Cosmological perturbation theory

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Lecture 4

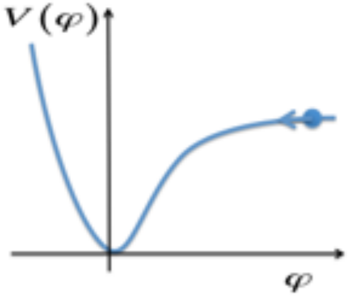
- Perturbation theory building-blocks
  - Scalar/vector/tensor decomposition
- Metric perturbations
  - Gauge-invariant perturbations
- Energy-momentum conservation
  - Conserved quantities on large scales
- Einstein equations
  - Recovering Newtonian growth of structure in ΛCDM

- Primordial perturbations from inflation
  - Scalar field Klein-Gordon equation
  - Gauge-invariant perturbations
  - Quantum fluctuations
  - Power spectra and Bispectra
  - Stochastic inflation

Homogeneous scalar field cosmology

- FLRW spacetime

$$ds^2 = -dt^2 + a^2(t)dX^2$$



- Spatially homogeneous scalar field: $\varphi(t)$
  - Klein-Gordon equation:

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'(\varphi)$$

rolling down gradient, V', Hubble damping, $H = \dot{a}/a$

  - Energy density (kinetic+potential):

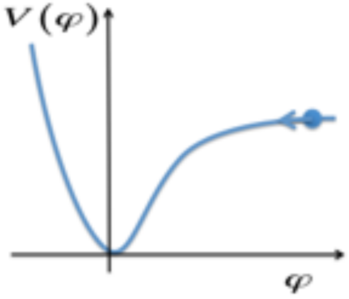
$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

  - Pressure (kinetic-potential):

$$P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

Slow-roll inflation

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left(V + \frac{1}{2}\dot{\varphi}^2\right) \\ \ddot{\varphi} + 3H\dot{\varphi} &= -V' \end{aligned}$$



- Inflation:

$$\ddot{a} > 0 \quad \Rightarrow \quad \dot{\varphi}^2 < V \quad \Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$

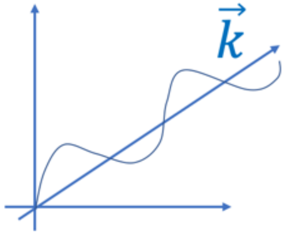
- Slow-roll inflation:

$$\begin{aligned} H^2 \simeq \frac{8\pi G}{3} V \quad &\Rightarrow \quad \dot{\varphi}^2 \ll V \quad &\Rightarrow \quad \epsilon \ll 1 \\ 3H\dot{\varphi} \simeq -V' \quad &\Rightarrow \quad |\ddot{\varphi}| \ll 3H|\dot{\varphi}| \quad &\Rightarrow \quad |\eta| \ll 1 \end{aligned}$$

  - integrated logarithmic expansion (or "e-folds")

$$N = \ln\left(\frac{a_{end}}{a_*}\right) = \int_{t_*}^{t_{end}} H\, dt \simeq \int_{\varphi_*}^{\varphi_{end}} \frac{H}{\dot{\varphi}}\, d\varphi$$

Inhomogeneous fluctuations



- Wave equation for scalar field perturbations in *unperturbed* FLRW cosmology (neglecting self-gravity):

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \left[\frac{k^2}{a^2} + V''\right]\delta\varphi = 0$$

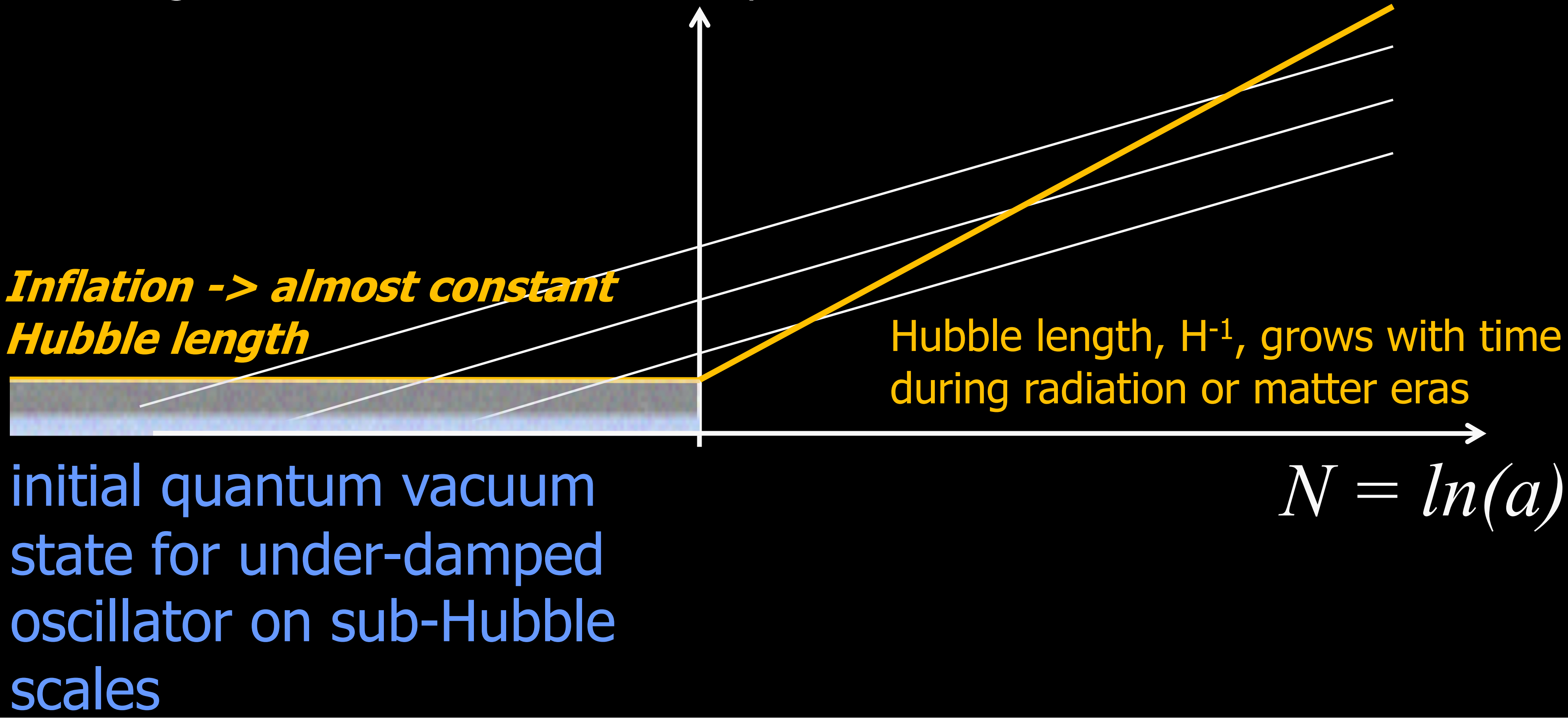
- Characteristic length-scales ($k = 2\pi/\lambda$):
  - $k \gg aH$ small-scales (sub-Hubble)
    underdamped quantum vacuum fluctuations
  - $k \ll aH$ large-scale (super-Hubble)
    overdamped squeezed state $\rightarrow$ "frozen-in"

Wave equation for massless field in FLRW cosmology:

$$\delta\ddot{\varphi} + 3H\,\delta\dot{\varphi} + (k/a)^2\,\delta\varphi = 0$$

Characteristic timescales for waves, comoving wavemode k: oscillation vs damping

- *small-scales , k/a > H, under-damped oscillator*
- *large-scales , k/a < H, over-damped "frozen-in"*



Gauge-dependence of scalar field perturbations

- FLRW cosmology has preferred time-slicing:

$$\varphi = \varphi_0(t)$$

but perturbed field does not:

$$\varphi = \varphi_0(t) + \delta\varphi(t, x^i)$$

- Time-slicing (gauge) freedom:

$$t \to \tilde{t} = t + \delta t(t, x^i)$$

- Scalar field $\tilde{\varphi} = \varphi$ invariant at same physical point:

$$\varphi_0(\tilde{t}) + \delta\tilde{\varphi}(\tilde{t}, x^i) = \varphi_0(t) + \delta\varphi(t, x^i)$$

- but field perturbation transforms at first order:

$$\delta\tilde{\varphi} = \delta\varphi - \dot{\varphi}_0 \delta t$$

Gauge invariant scalar perturbations

- Gauge-dependent scalar field perturbation:

$$\delta\tilde{\varphi} = \delta\varphi - \dot{\varphi}_0 \delta t$$

- Gauge-dependent metric/curvature perturbation:

$$\tilde{C} = C - H\delta t$$

- Gauge-invariant field in spatially-flat gauge:

$$Q = \delta\varphi - \frac{\dot{\varphi}_0}{H} C$$

- Gauge-invariant curvature in uniform-field gauge:

$$R = C - \frac{H}{\dot{\varphi}_0}\delta\varphi = -\frac{H}{\dot{\varphi}_0} Q$$

δN formalism

- *e*-folds:

$$N = \int_{t_*}^{t_{end}} H\, dt = \int_{\varphi_*}^{\varphi_{end}} \frac{H}{\dot{\varphi}}\, d\varphi$$

- δN on uniform-field hypersurface ($\varphi = \varphi_{end}$) due to $\delta\varphi_* = Q$ during inflation:

$$\delta N = \frac{\partial}{\partial \varphi_*} \left(\int_{\varphi_*}^{\varphi_{end}} \frac{H}{\dot{\varphi}}\, d\varphi \right) \delta\varphi_* = -\frac{H}{\dot{\varphi}} Q$$

- Thus gauge-invariant curvature:

