Primordial black holes and small-scale gravitational waves in two-field models of inflation

Dhiraj Kumar Hazra, IMSc, Chennai, India

November 11, 2020

with Braglia, Finelli, Smoot, Sriramkumar, Starobinsky

What we know

Primordial black holes can contribute to the energy density of CDM

However constraints indicate that only PBHs of mass range of 10^{-16} - 10^{-12} solar mass can contribute completely to the CDM density

Inflation provides the mechanism to generate large bump in the spectrum of curvature perturbations

The large perturbations also source the generation of gravitational waves by second order effects

Inflationary models

Single field local peak, Starobinsky 1992, Ivanov et. al. 1994, Motohashi et. al. 2019

Inflection point, Garcia-Bellido et. al. 2017, Germani et. al. 2017, Bhaumik et. al. 2019

Non-trivial sound speed, Kamenshchik et. al. 2018, Cai et. al. 2018

Hybrid inflation, Garcia-Bellido et. al. 1994, Kawaguchi et. al. 2008, Clesse et. al. 2015

Why multi-fields

Two field model can provide the general description of effective single field models

A turn in the field trajectory leads to amplification in the primordial spectrum

Simpler potential compared to single field models

Two stage inflation provides richer dynamics

Difficult to solve

More Parameters

Can give rise to large non-Gaussianities

The Institute of Mathematical Sciences

Framework

We work with the following action:

$$S[\phi,\chi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{{M_{\rm Pl}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{f(\phi)}{2}(\partial\chi)^2 - V(\phi,\chi)\right]$$

The non-canonical coupling: $f(\phi) \equiv \mathrm{e}^{2b(\phi)}$

Background evolution:

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = b_\phi e^{2b}\dot{\chi}^2$$
$$\ddot{\chi} + (3H + 2b_\phi\dot{\phi})\dot{\chi} + \mathrm{e}^{-2b}V_\chi = 0,$$

Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020

The Institute of Mathematical Sciences

Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020

Background evolution

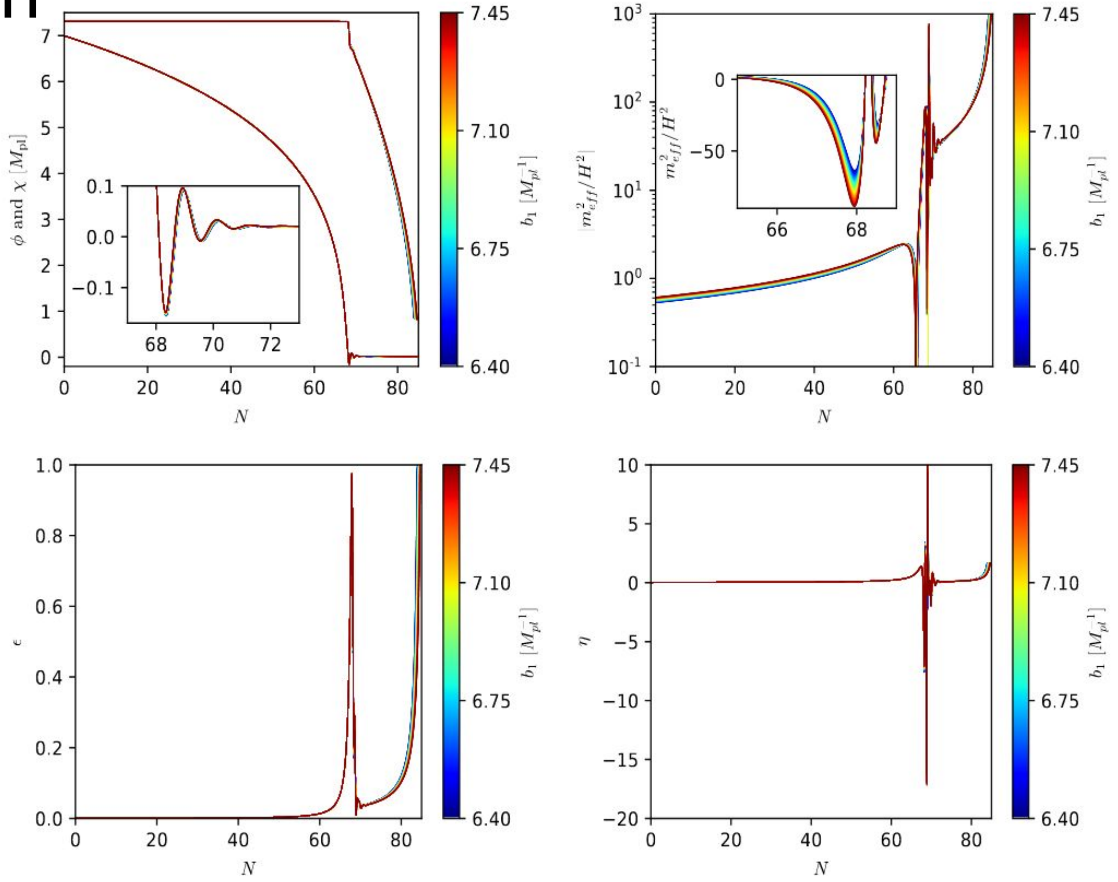
Toy model: KKLTI form for φ

$$V(\phi,\,\chi) = V_0\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{m_\chi^2}{2}\chi^2$$

$$H^2 = \frac{1}{3{M_{\rm Pl}}^2}\left[\frac{\dot{\phi}^2}{2} + e^{2b}\frac{\dot{\chi}^2}{2} + V\right]$$

$$\dot{H} = -\frac{1}{2{M_{\rm Pl}}^2}\left[\dot{\phi}^2 + e^{2b}\dot{\chi}^2\right].$$

