Floquet Fermi and Non-Fermi Liquids

(new non-equilibrium states of quantum matter)

Stability of quantum matter in and out of equilibrium at various scales
ICTS - TIFR, Bangalore, January 14, 2024

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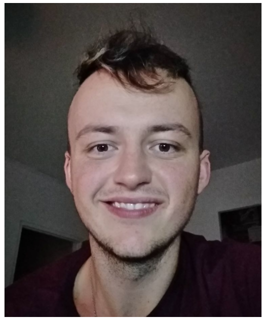

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arXiv:2309.03268 (2023)
Phys. Rev. B **107**, 195135 (2023)

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Heisenberg, Professor 1927 - 1942

Felix Bloch, Rudolf Peierls, Edward Teller ..

Debye, Professor 1927 - 1934

Nernst, Professor
Arrhenius, Professor..

Institute for Theoretical Physics

University Library

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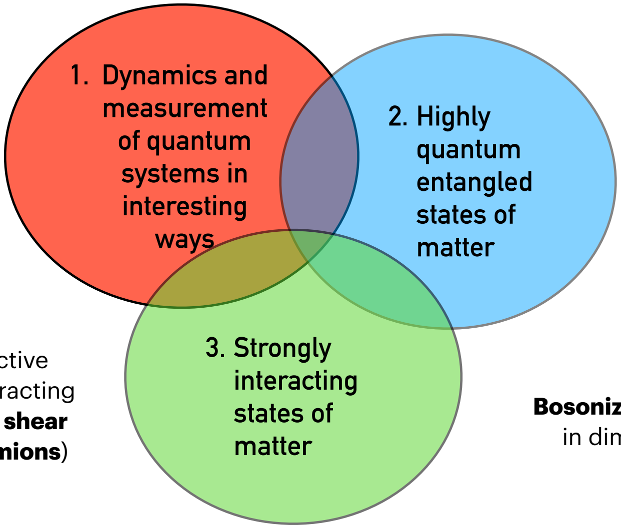

1. Dynamics and measurement of quantum systems in interesting ways

2. Highly quantum entangled states of matter

3. Strongly interacting states of matter

Berry phase effects (e.g. **Nonlinear Hall effect)**

Novel collective modes of interacting systems (e.g. **shear sound of fermions**)

Spin liquids

Quantum Hall liquids

Theory of novel probes (e.g. NV center spin qubits)

Bosonization techniques in dimensions 2 and higher

Looking for Ph.D. or Postdoc position? email me @:

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More on our research? check:

https://home.uni-leipzig.de/qcmt/home.html

Floquet Fermi Liquid

arXiv:2309.03268 (2023)
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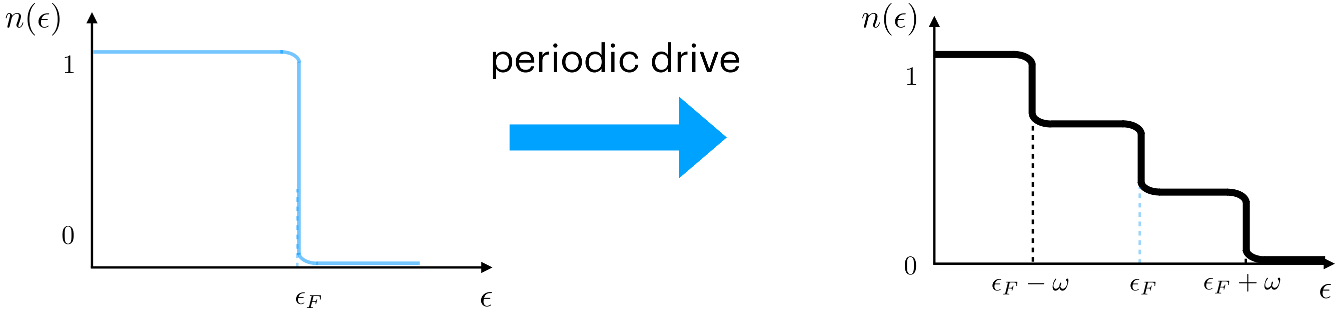
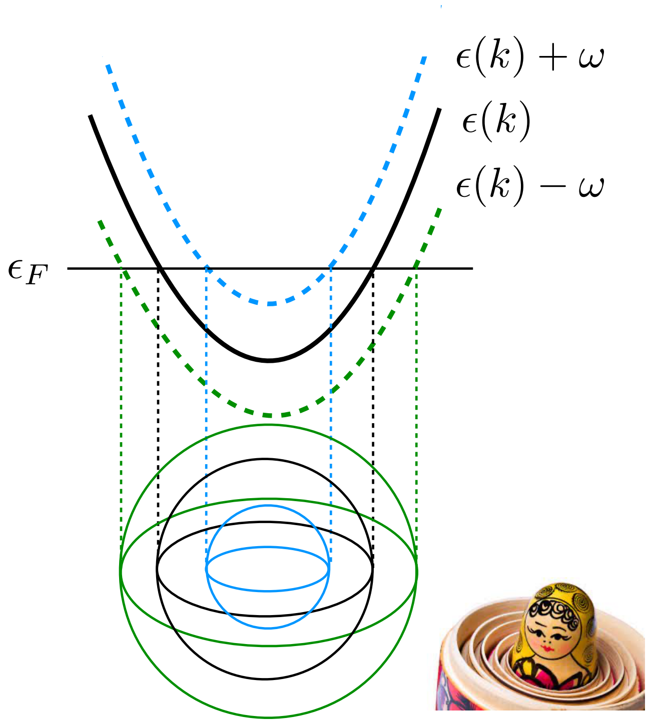
How fermions fill a Floquet band when coupled to a fermion bath?

“Fermi-Dirac” becomes “***Fermi-Dirac Staircase***”



periodic drive

Fermi surface replaced by collection of enclosed surfaces:
“***Layered Fermi Surfaces***”



periodic drive

Fermi Liquid → ***Floquet Fermi Liquid***

Phenomena of Floquet Fermi Liquids

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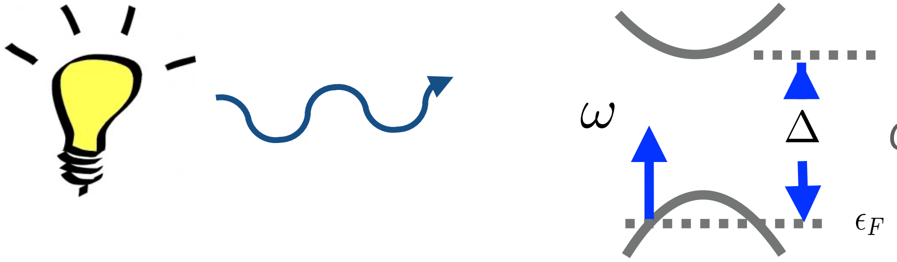
Can photo-current be generated within the optical gap?

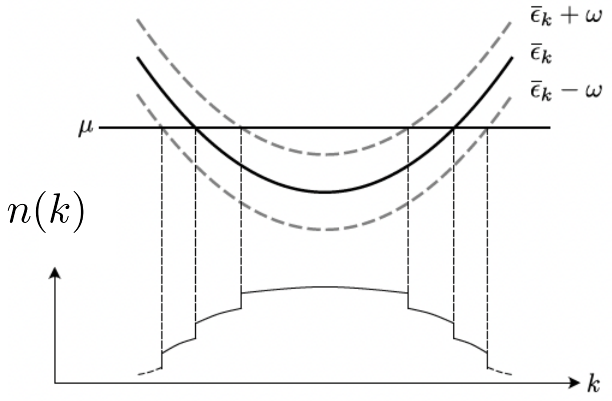


Yes ! Even when $\omega < \Delta$

Finite photocurrent

$$\boxed{\mathbf{j} \neq 0}$$

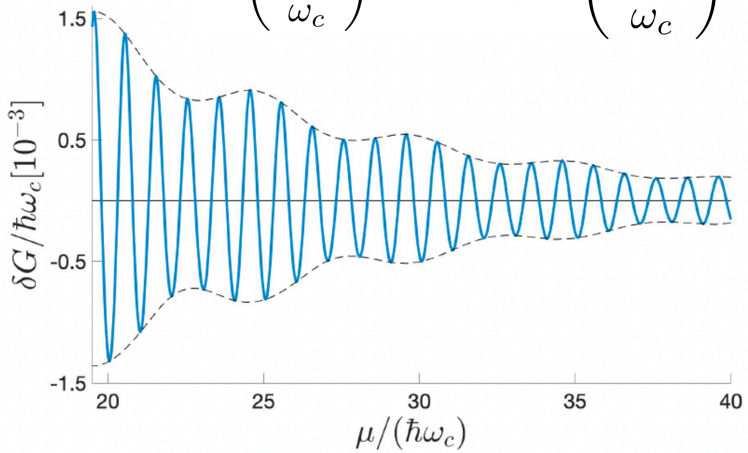
Additional low frequency quantum oscillations:



Magnetic field applied

Picture for Microwave Induced Resistance Oscillation (MIRO) Phenomena

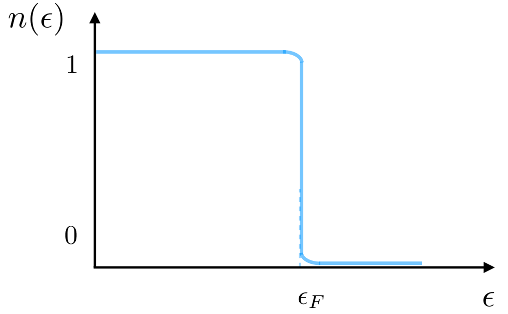
$$Cos\left(\frac{2\pi\mu}{\omega_c}\right) \qquad \sim Cos\left(\frac{2\pi\omega}{\omega_c}\right)$$



“Ultra-quantum-critical” Floquet “Non-Fermi Liquid”

unpublished

How fermions fill a Floquet band when coupled to a boson bath?

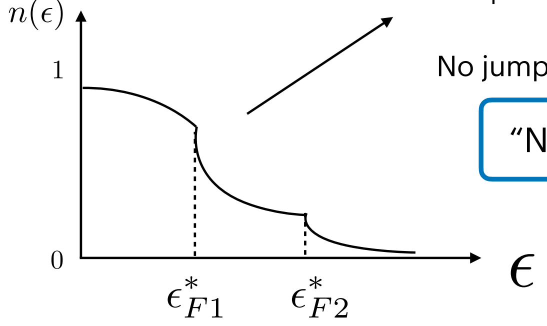


periodic drive

Boson bath

$T_{\text{bath}} = 0$

$T_{\text{bath}} = 0$



Sharp non-analyticities in occupation

No jumps (continuous occupation)

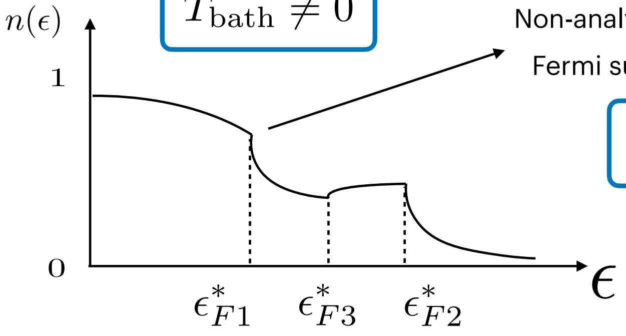
“Non-Fermi Liquid”

periodic drive

Boson bath at finite temperature

$T_{\text{bath}} \neq 0$

$T_{\text{bath}} \neq 0$



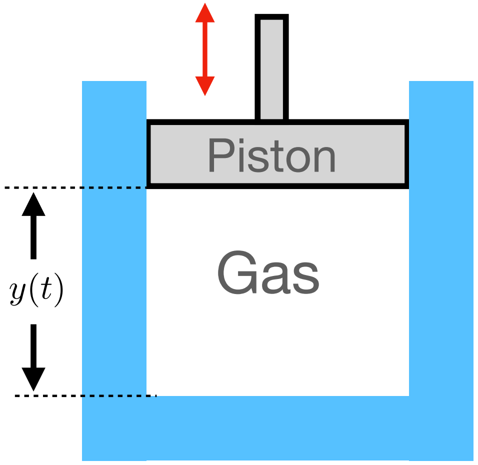
Non-analyticities remain intact at finite temperature

Fermi surface remains sharp at finite temperature !

“Ultra-Quantum critical”

Power law correlations survive to finite temperature

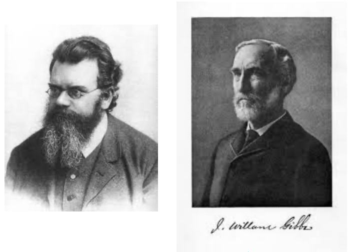
Non-equilibrium physics as an open frontier



$$H(y(t))$$

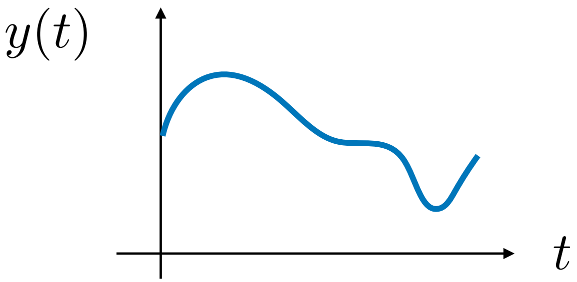
Hamiltonian depends on time


Equilibrium



Maximize entropy

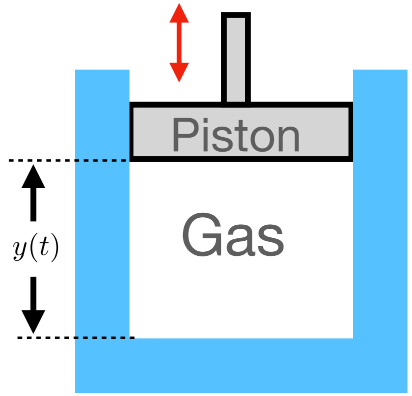
$$\rho_{\text{equil}} = \frac{e^{-\beta H(y)}}{Z}$$

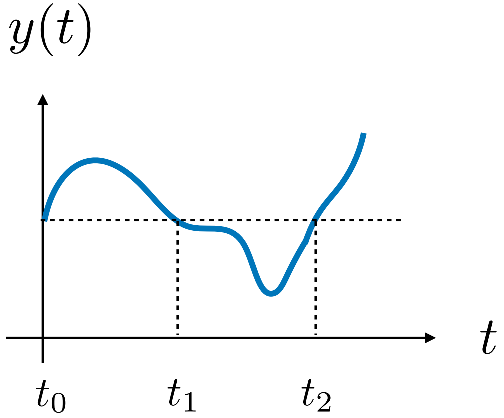

Non equilibrium

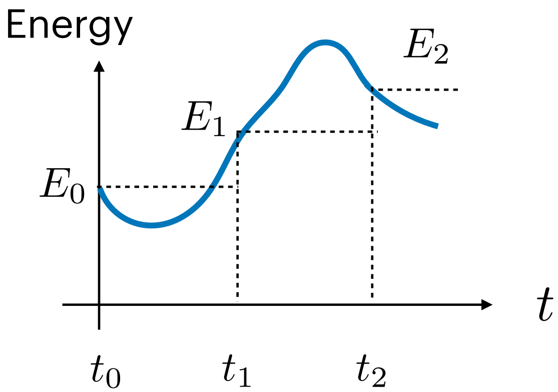
$$\rho_{\text{non}-\text{eq.}}(t) = ???$$

Baths avoid “thermal death” from the second Law

Closed driven systems approach infinite temperature at late times



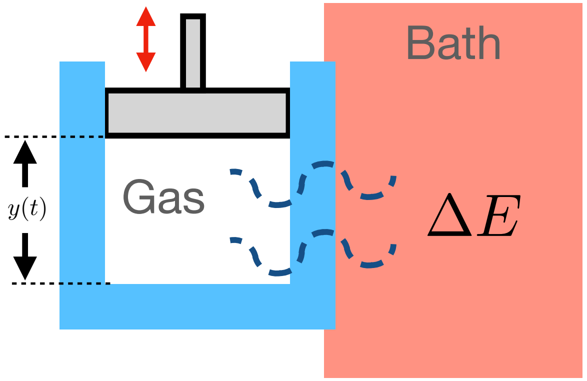


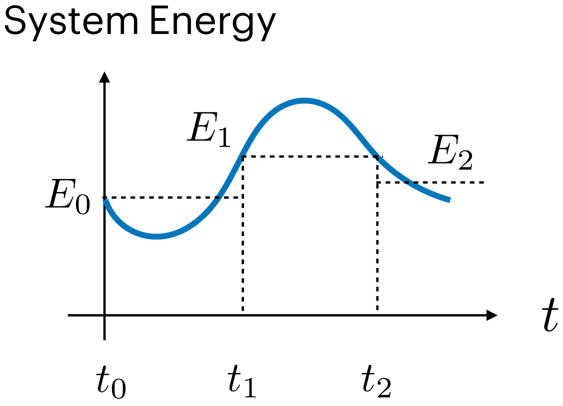


$$E_2 > E_1 > E_0$$

$$\rho_{\mathrm{non-eq.}}(\infty) \rightarrow \frac{1}{Z}$$

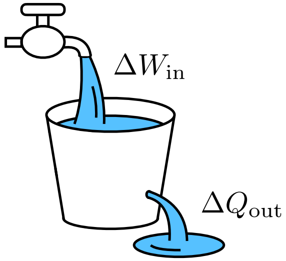
Open driven systems approach non-trivial steady states at late times