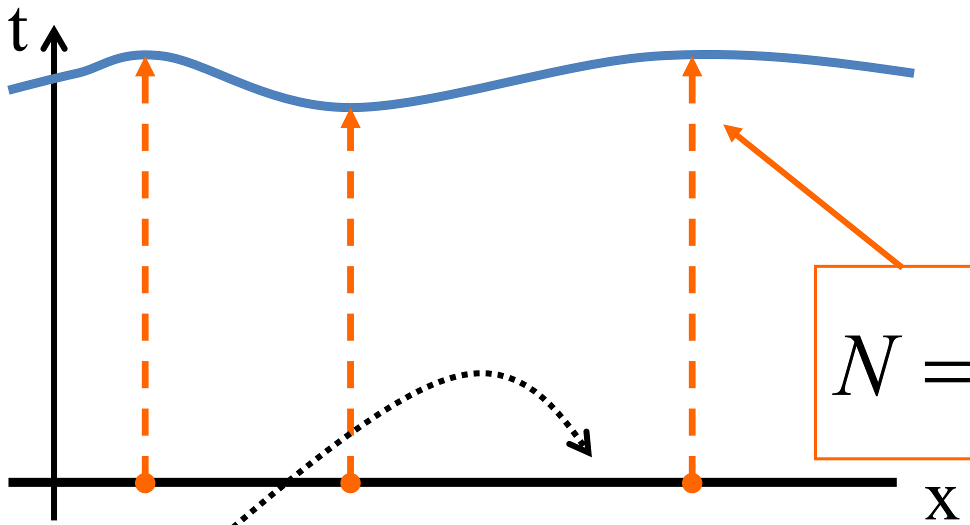
$$R = \delta N = -\frac{H}{\dot{\varphi}_0} Q$$

  - R remains constant on super-Hubble scales during single-field slow roll
  - in slow-roll, uniform-field $\Rightarrow$ uniform-density $\Rightarrow$
  - $R = \zeta$ remains constant for adiabatic (single clock) perturbations on super-Hubble scales after inflation

δN formalism for primordial density fluctuations

Starobinsky '85; Sasaki & Stewart '96; Lyth & Rodriguez '05

after inflation: curvature perturbation ζ on uniform-density hypersurface



$$N = \int_{initial}^{final} H\,dt$$

during inflation: field perturbations $\phi(x,t_i)$ on initial spatially-flat hypersurface

$$\zeta = N\left(\phi_{initial}\right) - \overline{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Scalar field perturbations

- Klein-Gordon equation for Sasaki-Mukhanov variable, $v = aQ = zR$, in terms of conformal time:

$$v'' + \left(k^2 - \frac{z''}{z}\right) v = 0$$

where $z = a\dot{\varphi}/H$ and $z''/z \simeq (2 + 5\epsilon - 3\eta)a^2H^2$

  - small scales (early times during inflation): $k^2 \gg (aH)^2$

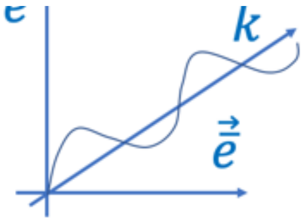
$$v''_{\vec{k}} + k^2 v_{\vec{k}} = 0$$

    oscillations $v_{\vec{k}} \propto e^{\pm ik\tau}$
  - large scales (late times during inflation): $k^2 \ll (aH)^2$

$$v''_{\vec{k}} - \frac{z''}{z} v_{\vec{k}} = 0$$

    growing mode (squeezed state) $v_{\vec{k}} \propto z \;\Rightarrow\; R_{\vec{k}} \to \text{const}$

Vector perturbations



- Metric perturbations transverse to propagation along $\vec{k}$:

$$F^i(t,\vec{x}) = \left[F_{\vec{k}}(t)e^i + \bar{F}_{\vec{k}}(t)\bar{e}^i\right]\exp(i\vec{k}.\vec{x})$$

where two polarisation vectors are orthonormal

$$\vec{e}\cdot\vec{k} = 0\,, \quad \vec{\bar{e}}\cdot\vec{k} = 0\,, \quad \vec{\bar{e}}\cdot\vec{e} = 0$$

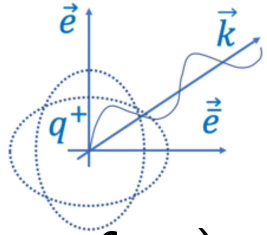
- No transverse vector perturbation for scalar fields:

$$\nabla^i\varphi(t,\vec{x}) = \nabla^i\left[\varphi_k(t)\exp(i\vec{k}.\vec{x})\right] = ik^i\left[\varphi_k(t)\exp(i\vec{k}.\vec{x})\right]$$

- momentum constraint then requires gauge-invariant vector metric perturbations to vanish

$$\nabla^2\left(F_i' + S_i\right) = 0\,.$$

Tensor perturbations



- Tensor metric perturbations (transverse and tracefree):

$$h_{ij}(t,\vec{x}) = \left[h^{+}_{\vec{k}}(t)q^{+}_{ij}(\vec{k}) + h^{\times}_{\vec{k}}(t)q^{\times}_{ij}(\vec{k})\right] e^{i\vec{k}.\vec{x}}$$

  where two polarisation tensors constructed from transverse vectors $\vec{e}$ and $\vec{\bar{e}}$

$$q^{+}_{ij} = \frac{1}{\sqrt{2}}\left(e_i e_j - \bar{e}_i \bar{e}_j\right), \quad q^{\times}_{ij} = \frac{1}{\sqrt{2}}\left(e^i \bar{e}^j + \bar{e}^i e^j\right)$$

$$q^{+}_{ij} q^{+\,ij} = 1\,, \quad q^{\times}_{ij} q^{\times\,ij} = 1\,, \quad q^{+}_{ij} q^{\times\,ij} = 0$$

- Gauge-independent at first order and no constraint
- Each mode amplitude obeys wave equation for massless field:

$$\ddot{h}_{\vec{k}} + 3H\dot{h}_{\vec{k}} + \frac{k^2}{a^2}h_{\vec{k}} = 0$$

Tensor perturbations

- Simple equation for canonically normalised field $u_{\vec{k}} = ah_{\vec{k}}/\sqrt{32\pi G}$ in terms of conformal time

$$u''_{\vec{k}} + \left(k^2 - \frac{a''}{a}\right) u_{\vec{k}} = 0$$

where $a''/a \simeq (2-\epsilon)a^2H^2$
  - small scales (early times during inflation): $k^2 \gg (aH)^2$

$$u''_{\vec{k}} + k^2 u_{\vec{k}} = 0$$

    oscillations $u_{\vec{k}} \propto e^{\pm ik\tau}$
  - large scales (late times during inflation): $k^2 \ll (aH)^2$

$$u''_{\vec{k}} - \frac{a''}{a} u_{\vec{k}} = 0$$

    growing mode (squeezed state) $u_{\vec{k}} \propto a \;\Rightarrow\; h_{\vec{k}} \to \text{const}$

Power spectra

- quantise canonically normalised fields on small scales $(k^2 \gg (aH)^2)$:

$$\langle \hat{u}_{\vec{k}_1} \hat{u}'_{\vec{k}_2} \rangle = \langle \hat{v}_{\vec{k}_1} \hat{v}'_{\vec{k}_2} \rangle = \frac{i\hbar}{2} (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2)$$

- large scales $(k^2 \ll (aH)^2)$ described by *classical* fields:

$$\langle X_{\vec{k}_1} X_{\vec{k}_2} \rangle = \frac{P_X(k_1)}{4\pi k_1^3} (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2)$$

  - tensors:

$$P_T(k) \simeq 64\pi G \left(\frac{H}{2\pi} \right)^2_{k=aH}$$

  - scalars:

$$P_R(k) \simeq \left[\left(\frac{H}{\dot{\varphi}} \right)^2 \left(\frac{H}{2\pi} \right)^2 \right]_{k=aH}$$

  - tensor-scalar ratio: $r \equiv P_T(k)/P_R(k) = 16\epsilon$

Weak scale dependence from slow roll

- Tensor tilt:

$$n_T \equiv \frac{d \ln P_T}{d \ln k} \simeq \left(\frac{d \ln H^2}{d \ln(aH)} \right)_{k=aH}$$

slow-roll time-dependence

$$\frac{d \ln H}{dt} = -\epsilon H \quad , \quad \frac{d \ln(aH)}{dt} = (1-\epsilon)H$$