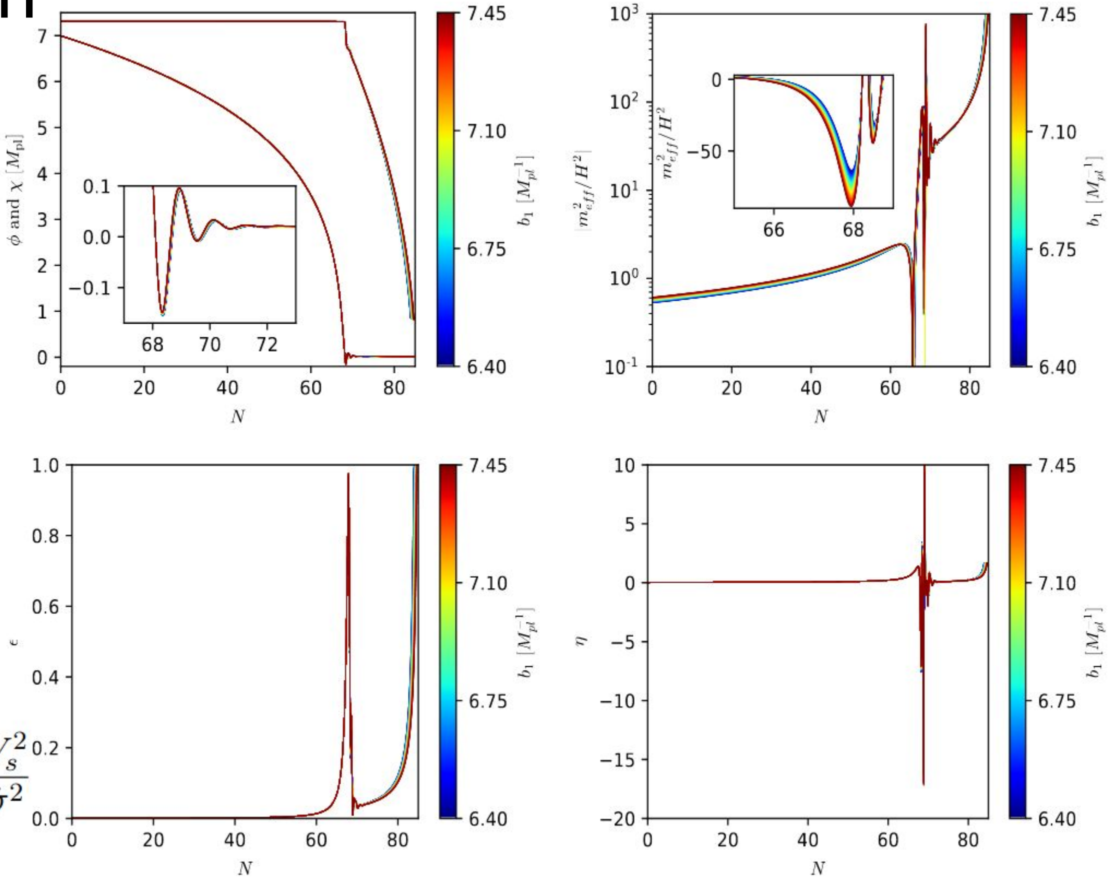
$$\epsilon_{i+1} \equiv \frac{d\ln\epsilon_i}{dN}$$



The Institute of Mathematical Sciences

Background evolution

Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020



Heavier field φ drives the first phase of inflation and rolls down the potential and χ field stays frozen

φ oscillates at the minimum and the second phase of inflation begins with χ rolling down to the bottom

Intermediate fast-roll takes place

Isocurvature mass:

$$m_{\rm eff}^2 \equiv V_{ss} + 3\dot{\theta}^2 + b_\phi^2 g(t) + b_\phi f(t) - b_{\phi\phi}\dot{\sigma}^2 - 4\frac{V_s^2}{\dot{\sigma}^2}$$

Marco, Finelli and Brandenberger, 2003

Perturbations

Metric perturbation: $$\mathrm{d}s^2 = -(1+2\Phi)\,\mathrm{d}t^2 + a^2(t)\,(1-2\Phi)\,\mathrm{d}\boldsymbol{x}^2$$

Mukhanov-Sasaki equations:

