Nucleosynthesis in Massive Stars and Core-Collapse Supernovae

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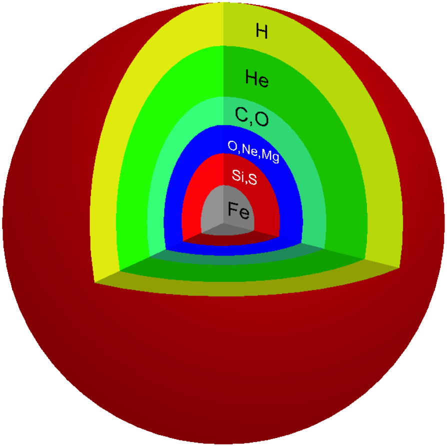
Lectures on Core-Collapse Supernovae

From Massive Stars to Supernovae



Onion shell structure of massive stars

Outline of Nucleosynthesis in Massive Stars: Helium Burning

He burning works by maintaining an extremely small equilibrium abundance of the very unstable $^{8}\mathrm{Be}$. There is a small chance that this nucleus captures another α-particle before decaying and reacts resonantly to form C:

$$\begin{aligned} {}^{4}\mathrm{He} + {}^{4}\mathrm{He} &\leftrightharpoons {}^{8}\mathrm{Be} \\ {}^{8}\mathrm{Be} + {}^{4}\mathrm{He} &\rightarrow {}^{12}\mathrm{C} + \gamma. \end{aligned}$$

Due to the short lifetime of $^{8}\mathrm{Be}$, this is essentially a three-body reaction. The approximate energy generation rate is:

$$\epsilon = 5.09 \times 10^{11} \mathrm{erg\,g^{-1}\,s^{-1}} \left(\frac{\rho}{\mathrm{g\,cm^{-3}}}\right)^2 X_\alpha^3 T_8^{-3} e^{-44.027/T_8},$$

Once some C has been made, it can also capture αs. Hence O (through $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$) and even some Ne can also be made by He burning. High densities (three-body reaction) favour C production over subsequent α captures. Because of this, and because of the preponderance of low-mass stars, most of the C in the universe originates from He burning in low-mass stars. Much of the O comes from He burning in high mass stars.

He Burning – Continued

He burning also makes some trace elements/isotopes. Processing of the $^{14}\mathrm{N}$ from CNO burning results in:

$$^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(e^{+}\nu)^{18}\mathrm{O}.$$

Although most of the $^{18}\mathrm{O}$ is destroyed later by $^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne}$, this is important because it imprints a small **neutron excess** on the composition and enables the production of non-α nuclei during subsequent burnning stages.
Moreover, one can start neutron capture reactions, e.g., by continuing α captures: $^{22}\mathrm{Ne}(\alpha,n)^{25}\mathrm{Mg}$. Massive stars contribute somewhat to the production of slow neutron capture process (s-process) elements (so called "weak s-process" up to mass number $A=90$).

C, Ne, O, Si Burning

C burning occurs almost exclusively in massive stars, i.e. if C burning is ignited, the star will eventually go all the way to Si burning, which produces iron group elements. There is probably a narrow mass window, where only C burning is ignited, and the star ends in an O-Ne white dwarf, however.
These advanced burning stages share a number of characteristics:

- Reaction branchings become more and more important,
- Neutrino emission become the dominant energy loss channel from stellar interiors
- Active cores/shells adjust to a state of "balanced power" where the integrated nuclear energy generation rate and neutrino losses are roughly equal.
- Because of the steep temperature dependence of the burning rates, the condition of balanced power roughly fixes the temperatures for steady burning.

C Burning

C burning starts at about $0.8\,\mathrm{GK}$. The main reactions of C burning are

$$\begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{23}\mathrm{Mg} + n - 2.62\,\mathrm{MeV} \\ &\rightarrow {}^{20}\mathrm{Ne} + \alpha + 4.62\,\mathrm{MeV} \\ &\rightarrow {}^{23}\mathrm{Na} + p - 2.24\,\mathrm{MeV}, \end{aligned}$$

but there are more side-chains and follow-up reactions. The principal products are ${}^{20,21}\mathrm{Ne}$, ${}^{23}\mathrm{Na}$, ${}^{24,25,26}\mathrm{Ne}$, ${}^{26,27}\mathrm{Ne}$, and some ${}^{29,30}\mathrm{Si}$ and ${}^{31}\mathrm{P}$. In addition, one has a lot of left-over O from the previous He burning stage.

