Dynamical Systems in Neuroscience

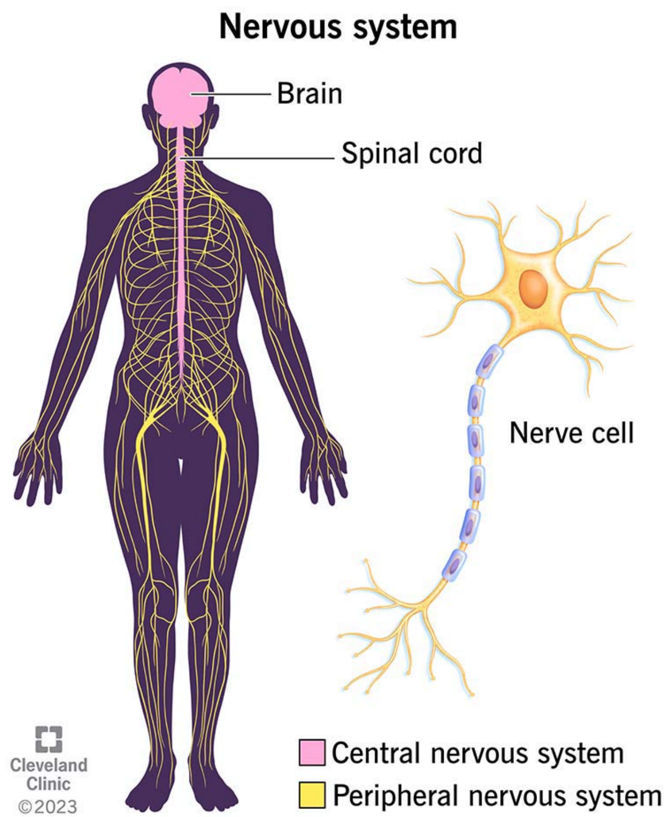
2026-05-18

Textbooks

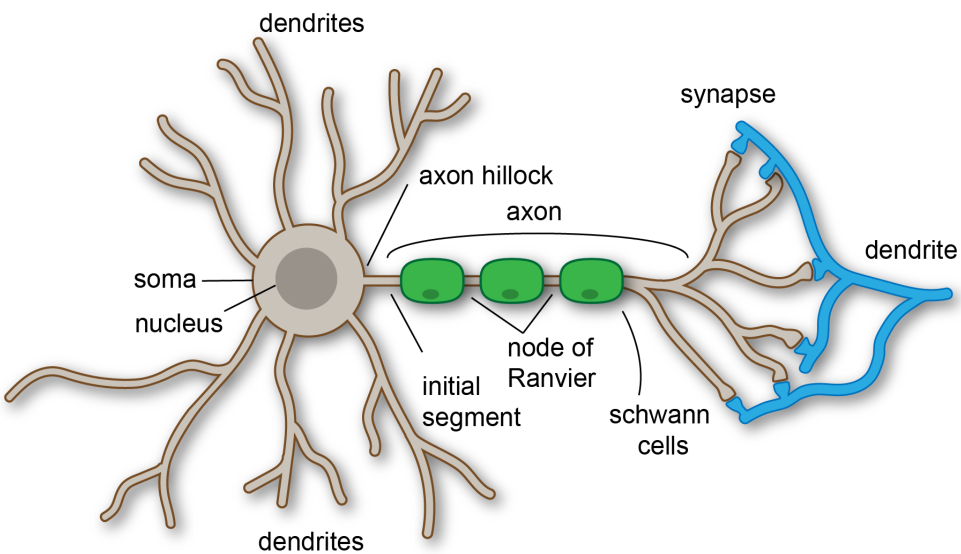
- Nonlinear Dynamics and Chaos - Steven Strogatz
- Chaos in Dynamical Systems - Edward Ott
- Dynamical Systems in Neuroscience - Eugene M. Izhikevich
- Mathematical Foundations of Neuroscience - Bard Ermentrout and David Terman
- Neuronal Dynamics - Gerstner, Kistler, Naud and Paninski

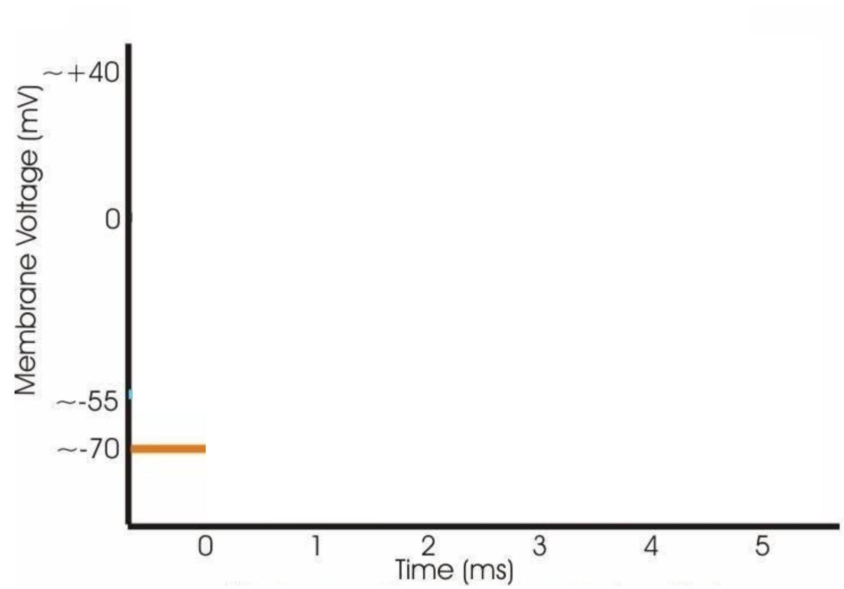
What is neuroscience?

Study of the nervous system



What is a neuron?





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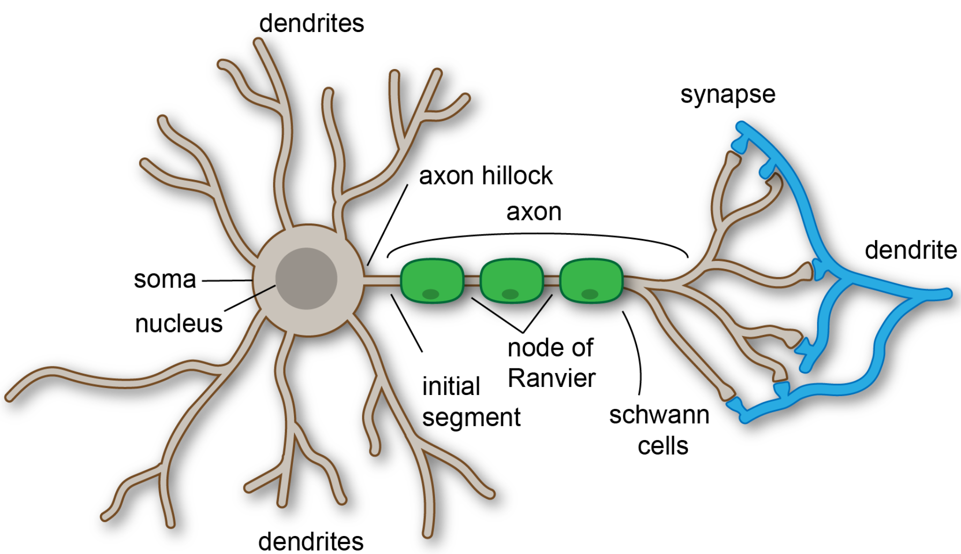

