

Less Travelled Path of Dark Matter: Axions and PBHs
ICTS-TIFR, Bengaluru

INTRODUCTION TO DARK MATTER

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THIS IS WHAT'S SO COOL ABOUT IT...

IT'S A CLASH OF WORLD VIEWS:

FOR A LONG TIME, PARTICLE PHYSICISTS

HAVE BEEN DRILLING DOWN INTO SMALLER AND SMALLER LENGTH SCALES.

atom



nucleus



quarks



solar system

milky-way

the whole universe

ASTRONOMERS

LOOKING FURTHER AND FURTHER OUT...

THEN COMPLETELY SEPARATE, YOU HAD

JUST RECENTLY, THESE TWO SUCCESSFUL WAYS OF DOING SCIENCE SORT OF... CLASHED.

WHAT ARE YOU LOOKING AT?

UNDERSTANDING OF “DARK AIR”



Lord Rayleigh
(Image: Nobel Prize Foundation)

When he compared nitrogen extracted from air with nitrogen extracted from chemical compounds, he found that the nitrogen from air was heavier. He concluded that the air must contain another, previously unknown substance.

Now called Argon.

THE 3 FOLD WAY

Thermodynamics

Cosmology

Kinetic Theory



Astrophysics

Chemistry


Particle Physics

REFERENCES

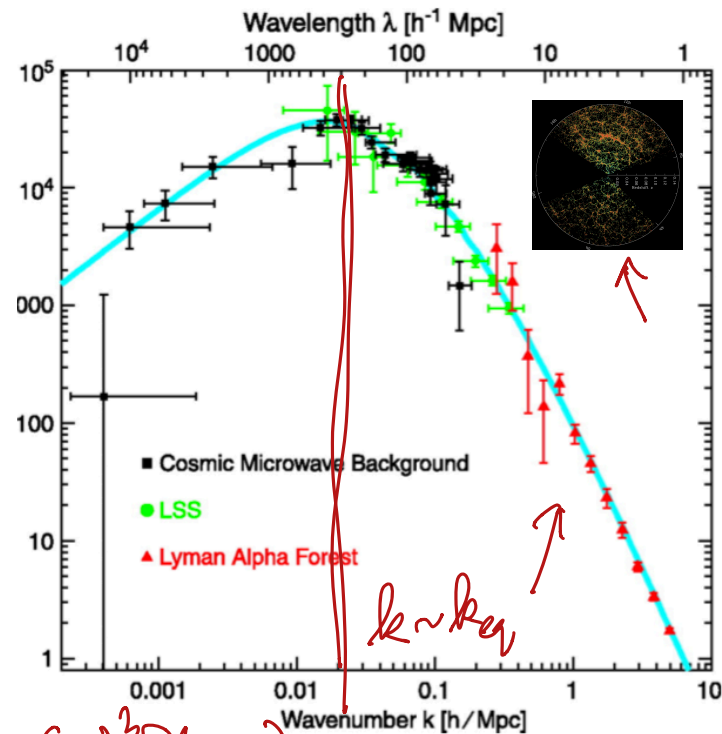
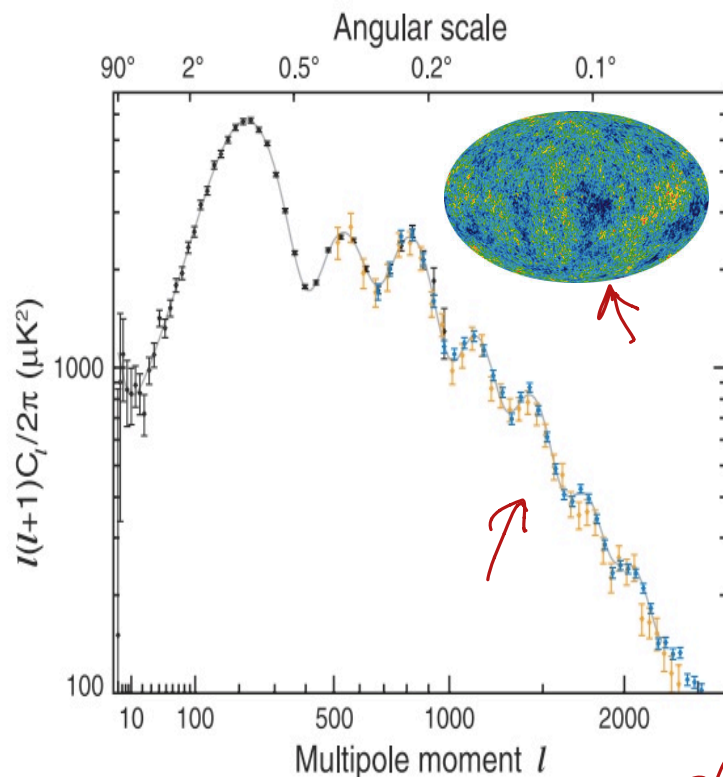
Textbooks:

1. Kolb and Turner (The Holy Bible)
2. Dodelson (Cosmology)
3. Profumo (DM)
4. Bernstein (Thermodynamics in Expanding Universe)

Notes/Papers:

1. BD/Paranjape/Mohanty (SERC School 2019)
2. Bauer and Plehn (DM) 
3. Steigman, BD, Beacom (1204.3622, relic density)

COSMOLOGICAL PROBES OF DM



$$P(k) = (2\pi)^3 \delta(k-k')$$

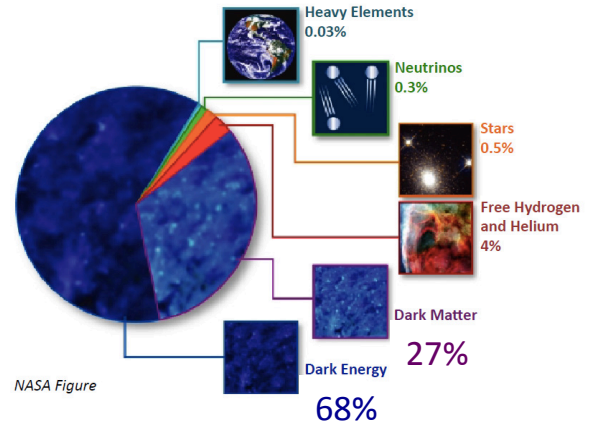
$$= \langle \delta(k) \delta(k') \rangle$$

LSS

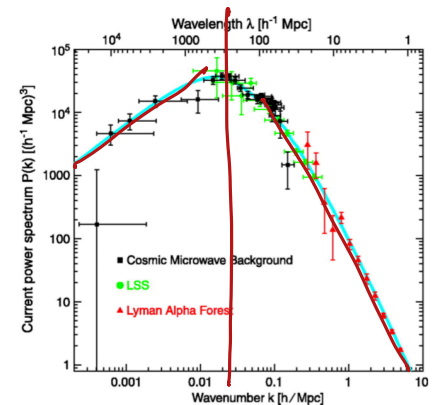
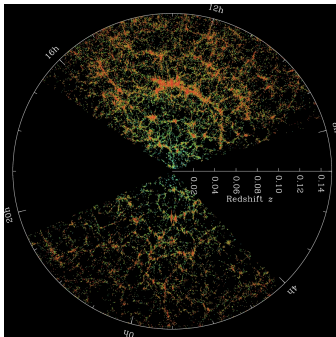
ABUNDANCE AND DISTRIBUTION

How much DM?

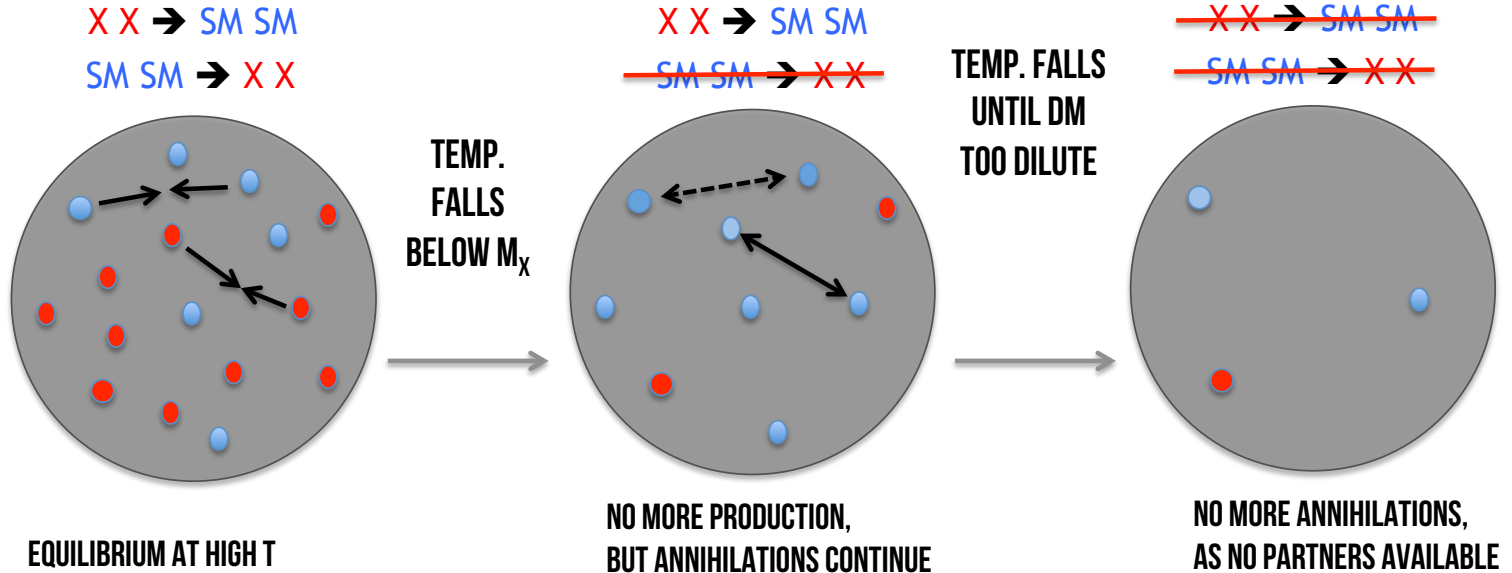
$$\frac{\rho_{dm}}{\rho_c} h^2 \approx \int \frac{dm}{m} h^2 \approx 0.12$$



How clumped?

