Université de Nice Sophia Antipolis - CNRS

Institut de Physique de Nice

# Resonant dipole-dipole interactions:
# Dicke subradiance and Anderson localisation

**Robin KAISER**

**Nice, France**

**Trapped Atoms, Molecules and Ions Meeting**
**Bangalore (online) 10 May 2021 to 22 May 2021**

# INPHYNI Science Team :

**M. Hugbart** **W. Guerin** **G. Labeyrie**

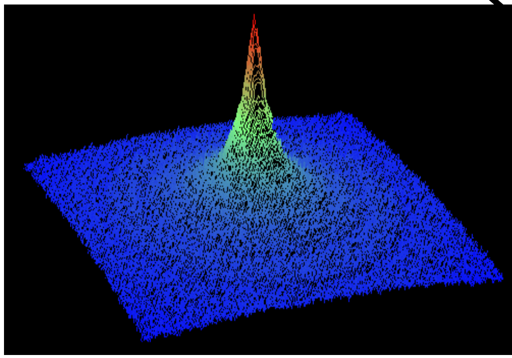
A. Kastberg, S. Asselie , P. Azam, A. Cipris,
M. Gaudesius, P. Lassegues, H. Letellier, A. Mitchell,
M. Morisse, A. Siciak, R. Giampaoli

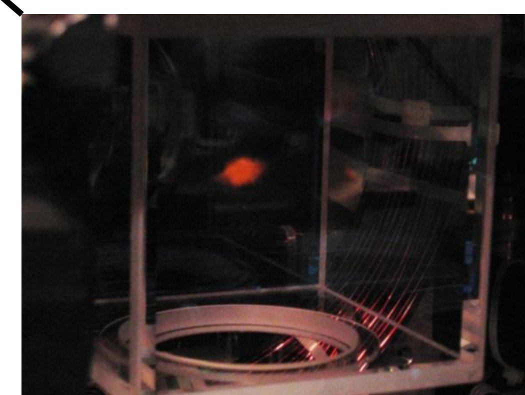
# Collaborations :

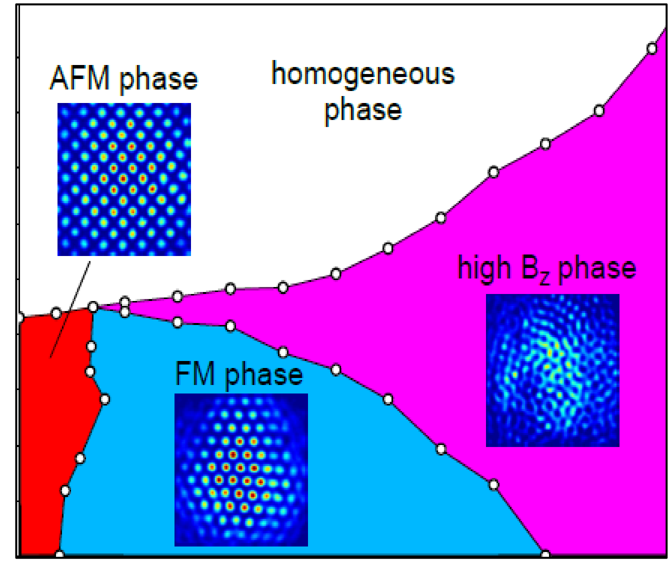
R. Bachelard, L. Celardo, I. Lesanovsky, I. Sokolov
P. Courteille, N. Piovella, J. Barré, M. Kastner,
E. Akkermans, J. Ye, A.M. Rey, J. Schachenmayer,
T. Ackemann, G.-L. Oppo, G. Robb, S. Barland, R. Nyman
N. Cherroret, D. Delande, J. Walraven, Q. Glorieux, T. Bienaimé
F. Vakili, J.P. Rivet, G. Alvarez, L. Orozco, …

# Dicke vs Anderson

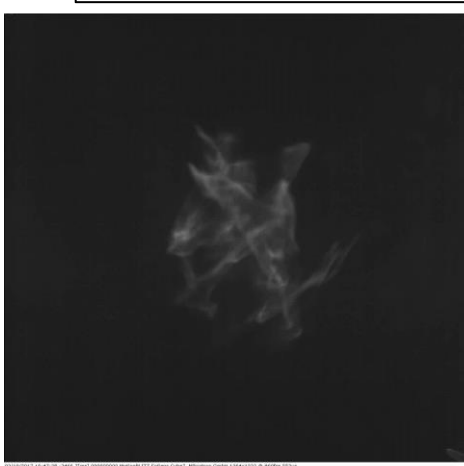
# plasma physics / pattern formation

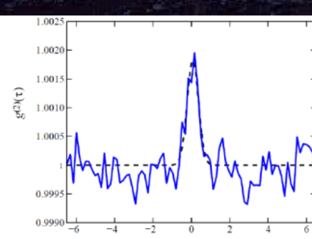


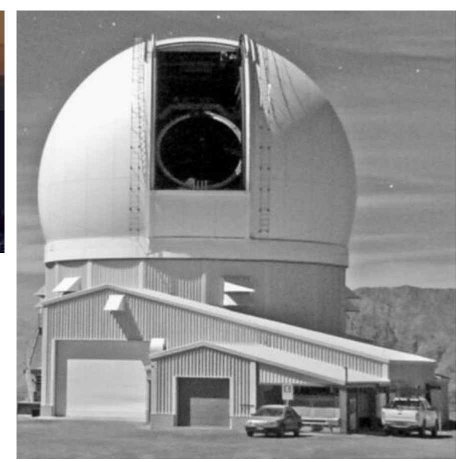


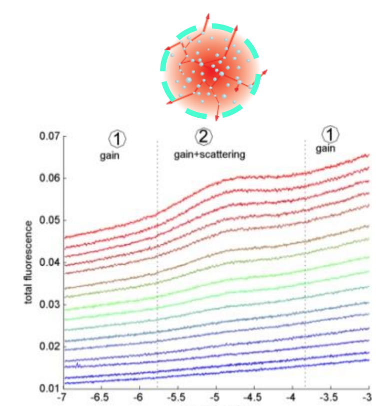



# plasma physics / astrophysics :
# self-oscillations, HBT, random lasing, Lévy flight of photons





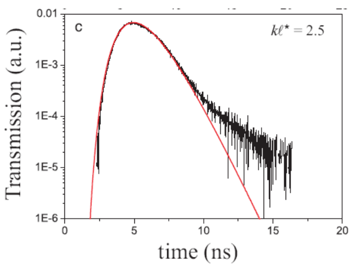






# + quantum fluids of light (photon thermalization)

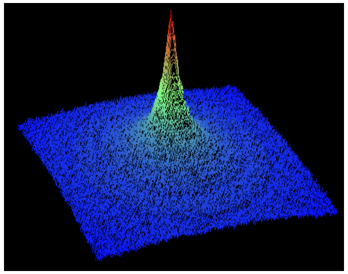
# Outline of the talk

Context




**Anderson Localisation of light**

From Anderson to Dicke



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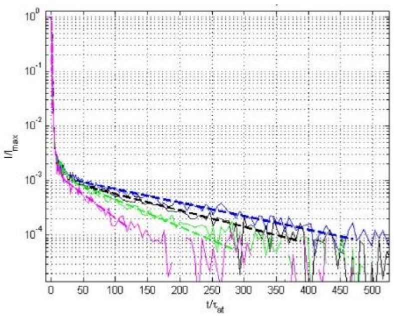
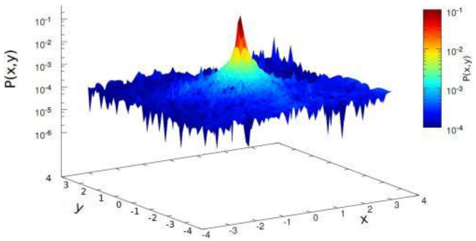



**Weak Localisation** **Subradiance**

And back

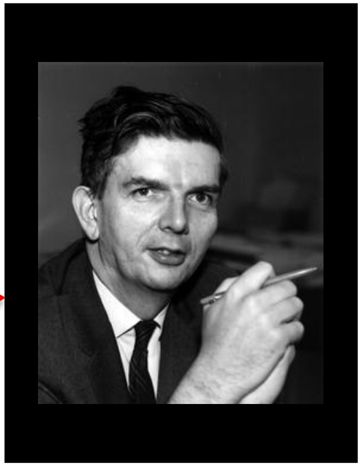


# Multiple Scattering of Light in Atomic samples : Disorder vs cooperative effects

**Anderson Localization**

**Multiple Scattering**

**Dicke States**



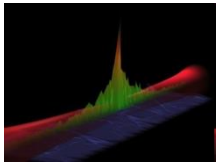
**"Local"**

**"Global"**

**Interferences**

- **Solid State Physics :** electrons

- **Matter Waves :**
  BEC in Disordered Potential,
  Kicked Rotator



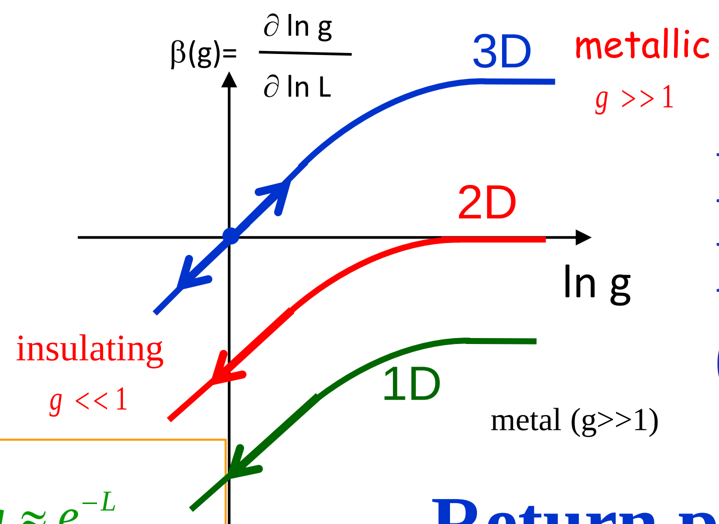
- **Accoustics :** Aluminium Beads

- **Light Scattering :**
  Semiconductor powder, White Paint,
  **Atoms**

# Anderson Localization of non interacting waves in 1,2 and 3D

- **Scaling theory of localization :** Abrahams et al., PRL **42**, 673 (1979)

g : dimensionless conductance




$g \approx e^{-L}$

$$\frac{\partial \ln g}{\partial \ln L} \approx \ln g$$

**In 3D : threshold for disorder**
**In 1&2D : all states are localized**
**(in infinite system, ε disorder)**

**Return probability needed for interference**

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$

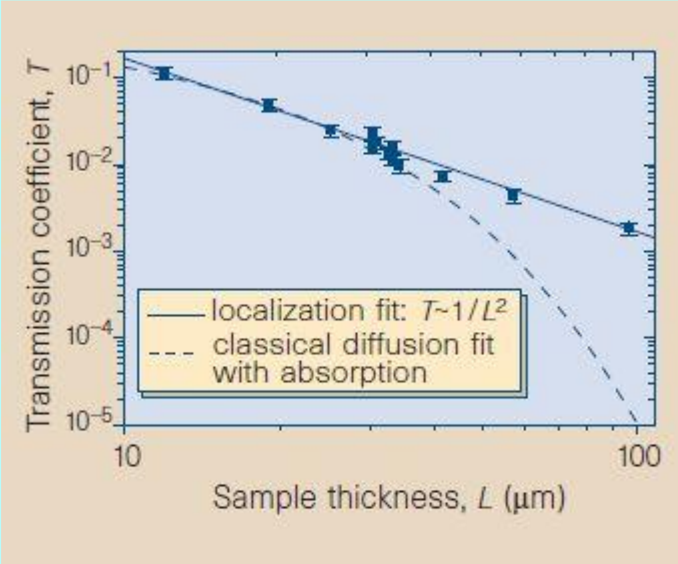
- **No microscopic theory**
  **self consistent theory of localization,**
  **numerical simulations of toy systems**

$W_{cr} \sim 16.5$

# Anderson Localization of Light in 3D :

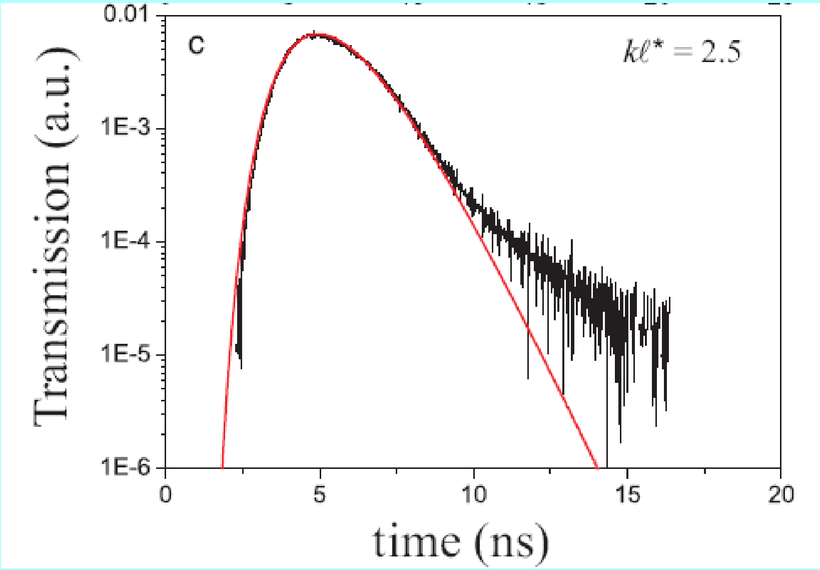
phase transition ⇒ strong scattering required

## Semi-conductor powder




D.Wiersma et al., Nature 1997

## White Paint



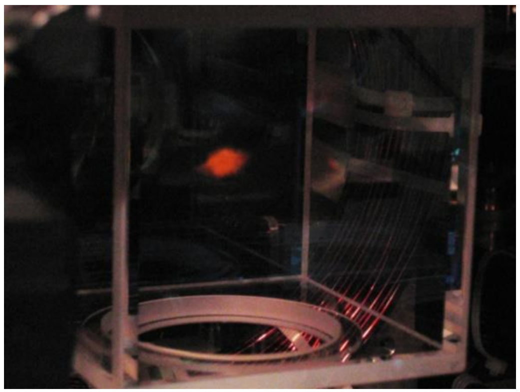

C.Aegerter et al., EPL 2006

F. Scheffold et al., Nature 398, 206(1999)
T. v. der Beek et al., PRB 85 115401 (2012)

F. Scheffold et al., Nat. Photon. 7, 934 (2013)
T Sperling et al., New J. Phys. 18, 013039 (2016)

# => Not observed so far

# Previous work



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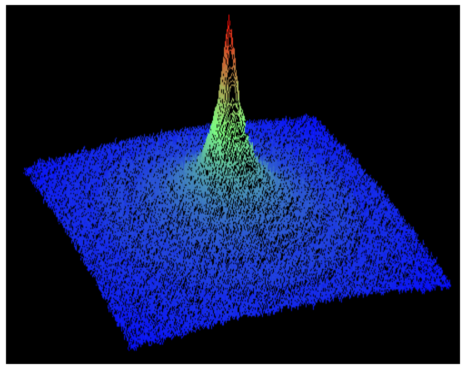

