

# Simulations of the Glasma in 2+1D and 3+1D

**David I. Müller**

Institute for Theoretical Physics  
TU Wien, Austria

*dmueller@hep.itp.tuwien.ac.at*

October 5, 2020

EXTREME NONEQUILIBRIUM QCD 2020

# Overview

- ▶ Introduction
- ▶ Glasma in 2+1D
- ▶ Glasma in 3+1D
- ▶ Jets in the Glasma
- ▶ Outlook

# Literature

## Collisions of 3D nuclei: 3+1D Glasma

- ▶ D. Gelfand, A. Ipp, D. Müller, PRD (2016) [[1605.07184](#)]  
numerical method, proof of concept
- ▶ A. Ipp, D. Müller, PLB (2017) [[1703.00017](#)]  
rapidity profiles at RHIC
- ▶ A. Ipp, D. Müller, EPJC (2018) [[1804.01995](#)]  
numerical improvements
- ▶ D. Müller, PhD thesis (2019) [[1904.04267](#)]  
lots of technical details
- ▶ A. Ipp, D. I. Müller, EPJA (2020) [[2009.02044](#)]  
recent review

## Jets in 2+1D Glasma

- ▶ A. Ipp, D. I. Müller, D. Schuh, PRD (2020) [[2001.10001](#)]  
methods, numerical checks
- ▶ A. Ipp, D. I. Müller, D. Schuh, PLB (2020) [[2009.14206](#)]  
phenomenological results, jet broadening parameter  $\hat{q}(\tau)$

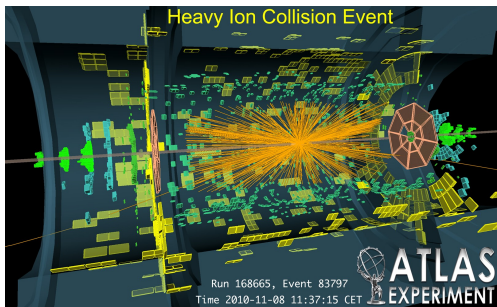
# Introduction

# Relativistic heavy ion collisions

Heavy-ion collision experiments as a means to study the properties of nuclear matter at extremely high energies

Examples:

- ▶ Au+Au at RHIC, BNL with  $\sqrt{s_{NN}}$  up to 200 GeV.
- ▶ Pb+Pb at LHC, CERN with  $\sqrt{s_{NN}}$  up to 5 TeV.



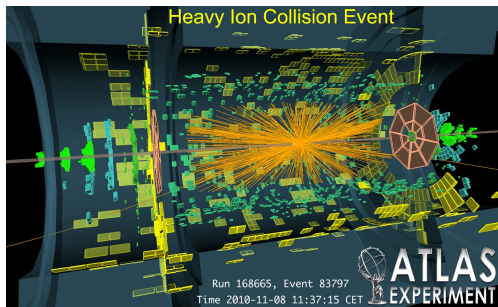
# Relativistic heavy ion collisions

Collect experimental data:

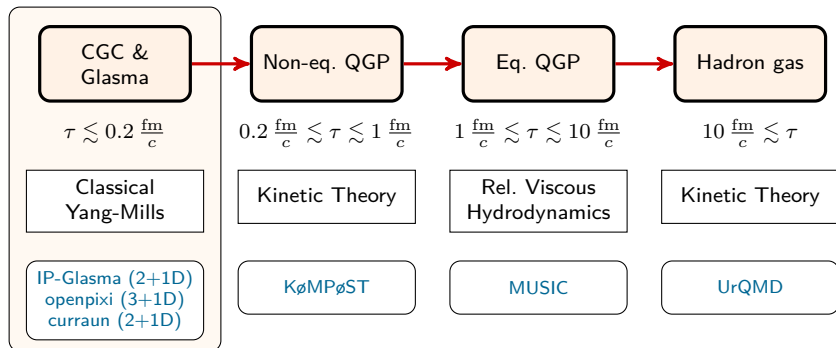
Number of produced particles,  $E$  and  $\mathbf{p}$  distributions, particle species, ...

Flow coefficients  $v_n$ , correlations, ...

All observations should be explainable using theory!



# State of the art: a chain of simulations



## Input:

- ▶ knowledge about nuclei (deep inelastic scattering, CGC models)
- ▶ properties of the QGP (collision integrals, viscosity, EOS)

## Output:

- ▶ predictions for observables (multiplicities, flow coefficients, ...)

# Color glass condensate

CGC is an effective theory for high energy QCD at weak coupling

Nuclei at high energies:

- ▶ Lorentz-contracted along collision axis
- ▶ Time-dilated dynamics
- ▶ **hard partons**: carry most of the momentum of the nucleus
- ▶ **soft partons**: high occupation number, (near-classical) coherent state

Split degrees of freedom:

- ▶ quarks, high momentum gluons: **classical color currents**  $J^\mu$
- ▶ low momentum gluons: **classical color fields**  $A_\mu$

$$D_\mu F^{\mu\nu} = J^\nu$$

CGC allows an effectively classical treatment of high energy nuclei

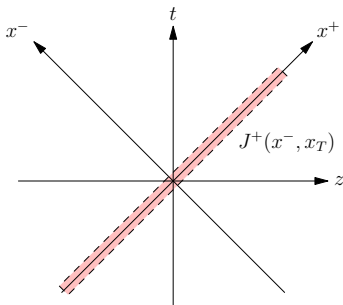


# Color glass condensate

Nucleus “A” described by color current

$$J^+(x^-, x_T) = \rho_A(x^-, x_T)$$

in terms of light cone coordinates  $x^\pm = (x^0 \pm x^3)/\sqrt{2}$  and transverse coordinates  $x_T = (x, y)$



