Wednesday, September 20, 2023 3:43 PM

3. Edge Modes in the IQHE

So far we have been focusing on the bulk. Now we consider the edge.

Once again, let us put periodic b.c. in the y-direction. This time we put a hard edge in x. By "hard" we mean that the potential felt by the electron is infinite

$$V_e(x) = 0 \qquad 0 < x < L_x \qquad (3.1)$$

$$V_e(x) = \infty \qquad x < 0 \quad \text{or} \quad x > L_x.$$

It will almost always be the case in real samples that

$$L_x, L_y \gg \ell \qquad (3.2)$$

For a $\perp$ field of 1 Tesla $\ell \cong 25\,\text{nm}$. The typical sizes of samples are several μm. This allows us to focus on one edge, say. $x \simeq L_x$. In Landau gauge, there is translation invariance in the y-direction, so

(3.3)

$$\psi_k(x,y) = e^{iky} f_k(x)$$

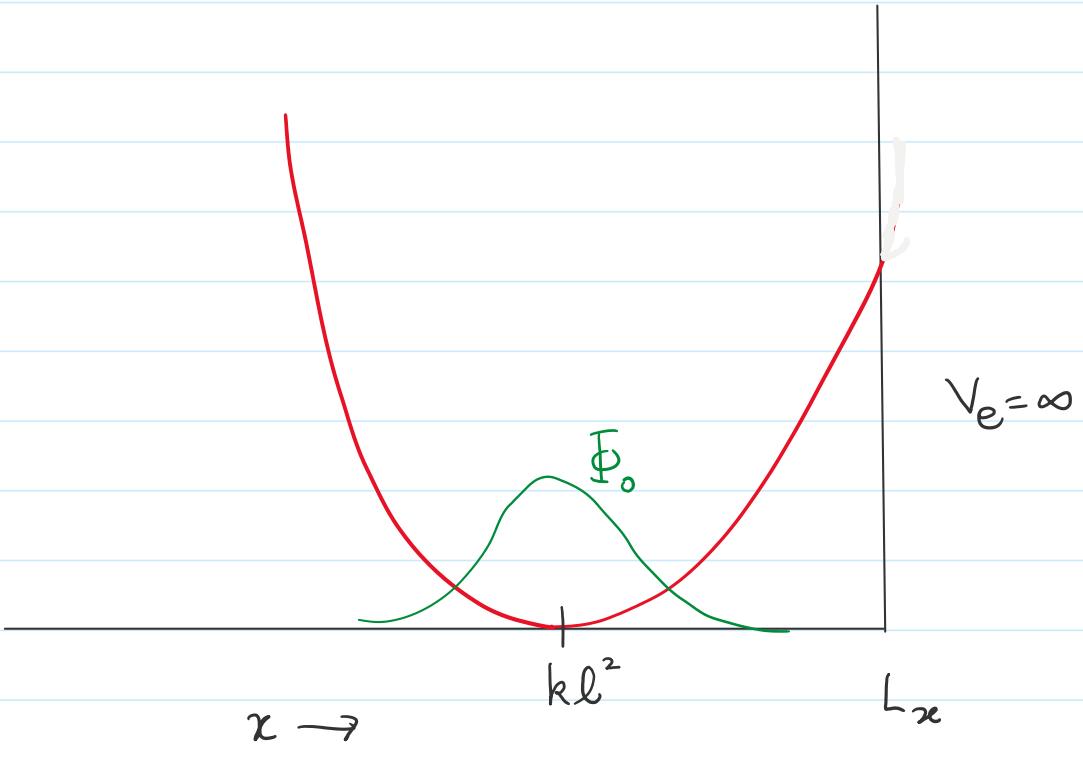
$$\psi_{nk}(x,y) = e^{iky}\,\Phi_{nk}(x) \quad ; \Phi_{nk}(0) = \Phi_{nk}(L_x) = 0$$

The equation satisfied by Φ_{nk} is

$$\frac{\hbar\omega_c}{2}\left\{-l^2\frac{d^2}{dx^2} + \frac{(x-kl^2)^2}{l^2} + V_e(x)\right\}\Phi_{nk}(x) = E_n(k)\,\Phi_{nk} \qquad (3.4)$$

In addition to the Harmonic potential centered at kl^2, the electron experiences the edge potential.

