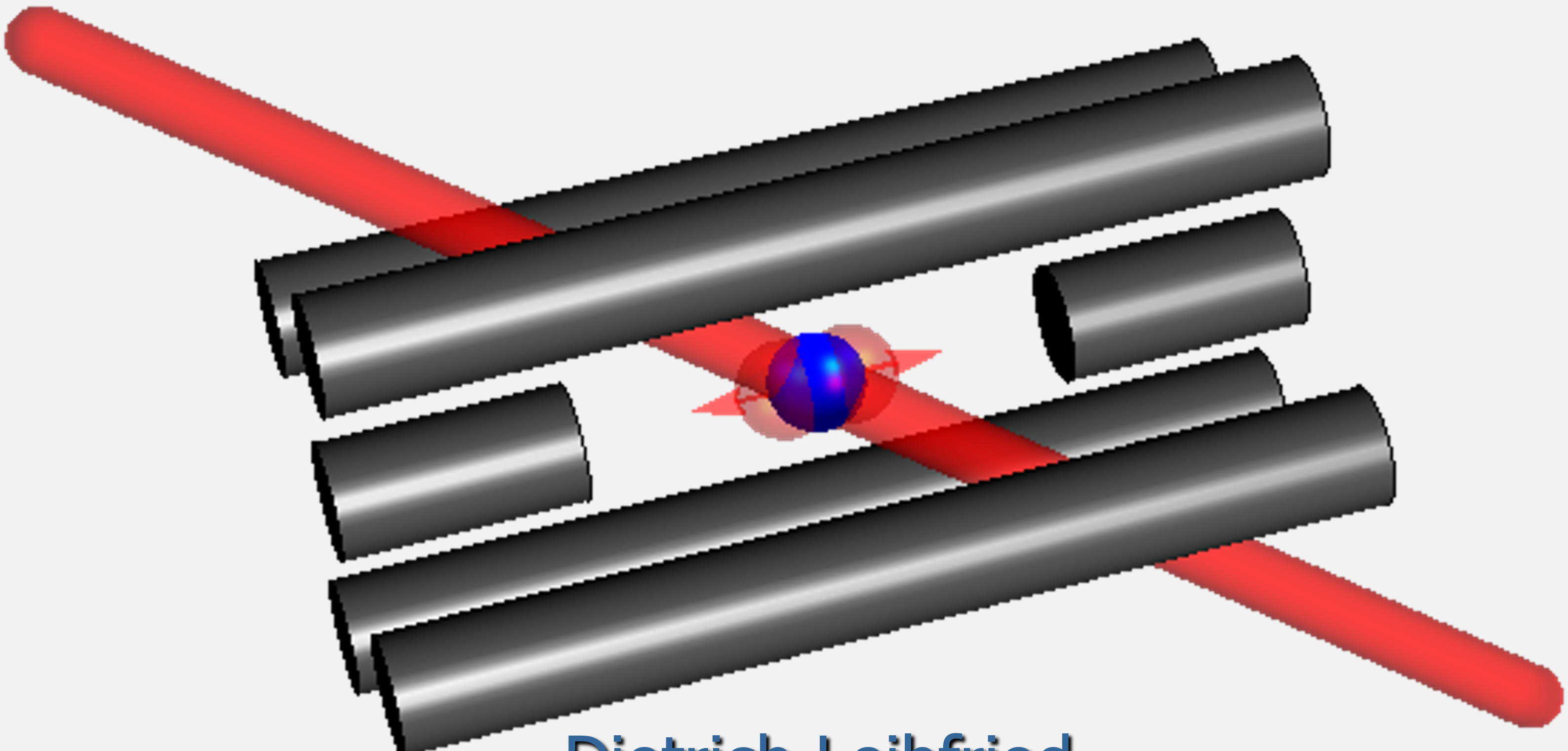
Trapping and Cooling of Atomic Ions



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Overview

- Part 1: Trapped two-level atoms coupled to light fields
- Part 2: Laser cooling of ions
- Part 3: Cooling of ion crystals
- Part 4: Ion transport and separation

Please ask questions!

(incomplete list of) reviews:

D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. E. King, and D. M. Meekhof, J. Res. Nat. Inst. Stand. Tech. 103, 259 (1998)
M. Sasura and V. Buzek, J. Mod. Opt. 49, 1593 (2002)
D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. 75, 281 (2003)
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D. Kielpinski, Front. Phys. China 3, 365 (2008)
D.J. Wineland, and D. Leibfried, Laser Phys. Lett. 8, No. 3, 175–188 (2011)
C. D. Bruzewicz, J. Chiaverini, R. McConnell, and J. M. Sage, Appl. Phys. Rev. 6, 021314 (2019).

Overview

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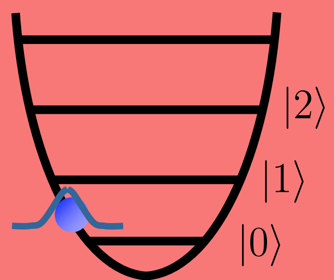
The basic Hamiltonian

$$\hat{H} = \hat{H}^{(m)} + \hat{H}^{(e)} + \hat{H}^{(i)}$$

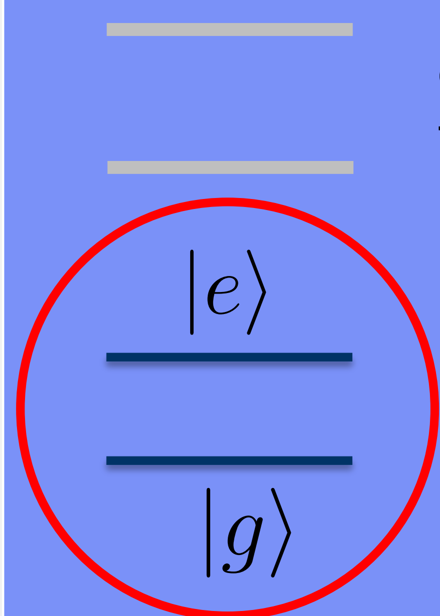
ion motion:



harmonic oscillators

$$\hat{H}^{(m)} = \sum_i \hbar\omega_i \hat{a}^\dagger \hat{a}$$

ion internal states:



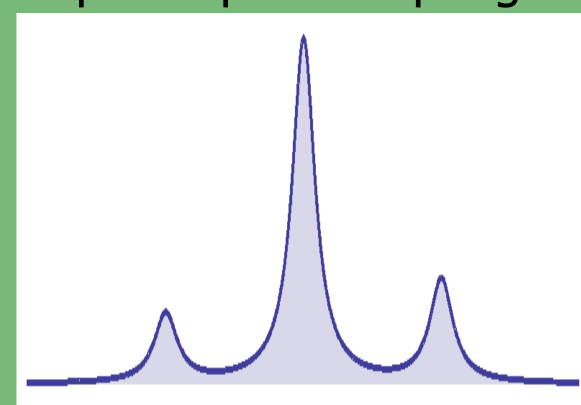
other states typically unimportant

selected levels

light interactions:

classical plane wave field(s)