- ▶ YM eqs.  $D_\mu F^{\mu\nu} = J^\nu$
- ▶ Use covariant gauge  $\partial_\mu A^\mu = 0$
- ▶ YM eqs. reduce to 2D Poisson eq.

$$-\Delta_T A^+(x^-, x_T) = \rho_A(x^-, x_T)$$

- ▶ Solve in Fourier space with infrared regulator  $m$

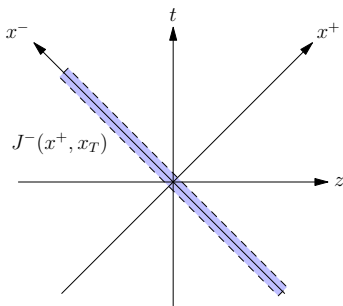
$$A^+(x^-, x_T) = \int \frac{d^2 k_T}{(2\pi)^2} \frac{\widetilde{\rho}_A(x^-, k_T)}{k_T^2 + m^2} e^{-ik_T \cdot x_T}$$

# Color glass condensate

Nucleus “B” described by color current

$$J^-(x^+, x_T) = \rho_B(x^+, x_T)$$

in terms of light cone coordinates  $x^\pm = (x^0 \pm x^3)/\sqrt{2}$  and transverse coordinates  $x_T = (x, y)$



- ▶ YM eqs.  $D_\mu F^{\mu\nu} = J^\nu$
- ▶ Use covariant gauge  $\partial_\mu A^\mu = 0$
- ▶ YM eqs. reduce to 2D Poisson eq.

$$-\Delta_T A^-(x^+, x_T) = \rho_B(x^+, x_T)$$

- ▶ Solve in Fourier space with infrared regulator  $m$

$$A^-(x^+, x_T) = \int \frac{d^2 k_T}{(2\pi)^2} \frac{\widetilde{\rho}_B(x^+, k_T)}{k_T^2 + m^2} e^{-ik_T \cdot x_T}$$

# Color glass condensate

Classical color currents  $J^\mu$  or charge densities  $\rho$  are random fields distributed according to some **probability functional**  $W[\rho]$ . This requires models.

Observables are computed as expectation values using functional integration:

$$\langle \mathcal{O}(A_\mu) \rangle = \int \mathcal{D}\rho \mathcal{O}(A_\mu[\rho]) W[\rho]$$

A simple model: **McLerran-Venugopalan (MV) model**

$$W[\rho] = Z^{-1} \exp \left( - \int d^2x_T dx^- \frac{\rho^a(x^-, x_T) \rho^a(x^-, x_T)}{2g^2 \mu^2 \lambda(x^-)} \right)$$

$$\langle \rho^a(x^-, x_T) \rangle = 0$$

$$\langle \rho^a(x^-, x_T) \rho^b(y^-, y_T) \rangle = g^2 \mu^2 \lambda(x^-) \delta^{ab} \delta(x^- - y^-) \delta^{(2)}(x_T - y_T)$$

No notion of finite radius  $\Rightarrow$  suitable for central collisions of very large nuclei

# Color glass condensate

Covariant gauge solutions:

$$J^+(x) = \rho_A(x) \quad \Rightarrow \quad A^+(x) = \int \frac{d^2 k_T}{(2\pi)^2} \frac{\widetilde{\rho}_A(x^-, k_T)}{k_T^2 + m^2} e^{-ik_T \cdot x_T}$$

$$J^-(x) = \rho_B(x) \quad \Rightarrow \quad A^-(x) = \int \frac{d^2 k_T}{(2\pi)^2} \frac{\widetilde{\rho}_B(x^+, k_T)}{k_T^2 + m^2} e^{-ik_T \cdot x_T}$$

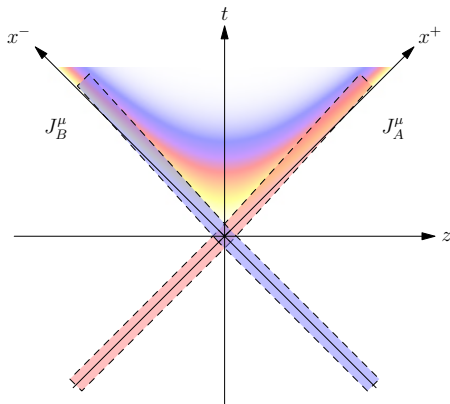
Solutions to the YM eqs. are simple for single nuclei, even though YM eqs. are non-linear.

