Glue and Sea Inside the Proton from a Light-Front Hamiltonian

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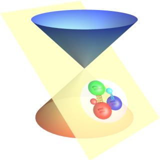


BLFQ Collaboration

Siqi Xu, Bolang Lin, Hongyao Yu, Zhimin Zhu, Sreeraj Nair. . . ,
Xingbo Zhao (IMP) and James P. Vary (ISU)

Workshop on probing hadron structure at EIC

ICTS International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

February 08, 2024

Overview

Introduction

Basis Light-Front Quantization (BLFQ) to

Proton : ($|qqq\rangle + |qqqg\rangle$)

Proton : ($|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$)

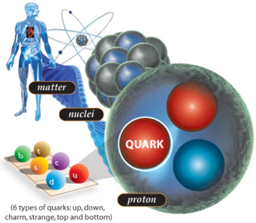
Conclusions

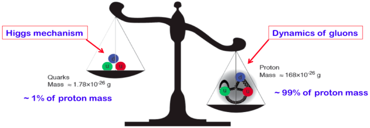
(PRD 108 094002 (2023), PLB 847 138305 (2023), work in progress)

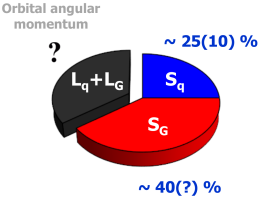
(Satvir Kaur : Valence quark and gluon TMDs of spin-1 QCD system)

Fundamental Properties: Mass and Spin

- About 99% of the visible mass is contained within nuclei
- Nucleon: composite particles, built from nearly massless quarks ($\sim 1\%$ of the nucleon mass) and gluons
- *How does 99% of the nucleon mass emerge?*
- Quantitative decomposition of *nucleon spin* in terms of quark and gluon degrees of freedom is not yet fully understood.
- *To address these fundamental issues → nature of the subatomic force between quarks and gluons, and the internal landscape of nucleons.*

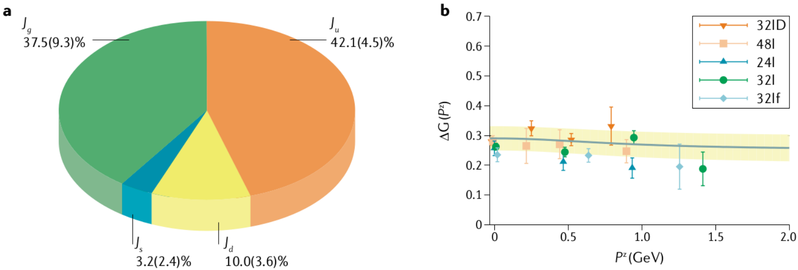






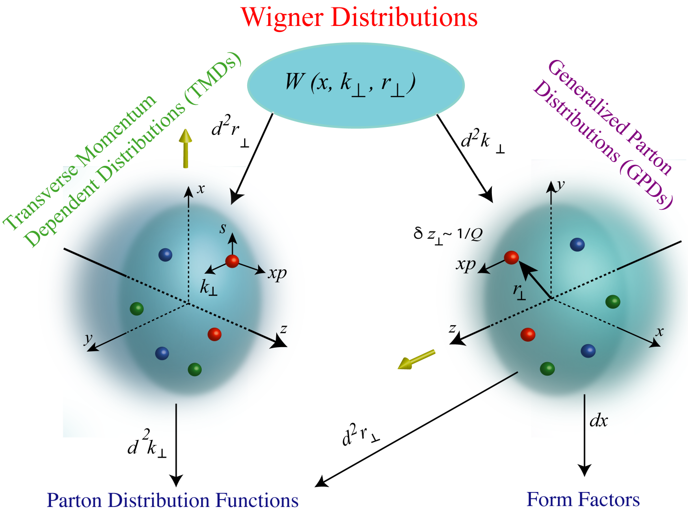
[1] Pictures (top to bottom) adopted from A. Signori, J. Qiu, C. Lorce

Spin sum rule	Formula	Terms	Characteristics
Frame independent (Ji)[30]	$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$	$\Delta\Sigma/2$ is the quark helicity L_q^z is the quark OAM J_g is the gluon contribution	The quark and gluon contributions, J_q and J_g, can be obtained from the GPD moments. A similar sum rule also works for the transverse angular momentum and has a simple parton interpretation
Infinite-momentum frame (Jaffe–Manohar)[31]	$\frac{1}{2}\Delta\Sigma + \Delta G + \ell_q + \ell_g = \frac{\hbar}{2}$	ΔG is the gluon helicity ℓ_q and ℓ_g are the quark and gluon canonical OAM, respectively	All terms have partonic interpretations; ℓ_q and ℓ_g are twist-three quantities. ΔG is measurable in experiments, including the RHIC spin and the EIC; ℓ_q and ℓ_g can be extracted from twist-three GPDs



[1] X. Ji, F. Yuan and Y. Zhao, Nature Reviews Physics 3, 65 (2021)
[2] Y.-B. Yang, R.S. Sufian, A. Alexandru et al., Phys. Rev. Lett. 118, 102001 (2017)
[3] Aidala's, Hatta's, Mathur's... talks,

Hadron tomography



- $x \to$ longitudinal momentum fraction; $k_\perp \to$ parton transverse momentum; $r_\perp \to$ transverse distance from the center.

Basis Light-Front Quantization (BLFQ)

A computational framework for solving relativistic many-body bound state problems in quantum field theories

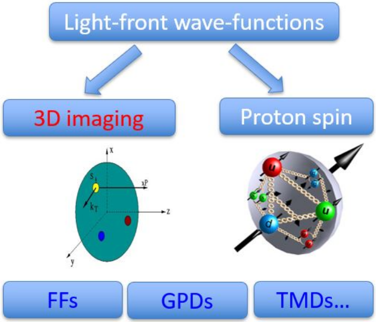
$$P^-P^+|\Psi\rangle = M^2|\Psi\rangle$$

- $P^- \equiv P^0 - P^3$: light-front Hamiltonian
- $P^+ \equiv P^0 + P^3$: longitudinal momentum
- $|\Psi\rangle$ mass eigenstate
- M^2 : mass squared eigenvalue for eigenstate $|\Psi\rangle$
- First-principle / effective Hamiltonian as input
- Evaluate observables

$$O \sim \langle\Psi|\hat{O}|\Psi\rangle$$

- direct access to light-front wavefunction of bound states

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

- Fock expansion of baryonic bound states:

$$|\text{Proton}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots ,$$

Solution proposed by BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^\perp)$ in the transverse plane	$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	Fock sector truncation