- dipole
- Raman
- quadrupole coupling

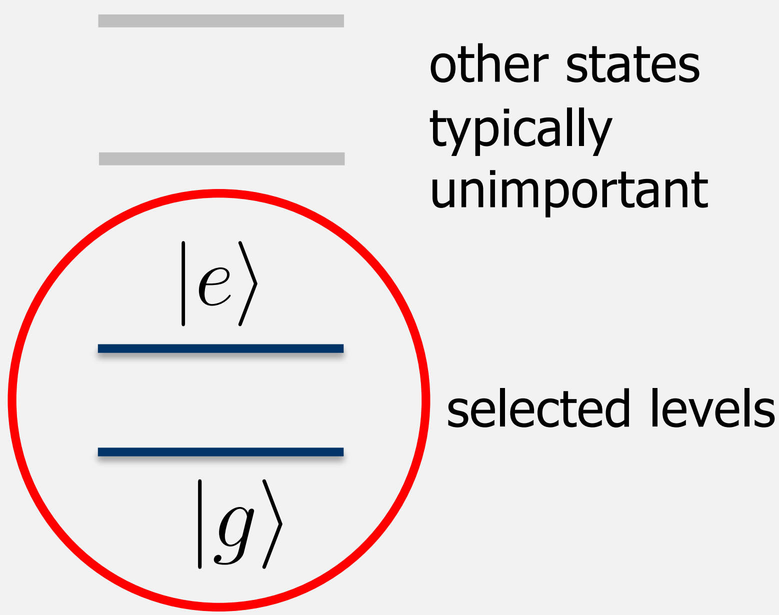


carrier

sidebands

coherent drive

Two-level Hamiltonian



$$|g\rangle\langle g| + |e\rangle\langle e| \mapsto \hat{I};$$
$$i(|g\rangle\langle e| - |e\rangle\langle g|) \mapsto \hat{\sigma_y};$$
$$|g\rangle\langle e| + |e\rangle\langle g| \mapsto \hat{\sigma}_x;$$
$$|e\rangle\langle e| - |g\rangle\langle g| \mapsto \hat{\sigma_z}.$$

mapping on Pauli matrices

$$\hat{H}^{(e)} = \hbar(\omega_g|g\rangle\langle g| + \omega_e|e\rangle\langle e|)$$

$$= \hbar\frac{\omega_e + \omega_g}{2}(|g\rangle\langle g| + |e\rangle\langle e|) + \hbar\frac{\omega}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$$

remove constant energy term

$$\hat{H}^{(e)} = \hbar\frac{\omega}{2}\;\sigma_z$$

Popular two-level systems

(list is not exhaustive)

	hyperfine qubit	hyperfine+ optical qubit	optical qubit
degrees of freedom	nuclear spin electron spin	nuclear spin electron spin electron energy	electron energy
species	$^{9}Be^{+}$, $^{25}Mg^{+}$, $^{111}Cd^{+}$, $^{67}Zn^{+}$ $^{137}Ba^{+}$,$^{171}Yb^{+}$	$^{43}Ca^{+}$, $^{87}Sr^{+}$, $^{199}Hg^{+}$	$^{40}Ca^{+}$, $^{88}Sr^{+}$

light-atom interaction

$$i(|g\rangle\langle e| - |e\rangle\langle g|) \mapsto \hat{\sigma}_y;$$
$$|g\rangle\langle e| + |e\rangle\langle g| \mapsto \hat{\sigma}_x;$$

$|e\rangle$

$|g\rangle$

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_0(e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} + \text{c.c.})$$

plane wave field

$$H^{(i)} = (|g\rangle\langle e| + |e\rangle\langle g|)\langle g|H^{(i)}|e\rangle$$

assume diagonal elements (AC Stark shifts) lumped into energy of states

matrix element determines Rabi frequency

matrix elements I

$$H^{(i)} = (|g\rangle\langle e| + |e\rangle\langle g|)\langle g|H^{(i)}|e\rangle$$

$$H_D = e_- \ \mathbf{x}_e \cdot \mathbf{E}_0 (e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} + \mathrm{c.c.})$$

dipole coupling
odd parity (S-P)

$$(\hbar/2)\Omega_D = e_- \ \langle g|(\mathbf{E}_0 \cdot \mathbf{x}_e)|e\rangle$$

Rabi-frequency

$$H_Q = \frac{1}{2} e_- \ \mathbf{k} \cdot \mathbf{x}_e (\mathbf{E}_0 \cdot \mathbf{x}_e)(i e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} + \mathrm{c.c.})$$

quadrupole coupling
even parity (S-D)

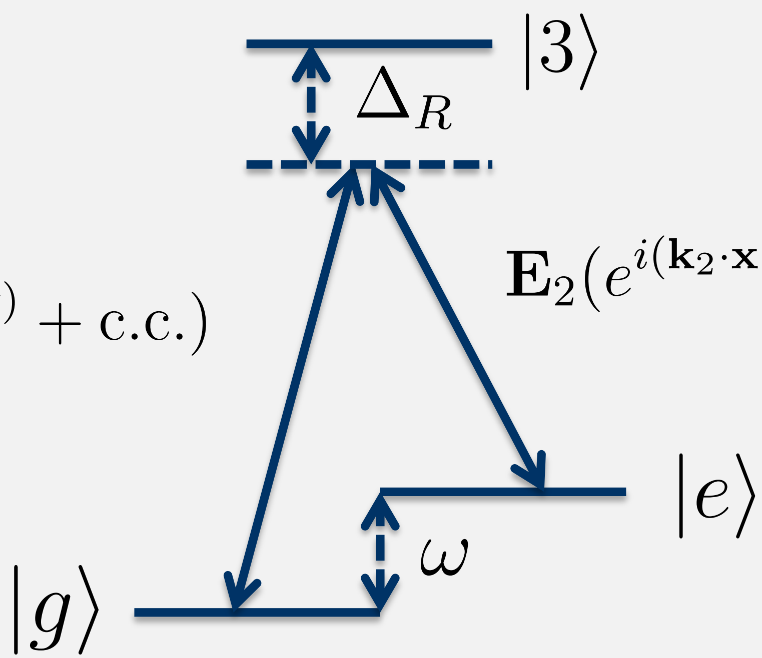
$$(\hbar/2)\Omega_Q = (e_-/2) \ \mathbf{k} \cdot \langle g|\mathbf{x}_e (\mathbf{E}_0 \cdot \mathbf{x}_e)|e\rangle$$

Rabi-frequency

matrix elements II

$$H^{(i)} = (|g\rangle\langle e| + |e\rangle\langle g|)\langle g|H^{(i)}|e\rangle$$