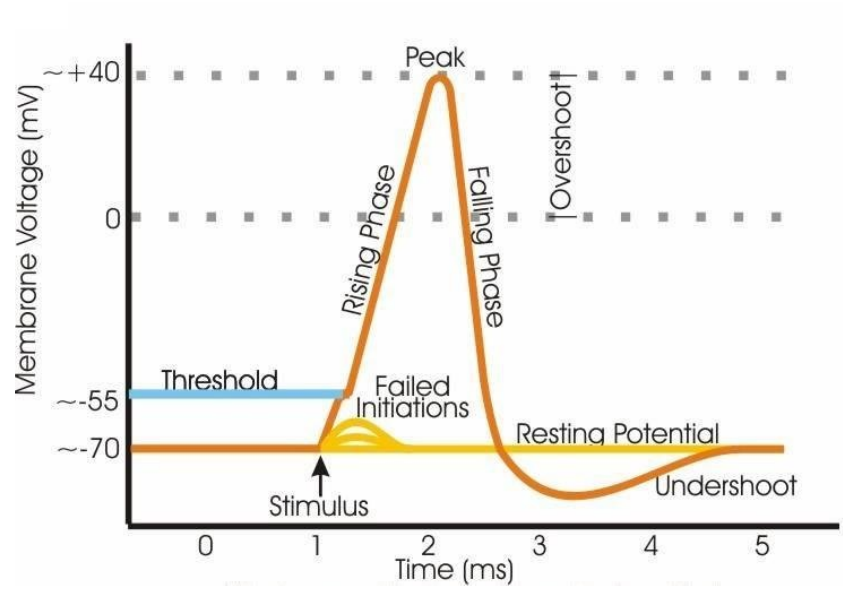

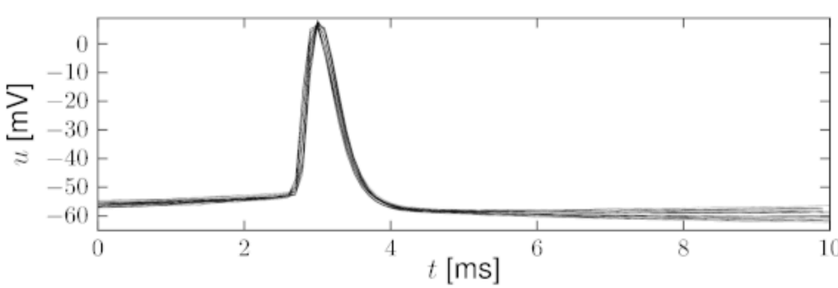

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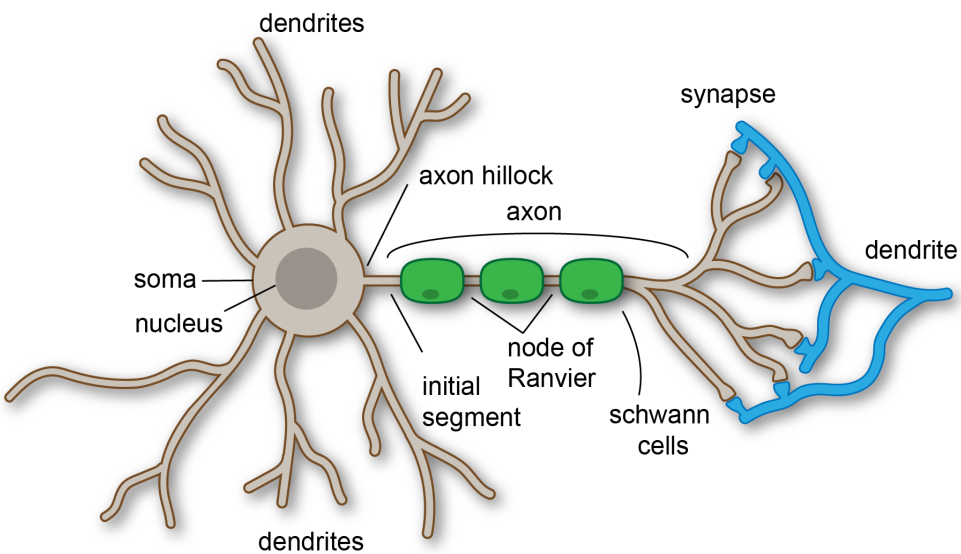
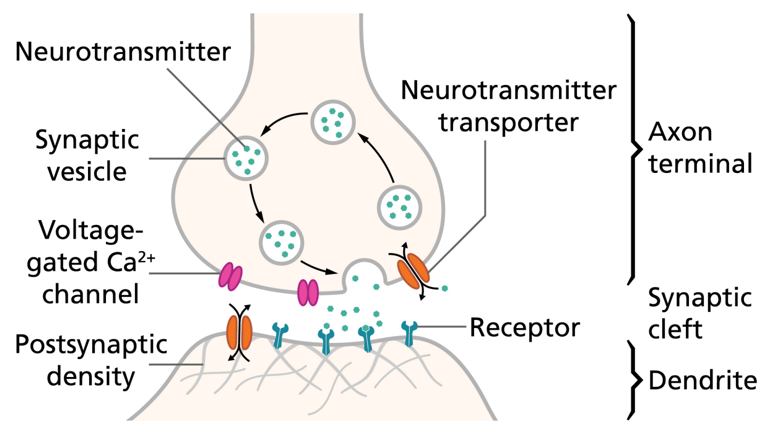
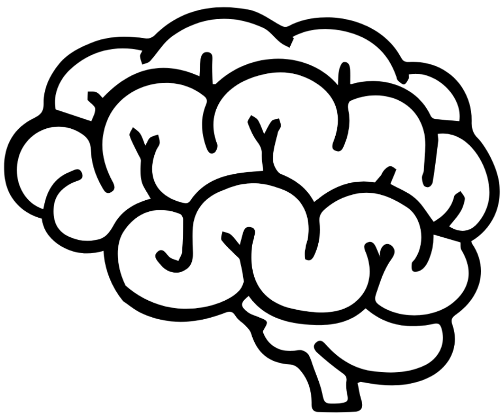