What about collisions? How can we solve

$$D_\mu F^{\mu\nu} = J_A^\nu + J_B^\nu ?$$

## The Glasma in 2+1D

# General collision scenario

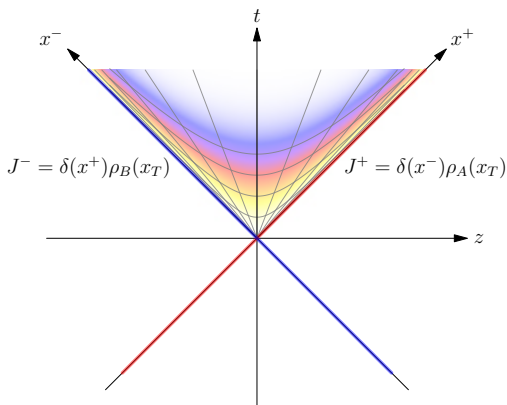


- ▶ **Glasma**: field produced from the collision of two CGCs
- ▶ Solution to the Yang-Mills equations

$$D_\mu F^{\mu\nu} = J_A^\nu + J_B^\nu$$

- ▶ Fields are non-perturbative
- ▶ Recoil of nuclei?
- ▶ No analytic solutions in the forward light cone
- ▶ Need approximations to make progress

# Ultra-relativistic collision scenario



- Assumption 1: no recoil
- Assumption 2: infinitesimally thin along  $z$

$$J^+(x) = \delta(x^-)\rho_A(x_T)$$

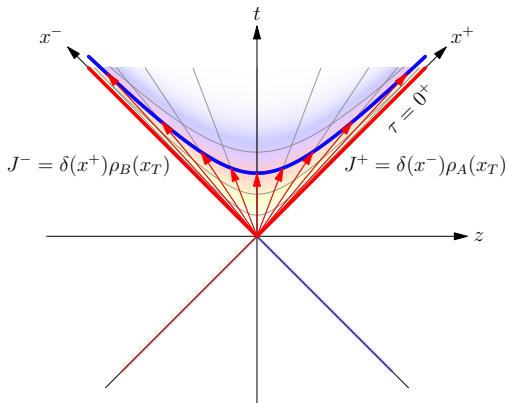
$$J^-(x) = \delta(x^+)\rho_B(x_T)$$

- Milne coordinates

$$\tau = \sqrt{2x^+x^-}, \quad \eta = \frac{1}{2} \ln \left( \frac{x^+}{x^-} \right)$$

- Invariance under boosts along  $z$  (rapidity  $\eta$  independent)
- Glasma is effectively 2+1D

# Ultra-relativistic collision scenario



- ▶ Initial conditions at boundary  $\tau = 0^+$  (analytic result)
- ▶ Evolution in future light cone determined by source-free Yang-Mills eqs.

$$D_\mu F^{\mu\nu} = 0$$

- ▶ In practice: **real-time lattice gauge theory** for time evolution along proper time  $\tau$



# The boost-invariant Glasma

Initial conditions:

$$\begin{aligned}\alpha^i(\tau \rightarrow 0^+, x_T) &= \alpha_A^i(x_T) + \alpha_B^i(x_T), \\ \alpha^\eta(\tau \rightarrow 0^+, x_T) &= \frac{ig}{2} [\alpha_A^i(x_T), \alpha_B^i(x_T)],\end{aligned}$$

with the light cone gauge solutions

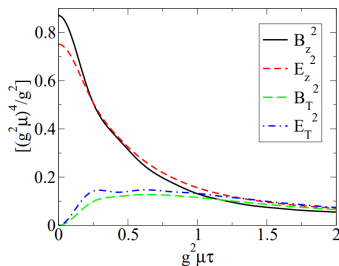
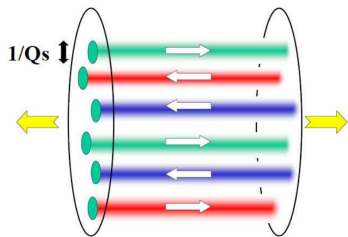
$$\alpha_{(A,B)}^i(x_T) = \frac{1}{ig} V_{(A,B)}(x_T) \partial^i V_{(A,B)}^\dagger(x_T).$$

Wilson lines computed from covariant gauge solutions:

$$V_{(A,B)}^\dagger(x_T) = \mathcal{P} \exp \left( -ig \int_{-\infty}^{+\infty} dx'^\mp A^\pm(x'^\mp, x_T) \right)$$

# The boost-invariant Glasma

Glasma at  $\tau = 0^+$  consists of **color-electric and -magnetic longitudinal flux tubes** with typical transverse size of  $Q_s^{-1}$  (saturation momentum,  $Q_s \approx g^2 \mu$ )



Figs. from H. Fujii, K. Itakura, Nucl. Phys. A (2008), [\[0803.0410\]](#)

T. Lappi, L. McLerran, Nucl. Phys. A (2006) [\[hep-ph/0602189\]](#)

Flux tubes expand and decay according to the YM eqs. until system reaches **free streaming state**

# Energy-momentum tensor

One of the main observables is the **energy-momentum tensor**  $T^{\mu\nu}$

$$T^{\mu\nu} = \frac{2}{g^2} \text{Tr} \left[ F^{\mu\rho} F^\nu{}_\rho - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right].$$

Expectation value in the MV model (Glasma rest frame):