$\mathbf{E}_1(e^{i(\mathbf{k}_1\cdot\mathbf{x}-\omega_1 t)} + \text{c.c.})$



$\mathbf{E}_2(e^{i(\mathbf{k}_2\cdot\mathbf{x}-\omega_2 t+\Delta\phi)} + \text{c.c.})$

two plane wave fields
off-resonant dipole coupling
adiabatically eliminate third level

$\omega \leftrightarrow \omega_1 - \omega_2,\ \mathbf{k} \leftrightarrow \Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ Raman coupling

$$(\hbar/2)\Omega_R = -\hbar\,\frac{|\Omega_{g3}\Omega_{e3}|}{\Delta_R}\,e^{i\Delta\phi}$$

phase between fields

Rabi-frequency

$0 \approx |\mathbf{k}_2| - |\mathbf{k}_1| \leq |\mathbf{k}_2 - \mathbf{k}_1| \leq |\mathbf{k}_2| + |\mathbf{k}_1| \approx 2|\mathbf{k}_1|$ widely variable eff. k

interaction picture

$$\hat{H}^{(i)} = (\hbar/2)\Omega(|g\rangle\langle e| + |e\rangle\langle g|)(e^{i(k\hat{x}_S-\omega t+\phi)} + e^{-i(k\hat{x}_S-\omega t+\phi)})$$

$$|e\rangle\langle g| \mapsto \hat{\sigma}_+ = 1/2(\hat{\sigma}_x + i\hat{\sigma}_y); \qquad |g\rangle\langle e| \mapsto \hat{\sigma}_- = 1/2(\hat{\sigma}_x - i\hat{\sigma}_y)$$

go into interaction picture: $\hat{H}_0 = \hat{H}^{(m)} + \hat{H}^{(e)} \quad \hat{U}_0 = \exp(-\frac{i}{\hbar}\hat{H}_0 t)$

$$\hat{H}_{\text{int}} = \hat{U}_0^\dagger \hat{H}^{(i)} \hat{U}_0$$

$$= (\hbar/2)\Omega(\sigma_+ e^{i\omega_0 t} + \sigma_- e^{-i\omega_0 t})\ e^{\frac{i}{\hbar}\hat{H}^{(m)}t}(e^{i(k\hat{x}-\omega t+\phi)} + e^{-i(k\hat{x}-\omega t+\phi)})e^{-\frac{i}{\hbar}\hat{H}^{(m)}t}$$

lengthy, but half the terms rotate fast at $\omega + \omega_0$ and can be neglected (rotating wave approximation, RWA). Other half rotates slowly at $\delta = \omega - \omega_0$.

$\hat{H}^{(m)}$ transforms ladder operators from Schrödinger to Heisenberg picture:

$e^{i/\hbar \hat{H}^{(m)}}\ \hat{a}\ e^{-i/\hbar \hat{H}^{(m)}} = \hat{a}e^{-i\nu t}$ where ν is the harmonic oscillator frequency

interaction Hamiltonian after RWA

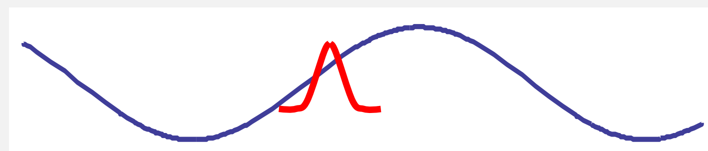
$$\mathbf{k}\cdot\hat{\mathbf{x}} = |\mathbf{k}||\hat{\mathbf{x}}|\cos\theta_k = kx_0(\hat{a}^\dagger e^{i\nu t} + \hat{a}e^{-i\nu t}) = \eta(\hat{a}^\dagger e^{i\nu t} + \hat{a}e^{-i\nu t})$$

$x_0 = \sqrt{\hbar/(2m\nu)}$ ground state extend

Hamiltonian in interaction picture, no terms rotating at optical frequencies

$$\hat{H}_{\text{int}} = (\hbar/2)\Omega\sigma_+ \exp\left[i(\phi - \delta t + \eta(\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t}))\right] + \text{h.c.}$$

expand in LD-parameter (typically $x_0 << \lambda$)



$$\hat{H}_{\text{int}} = (\hbar/2)\Omega\sigma_+ e^{i(\phi-\delta t)} \sum_{m=0}^{\infty} \frac{(i\eta)^m}{m!}(\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t})^m + \text{h.c.}$$

$m\nu - \delta \approx 0$ (m >0) m-th blue sideband (sideband attenuation $\frac{(\eta)^m}{m!}$)

$-m\nu - \delta \approx 0$ m-th red sideband

higher orders in η quickly drop, typically $m \leq 3$

Fock state matrix elements

detuning $\delta \approx s\nu$, (s integer) couples state-pairs $|g\rangle|n\rangle$ and $|e\rangle|n+s\rangle$

$$\Omega_{n,n+s} = \Omega_{n+s,n} = \Omega_0 |\langle n+s| e^{i\eta(a+a^\dagger)} |n\rangle| = \Omega_0 e^{-\eta^2/2} \eta^{|s|} \sqrt{\frac{n_<!}{n_>!}}\, L_{n_<}^{|s|}(\eta^2)$$

$n_<$ ($n_>$) is the lesser (greater) of $n+s$ and n

$$L_n^\alpha(X) = \sum_{m=0}^{n} (-1)^m \binom{n+\alpha}{n-m} \frac{X^m}{m!}$$

generalized Laguerre-polynomial

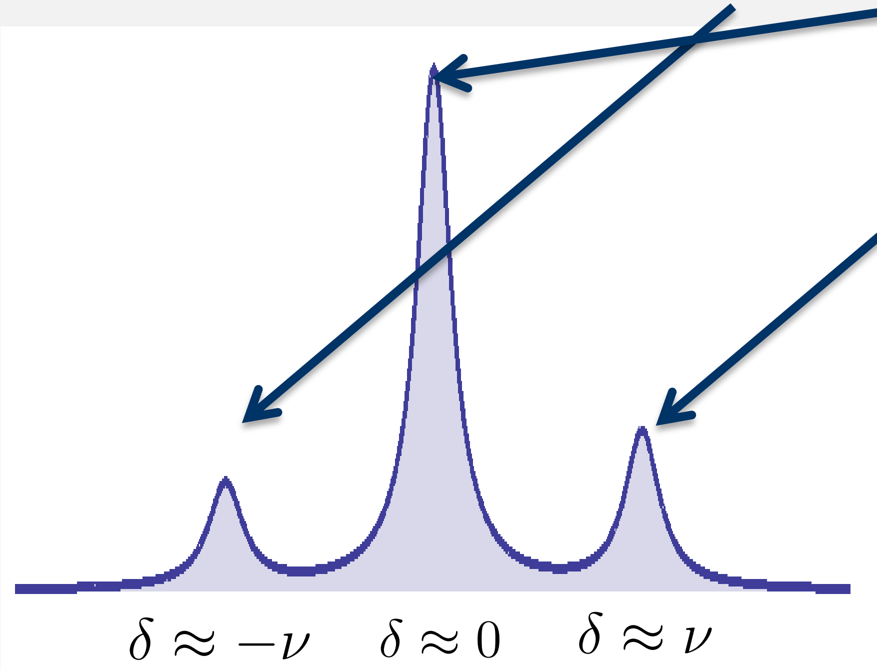
Lamb-Dicke regime

ion confined to region much smaller than optical wavelength
(typical ground state extension <10 nm, wavelength ≈ 280-900 nm)