For $L_x - X_k = L_x - kl^2 \gg l$, the part of the potential experienced by the wave function for n not too large is simply the Harmonic part



So for $L_x - kl^2 \gg l$ Φ_{nk} is the good old

Harmonic oscillator wavefunction and

$$E_n(k) \cong \hbar\omega_c\left(n+\tfrac{1}{2}\right)$$

When $L_x - k\ell^2 \simeq \ell$ the ∞ edge potential will make itself felt. Clearly the wave f^n will distort from the Harmonic oscillator form, and the energy will increase.

There is one special case we can solve, which is

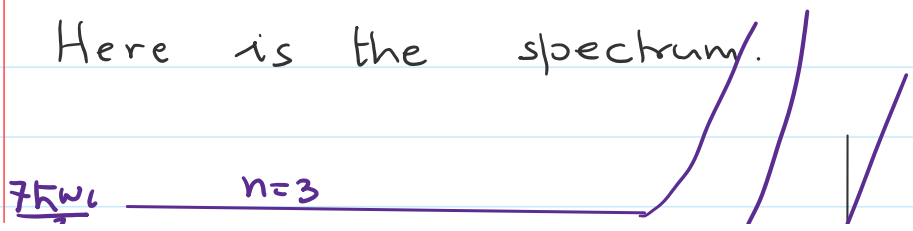
$$k\ell^2 = L_x.$$

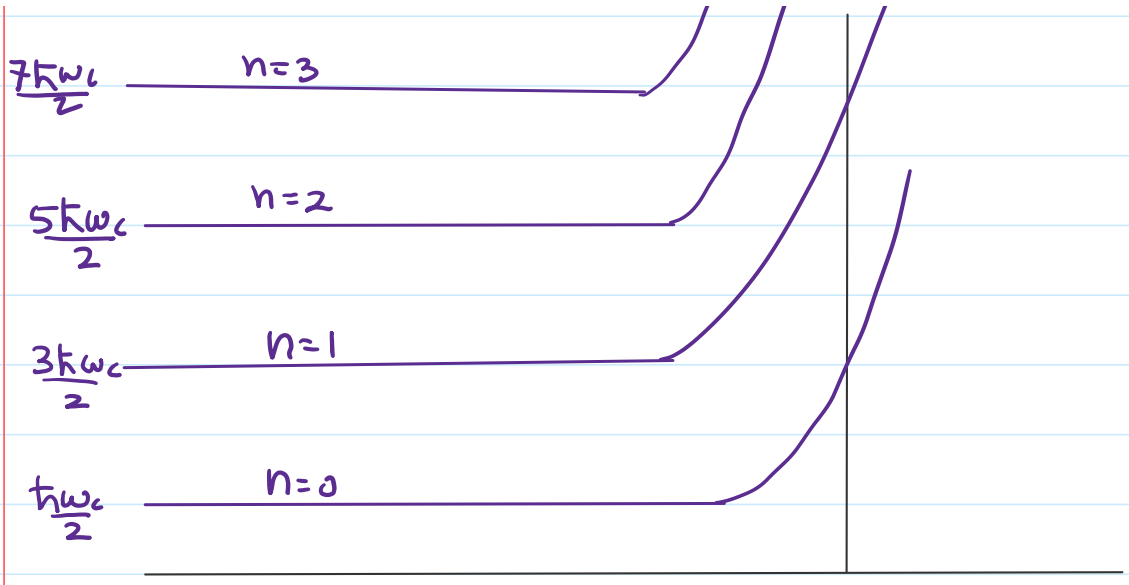
The wave f^n must vanish at the minimum of the Harmonic potential.