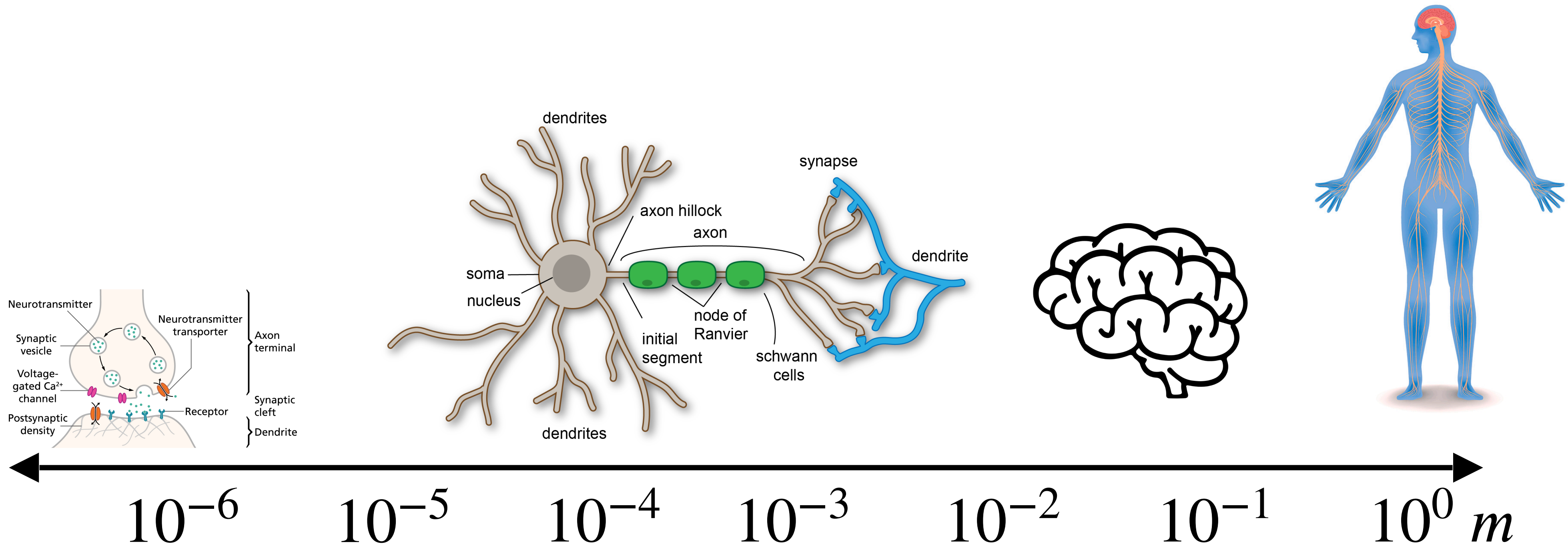
Neurons to neural circuits



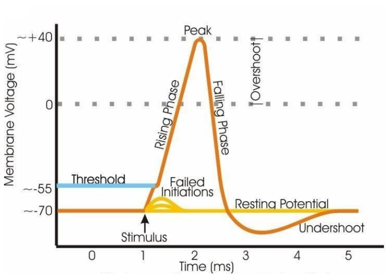
10^{11} neurons

10^{14} synapses

Relevant lengthscales



Relevant timescales

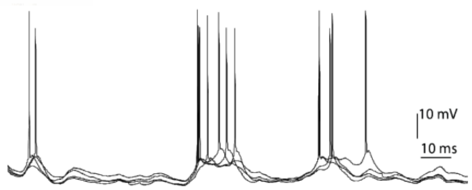


Synaptic plasticity

Sensory perception

Planning, reasoning

Long-term memory, learning

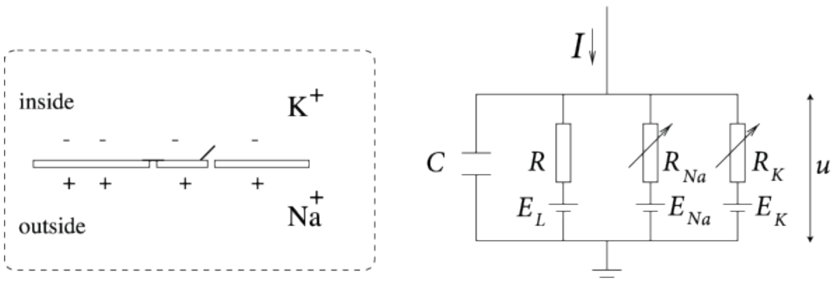


Short-term memory

10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} $10^{5} s$

Different levels of abstraction

- Model detailed biochemistry and biophysics
- Model individual spikes



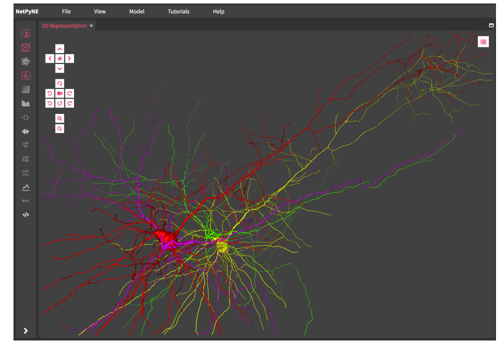
$$I = C_m \frac{\mathrm{d}V_m}{\mathrm{d}t} + \bar{g}_\mathrm{K} n^4 (V_m - V_K) + \bar{g}_\mathrm{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l),$$

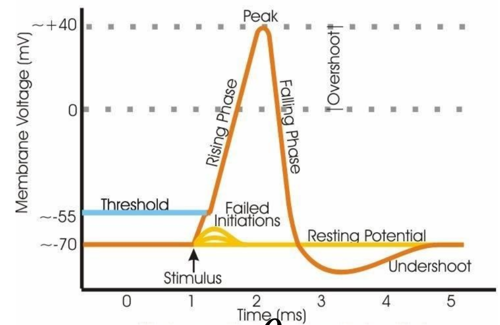
$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