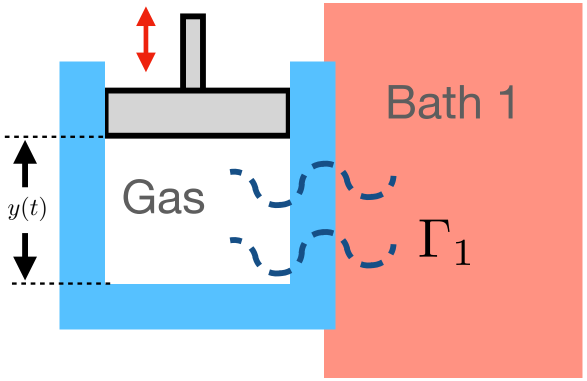


$$\rho_{\mathrm{non-eq.}}(t)$$

Uniquely determined by **state** and **nature** of bath and **y(t)**



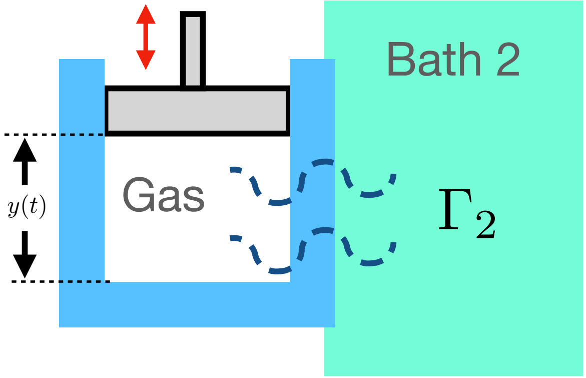
The nature of bath matters away from equilibrium



In equilibrium in limit of weak coupling to baths, nature of bath does not matter

$$\Gamma_1 \to 0 \qquad \Gamma_2 \to 0$$

$$\rho^{(1)}_{\text{equil}} = \rho^{(2)}_{\text{equil}} = \frac{e^{-\beta H}}{Z}$$



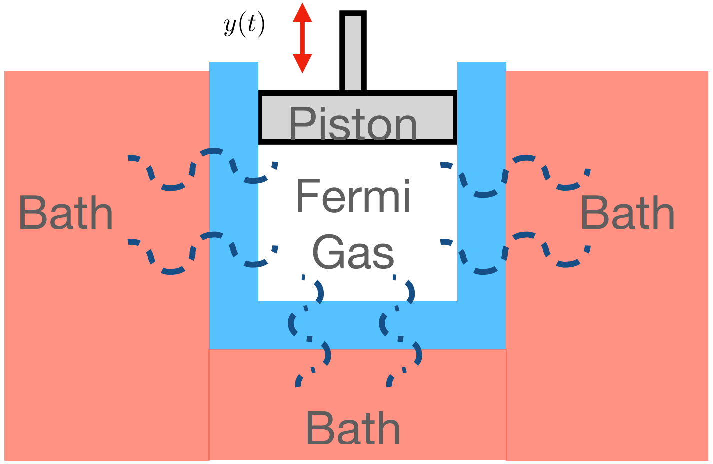
In **non-equilibrium** even in limit of weak coupling to baths, **nature of bath matters** for the steady state

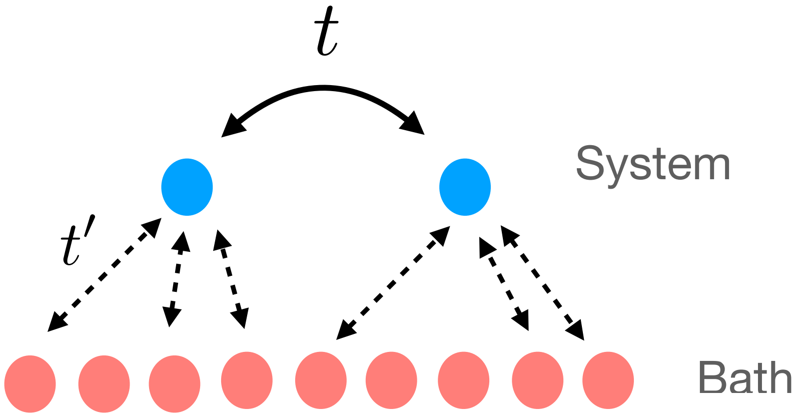
$$\rho^{(1)}_{\text{non}-\text{eq.}}(t) \neq \rho^{(2)}_{\text{non}-\text{eq.}}(t)$$

Non-interacting fermionic Baths & Open Schrödinger equation

arXiv:2309.03268 (2023)
Phys. Rev. B **107**, 195135 (2023)

Consider non-interacting fermions





One-fermion Hilbert space is a sum

$$\boxed{H^{\text{one body}} = H^{\text{system}} \bigoplus H^{\text{bath}}}$$

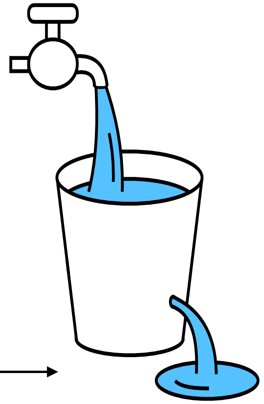
$$|\psi(t)\rangle = \begin{bmatrix} |\psi_S(t)\rangle \\ |\psi_B(t)\rangle \end{bmatrix}$$

$$H(t) = \begin{bmatrix} H_S(t) & H_{SB}(t) \\ H_{BS}(t) & H_B(t) \end{bmatrix}$$

Bath feedback effect

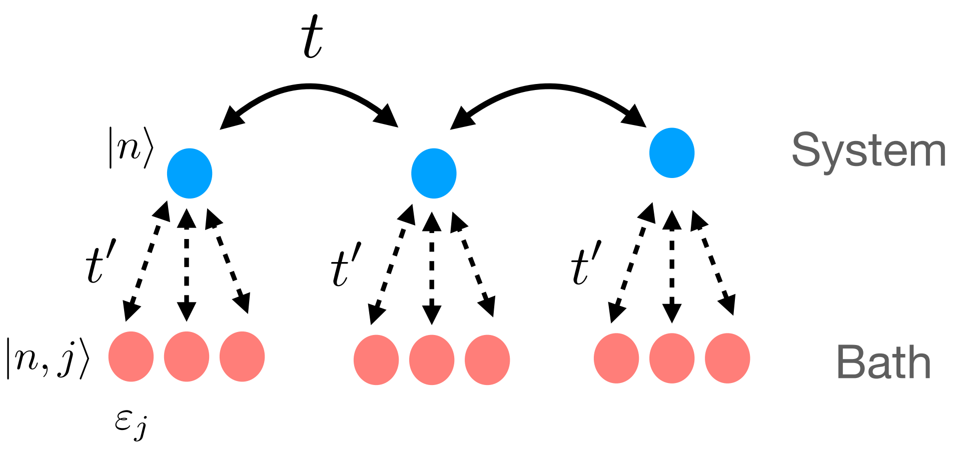
$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)U_B(t,t_0)|\psi_B(t_0)\rangle - iH_{SB}(t)\int_{t_0}^{t} \mathrm{d}t'\ U_B(t,t')H_{BS}(t')|\psi_S(t')\rangle$$

Decay, memory and energy renormalization effects



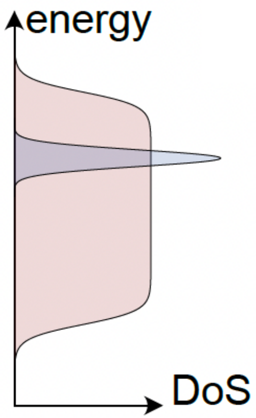
Featureless fermionic bath

arXiv:2309.03268 (2023)
Phys. Rev. B **107**, 195135 (2023)



$$\nu_B(\omega_b) = 2\pi \sum_j \delta(\omega_b - \varepsilon_j) \equiv \nu_0$$

Bath spectrum flat and infinitely broad



Initial condition: Bath equilibrium at $\rho(t_0) = \sum_{n,j} f_0(\epsilon_j)|n,j\rangle\langle n,j|$

μ_0 $\quad \beta_0 = \frac{1}{k_B T_{\text{bath}}}$

Non-hermitian but inhomogeneous Schrödinger

$$i\partial_t|\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)U_B(t,t_0)|\psi_B(t_0)\rangle - iH_{SB}(t)\int_{t_0}^{t} \mathrm{d}t'\ U_B(t,t')H_{BS}(t')|\psi_S(t')\rangle$$

Pure decay (no memory)

$$i\partial_t|\psi_n^{(j)}(t)\rangle = \left[H_S(t) - i\Gamma\right]|\psi_n^{(j)}(t)\rangle + t'\exp[-i\varepsilon_j(t-t_0)]\,|\,n\,\rangle$$

Bath feedback effect

Bath DOS

Decay rate: $\Gamma = \frac{\nu_0 t'^2}{2}$

Tunneling amplitude

Featureless Fermionic Bath

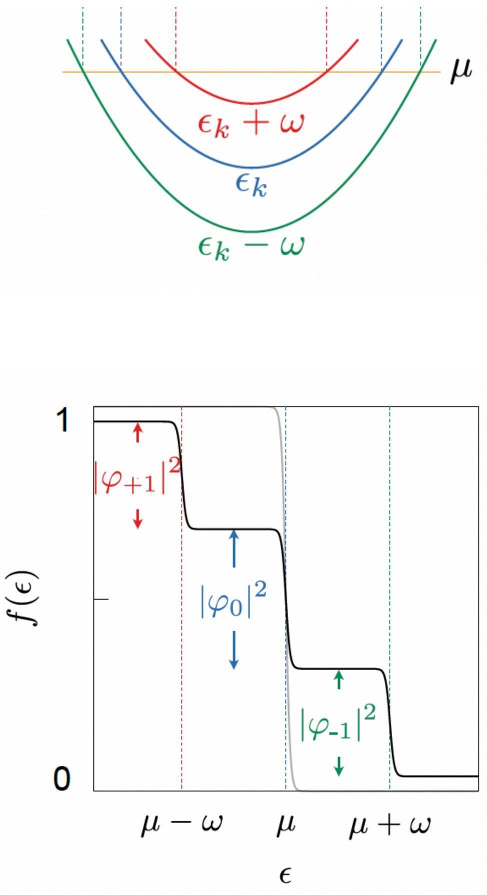
Decay rate: $\Gamma = \dfrac{\overset{\text{Bath DOS}}{\downarrow}\;\nu_0 t'^2}{2} \longleftarrow$ Tunneling amplitude

Exact non-equilibrium steady-state of **driven non-interacting fermions**

$$\rho_{\text{non-eq.}}(t) = \Gamma \int_{-\infty}^{+\infty} \frac{\mathrm{d}\epsilon}{\pi} f_0(\epsilon) U_\Gamma(t,\epsilon) U_\Gamma^\dagger(t,\epsilon)$$

$$U_\Gamma(t,\epsilon) = \int_{-\infty}^{t} \mathrm{d}t' e^{\Gamma(t'-t) - i\epsilon t'} U_S(t,t') \qquad i\partial_t U_S(t,t') = H_S(t) U_S(t,t')$$

$$f_0(\epsilon) = 1/[1 + e^{\beta}(\epsilon - \mu)]$$



Floquet Periodic Gibbs Ensemble with Fermi Dirac Staircase:

Floquet problem:

$$H_S(t) = H_S(t+T)$$

$$\Omega = 2\pi/T$$

$$|\psi_a^F(t)\rangle = \sum_n e^{-i\epsilon_a^F t - in\Omega t} |\varphi_{a,n}\rangle$$

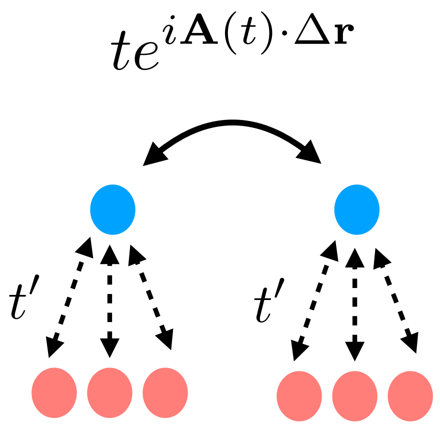
Weak coupling to bath: $\Gamma \to 0$

$$\lim_{\Gamma\to 0} \rho_S(t) = \sum_a p_a |\psi_a^F(t)\rangle \langle \psi_a^F(t)| ,$$

$$p_a = \sum_n |\varphi_{a,n}|^2 f_0(\epsilon_a^F + n\Omega),$$

Lazarides, Das, & Moessner, Phys. Rev. Lett. **112**, 150401 (2014).

Floquet Fermi Liquid in a simple Bloch band



Single Bloch band under
monochromatic field

$$\mathbf{A}(t) = -\frac{i}{\omega}\mathbf{E}_\omega \exp(-i\omega t) + \mathrm{c.\ c.}$$

$$H_S(t) = \int_{\mathbf{k}} \epsilon_{\mathbf{k}}(t) \left|\chi_{\mathbf{k}}\right\rangle \left\langle\chi_{\mathbf{k}}\right|,$$

$$\epsilon_{\mathbf{k}}(t) \equiv \epsilon(\mathbf{k} - \mathbf{A}(t)), \qquad \int_{\mathbf{k}} \equiv \int_{\mathrm{BZ}} \frac{\mathrm{d}\mathbf{k}}{(2\pi)^d}$$

Floquet Energy $\quad \bar{\epsilon}_{\mathbf{k}} \equiv \epsilon^{(0)}_{\mathbf{k}} = \int_0^T \frac{\mathrm{d}t}{T}\epsilon(\mathbf{k} - \mathbf{A}(t))$

Floquet
Wavefunction $\quad \phi_{\mathbf{k}}(t) = \sum_{l=-\infty}^{+\infty} \phi^{(l)}_{\mathbf{k}} \exp[-il\omega(t - t_0)]$

Steady state at finite Γ

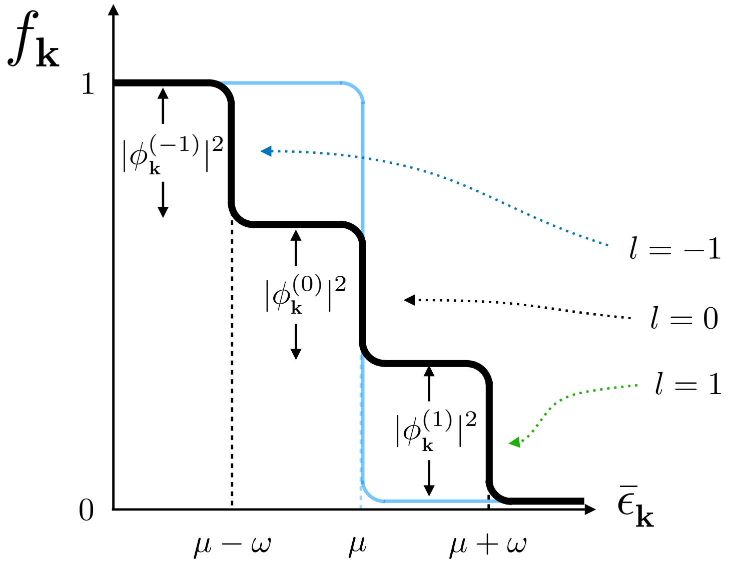
$$f_{\mathbf{k}}(t) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_b}{\pi} f_0(\omega_b) \ \Gamma \left| \sum_{l=-\infty}^{+\infty} \phi^{(l)}_{\mathbf{k}} \frac{\exp[-il\omega(t - t_0)]}{\bar{\epsilon}_{\mathbf{k}} - \omega_b - l\omega - i\Gamma} \right|^2$$

Ideal bath limit

Fermi Dirac Staircase

$$f_{\mathbf{k}}(t) \xrightarrow{\Gamma \to 0} \boxed{f_{\mathbf{k}} \equiv \sum_{l=-\infty}^{+\infty} |\phi^{(l)}_{\mathbf{k}}|^2 f_0(\bar{\epsilon}_{\mathbf{k}} - l\omega)}$$

arXiv:2309.03268 (2023)
Phys. Rev. B 107, 195135 (2023)



See e.g. also:

Seetharam, Bardyn, Lindner, Rudner & Refael. Physical Review X 5, 041050 (2015).