We know that n = odd wavefunctions do vanish at the center. This allows us to infer

$$E_n\left(k = L_x/\ell^2\right) = \hbar\omega_c\left(2n+1+\tfrac{1}{2}\right) \quad (3.5)$$

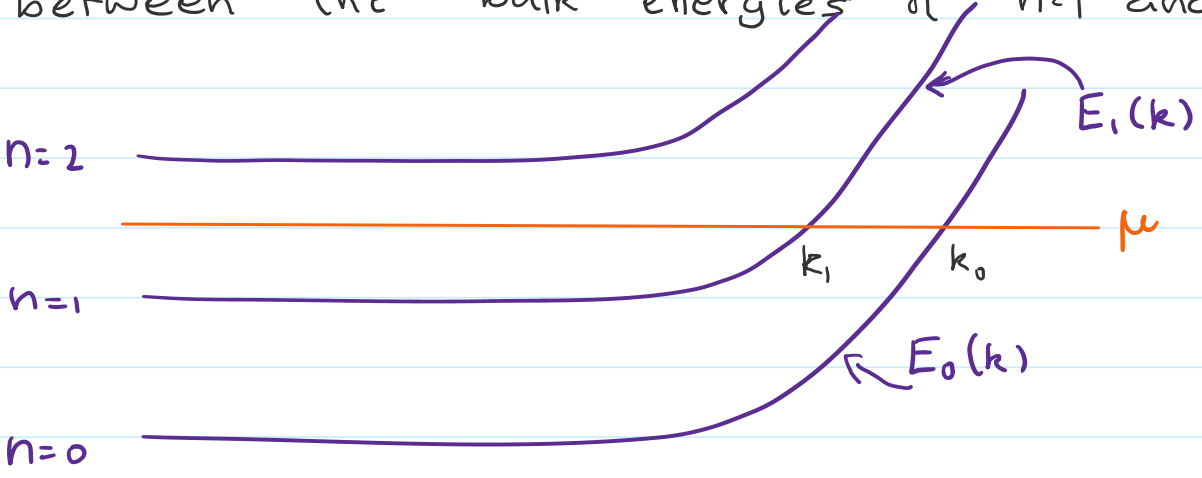
Here is the spectrum.





It is important to note that states with $kl^2 > L_x$ also exist. Of course, the wavefunction is always nonzero only for $x < L_x$.

Now suppose we put the chemical potential between the bulk energies of n=1 and n=2.



You can see that μ cuts the spectrum at two values of k, meaning two guiding center locations.

Recalling that $k = k_y$ we see that there

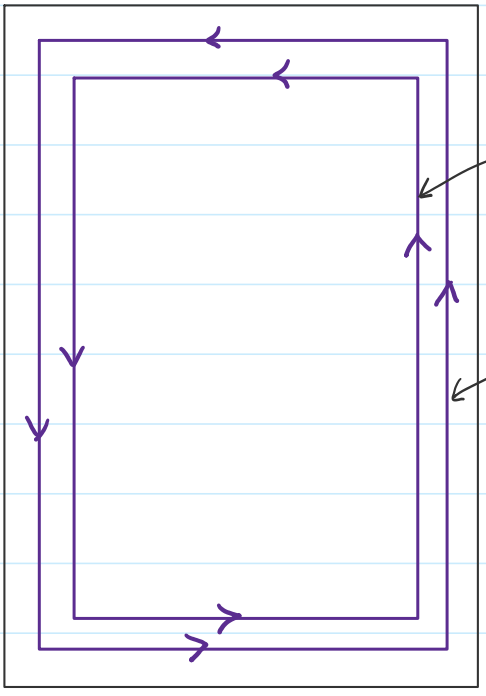
Recalling that $k = k_y$ we see that there is a nonzero velocity

$$v_0(k_0) = \left.\frac{dE_0(k)}{dk}\right|_{k_0} \qquad (3.6)$$

$$v_1(k_1) = \left.\frac{dE_1(k)}{dk}\right|_{k_1}$$

These edge modes travel in the y-direction.