Large $N_{\max}$ and K → High UV cutoff & low IR cutoff

- Exact factorization between center-of-mass motion and intrinsic motion

[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Nucleon within BLFQ

- The LF eigenvalue equation: $H_{\text{eff}}|\Psi\rangle = M^2|\Psi\rangle$

$$H_{\text{eff}} = \sum_a \frac{\vec{p}^2_{\perp a} + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b}\kappa^4\left[x_a x_b(\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a + m_b)^2}\right]$$
$$+ \frac{1}{2}\sum_{a\neq b}\frac{C_F 4\pi\alpha_s}{Q^2_{ab}}\bar{u}_{s'_a}(k'_a)\gamma^\mu u_{s_a}(k_a)\bar{u}_{s'_b}(k'_b)\gamma^\nu u_{s_b}(k_b)g_{\mu\nu}$$

Publications:

- Mondal et al., Phys. Rev. D 102, 016008 (2020) : Form Factors, PDFs,...
- Xu et al., Phys. Rev. D 104, 094036 (2021) : Nucleon structure,...
- Liu et al., Phys. Rev. D 105, 094018 (2022) : Angular Momentum,...
- Hu et al., Phys. Lett. B 2022, 137360 (2022) : TMDs,...
- Peng et al., Phys. Rev. D 106, 114040 (2022) : Λ and Λ_c PDFs,...
- Zhu et al., Phys. Rev. D 108, 036009 (2023) : Λ and Λ_c TMDs,...
- Kaur et al., Phys. Rev. D 109, 014015 (2024) : Chiral-odd GPDs,...
- Zhang et al., Phys. Rev. D ??? (2024) : Twist-3 GPDs...
- Nair et al., coming soon : GFFs,...
- Peng et al., coming soon : Double parton correlations,...

Proton with One Dynamical Gluon

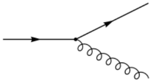
$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle$$

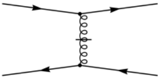
$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

$$P^- = P^-_{\text{QCD}} + P^-_C$$

QCD Interaction:

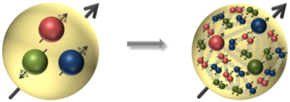
$$P^-_{\text{QCD}} = \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \Big\{ \frac{1}{2}\bar{\psi}\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2}A_a^i\left[m_g^2 + (i\partial^\perp)^2\right]A_a^i + g_s\bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2}g_s^2\bar{\psi}\gamma^+ T^a\psi\frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \Big\},$$





Confinement only in leading Fock:

$$P^-_C P^+ = \frac{\kappa^4}{2}\sum_{i\neq j}\left\{\{\vec{r}_{ij\perp}^{\,2} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i+m_j)^2}\right\}$$



Parameters:

Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016).

Proton with One Dynamical Gluon

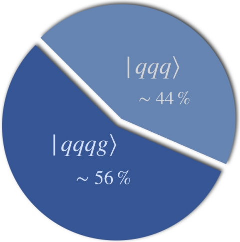
Fock expansion:

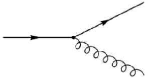
$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

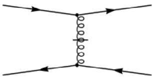
Light-front effective Hamiltonian :

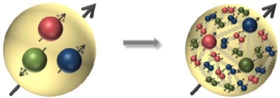
$$H_{\text{eff}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$

Fock Sector Decomposition





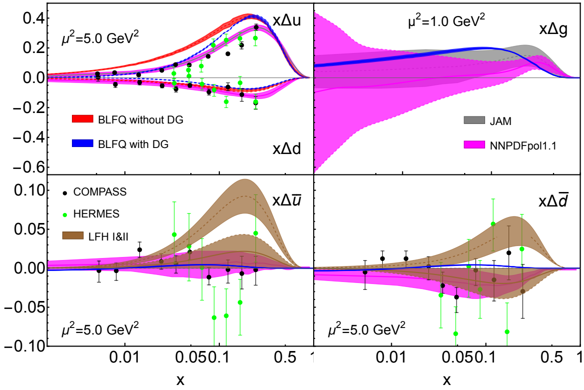




[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.

Helicity PDFs

BLFQ: PRD 108 (2023) 094002

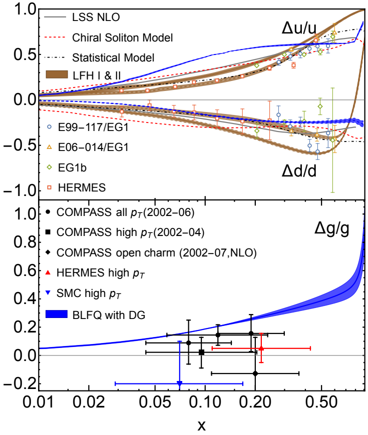


- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004$, $\frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002$.
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, sizeable to the proton spin.
- PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$.
- Sea quarks: solely generated from the QCD evolution.

[1] LFH: 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003].

Helicity Asymmetries

BLFQ: PRD 108 (2023) 094002



- Experimentally, the expected increase of $\Delta u/u$ is observed.
- For d quark: remains negative in the experimentally covered region.
- Global analyses favor negative values of $\Delta d/d$ at large-x.

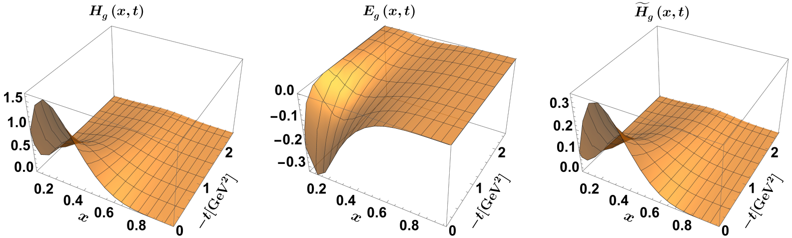
Gluon GPDs

BLFQ : PLB 847 (2023) 138305

$$F^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+\,H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\,E^g(x,\xi,t)\right)u(p,\lambda)\,,$$

$$\tilde{F}^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+\gamma_5\,\tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M}\,\tilde{E}^g(x,\xi,t)\right)u(p,\lambda)\,.$$

Non-skewed GPDs



- Model scale : $\mu_0^2 = 0.23 - 0.25$ GeV^2 (by matching $\langle x\rangle$ with global fit at 10 GeV^2 after scale evolution)
- Total Angular Momentum: $J = \frac{1}{2}\int \mathrm{d}x\, x[H(x,0)+E(x,0)]$; $J_g = 0.066$, 13.2% of the proton TAM.

