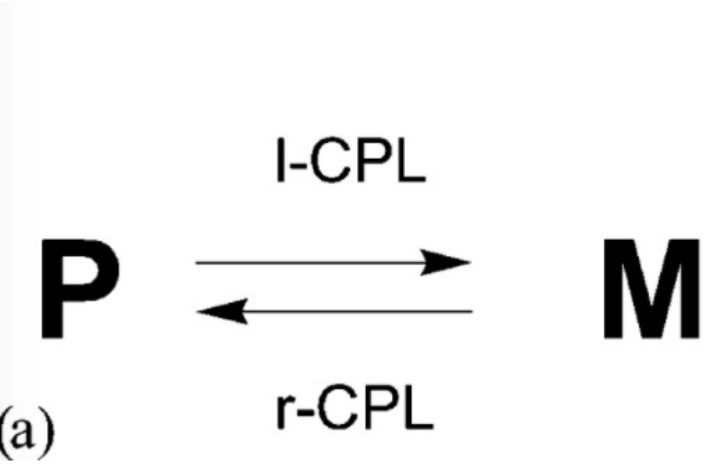
B. Feringa, W. Browne,
“Molecular switches”, 2011

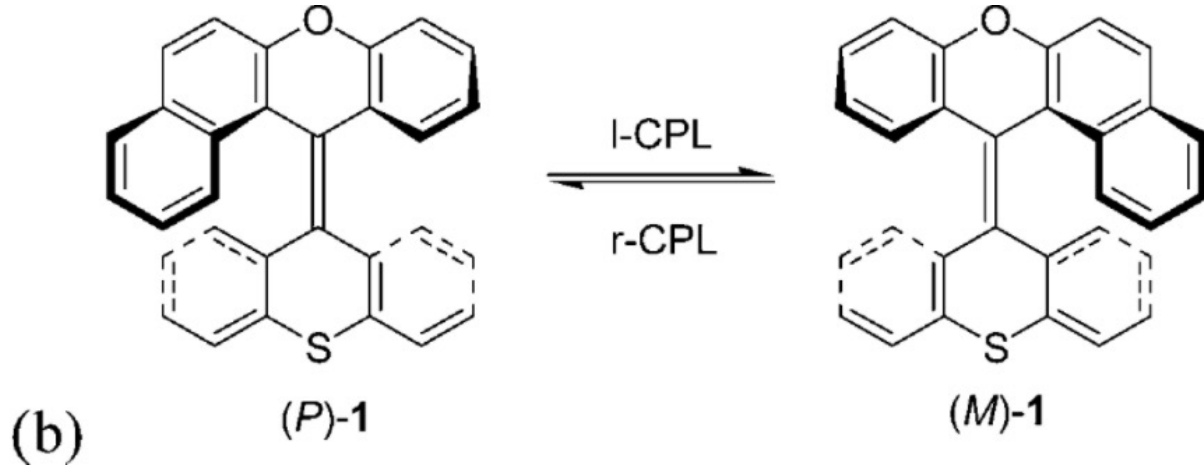


Protein-bound retinal molecule as a chiroptical switch: the mechanism of vision   chiral photo-pharmacology

Chiroptical switching of chiral molecules

Need circularly polarized light (CPL), with frequency optimized for inducing the tunnel transition between the enantiomers.





Review: B.Feringa, *J. Org. Chem.*, **2007**, *72* (18), pp 6635–6652

Applications: pharmaceutics, “chiral photomedicine”, optical data storage and processing, …