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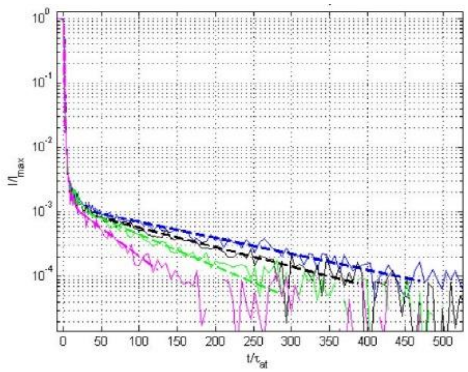
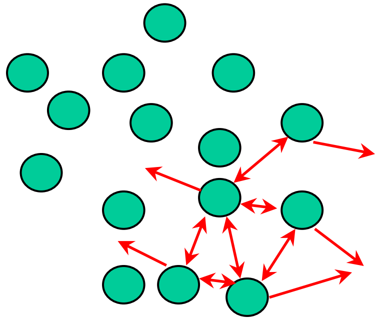


# Photons ...



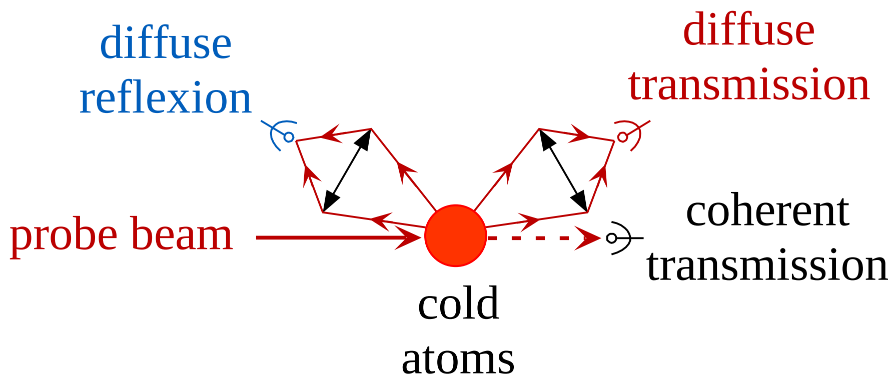
Random walk :

**Diffusion**

coefficient $D_0 \approx \ell^2/\tau$

$\ell = 1/n\,\sigma$

# Scattering Experiments with Cold Atomic



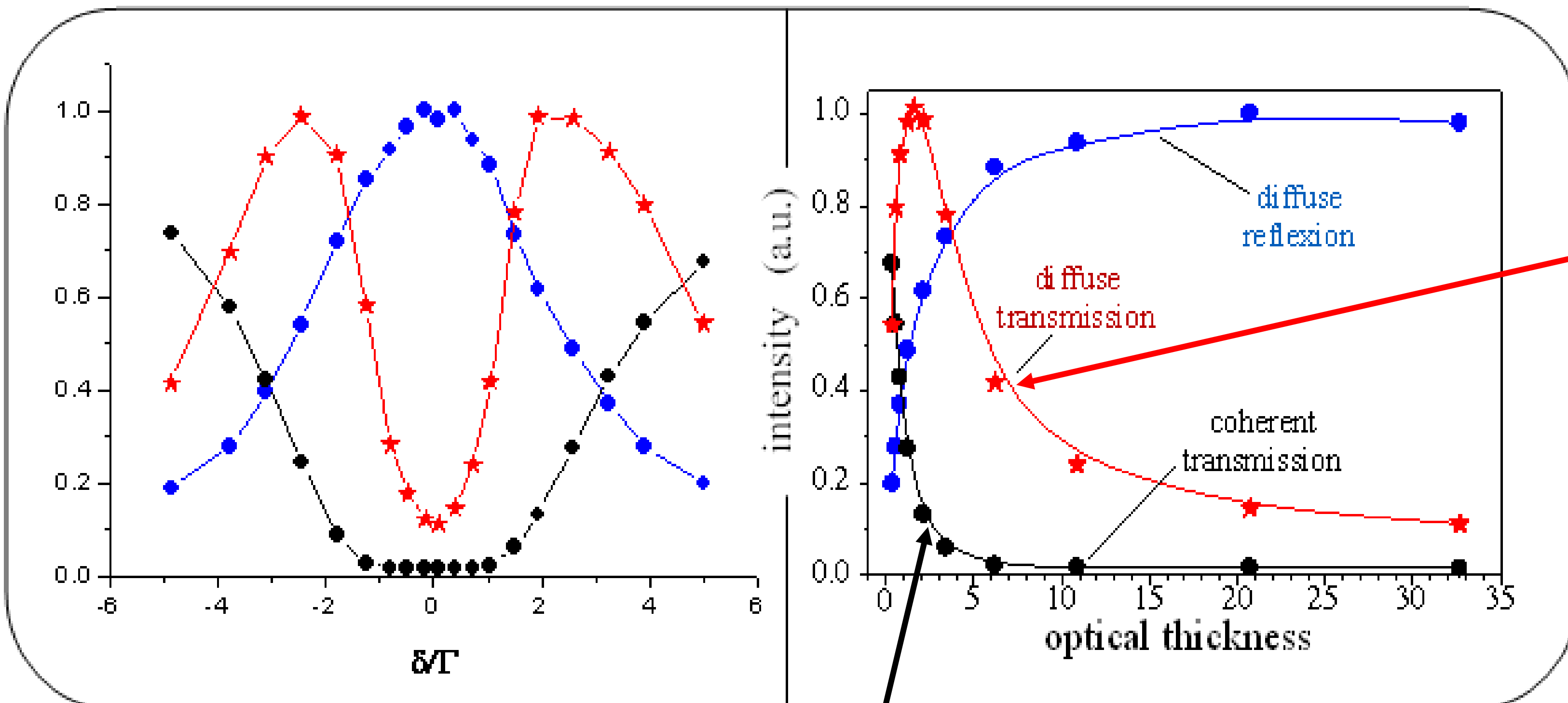

MOT parameters :

MOT diameter ≈ few mm

N ≈ few $10^{11}$

b ≈ 200

velocities ≈ 0.1m/s

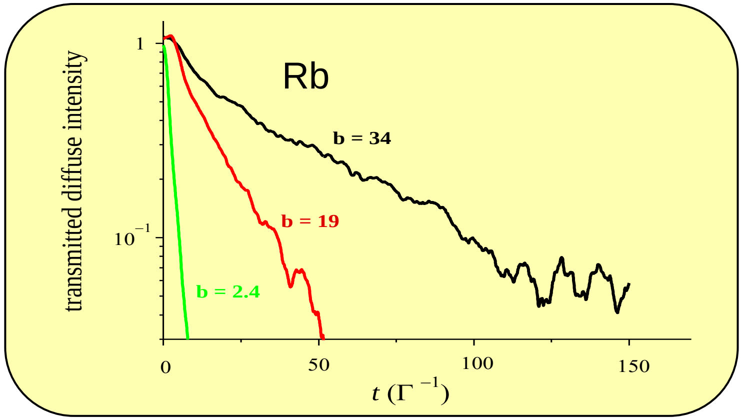


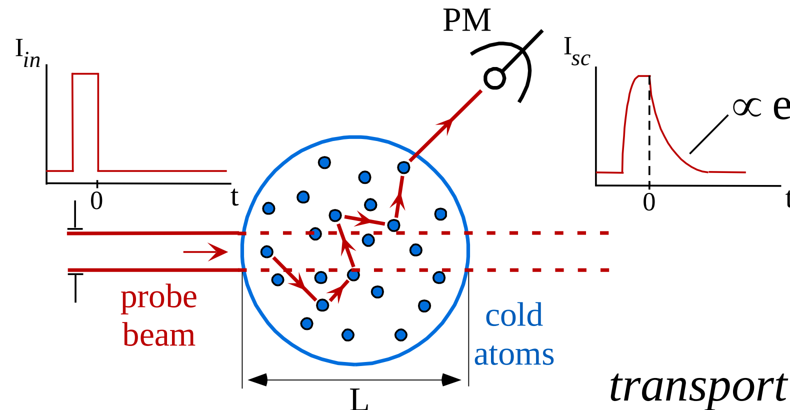

**Ohm's law for photons**

$\mathbf{T_{diff} \propto 1/L}$

**Beer-Lambert law :** $\mathbf{T_{coh} \propto e^{-L/\ell}}$

# Time Resolved Experiments




$$D = \frac{1}{3}\frac{l_{tr}^2}{\tau_{tr}}$$

*transport mean-free path*

*transport time*

$$= \frac{1}{3} l_{tr} v_{tr}$$

*transport velocity*

$$\frac{v_{tr}}{c_0} = \frac{\ell_{tr}}{c_0 \tau_{tr}} \approx 3 \cdot 10^{-5}$$

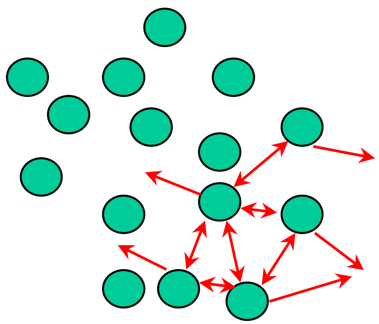
$$\tau_0 \approx \frac{L^2}{\pi^2 D} \Rightarrow \boxed{D \approx 0.66 m^2/s}$$

**NO** interference effect !

$\neq$ Localization

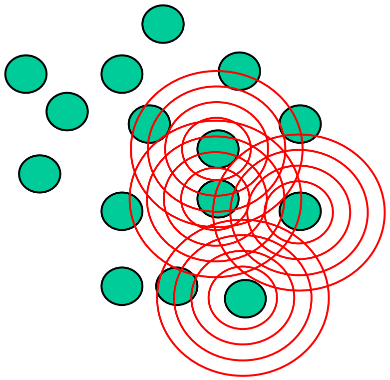
Phys. Rev. Lett. **91**, 223904 (2003)

# Photons ... ... are waves

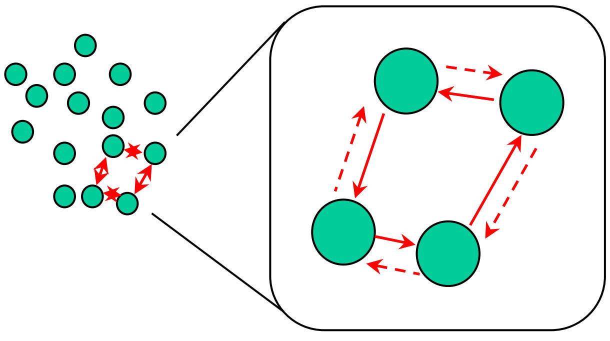


Random walk :
**Diffusion**
coefficient $D_0 \approx \ell^2/\tau$

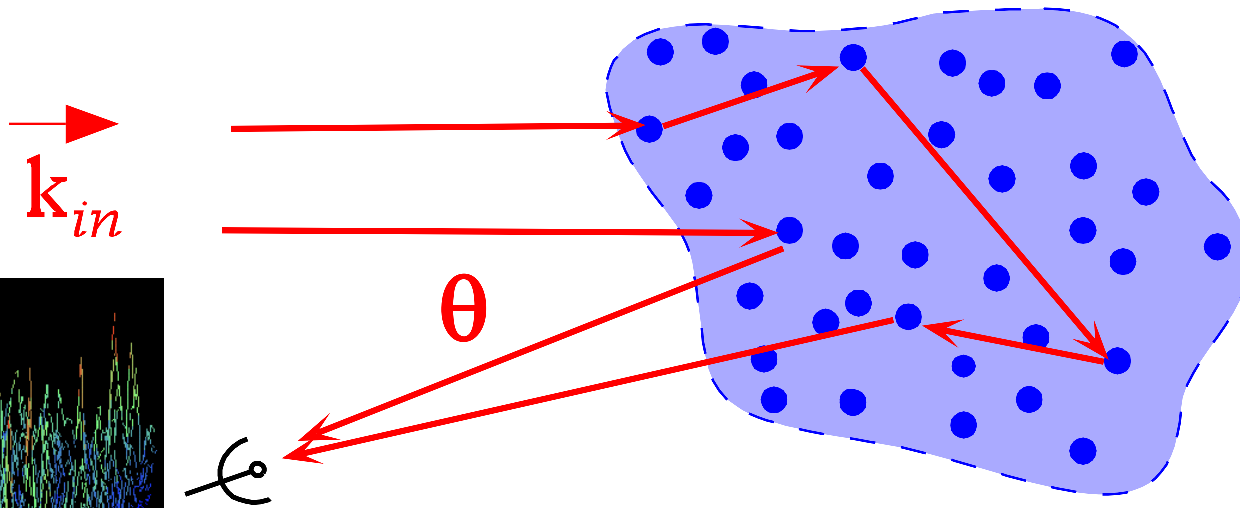
$\ell = 1/n\,\sigma$

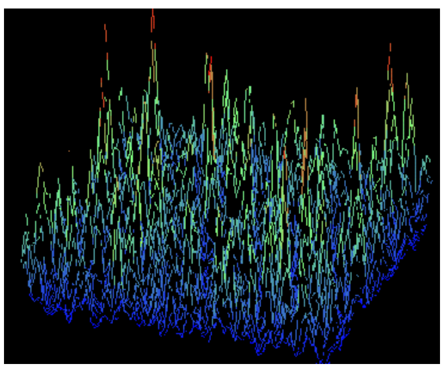


**Interference correction** to
Diffusion coefficient
$D \approx D_0\,[1- 1/(k\ell)^2]$
**Strong Localization** (D=0) :
Ioffe-Regel criterium : $k\ell \approx 1$
(near field scattering $\ell \approx$ l)