$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot{\theta}^2 - \frac{1}{a^3 {M_{\mathrm{Pl}}}^2}\left(\frac{a^3\dot{\sigma}^2}{H}\right)^{\cdot} + b_\phi u(t)\right] Q_\sigma$$
$$= 2\left(\dot{\theta}\delta s\right)^{\cdot} - 2\left(\frac{\dot{H}}{H} + \frac{V_\sigma}{\dot{\sigma}}\right)\dot{\theta}\delta s + b_{\phi\phi}\dot{\sigma}^2 \sin 2\theta \delta s + 2b_\phi h(t)$$

$$\ddot{\delta s} + 3H\dot{\delta s} + \left[\frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2 + b_\phi^2 g(t) + b_\phi f(t) - b_{\phi\phi}\dot{\sigma}^2 - 4\frac{V_s^2}{\dot{\sigma}^2}\right]\delta s$$
$$= 2\frac{V_s}{H}\left(\frac{H}{\dot{\sigma}}Q_\sigma\right)^{\cdot},$$

$$\dot{\sigma}^2 = \dot{\phi}^2 + \mathrm{e}^{2b}\dot{\chi}^2$$

$$\delta\sigma = \cos\theta\,\delta\phi + \sin\theta\,\mathrm{e}^b\,\delta\chi,$$
$$\delta s = -\sin\theta\,\delta\phi + \cos\theta\,\mathrm{e}^b\,\delta\chi$$

$$Q_\sigma = \delta\sigma + \dot{\sigma}/H\Phi$$

Perturbations

Change of variables: $\mathcal{R} = \frac{H}{\dot\sigma} Q_\sigma, \quad \mathcal{S} = \frac{H}{\dot\sigma}\delta s.$

$$\ddot{\mathcal{R}} + \left(H + 2\frac{\dot z}{z}\right)\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = -\frac{2V_s}{\dot\sigma}\dot{\mathcal{S}} - 2\left(-\mathrm{e}^{-b} b_\phi \cos^2\theta\, V_\chi + \sin\theta b_\phi V_\sigma + V_{\sigma s} + \frac{\dot\sigma}{H M_{\rm pl}^2} V_s\right)\mathcal{S}$$

$$\ddot{\mathcal{S}} + \left(H + 2\frac{\dot z}{z}\right)\dot{\mathcal{S}} + \left\{\frac{k^2}{a^2} - 2H^2 - \dot H + \frac{H\dot z}{z} + \frac{\ddot z}{z} - \dot\theta^2 - \dot\sigma^2 b_\phi^2 \cos^2\theta - \dot\sigma^2 b_{\varphi\varphi} + V_{ss} \right.$$
$$\left. + \; b_\phi\left[4\sin\theta\, V_s + (1+\sin^2\theta)V_\phi\right]\right\}\mathcal{S} = \frac{2V_s}{\dot\sigma}\dot{\mathcal{R}}$$

Power spectrum

At the end of inflation, the power spectra are evaluated as:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left(|\mathcal{R}_1|^2 + |\mathcal{R}_2|^2\right) = \mathcal{P}_{\mathcal{R}_1}(k) + \mathcal{P}_{\mathcal{R}_2}(k),$$

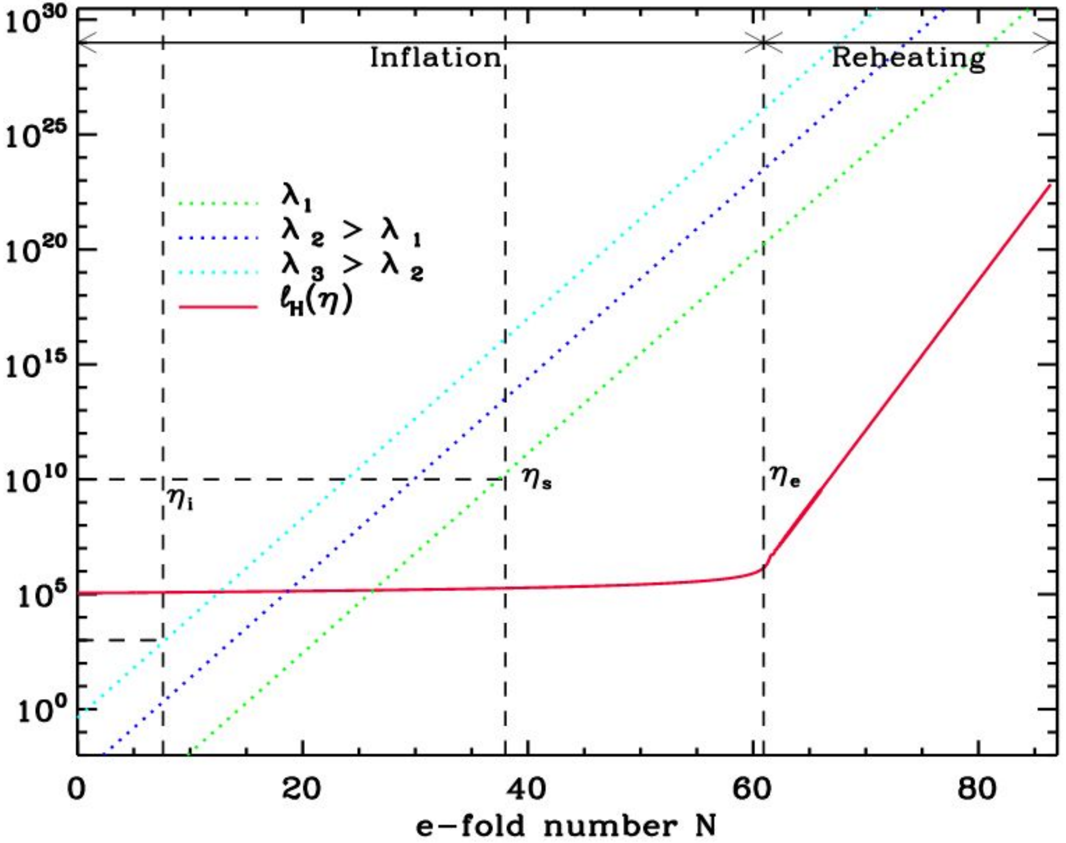
$$\mathcal{P}_{\mathcal{S}}(k) = \frac{k^3}{2\pi^2}\left(|\mathcal{S}_1|^2 + |\mathcal{S}_2|^2\right),$$

$$\mathcal{C}_{\mathcal{R}\mathcal{S}}(k) = \frac{k^3}{2\pi^2}\left(\mathcal{R}_1^*\mathcal{S}_1 + \mathcal{R}_2^*\mathcal{S}_2\right).$$