$$\frac{d\theta}{dt} = 1 - \cos\theta + (1 + \cos\theta) I(t),$$





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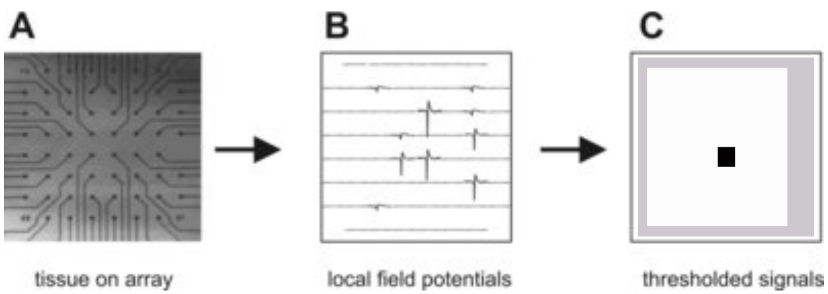
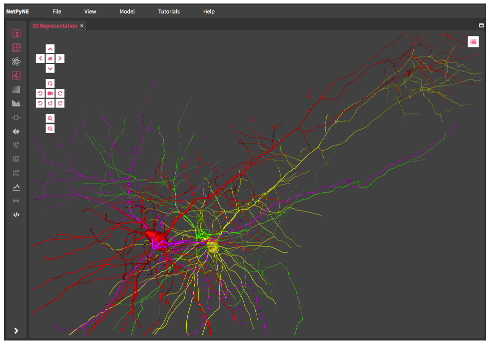
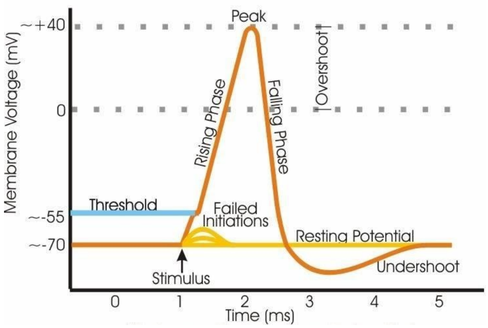
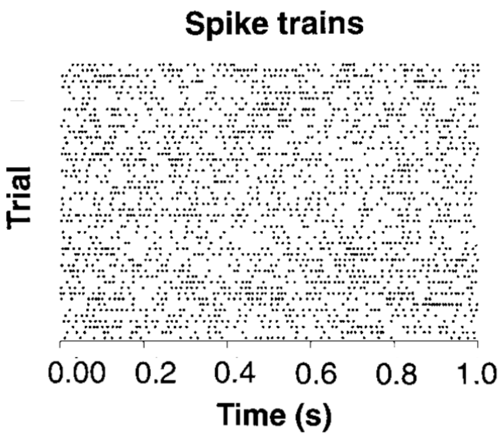


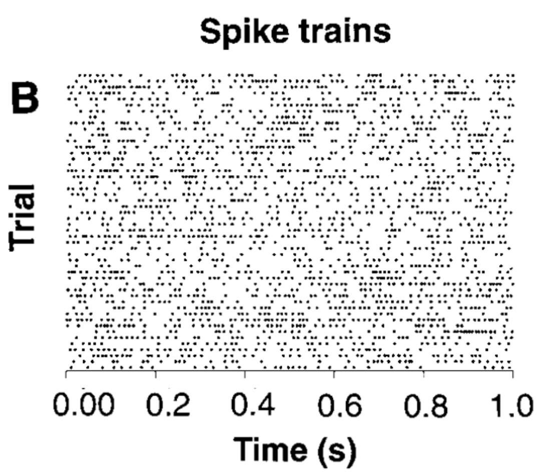


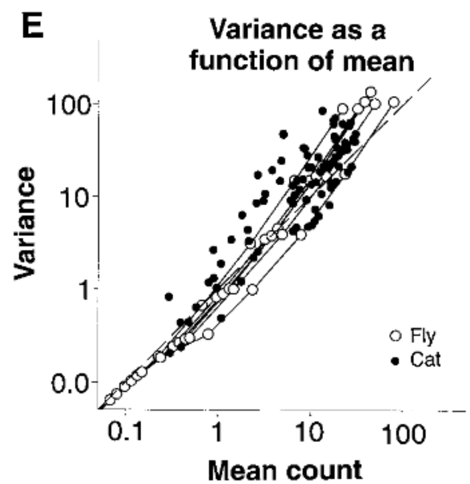




Different levels of abstraction

- Neuron spiking can seem stochastic
- Poisson spiking model: neuron action potential spike distribution





Ruyter van Steveninck et al 1997

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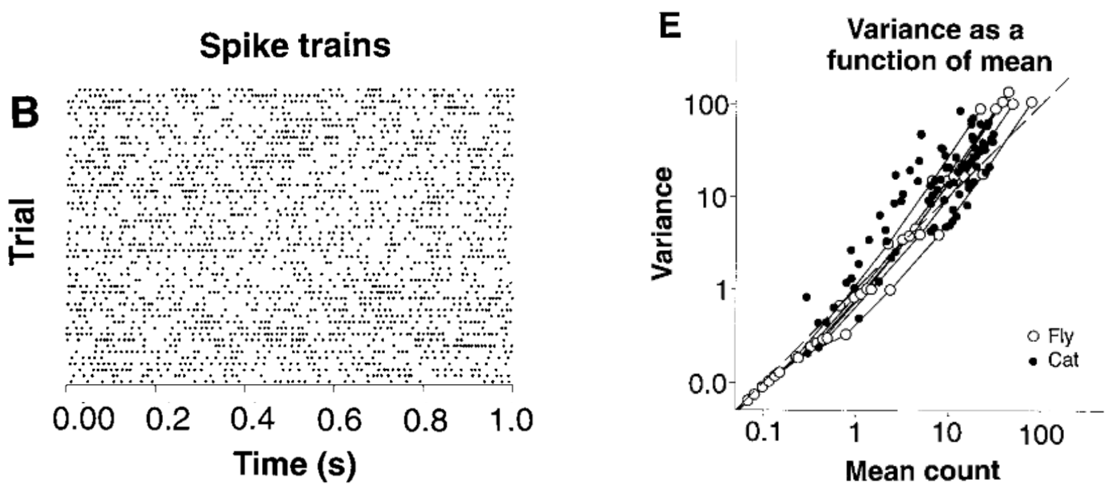

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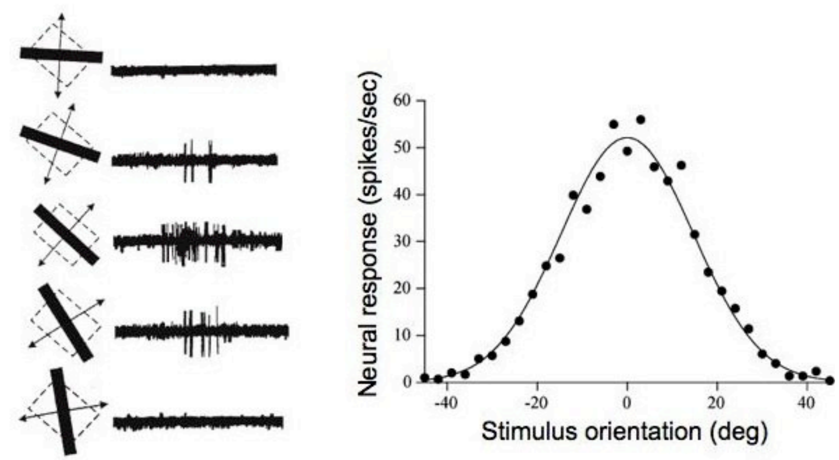