BLFQ Predictions for Spin Decomposition

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\, \Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\, \Delta G(x)$$

Total AM :

$$J_i = \int \mathrm{d}x\, x\, [H_i(x,0,0) + E_i(x,0,0)]$$

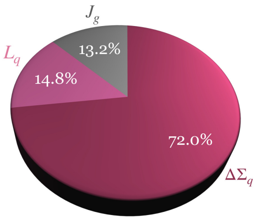
Kinetic OAM :

$$L_q = \int \mathrm{d}x\, [x\, \{H_q(x,0,0) + E_q(x,0,0)\} - \widetilde{H}_q(x,0,0)]$$

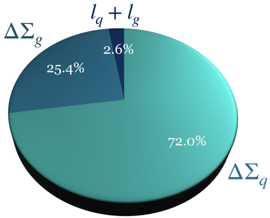
Canonical OAM :

$$l_i^z = -\int \mathrm{d}x\, \mathrm{d}^2\vec{p}_\perp\, \frac{\vec{p}_\perp^{\,2}}{M^2}\, F^i_{1,4}(x,0,\vec{p}_\perp^{\,2},0,0)$$

(a) Kinetic



(b) Canonical



[1] Hatta's talk: 6th Feb.
[2] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023), 094002.

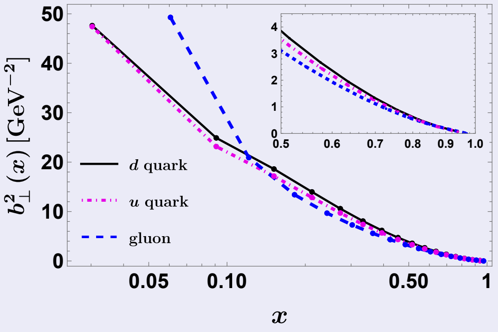
x-Dependent Squared Radius

$$\langle b_\perp^2\rangle^i(x) = \frac{\int d^2\vec{b}_\perp b_\perp^2 H^i(x,b_\perp)}{\int d^2\vec{b}_\perp H^i(x,b_\perp)},$$

- Transverse squared radius:

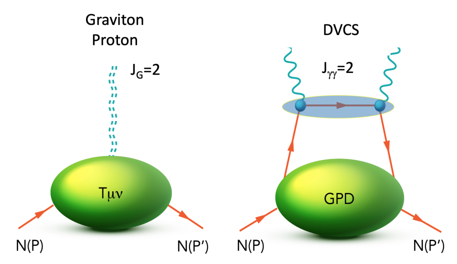
$$\langle b_\perp^2\rangle = \sum_i e_q \int_0^1 \mathrm{d}x f^i(x)\langle b_\perp^2\rangle^i(x)$$

- BLFQ: $\langle b_\perp^2\rangle = 0.47 \pm 0.04\ \text{fm}^2$
- Experimental data [2]: $\langle b_\perp^2\rangle_{\text{exp}} = 0.43 \pm 0.01\ \text{fm}^2$



[1] B. Lin, S. Nair, S.Xu, CM, X. Zhao, J. P. Vary, 2308.08275 [hep-ph].
[2] R. Dupre, M. Guidal and M. Vanderhaeghen, PRD 95, 011501 (2017).

Gravitational Form Factors



- Interaction with gravitons
- Encode information: momentum densities, energy densities, spin angular momentum, mechanical properties : pressure and force distributions, radius, etc.

- Gravitons not feasible in collider yet
- The graviton-proton coupling is mimicked with a pair of vector bosons interacting with quark and gluon (in DVCS process)

[Fig: Burkert *et. al.*: 2310.11568]

Nucleon Gravitational Form Factors

- Parametrization of matrix element in terms of GFFs

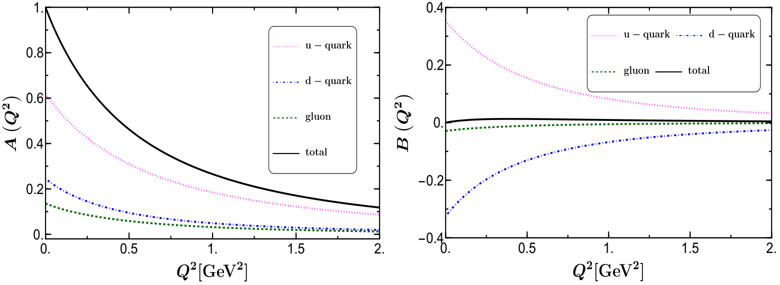
$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)Mg^{\mu\nu}\Big]U$$

- Momentum sum rule : $\sum_i A^i(0) = 1$
- Gravitomagnetic moment sum rule : $\sum_i B^i(0) = 0$
- Spin sum rule: $J^i = \frac{1}{2}\left[A^i(0) + B^i(0)\right]$
- $4C(q^2) = D(q^2)$ provides shear forces and the pressure distributions

[Burkert *et. al.*: Rev. Mod. Phys. 95, 041002 (2023)]
[Ji, Phys. Rev. Lett. 78, 610 (1997)]

[1] Keh-fei Liu talk's

$A(Q^2)$ and $B(Q^2)$

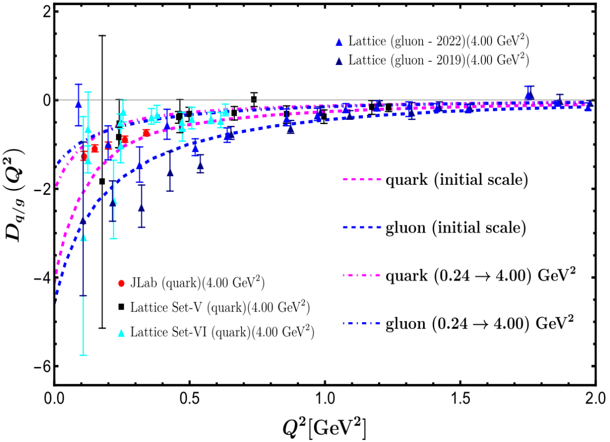


- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- Spin sum rule: $J^i = \frac{1}{2}\left(A^i(0) + B^i(0)\right)$

$$\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0$$

[1] S. Nair, CM, *et. al.* coming soon...

$D(Q^2)$



- $D(Q^2) = 4C(Q^2)$: T^{ij} components

[1] S. Nair, CM, *et. al.* coming soon...