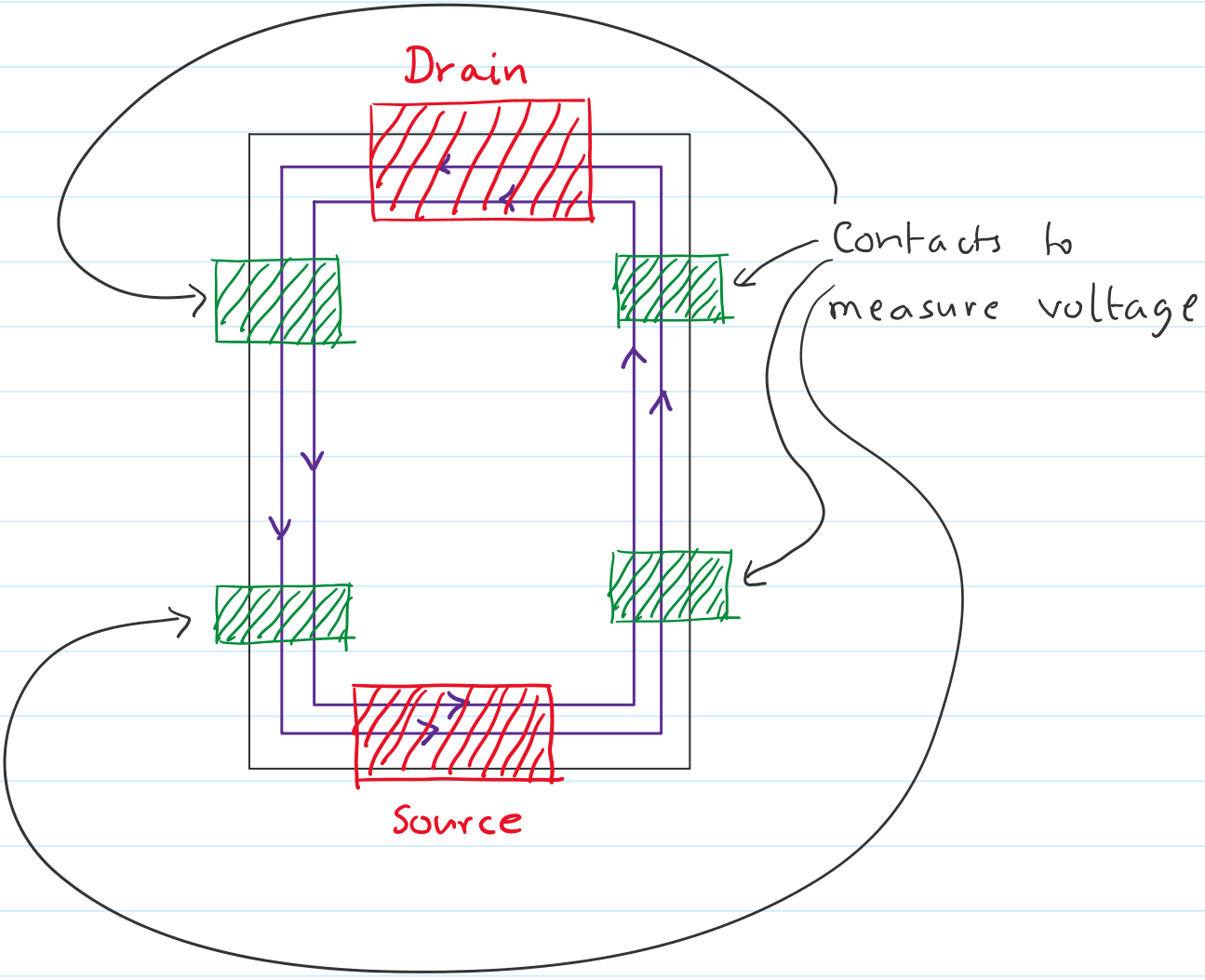
Let us remove the periodic b.c. and consider a rectangular sample.