Scientific Background on the Nobel Prize in Chemistry 2016

MOLECULAR MACHINES

J.-P.Sauvage

Sir J.F.Stoddart

B.L. Feringa

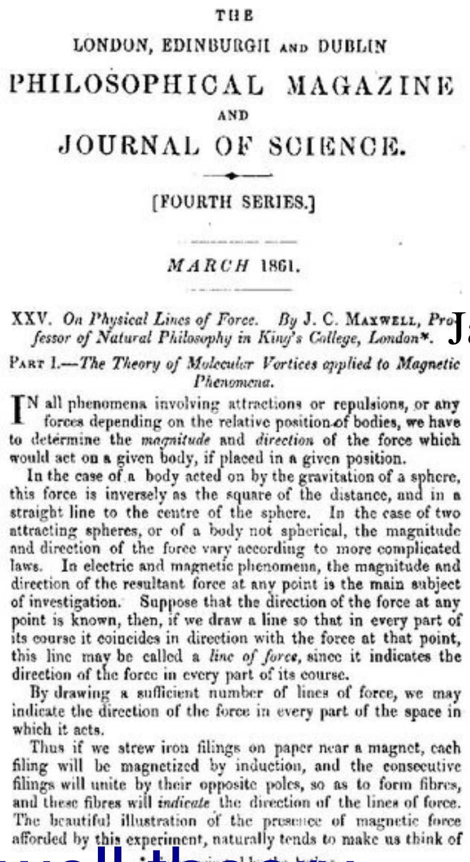
Light and electromagnetism

James C. Maxwell, 1831-1879

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$



THE
LONDON, EDINBURGH AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FOURTH SERIES.]

MARCH 1861.

XXV. *On Physical Lines of Force.* *By* J. C. MAXWELL, *Professor of Natural Philosophy in King's College, London*.*

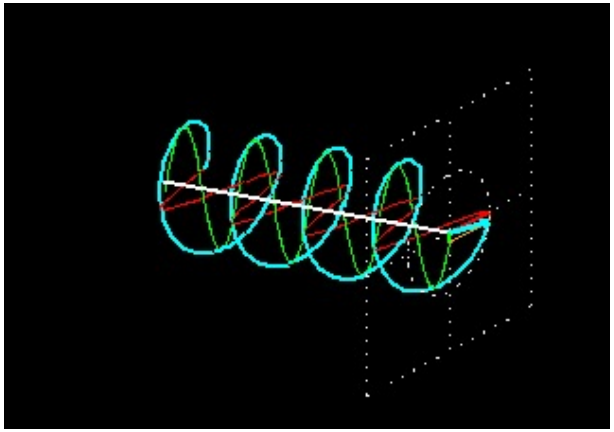
PART I.—*The Theory of Molecular Vortices applied to Magnetic Phenomena.*

IN all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the *magnitude* and *direction* of the force which would act on a given body, if placed in a given position.

In the case of a body acted on by the gravitation of a sphere, this force is inversely as the square of the distance, and in a straight line to the centre of the sphere. In the case of two attracting spheres, or of a body not spherical, the magnitude and direction of the force vary according to more complicated laws. In electric and magnetic phenomena, the magnitude and direction of the resultant force at any point is the main subject of investigation. Suppose that the direction of the force at any point is known, then, if we draw a line so that in every part of its course it coincides in direction with the force at that point, this line may be called a *line of force*, since it indicates the direction of the force in every part of its course.

By drawing a sufficient number of lines of force, we may indicate the direction of the force in every part of the space in which it acts.