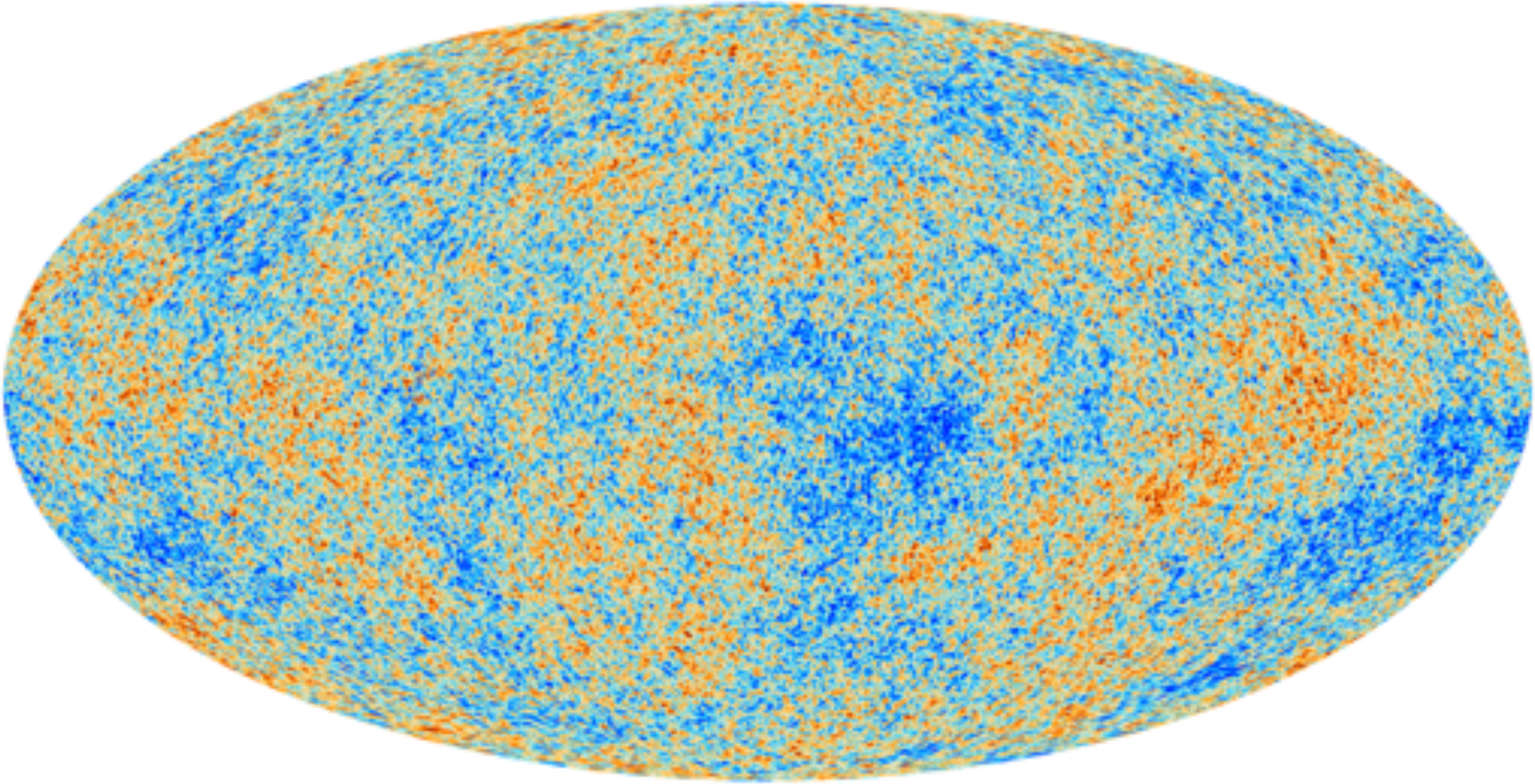
gives

$$n_T \simeq -2\epsilon$$

- Scalar tilt:

$$n_R - 1 \equiv \frac{d \ln P_R}{d \ln k} \simeq -6\epsilon + 2\eta$$

Precision cosmology Ade et al 2015



CMB temperature and polarization anisotropies well-described by Gaussian, adiabatic, but not-quite scale-invariant distribution of primordial density perturbations

Constraints from Planck 2018

- Scalar power spectrum

$$P_R(k = 0.05\ \mathrm{Mpc}^{-1}) = (2.2 \pm 0.8) \times 10^{-9}\ (68\%\ \mathrm{c.l.})$$

- Scalar index

$$n_R = 0.9649 \pm 0.0042$$

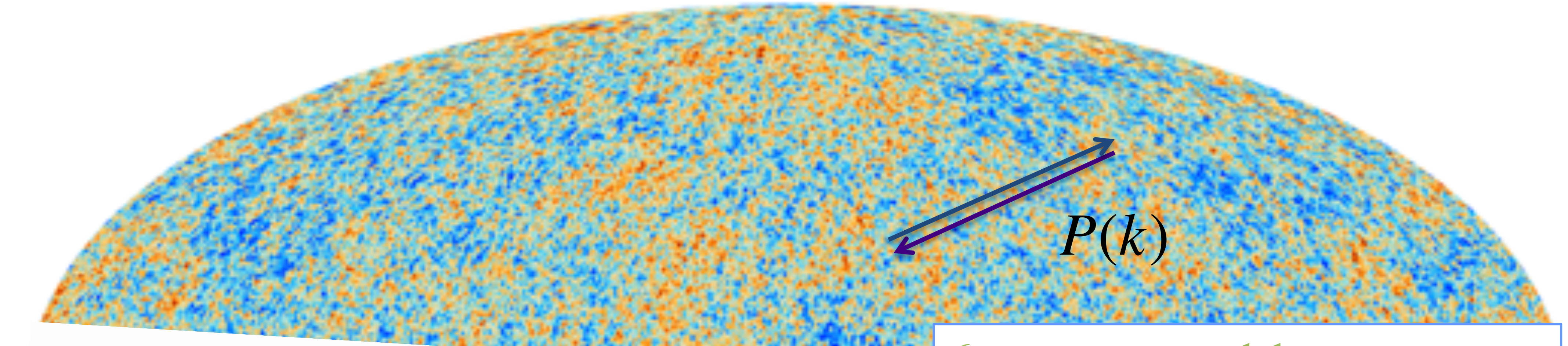
- Scalar running $(dn_R/d\ln k)$

$$\alpha_R = -0.0045 \pm 0.0067$$

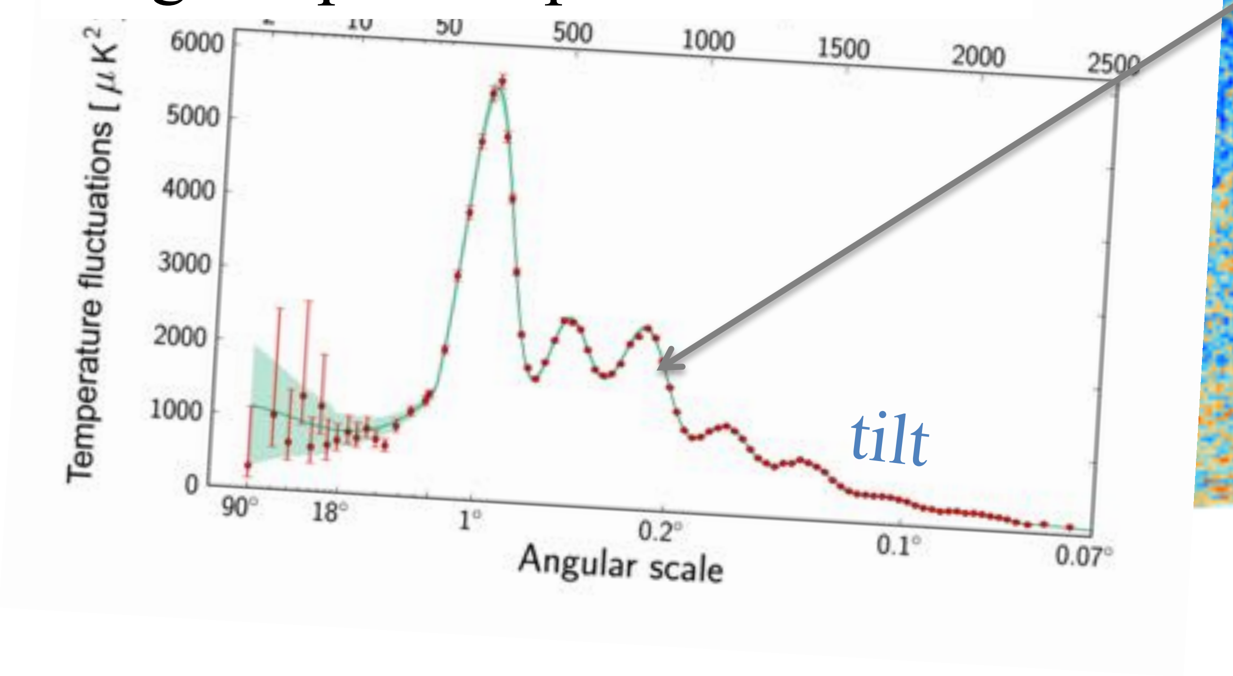
- Tensor-scalar ratio (+BICEP/Keck 2021)

$$r < 0.036\ (95\%\ \mathrm{c.l.})$$

Precision cosmology Ade et al 2015



Angular power spectrum:



6-parameter model:

4-parameter cosmology:

- $H_0 = 67.8 \pm 0.9$
- $\Omega_{matter} = 0.308 \pm 0.012$
- $\Omega_\Lambda = 0.692 \pm 0.012$
- Reionisation $z = 8.8 \pm 1.7$

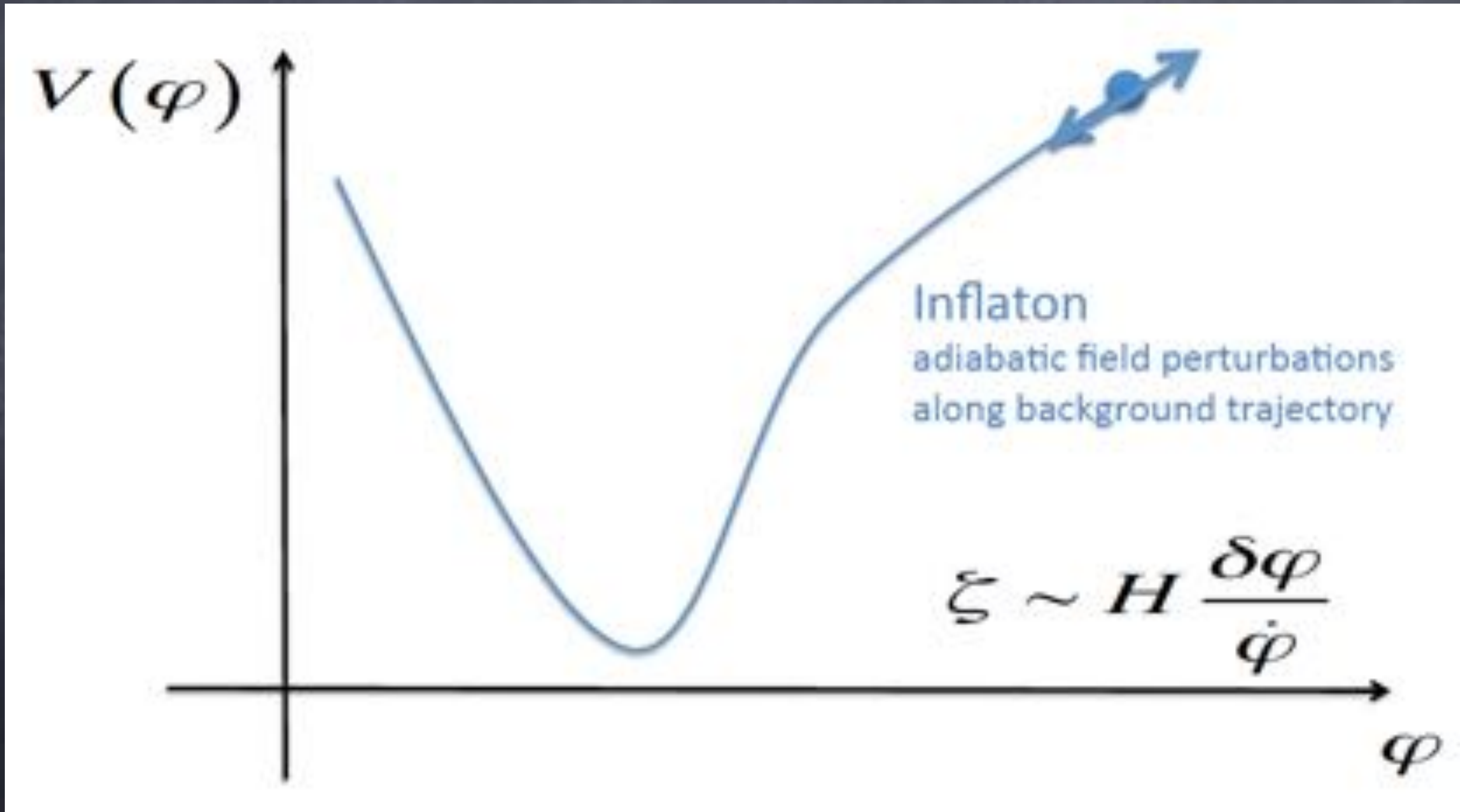
2-parameter ***initial fluctuations***:

