

Plan for the talk:

- δ -median spaces : definitions & examples
- a conjecture & evidences
- some known cases

Baby case : If G acts properly on a tree
then G acts properly on a Hilbert space H

$$\alpha : G \times H \longrightarrow H \quad \text{s.t}$$

$$\alpha(g) \in \text{Isom}_{\text{Aff}}(H) = \mathcal{U}(H) \ltimes H$$

$$\| \alpha(g)(x) - x \|_H \xrightarrow{g \rightarrow \infty} \infty \quad \forall x \in H$$



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$$\alpha : G \times H \longrightarrow H \quad \text{s.t}$$

$$\alpha(g) \in \text{Isom}_{\text{Aff}}(H) = \mathcal{U}(H) \ltimes H$$

$$\Rightarrow \alpha(g)(x) = \pi(g)(x) + b(g)$$

where $b : G \longrightarrow H$ 1-cocycle

$$\text{and } \|b(g)\| \xrightarrow{g \rightarrow \infty} \infty$$

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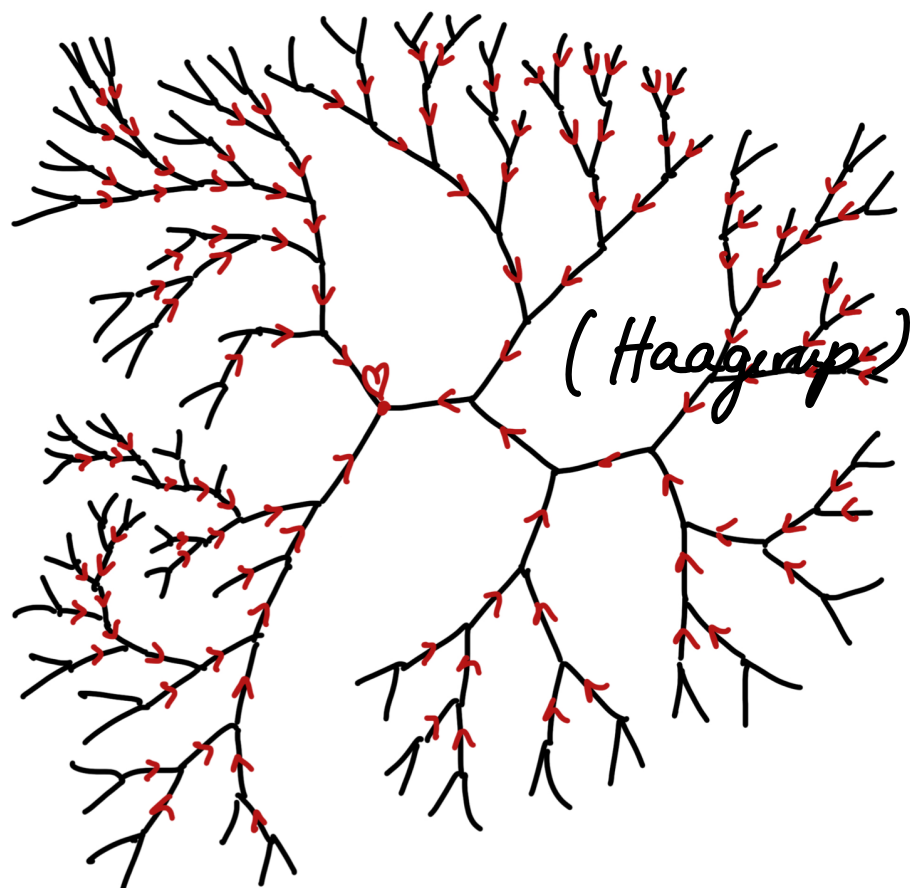
$$H = \ell^2(T \times T) \quad T = \text{vertices of the tree}$$

