Plan for the talk:

- δ-median spaces: definitions & examples
- a conjecture & evidences
- some known cases

Rank one & product of rank one Lie groups

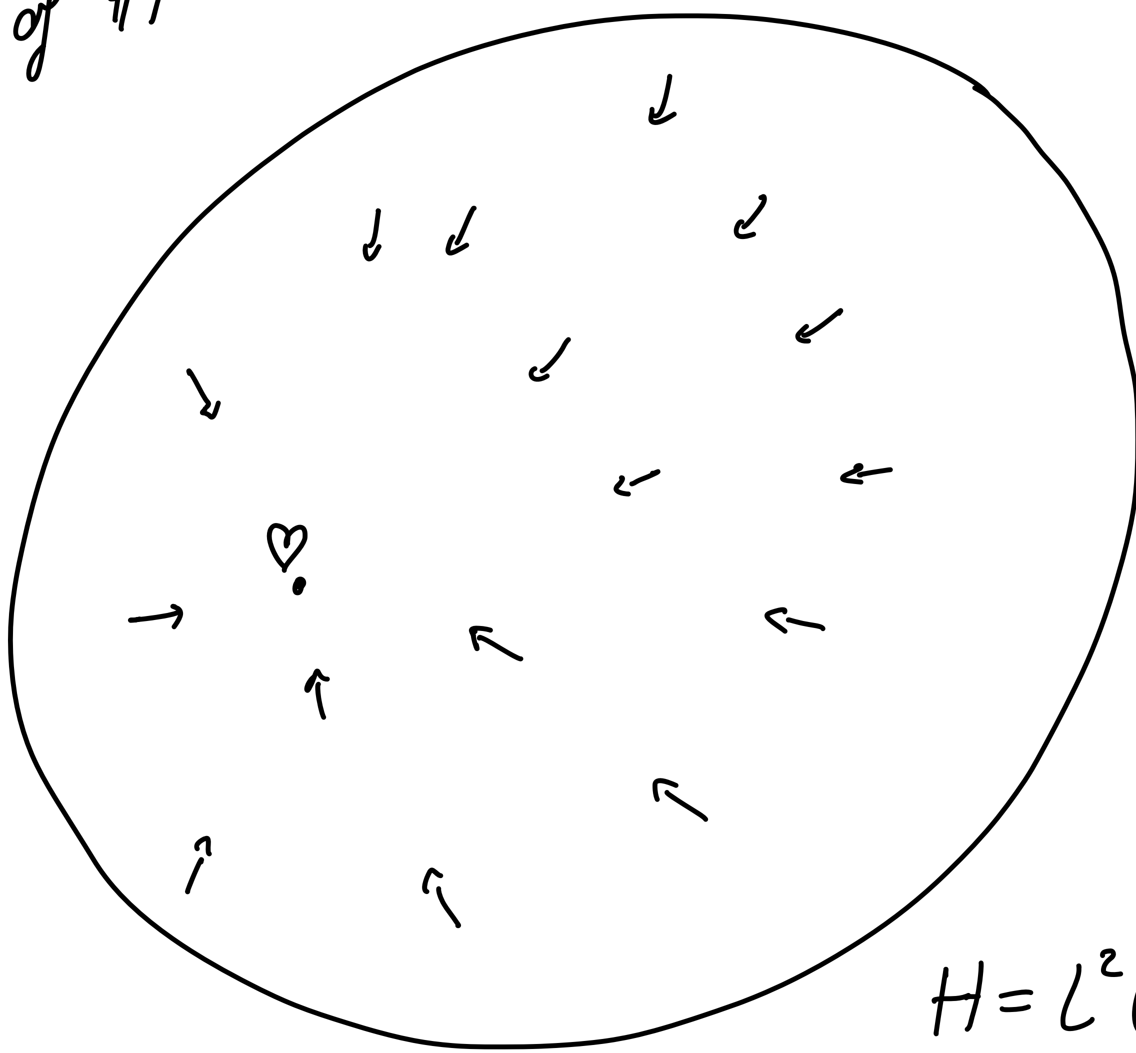
$SO(n,1)$, $SU(n,1)$ } act properly on a Hilbert space & on a median space

$Sp(n,1)$, $F_{4(-20)}$ } have property (T), so any action on a median space has a bounded orbit.