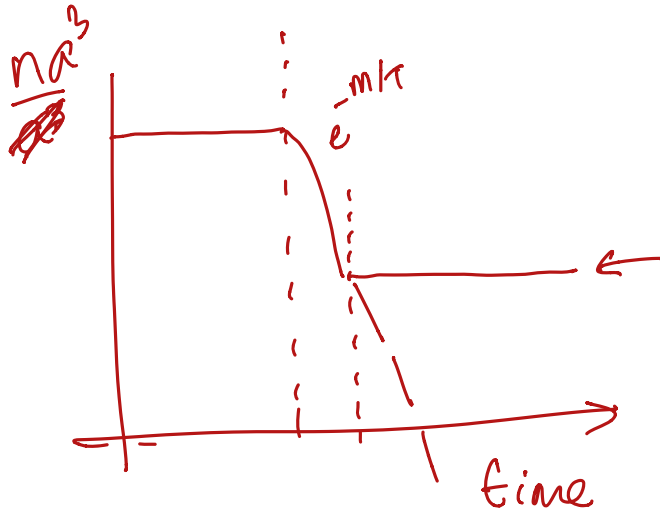


IDEA 1 : THERMAL FREEZEOUT



Chemical Freeze-out sets DM Density

BOLTZMANN EQN



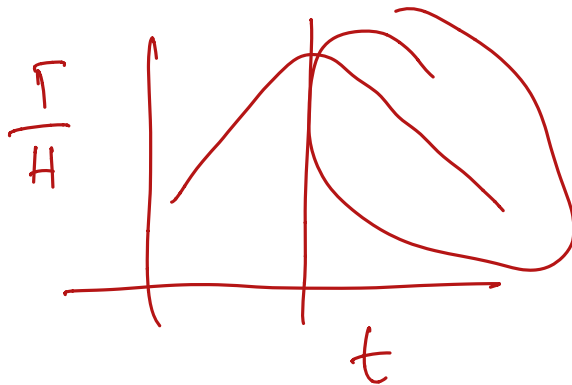
$$\Gamma \approx H$$

$$\Gamma \approx n \sigma v$$

$$n \sim T^3$$

$$\sigma \sim \begin{cases} (9/4) 1/T^2 & T \gg m \\ \frac{m^2}{M^4} & T \ll m \end{cases}$$

$$T \sim m_x$$



$$n \sigma v \sim \begin{cases} T & T \gg m \\ \frac{m^2 T^3}{M_{\text{mic}}^4} & T \ll m \end{cases}$$

$$H \sim T^2 / M_{\text{pl}}$$

$$f(\bar{x}, \bar{p})$$

$$dx^3 dy^3 dz^3 \sim h^3$$

$$\int f d^3 p \rightarrow n(x) = f_1 f_2 - f_3 f_4$$

$$\int d^3 p \left(\partial_t + \vec{v} \cdot \vec{\partial}_x \right) f(\bar{x}, \bar{p}, t) = 0$$

$$\downarrow$$

$$\left(\partial_t + \vec{v} \cdot \vec{\partial}_x \right) n = 0$$

$$\frac{d}{dt}$$

$$\frac{d}{dt} (nV) = 0$$

$$\left(\frac{d}{dt} + 3 \frac{\dot{a}}{a} \right) n = 0 \approx \frac{d}{dt} (na^3) = 0$$