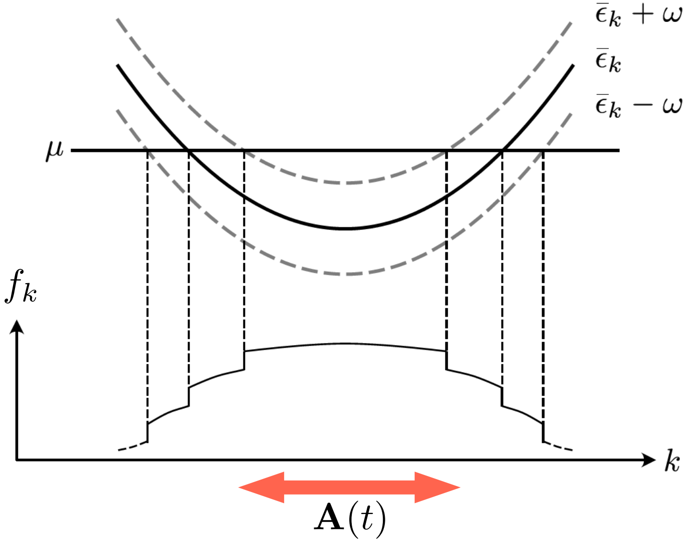
Ratchet effect (rectified current) in Bloch bands

$$\epsilon_{\mathbf{k}}(t) \equiv \epsilon(\mathbf{k} - \mathbf{A}(t))$$

$$\boxed{f_k \equiv \sum_{l=-\infty}^{+\infty} |\phi_{\mathbf{k}}^{(l)}|^2 f_0(\bar{\epsilon}_{\mathbf{k}} - l\omega)}$$

Floquet Fermi Liquid

Physical momentum still oscillates



Electric current

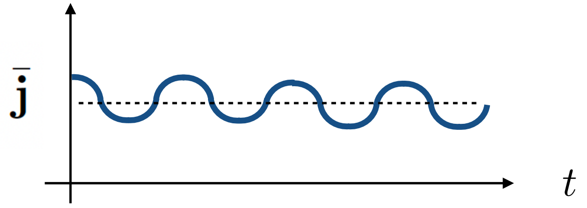
$$\mathbf{j}(t) = \int_{\mathbf{k}} f_k \ \nabla_{\mathbf{k}} \epsilon_{\mathbf{k}}(t)$$

Time-average (rectified) current

$$\bar{\mathbf{j}} = \int_0^T \frac{\mathrm{d}t}{T} \mathbf{j}(t) = \int_{\mathbf{k}} f_k \ \nabla_{\mathbf{k}} \bar{\epsilon}_{\mathbf{k}}$$

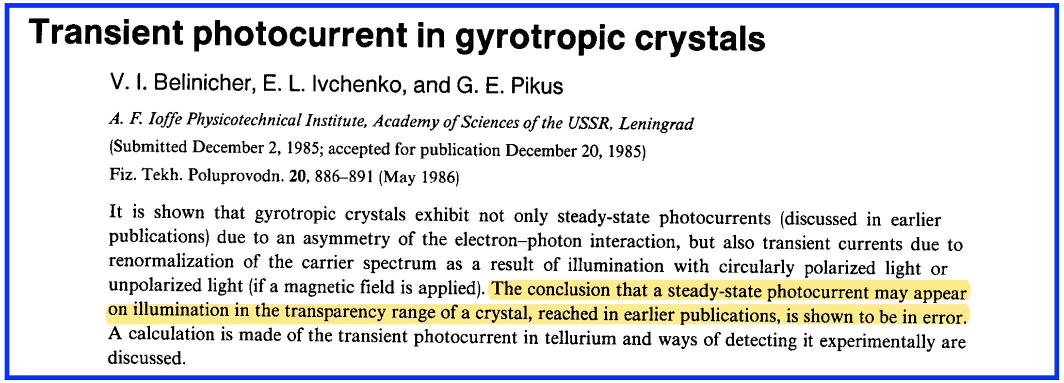
In contrast to equilibrium:

$$f_k \neq f(\bar{\epsilon}_k) \quad \Rightarrow \quad \bar{\mathbf{j}} \neq 0$$



Longstanding confusion in optoelectronics

Is rectification allowed in the optical gap in the ideal limit of vanishingly small relaxation rates?



Transient photocurrent in gyrotropic crystals

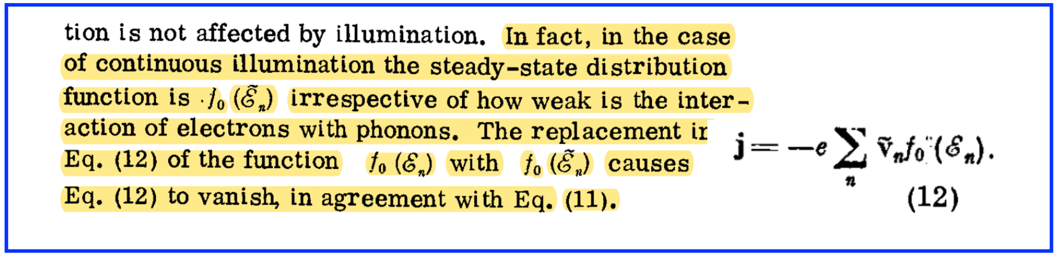
V. I. Belinicher, E. L. Ivchenko, and G. E. Pikus

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

(Submitted December 2, 1985; accepted for publication December 20, 1985)

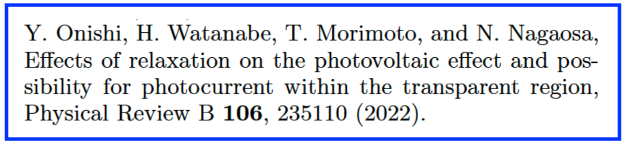
Fiz. Tekh. Poluprovodn. **20**, 886–891 (May 1986)

It is shown that gyrotropic crystals exhibit not only steady-state photocurrents (discussed in earlier publications) due to an asymmetry of the electron–photon interaction, but also transient currents due to renormalization of the carrier spectrum as a result of illumination with circularly polarized light or unpolarized light (if a magnetic field is applied). The conclusion that a steady-state photocurrent may appear on illumination in the transparency range of a crystal, reached in earlier publications, is shown to be in error. A calculation is made of the transient photocurrent in tellurium and ways of detecting it experimentally are discussed.

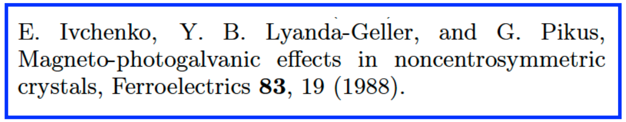


tion is not affected by illumination. In fact, in the case of continuous illumination the steady-state distribution function is $f_0(\tilde{\mathscr{E}}_n)$ irrespective of how weak is the interaction of electrons with phonons. The replacement in Eq. (12) of the function $f_0(\mathscr{E}_n)$ with $f_0(\tilde{\mathscr{E}}_n)$ causes Eq. (12) to vanish, in agreement with Eq. (11).

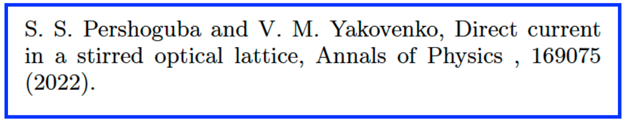
$$\mathbf{j} = -e\sum_n \tilde{\mathbf{v}}_n f_0(\mathscr{E}_n). \qquad (12)$$



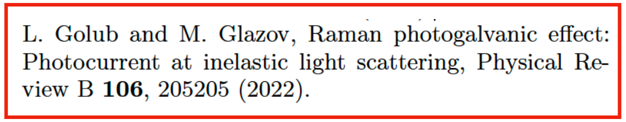
Y. Onishi, H. Watanabe, T. Morimoto, and N. Nagaosa, Effects of relaxation on the photovoltaic effect and possibility for photocurrent within the transparent region, Physical Review B **106**, 235110 (2022).



E. Ivchenko, Y. B. Lyanda-Geller, and G. Pikus, Magneto-photogalvanic effects in noncentrosymmetric crystals, Ferroelectrics **83**, 19 (1988).



S. S. Pershoguba and V. M. Yakovenko, Direct current in a stirred optical lattice, Annals of Physics , 169075 (2022).

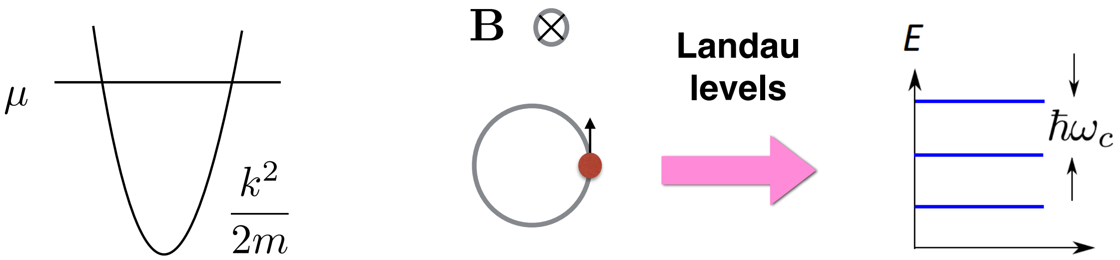


L. Golub and M. Glazov, Raman photogalvanic effect: Photocurrent at inelastic light scattering, Physical Review B **106**, 205205 (2022).

What are we NOT talking about:

1. *NOT Frequency difference effect. Our light is exactly monochromatic.*
2. *NOT a high order multi-photon absorption. Our effect occurs at the leading perturbative order* $\mathbf{j} \propto \mathbf{E}^2$
3. *Clean limit. E.g. NO in-gap impurities and NOT the "tail" of Drude or inter-band absorptions.*

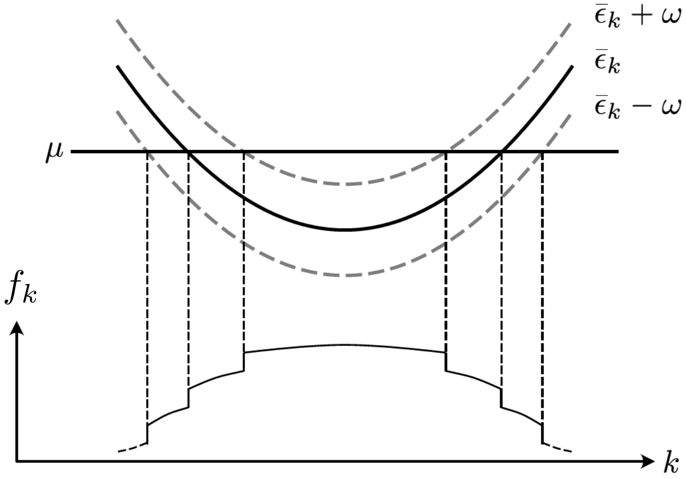
Quantum Oscillations of Floquet Fermi Liquids



$$\omega_c = \frac{B}{m}$$

Quantum oscillations

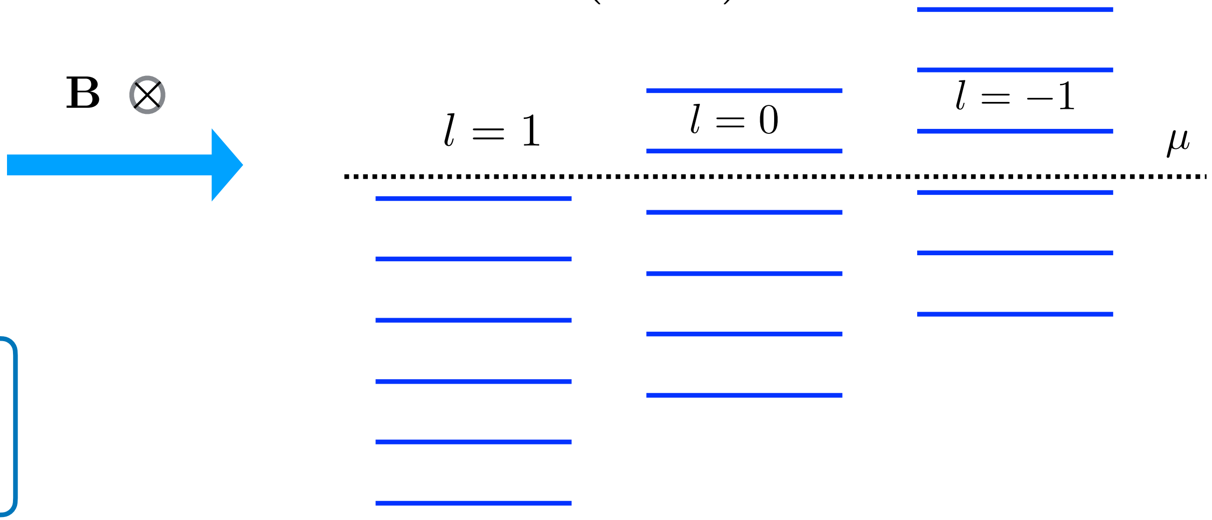
$$\sim Cos\left(\frac{2\pi\mu}{\omega_c}\right)$$



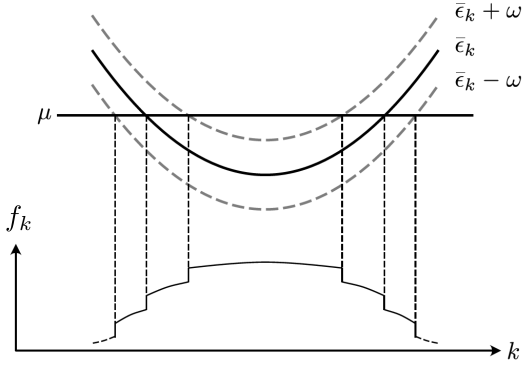
$$\boxed{f_k \equiv \sum_{l=-\infty}^{+\infty} |\phi_{\mathbf{k}}^{(l)}|^2 f_0(\bar{\epsilon}_{\mathbf{k}} - l\omega)}$$

Floquet Landau level spectrum

$$\omega_c\left(N + \frac{1}{2}\right) + \delta E - l\omega$$



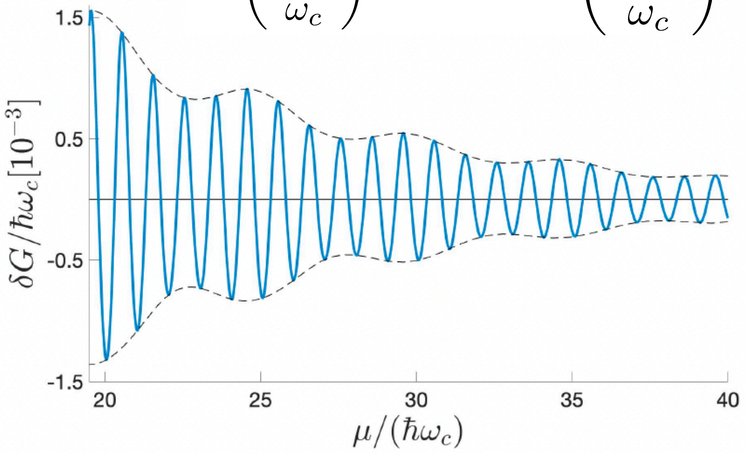
Quantum Oscillations of Floquet Fermi Liquids



Same frequency as MIRO

Lower frequency beating

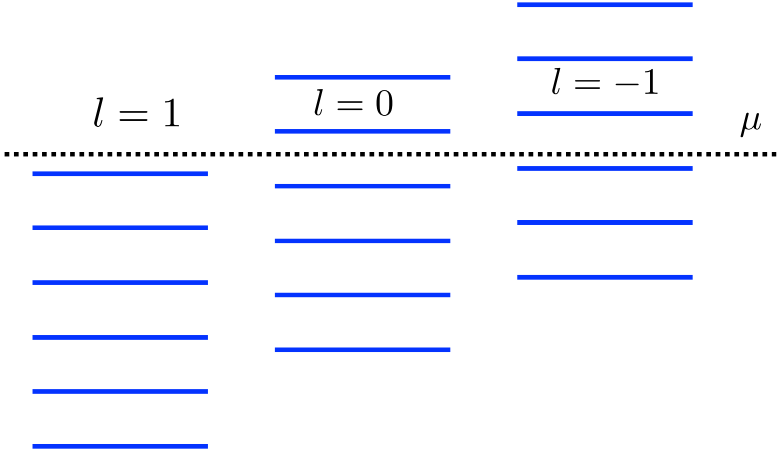
$$Cos\left(\frac{2\pi\mu}{\omega_c}\right) \qquad \sim Cos\left(\frac{2\pi\omega}{\omega_c}\right)$$



$\mathbf{B} \otimes$

Floquet Landau level spectrum

$$\omega_c\left(N+\frac{1}{2}\right)+\delta E-l\omega$$



Oscillations as if there were three Fermi surfaces of different area

Oscillations of DOS

$$\delta\nu \approx 2\sum_{k=1}^{\infty}(-1)^k R_T(k) F_E \cos\left(k\frac{S}{B}\right)$$

Agrees with MIRO resistivity oscillations theory:

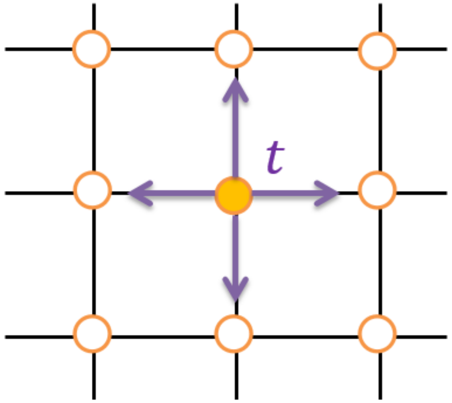
I. A. Dmitriev, Nonequilibrium magnetooscillations in spatially non-uniform quantum hall systems, Journal of Physics: Conference Series **334**, 012015 (2011).

