Actions S-me diau Spaces

India CHATTERJI

Plan for the talk:

-8-wedian spaces: définitions & examples

- a conjecture & evidences

- some known cases

S-median spaces

(X,d) geodesic metric space:

y:

y

d

isometry

 $I(x,y) = \{t \in X | d(x,t) + d(t,y) = d(x,y)\}$ the interval between z and y

S-median spaces (X,d) geodésic metric space:  $J(x,y) = \{t \in X \mid d(x,t) + d(t,y) \leq d(x,y) + \delta\}$ the f-interval between a and y

S-median Spaces: (X,d) geodesic metric space is called median if for any  $x,y,z\in X$   $T(x,y) \cap T(y,z) \cap T(z,z) = 1pt3$ 

S-median spaces: (X,d) geodesic metric space is called median if for any  $X,Y,Z \in X$   $T(X,Y) \cap T(Y,Z) \cap T(Z,Z) = 1pt3$ Ex: R

S\_median spaces: (X,d) geodésic metric space is called median if for any x, y, z ∈ X  $I(x,y) \cap I(y,z) \cap I(z,z) = \{pt\}$ Ex: R Z IR2

S-median spaces: (X,d) geodésic metric space is called median if for any x,y, z ∈ X

 $T(x,y) \cap T(y,z) \cap T(z,z) = \{pt\}$ 

S-median spaces: (X,d) geodésic metric space is called median if for any x,y, z ∈ X  $I(x,y) \cap I(y,z) \cap I(z,z) = \{pt\}$ NOT (R2, enclidean)

S-median Spaces: (X,d) geodesic metric space is called median if for any  $X,y,z\in X$   $T(x,y) \cap T(y,z) \cap T(z,x) = 1pt3$ 

S-median Spaces: (X,d) geodesic metric space S>0 is called S-median if for any  $X,y,z\in X$  $d_{H}(T_{S}(X,y)\cap T_{S}(y,z)\cap T_{S}(z,z), 1pt3)\leq C=C(S)$ 

 $J_s(x,y) = \{t \in X | d(x,t) + d(t,y) \leq d(x,y) + s\}$ the s-interval between z and y S-median spaces: (X,d) geodesic metric space S>0 is called S-median if for any  $x,y,z\in X$  $d_{H}(I_{S}(x,y)\cap I_{S}(y,z)\cap I_{S}(z,z), 1pt3)\leq C=C(S)$ 

Examples: median spaces are 0-median

· symmetric spaces associated to rank one lie groups

 $J_s(x,y) = \{t \in X | d(x,t) + d(t,y) \leq d(x,y) + \delta \}$ the s-interval between z and y

S-median spaces: (X,d) geodésic metric space S>0 is called 5-nection if for any x,y, z ∈ X  $d_{\mu}\left(\overline{I_{\delta}(x,y)} \cap \overline{I_{\delta}(y,z)} \cap \overline{I_{\delta}(z,z)}, \mathcal{I}_{\rho}t^{3}\right) \leq C = C(\delta)$ Examples: H/2 (or any Granor hyperbolic space)

S-median spaces: (X,d) geodésic metric space S>0 is called 5-nection if for any x,y, z ∈ X  $d_{H}\left(\mathcal{I}_{\delta}(x,y) \cap \mathcal{I}_{\delta}(y,z) \cap \mathcal{I}_{\delta}(z,z), 1pt3\right) \leq C = C(\delta)$ Examples: H/2, (H/2xH/2, d=dy2+dy2) products of Ha, He, HH NOT: symmetric spaces associated to higher rank lie groups.

Plan for the talk:

-8-wedian spaces: définitions & examples

- a conjecture & evidences

- some known cases

## Carjechure: let a be a finitely generated group

acts property by isometries an a uniformly locally finite f-median space.



ack properly by affine isometries at some  $\ell^p$  space.

Carjechure: let a be a finitely generated grap has bounded orbits for actions a uniformly locally finite f-median space.