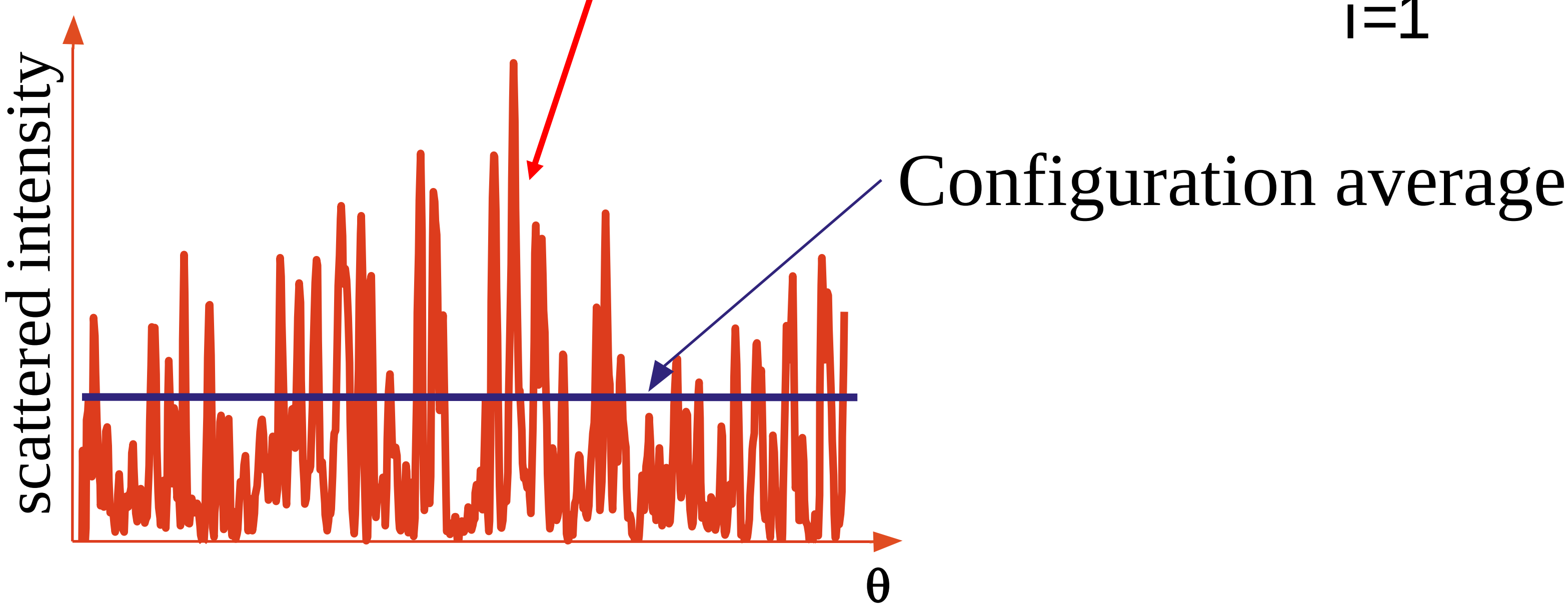


# Wave effects : Interferences and speckle



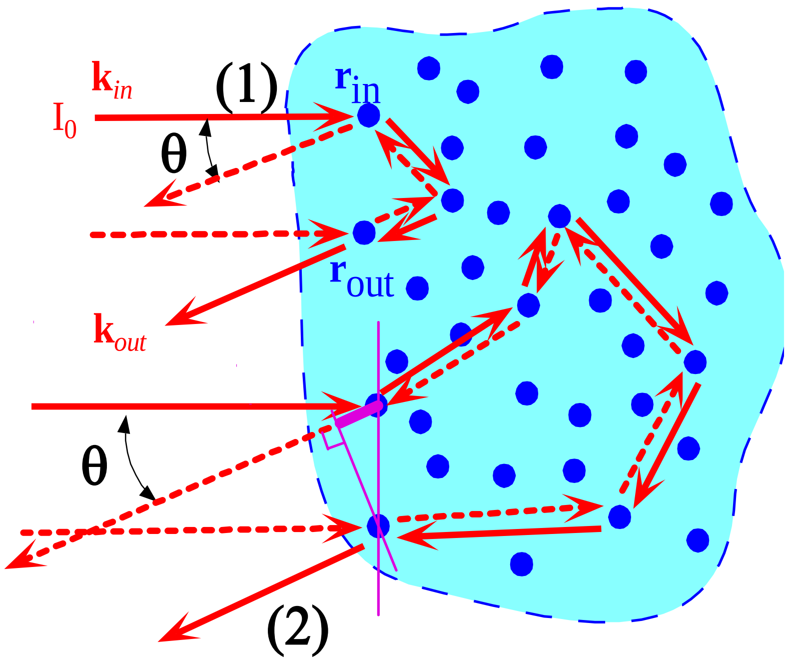



fixed scatterers : speckle pattern $\vec{E} = \sum_{i=1}^{N} \vec{E}_i$



# Configuration Averaged Intensity

- **uncorrelated** paths add incoherently
- **correlated** (i.e. reciprocal) paths
  add coherently

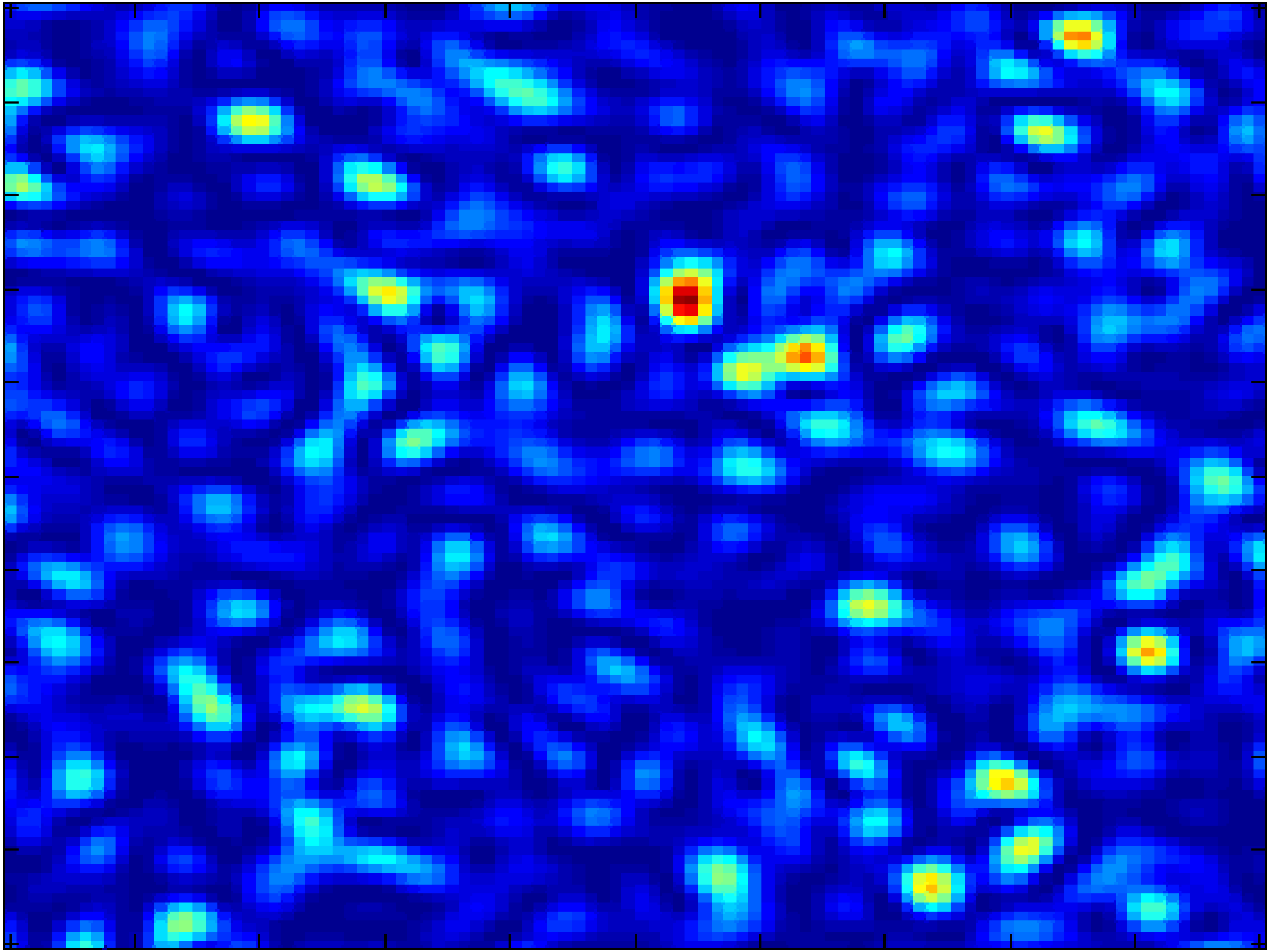



$\Delta\varphi=(k_{in}+k_{out}).(r_{in}-r_{out})$ $\theta=0 \Rightarrow \Delta\varphi = 0$ for any path

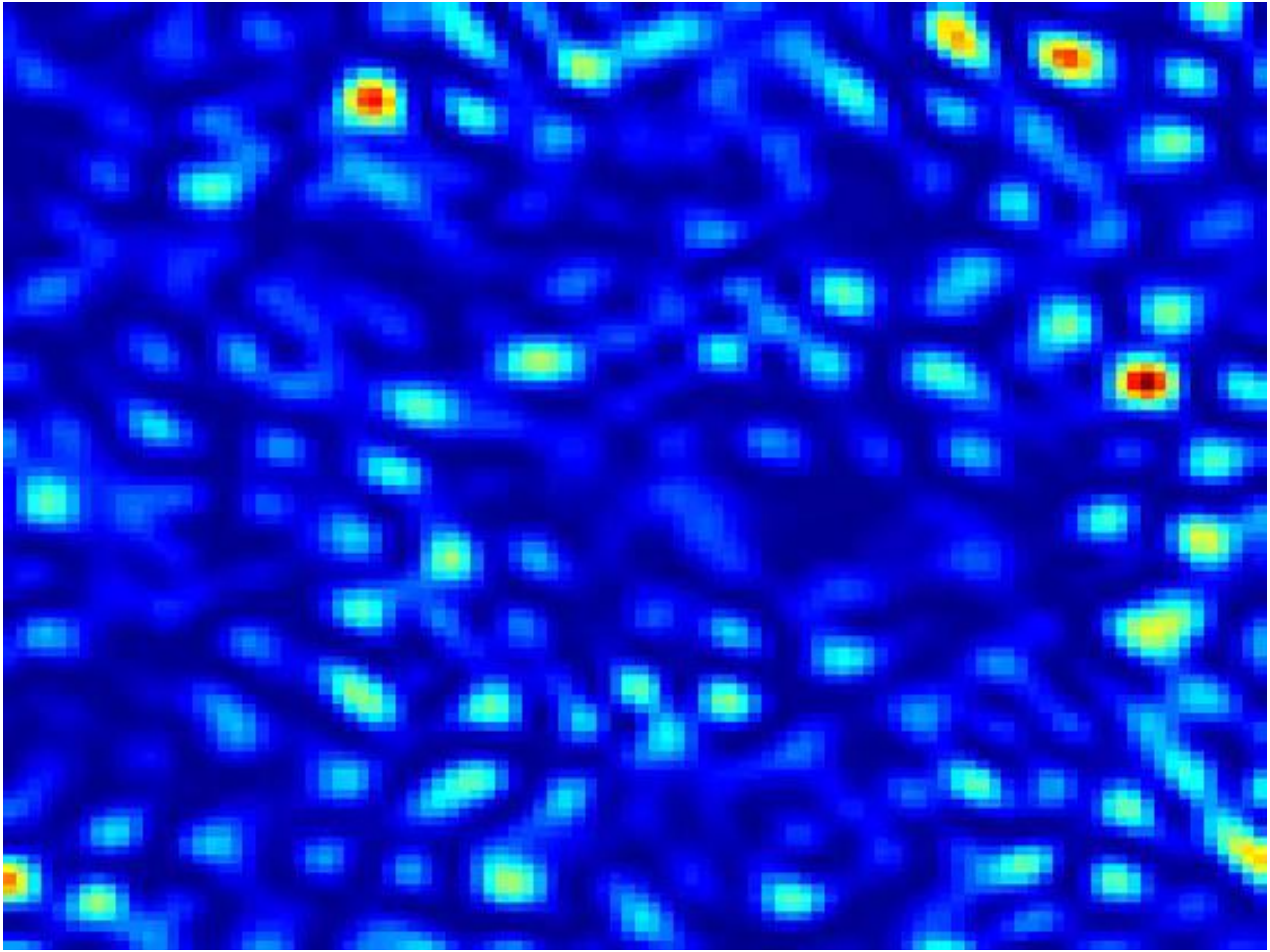
Coherent Backscattering $\dfrac{\langle I(0)\rangle}{\langle I(\theta)\rangle} = 2$

**multiple self-aligned Sagnac interferometer**

Fluctuating Speckle Pattern

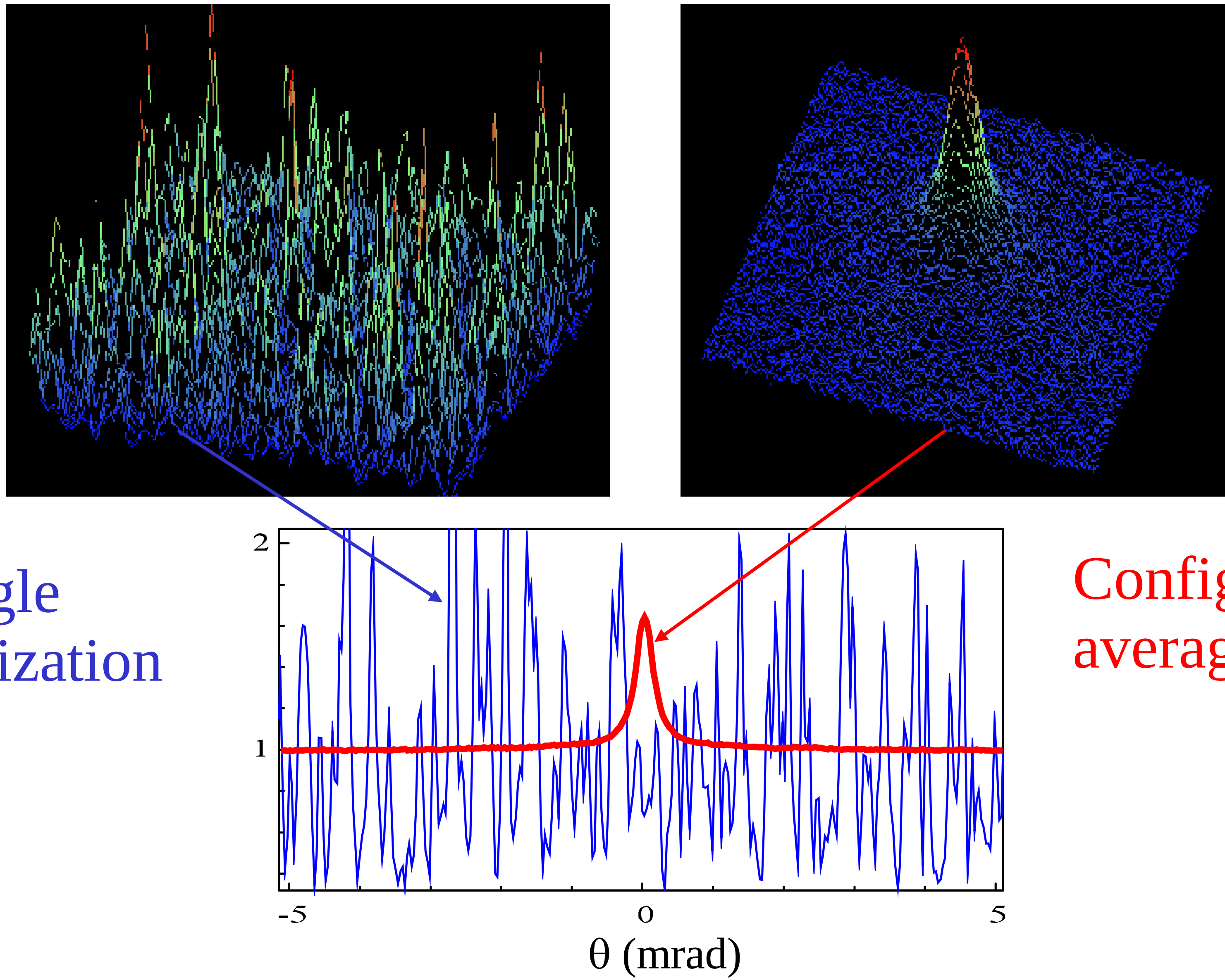


Integrated signal (configuration average)