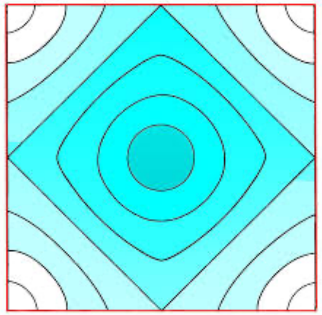
See however phase puzzle:

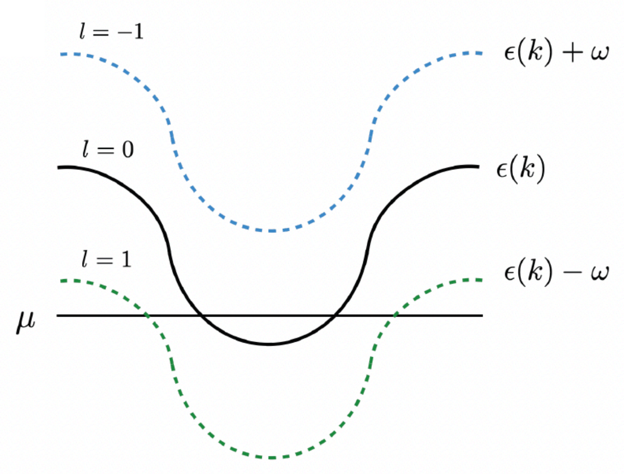
Q. Shi, P. D. Martin, A. T. Hatke, M. A. Zudov, J. D. Watson, G. C. Gardner, M. J. Manfra, L. N. Pfeiffer, and K. W. West, Shubnikov–de haas oscillations in a two-dimensional electron gas under subterahertz radiation, Phys. Rev. B **92**, 081405 (2015).

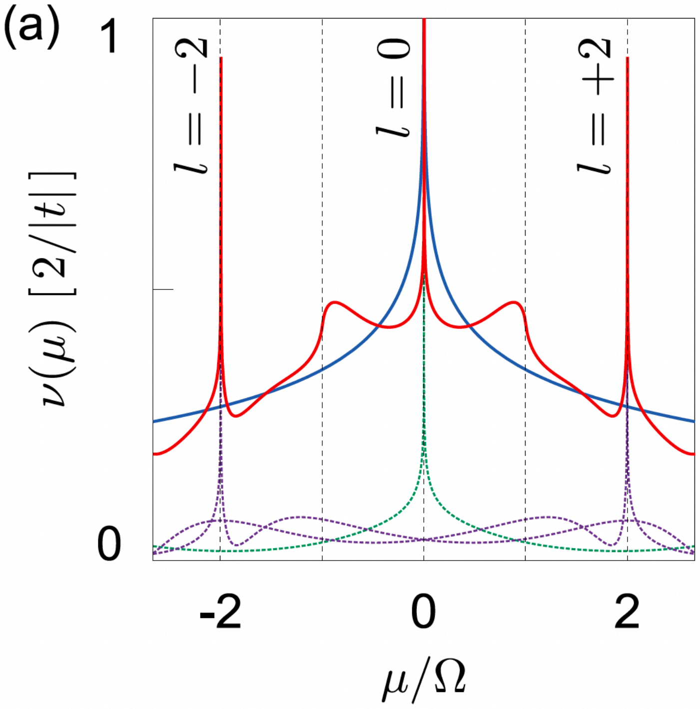
DoS and Floquet Van Hove singularities

Square lattice: van hove at 1/2-filling







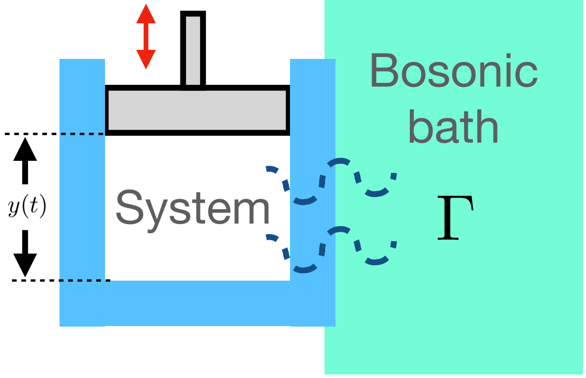


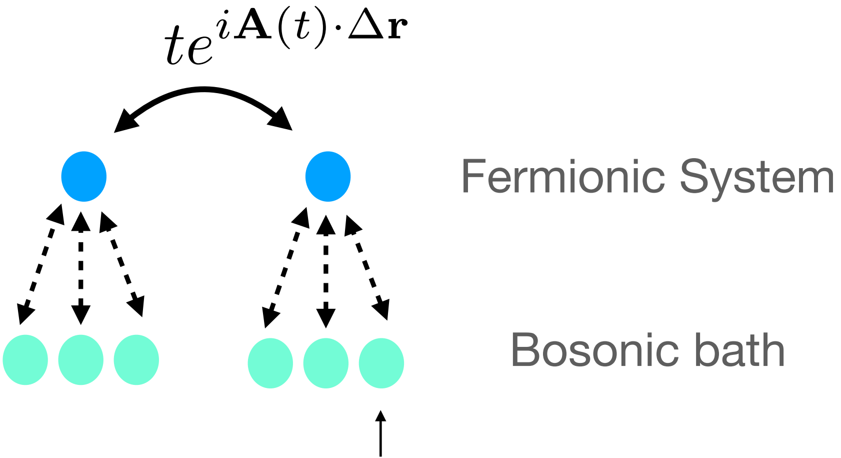
System will have weak coupling instability to try to gap out one of Floquet Fermi surfaces

arXiv:2309.03268 (2023)
Phys. Rev. B 107, 195135 (2023)

Floquet Non-Fermi liquid for ideal bosonic baths

Shi, Matsyshyn, Song, Sodemann unpublished





$\chi c_{\mathbf{r}}^{\dagger} c_{\mathbf{r}} b_{\mathbf{r},\lambda} \longrightarrow$

$\omega_\lambda b_{\mathbf{r},\lambda}^{\dagger} b_{\mathbf{r},\lambda}$ Collection of local harmonic oscillators

Boltzmann equation for one-body density matrix ρ

$$\partial_t \rho + i[h_{sys}(t), \rho] = J[\rho, t]$$

Periodic driving

Bath induced “transitions“

$$J[\rho, t] = J_{ems}[\rho, t] + J_{abs}[\rho, t]$$

Emission “transitions“

Absorption “transitions“

Fedir T. Vasko • Oleg E. Raichev

Quantum Kinetic Theory and Applications

Electrons, Photons, Phonons

$$\widehat{J}_{e,b}^{(e)}(\hat{\rho}|t) = \frac{1}{\hbar^2}\sum_q \int_{-\infty}^{t} dt' e^{\lambda t'} (N_{qt'}+1)$$

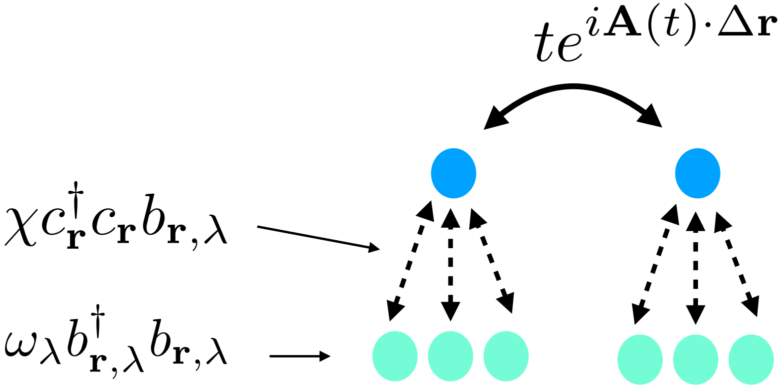
$$\times \left\{ e^{-i\omega_q(t-t')} \left[\widehat{S}(t,t') \left((1-\hat{\rho}_{t'})\hat{\chi}_{qt'}^{+}\hat{\rho}_{t'} + \hat{\rho}_{t'}\mathrm{sp}\hat{\chi}_{qt'}^{+}\hat{\rho}_{t'} \right) \widehat{S}^{+}(t,t'), \hat{\chi}_{qt} \right] \right.$$

$$\left. - e^{i\omega_q(t-t')} \left[\widehat{S}(t,t') \left(\hat{\rho}_{t'}\hat{\chi}_{qt'}(1-\hat{\rho}_{t'}) + \hat{\rho}_{t'}\mathrm{sp}\hat{\chi}_{qt'}\hat{\rho}_{t'} \right) \widehat{S}^{+}(t,t'), \hat{\chi}_{qt'}^{+} \right] \right\}.$$

$$(N_{qt'}+1) \to N_{qt'}, \quad \omega_q \to -\omega_q, \quad \hat{\chi}_{qt}^{+} \to \hat{\chi}_{qt}.$$

$$\widehat{J}_{e,b}^{(e)}(\hat{\rho}|t) \longrightarrow \widehat{J}_{e,b}^{(a)}(\hat{\rho}|t)$$

Floquet Non-Fermi liquid for ideal bosonic baths



Single Bloch band under monochromatic field

$$\mathbf{A}(t) = -\frac{i}{\omega}\mathbf{E}_\omega \exp(-i\omega t) + \text{c. c.}$$

$$H_S(t) = \int_{\mathbf{k}} \epsilon_{\mathbf{k}}(t)\ \ |\mathbf{k}\rangle\langle\mathbf{k}|$$

$$\epsilon_{\mathbf{k}}(t) \equiv \epsilon(\mathbf{k} - \mathbf{A}(t)), \qquad \int_{\mathbf{k}} \equiv \int_{\mathrm{BZ}} \frac{\mathrm{d}\mathbf{k}}{(2\pi)^d}$$

$$\rho = \int_{\mathbf{k}} f_{\mathbf{k}}(t)|\mathbf{k}\rangle\langle\mathbf{k}|$$

Boltzmann Equation

$$\partial_t \rho + i[h_{sys}(t), \rho] = J[\rho, t]$$

$$f_{\mathbf{k}}(t) \xrightarrow{\chi \to 0} f_{\mathbf{k}}$$

Ideal bath limit

$$0 = J[f_{\mathbf{k}}] = \sum_{\mathbf{k}'} W_{\mathbf{k}\to\mathbf{k}'} f_{\mathbf{k}}(1 - f_{\mathbf{k}'}) - W_{\mathbf{k}'\to\mathbf{k}} f_{\mathbf{k}'}(1 - f_{\mathbf{k}})$$

Floquet Non-Fermi Liquid for ideal bosonic baths

Boltzmann Equation

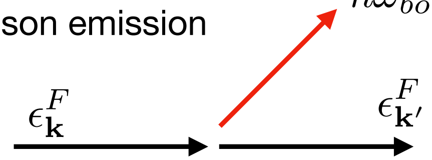
$$0 = J[f_{\mathbf{k}}] = \sum_{\mathbf{k}'} W_{\mathbf{k}\to\mathbf{k}'} f_{\mathbf{k}}(1 - f_{\mathbf{k}'}) - W_{\mathbf{k}'\to\mathbf{k}} f_{\mathbf{k}'}(1 - f_{\mathbf{k}})$$

$$W_{\mathbf{k}'\to\mathbf{k}} = W^{emi}_{\mathbf{k}'\to\mathbf{k}} + W^{abs}_{\mathbf{k}'\to\mathbf{k}}$$

$$W^{emi}_{\mathbf{k}\to\mathbf{k}'} = \sum_{l} \Theta(\epsilon^F_{\mathbf{k}} - \epsilon^F_{\mathbf{k}'} - l\omega)\Gamma^{\mathrm{emi}}(\epsilon^F_{\mathbf{k}} - \epsilon^F_{\mathbf{k}'} - l\omega)\Big|\sum_{l'} \phi^*_{l+l'}(\mathbf{k}')\phi_l(\mathbf{k})\Big|^2$$

Many transition channels open up :

Boson emission



$\hbar\omega_{boson} = \epsilon^F_{\mathbf{k}} - \epsilon^F_{\mathbf{k}'} - l\omega$

Detailed balanced violated

$$\Gamma^{\mathrm{emi}}(\epsilon) = \chi^2(\epsilon)\nu_{\mathrm{DOS}}(\epsilon)(N_{\mathrm{Bose}}(\epsilon) + 1)$$

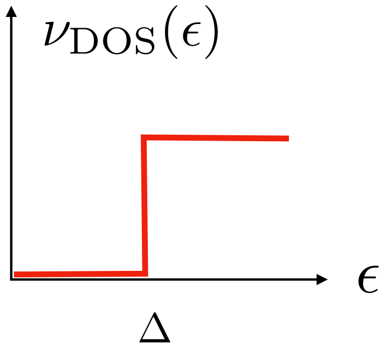
See e.g. also:

Seetharam, Bardyn, Lindner, Rudner & Refael. Physical Review X 5, 041050 (2015).

Esin, Rudner, Refael, Lindner, Physical Review B, 97(24), p.245401 (2018).

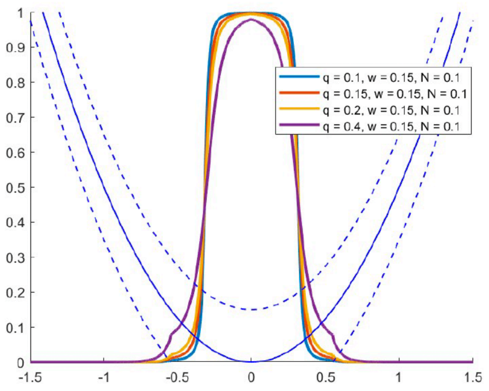
Floquet Non-Fermi Liquid for ideal bosonic baths

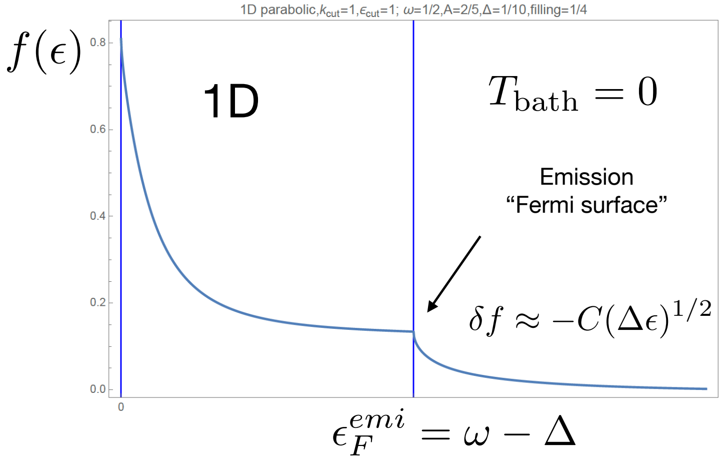
Case 1: gapped bath

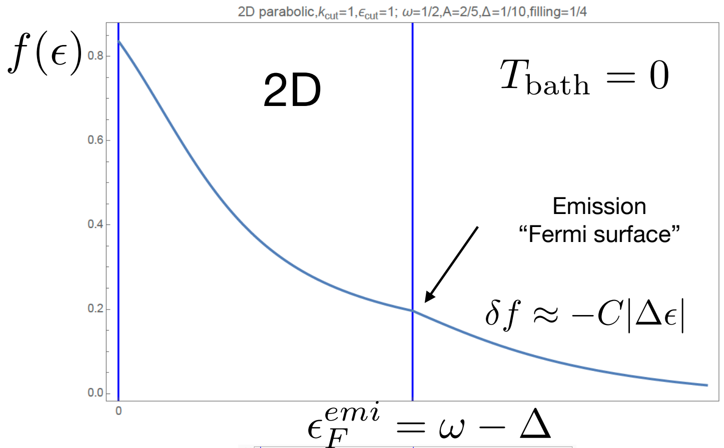


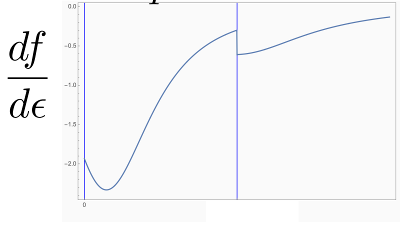
Boltzmann Equation

$$0 = \sum_{\mathbf{k}'} W_{\mathbf{k}\to\mathbf{k}'} f_{\mathbf{k}}(1 - f_{\mathbf{k}'}) - W_{\mathbf{k}'\to\mathbf{k}} f_{\mathbf{k}'}(1 - f_{\mathbf{k}})$$



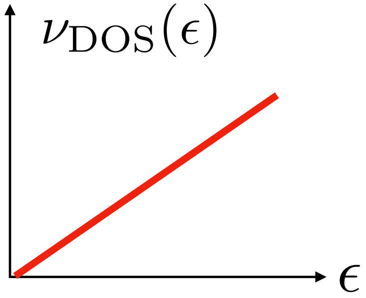






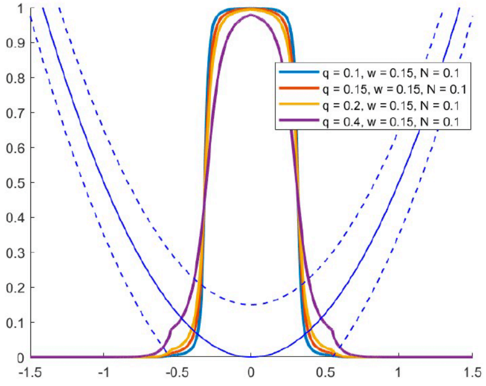
Floquet Non-Fermi Liquid for ideal bosonic baths