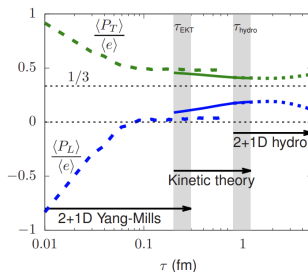
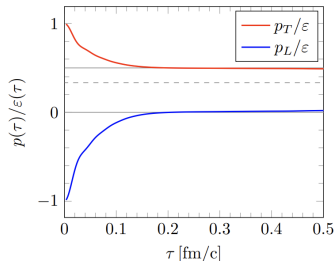
$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} \langle \varepsilon \rangle & 0 & 0 & 0 \\ 0 & \langle p_T \rangle & 0 & 0 \\ 0 & 0 & \langle p_T \rangle & 0 \\ 0 & 0 & 0 & \langle p_L \rangle \end{pmatrix}$$

Event-by-event: all components of  $T^{\mu\nu}$  relevant

# Energy-momentum tensor

One of the main observables is the **energy-momentum tensor**  $T^{\mu\nu}$

- ▶ **Problem 1: pressure anisotropy**  
(solved by KØMPØST)
- ▶ Problem 2: boost invariance
- ▶  $T^{\mu\nu}$  input for hydrodynamics/kinetic theory



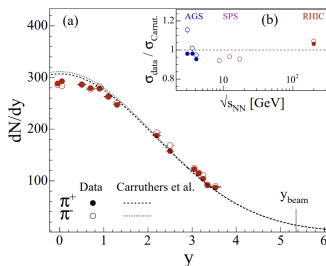
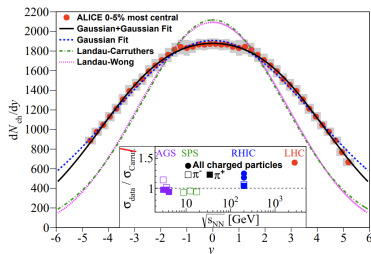
Figs. from: D. Müller, PhD thesis (2019) [\[1904.04267\]](#)

A. Kurkela, A. Mazeliauskas, JF Paquet, S. Schlichting, D. Teaney, PRL (2019) [\[1805.01604\]](#)

# Energy-momentum tensor

One of the main observables is the energy-momentum tensor  $T^{\mu\nu}$

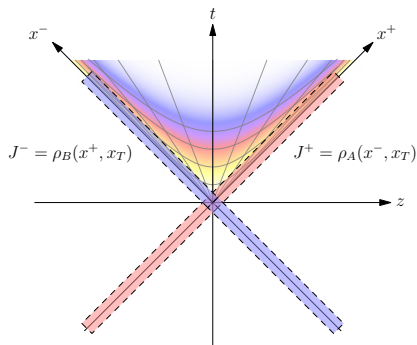
- ▶ Problem 1: pressure anisotropy (solved by KØMPØST)
- ▶ Problem 2: boost invariance
- ▶  $T^{\mu\nu}$  input for hydrodynamics/kinetic theory



Figs. from ALICE, PLB (2013) [1304.0347] and BRAHMS, PRL (2005) [nucl-ex/0403050]

## The Glasma in 3+1D

# Relativistic collision scenario with finite widths



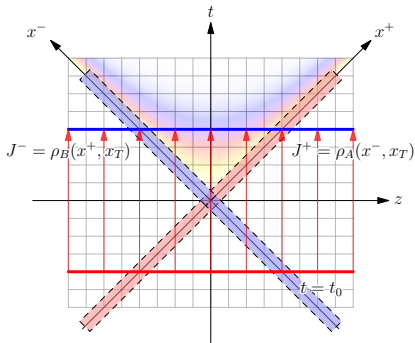
- ▶ Assumption: no recoil
- ▶ Keep width along  $z$  finite

$$J^+(x) = \rho_A(x^-, x_T)$$

$$J^-(x) = \rho_B(x^+, x_T)$$

- ▶ Finite interaction time
- ▶ No boost invariance
- ▶ Glasma is 3+1D
- ▶ Problem: rapidity dependent initial conditions in Milne coordinates?

## Relativistic collision scenario with finite widths



- ▶ Give up description in Milne coordinates  $(\tau, \eta)$
- ▶ Describe collision in laboratory frame  $(t, z)$
- ▶ Full 3+1D Yang-Mills eqs. with current

$$D_\mu F^{\mu\nu} = J_A^\nu + J_B^\nu$$

- ▶ Initial conditions at  $t = t_0$  (analytic result)
- ▶ Simulate full collision with **real-time lattice gauge theory** and **colored particle-in-cell**

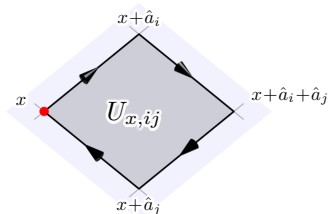


# Real-time lattice gauge theory

Lattice gauge theory is a gauge covariant discretization of Yang-Mills theory on a lattice with lattice spacings  $a^\mu$

Degrees of freedom: **gauge links**

$$U_{x,\mu} = \bar{\mathcal{P}} \exp \left( ig \int_x^{x+\hat{a}^\mu} dx'^\nu A_\nu(x') \right) \\ \approx \exp \left( ig a^\mu A_\mu \left( x + \frac{1}{2} \hat{a}^\mu \right) \right)$$



**Plaquettes** to approximate field strength tensor

$$U_{x,\mu\nu} = U_{x,\mu} U_{x+\mu,\nu} U_{x+\nu,\mu}^\dagger U_{x,\nu}^\dagger \\ \approx \exp \left( ig a^\mu a^\nu F_{\mu\nu} \left( x + \frac{1}{2} \hat{a}^\mu + \frac{1}{2} \hat{a}^\nu \right) \right)$$