TMDs of Spin-1/2 Target

Gluon TMDs correlator :

$$\Phi^{g[ij]}(x,\vec{k}_\perp;S) = \frac{1}{xP^+}\int \frac{dz^-}{2\pi}\frac{d^2\vec{z}_\perp}{(2\pi)^2}e^{ikz}\langle P;S|F_a^{+j}(0)\mathcal{W}_{+\infty,ab}(0;z)F_b^{+i}(z)|P;S\rangle\,|_{z^+=0^+}$$

Parametrization

$$\Phi^g(x,\vec{k}_\perp;S) = \delta_\perp^{ij}\Phi^{g[ij]}(x,\vec{k}_\perp;S)$$
$$= f_1^g(x,\vec{k}_\perp^2) - \frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M}f_{1T}^{\perp g}(x,\vec{k}_\perp^2)$$

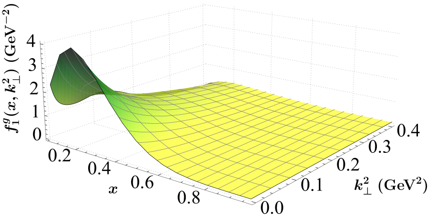
$$\tilde{\Phi}^g(x,\vec{k}_\perp;S) = i\epsilon_\perp^{ij}\Phi^{g[ij]}(x,\vec{k}_\perp;S)$$
$$= S^3 g_{1L}^g(x,\vec{k}_\perp^2) + \frac{\vec{k}_\perp\cdot\vec{S}_\perp}{M}g_{1T}^g(x,\vec{k}_\perp^2)$$

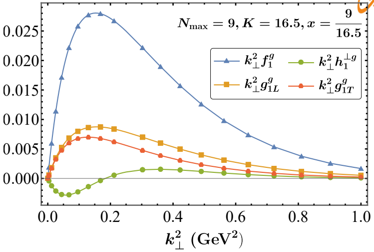
$$\Phi_T^{g,ij}(x,\vec{k}_\perp;S) = -\hat{\mathbf{S}}\Phi^{g[ij]}(x,\vec{k}_\perp;S)$$
$$= -\frac{\hat{\mathbf{S}}k_\perp^i k_\perp^j}{2M^2}h_1^{\perp g}(x,\vec{k}_\perp^2) + \frac{S^3\hat{\mathbf{S}}k_\perp^i\epsilon_\perp^{jk}k_\perp^k}{2M^2}h_{1L}^{\perp g}(x,\vec{k}_\perp^2)$$
$$+ \frac{\hat{\mathbf{S}}k_\perp^i\epsilon_\perp^{jk}S_\perp^k}{2M}\left(h_{1T}^g(x,\vec{k}_\perp^2) + \frac{\vec{k}_\perp^2}{2M^2}h_{1T}^{\perp g}(x,\vec{k}_\perp^2)\right)$$
$$+ \frac{\hat{\mathbf{S}}k_\perp^i\epsilon_\perp^{jk}(2k_\perp^k\vec{k}_\perp\cdot\vec{S}_\perp - S_\perp^k\vec{k}_\perp^2)}{4M^3}h_{1T}^{\perp g}(x,\vec{k}_\perp^2),$$

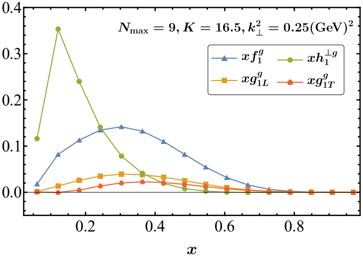
TARGET SPIN \ PARTON SPIN — GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$

[1] A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268.
[2] Meißner, et. al. PRD D 76 (2007), 034002.
[3] Pisano's, Khatiza's...talks

Gluon TMDs







- Positivity bounds

$$f_1^g(x, \boldsymbol{k}_\perp^2) > 0, \quad f_1^g(x, \boldsymbol{k}_\perp^2) \geq |g_{1L}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|}{M} |g_{1T}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|^2}{2M^2} |h_1^{\perp g}(x, \boldsymbol{k}_\perp^2)|$$

- Satisfies Mulders-Rodrigues relations

[1] Hongyao Yu, *et. al.* coming very soon...

Gluon TMDs

- Small-x limit

$$\lim_{x\to 0} \frac{\int d\boldsymbol{k}_\perp^2 |\boldsymbol{k}_\perp^2| h_1^{\perp g}(x, \boldsymbol{k}_\perp^2)}{2M^2 \int d\boldsymbol{k}_\perp^2 f_1^g(x, \boldsymbol{k}_\perp^2)} = 1$$

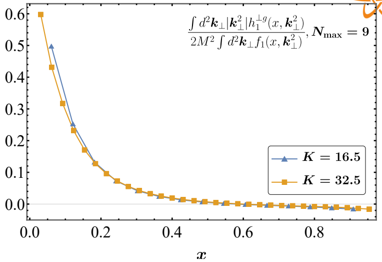
- Helicity asymmetry:

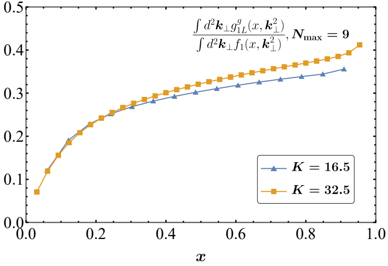
$$\lim_{x\to 0} \frac{\int d\boldsymbol{k}_\perp^2 g_{1L}^g(x, \boldsymbol{k}_\perp^2)}{\int d\boldsymbol{k}_\perp^2 f_1^g(x, \boldsymbol{k}_\perp^2)} = 0,$$

$$\lim_{x\to 1} \frac{\int d\boldsymbol{k}_\perp^2 g_{1L}^g(x, \boldsymbol{k}_\perp^2)}{\int d\boldsymbol{k}_\perp^2 f_1^g(x, \boldsymbol{k}_\perp^2)} = 1$$

.

- With larger truncation K, satisfies the limiting cases.





[1] Hongyao Yu, *et. al.* coming very soon...