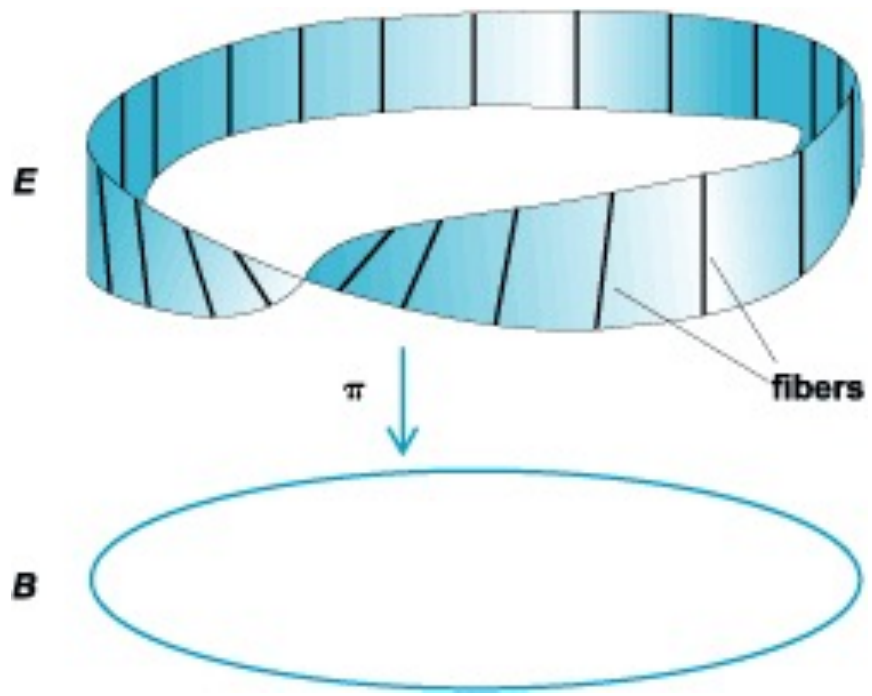
Thus if we strew iron filings on paper near a magnet, each filing will be magnetized by induction, and the consecutive filings will unite by their opposite poles, so as to form fibres, and these fibres will *indicate* the direction of the lines of force. The beautiful illustration of the presence of magnetic force afforded by this experiment, naturally tends to make us think of



Maxwell theory is left-right symmetric

Michael Faraday, 1791-1867

Gauge fields and topology

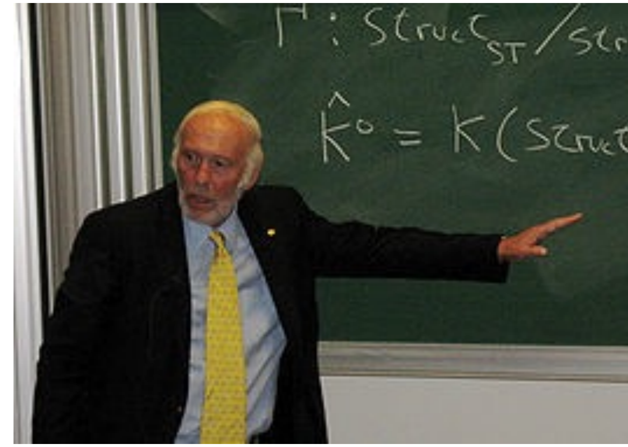


NB: Maxwell electrodynamics as a curvature of a line bundle

Möbius strip, the simplest nontrivial example of a fiber bundle

Gauge theories "live" in a fiber bundle space that possesses non-trivial topology (knots, links, twists,...)

Chern-Simons forms



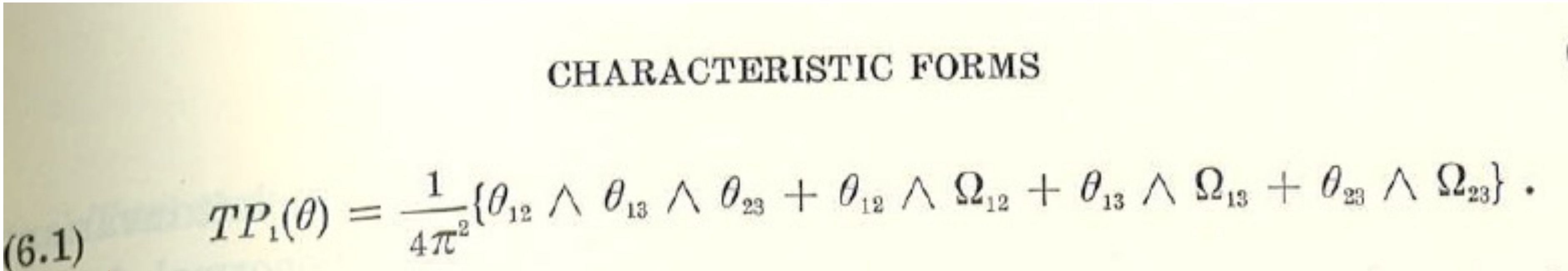
6. Applications to 3-manifolds

In this section M will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection θ and curvature tensor Ω. For A, B skew symmetric matrices, the specific formula for P_1 shows $P_1(A \otimes B) = -(1/8\pi^2)\ \mathrm{tr}\ AB$. Calculating from (3.5) shows