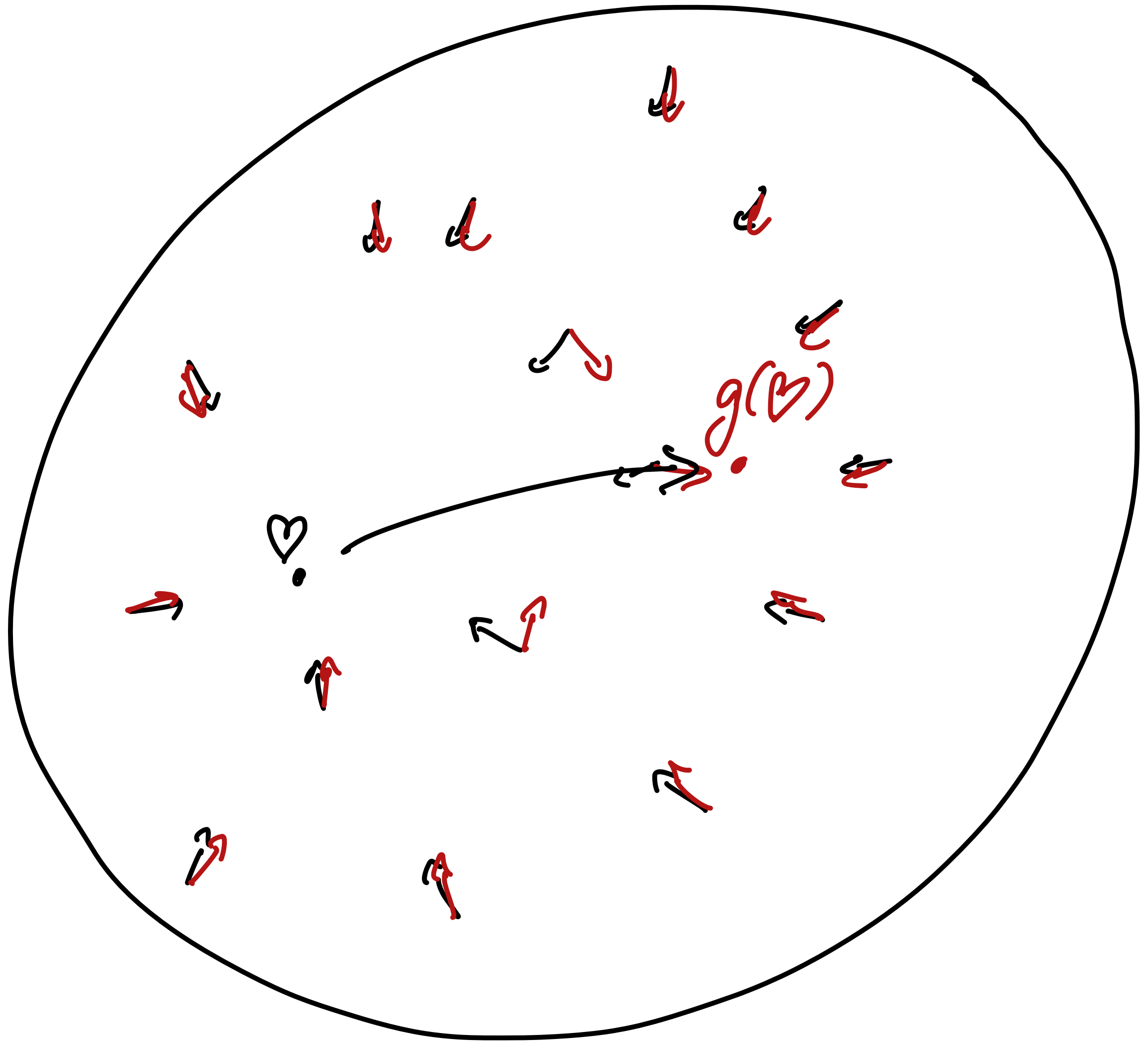
But act nicely on a δ-median space even δ-hyperbolic.

Similar proof as for the tree will work!