Ne Burning

At about 1.5 GK, we start to burn Ne. This is a **photodisintegration reaction**:

$$^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}.$$

Sometimes the α is captured, and we make Mg:

$$^{20}\mathrm{Ne}(\alpha,\gamma)^{24}\mathrm{Mg}.$$

These are the main products. In addition, one again gets some $^{26}\mathrm{Al}$, $^{29,30}\mathrm{Si}$, and $^{31}\mathrm{P}$.

Oxygen Burning

O burning occurs at about $2\,\mathrm{GK}$. Although it starts out as as two particle reaction $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow \ldots$, there are many secondary reactions. Thus, isotopes up to A=40 can be produced, including isotopes of Si, S, Cl, Ar, K, and Ca. O burning in massive stars is the primary source in the universe for these elements (for Ca, type Ia supernova are also important).

Silicon Burning

Silicon burning (at about $3.5\,\mathrm{GK}$) proceeds differently than the previous burning stages. The production of heavier elements does not occur by heavy-particle collisions any longer (i.e. no $^{28}\mathrm{Si} + {}^{28}\mathrm{Si} \rightarrow {}^{56}\mathrm{Ni}$. Instead, photodisintegration provides a pool of α-particles (and n,p), that react with the heavy nuclei. **Quasi-equilibrium** clusters are formed, in which the heavy nuclei are in equilibrium with respect to light particle reactions, e.g.

$$^{24}\mathrm{Mg} \stackrel{a}{\leftrightharpoons} {}^{28}\mathrm{Si} \stackrel{a}{\leftrightharpoons} {}^{32}\mathrm{S} \stackrel{a}{\leftrightharpoons} {}^{36}\mathrm{Ar} \stackrel{a}{\leftrightharpoons} {}^{40}\mathrm{Ca}\ldots.$$

Within such a cluster, the abundances of the various species are linked by a Saha equation of the type

$$Y(^{A}\mathrm{Z}) \propto Y(^{28}\mathrm{Si}) Y(\alpha)^{Z/2-7} Y(n)^{A-2Z}$$

Note that the light particle abundances (p, n, α) are *not* set by this Saha eqution; they are determined by the photodisintegration flow from $^{24}\mathrm{Mg}$ downwards.

Initially, one has two distinct quasi-equilibrium clusters, one between A=24 and A=45, and another at $A > 45$. The abundance within these clusters shifts to higher A as Y_α increases. Eventually the clusters merge and the abundances all shift towards the iron group. During silicon core burning, there is sufficient time for deleptonisation by electron captures, so one gets a neutron excess, and the quasi-equilibrium abundance shifts towards $^{56}\mathrm{Fe}$ instead of $^{56}\mathrm{Ni}$. Note, however, that the ashes of silicon core burning will eventually end up in the central remnant in a supernova explosion. Si burning also occurs during the explosion itself, however (Lecture 4), and produces iron group elements (complete Si burning), or intermediate-mass elements (incomplete Si burning).

Summary: Nucleosynthesis in Massive Stars

Most important ashes of hydrostatic burning procceses:

- **H burning:** He, also reshuffles C, O into N (secondary nucleosynthesis) in CNO cycle
- **He burning:** produces most of the C (low-mass stars) and much of the O in the universe (He burning in high-mass stars)
- **C burning (massive stars):** dominant production site for Ne, Mg, Na, some Al, P, leftover O from He burning
- **Ne burning:** O, Mg, some Al, Si, P
- **O burning:** dominant production site for Si, S, Cl, Ar, K, also Ca
- **Complete Si burning:** Iron group
- **Incomplete Si burning:** Iron group down through Ca, Ar, S, Si

Massive stars also contribute some slow-neutron capture elements (weak s-process). During the supernova, some rate isotopes are also produced by neutrino capture or spallation in the envelope (aside from the genuine supernova nucleosynthesis, which will be discussed in another lecture).