Gluon TMDs

- To check compatibility of BLFQ results with the Gaussian ansatz :

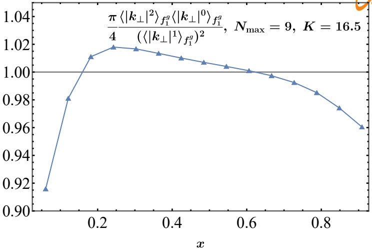
$$f_1^g(x, k_\perp^2) \approx a \frac{\exp\left(-\frac{|k_\perp|^2}{r}\right)}{\pi r}$$

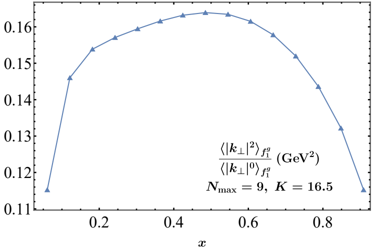
where $a = \langle |k_\perp|^0 \rangle_{f_1^g}$ and $r = \langle |k_\perp|^2 \rangle_{f_1^g}$

- If the Gaussian ansatz holds :

$$\frac{\langle |k_\perp|^2 \rangle_{f_1^g} \times \langle |k_\perp|^0 \rangle_{f_1^g}}{(\langle |k_\perp|^1 \rangle_{f_1^g})^2} \times \frac{\pi}{4} = 1$$

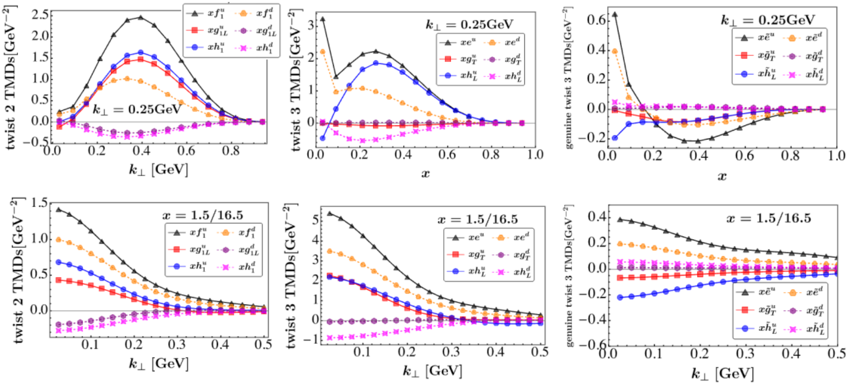
BLFQ results do not support Gaussian ansatz





[1] Hongyao Yu, *et. al.* in preparation

Twist-2 vs Twist-3 Quark TMDs



Twist-3 TMDs:

- more concentrating in small x and k_T
- large than twist-2 TMDs

$$e\left(x, k_\perp^2\right) = \frac{m}{M}\frac{f_1\left(x, k_\perp^2\right)}{x} + \tilde{e}\left(x, k_\perp^2\right)$$

$$g_T(x, k_\perp^2) = \frac{m}{M}\frac{h_1(x, k_\perp^2)}{x} - \frac{k_\perp^2}{2M^2}\frac{g_{1T}(x, k_\perp^2)}{x} - \tilde{g}_T(x, k_\perp^2)$$

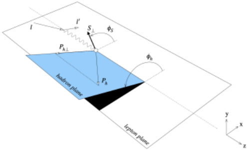
$$h_L\left(x, k_\perp^2\right) = \frac{m}{M}\frac{g_{1L}\left(x, k_\perp^2\right)}{x} - \frac{k_\perp^2}{M^2}\frac{h_{1L}^\perp\left(x, k_\perp^2\right)}{x} + \tilde{h}_L\left(x, k_\perp^2\right)$$

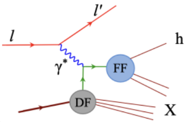
[1] Zhimin Zhu, *et. al.* in preparation

Semi-inclusive DIS

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\varphi_h\,d\psi} = \left[\frac{\alpha}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\times$$

$$\left\{\begin{aligned}
&1+\cos\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UU}^{\cos\phi_h}\right)+\cos 2\phi_h\left(\varepsilon A_{UU}^{\cos 2\phi_h}\right)\\
&+\lambda\sin\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LU}^{\sin\phi_h}\right)\\
&+S_L\left[\sin\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UL}^{\sin\phi_h}\right)+\sin 2\phi_h\left(\varepsilon A_{UL}^{\sin 2\phi_h}\right)\right]\\
&+S_L\lambda\left[\sqrt{1-\varepsilon^2}A_{LL}+\cos\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LL}^{\cos\phi_h}\right)\right]\\
&+S_T\left[\begin{aligned}&\sin(\phi_h-\phi_S)\left(A_{UT}^{\sin(\phi_h-\phi_S)}\right)\\&+\sin(\phi_h+\phi_S)\left(\varepsilon A_{UT}^{\sin(\phi_h+\phi_S)}\right)\\&+\sin(3\phi_h-\phi_S)\left(\varepsilon A_{UT}^{\sin(3\phi_h-\phi_S)}\right)\\&+\sin\phi_S\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin\phi_S}\right)\\&+\sin(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin(2\phi_h-\phi_S)}\right)\end{aligned}\right]\\
&+S_T\lambda\left[\begin{aligned}&\cos(\phi_h-\phi_S)\left(\sqrt{(1-\varepsilon^2)}A_{LT}^{\cos(\phi_h-\phi_S)}\right)\\&+\cos\phi_S\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos\phi_S}\right)\\&+\cos(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos(2\phi_h-\phi_S)}\right)\end{aligned}\right]
\end{aligned}\right\}$$





Factorization Theorem:

$$A_{UT}^{\sin(\phi_h-\phi_S)}\propto f_{1T}^{\perp}\otimes D_1$$ Twist-2

$$A_{UT}^{\sin(\phi_h+\phi_S)}\propto h_1\otimes H_1^{\perp}$$

$$A_{UT}^{\sin(3\phi_h-\phi_S)}\propto h_{1T}^{\perp}\otimes H_1^{\perp}$$ Twist-3

$$A_{UT}^{\sin(\phi_S)}\propto\frac{M}{Q}\left(f_T\otimes D_1+h_1\otimes H_1^{\perp}+\cdots\right)$$

$$A_{UT}^{\sin(2\phi_h-\phi_S)}\propto\frac{M}{Q}\left(h_T\otimes H_1^{\perp}+h_T^{\perp}H_1^{\perp}+\cdots\right)$$

$$A_{LT}^{\cos(\phi_h-\phi_S)}\propto g_{1T}\otimes D_1$$

$$A_{LT}^{\cos(\phi_S)}\propto\frac{M}{Q}\left(g_T\otimes D_1+e_T\otimes H_1^{\perp}+\cdots\right)$$

$$A_{LT}^{\cos(2\phi_h-\phi_S)}\propto\frac{M}{Q}\left(e_T\otimes H_1^{\perp}+e_T^{\perp}\otimes H_1^{\perp}+\cdots\right)$$

