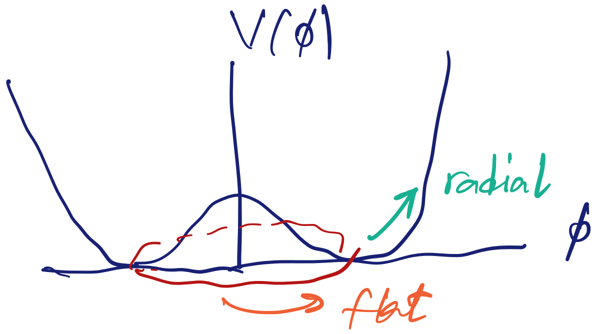
Chiral Perturbation Theory

Study the dynamics of Goldstone bosons (and ~~pseudo~~ GB).

$p \ll \Lambda_\chi \qquad \Lambda_\chi$ = symmetry breaking scale (scale of EFT expansion)

$\mathcal{L}$ analogous to M.

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) \qquad \phi = (\phi^1 \cdots \phi^N)$$



$V(\phi)$ has a flat direction

Then ϕ fluctuations along flat direction cost no potential energy and are massless.

Radial modes are massive. $\Lambda_\chi \sim$ mass of radial mode.

Theory has a symmetry group G, ground state $|\Omega_0\rangle$

Then $g|\Omega_0\rangle$ is also a ground state. Action of G does not change energy. If the symmetry is <u>unbroken</u>
$g|\Omega_0\rangle = |\Omega_0\rangle$ and there is a unique vacuum.

If the symmetry is <u>broken</u> then there are inequivalent degenerate vacua: $g|\Omega_0\rangle \neq |\Omega_0\rangle$

$H \subset G$ is a subgroup of G that leaves the vacuum invariant $h|\Omega_0\rangle = |\Omega_0\rangle$

Symmetry is broken $G \to H$ and vacua are in 1-1 correspondence with G/H (coset space, in general not a group)

$gh|\Omega_0\rangle = g|\Omega_0\rangle$ so gh, g give the same vacuum.

or $g_1|\Omega_0\rangle = g_2|\Omega_0\rangle \Rightarrow \underbrace{g_2^{-1}g_1}_{h}|\Omega_0\rangle = |\Omega_0\rangle$
$\Rightarrow g_1 = g_2 h$

If the invariance group of $|\Omega_0\rangle$ is H, of $g|\Omega_0\rangle$ is gHg^{-1}.

If G and H are continuous groups, then dim G fluctuations of $|\Omega_0\rangle$ of which dim H leave $|\Omega_0\rangle$ unchanged. dim G − dim H flat (massless) modes. This is Goldstone's theorem.

J. Goldstone, Nuov. Cim. 19 (1961) 154
Goldstone, Salam, Weinberg PR 127 (1965) 1962

Note that underlying theory can be anything and does not have to have scalar fields. There will still be dim(G/H) scalar Goldstone bosons.

General formalism developed by Coleman, Wess, Zumino PR 177 (1969) 2239, and Callan, Coleman, Wess, Zumino PR 177 (2247) 1969

coordinates for G/H

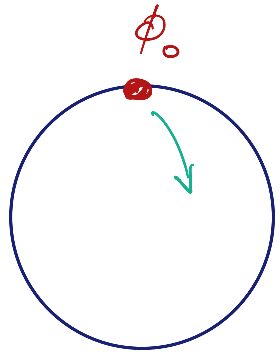
CCWZ

$O(N)$ model $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}(\phi\cdot\phi - v^2)^2$

as an example. $\vec{\phi} = (\phi^1, \ldots \phi^N)$. Theory has an $O(N)$ global symmetry, $\phi \to O\phi$

$O = N\times N$ orthogonal matrix (rotation)

Ground state : $\vec{\phi}\cdot\vec{\phi} = v^2$ = sphere S^{N-1}



any point on S^{N-1} is a ground state.

pick $\phi_0 = (0, \ldots, v)$ = "standard vacuum"

Get to any other vacuum by an $O(N)$ transformation.

$$O_N = \begin{bmatrix} O_{N-1} & 0 \\ 0 & 1 \end{bmatrix} = O(N-1) \text{ transformation}$$

leaves ϕ_0 invariant. $H = O(N-1)$

$S^{N-1} = O(N)/O(N-1)$

$$\dim S^{N-1} = \frac{N(N-1)}{2} - \frac{(N-1)(N-2)}{2} = N-1 \quad \checkmark$$

Want to study dynamics of angular fluctuations where ϕ lies on the sphere.

example Alonso, Jenkins, AM JHEP 08 (2016) 101

Switch to polar coordinates: radial coordinate and an angular coordinate.

$$\vec{\phi} = (v+\rho)\vec{n} \qquad \vec{n} = \text{unit vector.}$$

$$\partial_\mu \vec{\phi} = \partial_\mu \rho \, \vec{n} + (v+\rho)\partial_\mu \vec{n}$$

$$\partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi} = (\partial_\mu \rho)^2 + (v+\rho)^2 \partial_\mu \vec{n}^{\,2}$$

$$\vec{n} \cdot \partial_\mu \vec{n} = \frac{1}{2}\partial_\mu (\vec{n}\cdot\vec{n}) = 0$$

$$L = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(v+\rho)^2(\partial_\mu \vec{n}) - \frac{\lambda}{4}(\rho^2 + 2v\rho)^2$$

ρ is massive, so integrate it out,

$$m_\rho^2 = 2\lambda v^2$$

$$L = \frac{1}{2} v^2 (\partial_\mu \vec{n})^2$$ (higher order terms in $\partial\vec{n}$) from ρ exchange)