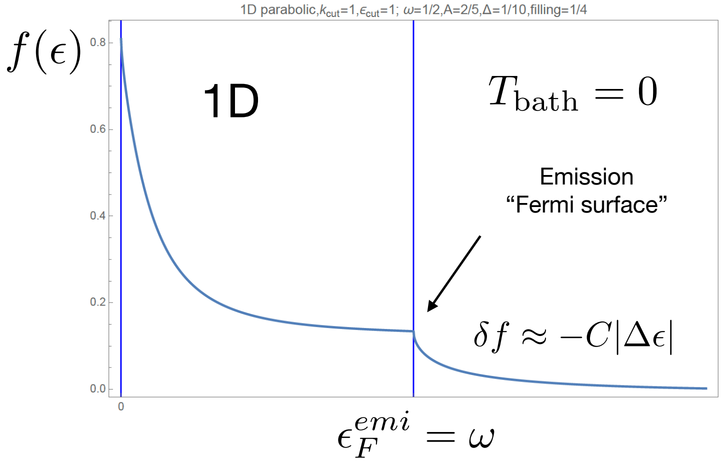
Case 2: Ohmic bath



Boltzmann Equation

$$0 = \sum_{\mathbf{k}'} W_{\mathbf{k}\to\mathbf{k}'} f_{\mathbf{k}}(1-f_{\mathbf{k}'}) - W_{\mathbf{k}'\to\mathbf{k}} f_{\mathbf{k}'}(1-f_{\mathbf{k}})$$





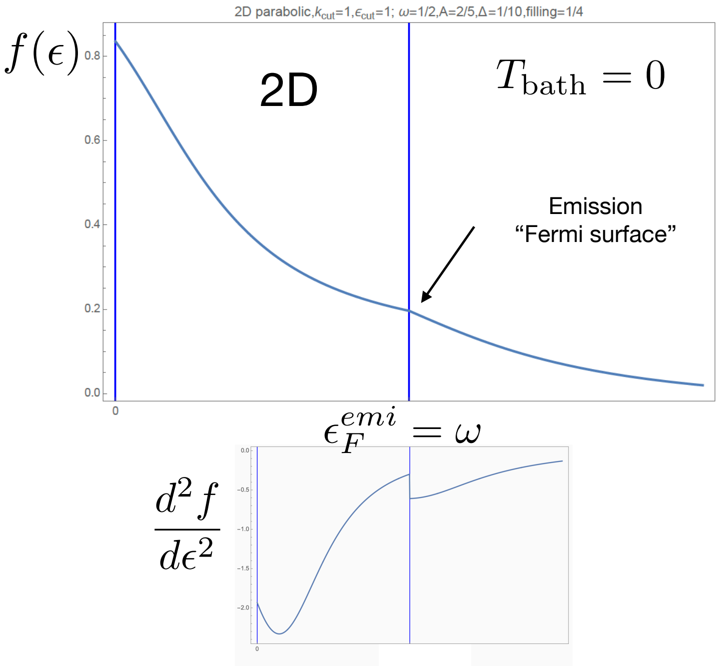
1D

$T_{\text{bath}} = 0$

Emission
"Fermi surface"

$\delta f \approx -C|\Delta\epsilon|$

$\epsilon_F^{emi} = \omega$



2D

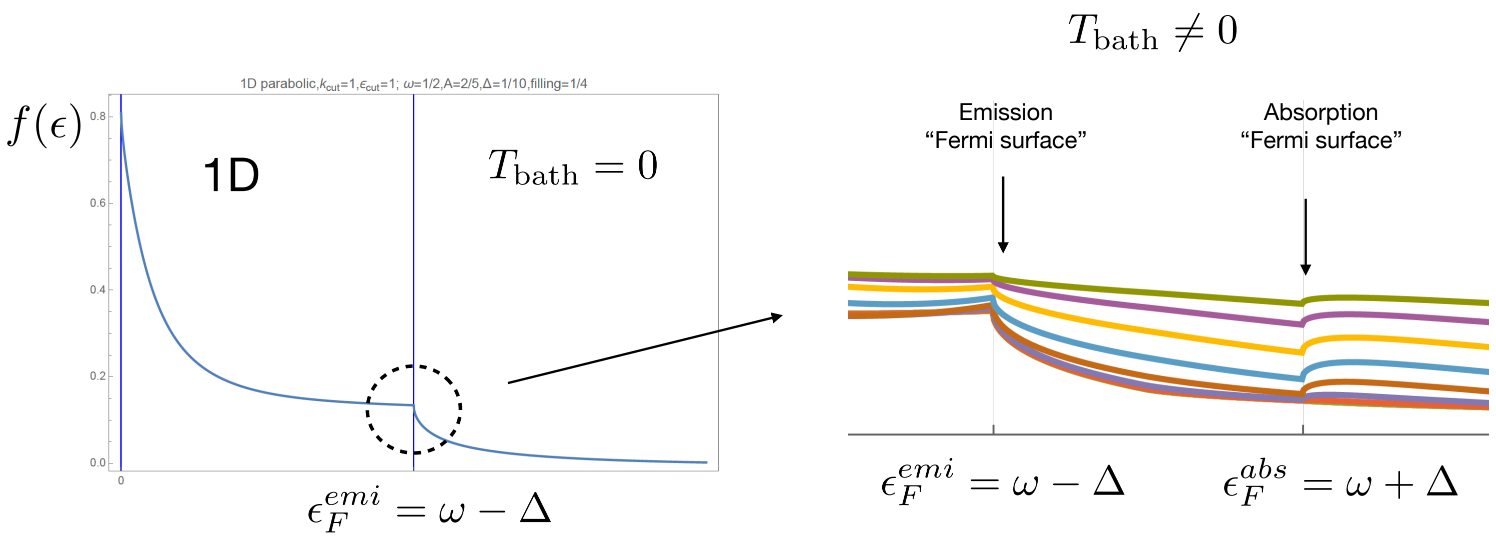
$T_{\text{bath}} = 0$

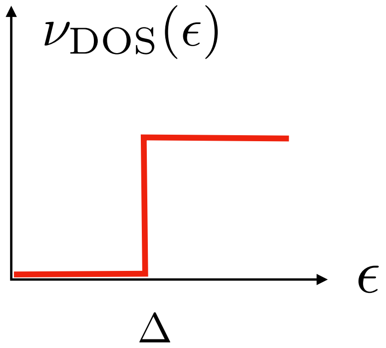
Emission
"Fermi surface"

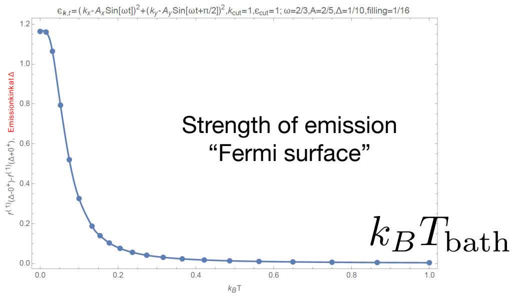
$\epsilon_F^{emi} = \omega$

Floquet Non-Fermi Liquid for ideal bosonic baths

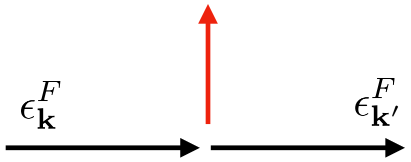
Case 1: gapped bath



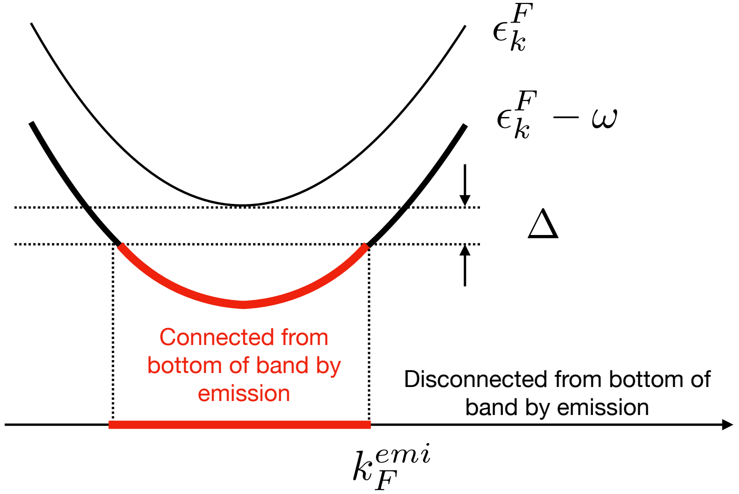


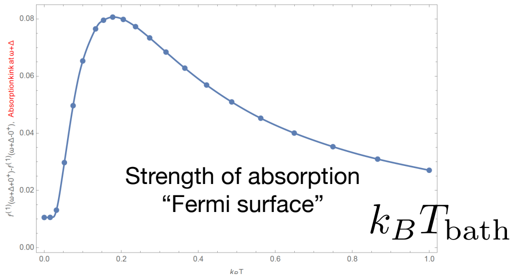


$$\hbar\omega_{boson}=\epsilon^F_{\mathbf{k}}-\epsilon^F_{\mathbf{k}'}-l\omega$$

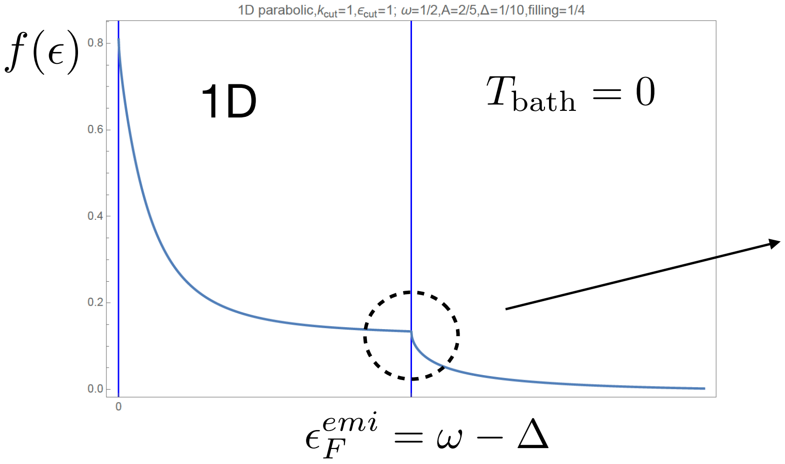


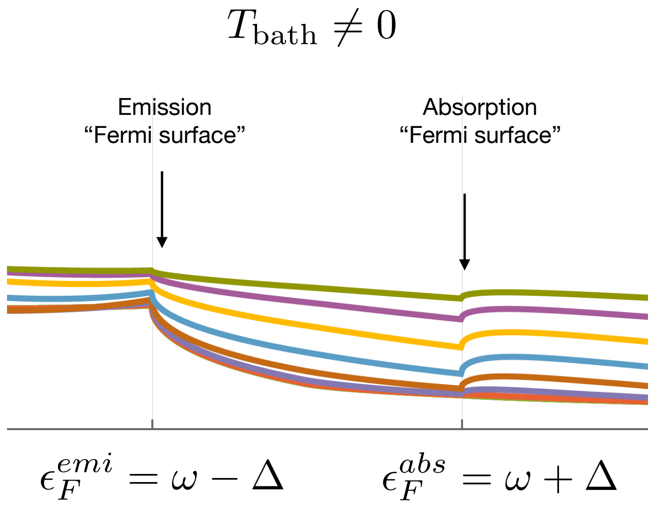
$$\hbar\omega_{boson}=\Delta=-\epsilon^F_{\mathbf{k}'}+\omega$$
$$l=-1$$



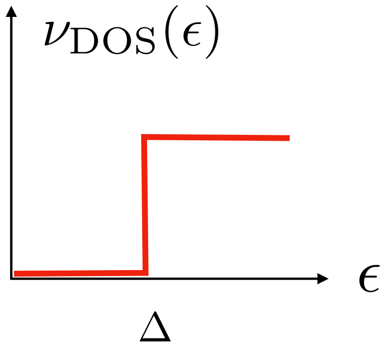


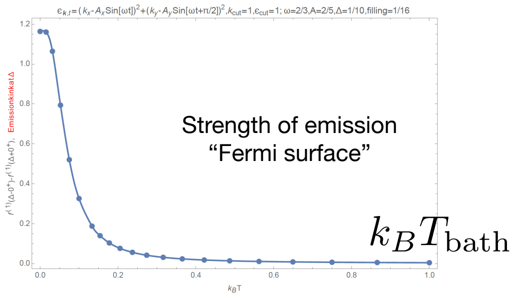
Floquet Non-Fermi Liquid for ideal bosonic baths



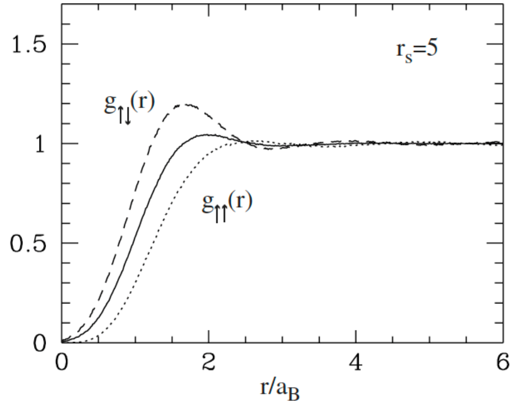


Case 1: gapped bath



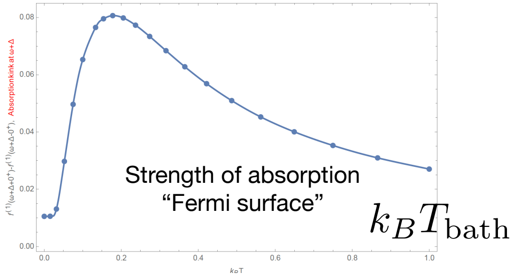


Two-point correlations Fermi fluids:



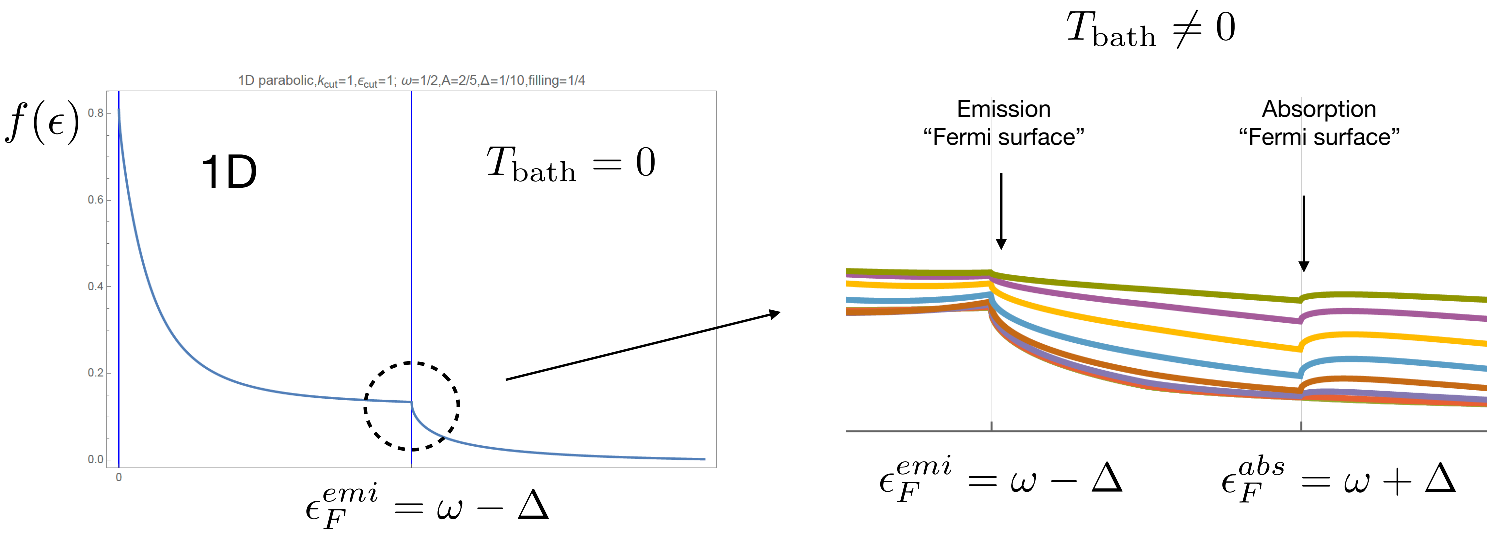
	Equilibrium	Ultra-quantum-critical Floquet Non-Fermi liquid
1D	$\frac{\sin(k_F r)^2}{r^2}$	$\frac{\sin(k_F r)^2}{r^4}$
2D	$\frac{\sin(k_F r)^2}{r^3}$	$\frac{\sin(k_F r)^2}{r^5}$

