COMQUI QHE Lecture III Part 3 Edge Theory

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Let us consider the theory of a single edge first. As we discovered at the end of Part 1, the action for $\nu = 1/k$ is

$$\boxed{S = -\frac{ke^2}{4\pi}\int_{-\infty}^{\infty} dt \int_0^L dx\, \partial_x\phi\,(\partial_t\phi + v\,\partial_x\phi) + \frac{e^2}{2\pi}\int_{-\infty}^{\infty} dt \int_0^L dx\,(A_0\partial_x\phi - A_x\partial_t\phi)} \quad (1)$$

The density is $\boxed{\rho = -\frac{e}{2\pi}\partial_x\phi}$ (2)

The canonical momentum is

$$\boxed{\Pi_\phi = -\frac{ke^2}{4\pi}\partial_x\phi} \quad (3)$$

Canonical quantization is implemented by

$$\boxed{[\phi(x), \Pi_\phi(x')] = \frac{i}{2}\delta(x-x')} \quad (4)$$

Normally a bosonic theory contains both right-movers and left movers. Together the canonical commutator should be $i\delta(x-x')$. Here we have a chiral boson, which has $\frac{1}{2}$ the number of a chiral boson, which has $\frac{1}{2}$ the number of degrees of freedom.

It will be useful to write down the commutators of the Fourier components of the density.

$$\boxed{\rho(x) = \frac{1}{L} \sum_k e^{iqx} \rho_q} \qquad (5)$$

Using the definition of $\rho = \frac{e}{2\pi} \partial_x \phi$ and the canonical commutator, we find

$$\boxed{[\rho_q, \rho_{q'}] = \frac{\nu}{2\pi} q \, \delta_{q+q',0}} \qquad (6)$$

This is known as the U(1) Kac-Moody algebra.

It turns out the electron operator can be expressed in terms of ϕ as well. One can see that

$$\rho(x) = \Psi_e^\dagger(x) \Psi_e(x) \qquad (7)$$

$$\Rightarrow \left[\rho(x), \Psi_e^\dagger(x')\right] = \delta(x-x') \Psi_e^\dagger(x)$$

Exercise: Use the commutators of ϕ to verify that

$$\boxed{\Psi_e(x) = e^{\frac{i\phi}{\nu}} = e^{ik\phi}} \qquad (8)$$

$$\boxed{\Psi_e(x) = e \quad = e} \qquad (8)$$

satisfies Eq (7) and

$$\boxed{\begin{aligned} &\{\Psi_e(x), \Psi_e(x')\} = 0 \qquad (9) \\ &\{\Psi_e(x), \Psi_e^{+}(x')\} = \delta(x-x') \end{aligned}}$$

Since ϕ is a compact field descended from a U(1) angle, we can define

$$\boxed{\Psi_m = e^{im\phi} \qquad m \in \mathbb{Z}} \qquad (10)$$

It turns out that $e^{i\phi}$ corresponds to the destruction of a Laughlin qp at the edge.

Now let us generalize to a multicomponent abelian QH CS theory. As we learned in Part 2, such a theory is described by a K-matrix and a t-vector

$$\boxed{\mathcal{L} = \frac{1}{4\pi} K_{IJ}\, \epsilon^{\mu\nu\lambda} a_{I\mu} \partial_\nu a_{J\lambda} + \frac{e}{2\pi} A_\mu\, \epsilon^{\mu\nu\lambda} t_I \partial_\nu a_{I\lambda}} \qquad (11)$$

It is straightforward to generalize the single component edge CS theory. Basically, every $a_{\mu I}$ leaves behind a chiral field ϕ_I at the edge.

$$\boxed{\mathcal{L}_{edge} = -\frac{e^2}{4\pi}\partial_x\phi_I K_{IJ}\partial_t\phi_J + \frac{e^2}{2\pi}t_I(A_0\partial_x\phi_I - A_x\partial_t\phi_I) - \frac{e^2}{4\pi}V_{IJ}\partial_x\phi_I\partial_x\phi_J} \quad (12)$$

V_{IJ} is a positive definite symmetric matrix (for stability) that depends on the details of the edge.

We can define the density of the I^{th} field

$$\rho_I = -\frac{e}{2\pi}\partial_x\phi_I \qquad \rho = \sum_I \rho_I \quad (13)$$

$$[\rho_{I,q}, \rho_{J,q'}] = (K^{-1})_{IJ}\frac{q}{2\pi}\delta_{q+q',0}$$

The generalized qp operators on the edge are

$$\boxed{\Psi_{\vec{l}} = e^{i l_I \phi_I}} \quad (14)$$

which satisfy the commutator

(15)

$$\boxed{[\rho_I(x), \Psi_{\vec{\ell}}(x')] = \ell_J (K^{-1})_{JI}\, \delta(x-x')\, \Psi_{\vec{\ell}}} \tag{15}$$

Operators that have a combination of ℓ_J such that

$$\boxed{[\rho(x), \Psi_{\vec{\ell}}(x')] = \delta(x-x')\, \Psi_{\vec{\ell}}(x')} \tag{16}$$

are candidates for the electron operator