# Real-time lattice gauge theory

Approximate Yang-Mills action with plaquettes: **Wilson action**

$$\begin{aligned} S &= -\frac{1}{2} \int d^4x \operatorname{Tr} [F_{\mu\nu} F^{\mu\nu}] \\ &\approx a^4 \sum_x \sum_i \frac{1}{(ga^\mu a^\nu)^2} \operatorname{Tr} \left[ \mathbf{1} - \frac{1}{2} U_{x,\mu\nu} - \frac{1}{2} U_{x,\mu\nu}^\dagger \right] \end{aligned}$$

- ▶ Discretization is correct up to  $\mathcal{O}(a^2)$
- ▶ Wilson action is symmetric under (space-)time reversal
- ▶ **Wilson action is gauge invariant** under lattice gauge transformations

$$\begin{aligned} U_{x,\mu} &\rightarrow \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger \\ U_{x,\mu\nu} &\rightarrow \Omega_x U_{x,\mu\nu} \Omega_x^\dagger \end{aligned}$$

- ▶ **Variation yields gauge covariant equations of motion**

# Real-time lattice gauge theory

Discrete Yang-Mills equations (continuous time  $t$ )

$$\dot{E}_i(t, \mathbf{x}) = - \sum_j \frac{1}{ga^i(a^j)^2} [U_{i,j}(t, \mathbf{x}) + U_{i,-j}(t, \mathbf{x})]_{\text{ah}} + J_i(t, \mathbf{x})$$

$$\dot{U}_i(t, \mathbf{x}) = ig a^i E_i(t, \mathbf{x}) U_i(t, \mathbf{x})$$

- ▶ Time evolution can be performed numerically
- ▶ External color current  $J_\mu(t, \mathbf{x})$
- ▶ Gauge covariant conservation

$$\dot{\rho}(t, \mathbf{x}) + \sum_i \frac{J_i(t, \mathbf{x}) - U_{x,-i}(t) J_i(t, \mathbf{x} - \hat{a}^i) U_{x,-i}^\dagger(t)}{a^i} = 0$$

- ▶ Gauss constraint

$$\sum_i \frac{E_i(t, \mathbf{x}) - U_{x,-i}(t) E_i(t, \mathbf{x} - \hat{a}^i) U_{x,-i}^\dagger(t)}{a^i} = \rho(t, \mathbf{x})$$

# Colored particle-in-cell method

- ▶ **Particle-in-cell (PIC)**: numerical method to simulate systems with large number of charged particles (plasma theory)
- ▶ Main concept: approximate continuous charge density with large number of charged particles

$$\rho(t, \mathbf{x}) \approx \sum_n Q_n \delta^{(3)}(\mathbf{x} - \mathbf{x}(t))$$

- ▶ **Colored particle-in-cell (CPIC)**: non-Abelian generalization

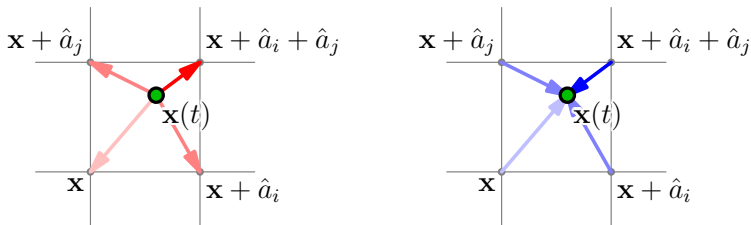
$$\rho(t, \mathbf{x}) \approx \sum_n \mathbf{Q}_n(t) \delta^{(3)}(\mathbf{x} - \mathbf{x}(t))$$

with time-dependent color charge  $\mathbf{Q}_n(t)$

- ▶ Allows gauge covariant treatment of color currents on the lattice

# Colored particle-in-cell method

- ▶ How to couple charged particles with continuous positions  $\mathbf{x}(t)$  with fields on a lattice?  $\Rightarrow$  **Interpolation methods**



- ▶ We use nearest grid point (NGP) interpolation
- ▶ Particles contribute to lattice charge density at NGP
- ▶ Particle movement generates current whenever NGP changes

Color rotation: particle moves from  $\mathbf{x}$  to  $\mathbf{y}$

$$\mathbf{Q}(t + \Delta t) = U_{\mathbf{x} \rightarrow \mathbf{y}}(t) \mathbf{Q}(t) U_{\mathbf{x} \rightarrow \mathbf{y}}^\dagger(t)$$

# McLerran-Venugopalan model in 3+1D

Relax ultrarelativistic approximation by allowing for finite width along  $z$

$$J_A^+(x^-, x_T) = \delta(x^-) \rho_A(x_T) \quad \Rightarrow \quad J_A^+(x^-, x_T) = f(x^-) \rho_A(x_T)$$

- ▶ Longitudinal profile  $f(x^-)$  or simply  $f(z)$
- ▶ Use Gaussian profile with longitudinal extent  $L \propto R_A/\gamma$

$$f(z) = \frac{1}{\sqrt{2\pi}L} \exp\left(-\frac{z^2}{2L^2}\right)$$

- ▶ Trivial color longitudinal structure (no fluctuations along  $z$ )