We can roughly distinguish three ejecta components in core-collapse supernovae:

- Material that makes it close to the proto-neutron star that is ejected because of neutrino heating,
- Material that is immediately accelerated to positive velocities as it is overrun by the shock, and undergoes **explosive burning** because of shock heating,
- Material that is shocked, but is not heated sufficiently for explosive burning. Here we may still have spallation reactions caused by neutrinos of sufficiently high energy (tens of MeV), like $^{12}\mathrm{C}(\nu, \nu' pp)^{10}\mathrm{Be}$.

We will consider the second and first components of the ejecta more closely.

Explosive Burning

The outcome of explosive burning is determined by:

- Initial composition in progenitor (elemental composition, also neutron excess if we're in the Si shell)
- Peak temperature: This is related to the explosion energy roughly as

$$T \approx 1.33 \times 10^{10}\,\mathrm{K} \times \left(\frac{E_{\mathrm{expl}}}{10^{51}\,\mathrm{erg}}\right)\left(\frac{r}{1000\,\mathrm{km}}\right)^{-3/4}$$

  ($\leftarrow$ equate explosion energy to energy of radiation gas behind shock).
- The post-shock entropy, which determines the trajectory of the expanding material in the ρ-T-plane ($s \propto T^3/\rho$). Equivalently, one needs the post-shock density, which is $\rho_{\mathrm{post}} \approx (3\dots10)\rho_{\mathrm{pre}}$.
- The expansion time scale τ which sets the rate of decrease of temperature and density after shock heating. This is not too far away from the free-fall time scale $\tau_{\mathrm{FF}} \approx 446\,\mathrm{s} \times \bar{\rho}_{\mathrm{CGS}}^{-1/2}$ ($\bar{\rho}_{\mathrm{CGS}}$: average density inside mass shell in cgs units). One can roughly extrapolate the decay of temperature and density as $T = T_0(1 + t/\tau)^{-1}$ and $\rho = \rho_0(1 + t/\tau)^{-3}$.

Explosive Burning

One key difference to hydrostatic burning in massive stars is that the ignition temperatures for the various burning processes are higher. This is due to the shorter time scale available for burning:

- 2–3 GK for explosive C/Ne burning
- 3–4 GK for explosive O burning
- 4–5 GK for incomplete Si burning
- $\gtrsim$5 GK for complete Si burning

The prodcuts of explosive C/Ne, O, and incomplete Si burning resemeble those from the hydrostatic burning stages. For complete Si burning into NSE, there can be important variations, which results, e.g., in different abundances compared to NSE freeze-out in type Ia supernovae.

NSE Composition

Let us consider some properties of nuclear statistical equilibrium as a background for nucleosynthesis in type Ia supernovae. Intuitively, it may be clear that if we burn a mixture of the symmetric nuclei $^{12}\mathrm{C}$ and $^{12}\mathrm{O}$ at sufficiently high temperature, we end up with $^{56}\mathrm{Ni}$. But on closer inspection, this is not so intuitive any more. First we have to avoid too high temperatures to avoid photodisintegration $^{56}\mathrm{Ni} \to 14^{4}\mathrm{He}$.

We can estimate quantitatively where this happens using the Saha equation

$$\frac{n_\alpha^{14}}{n_{^{56}\mathrm{Ni}}} = \theta^{13} \left(\frac{4^{3/2\times 14}}{56^{3/2}} \right)^3 \exp \frac{\Delta Q}{kT},$$

to determine when the abundances of $^{56}\mathrm{Ni}$ and $^{4}\mathrm{He}$ in NSE are equal. Note that $\theta = h/\sqrt{2\pi kT}$ here. The Q-value is $-87.853\,\mathrm{MeV}$.

Why $^{56}\mathrm{Ni}$?