……

[1] Bacchetta, et al, JHEP 02 (2007) 093

[1] Zhimin Zhu, *et. al.* in preparation

Spin asymmetry in SIDIS process

twist-2 $F_{UU,T} = \mathcal{C}[f_1 D_1]$ $F_{UU,L} = 0$ $F_{LT}^{\cos(\phi_h - \phi_S)} = \mathcal{C}\Big[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} g_{1T} D_1\Big]$ $\hat{\boldsymbol{h}} = \frac{\boldsymbol{P}_{h\perp}}{|\boldsymbol{P}_{h\perp}|}$

twist-3 $$F_{LT}^{\cos\phi_S} = \boxed{\frac{2M}{Q}}\mathcal{C}\Big\{-\Big(xg_T D_1 + \frac{M_h}{M}h_1\frac{\tilde{E}}{z}\Big) + \frac{\boldsymbol{k}_T \cdot \boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_T H_1^\perp - \frac{M_h}{M}g_{1T}\frac{\tilde{D}^\perp}{z}\Big) + \Big(xe_T^\perp H_1^\perp + \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\}$$

$$\sim -\frac{2M}{Q}\mathcal{C}[xg_T D_1]$$ supression factor

EOM relation: $$xg_T = x\tilde{g}_T - \frac{p_T^2}{2M^2}g_{1T} + \frac{m}{M}h_1$$

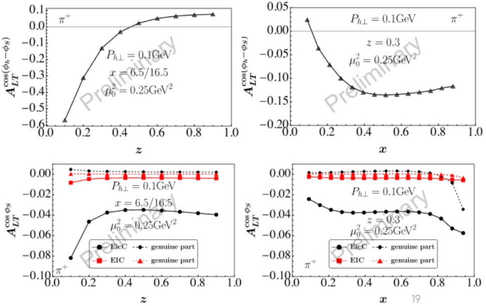
Kinematic parameters : M~1 GeV, Q_{EicC} ~ 10 GeV, Q_{EIC} ~ 100 GeV

Spin asymmetries :

twist-2 : $$A_{LT}^{\cos(\phi_h - \phi_S)} = \frac{F_{LT}^{\cos(\phi_h-\phi_S)}}{F_{UU,T} + \varepsilon F_{UU,L}}$$

twist-3 : $$A_{LT}^{\cos\phi_S} = \frac{F_{LT}^{\cos\phi_S}}{F_{UU,T} + \varepsilon F_{UU,L}}$$

- The twist-3 DSA, $A_{LT}^{\cos\phi_S}$, is smaller than the twist-2 DSA, $A_{LT}^{\cos(\phi_h-\phi_S)}$.
- Twist-3 spin asymmetries may be easier to measure in EicC than in EIC.



[1] Zhimin Zhu, *et. al.* in preparation

Effective Hamiltonian with Dynamical Gluon and Sea Quarks

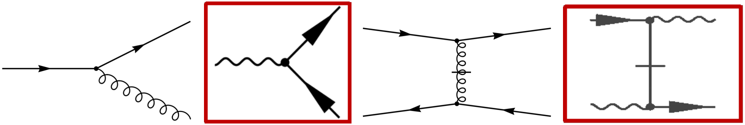
Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + c_1\,|\,uudu\bar{u}\rangle + c_2\,|\,uudd\bar{d}\rangle + c_3\,|\,uuds\bar{s}\rangle + \ldots$$

Light-front QCD Hamiltonian :

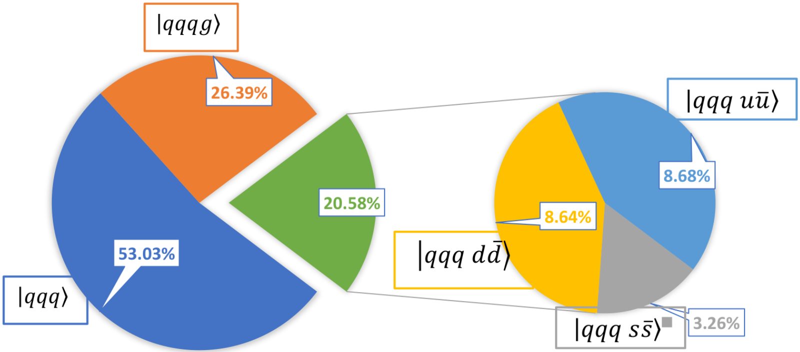
$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^{\,2} + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$

$$H_{\text{vertex}} + H_{\text{inst}} = g_s \bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi$$
$$+ \frac{1}{2} g_s^2 \bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{(i\partial^+)} A_\nu \gamma^\nu \psi$$



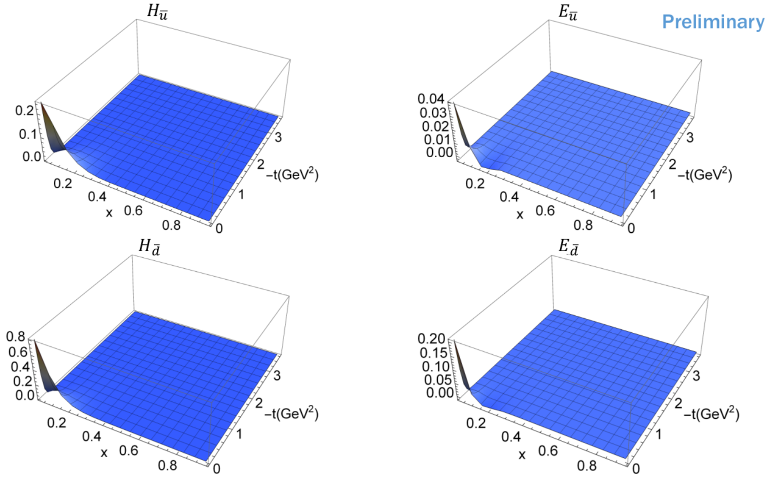
[1] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