Hubel & Wiesel 1968

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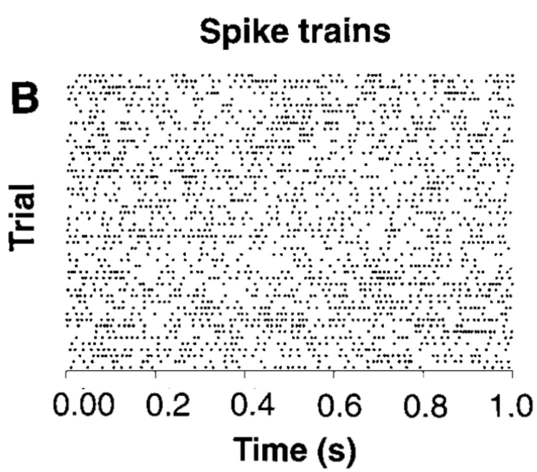

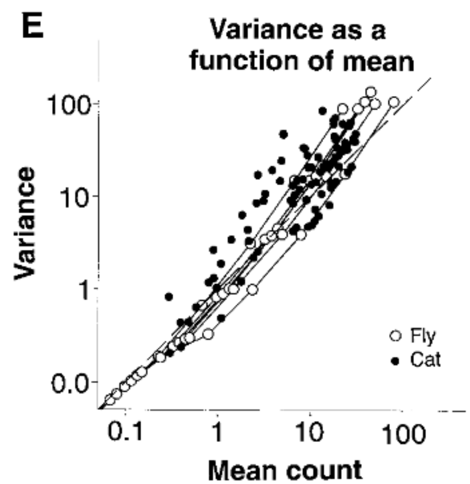

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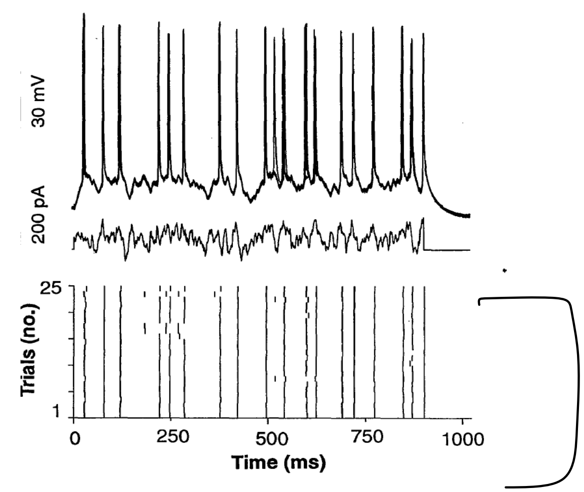

Mainen and Sejnowski 1995

Different levels of abstraction

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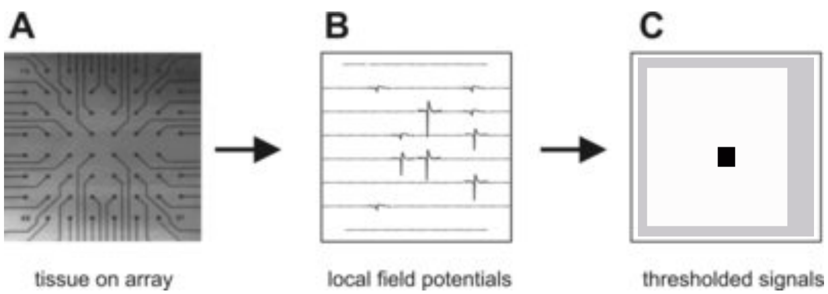
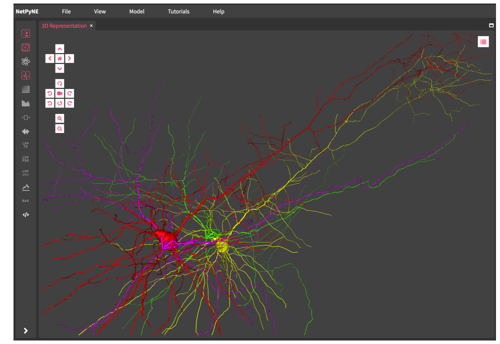
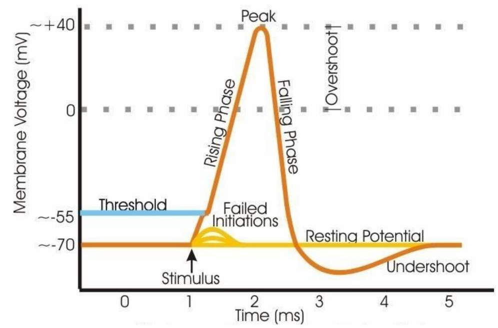
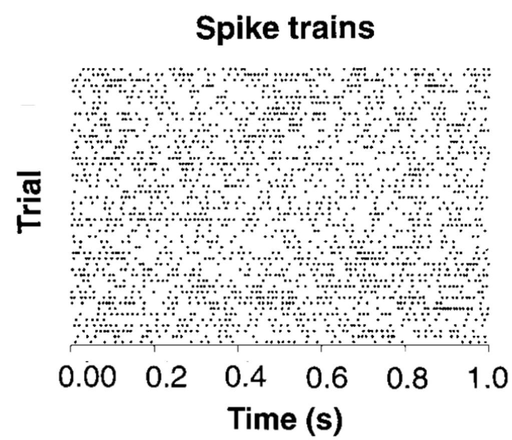
- Model individual spikes
- Model binary neurons
- Model firing rate dynamics



$$\frac{\partial s_i}{\partial t} + \frac{s_i}{\tau} = \phi\left[\sum_j W_{ij} s_j + B_i\right]$$