H

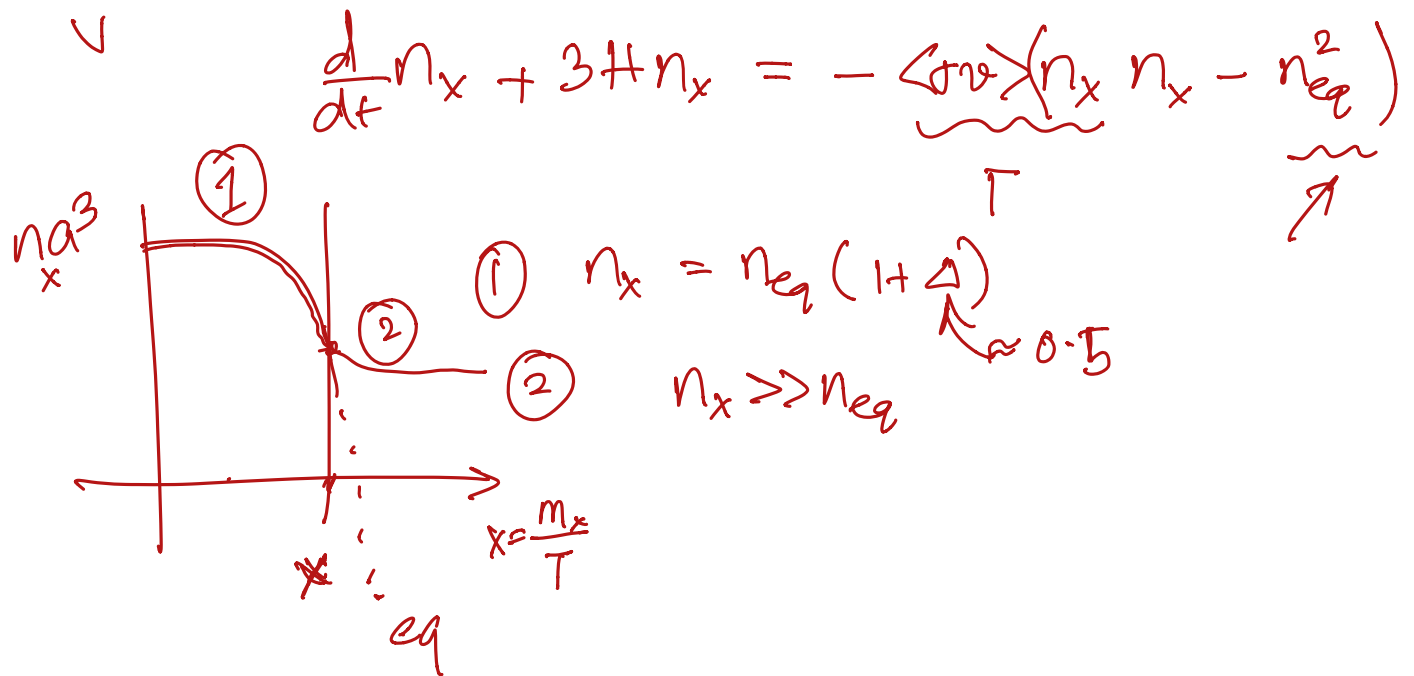
$$\boxed{\frac{dn_x}{dt} + 3Hn_x = 0}$$

$$XX \rightarrow SS$$

σ

$$\frac{\int d^3 p_1 d^3 p_2 e^{-(E_1 + E_2)/T}}{\int d^3 p_1 d^3 p_2 e^{-(E_1 + E_2)/T}} = \langle \sigma v \rangle$$

of p. 12 e 4 c 41



$$\Omega h^2 = 0.12 \quad \frac{x_{\star}}{23} \quad \frac{\sqrt{g_{\star}}}{10} \quad \frac{2 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma v \rangle}$$