- Amplitude $\Delta T/T = 0.000047$
- Tilt $= -0.032 \pm 0.006$

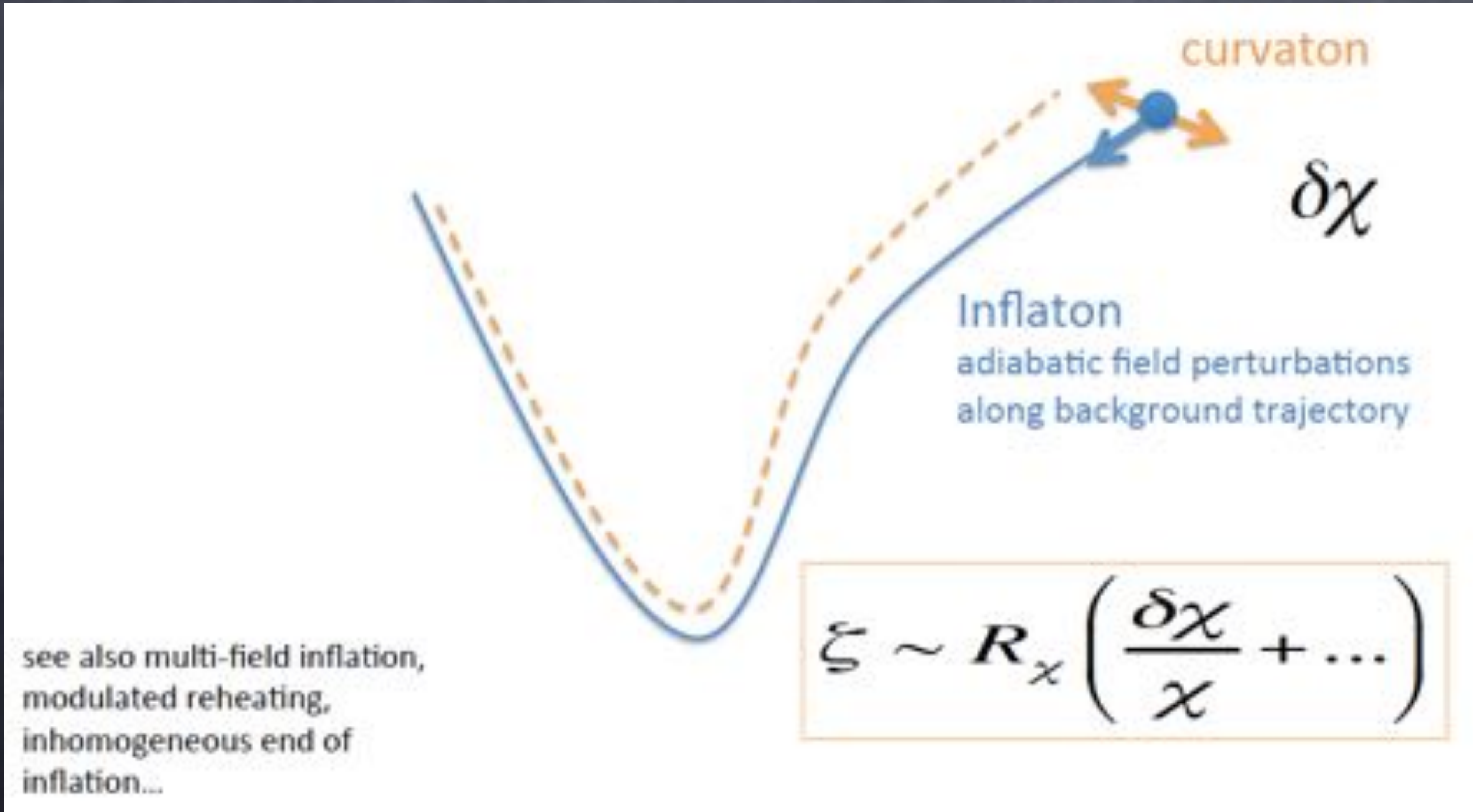
inflationary phenomenology

- Could inflation be very different from a simple single scalar field?
- and, if it was, how would we know?

single-field phenomenology



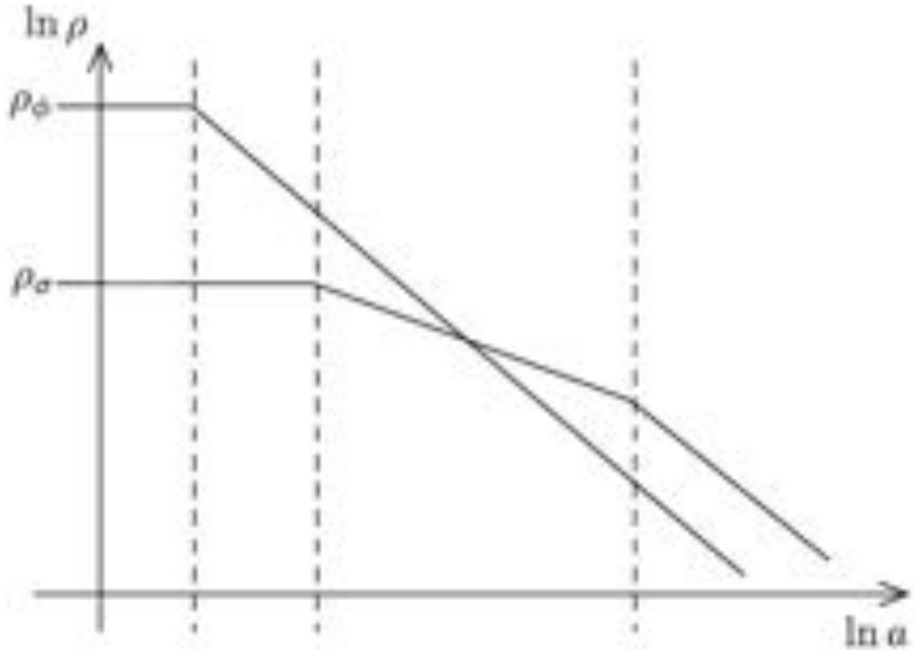
multi-field phenomenology



Inflaton models + curvaton field, χ

Curvaton scenarios with quadratic potential $V(\phi,\chi) = U(\phi) + m_\chi^2 \chi^2 / 2$

Linde and Mukhanov, 1997
Enqvist and Sloth, 2001
Lyth and Wands, 2001
Moroi and Takahashi, 2001
Langlois and Vernizzi, 2004