$^{56}\mathrm{Ni}$ is itself unstable. The reason it nonetheless dominates the ejecta composition of type Ia supernovae emerges from a detailed numerical solution of the full Saha equation

$$\frac{n_p^Z n_n^{A-Z}}{n_{^A\mathrm{Z}}} = \theta^{A-1} \left(\frac{1}{A^{3/2}}\right)^3 \exp\frac{\Delta Q}{kT},$$

if we keep the ratio of *all* (bound and unbound) protons to baryons fixed, $Y_e = \sum_i Z_i n_i / \sum_i A_i n_i = \mathrm{const.}$. (This ratio is also known as electron fraction; it is identical to the number of electrons per baryon because of charge conservation). Note that $\theta = h/\sqrt{2\pi k T}$.

Instead of Y_e, one sometimes uses the neutron excess η to parameterise the neutron-to-proton ratio:

$$\eta = 1 - 2Y_e.$$

It turns out that nuclei with Z/A deviating strongly from Y_e are strongly penalised (if $Y_e \leq 0.5$) even if they are strongly bound. This is because we can't form them without extra free protons and neutrons (or very unstable nuclei with opposite η). In the regime where nuclei dominate, the most abundant nuclei will have $Z/A \approx Y_e$.

Why ^{56}Ni?

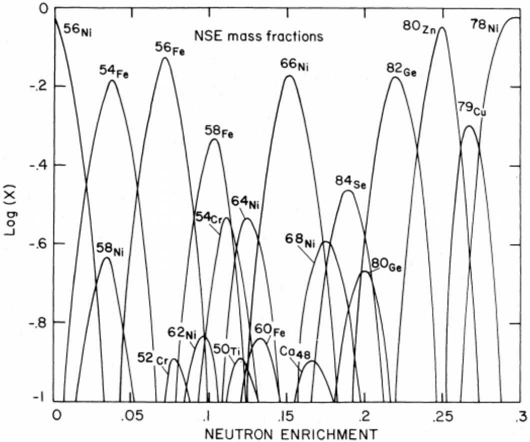


FIG. 2.—Mass fractions obtained in NSE as a function of neutron enrichment η for fixed temperature $T = 3.5 \times 10^9$ K and density $\rho = 10^7$ g cm^{-3}

Hartmann, Woosley, & El Eid (1985).

Different from type Ia supernovae, there is sufficient time for weak processes to establish a neutron excess deep in the star. At the edge of the iron core one has $\eta \approx 0.04$. If material from the iron core or the deepest layer of the Si shell were ejected, the resulting abundances would be very different from solar. Nucleosynthesis thus places constraints on the explosion mechanism: No iron core material must be ejected.

The entropy during explosive burning in core-collapse supernovae is also higher than for NSE nucleosynthesis in thermonuclear supernovae. If the entropy is sufficiently high (or if the expansion time scale is short), we enter the regime of α-rich freeze-out.

α-rich Freeze-out

At low entropy, NSE only freezes out when almost all of the α particles are incoroporated into heavy nuclei. At high entropy, the triple-α link between light clusters and heavy nuclei freezes out while the α-abundance is still high.
The αs can then still be consumed slowly by capture reactions on heavy nuclei (balanced dissociation reactions), but their abundance will not be in equilibrium with the heavy nuclei. Instead we have a pool of light particles (α,n,p), and one (or more) equilibrium cluster(s) of heavy nuclei. Within such a cluster, equilibrium still obtains according to the Saha equation

$$\frac{n_{A+4n(\mathrm{Z+2n})}}{n_{A_{\mathrm{Z}}} n_\alpha^n} = \theta^n \left(\frac{A^{3/2}}{2^n}\right)^3 \exp \frac{\Delta Q}{kT},$$

but the abundances of the light particles are now a free parameter. Eventually, this equilibrium will also freeze-out
Compared to NSE, the abundances in this quasi-equilibrium (QSE) are tilted towards more α-rich nuclei, so one can get, e.g., $^{60-62}\mathrm{Ni}$ and $^{64}\mathrm{Zn}$. One also gets enhanced production of $^{44}\mathrm{Ti}$ (deacys to Ca) when the heavy QSE cluster breaks up an $^{44}\mathrm{Ti}$ is the upper "bottleneck" of the intermediate-mass QSE cluster $^{24}\mathrm{Mg} \leftrightharpoons {}^{28}\mathrm{Si} \leftrightharpoons {}^{32}\mathrm{S} \leftrightharpoons {}^{36}\mathrm{Ar} \leftrightharpoons {}^{40}\mathrm{Ca} \leftrightharpoons {}^{44}\mathrm{Ti}$.