cone

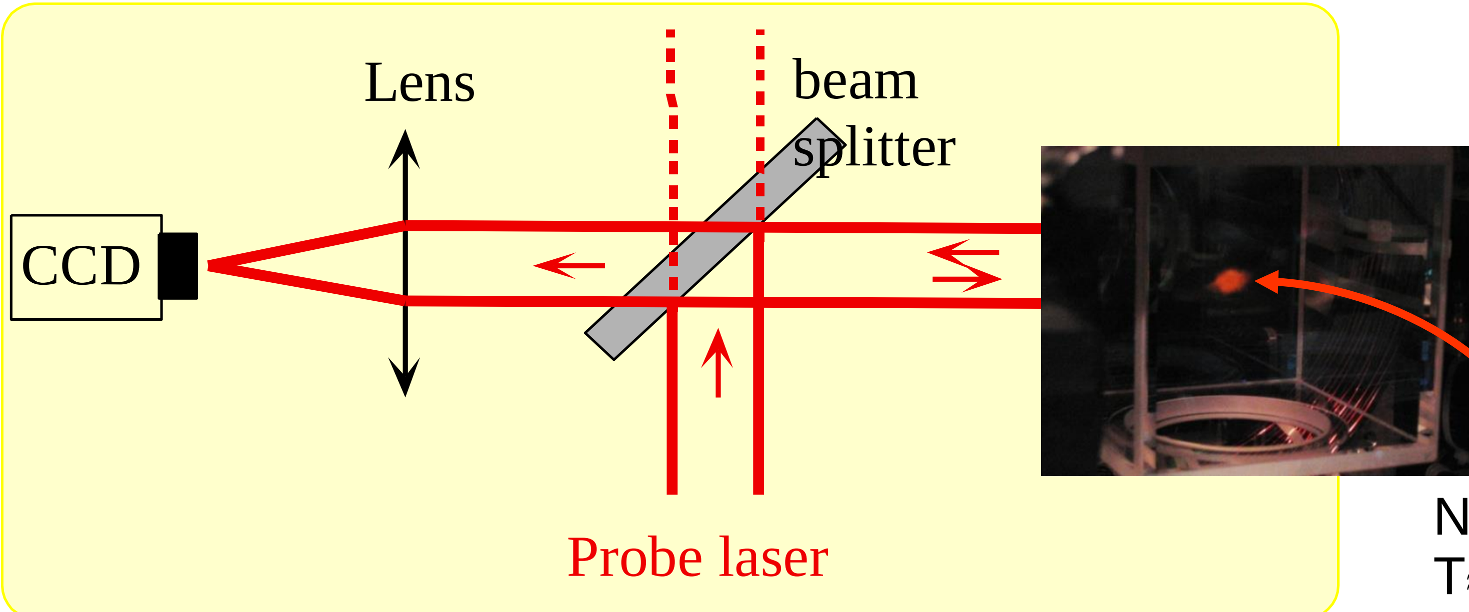
# Configuration Average



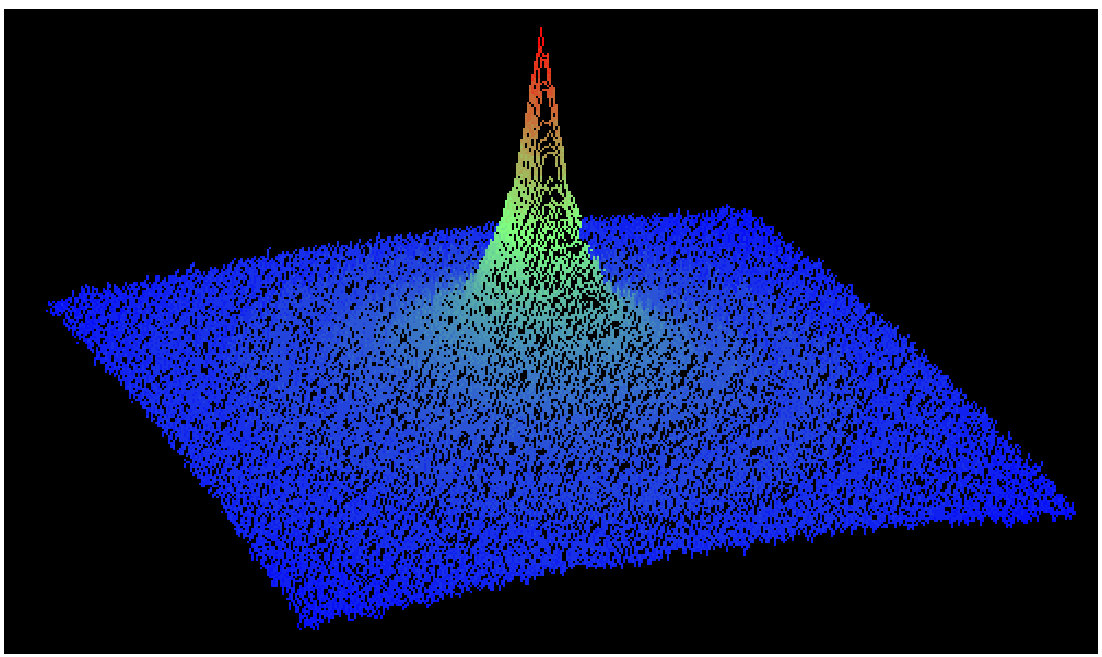

Single realization

Configuration average

# Weak Localisation = precursor of strong Localisation?




$N \approx 10^{10}$
$T \approx 100 \mu K$
$k\ell \approx 1000$



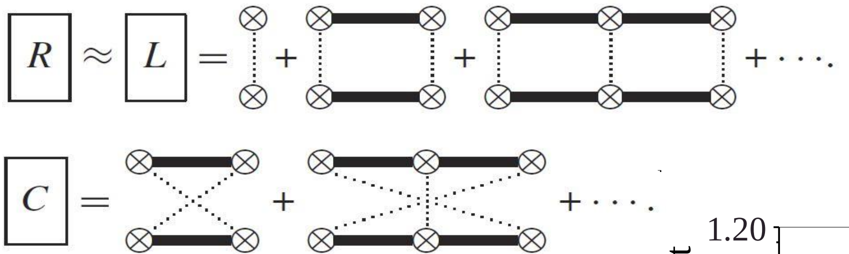
Phys. Rev. Lett., **83**, 5266 (1999)

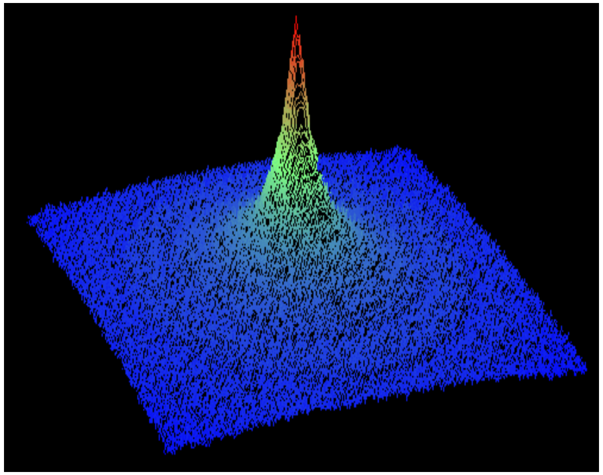
Coherence after resonant scattering with atoms !

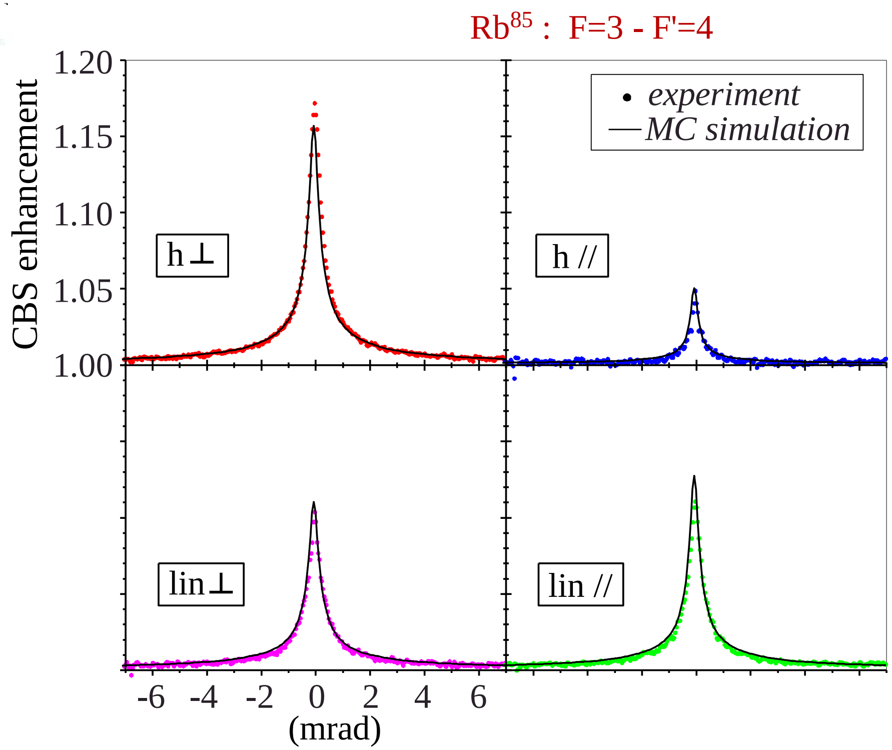
also : M. Havey et al.

# Theory :

- no “exact” solution
- diagrammatic approach






Excellent agreement
(no free parameter)

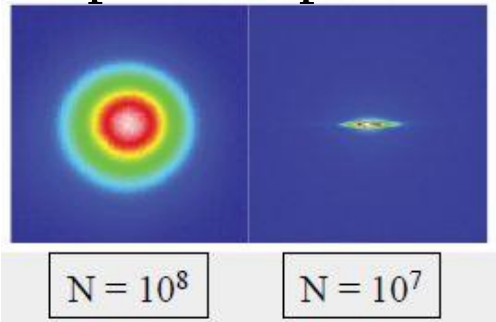
T. Jonckheere et al., Phys. Rev. Lett., **85**, 4269 (2000)

# Problems to face

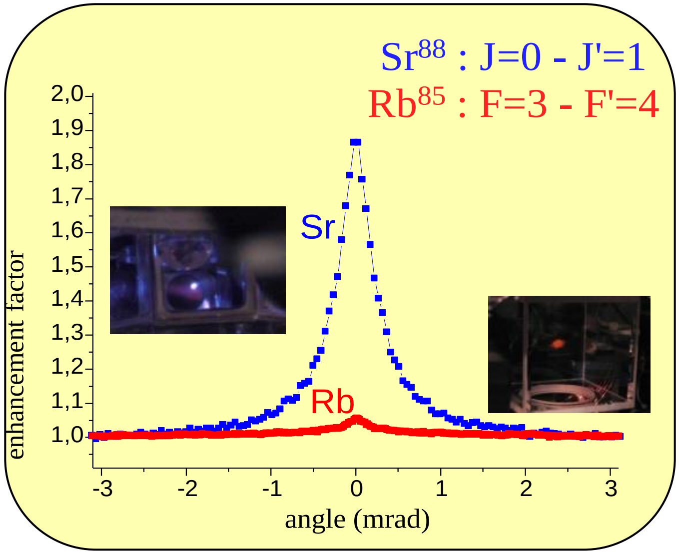
- High spatial densities → interactions :
  Ioffe-Regel criterion : $k\ell \approx 1$

M. Havey, A. Browaeys)

Dipole Trap



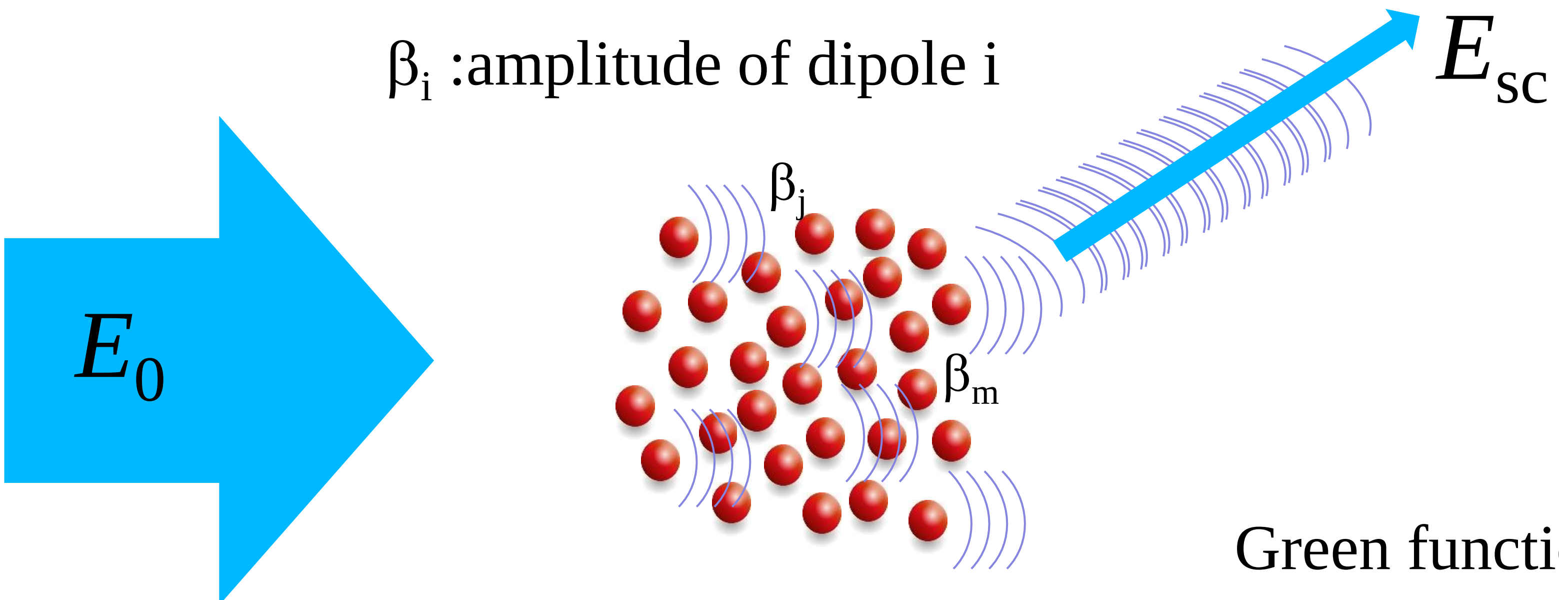

- Internal structure of Rb ?