Fock Sector Decomposition



Sea Quark GPDs

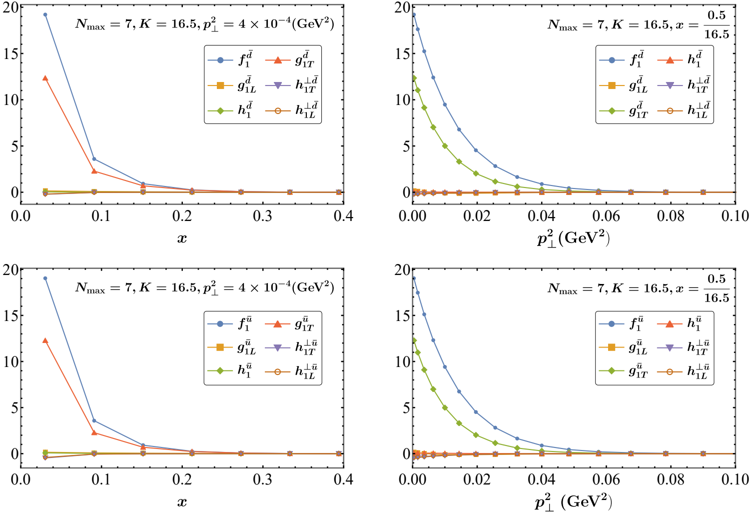
GPDs at $\xi = 0$



- $\bar{u}$ and $\bar{d}$ GPDs
- $\bar{u}$ and $\bar{d}$ GPD E have small negative region around $x \sim 0.2$

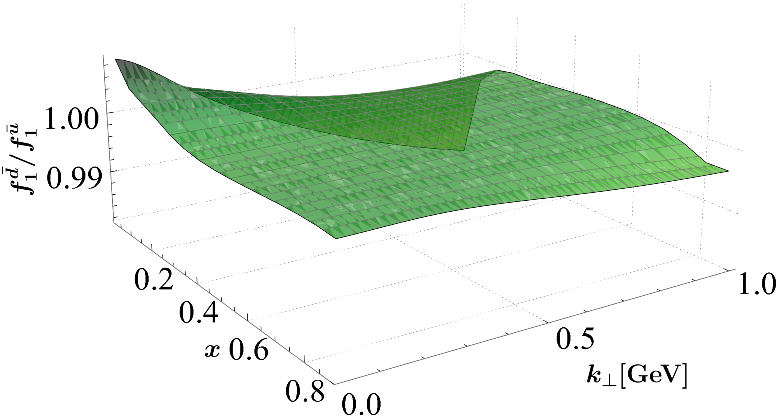
Sea Quark TMDs

Preliminary results



Sea Quark TMDs Asymmetries

Preliminary results



[0] Hongyao Yu, *et. al.*, in preparation

Conclusions

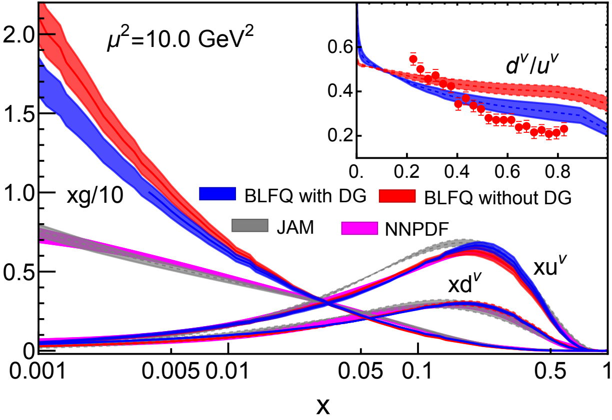
- Basis Light-front Quantization : A non-perturbative approach based on light-front QCD Hamiltonian
- LF Hamiltonian ⇒ Wavefunctions ⇒ Observables.
- Explored gluon and sea quarks within proton based on $|qqq\rangle + |qqqg\rangle$ and $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$, respectively.
- Provides good description of data/global fits for various observables.
- With one dynamical gluon, the quark spin contributes 70%; the gluon spin plays a substantial role (26%) in understanding the nucleon spin.

Outlook

- Include three-gluon and four-gluon interactions in the Hamiltonian.
- *This is not a complete picture . . . long way to go.*

Enormous amount of possibilities with future EICs . . . . . . Thank You

Unpolarized PDFs



Including dynamical gluon (DG):

- Model scale : $\mu_0^2 = 0.195$ GeV^2 $\Rightarrow$ $\mu_0^2 = 0.23 - 0.25$ GeV^2
- Gluon distribution: closer to global fits.

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.

Overview of TMDs for Spin-1/2 Target

Quark correlator

$$\Phi_q^{[\Gamma]}\left(P,S;x=\frac{k^+}{P^+},\vec{k}_\perp\right)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}z^\perp}{2(2\pi)^3}e^{ik\cdot z}\langle P,S|\,\bar{\Psi}_q(0)\Gamma\mathcal{W}(0^\perp,z^\perp)\Psi_q(z)\,|P,S\rangle\Big|_{z^+=0},$$

Parameterization:

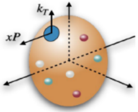
8 twist-2 TMDs:

- 6 T-even terms
- 2 T-odd terms

$$\Phi^{[\gamma^+]}=f_1-\frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M}f_{1T}^\perp,$$

$$\Phi^{[\gamma^+\gamma^5]}=\Lambda g_{1L}+\frac{k_\perp\cdot\boldsymbol{S}_\perp}{M}g_{1T},$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}=S_\perp^j h_1+\Lambda\frac{k_\perp^j}{M}h_{1L}^\perp+S_\perp^i\frac{2k_\perp^i k_\perp^j-(k_\perp)^2\delta^{ij}}{2M^2}h_{1T}^\perp+\frac{\epsilon_\perp^{ji}k_\perp^i}{M}h_1^\perp,$$



16 twist-3 TMDs:

- 8 T-even terms
- 8 T-odd terms

$$\Phi^{[1]}=\frac{M}{P^+}\left[e-\frac{\epsilon_T^{\rho\sigma}k_{\perp\rho}S_{T\sigma}}{M}e_T^\perp\right],$$

$$\Phi^{[i\gamma_5]}=\frac{M}{P^+}\left[S_L e_L-\frac{k_\perp\cdot S_T}{M}e_T\right],$$

$$\Phi^{[\gamma^\alpha]}=\frac{M}{P^+}\left[-\epsilon_T^{\alpha\rho}S_{T\rho}f_T-S_L\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}f_L^\perp-\frac{k_\perp^\alpha k_\perp^\rho-\frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}\epsilon_{T\rho\sigma}S_T^\sigma f_T^\perp+\frac{k_\perp^\alpha}{M}f^\perp\right],$$

$$\Phi^{[\gamma^\alpha\gamma_5]}=\frac{M}{P^+}\left[S_T^\alpha g_T+S_L\frac{k_\perp^\alpha}{M}g_L^\perp-\frac{k_\perp^\alpha k_\perp^\rho-\frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}S_{T\rho}g_T^\perp-\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}g^\perp\right],$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]}=\frac{M}{P^+}\left[\frac{S_T^\alpha k_\perp^\beta-k_\perp^\alpha S_T^\beta}{M}h_T^\perp-\epsilon_T^{\alpha\beta}h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}=\frac{M}{P^+}\left[S_L h_L-\frac{k_\perp\cdot S_T}{M}h_T\right].$$

Jaffe-Ji notation:

- f, e → unpolarized quarks
- g → longitudinally polarized quarks
- h → transversely polarized quarks

- 1 → the leading twist
- L → longitudinally polarized hadron
- T → transversely polarized hadron
- ⊥ → existing $k_\perp$ with a non-contracted index

[1] Meißner, et. al. JHEP08 (2009) 056.