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\}\,. \tag{6.1}$$

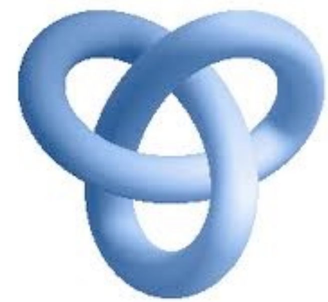
What does it mean for a gauge theory?

Chern-Simons theory


CHARACTERISTIC FORMS

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} . \quad (6.1)$$

What does it mean for electromagnetism?



Geometry		Physics
Riemannian connection	⟷	Gauge field
Curvature tensor	⟷	Field strength tensor

$$S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)$$

“magnetic helicity”

Chern-Simons form and circularly polarized light

How to describe the helicity of the circularly polarized light?

Magnetic helicity itself does not obey electric-magnetic symmetry of Maxwell equations in vacuum:

$$\mathbf{E} \to \cos\theta\ \mathbf{E} + \sin\theta\ \mathbf{B}$$

$$\mathbf{B} \to \cos\theta\ \mathbf{B} - \sin\theta\ \mathbf{E}$$

Heaviside, 1892
Larmor, 1897

We can however enforce this symmetry by introducing, in addition to the magnetic helicity, the dual pseudovector gauge potential C. In Coulomb gauge C is defined by:

Bateman, 1915

$$\nabla \mathbf{A} = \nabla \mathbf{C} = 0$$

$$\mathbf{E} = -\nabla \times \mathbf{C} = -\dot{\mathbf{A}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\dot{\mathbf{C}}$$

Optical helicity of the circularly polarized light

Electric-magnetic transformation

$$\mathbf{E} \to \cos\theta\ \mathbf{E} + \sin\theta\ \mathbf{B}$$

$$\mathbf{B} \to \cos\theta\ \mathbf{B} - \sin\theta\ \mathbf{E}$$

is induced by

$$\mathbf{A} \to \cos\theta\ \mathbf{A} + \sin\theta\ \mathbf{C}$$

$$\mathbf{C} \to \cos\theta\ \mathbf{C} - \sin\theta\ \mathbf{A}$$

We can now define the **optical helicity** by adding CS terms for A and C:

$$H \equiv \frac{1}{2}\int d^3x\,(\mathbf{A}\cdot(\nabla\times\mathbf{A}) + \mathbf{C}\cdot(\nabla\times\mathbf{C})) = \frac{1}{2}\int d^3x\,(\mathbf{A}\cdot\mathbf{B} - \mathbf{C}\cdot\mathbf{E})$$

Candlin, 1965; Trueba,Ranada, 1996; Afanasiev,Stepanovsky, 1996; Cameron, Barnett, Yao, 2012

Optical helicity of the circularly polarized light

The optical helicity

$$H = \frac{1}{2}\int d^3x\,(\mathbf{A}\cdot\mathbf{B} - \mathbf{C}\cdot\mathbf{E})$$