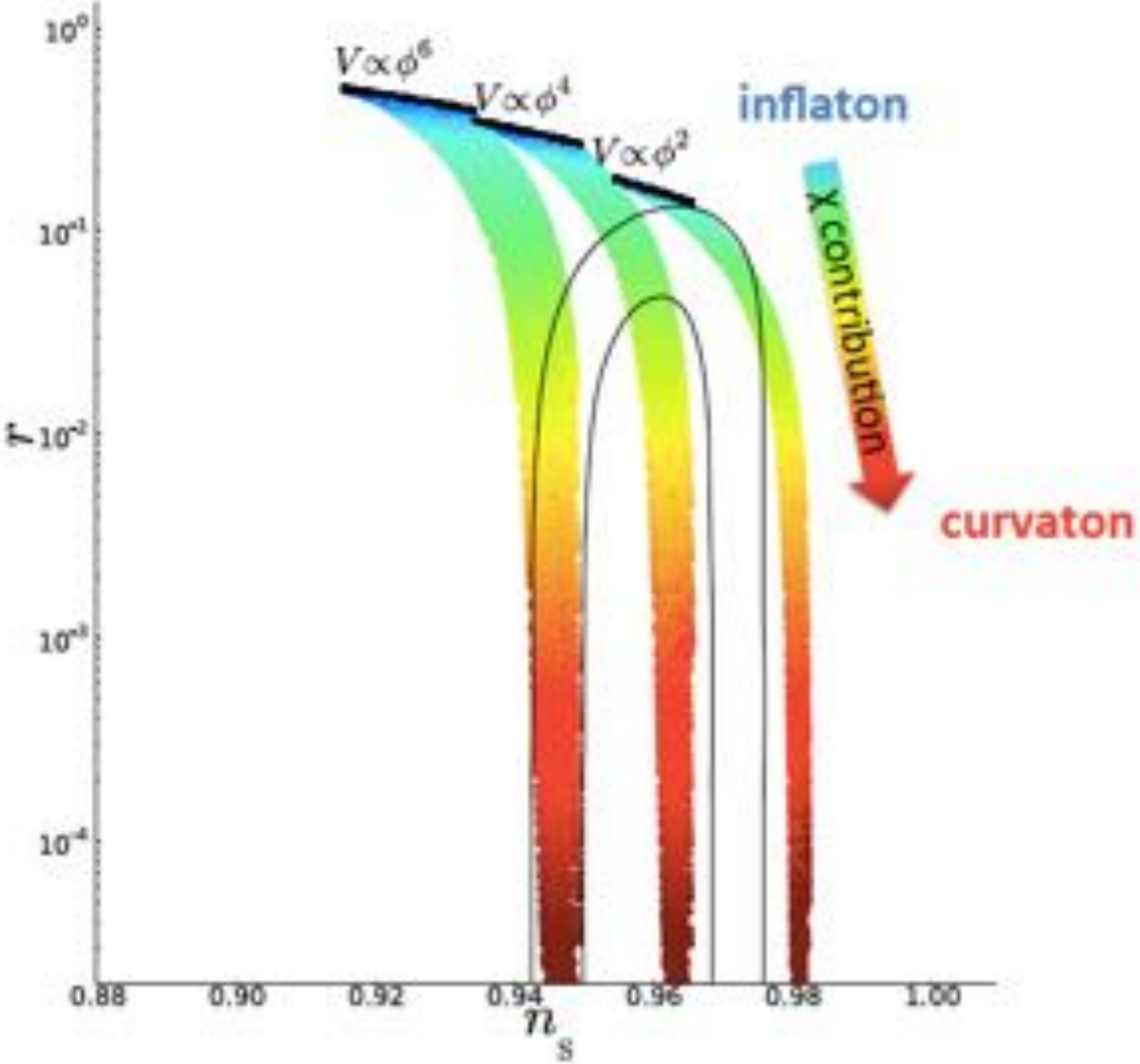


more reheating parameters: $\Gamma_\phi \rightarrow \Gamma_\phi\,,\ \Gamma_\chi\,,\ m_\chi\,,\ \chi_{\rm end}$

primordial perturbations directly dependent on reheating
(not just through the expansion N.)

large-field inflaton (LFI) plus quadratic curvaton, χ

Vennin, Koyama and Wands (2015)



residual isocurvature modes

Lyth, Ungarelli & Wands '02

Moroi & Takahashi '02

Gordon & Lewis '03

Gupta, Malik & Wands '04

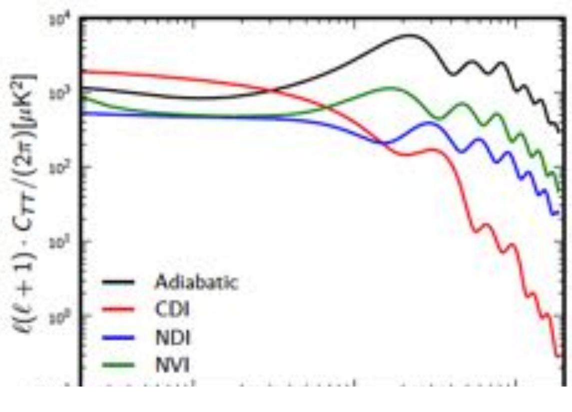
- cdm/baryon asymmetry created *by* curvaton decay

$$\zeta = \zeta_\gamma \approx R_\chi \zeta_\chi \quad , \quad \zeta_m = \zeta_\chi$$

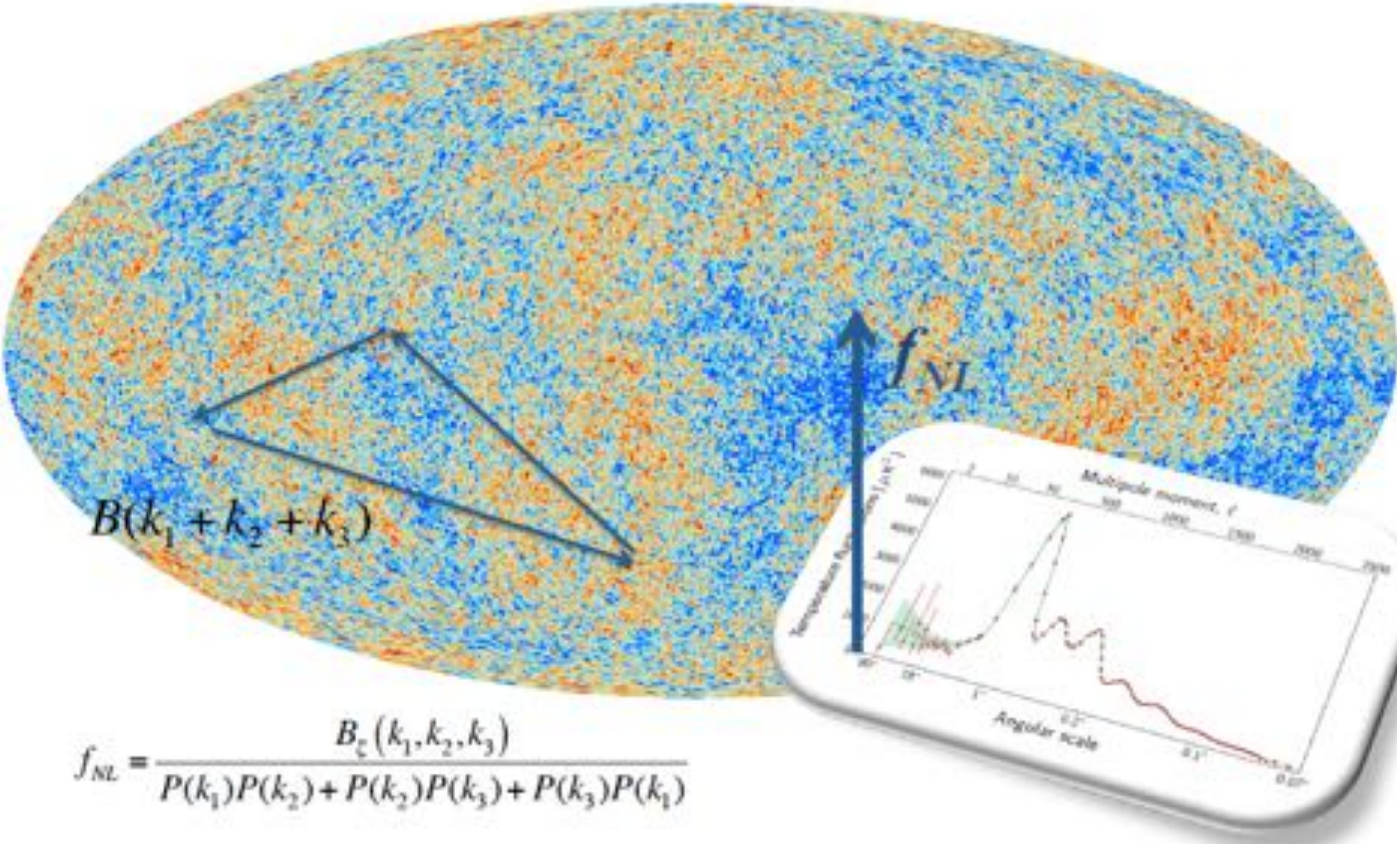
$$\Rightarrow \quad S_m \quad = \quad 3\left(1 - R_\chi\right)\zeta_\chi \quad = \quad 3\left(\frac{1 - R_\chi}{R_\chi}\right)\zeta$$

  - curvature and isocuravture perturbations naturally of same magnitude
  - *relative magnitude related to non-Gaussianity*

- Planck 2013:
  - $S_m < 0.05\,\zeta$
  - ➔ $R_\chi > 0.983$

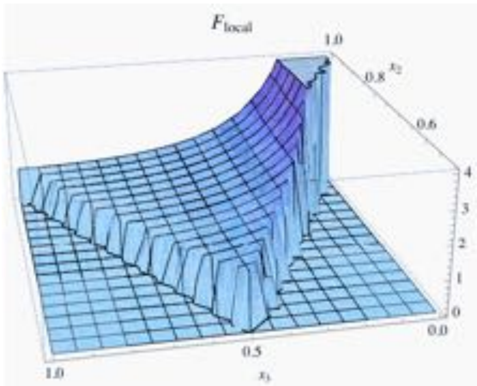


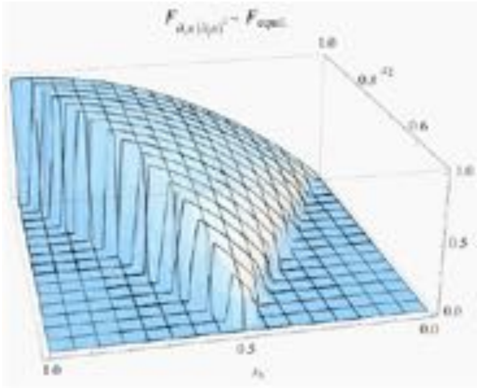
more information in higher-order correlators...