x_{\star} is NOT where $\Gamma = H$

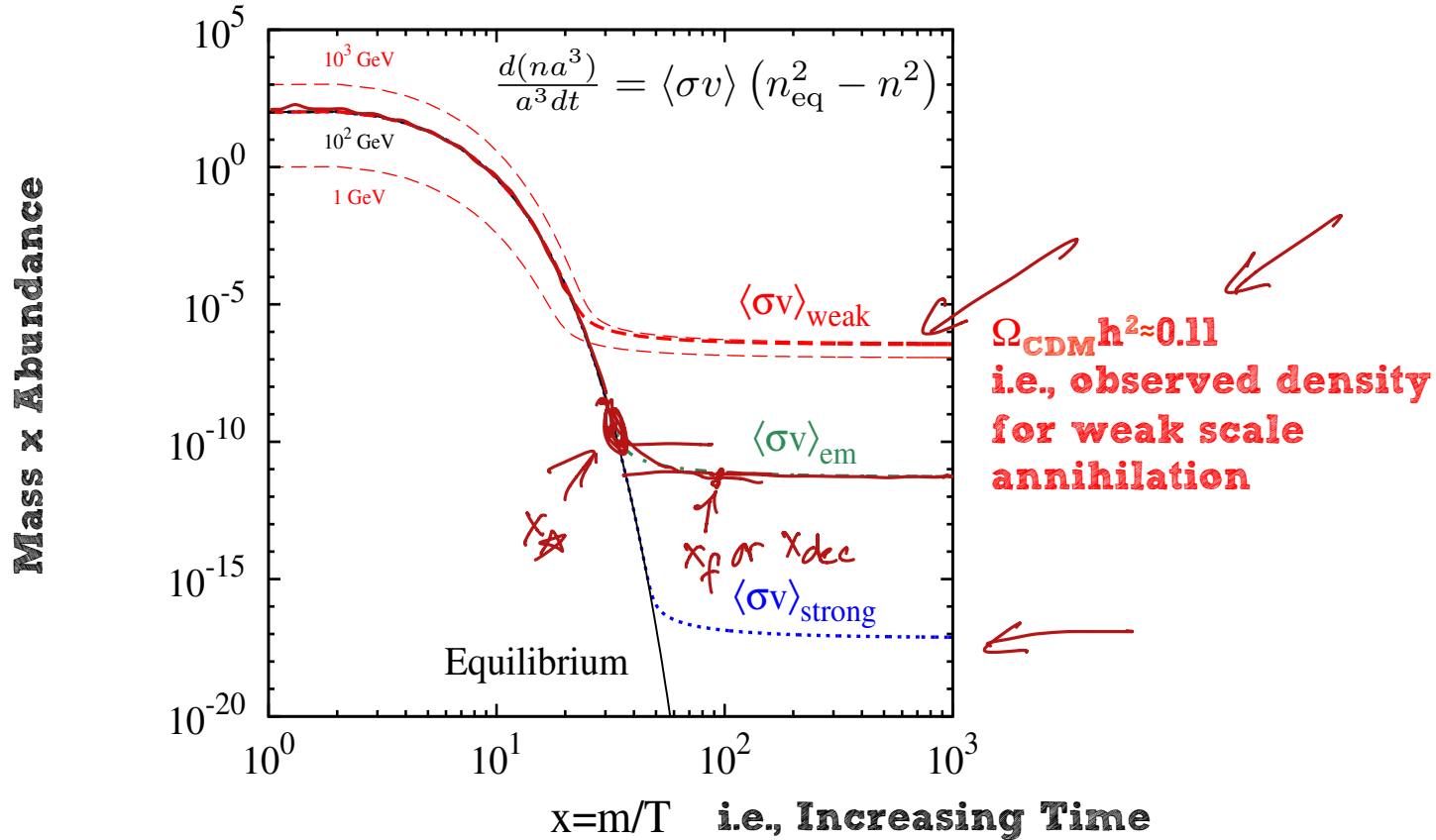
x_f is where $\Gamma = H$

$$x_{\star} \lesssim x_f$$

γ decreases by ~ 30

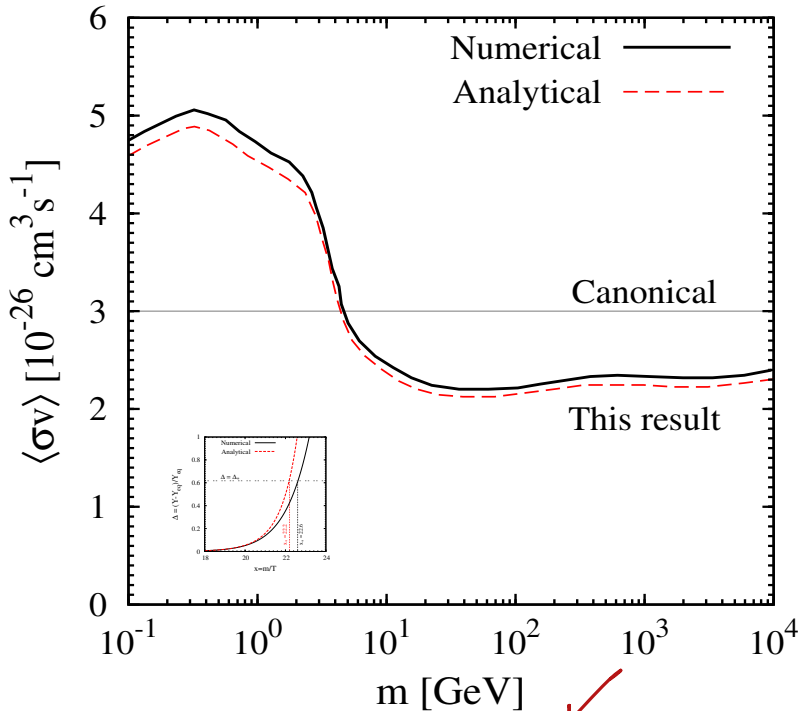
between x_{\star} & x_f

FREEZE-OUT



Zeldovich (1965); Chiu (1966); Lee and Weinberg (1977); Hut (1977); Wolfram (1979); Steigman (1979).

PRECISION COMPUTATION



**Depends modestly
on Mass**

**Must take into
account for low-
mass WIMPs, below
DM mass = 10 GeV**

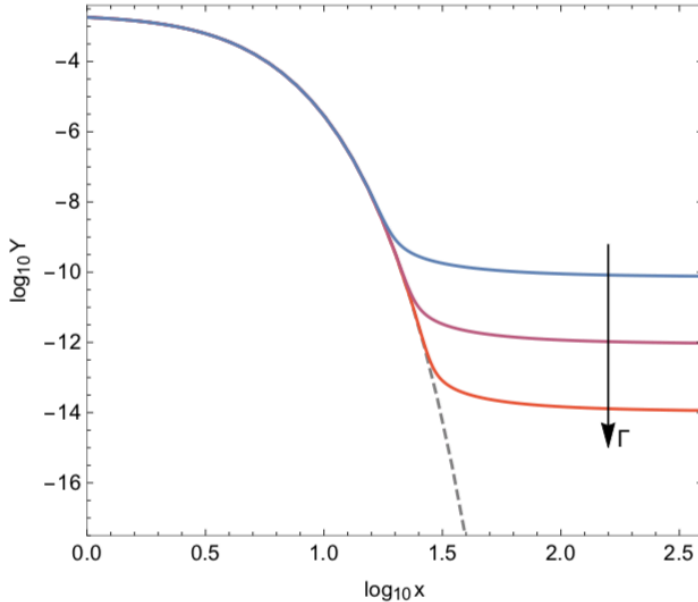
Steigman, BD, and Beacom (PRD, 2012)

$$10^{26} \langle\sigma v\rangle = 0.902 \left(\frac{0.11}{\Omega h^2} \right) \left(\frac{x_*}{g_*^{1/2}} \right) \left(\frac{(\Gamma/H)_*}{1 + \alpha_* (\Gamma/H)_*} \right)$$

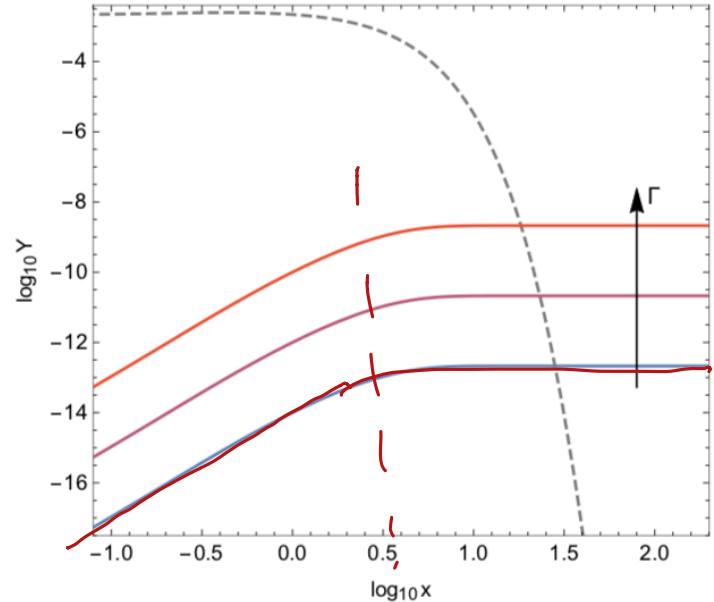


Gary
Steigman

IDEA 2: FREEZE IN



Freeze Out



Freeze In



Bernal et al (IJMP 2017)