Hamiltonian can be expanded to lowest order:

$$\hat{H}_{\mathrm{LD}}(t) = (\hbar/2)\Omega_0\sigma_+ \left\{1 + i\eta(\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t})\right\} e^{i(\phi-\delta t)} + \mathrm{h.c.}$$

contains three resonances: red sideband, carrier, blue sideband



$$\hat{H}_{\mathrm{rsb}} = (\hbar/2)\Omega_0\eta(\hat{a}\sigma_+ e^{i\phi} + \hat{a}^\dagger\sigma_- e^{-i\phi})$$

$$\hat{H}_{\mathrm{car}} = (\hbar/2)\Omega_0(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$

$$\hat{H}_{\mathrm{bsb}} = (\hbar/2)\Omega_0\eta(\hat{a}^\dagger\sigma_+ e^{i\phi} + \hat{a}\sigma_- e^{-i\phi})$$

Lamb-Dicke regime

ion confined to region much smaller than optical wavelength
(typical ground state extension <10 nm, wavelength ≈ 280-900 nm)

$$\hat{H}_{\mathrm{rsb}} = (\hbar/2)\Omega_0\eta(\hat{a}\sigma_+ e^{i\phi} + \hat{a}^\dagger\sigma_- e^{-i\phi})$$

$|n\rangle|g\rangle \rightarrow |n-1\rangle|e\rangle$ $\qquad \Omega_{n,n-1} = \Omega_0\sqrt{n}\eta$

$$\hat{H}_{\mathrm{car}} = (\hbar/2)\Omega_0(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$

$|n\rangle|g\rangle \leftrightarrow |n\rangle|e\rangle$ $\qquad \Omega_{n,n} = \Omega_0$

$$\hat{H}_{\mathrm{bsb}} = (\hbar/2)\Omega_0\eta(\hat{a}^\dagger\sigma_+ e^{i\phi} + \hat{a}\sigma_- e^{-i\phi})$$

$|n\rangle|g\rangle \rightarrow |n+1\rangle|e\rangle$ $\qquad \Omega_{n,n+1} = \Omega_0\sqrt{n+1}\eta$

State evolution (Rabi-flopping)

general state:

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} c_{n,g}(t)|n,g\rangle + c_{n,e}(t)|n,e\rangle$$

Schrödinger equation:

$$i\hbar\partial t|\Psi(t)\rangle = \hat{H}_{\text{int}}|\Psi(t)\rangle$$

l-th sideband: $\delta = l\nu + \delta'$; $\delta' \ll \nu$

$$\dot{c}_{n,g} = -i^{(1-|l|)} e^{i(\delta' t-\phi)} (\Omega_{n+l,n}/2) c_{n+l,e}$$

$$\dot{c}_{n+l,e} = -i^{(1+|l|)} e^{-i(\delta' t-\phi)} (\Omega_{n+l,n}/2) c_{n,g}$$

solution:

$$\begin{pmatrix} c_{n+l,e}(t) \\ c_{n,g}(t) \end{pmatrix} = T_n^l \begin{pmatrix} c_{n+l,e}(0) \\ c_{n,g}(0) \end{pmatrix}$$

$$f_n^l = \sqrt{\delta'^2 + \Omega_{n+l,n}^2}$$

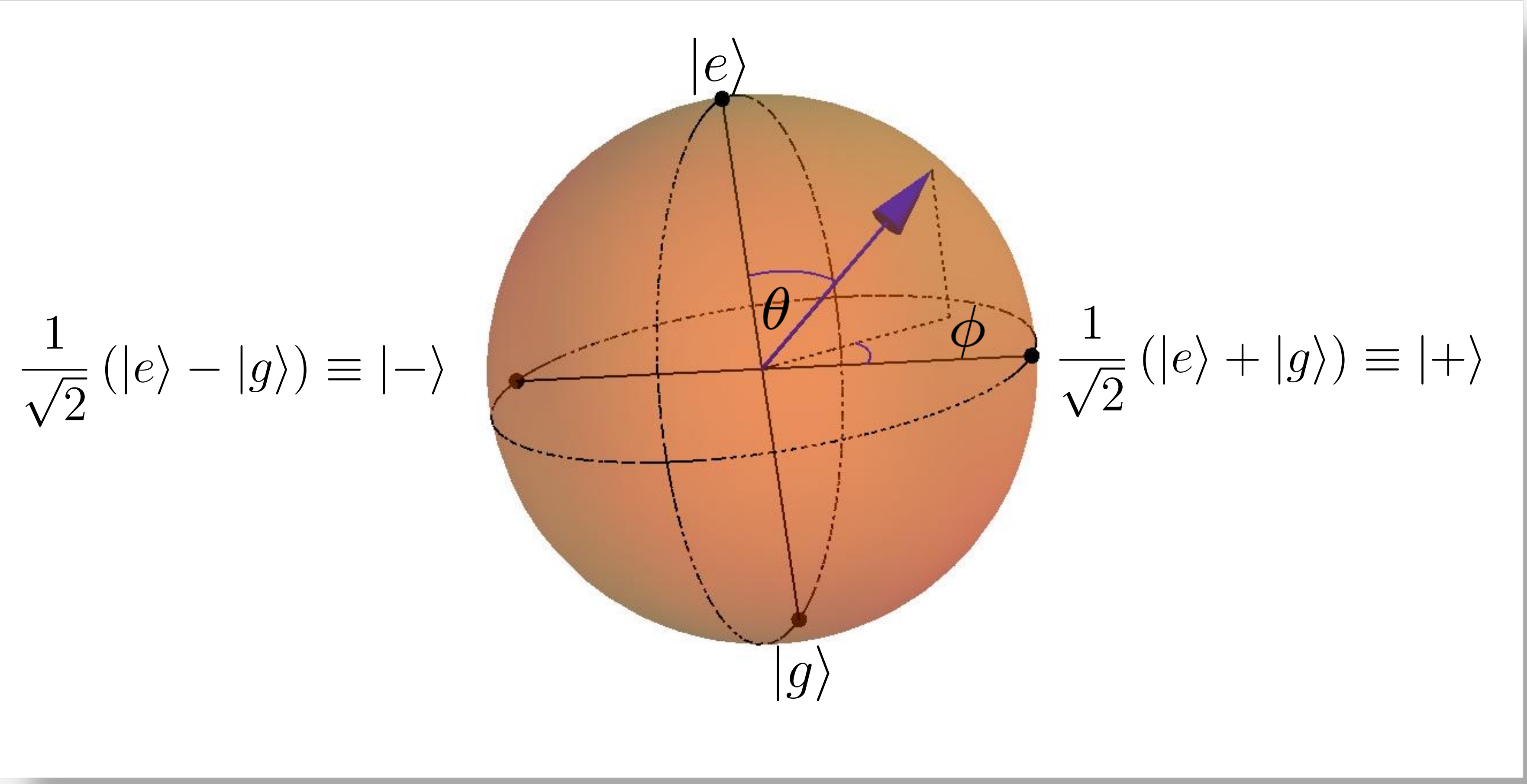
$$T_n^l = \begin{pmatrix} e^{-i(\delta'/2)t}\left(\cos(f_n^l t/2) + i\frac{\delta'}{f_n^l}\sin(f_n^l t/2)\right) & -i\frac{\Omega_{n+l,n}}{f_n^l}\, e^{i(\phi+|l|\pi/2-\delta' t/2)}\sin(f_n^l t/2) \\ -i\frac{\Omega_{n+l,n}}{f_n^l}\, e^{-i(\phi+|l|\pi/2-\delta' t/2)}\sin(f_n^l t/2) & e^{i(\delta'/2)t}\left(\cos(f_n^l t/2) - i\frac{\delta'}{f_n^l}\sin(f_n^l t/2)\right) \end{pmatrix}$$