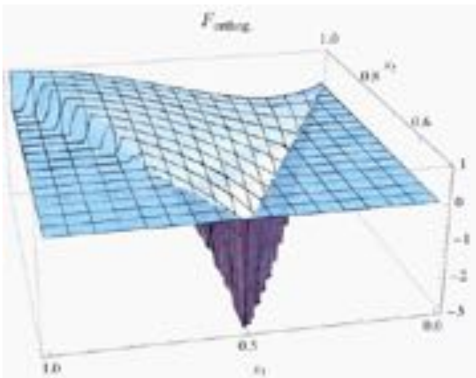


$$f_{NL} = \frac{B_\zeta(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$

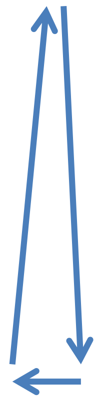
Many shapes for primordial bispectra





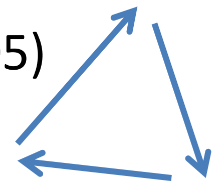


- local type (Komatsu&Spergel 2001)
  - local in real space
  - max for squeezed triangles: k<<k’,k’’



$$B_\zeta(k_1,k_2,k_3) \propto \left(\frac{1}{k_1^3k_2^3} + \frac{1}{k_2^3k_3^3} + \frac{1}{k_3^3k_1^3} \right)$$

- equilateral type (Creminelli et al 2005)
  - peaks for k1~k2~k3



$$B_\zeta(k_1,k_2,k_3) \propto \left(\frac{3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{k_1^3k_2^3k_3^3} \right)$$

- orthogonal type (Senatore et al 2009)
  - independent of local + equilateral shapes

$$B_\zeta(k_1,k_2,k_3) \propto \left(\frac{81}{k_1k_2k_3(k_1+k_2+k_3)^3} \right)$$

- separable basis (Ferguson et al 2008)

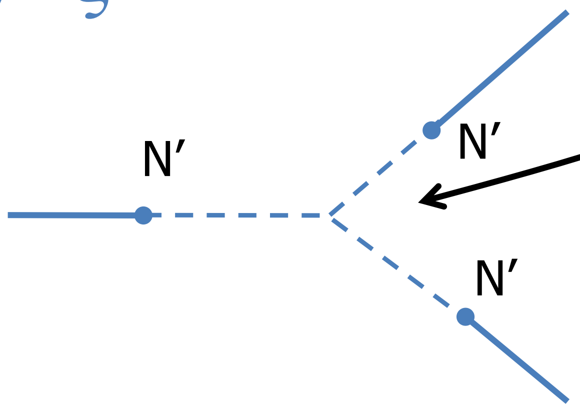
local type non-Gaussianity from non-linear δN

$$\delta\phi_I = \delta_1\phi_I + \frac{1}{2}\delta_2\phi_I + \ldots$$

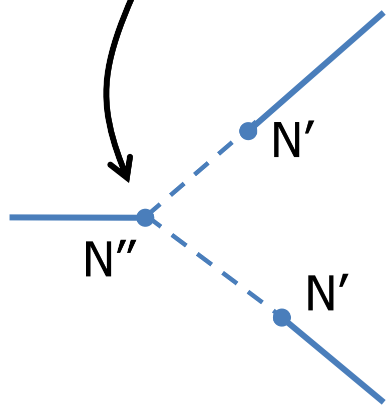
$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \ldots$$

$$= \left[\sum_I \frac{\partial N}{\partial\phi_I}\delta_1\phi_I\right] + \frac{1}{2}\left[\sum_I \frac{\partial N}{\partial\phi_I}\delta_2\phi_I + \sum_{I,J}\frac{\partial^2 N}{\partial\phi_I\partial\phi_J}\delta_1\phi_I\delta_1\phi_J\right] + \ldots$$

e.g., $<\zeta^3>$



sub-Hubble field interactions



super-Hubble classical evolution

Byrnes, Koyama, Sasaki & DW (arXiv:0705.4096)

simplest local form of non-Gaussianity

applies to many inflation models including curvaton, modulated reheating, etc

$\zeta = \zeta(\delta\phi)$ is ***local function of single Gaussian field,*** $\delta\phi(x)$

$$\zeta(x) = N'\delta\phi(x) + \frac{1}{2}N''\left(\delta\phi(x)\right)^2 + \ldots$$

$$\Rightarrow \quad \langle \zeta(x_1)\zeta(x_2)\rangle = N'^2\langle \delta\phi(x_1)\delta\phi(x_2)\rangle + \ldots$$

$$\langle \zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \frac{1}{2}N'^2 N''\langle \delta\phi(x_1)\delta\phi(x_2)\delta\phi^2(x_3)\rangle + \ldots$$

$$= \frac{3}{5}f_{NL}\langle \zeta(x_2)\zeta(x_3)\rangle\langle \zeta(x_1)\zeta(x_3)\rangle + \ldots$$

where $$\boxed{f_{NL} = \frac{5}{6}\frac{N''}{(N')^2}}$$

- odd factors of 3/5 because (Komatsu & Spergel, 2001, used) $\Phi_1 = (3/5)\zeta_1$

non-Gaussianity from inflation?

- **single slow-roll inflaton field**
  - during conventional slow-roll inflation $f_{NL}^{\ local} \approx N'' / N'^2 = O(\varepsilon) << 1$
  - ***adiabatic perturbations***

    => ζ constant on large scales => more generally: $f_{NL}^{\ local} \propto n_\zeta - 1$

- **sub-Hubble interactions** $L = (\nabla\varphi)^2 + (\nabla\varphi)^4 + \ldots$
  - e.g. DBI inflation, Galileon fields... $\Rightarrow \quad f_{NL}^{\ equil} \approx 1 / c_s^{\ 2}$

- **super-Hubble evolution**
  - *non-adiabatic perturbations* during inflation => $\zeta \neq$ constant
  - usually suppressed during slow-roll inflation
  - at/after end of inflation (modulated reheating, etc)
    - e.g., **curvaton** $f_{NL}^{local} \approx 1 / \Omega_{\chi,decay}$

curvaton scenario:

Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001

curvaton χ = weakly-coupled, late-decaying scalar field

- light field (m<H) during inflation acquires an almost scale-invariant, ***Gaussian distribution of field fluctuations*** on large scales
- **quadratic energy density** for free field, $\rho_\chi = m^2\chi^2/2$
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi} \approx \frac{1}{3}\left(\frac{2\chi\delta\chi + \delta\chi^2}{\chi^2}\right)$$

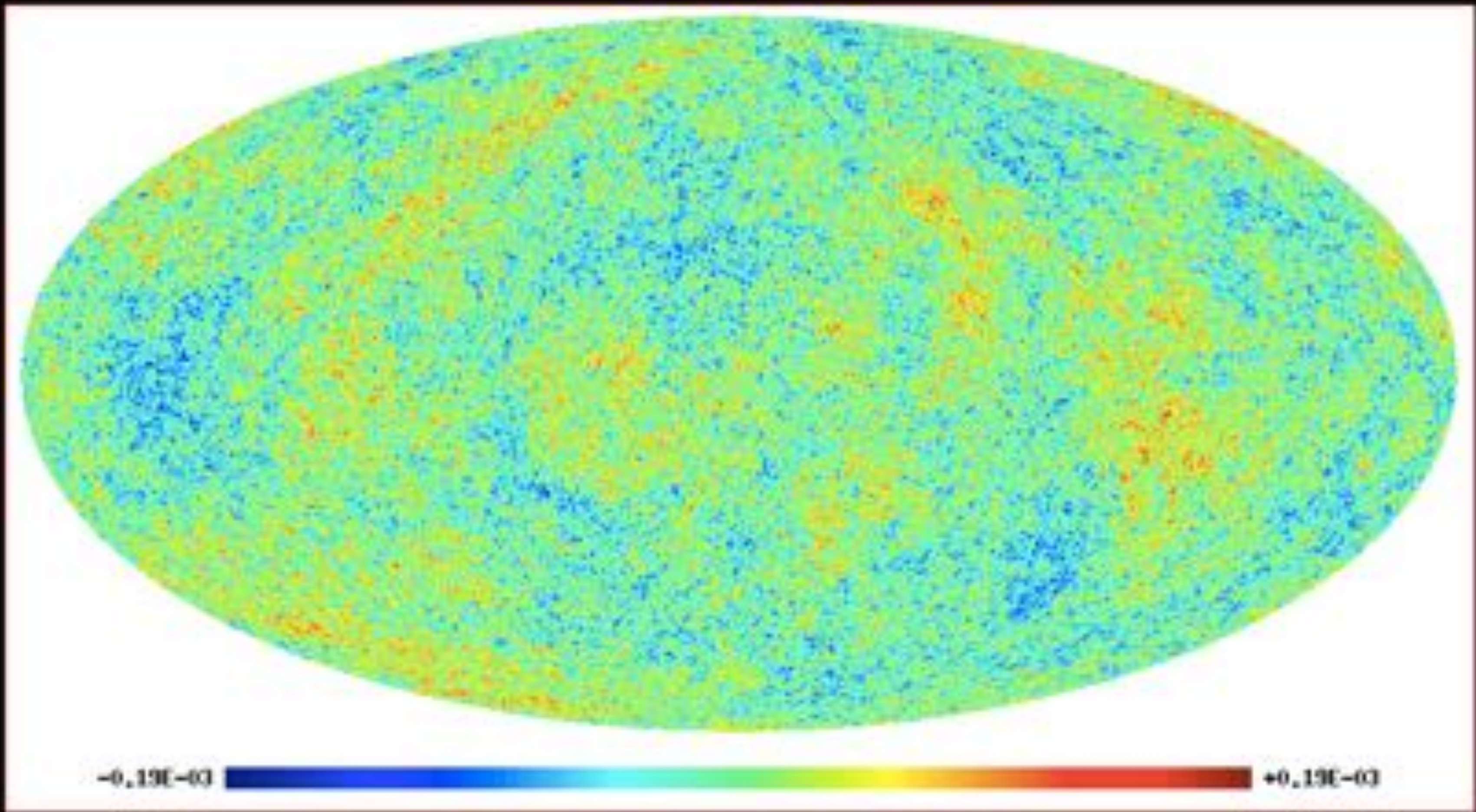
- **transferred to radiation when curvaton decays** after inflation with some **efficiency,** $0 < R_\chi < 1$, where $R_\chi \approx \Omega_{\chi,decay}$

$$\zeta = R_\chi \zeta_\chi \approx \frac{R_\chi}{3}\left(2\frac{\delta\chi}{\chi} + \frac{\delta\chi^2}{\chi^2}\right)$$

$$= \zeta_G + \frac{3}{4R_\chi}\zeta_G^2 \quad \Rightarrow \quad f_{NL} = 5/4R_\chi$$