STRUCTURE AS A PROBE OF DM

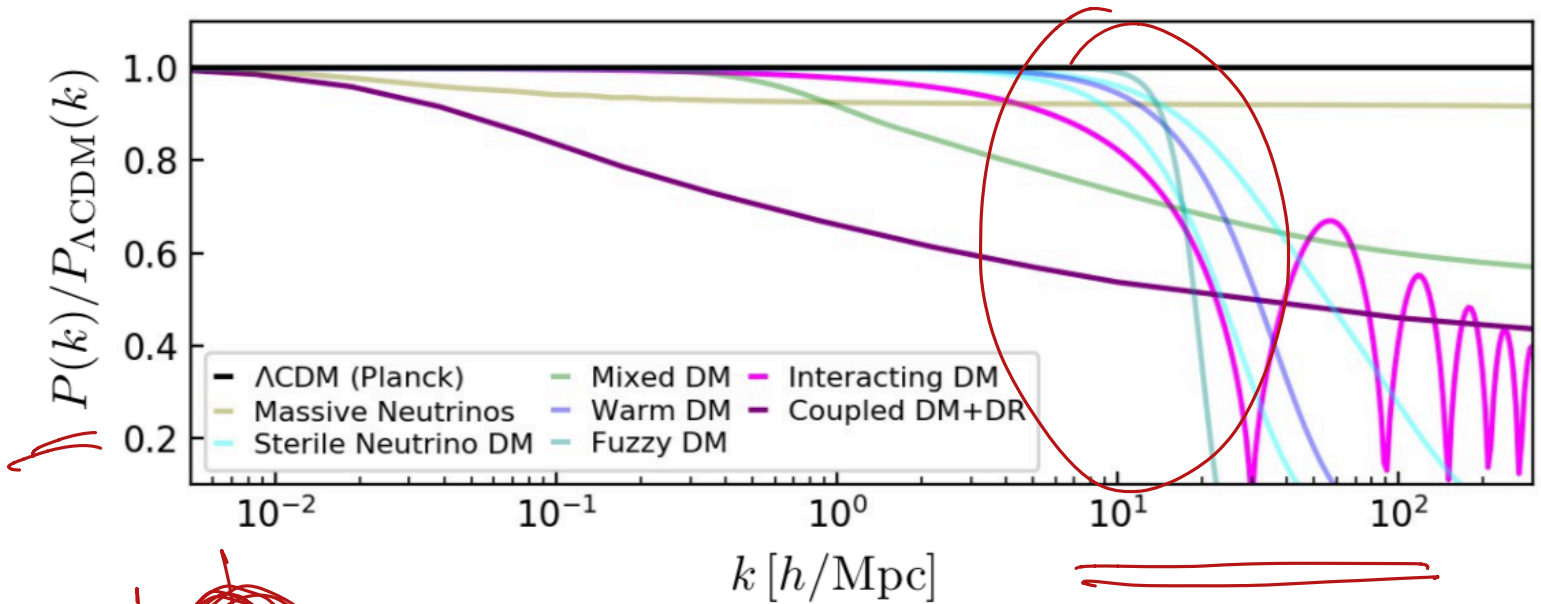
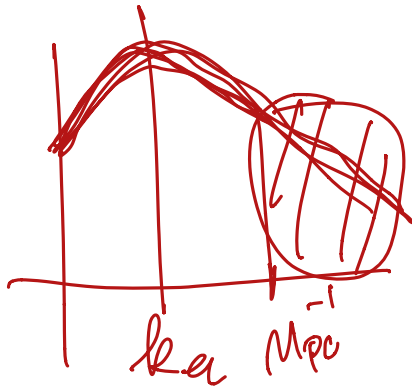
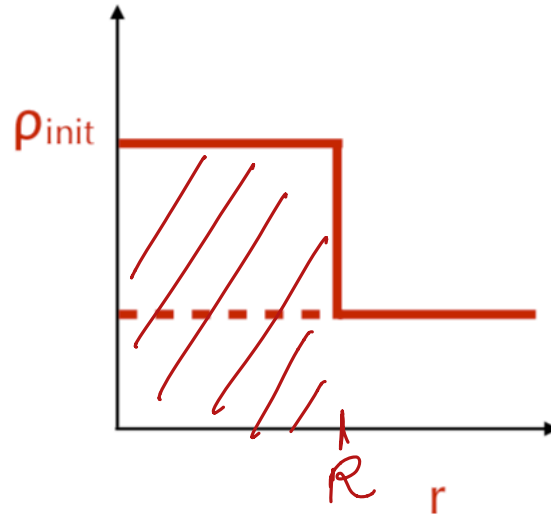
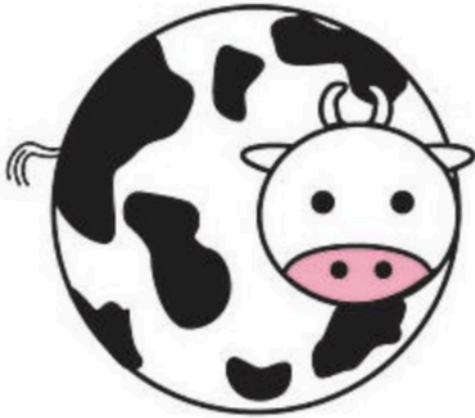


Image: Schneider (2018)



SPHERICAL COLLAPSE



$$\underbrace{d^2R / dt^2} = - \underbrace{GM(<R,t)} / R^2$$

Assume no shell-crossing: $M(<R,t) = M(<R_{\text{init}},t_{\text{init}}) = \text{constant}$