GPDs and GFFs

- The second Mellin's moment of GPDs:

$$\int \mathrm{d}x\, x\, H(x,\xi,t) = A(t) + \xi^2 D(t)$$
$$\int \mathrm{d}x\, x\, E(x,\xi,t) = B(t) - \xi^2 D(t)$$

- GPDs in terms of the Compton Form Factors :

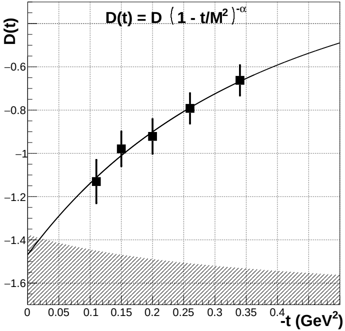
$$\mathrm{Re}\mathcal{H}(\xi,t) + i\,\mathrm{Im}\mathcal{H}(\xi,t) = \int_0^1 \mathrm{d}x \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x + i\epsilon}\right] H(x,\xi,t)$$

- Compton Form Factors are directly related to the observables we can experimentally determine in DVCS measurements.
- In DVCS experiments, GPDs are not directly accessible in the full x-space, but only at $x = \pm\xi$

D-term

- Only $D(t) = 4C(t)$ GFF can be extracted via DVCS
- $D(t)$ can be determined from the dispersion relation :

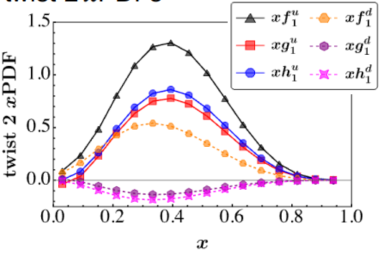
$$D(t) = \mathrm{Re}\mathcal{H}(\xi,t) - \frac{1}{\pi}\mathcal{P}\int_0^1 \mathrm{d}x \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] \mathrm{Im}\mathcal{H}(\xi,t)$$



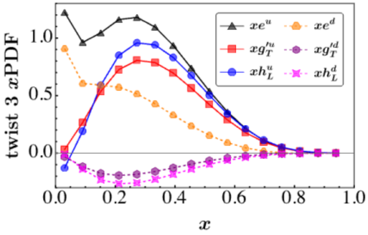
[Fig: Burkert *et. al.*: 2310.11568]

xPDFs: Twist-2 vs Twist-3

twist-2 xPDFs

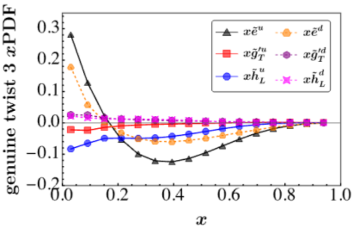


twist-3 xPDFs



$$\int \frac{d^2 k_\perp}{(2\pi)^2} f(x, k_\perp) = f(x)$$

genuine twist-3 xPDFs

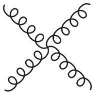
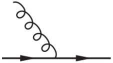
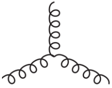


Twist-3 PDFs:

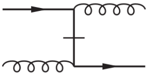
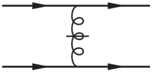
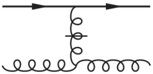
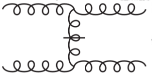
- more concentrating in small x
- similar magnitude to twist-2 PDFs

[1] Zhimin Zhu, *et. al.* in preparation

Light-Front QCD with Light-Cone Gauge ($A^+ = 0$)

$$
\begin{aligned}
\hat{P}^-_{\mathrm{LFQCD}} = & \frac{1}{2}\int \mathrm{d}x^- \mathrm{d}^2 x^\perp \;\; \overline{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\psi + A^{ia}(i\partial^\perp)^2 A^{ia} \\
& + g_s \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; \overline{\psi}\gamma_\mu A^{\mu a} T^a \psi \\
& + \frac{g_s^2}{2}\int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; \overline{\psi}\gamma_\mu A^{\mu a} T^a \frac{\gamma^+}{i\partial^+}\Big(\gamma_\nu A^{\nu b} T^b \psi\Big) \\
& + \frac{g_s^2}{2}\int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; \overline{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2}\Big(\overline{\psi}\gamma^+ T^a \psi\Big) \\
& - g_s^2 \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; i f^{abc}\, \overline{\psi}\gamma^+ T^c \psi \frac{1}{(i\partial^+)^2}\Big(i\partial^+ A^{\mu a} A^b_\mu\Big) \\
& + g_s \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; i f^{abc}\, i\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
& + \frac{g_s^2}{2}\int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; i f^{abc}\, i f^{ade}\, i\partial^+ A^{\mu b} A^c_\mu \frac{1}{(i\partial^+)^2}\Big(i\partial^+ A^{\nu d} A^e_\nu\Big) \\
& - \frac{g_s^2}{4}\int \mathrm{d}x^- \mathrm{d}^2 x^\perp \; i f^{abc}\, i f^{ade} A^{\mu b} A^{\nu c} A^d_\mu A^e_\nu .
\end{aligned}
$$

[1] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998).

Parameters

$$|P,\Lambda\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle$$

- **We use following observables to fix the parameters in the first two Fock sectors**
  - **Nucleon mass**
  - **Nucleon electromagnetic form factors**

m_u	m_d	m_f	g	b	b_{inst}
0.99 GeV	0.94 GeV	5.9 GeV	3.0	0.6 GeV	2.7 GeV

- **The parameters effectively parameterize certain non-perturbative dynamics**

- **In five-quark Fock component, the quark masses are equal to current quark masses**

m_u	m_d	m_s
0.00216 GeV	0.00467 GeV	0.0934 GeV

Truncation parameters: $N_{\max} = 7$ **and** $K_{\max} = 16$