Bloch sphere (here for carrier)

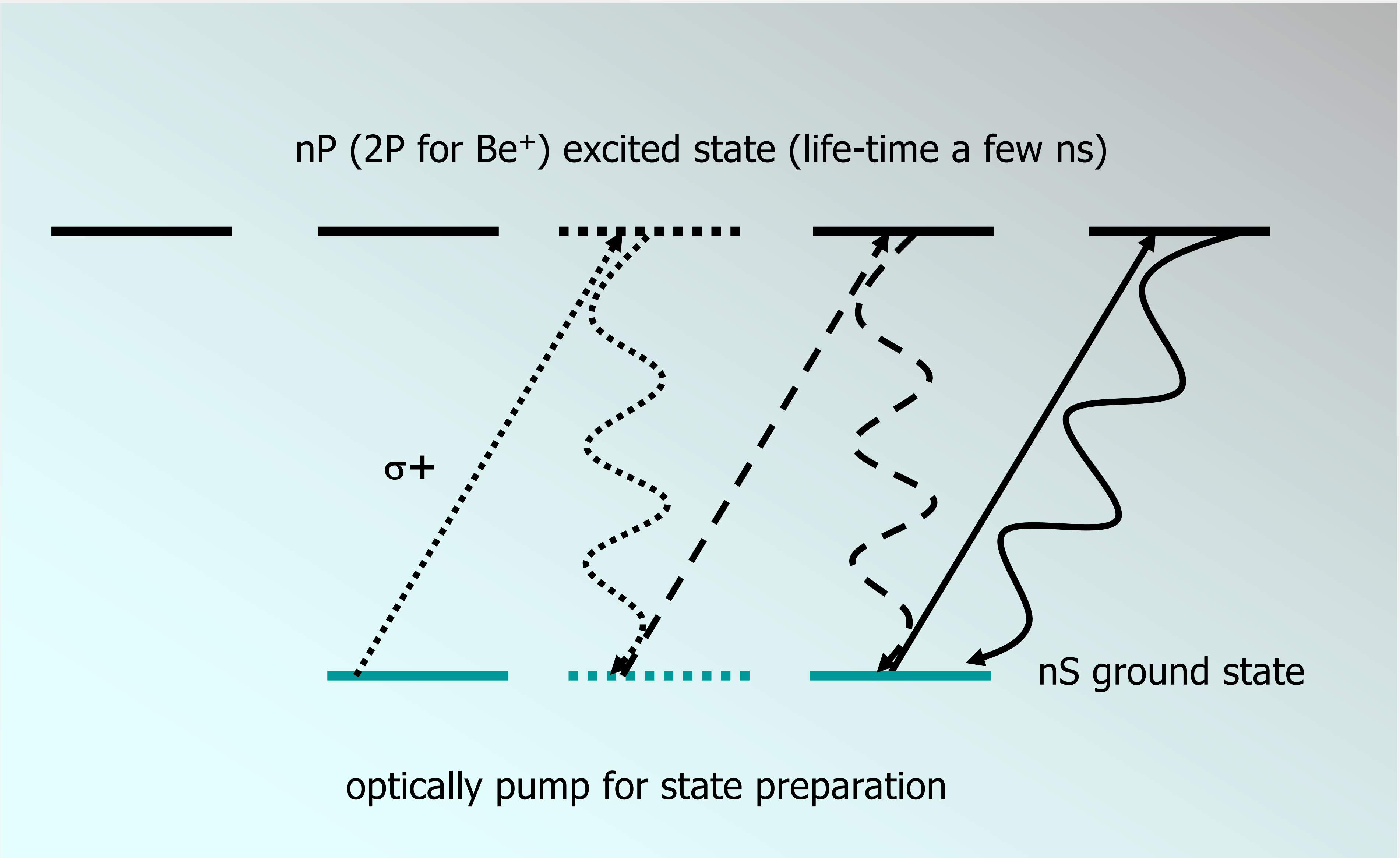
$$|\Psi\rangle = \alpha|e\rangle + \beta|g\rangle; \quad |\alpha|^2 + |\beta|^2 = 1$$