Neutrino-Heated Ejecta

Material that makes it close to the proto-neutron star is completely dissociated into neutrons and protons. When it is ejected, it also undergoes some kind of freeze-out nucleosynthesis, but there are two crucial differences compared to explosive burning:

- The entropy can be higher than for explosive nucleosynthesis.
- The neutron-to-proton ratio is reset considerably by the neutrino absorption and electron/positron capture reactions $n + \nu_e \leftrightharpoons p + e^-$ and $p + \bar{\nu}_e \leftrightharpoons n + e^+$.
- At low densities, the neutrino absorption dominates and drives the Y_e towards an equilibrium value

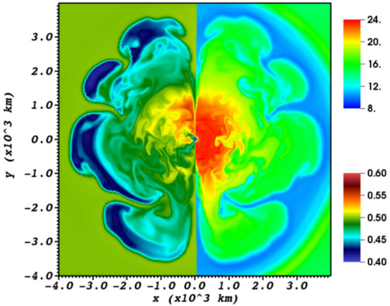
$$Y_e \approx \left(1 + \frac{L_{\bar{\nu}_e} E^2_{\bar{\nu}_e}}{L_{\nu_e} E^2_{\nu_e}}\right)^{-1}$$

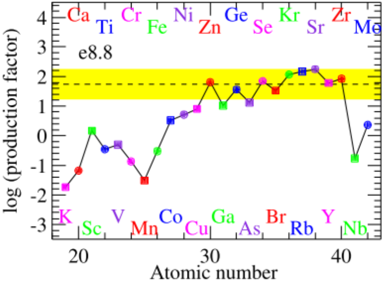
- The expansion time-scale $\tau_{\mathrm{exp}} = r/|v|$ of the material also plays a role for the final Y_e. Y_e is roughly determined when the neutrino absorption rate equals the expansion rate τ^{-1}_{exp}

The result is that one can get a complex distribution of s and Y_e in the ejecta due to asymmetries in the explosion.

Nucleosynthesis in Neutrino-Heated Ejecta

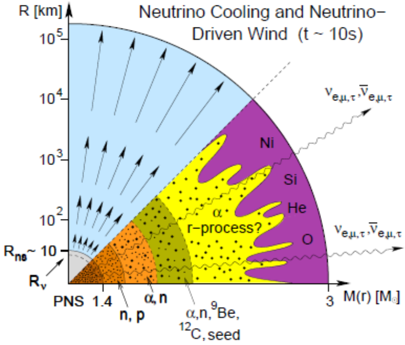
Some important trans-iron elements can be produced in neutrino-heated ejecta with $Y_e < 0.5$, resulting from a combination of freeze-out processes (neutrino-rich NSE and QSE), most notably Zn and Sr, Y, Zr, for which the contribution from supernovae may be very important.





Wanajo et al. (2018)

During the proto-neutron star cooling phase residual neutrino heating drives a wind from the surface. The mass lost in the wind is tiny $\sim 10^{-3} M_{\odot}$, but it may still play an important role for nucleosynthesis.



Other Nucleosynthesis Process in Core-Collapse Supernovae

- It has been proposed that the neutrino-driven wind could be a site for the rapid neutron capture process (r-process). Recent simulations do not bear this out, however. First, the neutrino-driven wind has recently been predicted to be proton-rich and not neutron-rich. Even if it were neutron-rich, the ratio of neutrons to seed nuclei, which is set by the entropy and expansion time-scale of the wind, may be too low.
- But one could have proton-capture reactions to make heavy nuclei beyond $A = 90$ in proton-rich outflows. These capture reactions on the proton-rich side of the valley of stability normally get stuck at β-decay waiting points. If neutrinos generate a few free neutrons, then these waiting points can be bridged by (n, p) reactions (νp-process).

R-process in Core-Collapse Supernovae

Many of the heavy isotopes beyond iron are formed by rapid neutron captures (r-process) in very neutron-rich environments. Compact object mergers are robust r-process sources.