Isocurvature sourced
curvature perturbation

The Institute of Mathematical Sciences

Mode evolution



The code: BINGO

Extended BINGO -- BI-spectra and Non-Gaussianity Operator for 2 field models with non-canonical Lagrangians

Solve the Background and Curvature perturbation equations without analytical approximations

Impose Bunch-Davies initial conditions on the Mukhanov-Sasaki variables deep inside the Hubble radius in a way that the curvature and isocurvature perturbations stay decoupled initially

We evaluate the spectra at the end of inflation. There can be super Hubble evolution of curvature perturbations

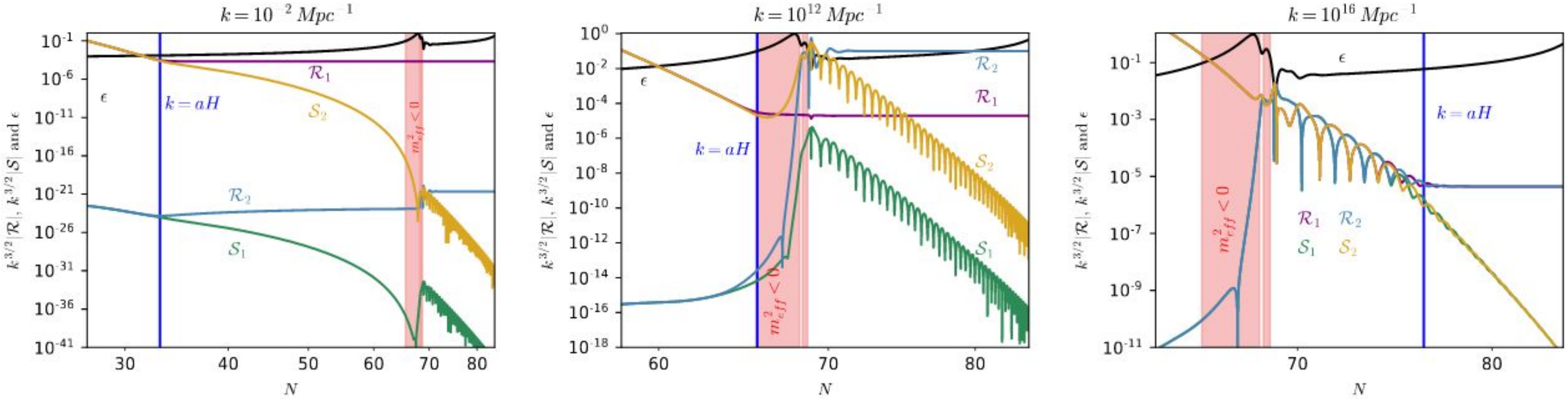
Tilt and tensor to scalar ratios

	ϕ_i [$M_{\rm pl}$]	χ_i [$M_{\rm pl}$]	n_s	r
SKA	7.0	9.3	0.9184	0.042
LISA	7.0	7.31	0.9537	0.020
BBO	7.0	6.55	0.9601	0.017
ET	7.0	5.6	0.9640	0.014

Initial values of the scalar fields and the spectral tilt and tensor to scalar ratios corresponding to the peaks of our primordial spectrum that falls in the observation band of SKA, LISA, BBO and ET

Mode evolution

$$m_{\rm eff}^2 \equiv V_{ss} + 3\dot{\theta}^2 + b_\phi^2 g(t) + b_\phi f(t) - b_{\phi\phi}\dot{\sigma}^2 - 4\frac{V_s^2}{\dot{\sigma}^2}$$