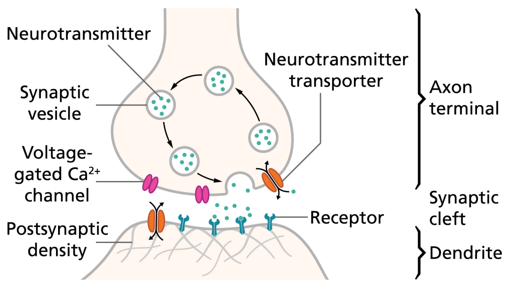


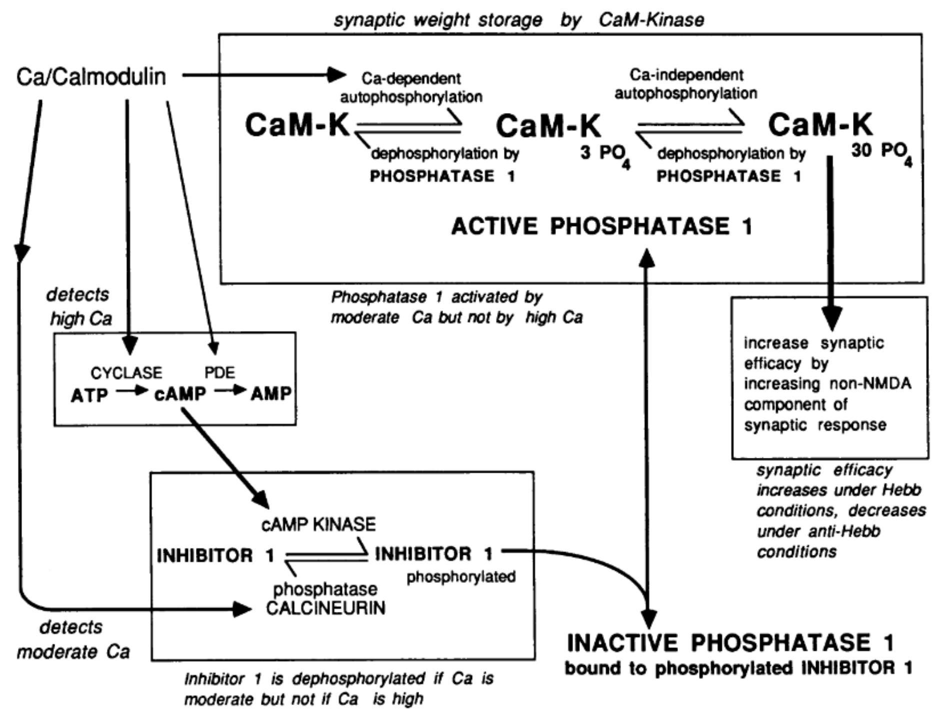




Dynamics of the circuits themselves

Neural plasticity modeled at various levels of abstraction





Proc. Natl. Acad. Sci. USA
Vol. 86, pp. 9574–9578, December 1989
Neurobiology

A mechanism for the Hebb and the anti-Hebb processes underlying learning and memory

(calmodulin/calcium/calmodulin-dependent protein kinase II/protein phosphatase 1/adenylate cyclase/calcineurin)

JOHN LISMAN
Department of Biology, Brandeis University, Waltham, MA 02254

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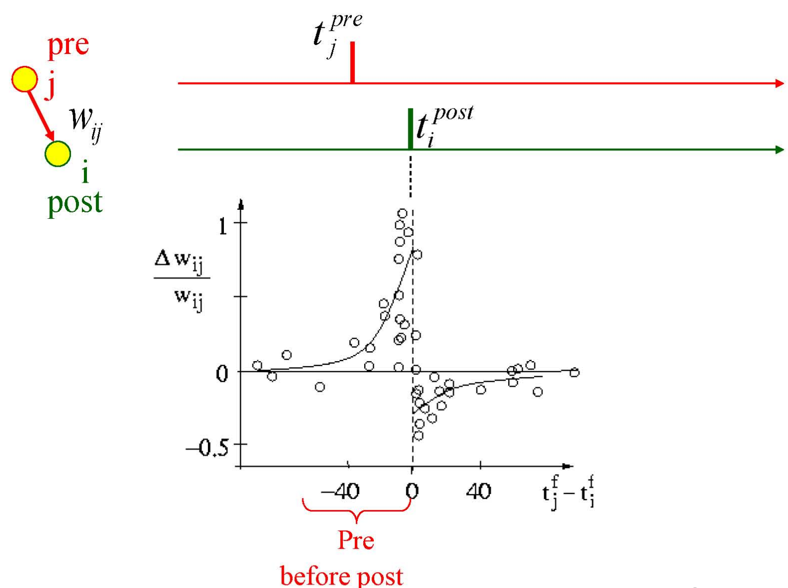

Bi & Poo (1998)

The Spike-Timing Dependence of Plasticity

Daniel E. Feldman[1,*]
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*Correspondence: dfeldman@berkeley.edu
http://dx.doi.org/10.1016/j.neuron.2012.08.001

Timescales of plasticity

- STDP ~10 milliseconds
- BTSP ~1-10 seconds
- LTP, LTD ~ 1-1000 minutes
- Neurodevelopment ~ hours to months
- Evolutionary?

Nonlinear dynamical systems

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- Broad goal:
  Understand and analyze nonlinear dynamics **without** solving the entire system

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$\sigma = 10$, $\rho = 28$, and $\beta = \frac{8}{3}$

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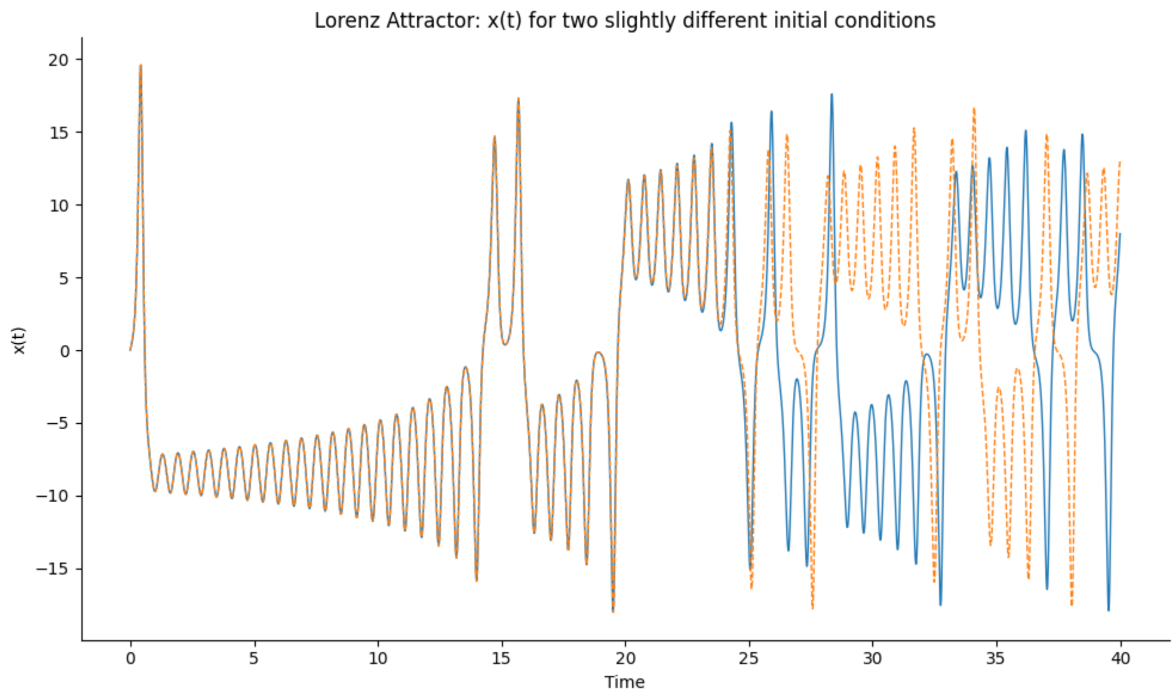