$\heartsuit \in T$ base point

$$\xi_{\heartsuit} \in \ell^{\infty}(T \times T)$$
$$\xi_{\heartsuit}(x, y) = \begin{cases} 1 & \text{if } \{x, y\} \text{ is an} \\ & \text{edge} \rightarrow \heartsuit \\ 0 & \text{otherwise} \end{cases}$$

$$\notin \ell^2(T \times T)$$

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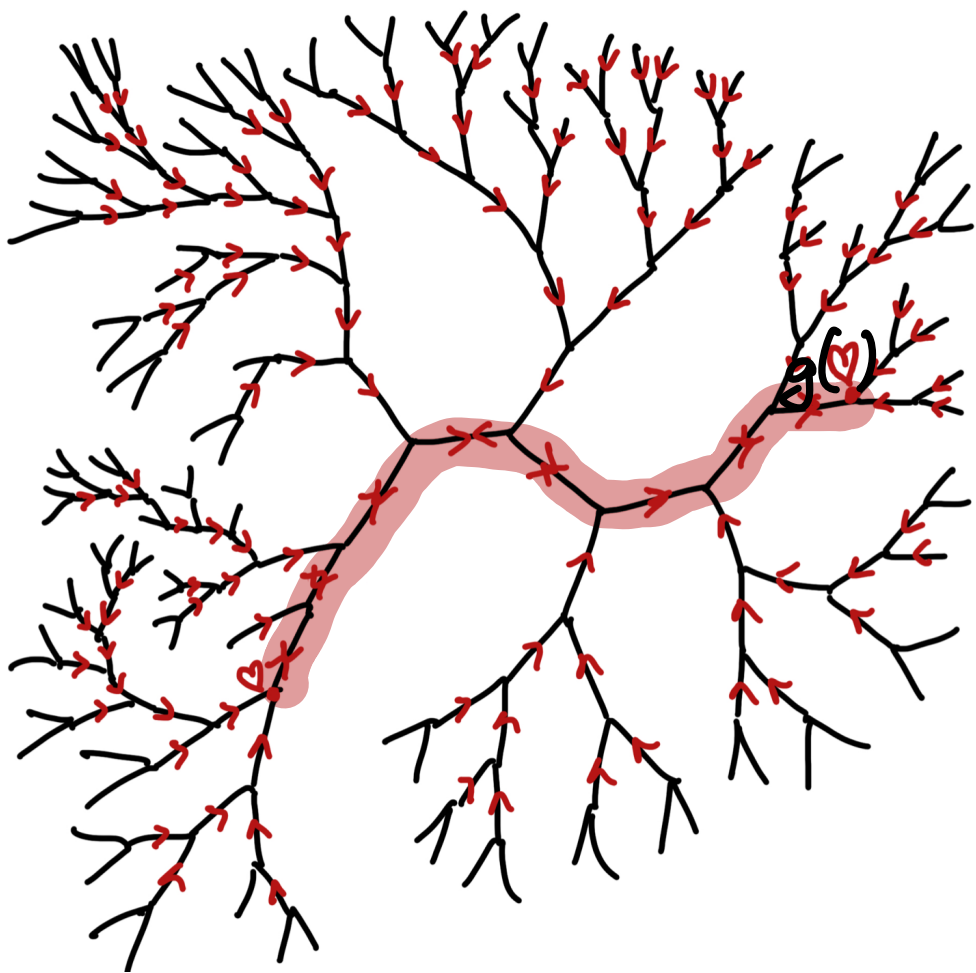


$$\xi_{\heartsuit}(x, y) =$$

$$= \begin{cases} 1 & \text{if } \{x, y\} \text{ is an} \\ & \text{edge} \rightarrow \heartsuit \\ 0 & \text{otherwise} \end{cases}$$

$$\in \ell^2(T \times T)$$

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$$\xi_{\heartsuit} - \xi_{g(\heartsuit)}$$

$$\in \ell^2(T \times T) = H$$

$$\|\xi_{\heartsuit} - \xi_{g(\heartsuit)}\|$$

$$\longrightarrow \infty$$

$$\text{as } g \rightarrow \infty$$