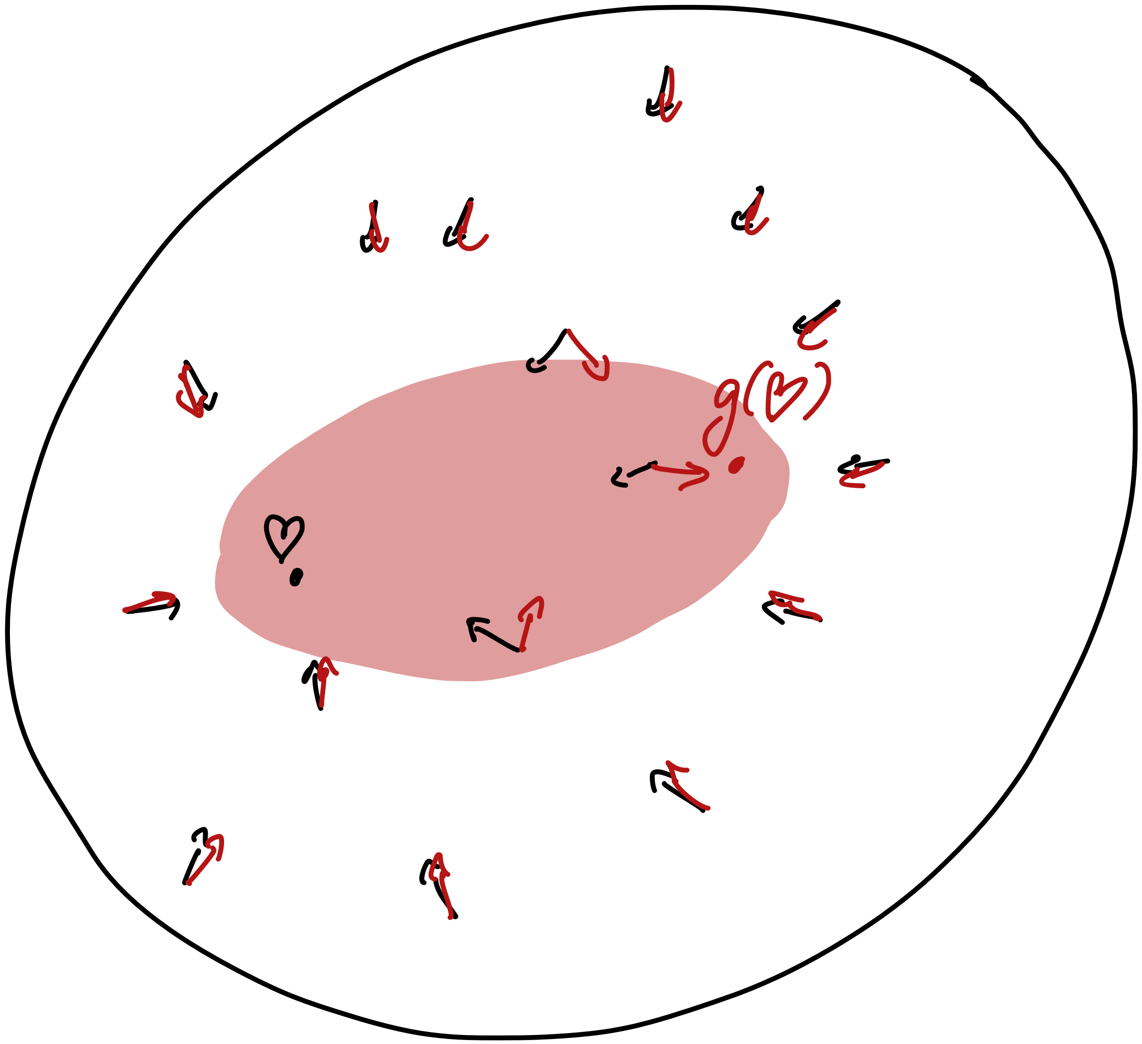
Think of $\mathbb{H}^2$



$H = L^2(T\mathbb{H}^2)$

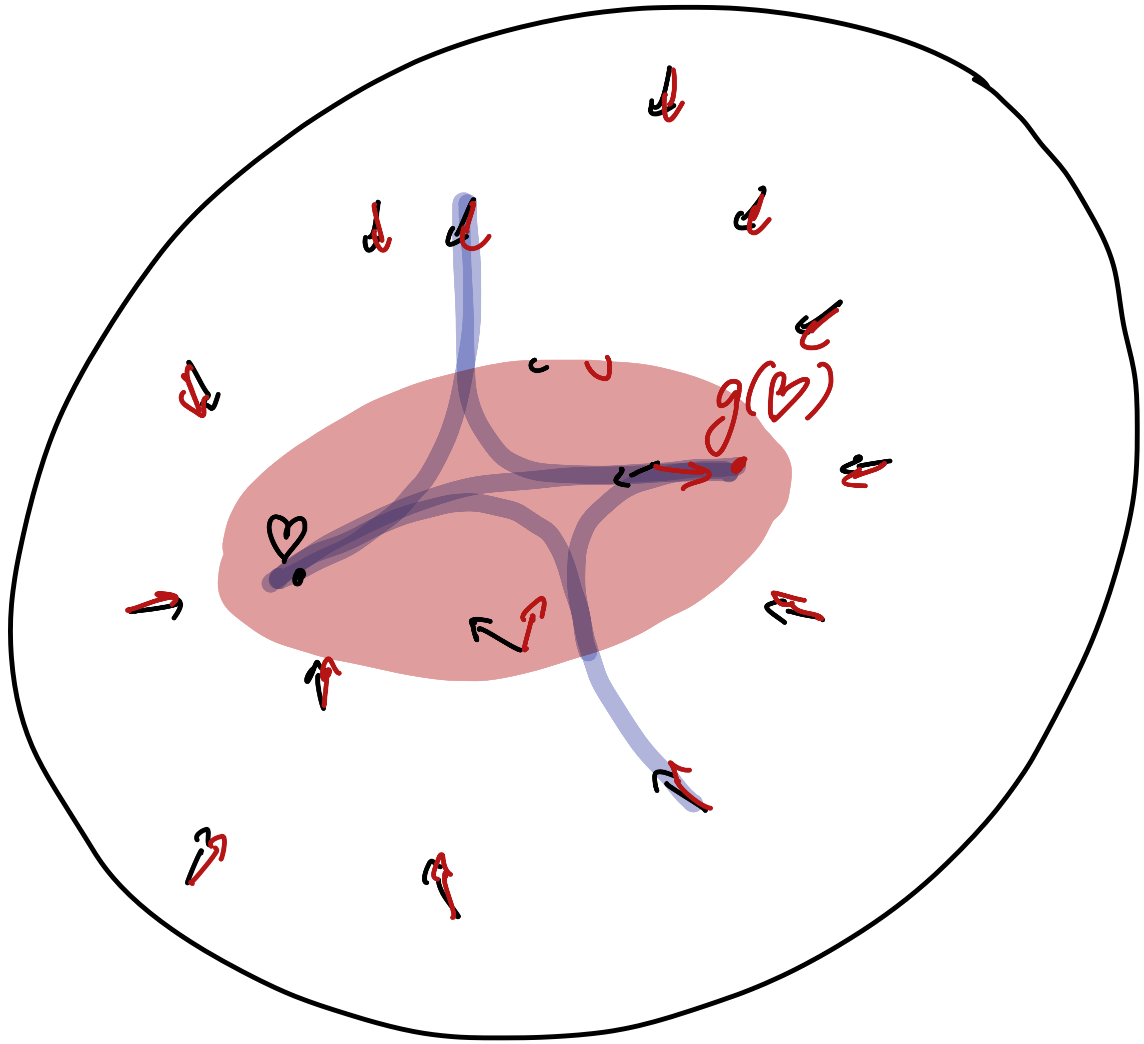