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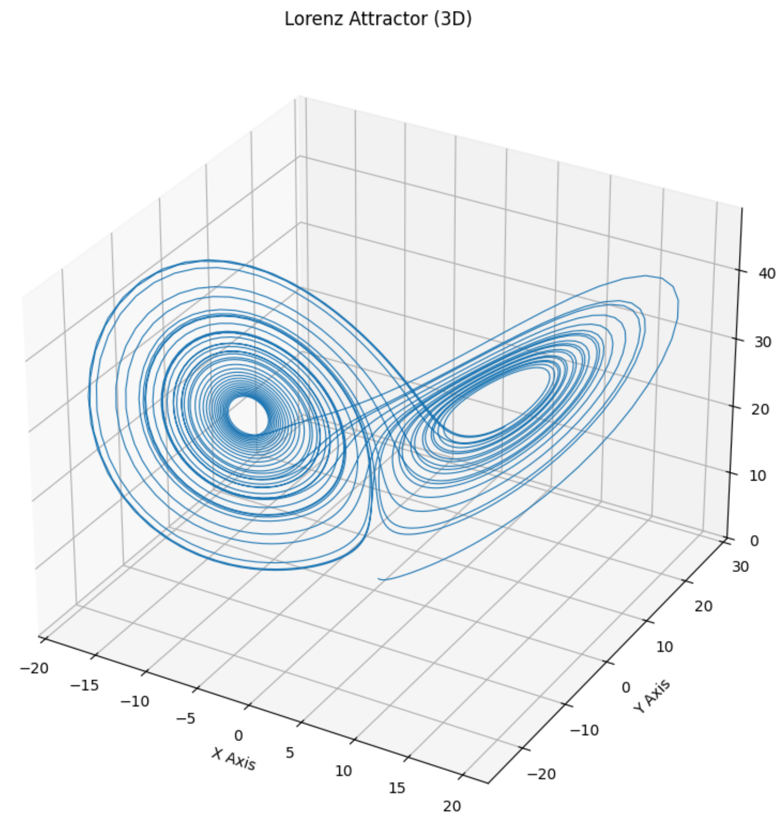

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Edward Lorenz

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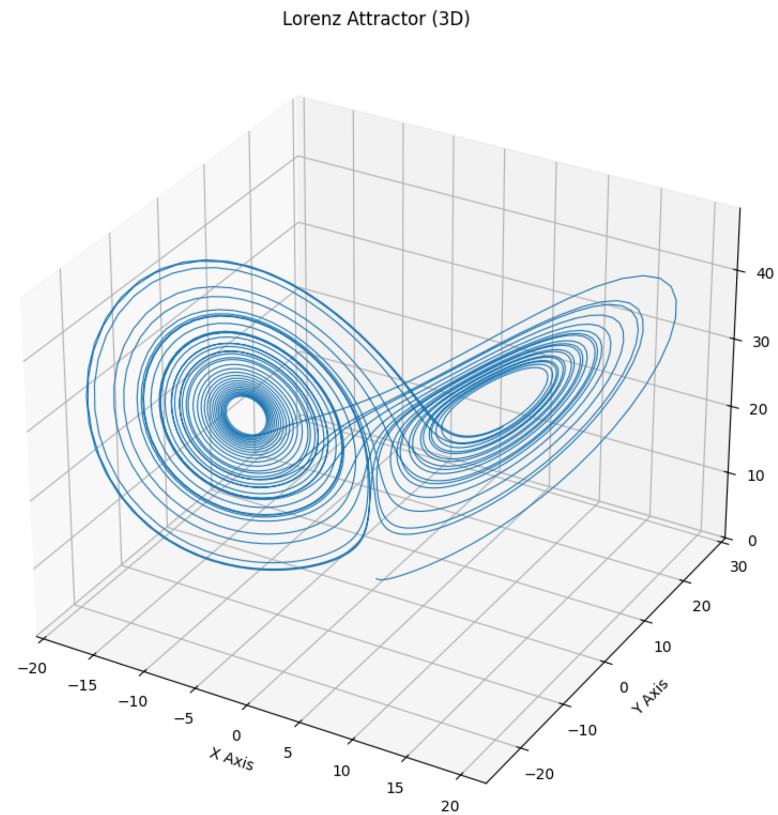

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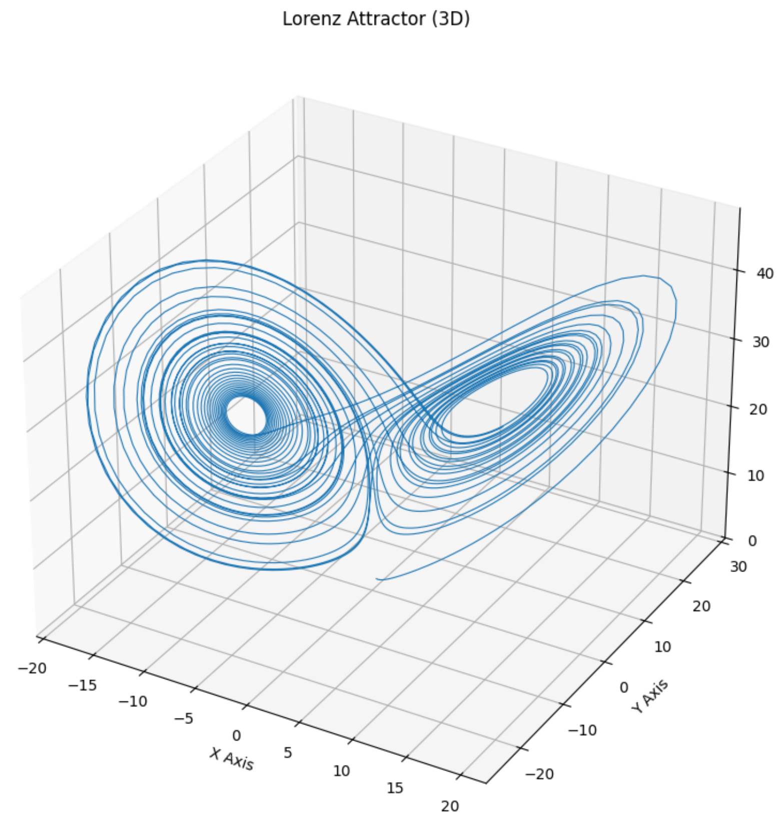
Edward Lorenz

Ellen Fetter

Margaret Hamilton



What is a Dynamical System?

$\dot{\bar{x}} = F(\bar{x})$ Autonomous D.S.