# Glasma in 3+1D

3D density plot of energy density  $\varepsilon(x)$

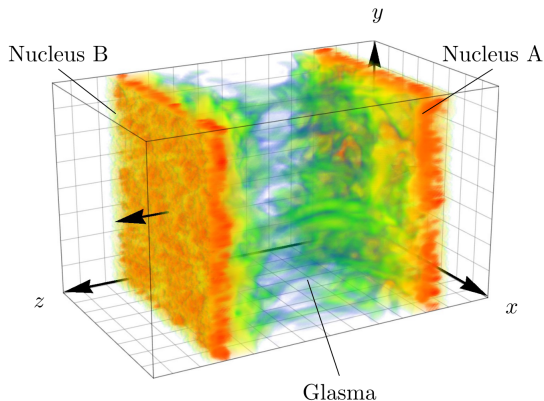


Fig. adapted from A. Ipp, D. Müller, PLB (2017) [\[1703.00017\]](#)

# Energy-momentum tensor

Expectation value in the 3+1D MV model (laboratory frame):

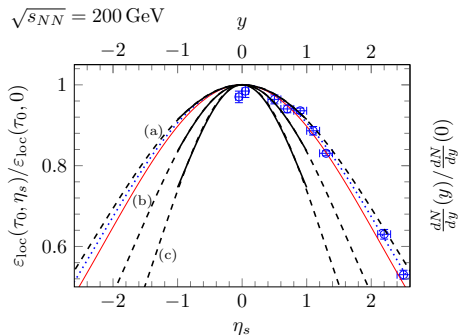
$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} \langle \varepsilon \rangle & 0 & 0 & \langle S_L \rangle \\ 0 & \langle p_T \rangle & 0 & 0 \\ 0 & 0 & \langle p_T \rangle & 0 \\ \langle S_L \rangle & 0 & 0 & \langle p_L \rangle \end{pmatrix}$$

- ▶ Longitudinal energy flux (Poynting vector)  $\langle S_L \rangle$
- ▶ What is the rapidity dependence of the produced Glasma?
- ▶ Observable of interest: **local rest frame energy density**  $\langle \varepsilon_{\text{loc}} \rangle$
- ▶ Choose local frame s.t.  $\langle S_L \rangle$  vanishes and obtain diagonal  $\langle T^{\mu\nu} \rangle$
- ▶ Use Milne coordinates to obtain  $\langle \varepsilon_{\text{loc}} \rangle(\tau, \eta)$



# Rapidity profiles

Comparison of space-time rapidity  $\eta$  dependence of  $\langle \varepsilon_{\text{loc}} \rangle$  at  $\tau = 1 \text{ fm}/c$  to charged particle multiplicity as a function of momentum rapidity  $y$  from BRAHMS experiment at RHIC



IR regulator  $m$  dependence: (a)  $m = 0.2 \text{ GeV}$ , (b)  $m = 0.4 \text{ GeV}$ , (c)  $m = 0.8 \text{ GeV}$

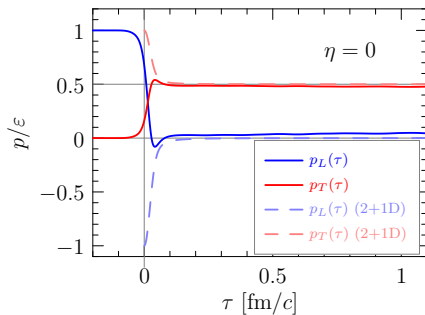
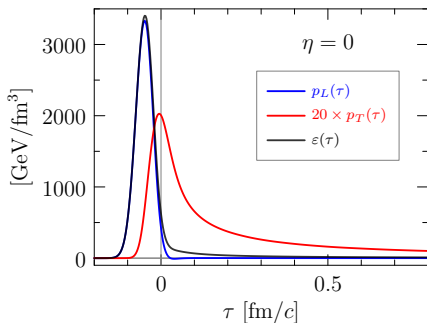
Red line: **Landau model**

Fig. from A. Ipp, D. Müller, PLB (2017) [[1703.00017](#)]

Data from BRAHMS, PRL (2005) [[nucl-ex/0403050](#)]

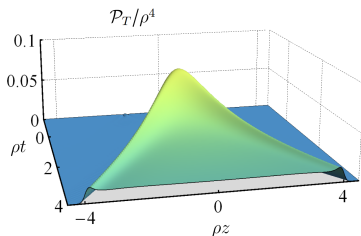
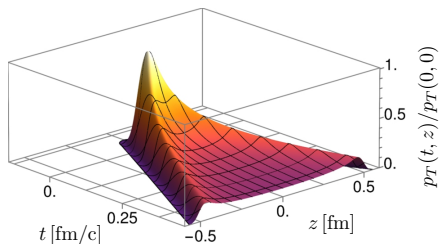
# Pressure anisotropy

- ▶ At early times: color fields of the nucleus dominate over Glasma
- ▶ At late times: free-streaming limit  $\langle p_L \rangle \approx 0$
- ▶ Same anisotropy issue as in 2+1D Glasma
- ▶ Full space-time distribution of e.g. transverse pressure  $p_T$