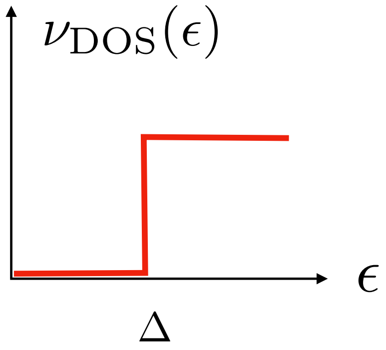
$$g(\vec{r}_2, \vec{r}_1) \equiv \frac{1}{n(\vec{r}_2)n(\vec{r}_1)} \left\langle \sum_{i \neq j} \delta(\vec{r}_1 - \vec{r}_i)\delta(\vec{r}_2 - \vec{r}_j) \right\rangle$$

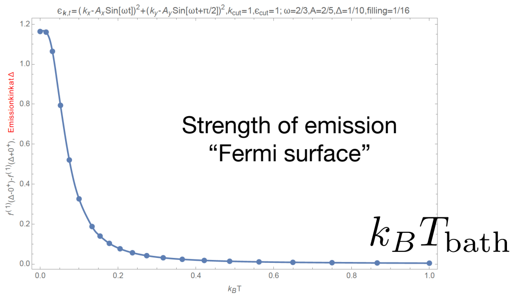


Floquet Non-Fermi Liquid for ideal bosonic baths

Case 1: gapped bath





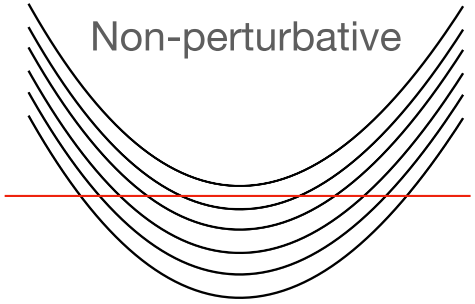


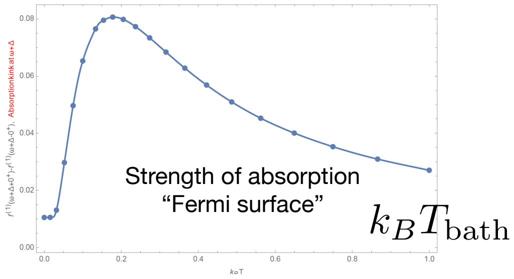
How can the system remain quantum ?

$$k_B T_{\text{bath}} \gg E_F \qquad k_B T_{\text{bath}} \gg \hbar\omega$$

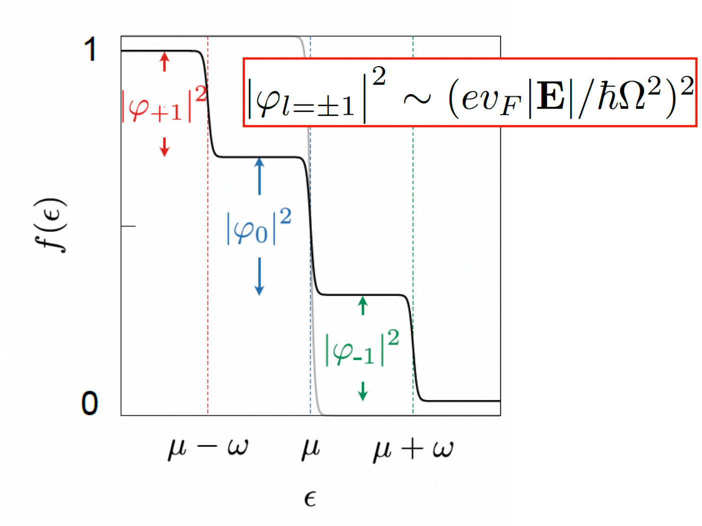
Work per cycle remains quantum

$$\hbar\omega \gg \Delta W_{\text{cycle}} \sim \frac{vE}{\omega}$$





Experimental realization



$$\left|\varphi_{l=\pm 1}\right|^2 \sim (ev_F|\mathbf{E}|/\hbar\Omega^2)^2$$

1) Frequency needs to exceed thermal and other broadening scales

$$\hbar\omega > k_B T \qquad \hbar\omega > \Gamma_0$$

2) Light intensity controls height of jumps. Intensity needs to be comparable to intensity scale:

$$\boxed{I \sim I_0 = \frac{\hbar\omega^4}{8\pi\alpha v_F^2}} \qquad \alpha \approx 1/137$$

Pump probe mid infrared Floquet Physics

Y. Wang, H. Steinberg, P. Jarillo-Herrero, and N. Gedik, Observation of floquet-bloch states on the surface of a topological insulator, Science **342**, 453 (2013).

$$\hbar\omega = 120 \text{ meV}$$

$$I_0 \approx 4 \times 10^{12}\text{W}/\text{m}^2$$

J. W. McIver, B. Schulte, F.-U. Stein, T. Matsuyama, G. Jotzu, G. Meier, and A. Cavalleri, Light-induced anomalous hall effect in graphene, Nature physics **16**, 38 (2020).

$$\left|\varphi_{l=\pm 1}\right|^2 \sim (ev_F|\mathbf{E}|/\hbar\Omega^2)^2 \sim 0.25$$

Floquet Physics with microwaves!

$$\frac{\omega}{2\pi} = 10 \text{ GHz}$$

$$I_0 \approx 0.2\text{W}/\text{m}^2$$

I. Dmitriev, A. Mirlin, D. Polyakov, and M. Zudov, Nonequilibrium phenomena in high landau levels, Reviews of Modern Physics **84**, 1709 (2012).

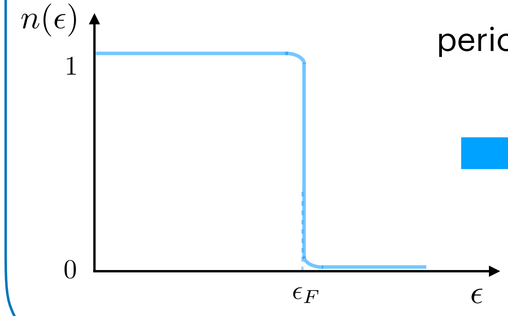
$\hbar\omega/k_BT \sim 0.5K$

THANKS !

arXiv:2309.03268 (2023)
Phys. Rev. B **107**, 195135 (2023)

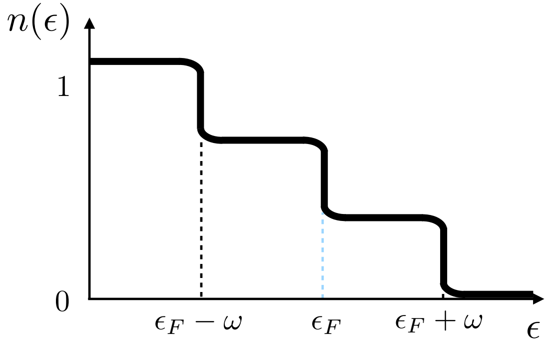
Fermionic bath

Fermi-Dirac

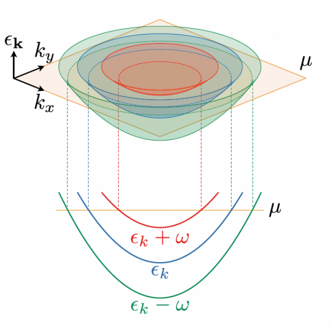


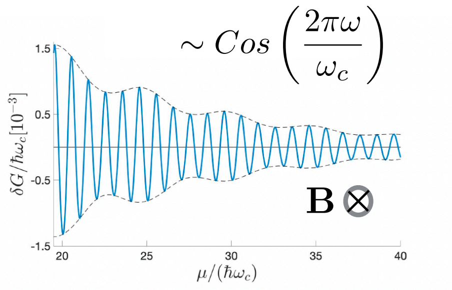
periodic drive

Fermi-Dirac **Staircase**



Floquet Fermi Liquids



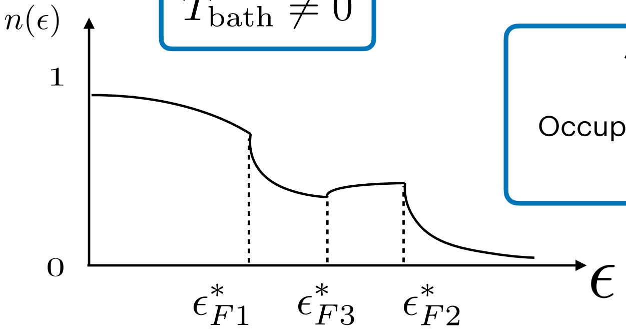


Low frequency Q-Os.

Bosonic bath

periodic drive

$T_{\text{bath}} \neq 0$



Ultra-Critical Floquet Non-Fermi Liquids

“Non-Fermi Liquid”

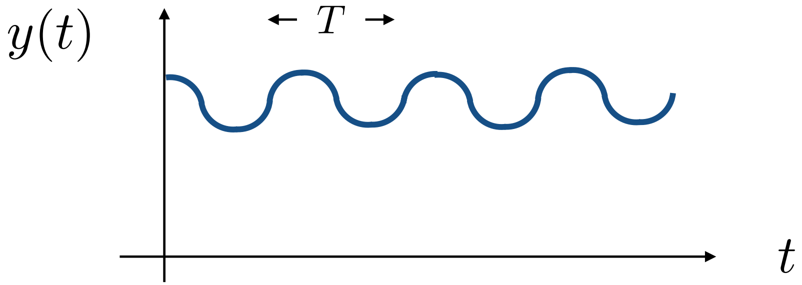
Occupation has non-analyticities but no jumps.

“Ultra-critical”

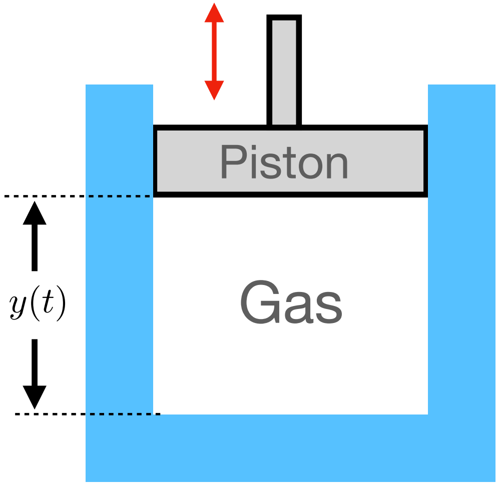
Power law correlations and Fermi surfaces survive to finite temperature

“Thermal death” of closed Floquet systems

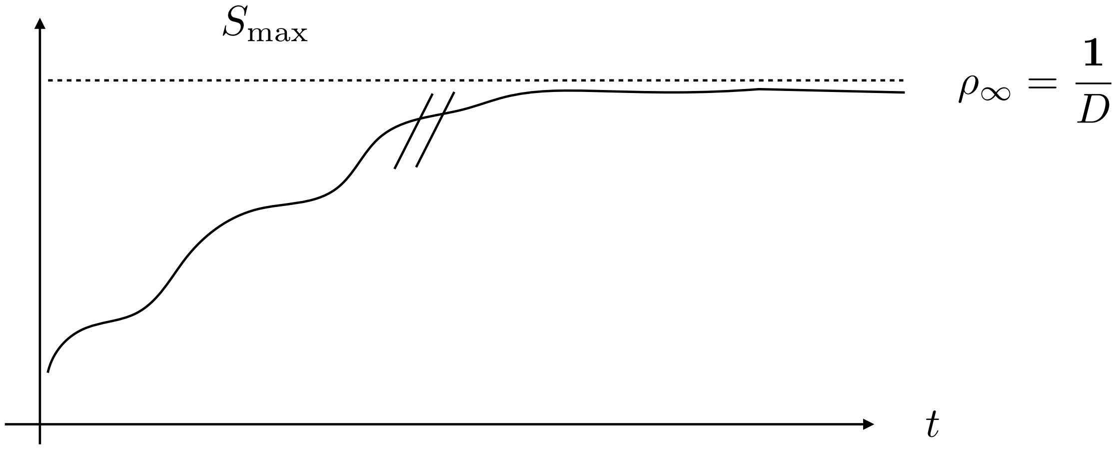
Floquet system $H(t) = H(t+T)$



Second law of thermodynamics implies closed periodically driven systems approach infinite temperature at late times



Entropy



D’Alessio & Rigol, 2014. Physical Review X, 4(4), p.041048

Lazarides, Das, & Moessner, 2014. Physical Review E, 90(1), p.012110.

Ponte, Chandran, Papić, & Abanin, 2015. Annals of Physics, 353, pp.196-204

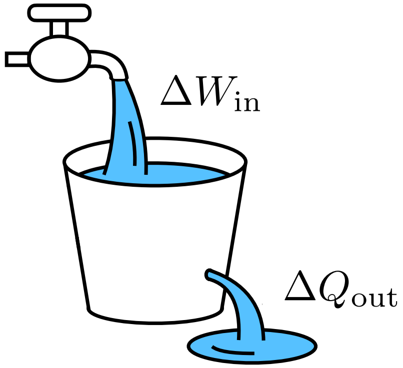
“Cold” non-equilibrium quantum states

How to avoid “thermal death”?

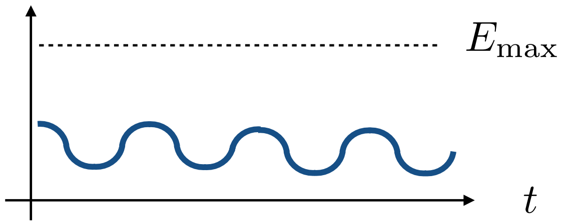
Couple to **heat bath**

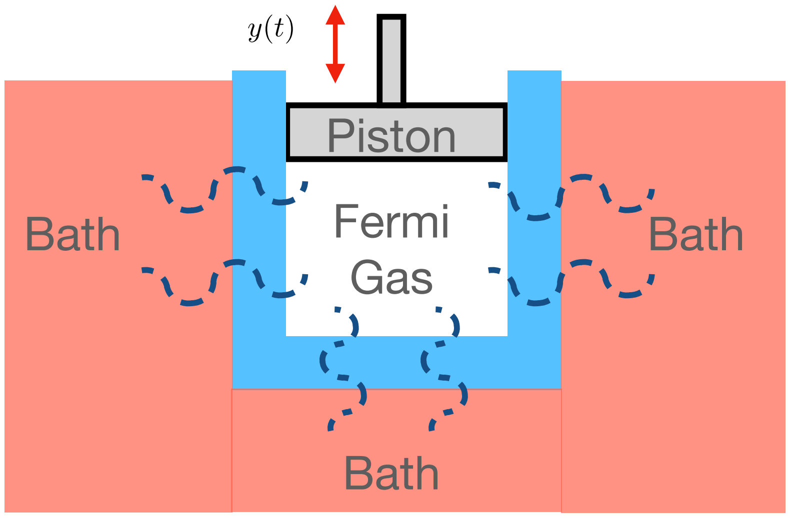
↓

Non-trivial steady state



System energy density

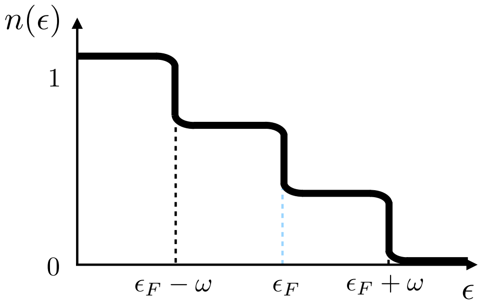




→

Floquet Fermi Liquid

Non-trivial steady state occupation of fermions



$$\omega = \frac{2\pi}{T}$$

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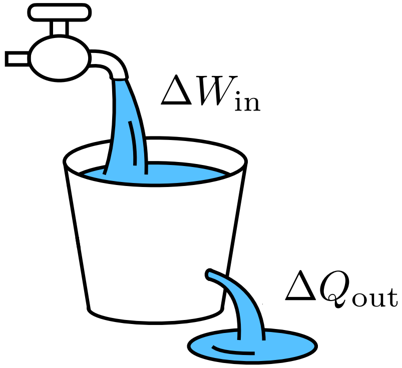
“Cold” non-equilibrium quantum states

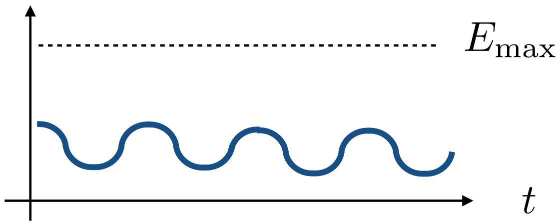
How to avoid “thermal death”?