$$\xi_{\heartsuit} - \xi_{g(\heartsuit)}$$



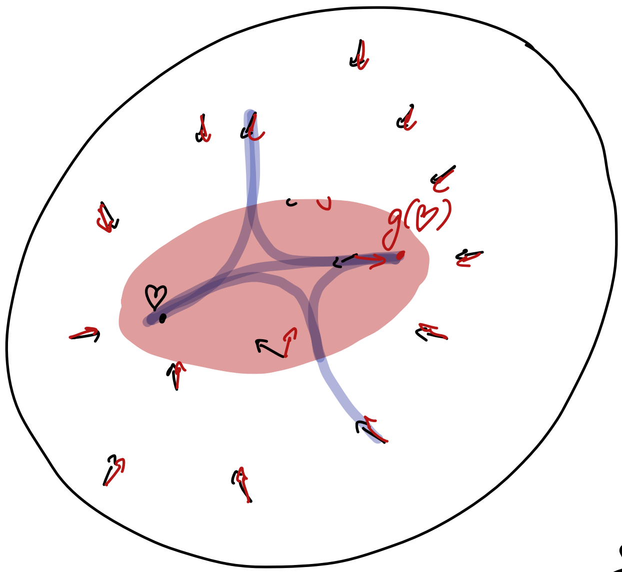
$$\xi_{\heartsuit} - \xi_{g(\heartsuit)} \sim 2 \text{ on } I_\delta(x,y)$$