Isocurvature sourcing occurs much after Hubble crossing

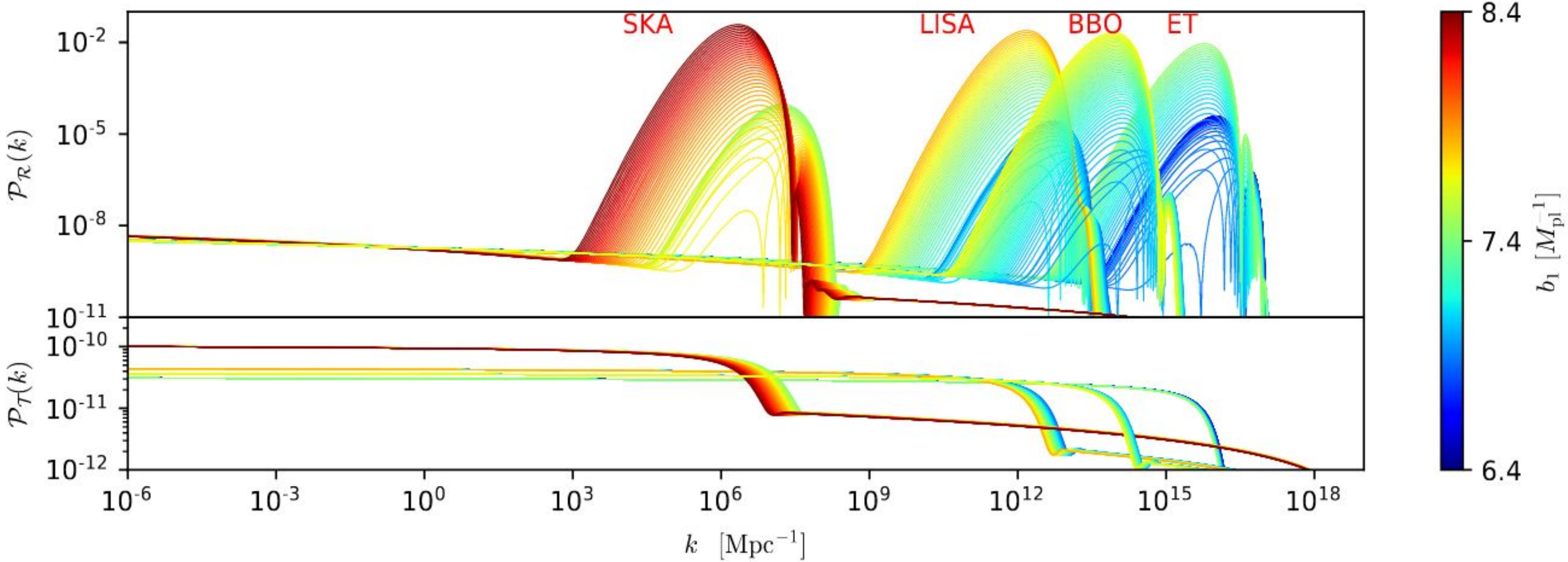
Occurs during Hubble crossing

Occurs much before Hubble crossing

Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020

Power spectra

Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020



Primordial scalar and tensor power spectra obtained from BINGO, spanning 25 decades

The Institute of Mathematical Sciences

PBH formation

Large primordial perturbations collapse to form primordial black holes when the modes re-enter the Hubble radius in the radiation dominated era

$$f_{\rm PBH}^{\rm tot} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{CDM}} = \int d\,\ln M \frac{df_{\rm PBH}(M)}{d\ln M},$$

$$\frac{df_{\rm PBH}(M)}{d\ln M} = \nu(M)^2 \left|\frac{d\ln \nu(M)}{d\ln M}\right| f_{\rm PBH}(M)$$

$$f_{\rm PBH}(M) = 2.7\times 10^8 \left(\frac{0.2}{\gamma}\frac{M}{M_\odot}\sqrt{\frac{g_{*,f}}{10.75}}\right)^{-1/2} \beta(M)$$

Scales of future observations

$$k_{\rm SKA} \sim 2 \times 10^6 \ {\rm Mpc}^{-1}, \ k_{\rm LISA} \sim 10^{12} \ {\rm Mpc}^{-1}, \ k_{\rm BBO} \sim 8 \times 10^{13} \ {\rm Mpc}^{-1} \ k_{\rm ET} \sim 6 \times 10^{15} \ {\rm Mpc}^{-1}$$

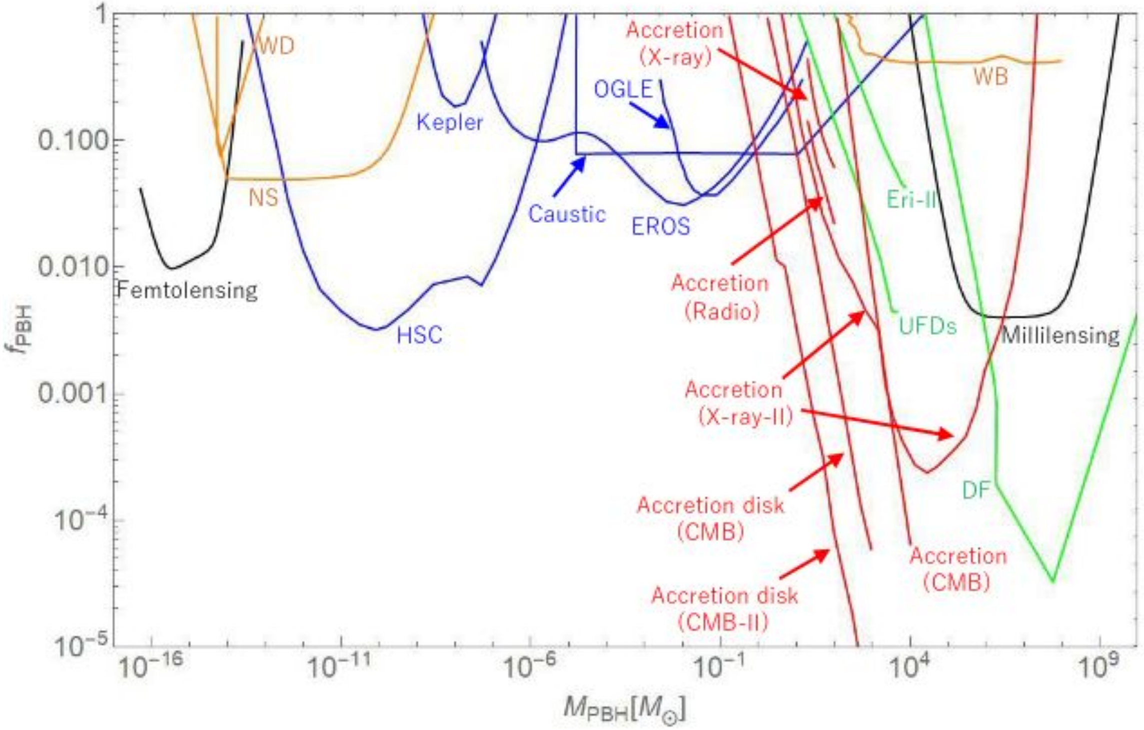
$$M_{\rm SKA} \sim 35 M_\odot, \ M_{\rm LISA} \sim \times 10^{-12} M_\odot, \ M_{\rm BBO} \sim 3 \times 10^{-16} M_\odot \ \text{and} \ M_{\rm ET} \sim 7 \times 10^{-20} M_\odot$$