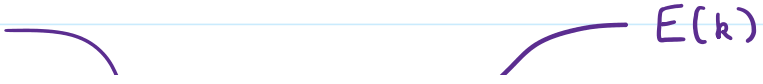
You can see that the modes are chiral, that

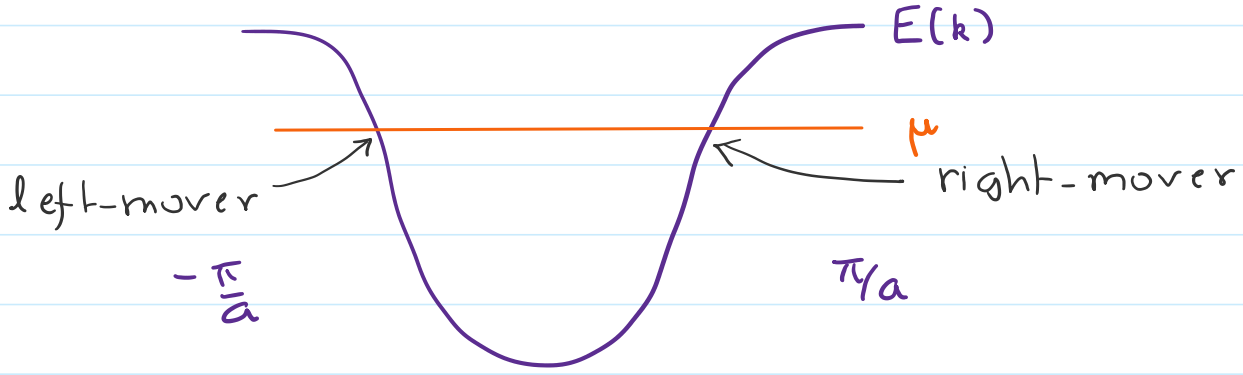
You can see that the modes are chiral, that is, they only go counterclockwise in this case.

The above is a sample without any contacts to measure anything. More typically, you have several contacts, and the sample looks like this.



In a one-dimensional metallic wire, μ is somewhere in the middle of a band.





There are both right-moving and left-moving modes in the same 1D wire. Disorder can scatter the modes into each other and localize. At the IQH edge disorder cannot scatter a right mover into a left mover, because they are far from each other.

The quantum Hall edge modes are protected against localization by disorder.

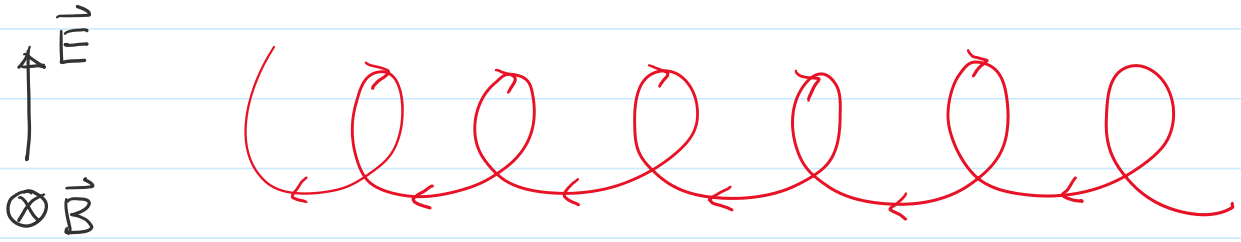
Quantum Hall states are one of the simplest examples of topological insulators.

4. Disorder and the Quantization of σ_{xy}

I mentioned before that we need disorder to see the quantization of σ_{xy}. Let us see how this works. We will take a limit where the

see the quantization of σ_{xy}. Let us see how this works. We will take a limit where the B is very large, so ℓ is much smaller than the scale over which the disorder potential varies.

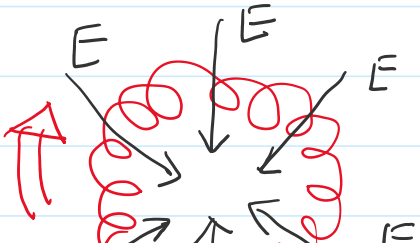
First consider a small constant (in space) $\vec{E}$ field in the plane. There is a large $\vec{B}$ field in the $-\hat{e}_z$ direction. The electrons undergo "guiding center drift".