$\vec{n}\cdot\vec{n} = 1$ need a parameterization of angular coordinate.

$$v\vec{n} = (\pi_1, \pi_2 \, -- \, \pi_N) \qquad \pi_1^2 + \,..\, + \pi_N^2 = v^2$$

$$L = \frac{1}{2}\sum \partial_\mu \pi_i \, \partial_\mu \pi_i$$

choose $(\pi_1 \,..\, \pi_{N-1}) = \vec{\pi} \longrightarrow N-1$ GB.

$$\pi_N = \sqrt{v^2 - \vec{\pi}\cdot\vec{\pi}}$$ in N hemisphere

$$\partial_\mu \pi_N = -\frac{\vec{\pi}\cdot\partial\vec{\pi}}{(v^2 - \vec{\pi}\cdot\vec{\pi})^{1/2}} \qquad \vec{\pi} = O(N-1) \text{ vector}$$

$$L = \frac{1}{2}\left[\partial_\mu \vec{\pi}\cdot\partial_\mu\vec{\pi} + \frac{(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2}{v^2 - \vec{\pi}\cdot\vec{\pi}}\right]$$

Used by Weinberg for $SU(2)$ χPT.

$$O(4)/O(3) \sim \frac{SU(2)\times SU(2)}{SU(2)}$$

π are Goldstone bosons and derivatively coupled

$O(p^2)$ interactions. $(\pi\cdot\partial\pi)^2$ etc. A constant π

is a change of vacuum

CCWZ is an alternate parameterization. (A field redefinition

(so Green's functions change but not S-matrix elements)

$$\phi_0 = v\begin{pmatrix}0\\ \vdots\\ 1\end{pmatrix} = v\,\vec{n}_0$$

$$\text{CCWZ : pick } \vec{n} = e^{i X^a \frac{\pi^a}{v}}\begin{pmatrix}0\\ \vdots\\ 1\end{pmatrix} = \underbrace{\xi}_{O(N)}\begin{pmatrix}0\\ \vdots\\ 1\end{pmatrix} = \xi\, n_0$$

$\hookrightarrow$ coordinate system on G/H
which is generally curved.

X^a are broken generators, ie rotations that move ϕ_0.

X^a imaginary antisymmetric matrices. $\xi^T = \xi^{-1}$

$$X^1 = -i\left[\begin{array}{c|c} 0 & \begin{matrix}1\\0\\0\end{matrix} \\ \hline \begin{matrix}-1 & 0\end{matrix} & 0\end{array}\right] \qquad X^2 = -i\left[\begin{array}{c|c} 0 & \begin{matrix}0\\1\\0\end{matrix} \\ \hline \begin{matrix}0 & -1 & 0\end{matrix} & 0\end{array}\right] \quad \cdots$$

X^a form representations of unbroken group

π^a transform like X^a the broken generators

$$\text{3-D}: \quad \vec{n} = e^{i(J_x \pi^x + J_y \pi^y)}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\vec{n} = \xi\, n_0 \qquad \partial_\mu \vec{n} = \partial_\mu \xi\, n_0$$

$$\mathcal{L} = \frac{1}{2}\, n_0^T\, \partial_\mu \xi^T\, \partial_\mu \xi\, n_0$$

$$\xi = e^{\frac{1}{v}\left[\begin{array}{ccc|c} & \cdots & & \pi^1 \\ & & & \vdots \\ & & & \pi^{N-1} \\ \hline -\pi^1 & \cdots & -\pi^{N-1} & 0 \end{array}\right]} = e^{\Pi} = 1 + \Pi + \frac{\Pi^2}{2} + \cdots$$

$$\partial_\mu \xi = \partial_\mu \Pi + \cdots$$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi \cdot \partial^\mu \pi\right) + \underbrace{\text{non-linear}}_{\text{determined by the symmetry.}} \text{ terms.}$$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi \cdot \partial_\mu \pi\right) + \frac{1}{6v^2}\left\{(\pi \cdot \partial\pi)^2 - (\pi \cdot \pi)(\partial\pi \cdot \partial\pi)\right\}$$

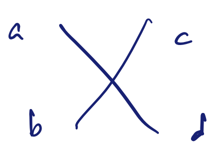
$$+ \cdots$$

vs previous coordinate choice

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi \cdot \partial\pi\right) + \frac{1}{2v^2}(\pi \cdot \partial\pi)^2 + \cdots$$

same on-shell :

p_i incoming



$$(\pi \cdot \partial\pi)^2 \longrightarrow -2\,\delta_{ab}\delta_{cd}\,(p_a + p_b)\cdot(p_c + p_d)$$
$$+ \delta_{ac}\delta_{bd} \cdots$$
$$+ \delta_{ad}\delta_{bc} \cdots$$
$$= +2\,\delta_{ab}\delta_{cd}\, s + \cdots$$

$$(\pi \cdot \pi)(\partial\pi \cdot \partial\pi) = -4\, \delta_{ab}\delta_{cd}\,(p_a \cdot p_b + p_c \cdot p_d)$$

$$= -4\, \delta_{ab}\delta_{cd} \left\{ \frac{1}{2}(p_a + p_b)^2 + \frac{1}{2}(p_c + p_d)^2 - \frac{1}{2}p_a^2 - \frac{1}{2}p_b^2 - \frac{1}{2}p_c^2 - \frac{1}{2}p_d^2 \right\}$$

$$= -4\, \delta_{ab}\delta_{cd} \cdot s \qquad \text{since } p_i^2 = 0$$

(on-shell)

check on-shell amplitudes same