We calculate the f_{PBH} around the cosmological scales expected to be probed by future observations

Obtain PBH fraction as a function of non-canonical coupling

Initial value of second field is adjusted to generate the PBH fraction at desired scales

Constraints on PBH fraction



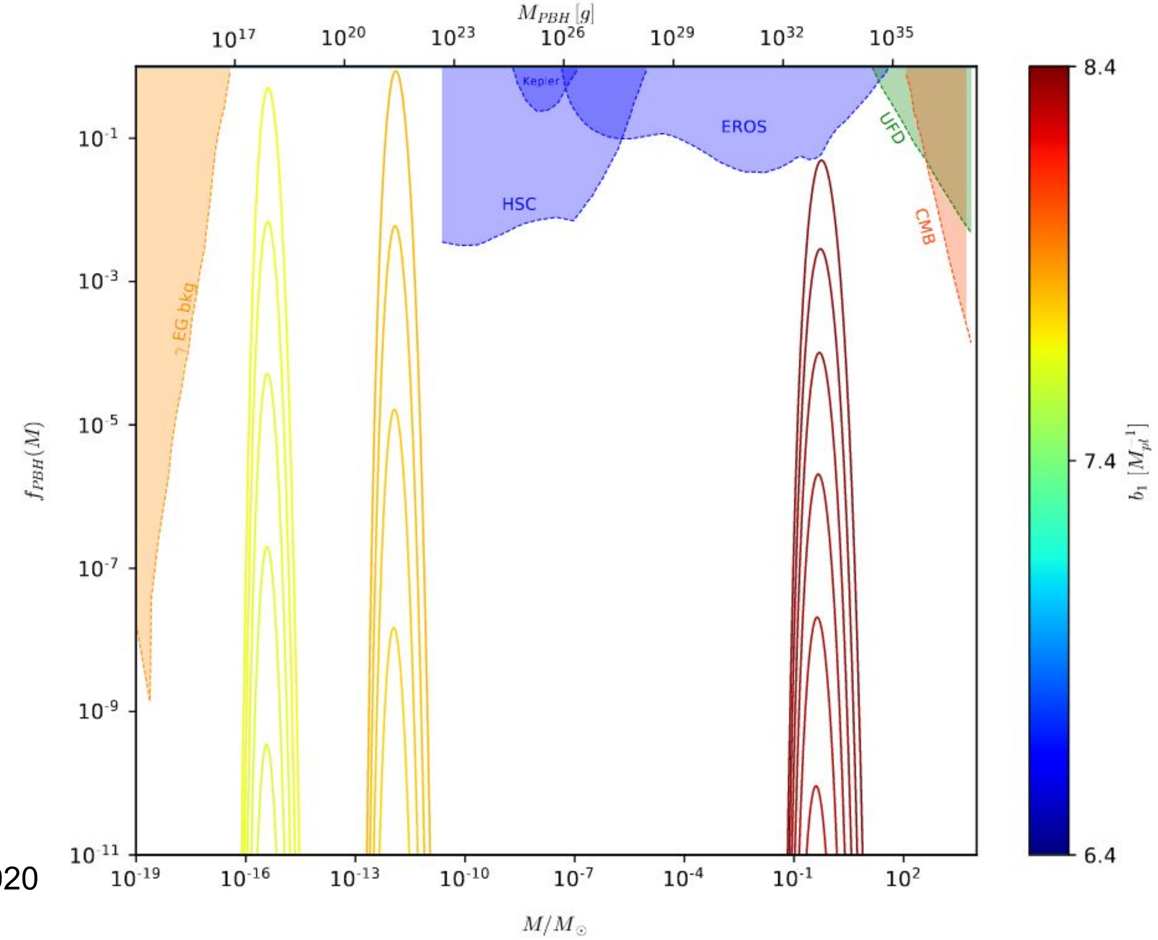
Sasaki et. al. 2018

PBH fraction

Observational constraints

$$f_{\rm SKA}^{\rm tot} = 0.01 \text{ and } f_{\rm LISA}^{\rm tot} = f_{\rm BBO}^{\rm tot} = 1$$

-- maximum fraction allowed



Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020

Gravitational waves

Large scalar overdensities behaves as a (second order) source for sbGW through second order perturbations (Acquaviva et. al. 2003, Inomata et. al. 2017):