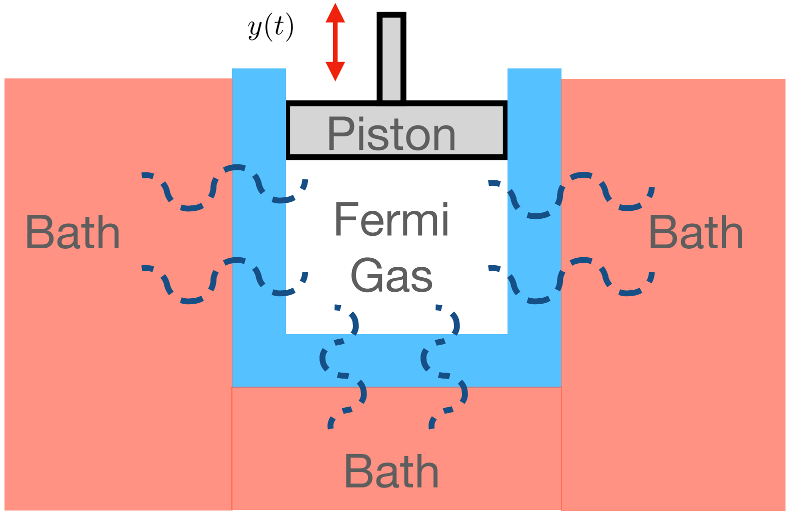
Couple to **heat bath**

Non-trivial steady state



System energy density





Floquet Fermi Liquid

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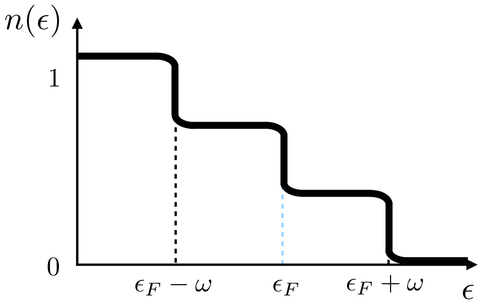


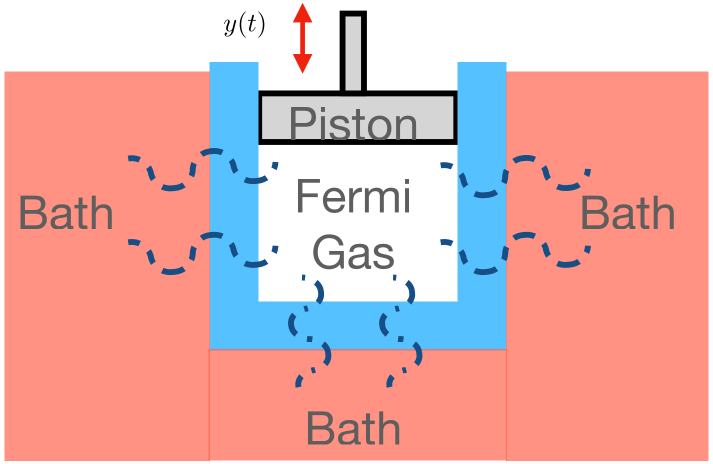
$$\omega = \frac{2\pi}{T}$$

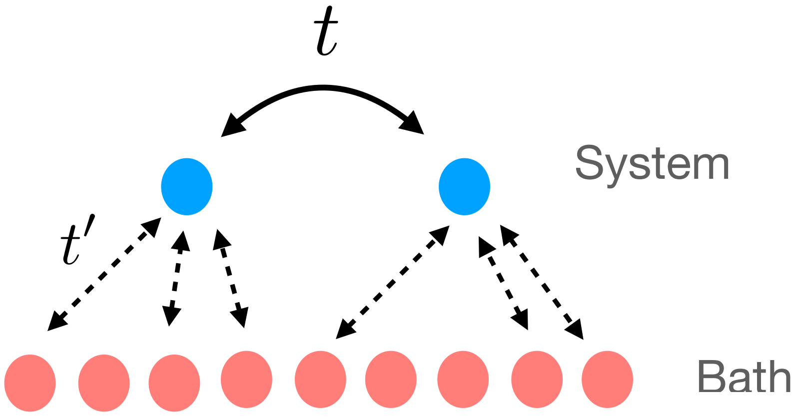
Phys. Rev. B **107**, 195135 (2023)

Open system Schrödinger Equation

Phys. Rev. B **107**, 195135 (2023)

Consider non-interacting fermions





One-fermion Hilbert space is a sum

$$\boxed{H^{\text{one body}} = H^{\text{system}} \bigoplus H^{\text{bath}}}$$

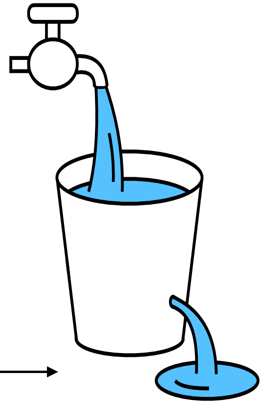
$$|\psi(t)\rangle = \begin{bmatrix} |\psi_S(t)\rangle \\ |\psi_B(t)\rangle \end{bmatrix}$$

$$H(t) = \begin{bmatrix} H_S(t) & H_{SB}(t) \\ H_{BS}(t) & H_B(t) \end{bmatrix}$$

Bath feedback effect

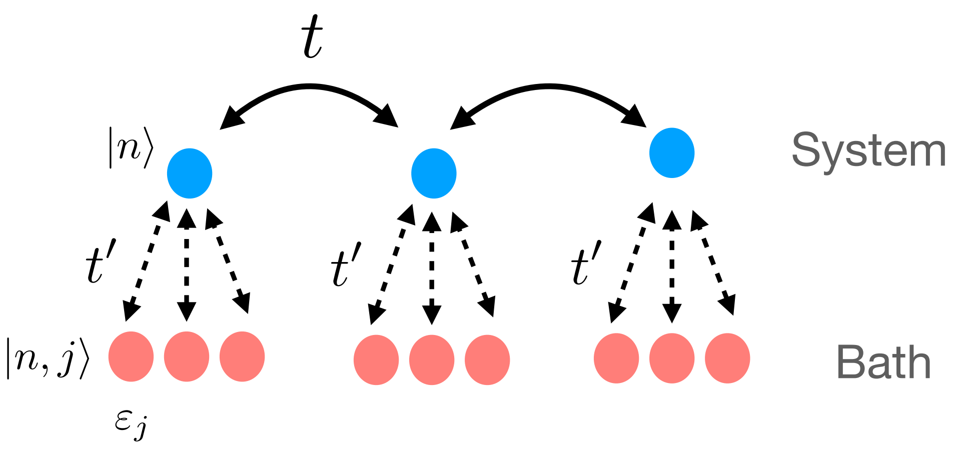
$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)U_B(t,t_0)|\psi_B(t_0)\rangle$$
$$- iH_{SB}(t)\int_{t_0}^{t} \mathrm{d}t' \; U_B(t,t')H_{BS}(t')|\psi_S(t')\rangle$$

Decay, memory and energy renormalization effects



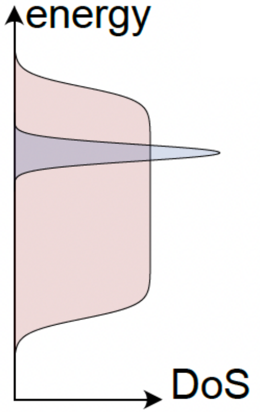
Featureless fermionic bath

Phys. Rev. B **107**, 195135 (2023)



$$\nu_B(\omega_b) = 2\pi \sum_j \delta(\omega_b - \varepsilon_j) \equiv \nu_0$$

Bath spectrum flat and infinitely broad



Initial condition: Bath equilibrium at $\rho(t_0) = \sum_{n,j} f_0(\epsilon_j)|n, j\rangle\langle n, j|$

μ_0 $\quad \beta_0 = \frac{1}{k_B T_{\text{bath}}}$

Non-hermitian but inhomogeneous Schrödinger

$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)U_B(t, t_0)|\psi_B(t_0)\rangle - iH_{SB}(t)\int_{t_0}^{t} \mathrm{d}t' \, U_B(t, t')H_{BS}(t')|\psi_S(t')\rangle$$

Pure decay (no memory)

$$i\partial_t |\psi_n^{(j)}(t)\rangle = \left[H_S(t) - i\Gamma\right] |\psi_n^{(j)}(t)\rangle + t' \exp[-i\varepsilon_j(t - t_0)] \, | \, n \, \rangle$$

Bath feedback effect

$$\Gamma = \frac{\nu_0 t'^2}{2}$$

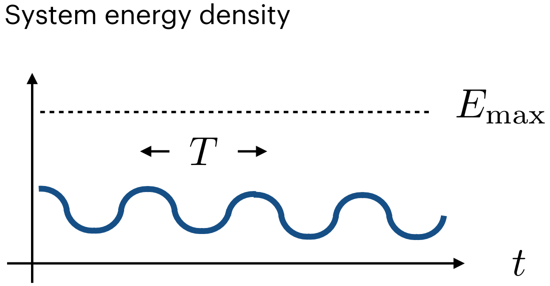
Periodic Gibbs Ensemble as a non-dissipative steady state

Floquet problem:

$$H(t) = H(t+T)$$

$$\Omega = 2\pi/T$$

$$|\psi_a^F(t)\rangle = \sum_n e^{-i\epsilon_a^F t - in\Omega t} |\varphi_{a,n}\rangle$$


System energy density

Weak coupling to bath: $\Gamma \to 0$

$$\lim_{\Gamma \to 0} \rho_S(t) = \sum_a p_a |\psi_a^F(t)\rangle \langle \psi_a^F(t)| ,$$

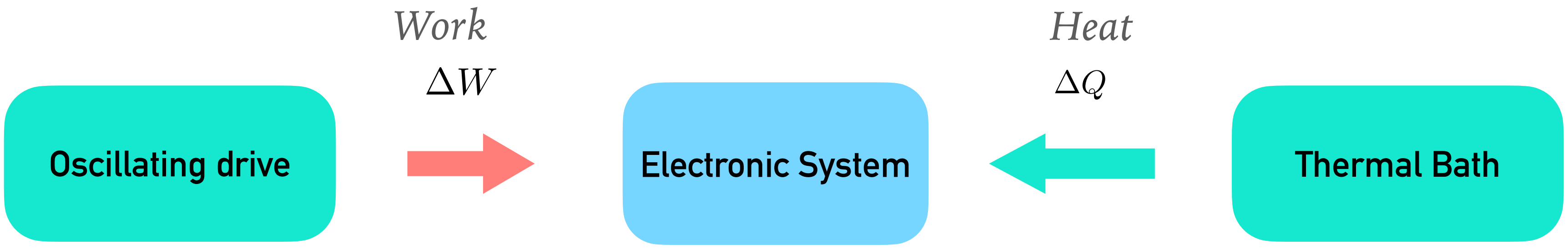
$$p_a = \sum_n |\varphi_{a,n}|^2 f_0(\epsilon_a^F + n\Omega),$$

In steady state heat and work are balanced per cycle:

$$\Delta E_{\text{system}} = \Delta W + \Delta Q = 0$$

For infinitesimal coupling to bath the instantaneous heating rate vanishes $\Gamma \to 0$

$$\dot{Q} = \mathrm{Tr}\left(\frac{d\rho_{\text{sys}}(t)}{dt} H_{\text{sys}}(t) \right) = 0$$


Work

ΔW

Heat

ΔQ

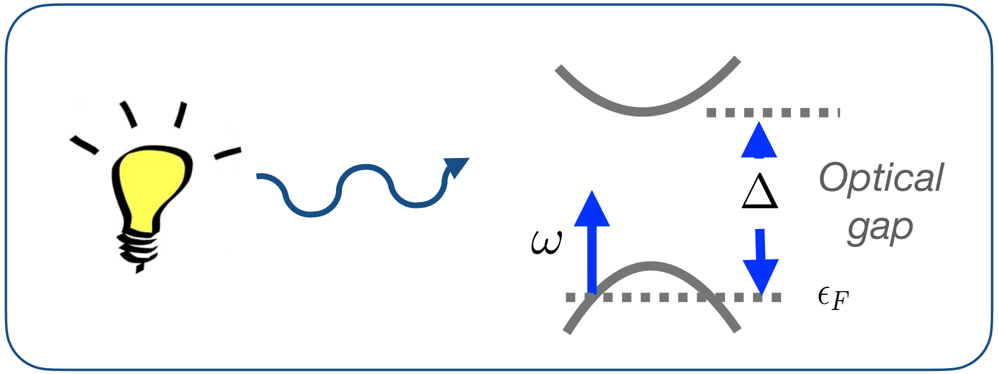
Oscillating drive

Electronic System

Thermal Bath

Photo-current generation below the optical gap of metals

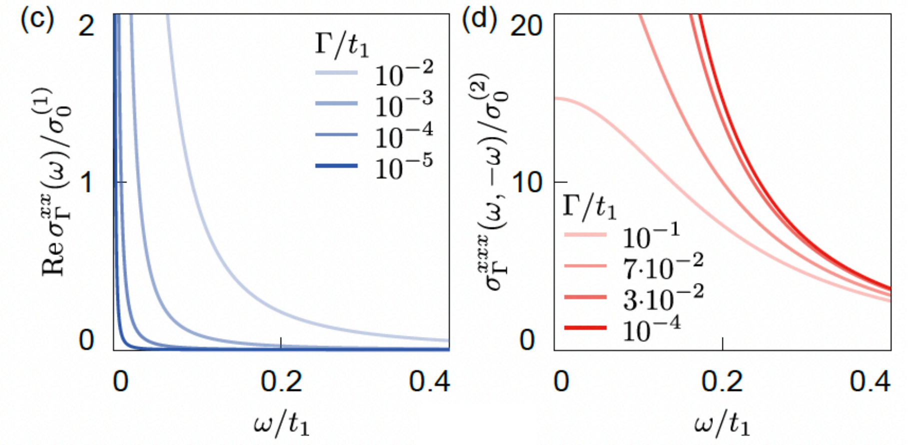
Phys. Rev. B **107**, 195135 (2023)
Phys. Rev. B **107**, 125151 (2023)



$$\text{Re}\ \sigma(\omega) \approx D\frac{\Gamma}{\omega^2}$$

$\Gamma \to 0$

Light absorption vanishes



Fermi Dirac Staircase to order $|E_\omega|^2$

$$f_k = \left(1 - \frac{2\left|\epsilon_{\mathbf{k}}^{(1)}\right|^2}{\omega^2}\right) f_0(\bar{\epsilon}_{\mathbf{k}}) + \frac{\left|\epsilon_{\mathbf{k}}^{(1)}\right|^2}{\omega^2} f_0(\bar{\epsilon}_{\mathbf{k}} - \omega) + \frac{\left|\epsilon_{\mathbf{k}}^{(-1)}\right|^2}{\omega^2} f_0(\bar{\epsilon}_{\mathbf{k}} + \omega)$$

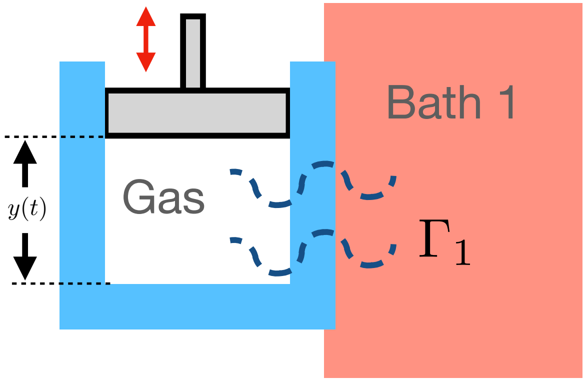
$$\epsilon_{\mathbf{k}}^{(1)} = \frac{i}{\omega}\sum_\alpha \partial_\alpha \epsilon(\mathbf{k})\, E_\omega^\alpha + O(|E_\omega|^3),$$

Finite in-gap rectification conductivity

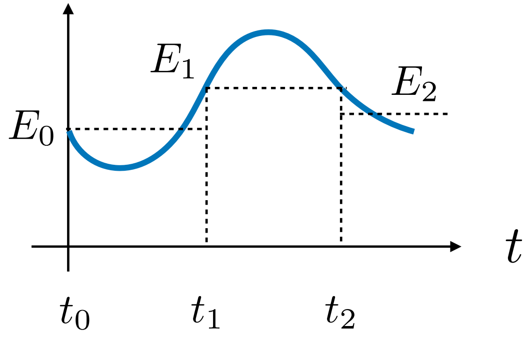
$$j_\gamma^{(0)} = \sigma_\Gamma^{\gamma\alpha\beta}(\omega, -\omega) E_\omega^\alpha (E_\omega^\beta)^*$$

$$\lim_{\Gamma\to 0} \sigma_\Gamma^{\gamma\alpha\beta}(\omega, -\omega) = \frac{1}{2\omega^4}\int_{\mathbf{k}} (\partial_\gamma \bar{\epsilon}_{\mathbf{k}})(\partial_\alpha \bar{\epsilon}_{\mathbf{k}})(\partial_\beta \bar{\epsilon}_{\mathbf{k}}) \times \left[f_0(\bar{\epsilon}_{\mathbf{k}} + \omega) + f_0(\bar{\epsilon}_{\mathbf{k}} - \omega) - 2f_0(\bar{\epsilon}_{\mathbf{k}})\right].$$