Newtonian metric potential a ***Gaussian random field***

$$\Phi(x) = \phi_G(x)$$

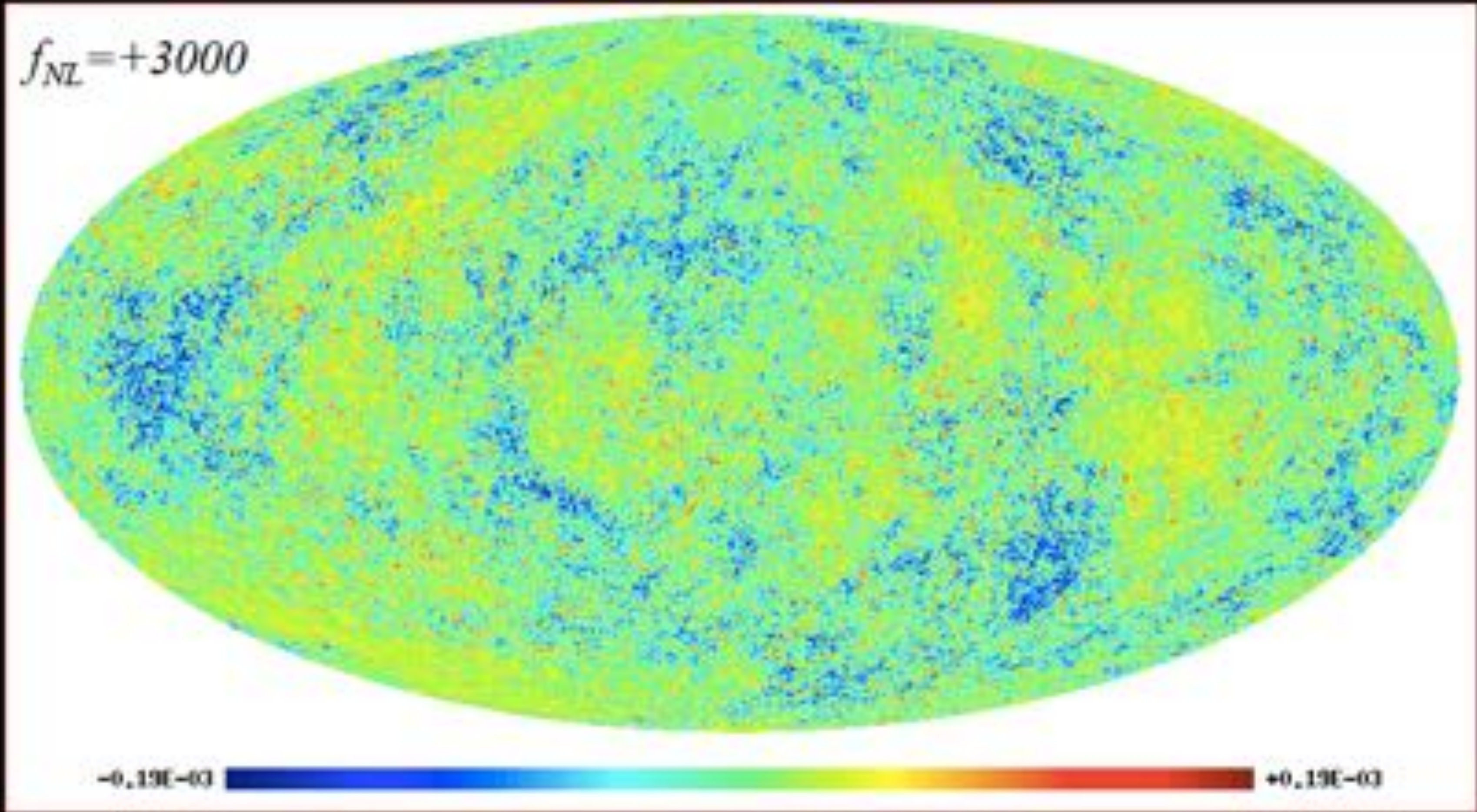


$$\Delta T/T \approx -\Phi/3 \approx -\zeta / 5$$

Liguori, Matarrese and Moscardini (2003)

Newtonian metric ***a local function of Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL}\,(\,\phi_G^2(x) - \langle\phi_G^2\rangle\,)$$

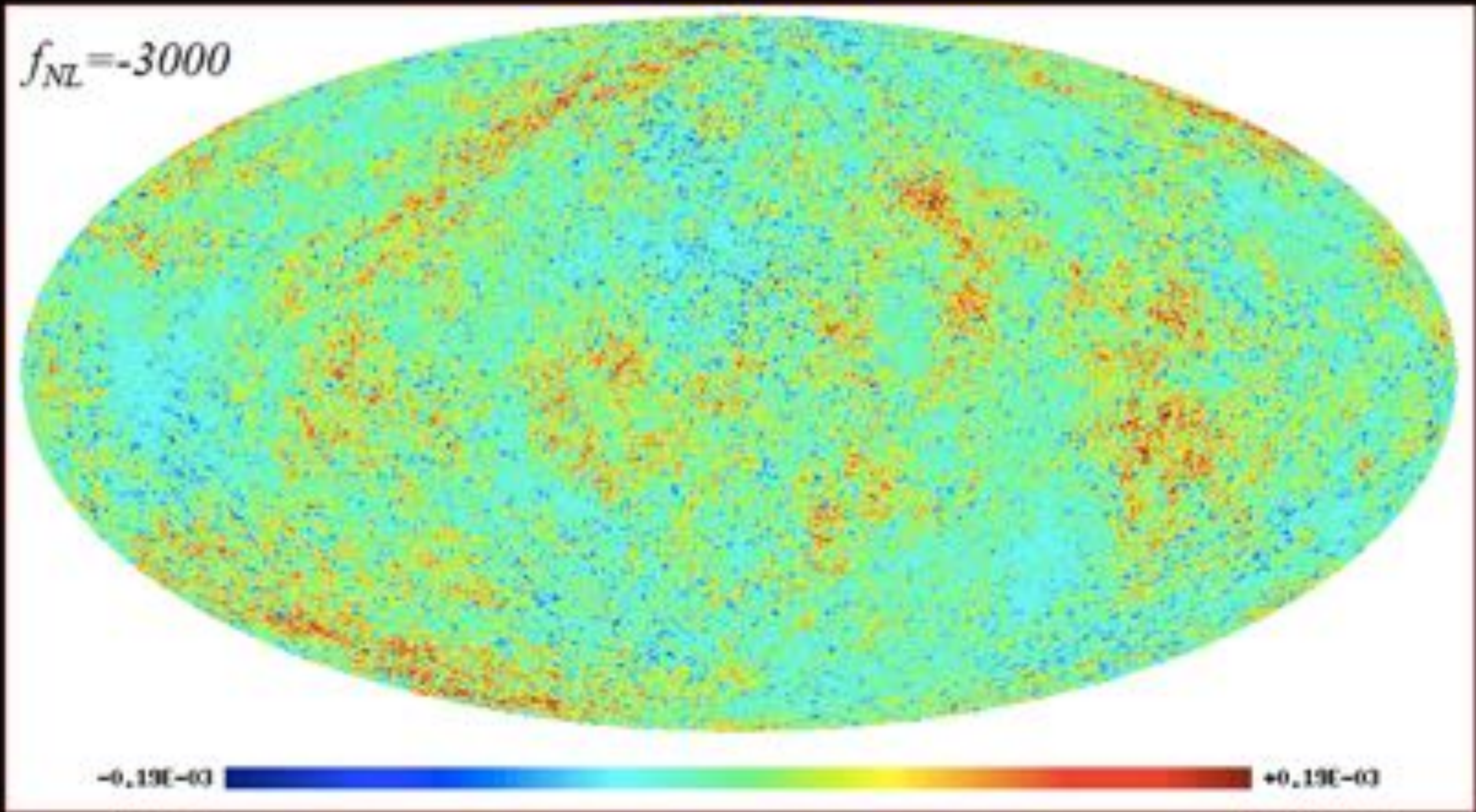


$\Delta T/T \approx -\Phi/3$, so positive $f_{NL} \Rightarrow$ more cold spots in CMB

Liguori, Matarrese and Moscardini (2003)

Newtonian potential ***a local function of Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL} \left(\phi_G^2(x) - \langle \phi_G^2 \rangle \right)$$



$\Delta T/T \approx -\Phi/3$, so negative $f_{NL} \Rightarrow$ more hot spots in CMB

Liguori, Matarrese and Moscardini (2003)