& very small outside



$\xi_{\heartsuit} - \xi_{g(\heartsuit)} \sim 2$ on $I_\delta(x,y)$

& very small outside



$\xi_{\heartsuit} - \xi_{g(\heartsuit)} \sim 2$ on $I_\delta(x,y)$

& very small outside

because of thin triangles

so that

$$\| \xi_{\heartsuit} - \xi_{g(\heartsuit)} \|_p < \infty \quad \text{for } p \text{ large enough}$$

and $\| \xi_{\heartsuit} - \xi_{g(\heartsuit)} \|_p \sim d(\heartsuit, g(\heartsuit)) \xrightarrow[g \to \infty]{} \infty$

Alvarez-Lafforgue: construct some kind of tangent vector using a flow on a uniformly locally finite hyperbolic graph.

Chatterji-Dahmani-Haettel-Lecureux: rephrase everything in terms of tangent bundles on a metric space.

Chatterji - Dahmani - Haettel - Lecureux : rephrase everything in terms of tangent bundles on a metric space -

<u>Definition</u>: For (X,d) a metric space with a nice enough measure, a tangent space on X is a Polish space TX, with :

(a) $\pi : TX \longrightarrow X$ a Borel map

(b) $\pi^{-1}(a)$ is a Banach space $\forall a \in X$

(c) $\exists\ X \times X \longrightarrow TX \quad (a,x) \longmapsto \vec{ax} \in T_a X$ measurable with $\vec{aa} = \vec{0} \in T_a X$

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The tangent space is negatively curved if

$$\|\vec{ax} - \vec{ay}\| \leq C e^{-cd(a,x)} \qquad C = C(d(x,y))$$

Proper if

$$\int_X \|\vec{ax} - \vec{ay}\|\, dx \geqslant K d(x,y)$$

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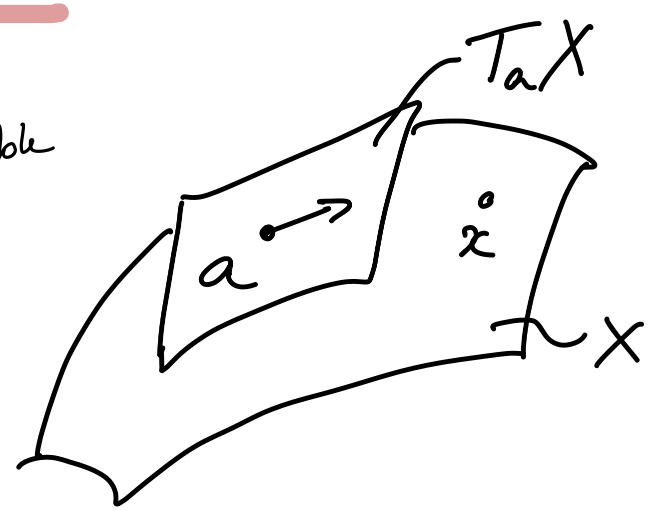

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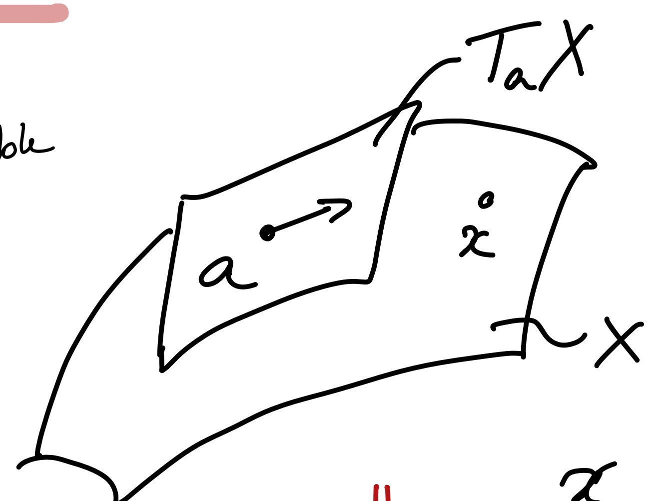
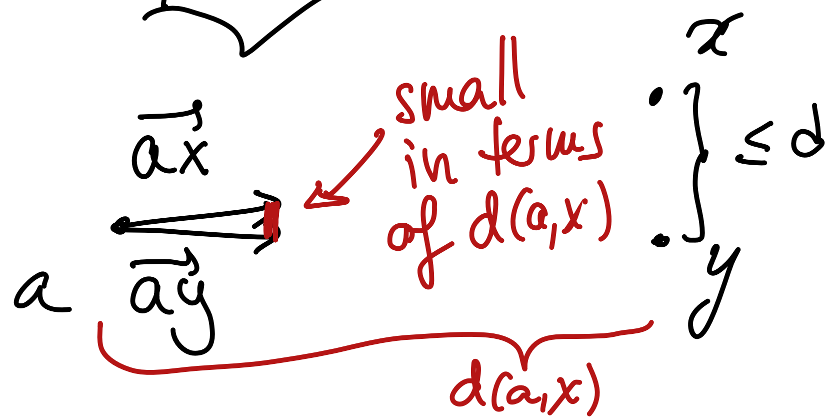