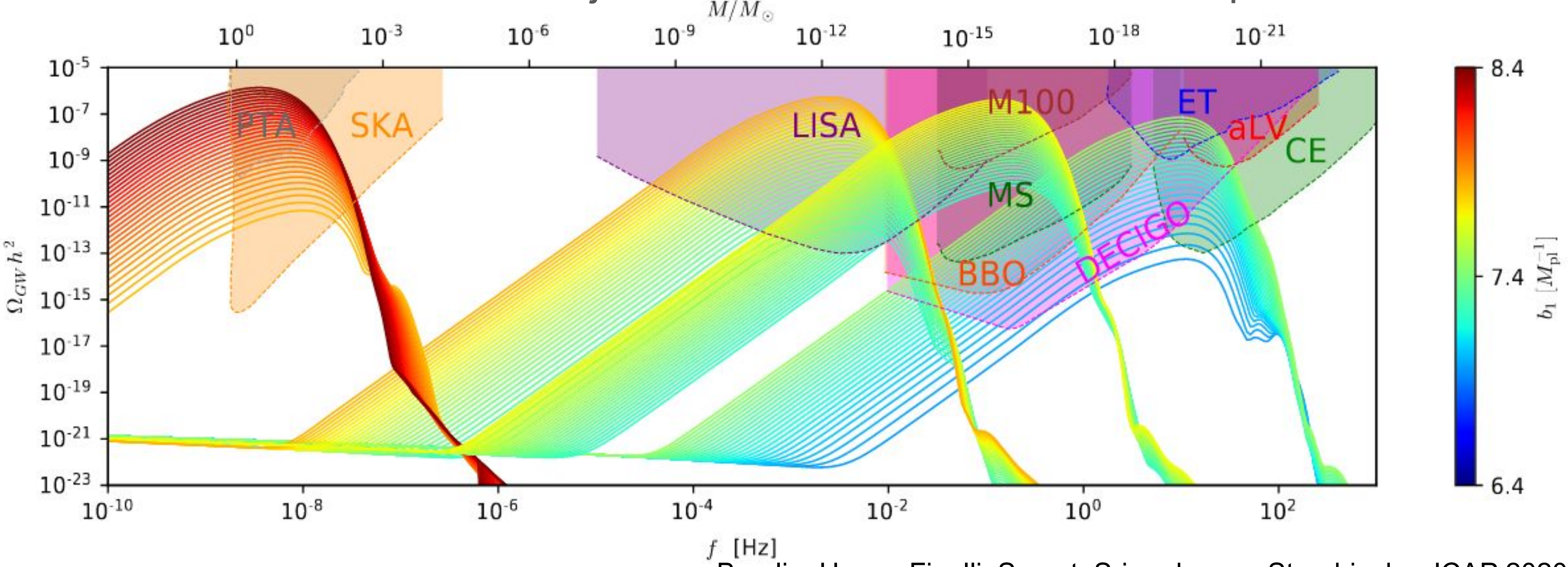
$$\Omega_{\mathrm{GW}} = \frac{\Omega_{r,0}}{36} \int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} ds \left[\frac{(d^2 - 1/3)(s^2 - 1/3)}{s^2 - d^2} \right]^2 \cdot \mathcal{P}_{\mathcal{R}} \left(\frac{k\sqrt{3}}{2}(s+d) \right) \mathcal{P}_{\mathcal{R}} \left(\frac{k\sqrt{3}}{2}(s-d) \right) \left[\mathcal{I}_c(d,s)^2 + \mathcal{I}_s(d,s)^2 \right]$$

We have built a pipeline to calculate the PBH fraction and relic density of gravitational waves using any model of inflation.

The Institute of Mathematical Sciences

Relic GW energy

Relic GW energy density as a function of the non-canonical coupling. Note that here also initial field value is adjusted to shift the scale of the GW peak



Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020

PBH and GW

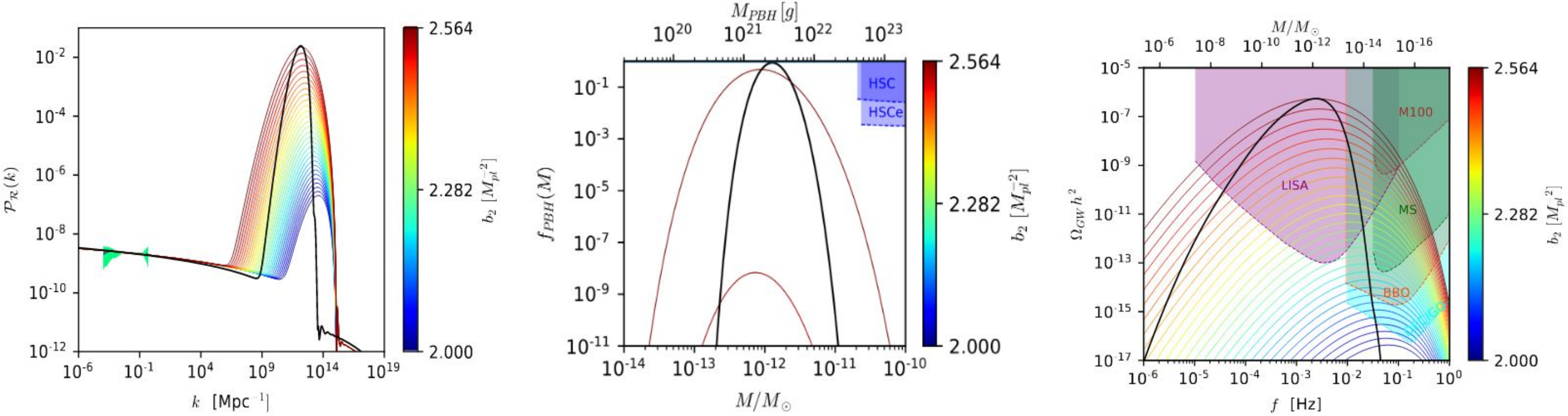
Primordial spectra that produce PBHs also generates GW's with amplitudes that are within observational threshold of future surveys