Credit: Aseem Paranjape

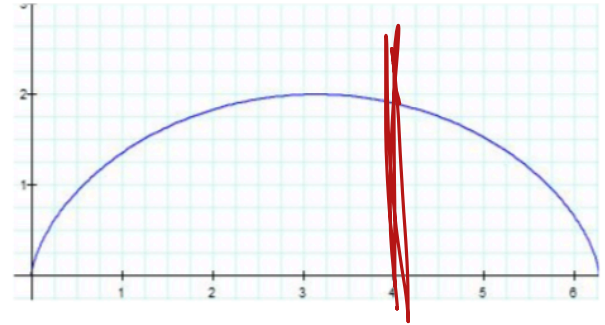
SPHERICAL COLLAPSE

Kolb & Turner

Nonlinear solution:

$$R \propto 1 - \cos \theta \quad ; \quad t \propto \theta - \sin \theta$$

$$\frac{\rho}{\bar{\rho}} = \frac{9 (\theta - \sin \theta)^2}{2 (1 - \cos \theta)^3}$$



Linearly extrapolated value:

$$\delta_L = \frac{3}{5} \left(\frac{3}{4} \right)^{2/3} (\theta - \sin \theta)^{2/3}$$

Value at collapse

$$\delta_c = 1.686$$

Virialisation:

$$2 (\text{K.E.}) + (\text{P.E.}) = 0$$

$$\text{Also } \text{K.E.} + \text{P.E.} = \text{constant}$$

$$R(t_{\text{vir}}) = R(t_{\text{ta}}) / 2$$

$$\Delta_{\text{vir}} = 18\pi^2 \approx 178$$

Erratum:

$\delta_c = 1.686$ is density in spherical collapse if
linear theory is (wrongly) extrapolated to time of collapse

Credit: Aseem Paranjape

LINEAR PERTURBATION THEORY

$f \rightarrow n$
:

$$\frac{\partial \rho_m}{\partial t} = -\nabla \cdot (\rho_m \vec{u})$$

continuity equation

$$\left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) \vec{u} = -\frac{\nabla p}{\rho_m} - \nabla \phi$$

Euler equation

$$\nabla^2 \phi = 4\pi G \rho_m$$

Poisson equation ,

$\delta \rightarrow \delta(0.3)$
 $G(1)$

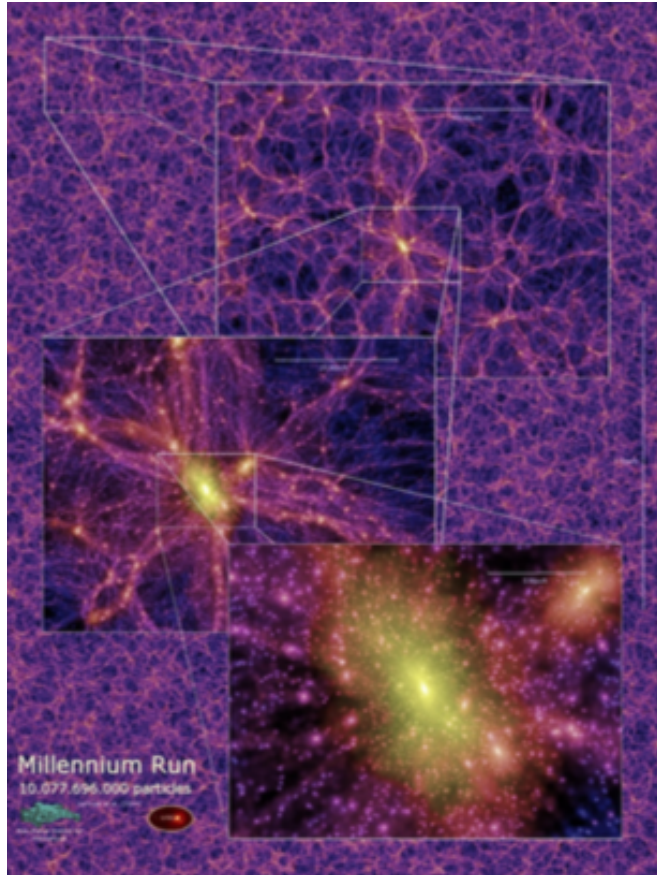


$$\rho_m = \bar{\rho} (1 + \delta_m)$$

Gravity Collapse
Pressure

$$\ddot{\delta}_k + 2H\dot{\delta}_k = \delta_k \left[\frac{\bar{\rho}}{2M_{pl}^2} - \left(\frac{c_s k a_0}{a} \right)^2 \right]$$

N BODY SIMULATIONS



Start with a box of particles, with initial conditions (x,v) consistent with CDM

Evolve their positions using collisionless Boltzmann equation

Various numerical issues/ techniques

One key issue: “particles” can be say, $10^6 - 10^7$ solar masses!

Identify halos, merger trees, etc.

Predict clustering, substructure, merger histories, etc.

Some clarifications:

① Notion of $\delta_c = 1.666$ comes from asking what would be the density contrast in linear theory if it were to be extrapolated to time of collapse in spherical top hat model.

For pbh δ_c is simply what is overdensity that causes BH-formation (in rad.-down phase). There is no linear theory assumption here.