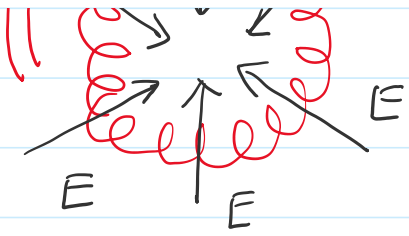
The electron decelerates when it travels in the direction of $\vec{E}$ and accelerates when it travels opposite to $\vec{E}$. The drift velocity is

$$\vec{v}_d = \frac{\vec{E} \times \vec{B}}{B^2} \qquad (4.1)$$

Now consider the region near a disorder minimum. The electron's semiclassical orbit looks like

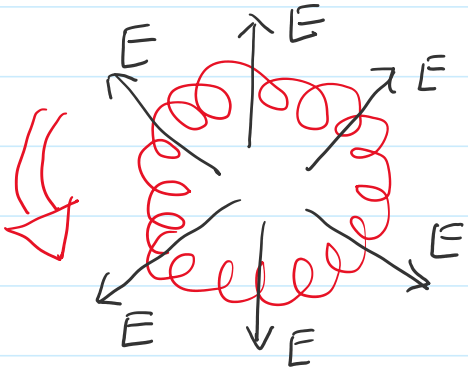


Near a minimum



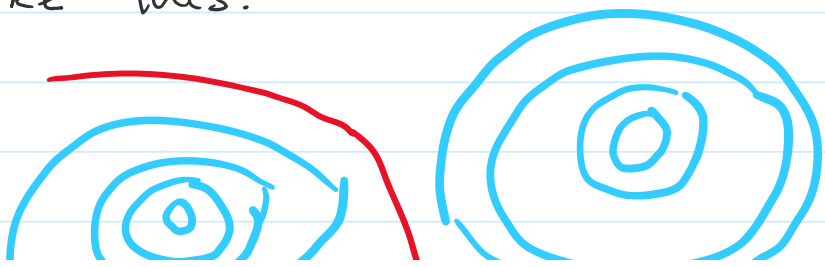
Quantizing this by Bohr-Sommerfeld, we get (somewhat smeared by ℓ) equal potential energy contours of the disorder potential as the place where the wavefunction lives.

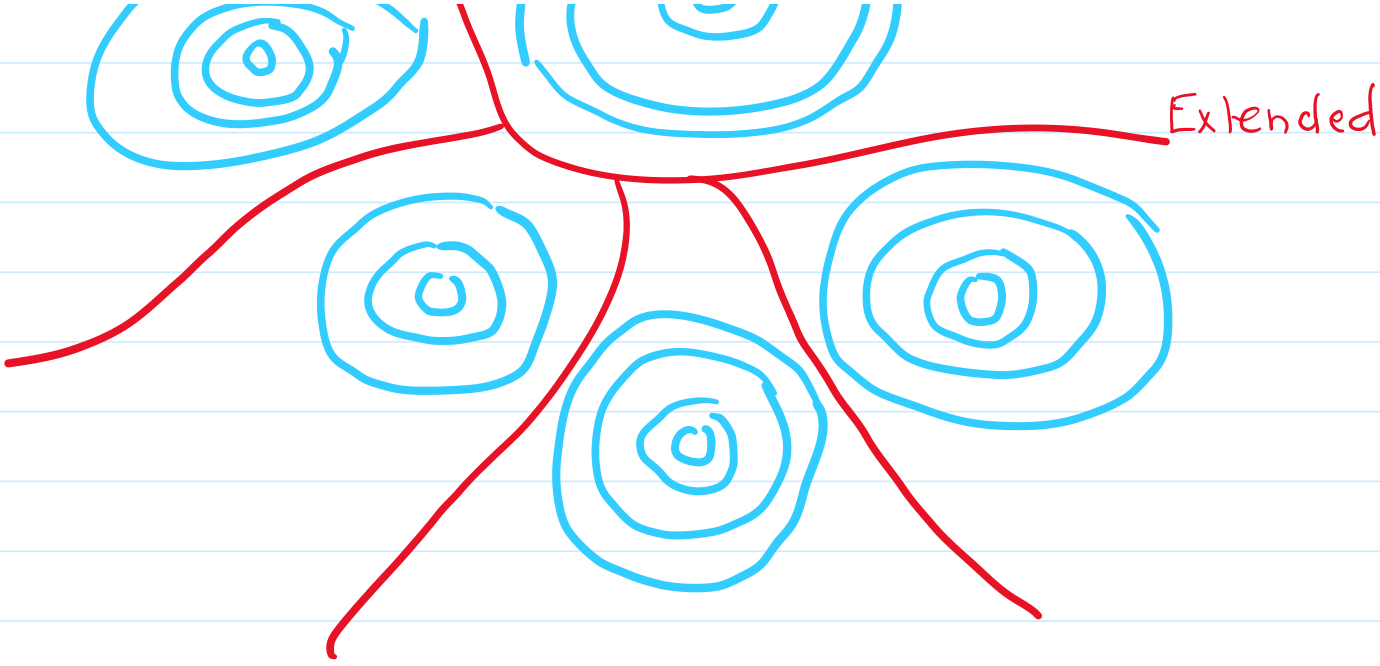
The wavefunction is localized around the minimum. This is expected, but what is surprising is that it is localized around maxima as well!