# Cooperative Scattering and Dicke states

- **Ab initio model for light scattering**
- **Experiments**
- **Effective Hamiltonian approach**

# Building up a refractive index « ab inito » (from individual atoms to macroscopic index)




Green function
dipole-dipole interactions

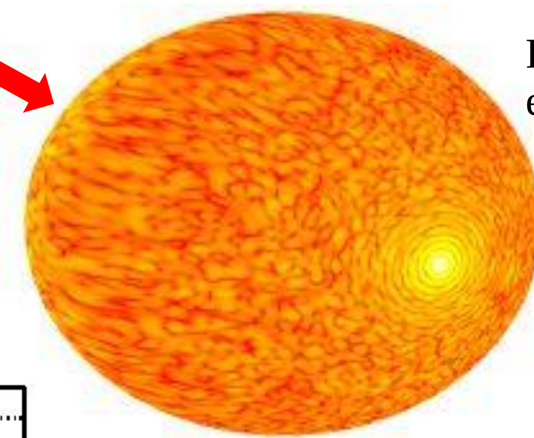
$$\dot{\beta}_j(t) = \boxed{-\frac{i}{2}\Omega e^{i\mathbf{k}_0\cdot\mathbf{r}_j} + \left(i\Delta - \frac{\Gamma}{2}\right)\beta_j(t)} - \boxed{\frac{\Gamma}{2}\sum_{m\neq j}^{N}\beta_m \frac{\exp(ik_0\mid \mathbf{r}_j - \mathbf{r}_m \mid)}{ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid}}$$

$$E_{\rm sc}(\mathbf{r}) = -\frac{\hbar\Gamma}{2d}\sum_{j=1}^{N}\beta_j \frac{e^{ik_0|\mathbf{r}-\mathbf{r}_j|}}{k_0|\mathbf{r}-\mathbf{r}_j|}$$

**Long Range**

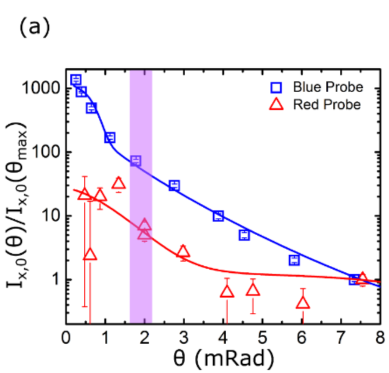
# Spherical gaussian cloud: steady state emission diagram

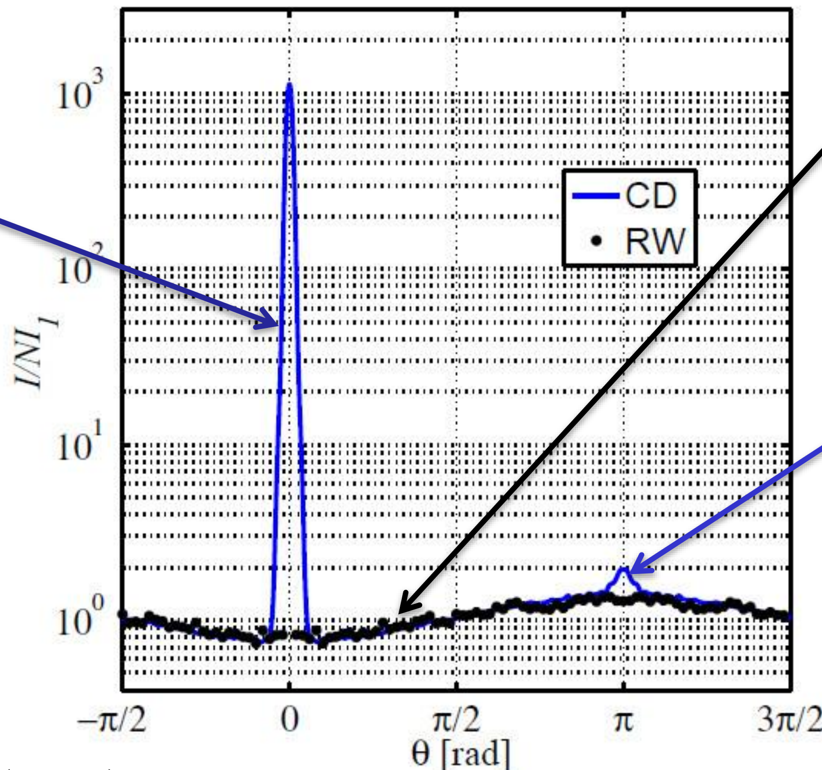
Cloud of atoms



Far field emission diagram

refractive index (mean field)




Incoherent model (particles trajectories, scattering in ‘empty modes’)

Mesoscopic physics: Weak localization (waves beyond mean field)

S. Bromley et al., Nat. Comm. 7, 11039 (2016)

# Time resolved light emission : Dicke 1954

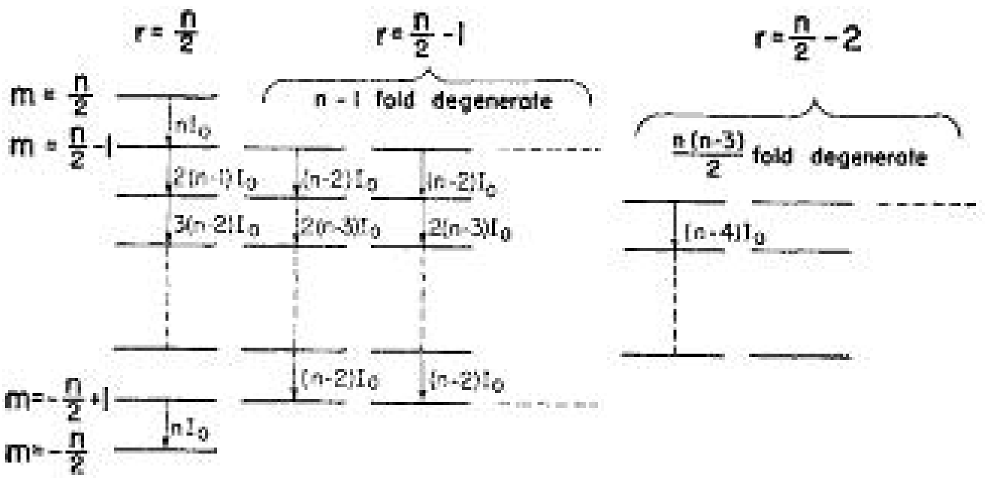
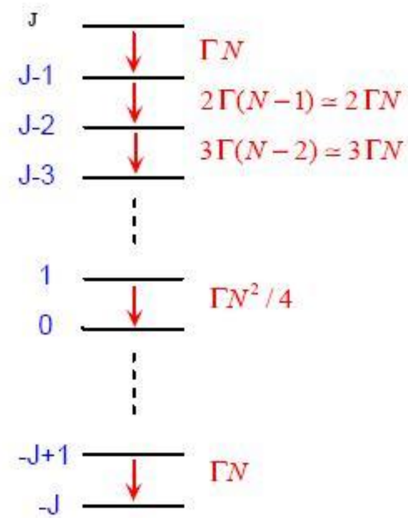
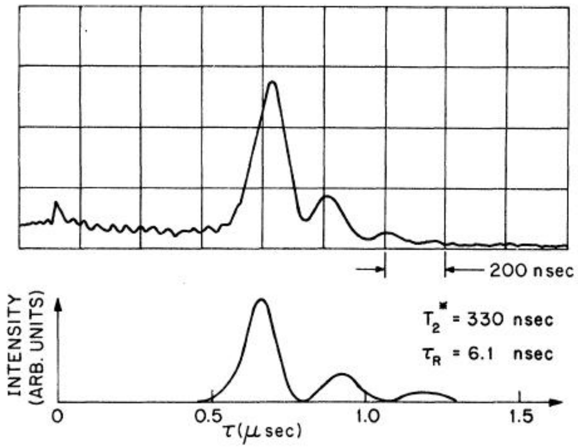



Fig. 1. Energy level diagram of an $n$-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.




**R. Dicke 1954**




**First experimental observation of 'superradiance'**

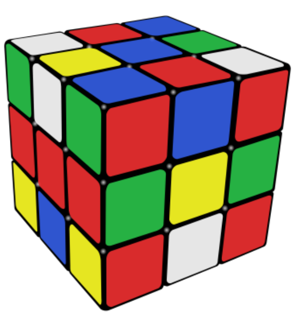
**Feld et al. 1973**

# Single photon excitation / low intensity limit

**Small Volume (L<<λ)**


| **Fast** and **slow** collective time scales emerging from **long range** coupling |
| --- |

**Extended Volume (L>>λ)** $\Gamma_{max} \sim C\,\Gamma_0 \sim b_0\,\Gamma_0$



$\boxed{b_0 = N_{at}/N_{modes}}$

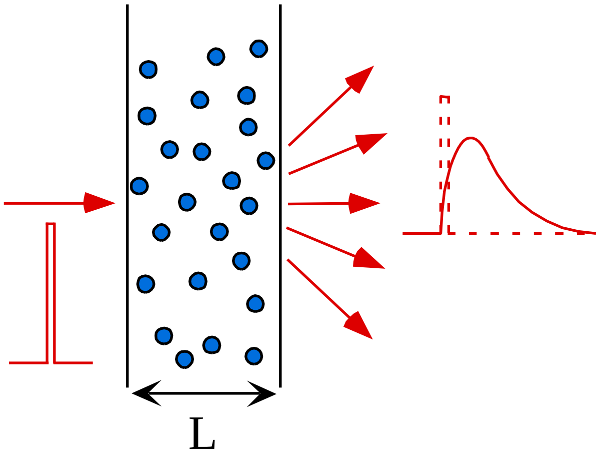
**Cooperativity without cavity**

$N_{modes} \sim (L/\lambda)^2$

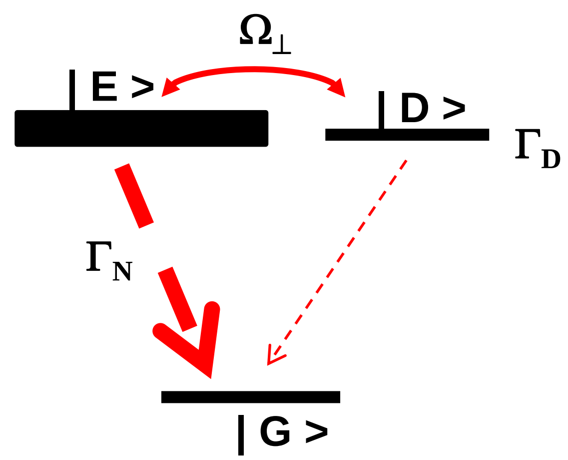
$b_0 \sim N_{at}/(kL)^2$

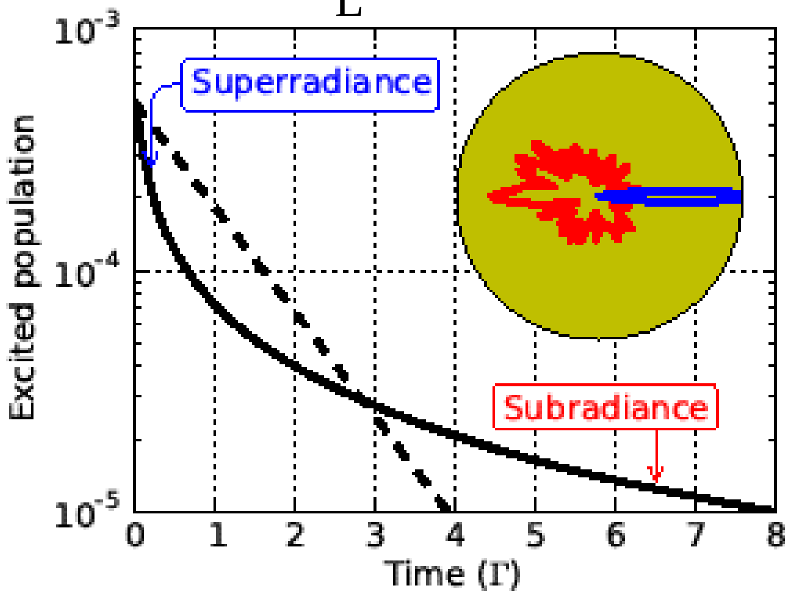
**Long Range :**
**Global effect / size dependant**