(=)

any action of G Gack property by affine isomehies an some l'espace has a fixed point. (strang property (T))

Carjechure: let a be a finitely generated grap acts properly by isometries an a uniformly locally finite f-median space. Lo balls intersected with uniform nets have uniformly bounded cardinality a ack properly by affine isometies as some l'epace.

## Carjechure: let a be a finitely generated group

acts proposty by isometries an a uniformly locally finite f-median space.

Lo not that:

(=)

"不不不

ack properly by affine isometries an some  $\ell^p$  space.

Plan for the talk:

\_8-wedian spaces: definitions & examples

- a conjecture & evidences

- some known cases

Status of that conjecture, evidences:

Yu, Alvarez-Caffergue, Panou, Baurdan (98 -...-2016)

If Gacks properly by isometries on a uniformly locally finite S-hyperbolic space, then Gacks on  $C^p(G \times G)$  for plarge enough.

Status of that oujecture, evidences: Yu, Alvarez-Cafforgne, Pausu, Bourdon (98 -...-2016) If Gacks properly by isometries an a uniformly locally finite S-hyperbolic space, then a acts on ep(GxG) for p large enough. Chatterji - Druhi - Haghard (2007) Gack proposly as a median space (=) Gacks properly as a Hilbert space

Status of that oujecture, evidences: Chatterji - Druhi - Haghad (2007) Yu, Alvarez-Cafforgue, Pansu, Barrolan (98 -..-2016) If G ach properly by isometimes an a uniformly Gack properly as a median space (=) locally finite s. hyperbolic space, then a acts Gacks properly as a Hilbert space on ep(G×G) for p large enough.

Chatterji - Druhi - Haghard (2007) Any action of a by isometies on a wedian space has a boundled orbit

(=) a has Kathdan properly (T).

## Status of that aujecture, evidences:

Yu, Alvarez-lafforgue, Pauvu, Baurdan (98 - ... 2016) If G ach properly by isometries an a weiformly locally fruite S. hyperbolic space, then a acts on eff(G×G) for \$ bigs enough.

Chatterji - Druhi - Haghard (2007) Gack properly on a median space (=) Any action of a by isometies on a wedian space how a bounded orbit. a a Hilbert space

(T).

Haethel: Cathices in higher rank simple lie groups Cannot act on a 8-median space without a bounded orbit.

Bader - Furman - Gelander - Monod: Same lattices Cannot act an an el space without fix paint.

Hoeld: Comme Composition of the group Live g Status of that oujecture, evidences: Lathical act on a S. median space without cannot act or his Any action of a by isometries on a median space Yu, Alvarez-Cafforgue, Pausu, Bairdan (98 -..-2016) Bader - Furman - plander - Honod: Hand on hard of some of the control of the cont on eff(GxG) for p large enough. Challering propuly on a Hilbert spa Cachs properly as a Hilbert space If G ach properly by isometries on a uniformly Bader - turman - yulander - 1 canoon : same pix point.

Cannot act on our locally finite 8-hyperbolic space, then a ach Koeldon property (T). SL31R

Hoeld: Cannot ark and a live group. Status of that oujecture, evidences: · Cannot act on a space without Any action of a by isometimes on a median space.

Any action of a by isometimes on a median space. Yu, Alvarez-Caffergue, Pausu, Bairdan (98 -..-2016) Cachs properly a a Hilbert space

Cachs properly

Cachs properly

Cachs properly Charley - Douber - Haghard (2007) If G ach properly by isometimes on a uniformly Wader - turman - your er space without fox point.

Counnot act on an locally fruite s. hyperbolic space, then a act w C has Kaehdan property (T). or l'e(G×G) for p large enaigh. open: Do mapping class groups have property (T)? Act properly as a Hilbert space? On same PP-space?

## Status of that conjecture, evidences: | Status | Status

Open: Do napping class groups have properly(T)?

Act properly as a Hilbert space? On some

eP-space?

Known (Bestrina-Branberg-Frijiwara, Petyt):

Happing class groups act properly by isauchies on a uniformly locally finite S-nedian space.