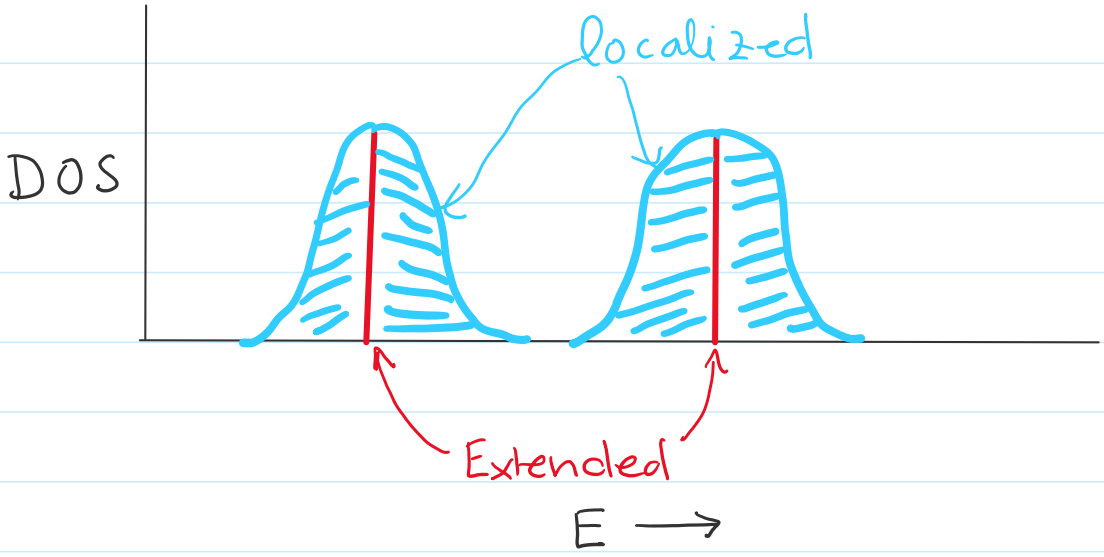
Near a maximum.

So on a big picture level what we have in a sample with both maxima and minima of the disorder potential looks something like this.





The key point is that there is an extended state that lies between the states localized around minima and the states localized around maxima.



Somewhere in every disorder-broadened LL, there is an extended state that connects the bulk to the edge.

The plateau transition between different quantized values of σ_{xy} occurs when the chemical potential crosses the extended state. This is strictly true at $T=0$. For $T>0$, the plateau transition is broadened to a width of $k_B T$ in μ.

From this picture, one can also understand the behavior of σ_{xx}. A nonzero σ_{xx} means nonzero dissipation. In order to have dissipation, there must be extended states near μ for the electrons to scatter into. There are no such states when μ is far from the extended states at the centers of the broadened Landau levels.