The tangent space is negatively curved if
$\|\vec{ax} - \vec{ay}\| \leq C e^{-c d(a,x)}$ $\qquad C = C(d(x,y))$



Proper if $$\int_X \|\vec{ax} - \vec{ay}\|\, dx \geq K d(x,y)$$

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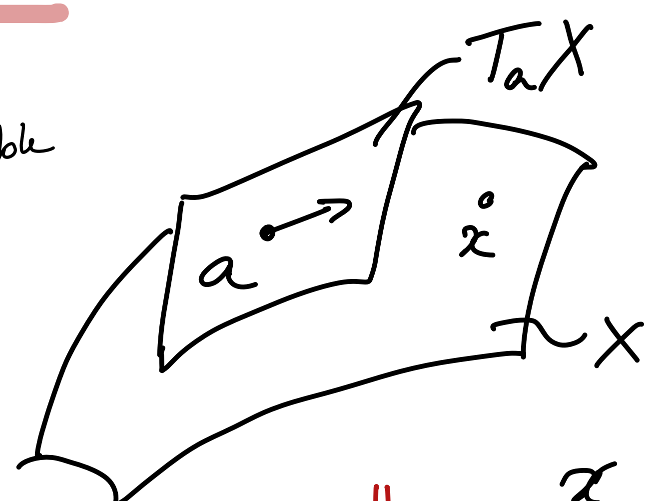

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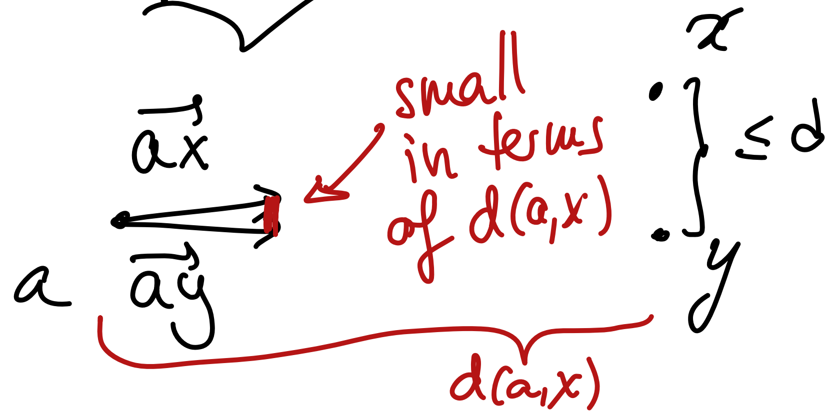
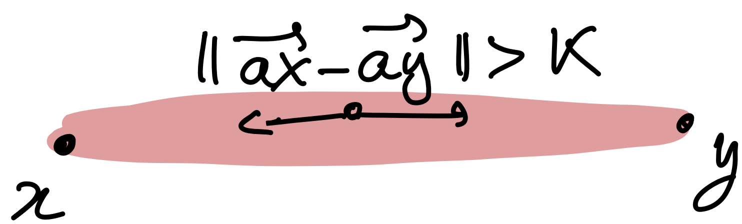
The tangent space is negatively curved if $\|\vec{ax} - \vec{ay}\| \le C e^{-c\, d(a,x)}$ $\quad C = C(d(x,y))$



Proper if $\int_X \|\vec{ax} - \vec{ay}\|\, dx \geqslant K\, d(x,y)$



! uniformly locally finite important in general: Minasyan-Osin produce groups that have fix points on any isometric action on an ℓ^p space, but admit an effective action on a quasi-tree.

THANK YOU

FOR YOUR

ATTENTION !