The r-process might still occur in magnetorotatial explosions, which probably explain so-called hypernovae with explosion energies of $\sim 10^{52}\,\mathrm{erg}$. Here strong magnetic fields and rotation lead to the formation of bipolar jets. Due to large outflow velocities, the material ejected from close to the neutron star stays very neutron-rich, so that r-process nucleosynthesis can occur.

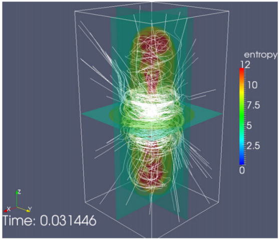


Figure 1. 3D entropy contours spanning the coordinates planes with magnetic field lines (white lines) of the MHD-CCSN simulation ~31 ms after bounce. The 3D domain size is 700 × 700 × 1400 km.

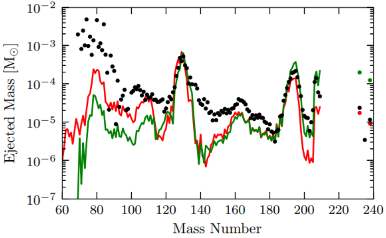


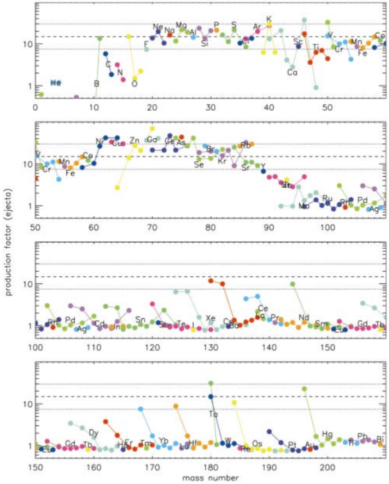
Figure 4. Integrated mass fractions for nucleosynthesis calculations with (red) and without (green) neutrino heating. Black dots represent solar *r*-process element abundances (Sneden et al. 2008) scaled to fit the red line at $A = 130$.

(Credit: Winteler et al. 2012)

Summary of Core-Collapse Supernova Nucleosynthesis

Core-collapse supernovae are

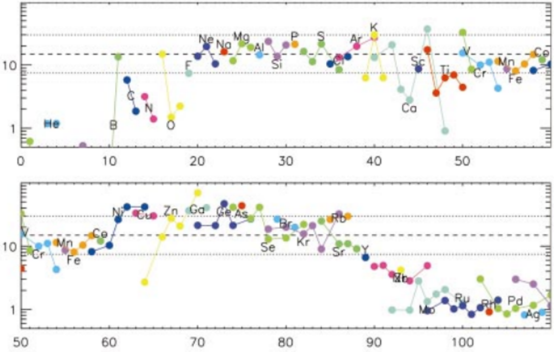
- dominant producers of elements between O and K (made mostly already by hydrostatic burning)
- contribute significantly to elements from Ca up to and including the iron group (made during the explosion)
- eject weak s-process elements made during hydrostatic burning (up to rubidium)
- may make a few other important trans-iron elements/isotopes



Woosley, Heger & Weaver 2002, Review of Modern Physics 74, 1015

Summary – Advanced Burning Stages in Massive Stars (bigger Figure)

Production factors (incl. hydrostatic burning & explosive burning in the supernova):



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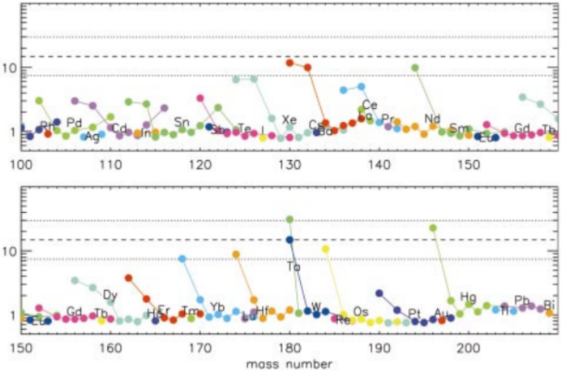
Production factors (bigger Figure)):



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