is invariant under electric-magnetic symmetry

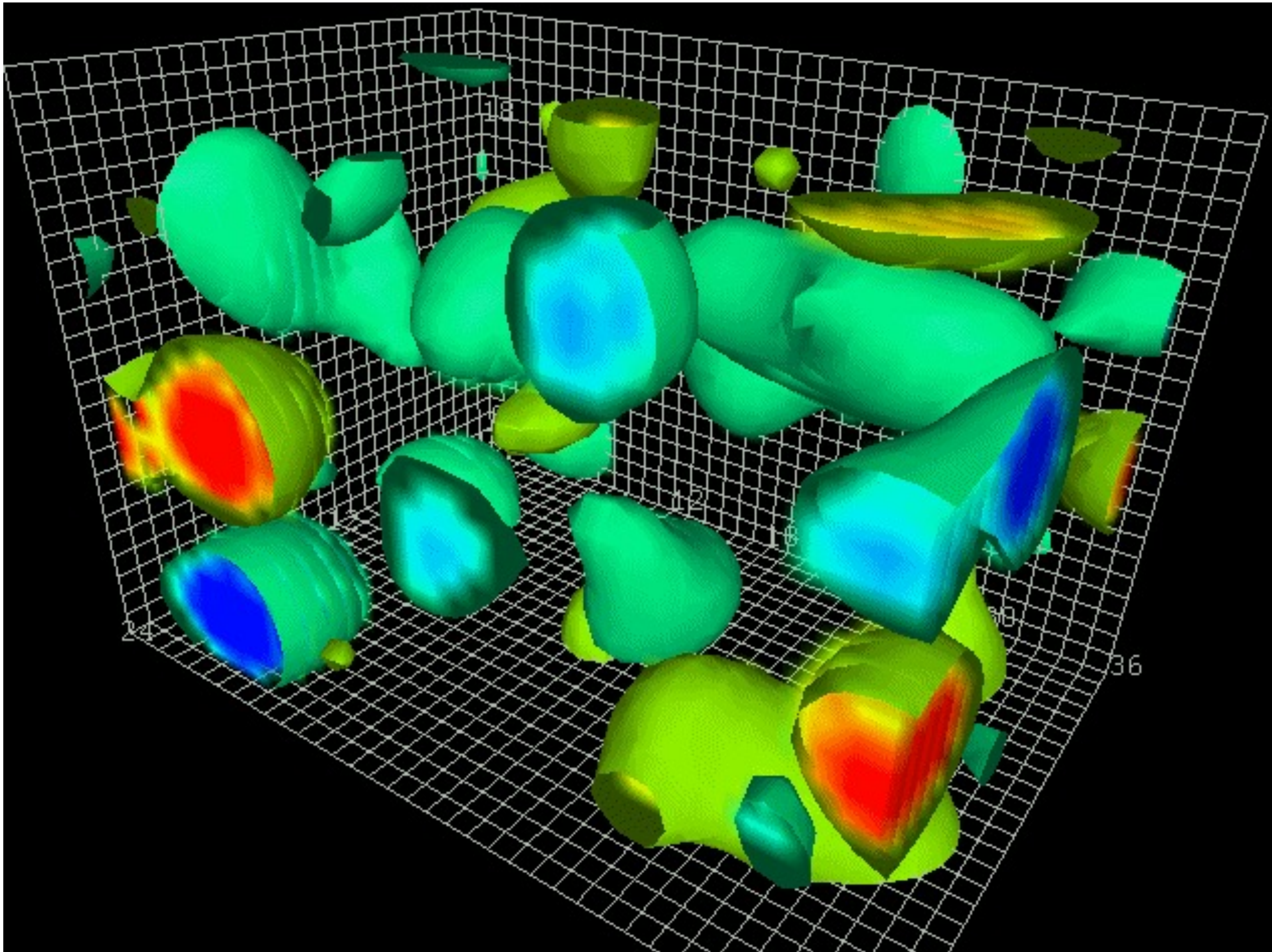
$$\mathbf{A} \to \cos\theta\ \mathbf{A} + \sin\theta\ \mathbf{C}$$

$$\mathbf{C} \to \cos\theta\ \mathbf{C} - \sin\theta\ \mathbf{A}$$

It is a T-even, P-odd quantity that is conserved *in the absence of interactions with chiral (P-odd) matter*:

$$\frac{dH}{dt} = 0$$

Topological number fluctuations in QCD vacuum (3+1)d
("cooled" configurations)



D. Leinweber

“Topological foam” in QCD vacuum, (3+1) Dimensions

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