rewrite as

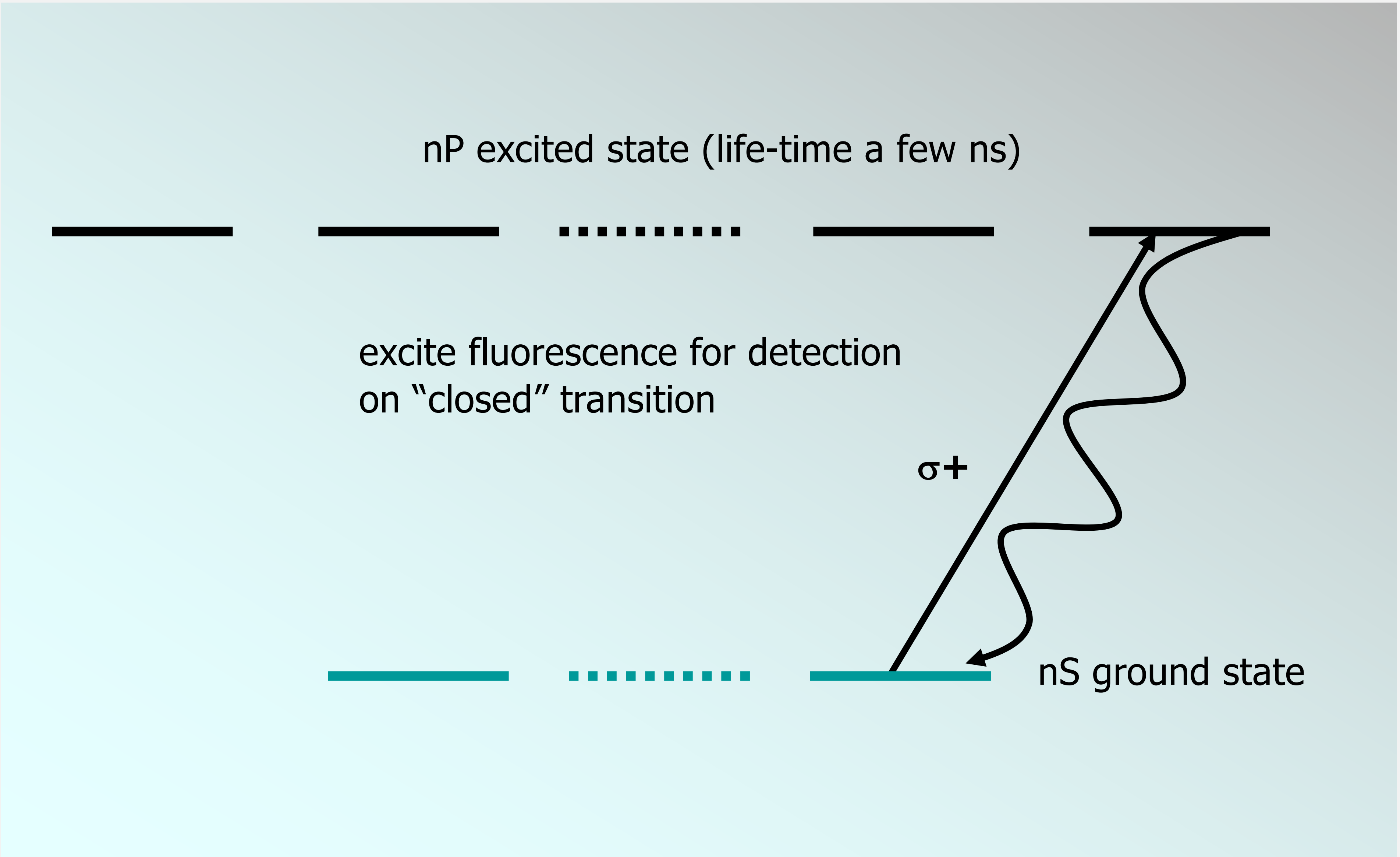
$$|\Psi\rangle = e^{i\gamma}\left(\cos(\theta/2)|e\rangle + e^{i\phi} sin(\theta/2)|g\rangle\right)$$