In fact, for very low mass PBH, that do not contribute to CDMs also produce GW's that can be detected

Therefore detection of sbGWs will provide us the mass window to look for PBHs

Effect of non-linear coupling

Non-vanishing second derivative in $f_B(\phi) = \exp(2b_2\phi^2)$ changes effective mass of isocurvature perturbations. Broadens the peak structures



Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky, JCAP 2020

The Institute of Mathematical Sciences

Differences between potentials

Parametrize the ratio of the potential with $R \equiv V_0/(m_\chi M_{\rm pl})^2$

To compare different scenarios we keep $f_{\rm PBH}^{\rm TOT} \simeq 1$

	$\chi_i\,[M_{\rm pl}]$	$V_0\,[10^{-10} M_{\rm pl}^4]$	R	$b_1\,[M_{\rm pl}]^{-1}$
1	3.2	6.4	30	9.466
2	7.31	7.08	500	7.837
3	8.1	7.6	1050	7.382
4	8.5	8.21	3800	6.233

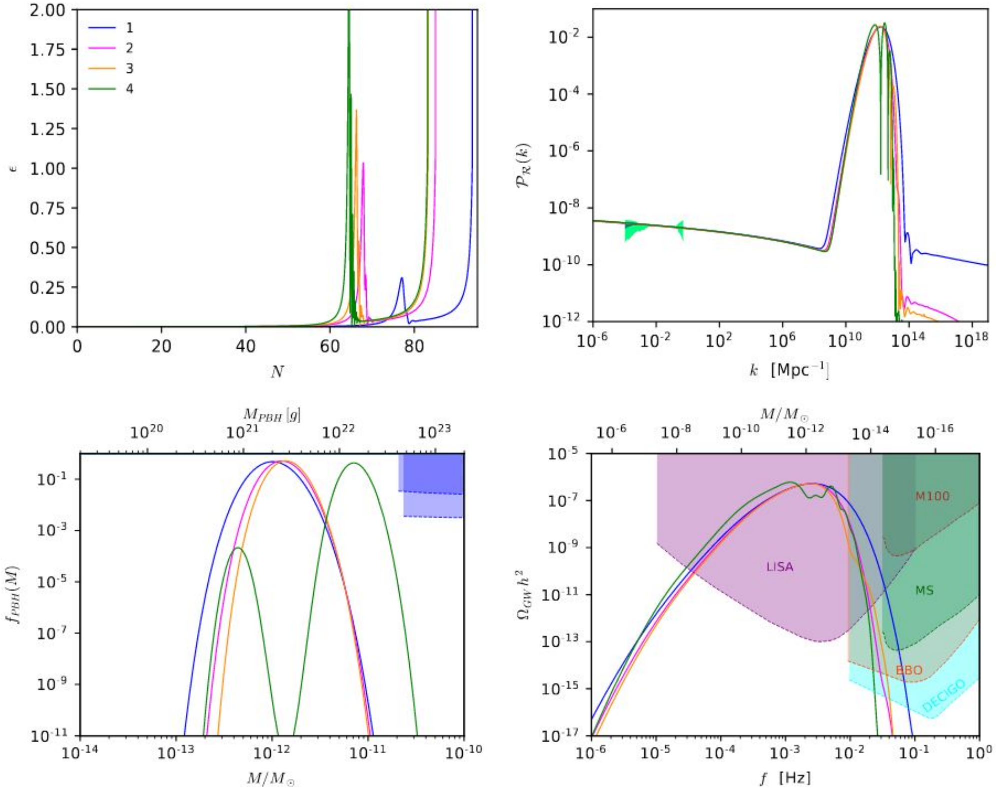
Differences between potentials

Higher differences in potential leads to multiple higher peaks in the slow roll parameters

Leads to multi-peak structure in the primordial spectrum

Generates 2 peaks in the PBH fraction. The green line shows the 2 peak structure

The peak structures in the relic density in the GW also gets modified



The Institute of Mathematical Sciences

To summarize

Two stage inflation driven by two fields with large non-canonical coupling gives rise to large amplification of primordial power spectrum

The maximal growth rate can occur with n_s=4

Beyond a threshold, large perturbations collapse into PBHs when re-enter the Hubble radius during radiation dominated era

Using evaluated primordial spectra we have computed the predicted mass fraction of the PBHs and and relic energy density of GWs

The Institute of Mathematical Sciences

To summarize

The mechanism is generic, though we have used particular potential here

A change in the non-canonical coupling such as quadratic in the field leads to widening of the bump and therefore in the PBH fraction and in peak in the sbGWs

We can constrain the model jointly with future PBH and GW observations

Non-Gaussianity affects the formation of PBH and sbGW strongly and we have not considered the effect, (Young et. al. 2013, Garcia-Bellido et. al. 2017)

Thank You

The Institute of Mathematical Sciences