# Time dependent experiments : coherent scattering

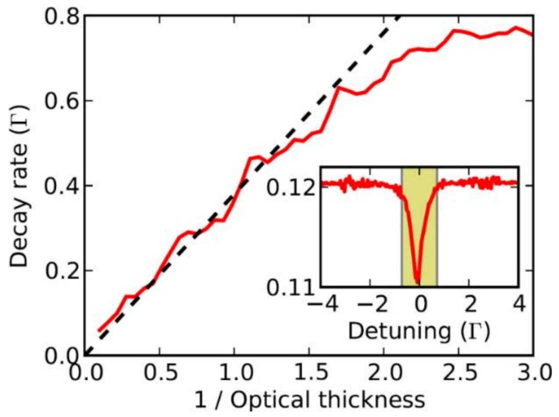



**Superradiance = bright state**
**Subradiance = metastable 'dark' states**



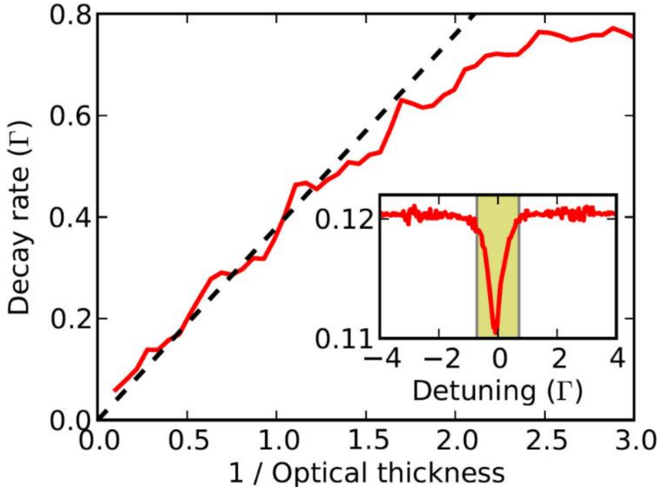

Numerical Simulation of N driven coupled dipoles




T. Bienaimé, N. Piovella, R.K., PRL **108** 123602 (2012)

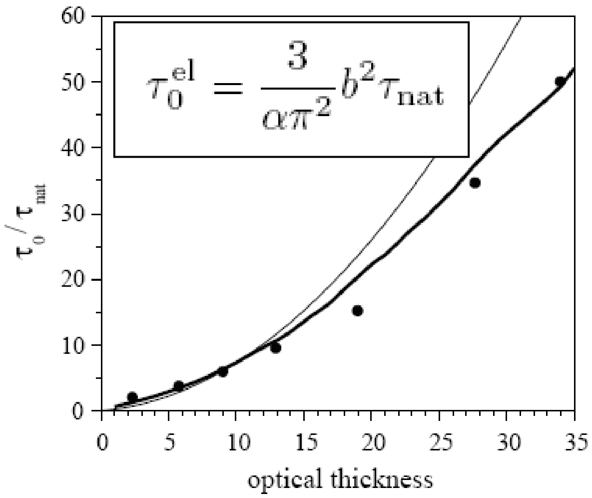
# Subradiance vs incoherent scattering

$t_{sub} \propto b_0$




- Does not require large spatial densities
- Requires large optical densities

$t_{Rad.Trap.} \propto b(\delta)^2$



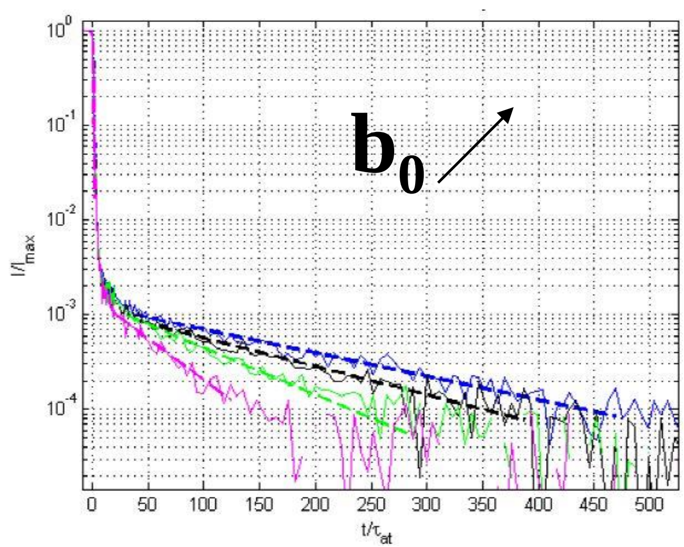

- Random walk of photons (without interference)
- Diffusion equation

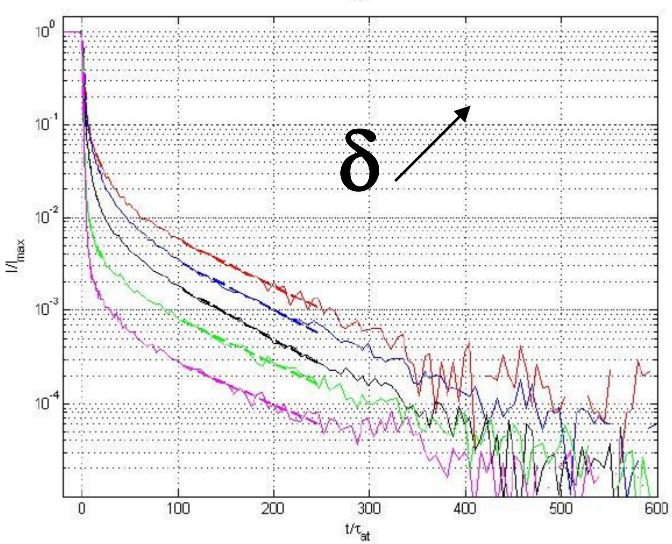
$t_{Anderson} \propto \exp\{b(\delta)\}$ ?

- Density Threshold ?

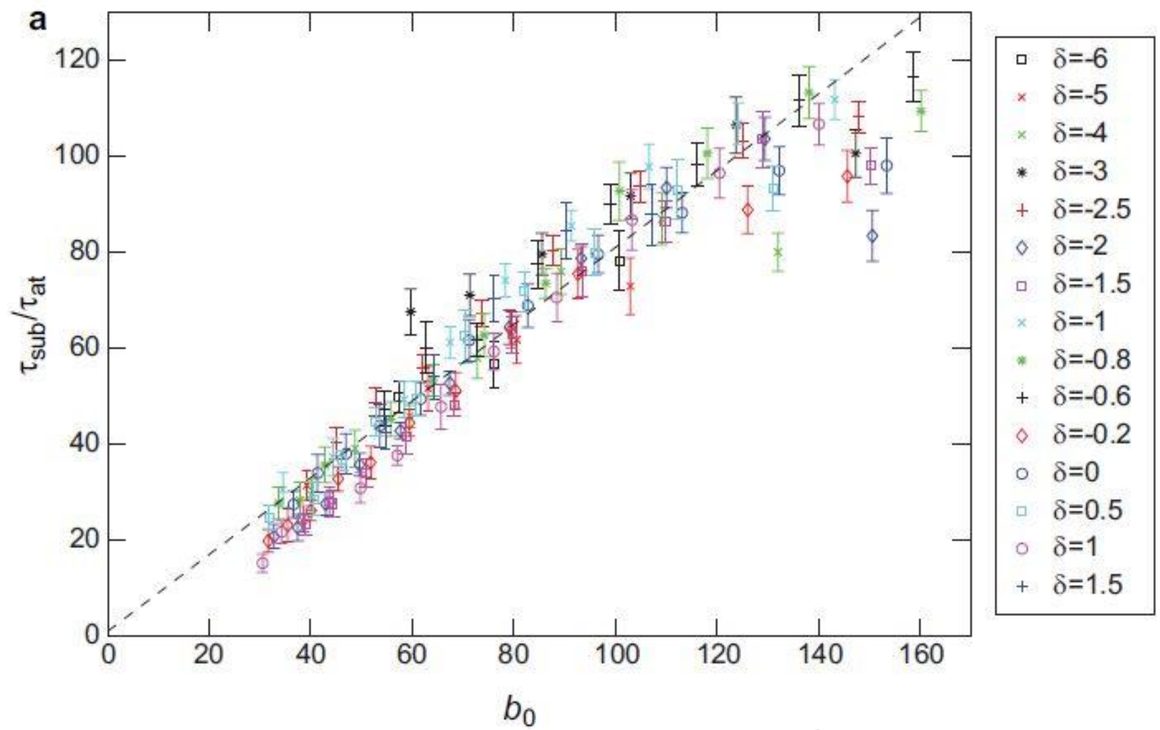
# Experimental results

## Long decay at b(δ)<1 ☺




## τ(b0)




## Increases as $b_0$ ☺


Selected for a Viewpoint in *Physics*

PRL 116, 083601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

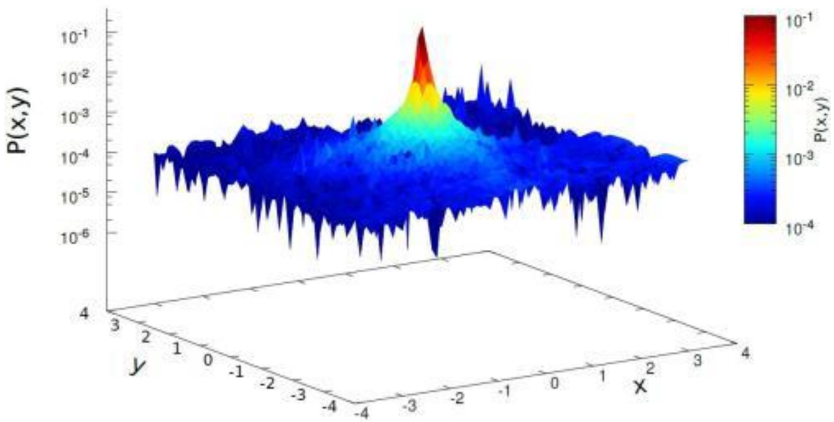

**Subradiance in a Large Cloud of Cold Atoms**

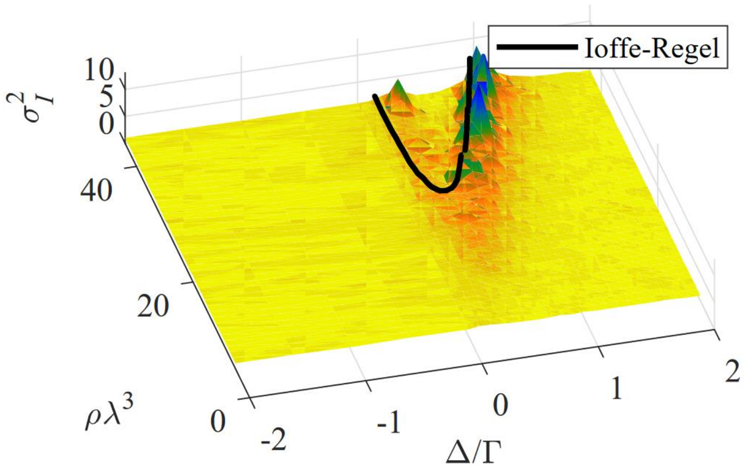

William Guerin,[1,*] Michelle O. Araújo,[1,2] and Robin Kaiser[1]

# Open questions on super-  and subradiance

- Relation to radiation trapping and pair physics with spectral broadening after switch-off
- How to exploit it as a memory
- Entanglement and possible signatures
- Photon blocade and non classical photons sources
- Impact of degenerate Zeeman sublevels
- Ordered vs disordered systems
- High vs low density limits, long range vs short range interactions
- 1, 2 vs 3 dimensions

# Towards
# Anderson Localisation of light by cold atoms



# Theory : Effective Hamiltonian in the single excitation manyfold

$$\boldsymbol{H_{eff} = (\hbar\omega_0 - \mathrm{i}\frac{\hbar\Gamma_0}{2})\sum_i S_i^z + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j} V_{ij} S_i^{\,+} S_j^{\,-}}$$

Diagonal :
On site energy

Off diagonal :
transport

$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

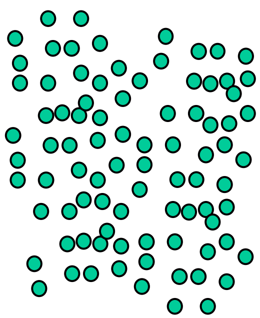
$$\beta_{ij} = \frac{3}{2}\left[-p\frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q\left(\frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

$$\gamma_{ij} = \frac{3}{2}\left[p\frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q\left(\frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

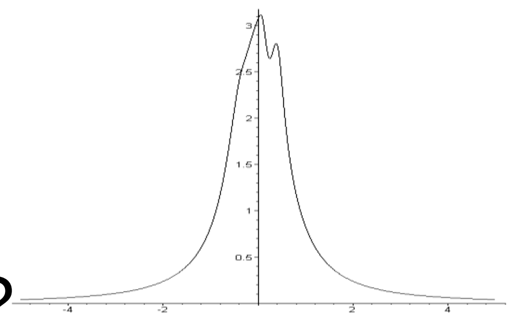
- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, …)

# Eigenvalues

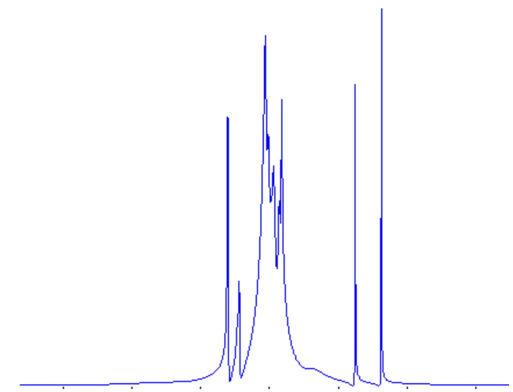
Cloud of Atoms = Large Molecule (with $10^{10}$ atoms)



'dilute' molecule




'dense' molecule



molecular spectrum ?

SHENG LI AND ERIC J. HELLER PHYSICAL REVIEW A 67, 032712 (2003)

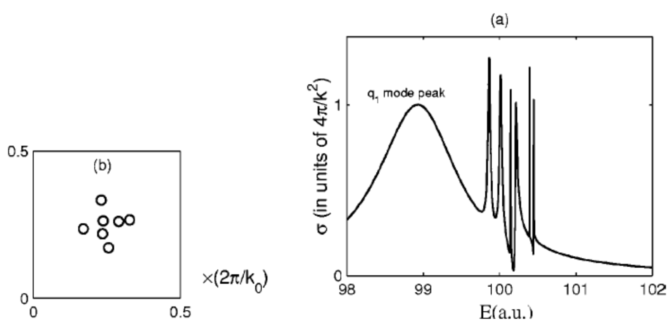
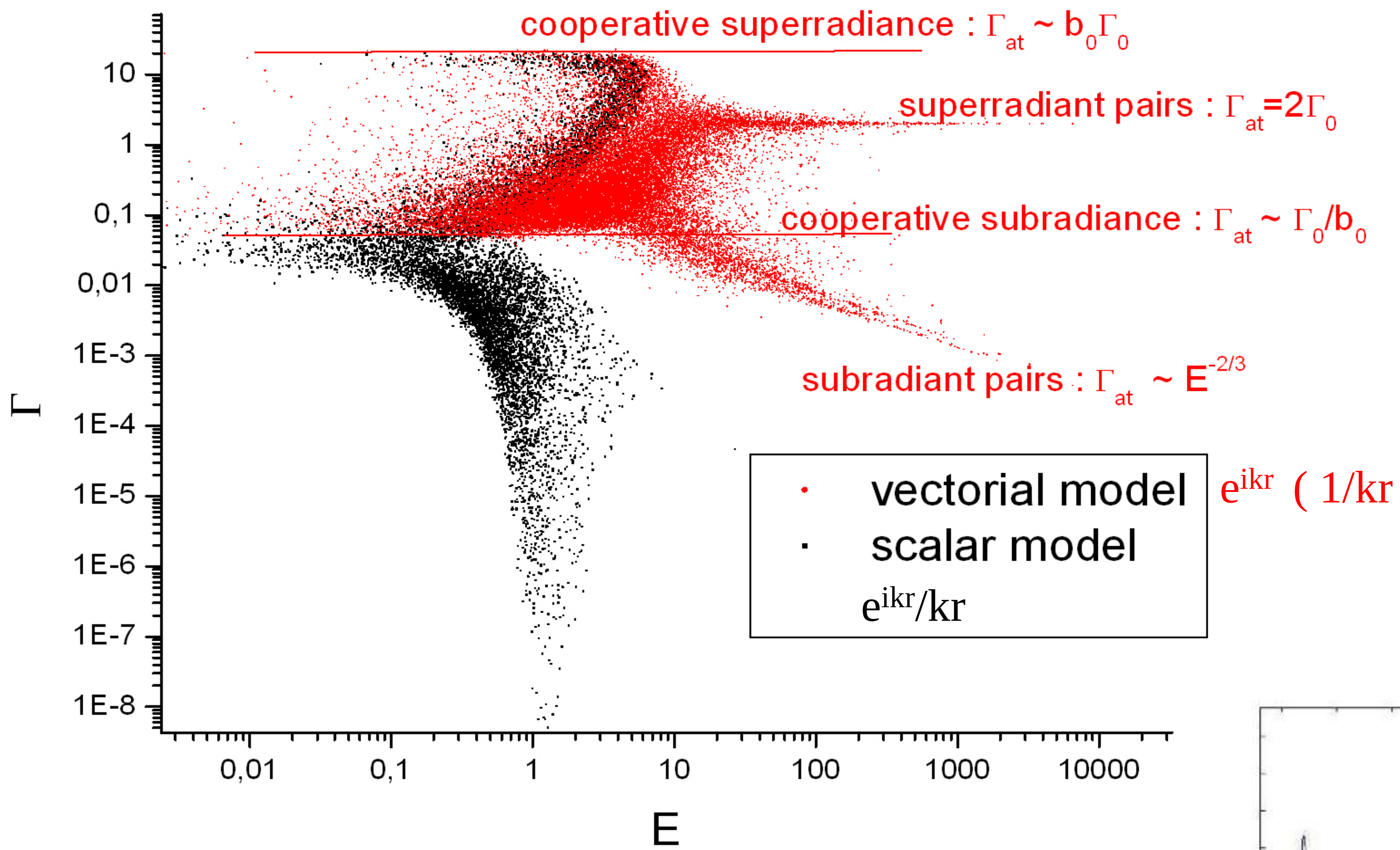