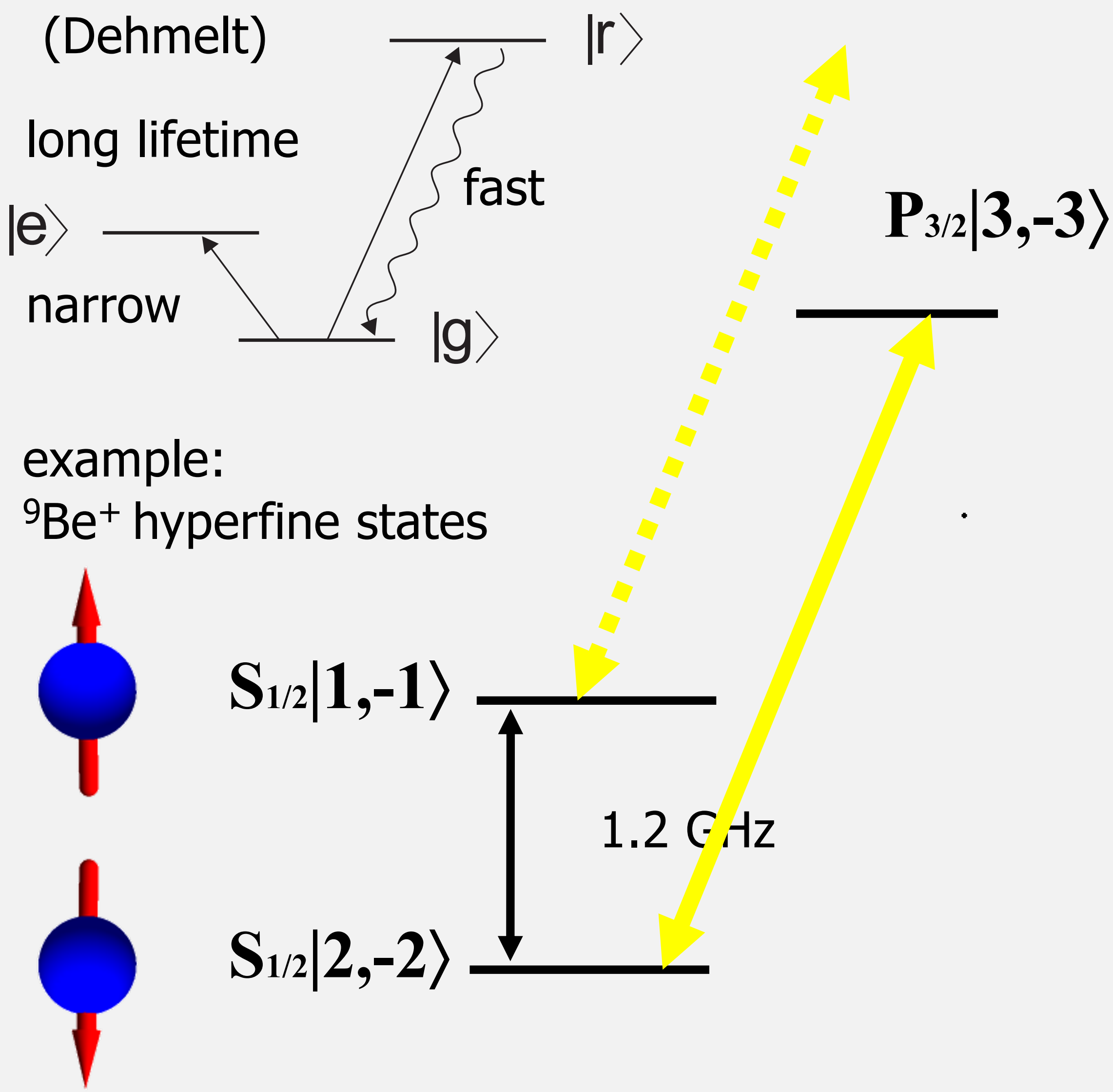
Resonant S → P transition

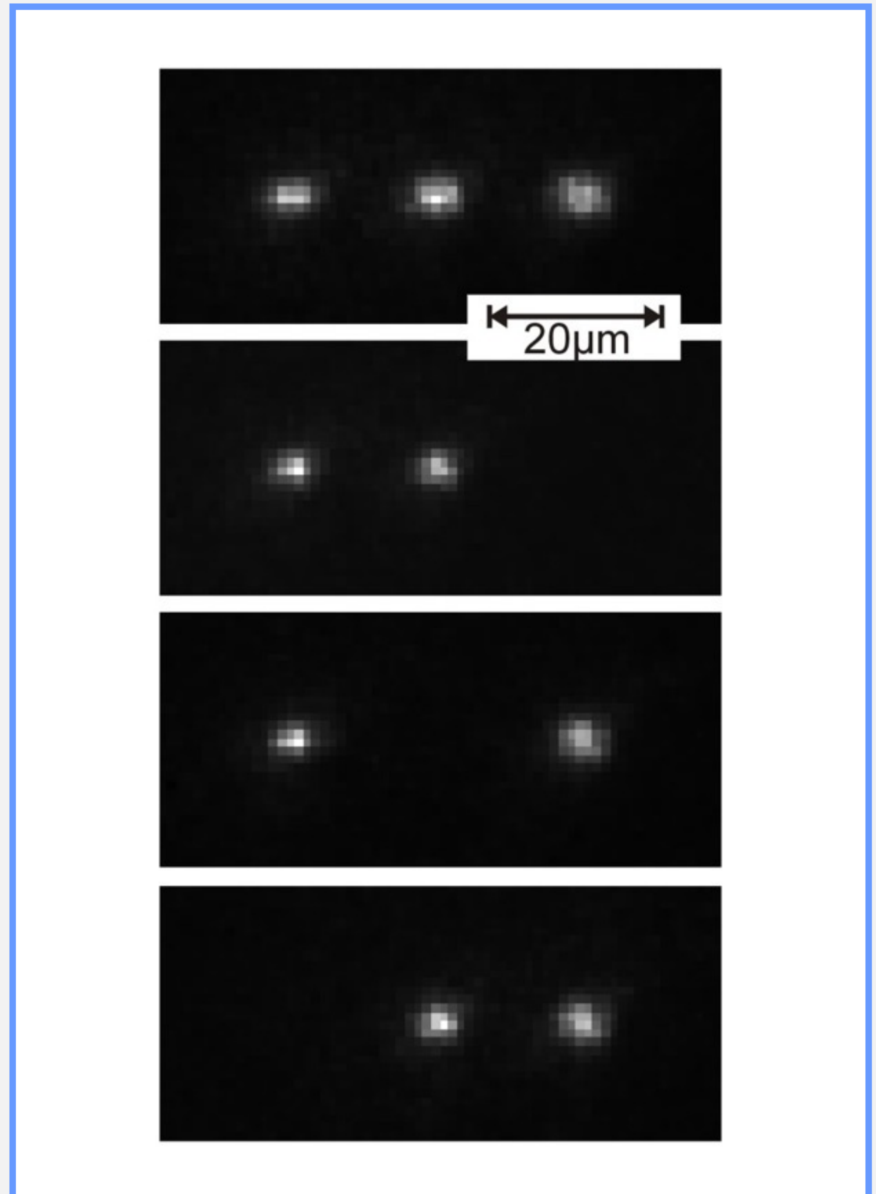


Resonant S $\rightarrow$ P transition



Electron shelving detection





3 Ca^+ ions (bright S, dark D state)
Univ. of Innsbruck [PRA **60**, 145 1999]

Overview

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- Part 2: Laser cooling of ions
- Part 3: Cooling of ion crystals
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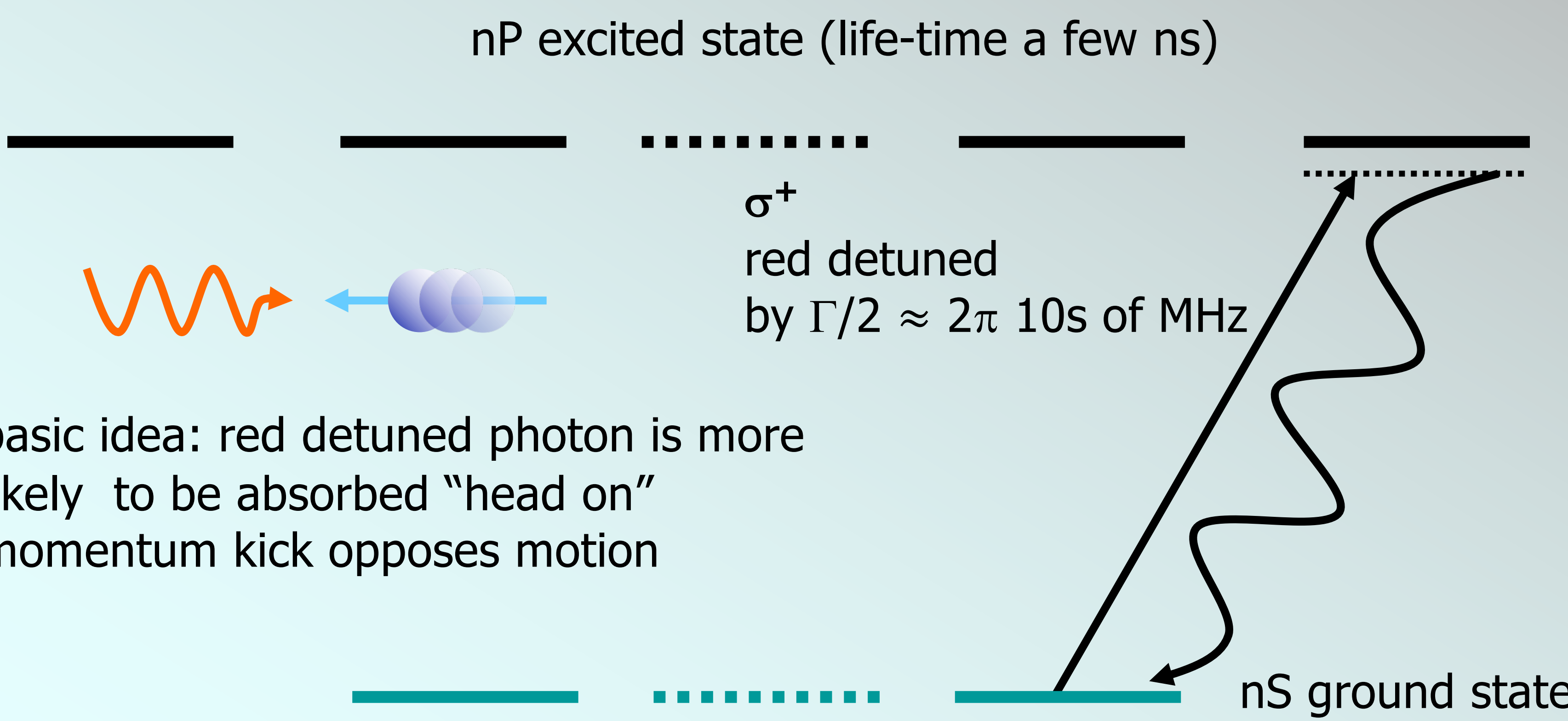
Doppler cooling



basic idea: red detuned photon is more likely to be absorbed "head on" momentum kick opposes motion