The nature of bath matters away from equilibrium



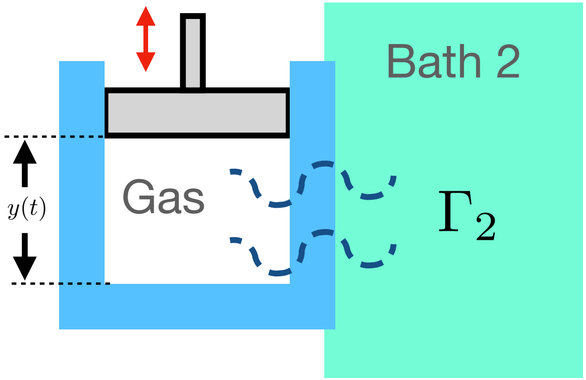
System Energy for bath 1



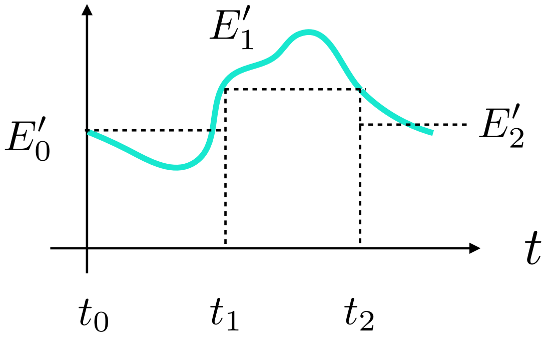
In equilibrium in limit of weak coupling to baths, nature of bath does not matter

$$\Gamma_1 \to 0$$
$$\Gamma_2 \to 0$$

$$\rho^{(1)}_{\mathrm{equil}} = \rho^{(2)}_{\mathrm{equil}} = \frac{e^{-\beta H}}{Z}$$



System Energy for bath 2

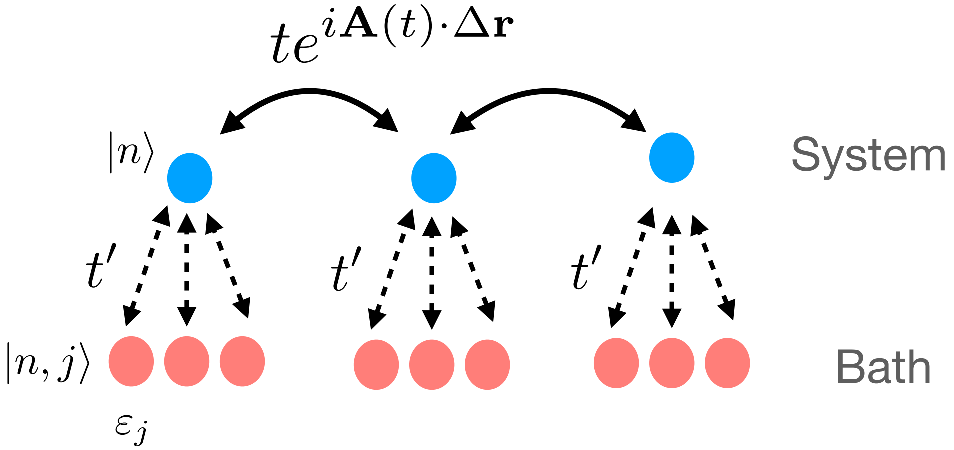


In **non-equilibrium** even in limit of weak coupling to baths, **nature of bath matters**

$$\rho^{(1)}_{\mathrm{non-eq.}}(t) \neq \rho^{(2)}_{\mathrm{non-eq.}}(t)$$

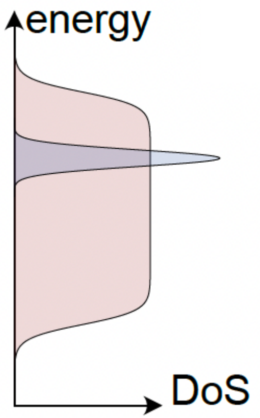
Featureless fermionic bath

Phys. Rev. B **107**, 195135 (2023)



$$\nu_B(\omega_b) = 2\pi\sum_j \delta(\omega_b - \varepsilon_j) \equiv \nu_0$$

Bath spectrum flat and infinitely broad



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μ_0 $\beta_0 = \frac{1}{k_B T_{\text{bath}}}$

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Pure decay (no memory)

$$i\partial_t|\psi_n^{(j)}(t)\rangle = \left[H_S(t) - i\Gamma\right]|\psi_n^{(j)}(t)\rangle + t'\exp[-i\varepsilon_j(t-t_0)]\,|\,n\,\rangle$$

Bath feedback effect

$$\Gamma = \frac{\nu_0 t'^2}{2}$$

Featureless Fermionic Bath

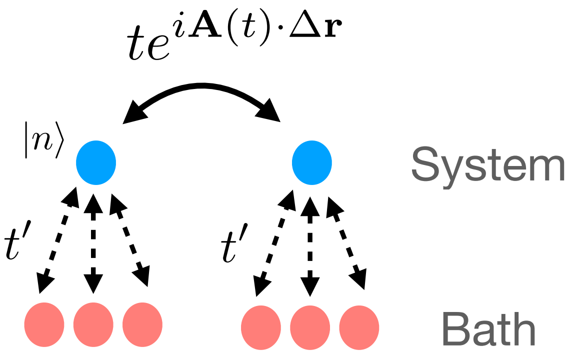
Exact non-equilibrium steady-state of **driven non-interacting fermions**

Decay rate:

Bath DOS

$$\Gamma = \frac{\nu_0 t'^2}{2}$$

Tunneling amplitude



$$\rho_{\text{non-eq.}}(t) = \Gamma \int_{-\infty}^{+\infty} \frac{d\epsilon}{\pi} f_0(\epsilon) U_\Gamma(t,\epsilon) U_\Gamma^\dagger(t,\epsilon)$$

$$U_\Gamma(t,\epsilon) = \int_{-\infty}^{t} dt' e^{\Gamma(t'-t) - i\epsilon t'} U_S(t,t')$$

$$i\partial_t U_S(t,t') = H_S(t) U_S(t,t')$$

$$f_0(\epsilon) = 1/[1 + e^\beta(\epsilon - \mu)]$$

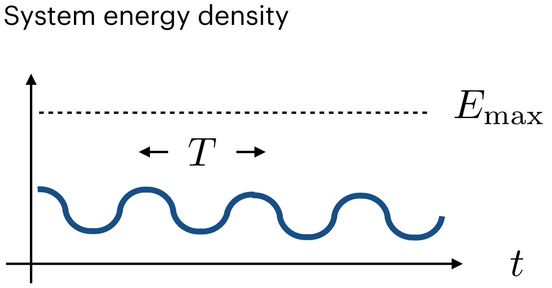
Floquet Periodic Gibbs Ensemble with Fermi Dirac Staircase:

Floquet problem:

$$H_S(t) = H_S(t+T)$$

$$\Omega = 2\pi/T$$

$$|\psi_a^F(t)\rangle = \sum_n e^{-i\epsilon_a^F t - in\Omega t} |\varphi_{a,n}\rangle$$



Weak coupling to bath: $\Gamma \to 0$

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Featureless Fermionic Bath

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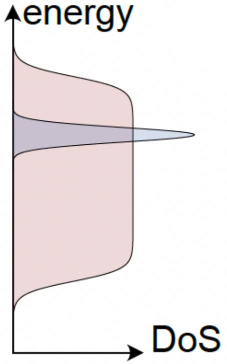

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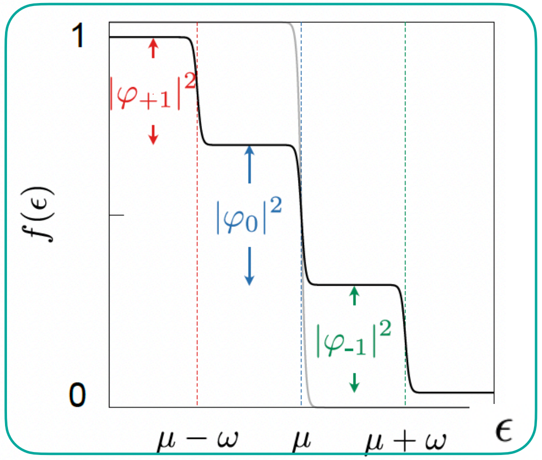
$$p_a = \sum_n \left|\varphi_{a,n}\right|^2 f_0(\epsilon_a^F + n\Omega),$$



Non-linear Hall effect Rectification Conductivity

Inti Sodemann and Liang Fu, PRL 115, 216806 (2015)

$$j_{DC} = \sigma^{(2)}_{ijk}(\omega, -\omega) E_i(\omega) E_k(\omega)^*$$

Berry curvature dipole

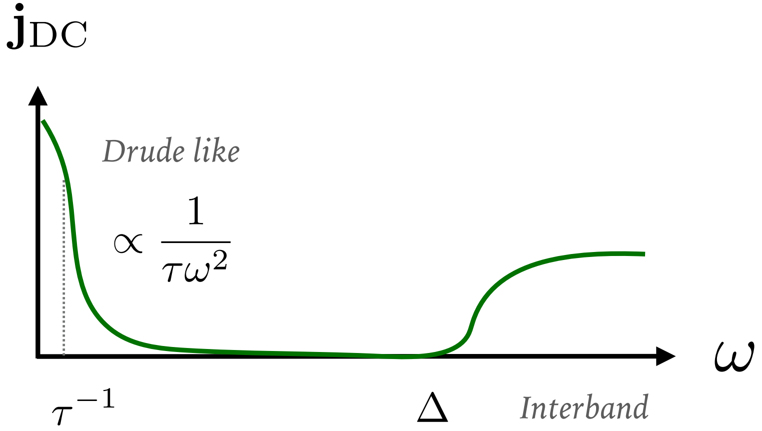
$$D_{ik} = \langle \partial_{\mathbf{k}_i} \mathbf{\Omega}_j \rangle$$

Drude-like form:

Matsyshyn & Sodemann, PRL 123, 246602 (2019)

$$\sigma^{(2)}_{ijk}(\omega, -\omega) = \frac{1}{i\omega + 1/\tau} \left(\frac{e^3}{\hbar^2} \epsilon_{ikl} D_{jl} \right)$$

Real part: *linear polarization*

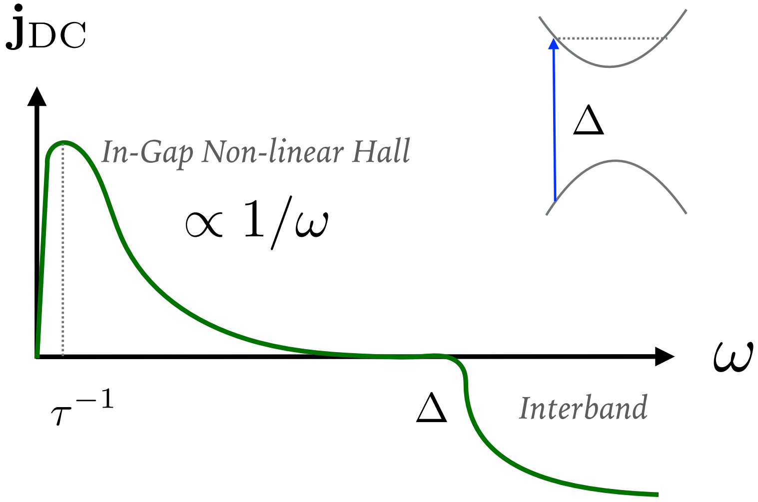


Qiong Ma, Su-Yang Xu et al. Nature 565, 337 (2019).

Kang, Kaifei, et al. *Nat. mater.* 18.4 324 (2019).

Imaginary part: *circular polarization*

(not yet seen experimentally!)

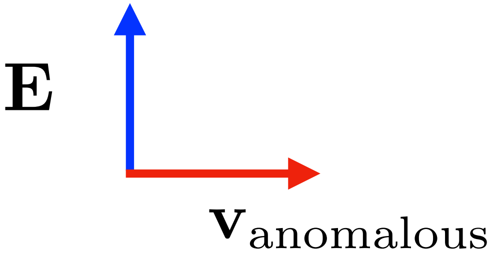


Berry Dipole “Demon”

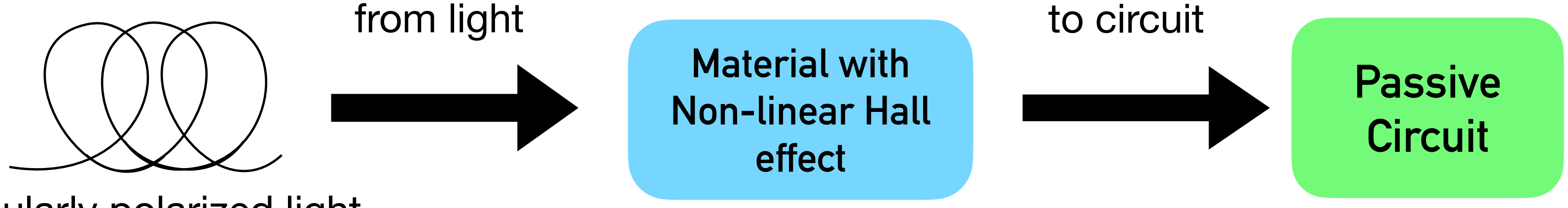
Ideal reversible photovoltaic mechanism based on Non-linear Hall effect

$$\mathbf{j}^{\mathrm{NLHE}}(t) \cdot (\mathbf{E}_1(t) + \mathbf{E}_0) = 0$$

$$\mathbf{v} = \frac{\partial \epsilon(\mathbf{k})}{\partial \mathbf{k}} + e\mathbf{\Omega}(\mathbf{k}) \times \mathbf{E}(t)$$



CPGE cell :



Energy received from light

Material with Non-linear Hall effect

Energy given to circuit

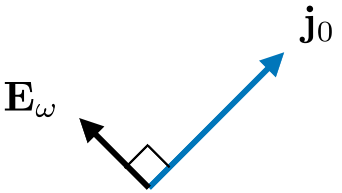
Passive Circuit

Circularly polarized light

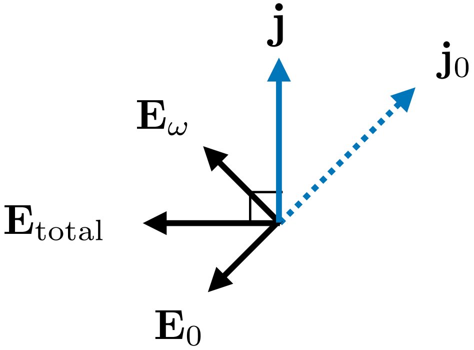
See also:

- Onishi, Fu, arXiv:2211.17219.
- Rappoport et al., PRL 130, 076901 (2023).

Under pure illumination



Illumination plus DC field



Circuit receives energy:

$$\mathbf{j} \cdot \mathbf{E}_0 < 0$$

Radiation gives energy:

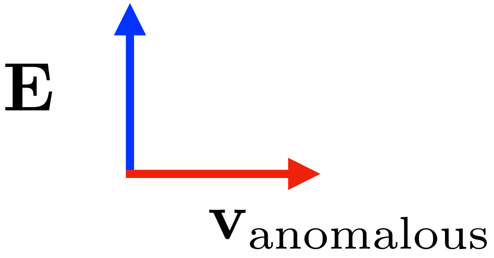
$$\mathbf{j} \cdot \mathbf{E}_\omega > 0$$

Berry Dipole “Demon”

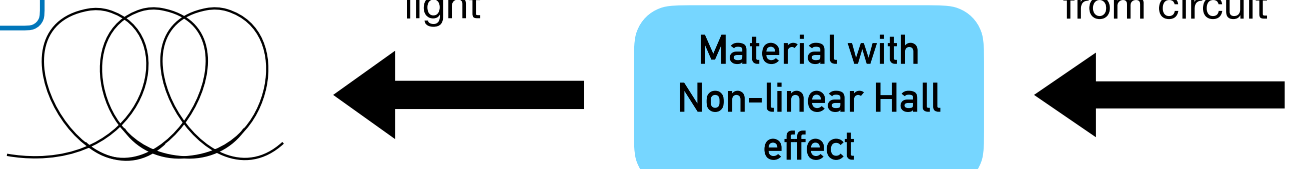
Ideal reversible photovoltaic mechanism based on Non-linear Hall effect

$$\mathbf{j}^{\mathrm{NLHE}}(t)\cdot(\mathbf{E}_1(t)+\mathbf{E}_0)=0$$

$$\mathbf{v}=\frac{\partial\epsilon(\mathbf{k})}{\partial\mathbf{k}}+e\boldsymbol{\Omega}(\mathbf{k})\times\mathbf{E}(t)$$



Amplifier of circular light



Energy given to light

Energy received from circuit

Material with Non-linear Hall effect

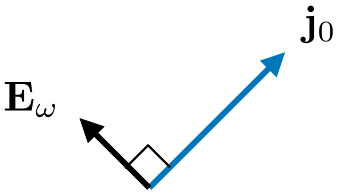
Active Circuit (Battery)

Circularly polarized light

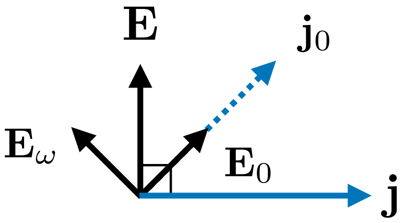
See also:

- Onishi, Fu, arXiv:2211.17219.
- Rappoport et al., PRL 130, 076901 (2023).

Under pure illumination



Illumination plus DC field



Circuit gives energy:

$$\mathbf{j}\cdot\mathbf{E}_0>0$$

Radiation receives energy:

$$\mathbf{j}\cdot\mathbf{E}_\omega<0$$