$\dot{\bar{x}} = A\bar{x}$

$$\begin{cases} \dot{x}_1 = F_1(x_1, \dots x_n) \\ \dot{x}_2 = F_2(\quad) \\ \vdots \\ \dot{x}_n = F_n(x_1 \dots x_n) \end{cases}$$

1) Time explicit ←
2) Higher order Deriv. ←
3) Implicit Dyn ←
4) Neural Fields
5) Discrete systems

Non. Aut. D.S. → $\dot{\bar{x}} = F(\bar{x}, t)$

$\bar{y} = \begin{pmatrix} \bar{x} \\ t \end{pmatrix}$ $\dot{\bar{y}} = \begin{pmatrix} F(\bar{x}) \\ 1 \end{pmatrix}$

$\dot{t} = 1$

$\ddot{x} = -x$

$y = \dot{x}$

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$

$\dot{x} = F(x) \leftarrow$

$\left[\dot{x} = F(x, t)\right]$

F is cts

DF or F' is cts

$\dot{\bar{x}} = A\bar{x}$ (non-normal dynamics ← V. V. Interesting)

Implicit

$\left[F(\dot{x}, x, t) = 0\right]$

$\bar{x}(t) = c_1 e^{\lambda_1 t}\hat{e}_1 + c_2 e^{\lambda_2 t}\hat{e}_2 + \dots$

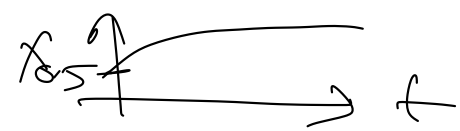
1D Flows

$x(0) = 0.5$

$\dot{x} = F(x) = -x(x-1)(x-2)$

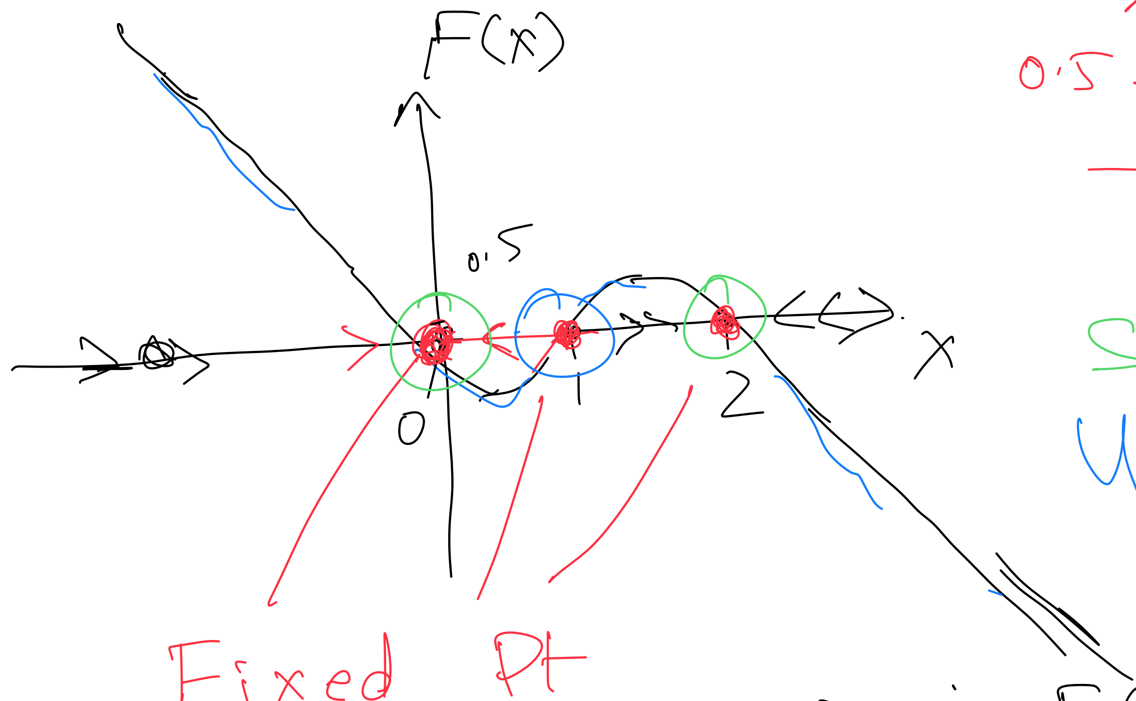
$$-\frac{dx}{x(x-1)(x-2)} = dt \longrightarrow t = -\frac{1}{2}\ln x + \ln(x-1) - \frac{1}{2}\ln(x-2) + C$$

$x = $ ~~~~

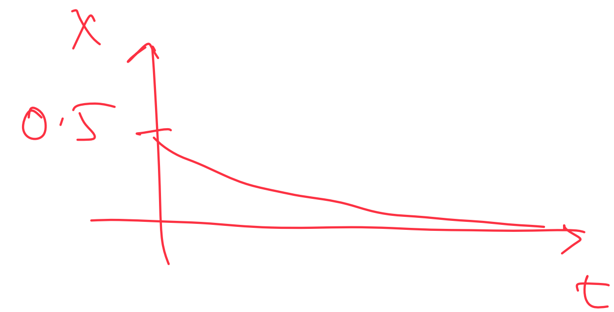


$$\dot{x} = -x(x-1)(x-2) + \varepsilon \sin x$$

$$= F(x)$$



$$\rightarrow F(x_*) = 0$$



Stable → $F' < 0$

Unstable → $F' > 0$

$$\dot{x} = F(x)$$

$$\dot{x}_* + \dot{\varepsilon} = F(x_* + \varepsilon), \quad \dot{x}_* = 0$$

$$\dot{\varepsilon} = F(x_*) + F'(x_*) \cdot \varepsilon$$

$$\dot{\varepsilon} = F'(x_*)\, \varepsilon$$

$$x = x_* + \varepsilon$$

$$\dot{x} = F(x)$$

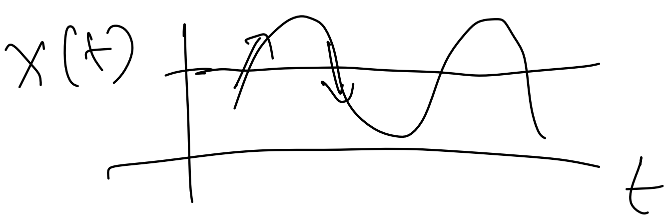
$$x(0) = x_0$$

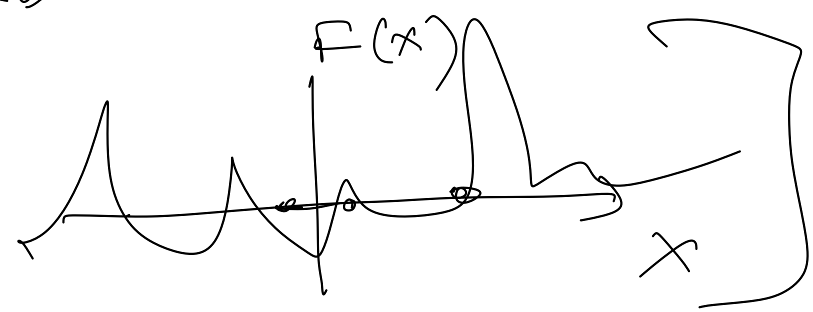
What happens ~~at~~ as $t \to \infty$

1) $x(t) \to x^*$ (goes to a F.P.) — can be multiple F.P. depending on init. cond.

2) $x(t) \to \infty$ or $x(t) \to -\infty$

~~Periodic $x(t+T) = x(t)$~~ ✗





1D Flow

↳ cannot have periodicity