# Pressure anisotropy

- ▶ At early times: color fields of the nucleus dominate over Glasma
- ▶ At late times: free-streaming limit  $\langle p_L \rangle \approx 0$
- ▶ Same anisotropy issue as in 2+1D Glasma
- ▶ Full space-time distribution of e.g. transverse pressure  $p_T$



Figs. from A. Ipp, D. Müller, PLB (2017) [\[1703.00017\]](#)

J. Casalderrey-Solana, M. P. Heller, D. Mateos, W. van der Schee, PRL (2013) [\[1305.4919\]](#)

# Extensions

Model for charge density  $\rho(x)$  is too simple:

- ▶ No finite radius
- ▶ No longitudinal color fluctuations

$$J_A^+(x^-, x_T) = f(x^-)\rho_A(x_T) \quad \Rightarrow \quad J_A^+(x^-, x_T) = \rho_A(x^-, x_T)$$

Numerical issues:

- ▶ 3+1D simulations are computationally expensive!
- ▶ High lattice resolution for numerical stability
- ▶ Large simulation volumes for realistic results
- ▶ Semi-implicit methods as an alternative to standard leapfrog approach  
For details, see: A. Ipp, D. Müller, EPJC (2018) [\[1804.01995\]](#)

# Other works on 3+1D Glasma

S. Schlichting, B. Schenke, PRC (2016) [[1605.07158](#)]

- ▶ Rapidity dependence from JIMWLK
- ▶ No full 3+1D dynamics, “stitched together” 2+1D
- ▶ LHC energies

S. McDonald, S. Jeon, C. Gale, QM 2018 & 2019 [[1807.05409](#)], [[2001.08636](#)]

- ▶ Rapidity dependence from JIMWLK
- ▶ 3+1D dynamics in  $(\tau, \eta)$  + hydrodynamics
- ▶ LHC energies
- ▶ No rigorous derivation of the initial conditions

3+1D CPIC approach (this talk)

- ▶ Rapidity dependence from finite width, no JIMWLK
- ▶ 3+1D dynamics in  $(t, z)$
- ▶ RHIC energies
- ▶ Rigorous initial conditions, but no longitudinal fluctuations (yet)

## Jets in the Glasma

# Jets in the Glasma

- ▶  $T^{\mu\nu}$  is not everything!
- ▶ **Jets**: highly energetic, focused particle “sprays” that originate from hard scatterings of partons **during the collision** at  $\tau \approx 0$
- ▶ Interactions with the medium
  - ▶ momentum broadening (opening angle)
  - ▶ energy loss (quenching)
- ▶ Jets interact with all stages of the medium
- ▶ Strong color fields of the Glasma might affect jets even before the hydrodynamical stage

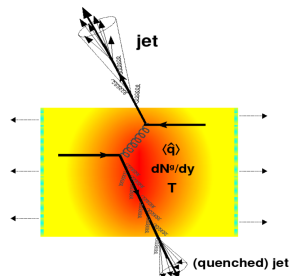


Fig. from D. d'Enterria, *Jet quenching*, Landolt-Bornstein (2010) [0902.2011]

# Jets in the Glasma

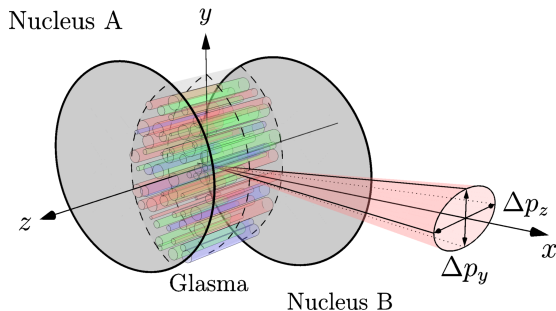


Fig. from A. Ipp, D. I. Müller, D. Schuh, PRD (2020) [[2001.10001](https://arxiv.org/abs/2001.10001)]



# Jets in the Glasma

- ▶ Model early-time jet as single quark (or gluon)
- ▶ Very high initial momentum, no deflection
- ▶ Interaction with the Glasma via the non-Abelian Lorentz force
- ▶ Accumulation of transverse momentum  $p_{\perp}$  over time
- ▶ 2+1D Glasma for simplicity

## Wong equations

$$\begin{aligned}\frac{dp_{\mu}}{dt} &= g Q^a \frac{dx^{\nu}}{dt} F_{\mu\nu}^a(x(t)) & \frac{dp_{\mu}}{dt} &= q \frac{dx^{\nu}}{dt} F_{\mu\nu}(x(t)) \\ \frac{dQ^a}{dt} &= g \frac{dx^{\mu}}{dt} f^{abc} A_{\mu}^b(x(t)) Q^c & \frac{dq}{dt} &= 0\end{aligned}$$

Background field  $A_{\mu}$  from 2+1D Glasma simulation

# Jet momentum broadening in the Glasma

- ▶ Integrate Wong's equations
- ▶ Average over color charges of the jet and background field