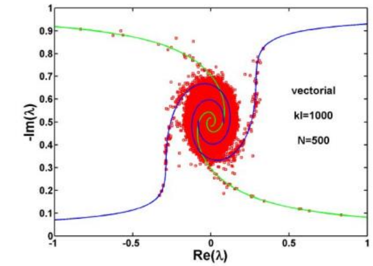

FIG. 4. (a) Total cross section as a function of energy for a system of seven identical scatterers randomly placed on a plane. Each scatterer is the same as used in Fig. 1. The positions of the scatterers are shown in (b).

proximity resonances
doorway states
giant oscillator strength

# Eigenvalues for N coupled dipoles

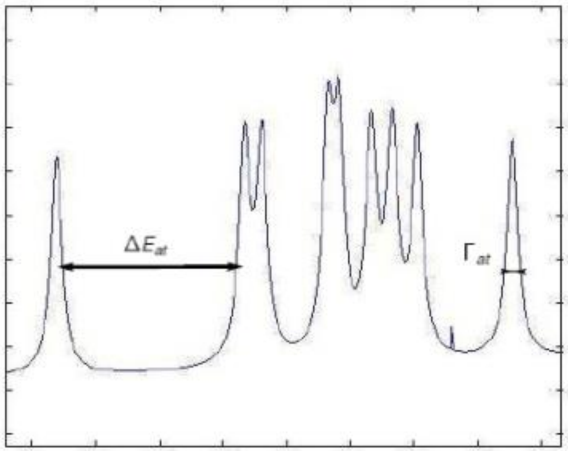
kl=0.1



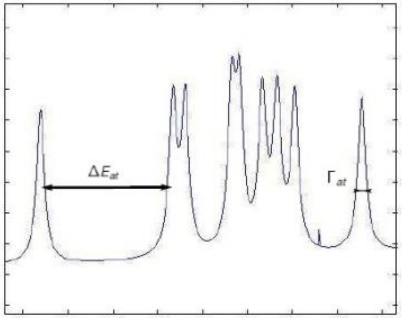




**Resonance Overlap (« Thouless »)**

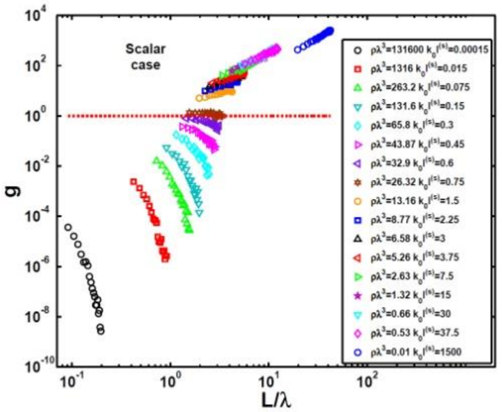
$$g = \left\langle \frac{1}{\langle 1/\Gamma\rangle\langle\Delta E\rangle} \right\rangle$$

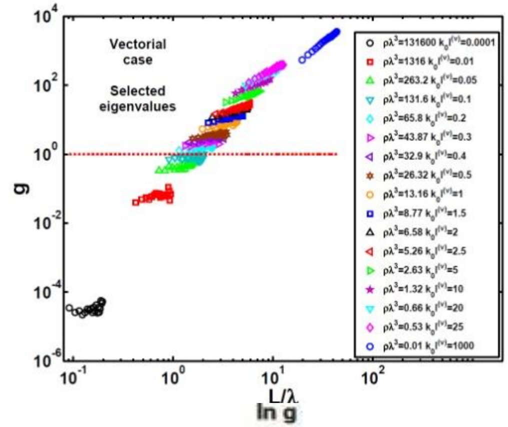


# Resonance Overlap (« Thouless »)

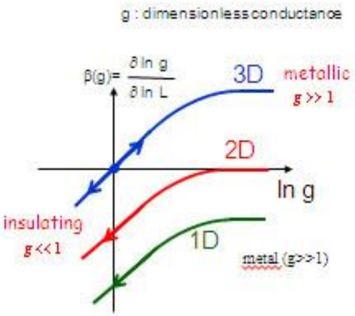


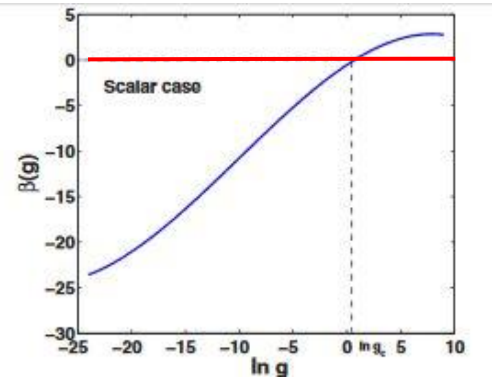

$$g = \left\langle \frac{1}{\langle 1/\Gamma \rangle \langle \Delta E \rangle} \right\rangle$$

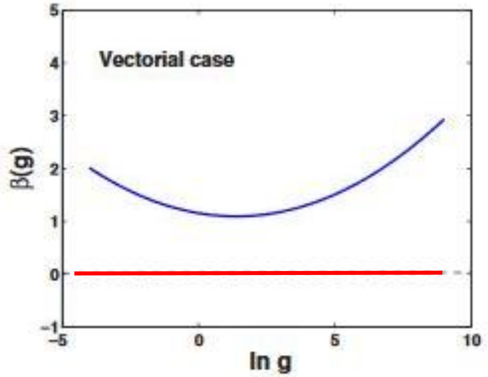



Scaling function β(g)




**S. Skipetrov, I. Sokolov, PRL 112, 023905 (2014)**

**Bellando et al,. Phys. Rev. A 90, 063822 (2014)**

**No Anderson Localization for Vectorial Light in 3D !**

**NOT captured by general scaling Ansatz by the gang of four nor by self-consistent theory of localization !**

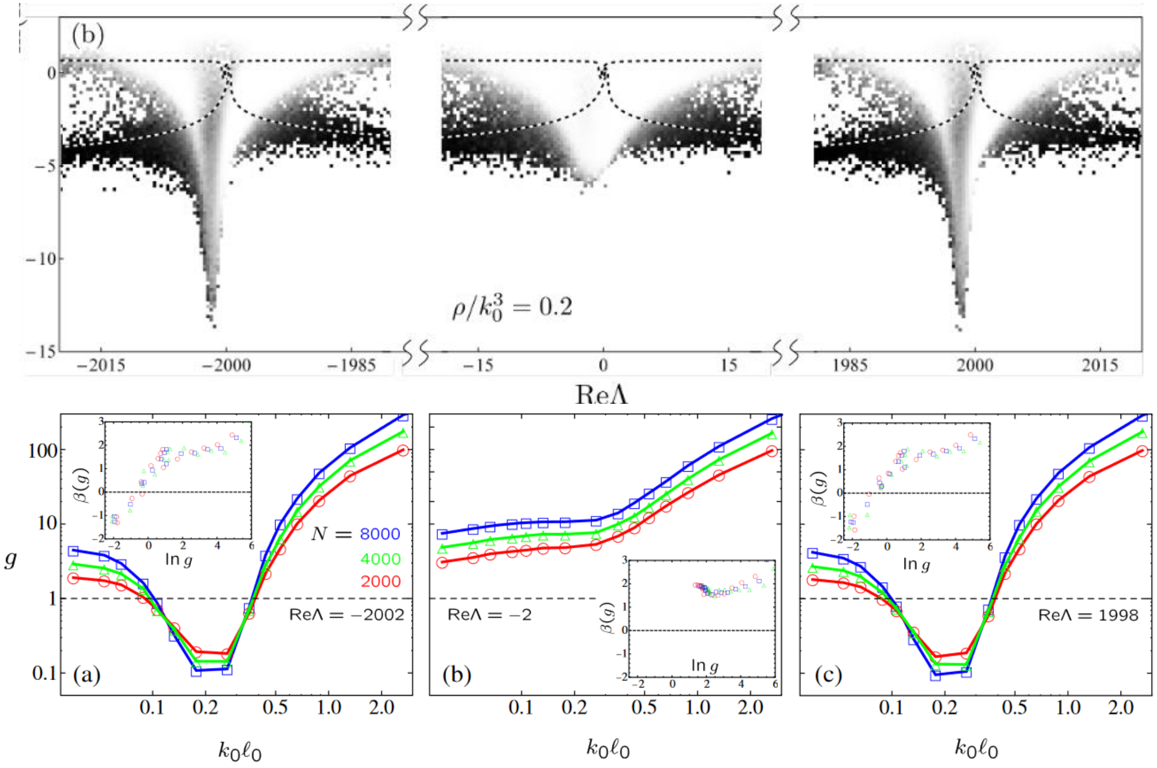
**Very bad news for Anderson localization of light by cold atoms!**

**BUT : there might be a solution ☺**

# Roadmap(s) towards
# Anderson Localisation of Light

## 1) Magnetic field assisted Anderson localization

Dense sample + magnetic field :
partial suppression of near field dipole dipole interactions

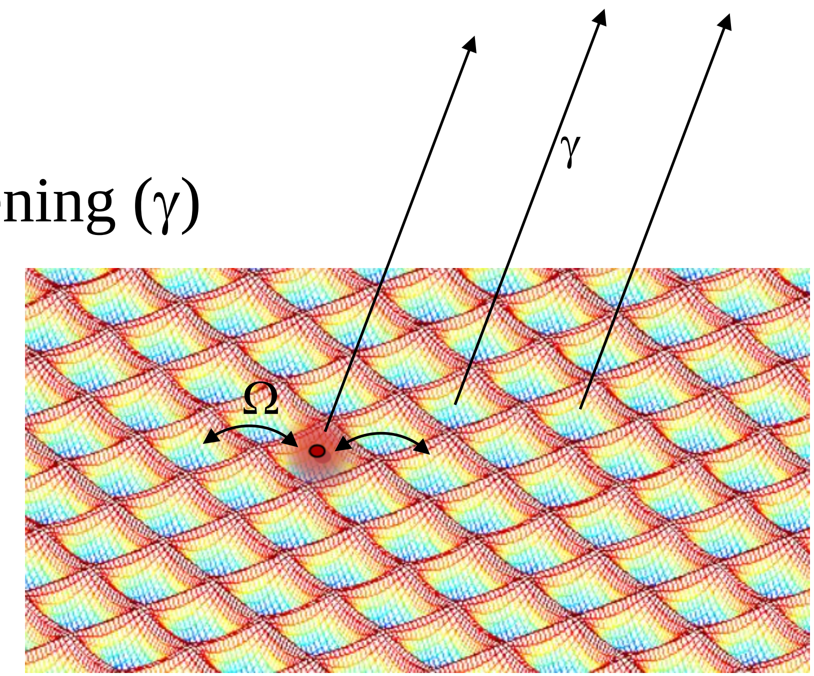



S. Skipetrov, I. Sokolov,
PRL **114**, 053902 (2015)

## 2) Combining Anderson and Dicke

## Toy Model : Open Disordered System:

3D Anderson model on 10 x10 x10 lattice
hopping (Ω) + on site disorder (W) + opening (γ)

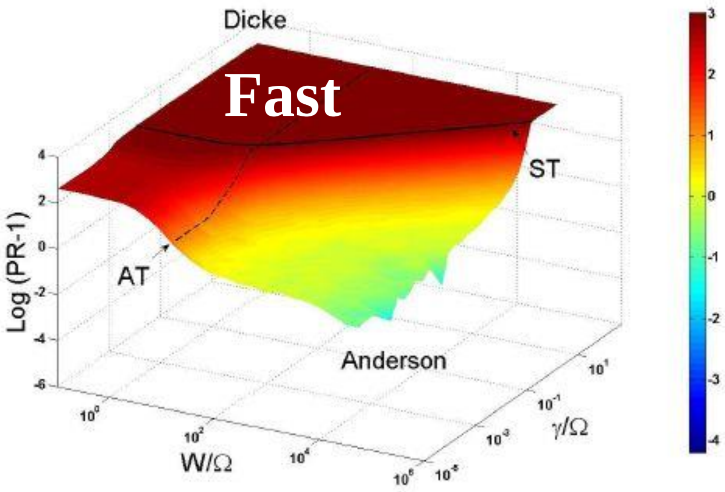



$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} \left(|j\rangle\langle i| + |i\rangle\langle j|\right)$$

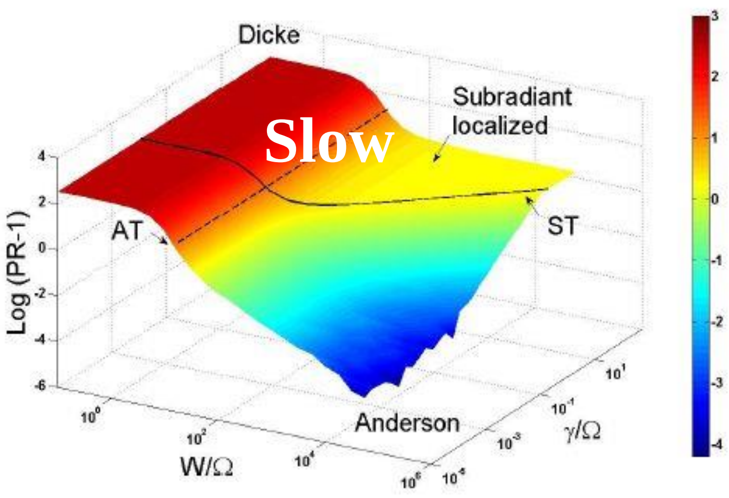
$$(H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2} \sum_c A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}$$