② $P(k)$ was wrongly interpreted as giving "which scales are most likely to have structure". This is not totally correct.

$$P(k) \sim \int e^{-ikx} \underbrace{\int \delta(x_0) \delta(x_0+x) dx_0}_L$$

Suppose $\langle \delta(x) \delta(x') \rangle \sim \cos(k'x) \langle \delta(x) \delta(0) \rangle$

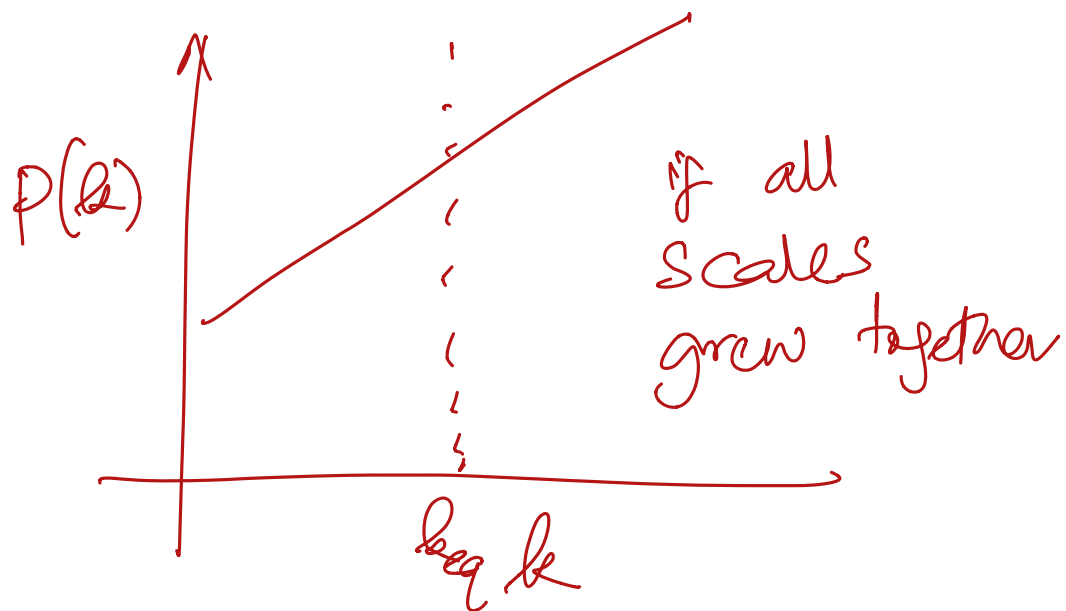
$$P(k) \sim \int e^{-ikx} e^{+ik'x} dx$$

$$\sim \delta(k-k')$$

So it is true that $P(k) \neq 0$ does say "where" there is power.

BUT $P(k)$ also depends on amplitude of the fluctuations at scale $2\pi/k$.

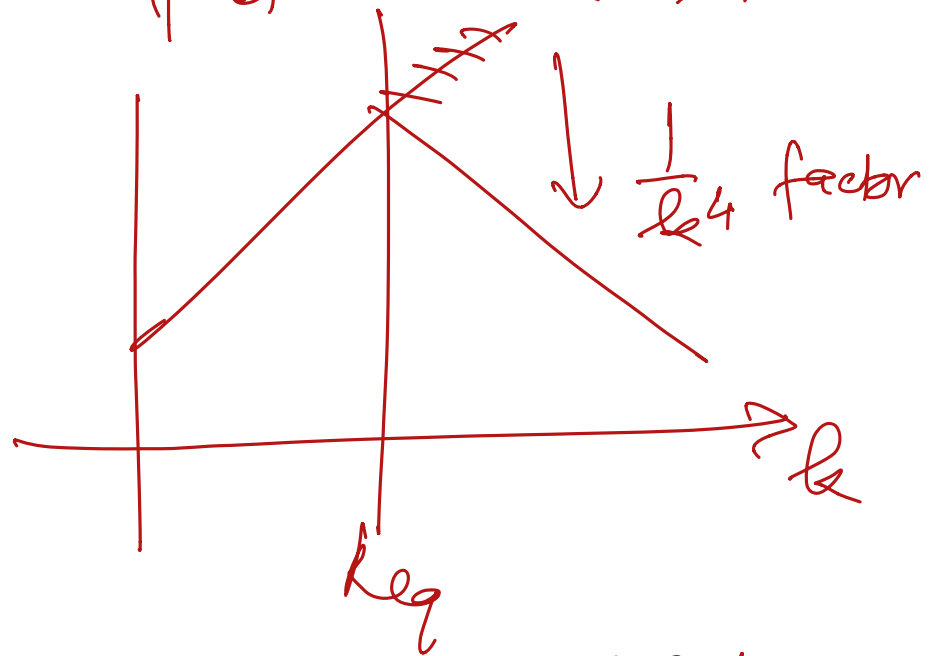
In cosmology $P(k)$ would have looked like



BUT small scales ($k > k_{eq}$) only grow after equality

So they have to wait by a factor $\left(\frac{a}{a_{eq}}\right)^2$, and $P(k)$

gets a factor $k^4 \dots$



Overall scaling with ' k '
depends on dimension of " $P(k)$ "
in 3 dimensions.

One can say $P(k)$ is
fourier transform of the
autocorrelation ~~fn~~.

In 3D:

$$\xi(x) = \int_{-L/2}^{L/2} dx_0 \delta(x_0) \delta(x_0 + x)$$

$$\Delta^2 \approx \frac{k^3 P(\vec{k})}{2\pi^2}$$

is dim-less

$P(\vec{k})$ has dimension of L^3 .