Main result for a quark moving along  $x$ -axis ( $i = y, z$ ):

$$\langle p_i^2(\tau) \rangle_q = \frac{g^2}{N_c} \int_0^\tau d\tau' \int_0^\tau d\tau'' \langle \text{Tr} [f^i(\tau') f^i(\tau'')] \rangle$$

$$f^y(\tau) = U(\tau) (E_y(\tau) - B_z(\tau)) U^\dagger(\tau)$$

$$f^z(\tau) = U(\tau) (E_z(\tau) + B_y(\tau)) U^\dagger(\tau)$$

Color rotation matrix in temporal gauge  $A^\tau = 0$ :

$$U(\tau) = \mathcal{P} \exp \left( -ig \int_0^\tau d\tau' A_x(\tau') \right)$$

# Jet momentum broadening in the Glasma

Simplified jet broadening picture in terms of Glasma flux tubes:

- ▶ Electric flux tubes:  
 $\Delta p_z$  (or rapidity) broadening
- ▶ Magnetic flux tubes:  
 $\Delta p_y$  (or azimuthal) broadening
- ▶ Electric and magnetic flux tubes  
are not created equally
- ▶ Momentum broadening is  
anisotropic

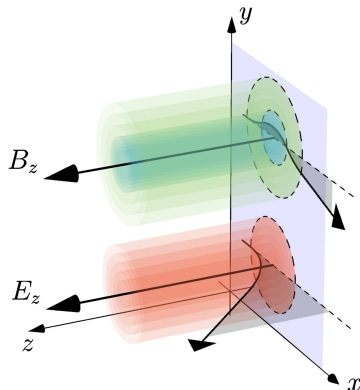
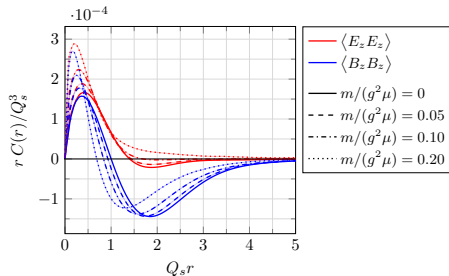


Fig. from A. Ipp, D. I. Müller, D. Schuh, PLB (2020) [2009.14206]

# Jet momentum broadening in the Glasma

Simplified jet broadening picture in terms of Glasma flux tubes:

- ▶ Electric flux tubes:  
 $\Delta p_z$  (or rapidity) broadening
- ▶ Magnetic flux tubes:  
 $\Delta p_y$  (or azimuthal) broadening
- ▶ Electric and magnetic flux tubes  
are not created equally
- ▶ Momentum broadening is  
anisotropic



Plot: initial correlation functions  
for  $E_z$  and  $B_z$

Fig. from A. Ipp, D. I. Müller, D. Schuh, PRD (2020) [[2001.10001](#)]

# Accumulated momenta at $\tau = 0.6 \text{ fm}/c$

- Accumulated momenta at transition time  $\tau_0 = 0.6 \text{ fm}/c$  for various  $Q_s$

$$\langle p_{\perp}^2 \rangle \approx Q_s^2$$

- Broadening is anisotropic

$$\frac{\langle p_z^2 \rangle}{\langle p_y^2 \rangle} \approx 2$$

- Infrared dependence  $m$
- Gluonic jets

$$\frac{\langle p_{\perp}^2 \rangle_g}{\langle p_{\perp}^2 \rangle_q} = \frac{C_A}{C_F} = \frac{2N_c^2}{N_c^2 - 1}$$

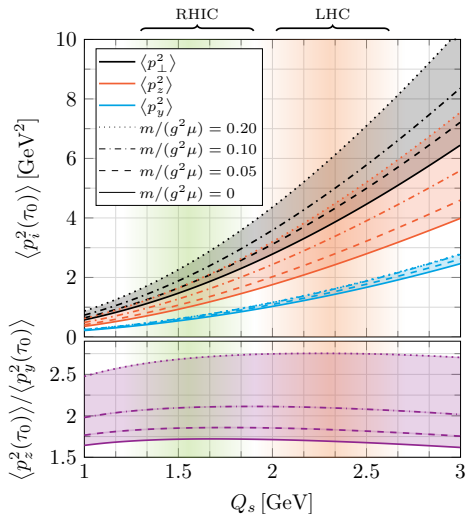
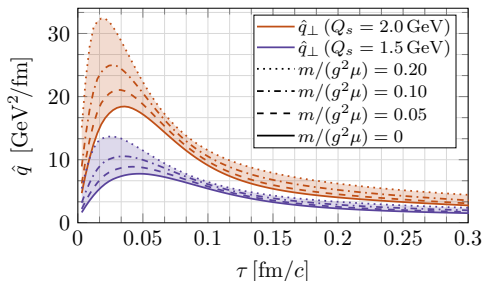


Fig. from A. Ipp, D. I. Müller, D. Schuh, PLB (2020) [2009.14206]

# Jet momentum broadening in the Glasma

Jet broadening parameter  $\hat{q}$ : accumulated (squared) momentum per unit time

$$\hat{q}_{\perp}(\tau) = \frac{d\langle p_{\perp}^2 \rangle}{d\tau}$$



Most momentum is accumulated in the earliest stages of the Glasma!

Fig. from A. Ipp, D. I. Müller, D. Schuh, PLB (2020) [\[2009.14206\]](#)

# Outlook

## 3+1D Glasma

- ▶ Collisions at higher energies (LHC)
- ▶ Include longitudinal structure

## Jets in the Glasma

- ▶ Energy loss
- ▶ CPIC simulations for jets
- ▶ Jets in 3+1D Glasma

# Thank you!