All sites coupled to one single decay channel : $Q_{ij}=1$

# Toy Model : Anderson lattice model + coupling to one open mode

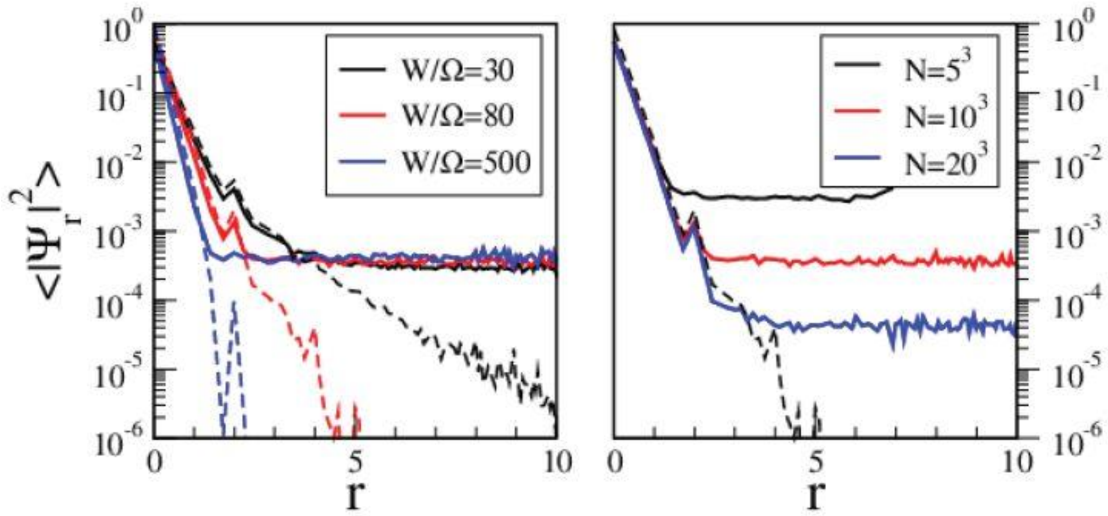



(a)




(b)

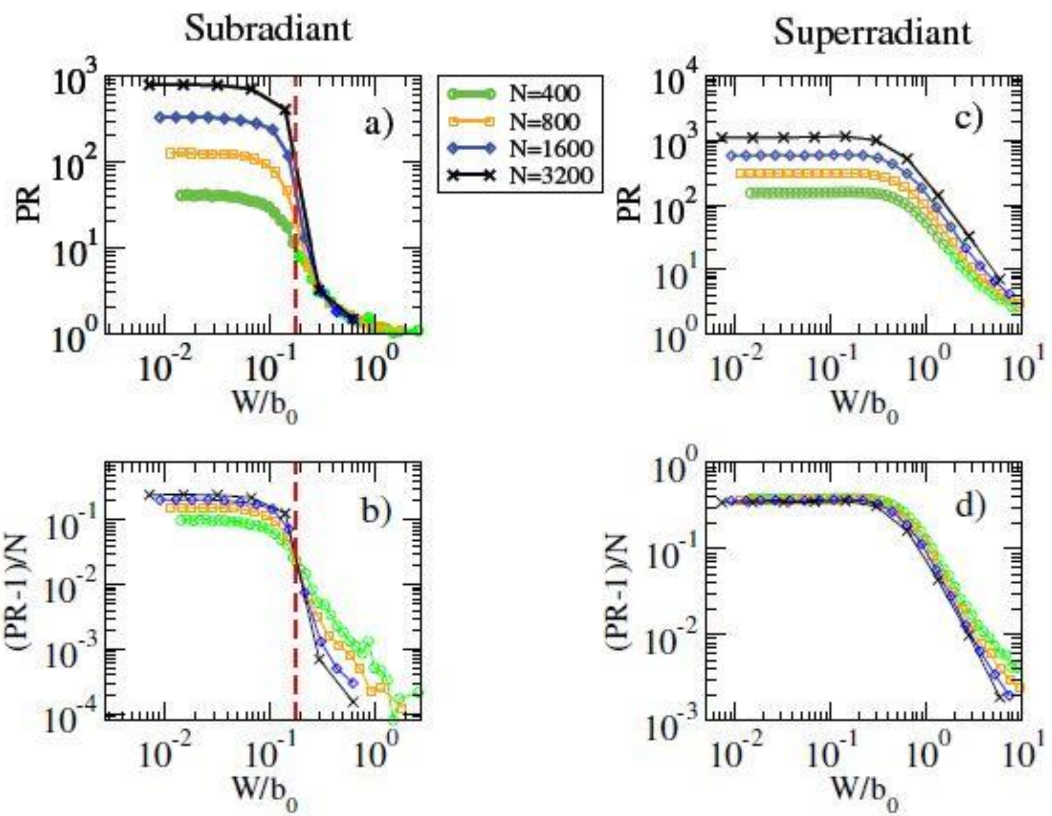
## Hybrid Subradiant States « decoupled » from outside world

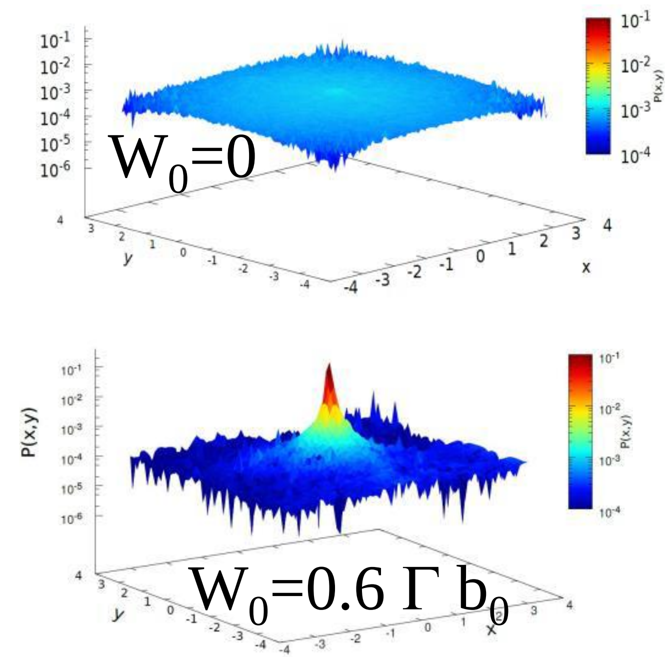



A.Biella, F.Borgonovi, R.K., G.L.Celardo, EPL, 103, 57009 (2013)

2) Apply **diagonal disorder** in coupled dipole model

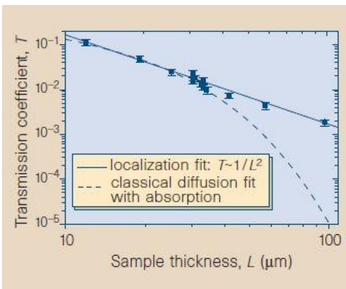
Numerical solution of **« coupled dipole » equations**

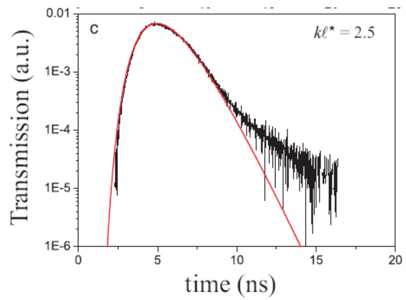


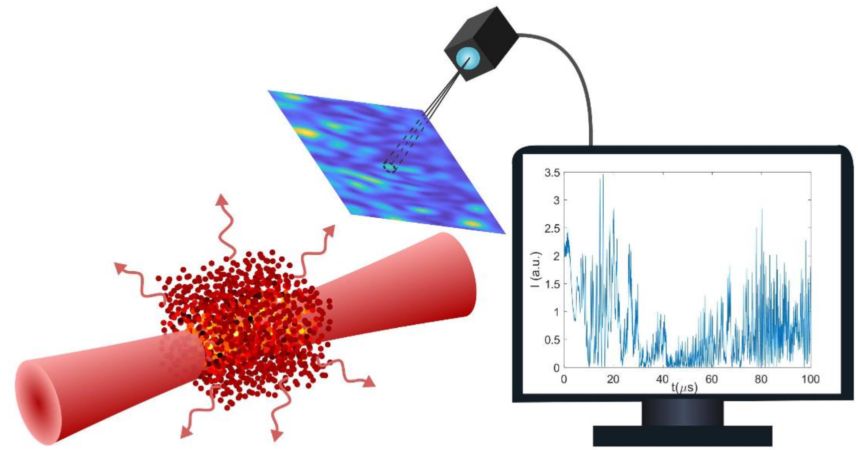
# Macroscopic observables of Anderson localisation

No insulating behaviour from
(time resolved) average transport ☹

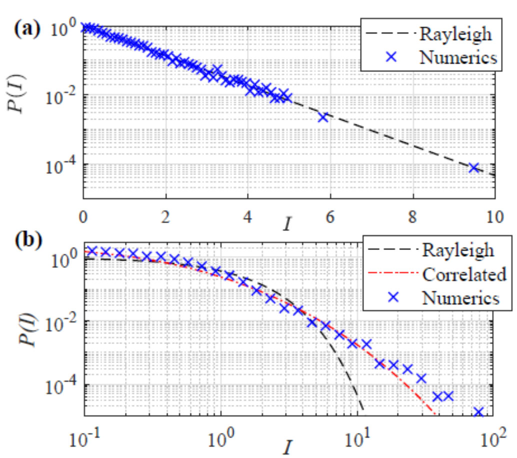


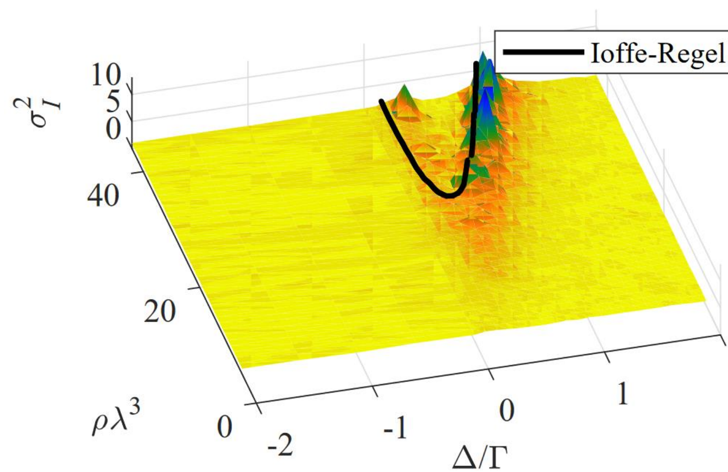




Noise is the signal ☺ :
intensity correlations : g(2)(τ)




# Microscopic (ab initio) model of observables of Anderson localisation

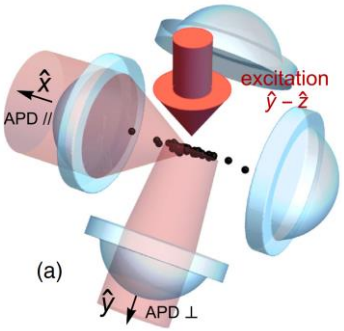


# Prospectives

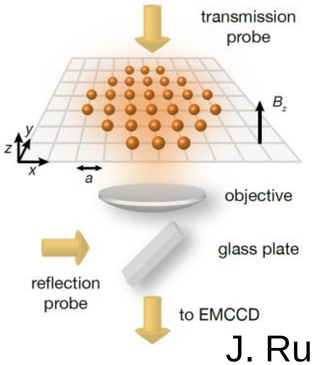
Towards Anderson localization with Yb atoms

High density samples (small / large volume)




G. Feriol et al.,
Phys. Rev. X 11, 021031 (2021)

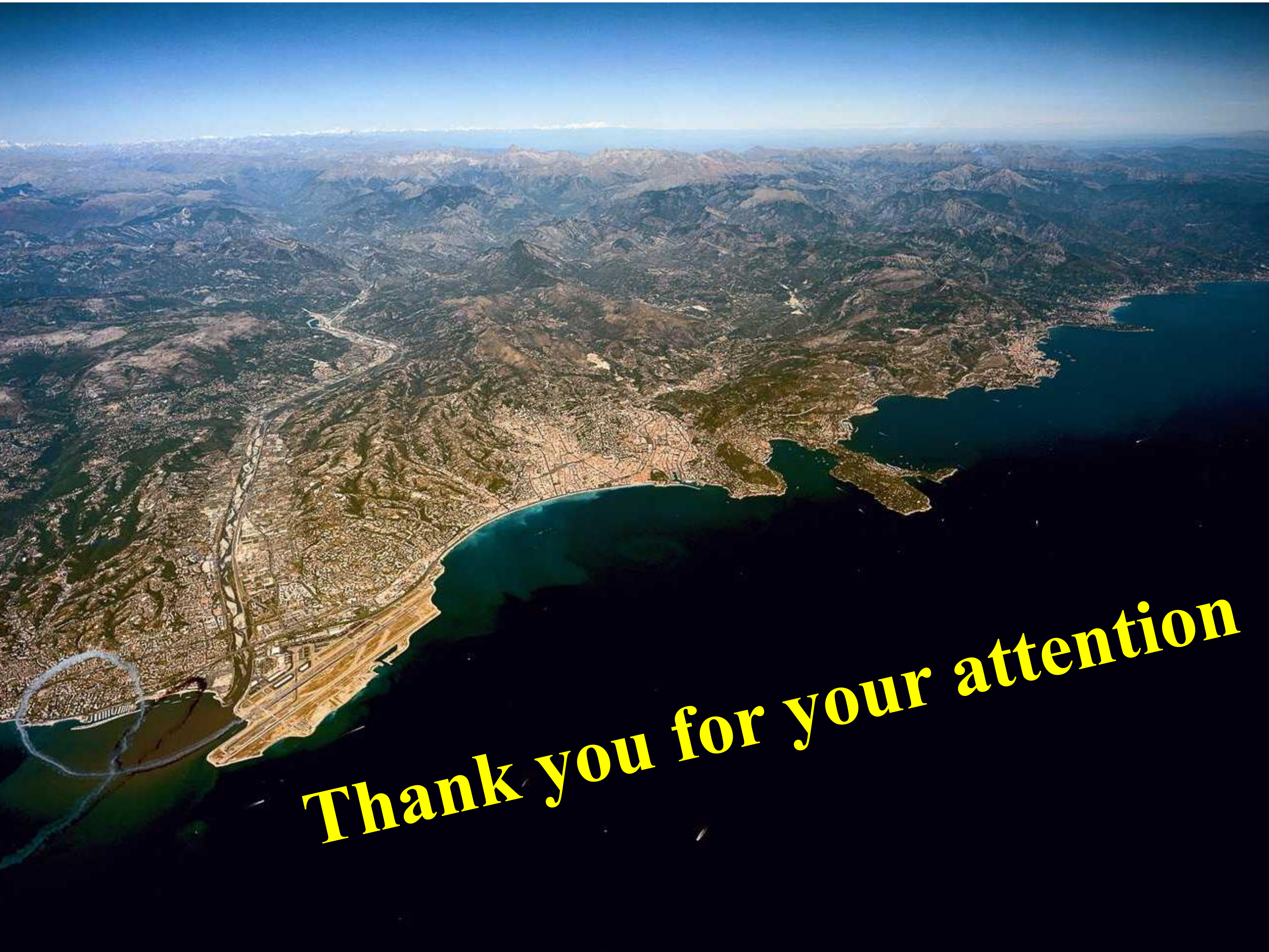
Ordered atomic arrays (1, 2, 3D)




J. Rui et al., Nature 583, 369 (2020)

Coupled dipole induced entanglement

… **+ your ideas !!!**


Thank you for your attention