Constraints from Planck 2018

Beyond single-field, slow-roll inflation

- primordial isocurvature fraction

$$\frac{\mathcal{P}_{S_m}}{\mathcal{P}_\zeta} < 0.025 \ (95\% \ \mathrm{c.l.})$$

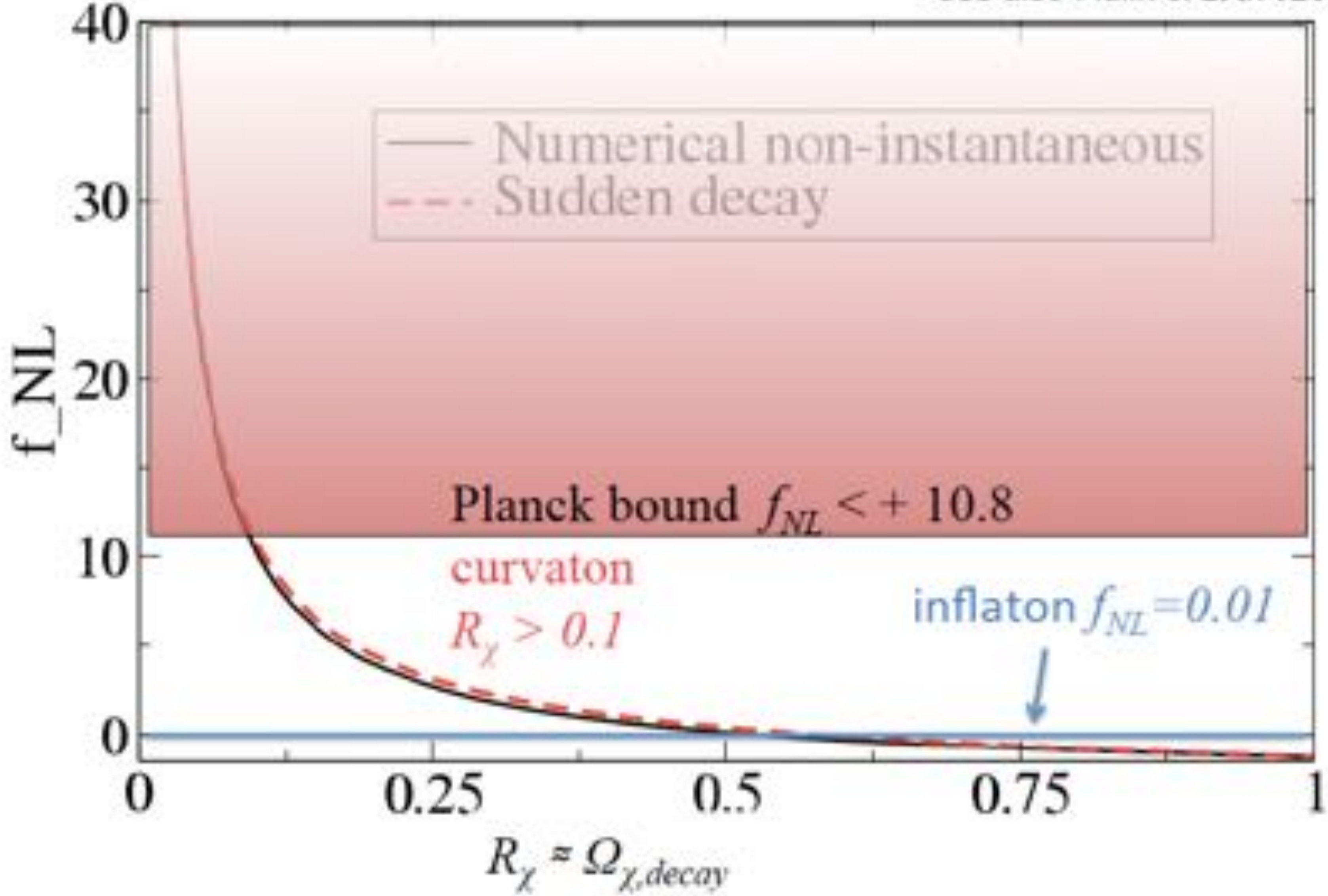
- primordial scalar bispectrum: $B \sim f_{NL} P^2$

$$f_{NL}^{local} = -1 \pm 5 \ (68\% \ \mathrm{c.l.})$$

$$f_{NL}^{equil} = -26 \pm 47 \ (68\% \ \mathrm{c.l.})$$

non-linearity parameter for quadratic curvaton

Sasaki, Valiviita & Wands (2006)
see also Malik & Lyth (2006)



Constraints from galaxy redshift surveys

- eBOSS survey (Mueller et al 2021)

$$f_{NL}^{local} = -12 \pm 21 \text{ (68\% c.l.)}$$

- future surveys
  - DESI *now taking data*

$$\Delta f_{NL}^{local} \approx 4$$

  - SKA radio galaxy surveys

$$\Delta f_{NL}^{local} \approx 1$$

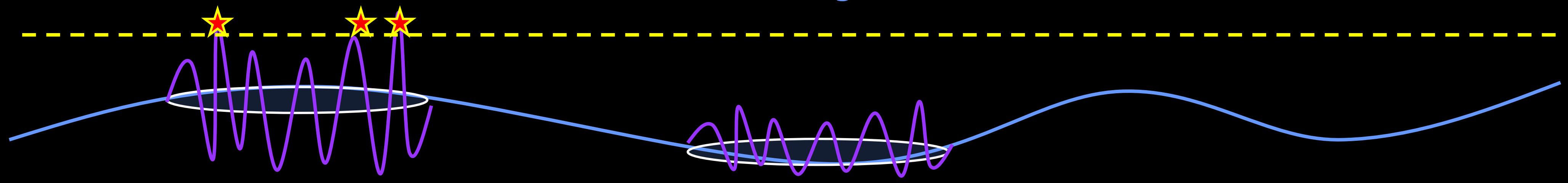
  - SphereX satellite proposal

$$\Delta f_{NL}^{local} \approx 0.2$$

peak – background split for galaxy bias

BBKS' 87

Local density of galaxies determined by number of peaks in density field above threshold
=> leads to galaxy bias: $b = \delta_g / \delta_m$



Poisson equation relates primordial density to Newtonian potential

$$\nabla^2 \Phi = 4\pi G \delta\rho \quad => \quad \phi_L = (3/2)\,(aH/k_L)^2\,\delta_L$$

so **local** $\Phi(x) \Rightarrow$ ***non-local form*** for primordial density field $\delta(x)$ from

+ inhomogeneous modulation of small-scale power

$$\sigma(X) = [\,1 + 6 f_{NL}\,(aH/k)^2\,\delta_L(X)\,]\,\sigma_s$$

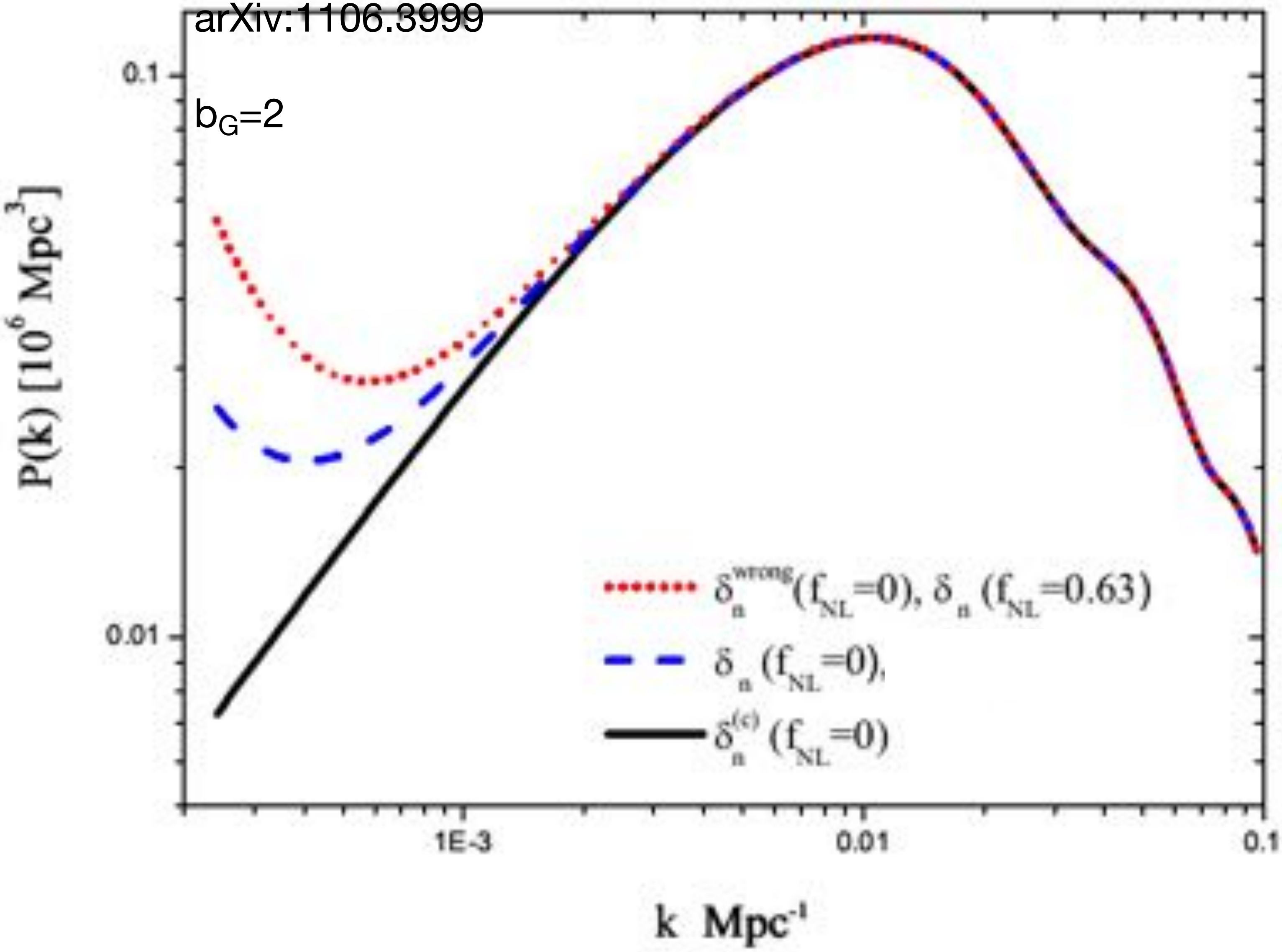
$\Rightarrow$ ***strongly scale-dependent bias on large scales***

Dalal et al, arXiv:0710.4560

Galaxy power spectrum at z=1

Bruni, Crittenden, Koyama, Maartens, Pitrou & Wands, arXiv:1106.3999

b_G=2



Conclusions

- Astrophysical surveys probe structure in our universe
  - growth of structure probes matter content and gravity
  - primordial structure probes the very early universe at ultra high energies
- Cosmological perturbation theory enables accurate and robust theoretical modelling
  - Linear theory of Gaussian perturbations provides a good description of current observations at high redshift and/or large scales
    - *golden age of precision cosmology*
  - Non-linear perturbations probe interactions
    - gravitational clustering at late times
    - primordial physics at ultra high energies

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