Doppler cooling

ion oscillates in harmonic trapping potential:

$$V_p(x) = \frac{1}{2}m\nu^2 x^2 \qquad v(t) = v_0 \cos(\nu t)$$

excitation and decay on dipole transition is much faster than oscillation period, therefore radiation pressure can be modeled as continuous friction force

$\Delta p = \hbar k$ momentum kick per absorbed photon along k-vector, emission random

$$\left(\frac{dp}{dt}\right)_a \approx F_a = \hbar k \Gamma \rho_{ee}$$

average force, proportional to decay rate Γ and excited state population ρ_{ee}

$$\rho_{ee} = \frac{s/2}{1 + s + (2\delta_{\mathrm{eff}}/\Gamma)^2}$$

ρ_{ee} has a Lorentzian lineshape, scattering depends on

$$s = 2|\Omega|^2/\Gamma^2$$

saturation

$$\delta_{\mathrm{eff}} = \Delta - \mathbf{k} \cdot \mathbf{v}$$

detuning from resonance

Doppler shift

Doppler cooling

$$F_a \approx F_0(1+\kappa v)$$

can linearize average force in velocity close to final state

$$F_0 = \hbar k \Gamma \frac{s/2}{1+s+(2\Delta/\Gamma)^2}$$

averaged radiation pressure due to scattering rate at zero velocity; displaces ion from equilibrium along k-vector

$$\kappa = \frac{8k\Delta/\Gamma^2}{1+s+(2\Delta/\Gamma)^2}$$

friction coefficient, is negative for negative detuning

on average over many periods energy decreases as:

$$\dot{E}_c = \langle F_a v \rangle = F_0(\langle v \rangle + \kappa \langle v^2 \rangle) = F_0 \kappa \langle v^2 \rangle$$

would go to zero, but absorption and emission are discrete causing diffusion due to random kicks

$$\dot{E}_h = \frac{1}{2m}\frac{d}{dt}\langle p^2 \rangle = \dot{E}_{abs} + \dot{E}_{em} = \dot{E}_{abs}(1+\xi) \simeq \frac{1}{2m}(\hbar k)^2 \Gamma \ \rho_{ee}(v=0)\ (1+\xi)$$

average component of emission kick along k, $\xi = 2/5$ for dipole radiation

Doppler cooling limit

equilibrium is reached when cooling and heating are equal $\dot{E}_c = \dot{E}_h$

$$m\langle v^2\rangle = k_B T = \frac{\hbar\Gamma}{8}(1+\xi)\left[(1+s)\frac{\Gamma}{2\Delta} + \frac{2\Delta}{\Gamma}\right]$$

minimum temperature ins reached at a detuning of (from $dT/d\Delta=0$)

$$T_{min} = \frac{\hbar\Gamma\sqrt{1+s}}{4k_B}(1+\xi) \qquad \text{for} \qquad \Delta = \Gamma\sqrt{1+s}/2$$

typically $\xi \approx 2/5$ and $T_{min} \approx \hbar\Gamma/(2k_B) \approx$ a few mK

low saturation yields better cooling limit, but reduces cooling rate

in practice some groups use pre-cooling at $s \gg 1, \Delta_p \gg \Gamma/2$
and only use low intensity ($s \leq 1/2$) at Doppler detuning for last few μs

resolved sideband cooling

to resolve sidebands one needs detuning $\delta \gg \tilde{\Gamma}$

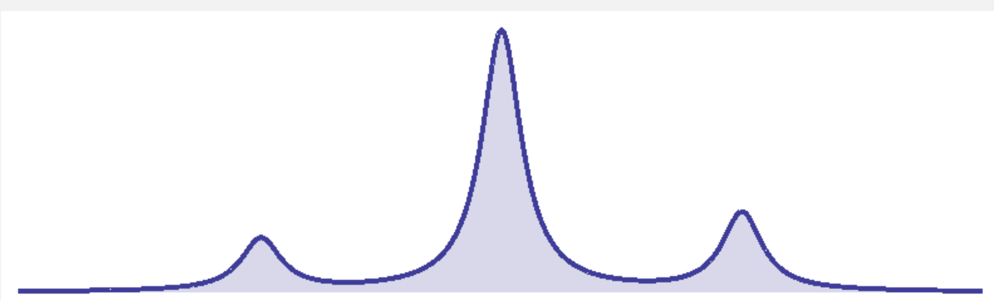
$\tilde{\Gamma}$ the decay of the excited state is not necessarily natural linewidth, often determined by extra repumper beams.

Typically sidebands are resolved on quadrupole and Raman transitions, not dipole transitions

assume pre-cooling to LD-limit and detuning set to resonance with red sideband

$$\hat{H}_{\rm int}^{\rm LD}(t) = (\hbar/2)\Omega[\boxed{\hat{\sigma}_+ e^{i\nu t} + \hat{\sigma}_- e^{-i\nu t}} + i\eta(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger) + i\eta(\hat{\sigma}_+ \hat{a}^\dagger e^{i2\nu t} + \hat{\sigma}_- \hat{a} e^{-i2\nu t}))]$$

carrier | res. red sideband | blue sideband

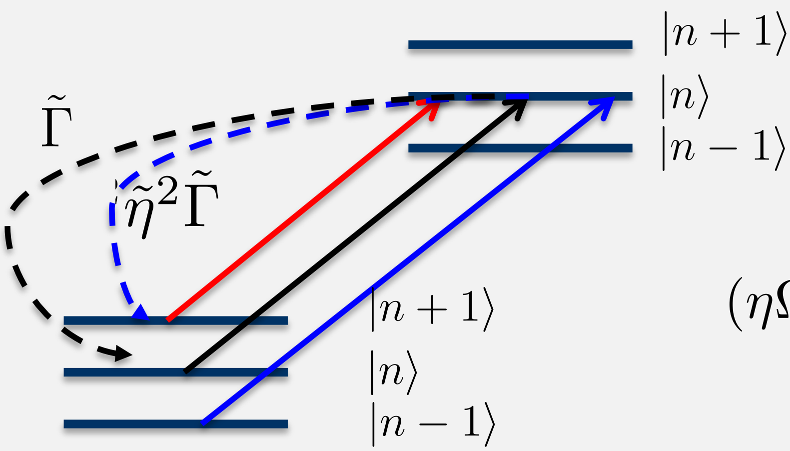


for finite linewidth, will scatter off-resonant on carrier and blue sideband

resolved sideband cooling

rates for $\delta \gg \tilde{\Gamma}$

$$R_n = \tilde{\Gamma} P_e(n) = \tilde{\Gamma} \frac{(\eta\sqrt{n}\Omega)^2}{2(\eta\sqrt{n}\Omega)^2 + \tilde{\Gamma}^2}$$



resonant red sideband

$[\Omega/(2\nu)]^2$ off-resonant carrier

$(\eta\Omega/[2(2\nu)])^2$ off-resonant blue sideband

assume cooling close to ground state, restrict rates to n=0,1:

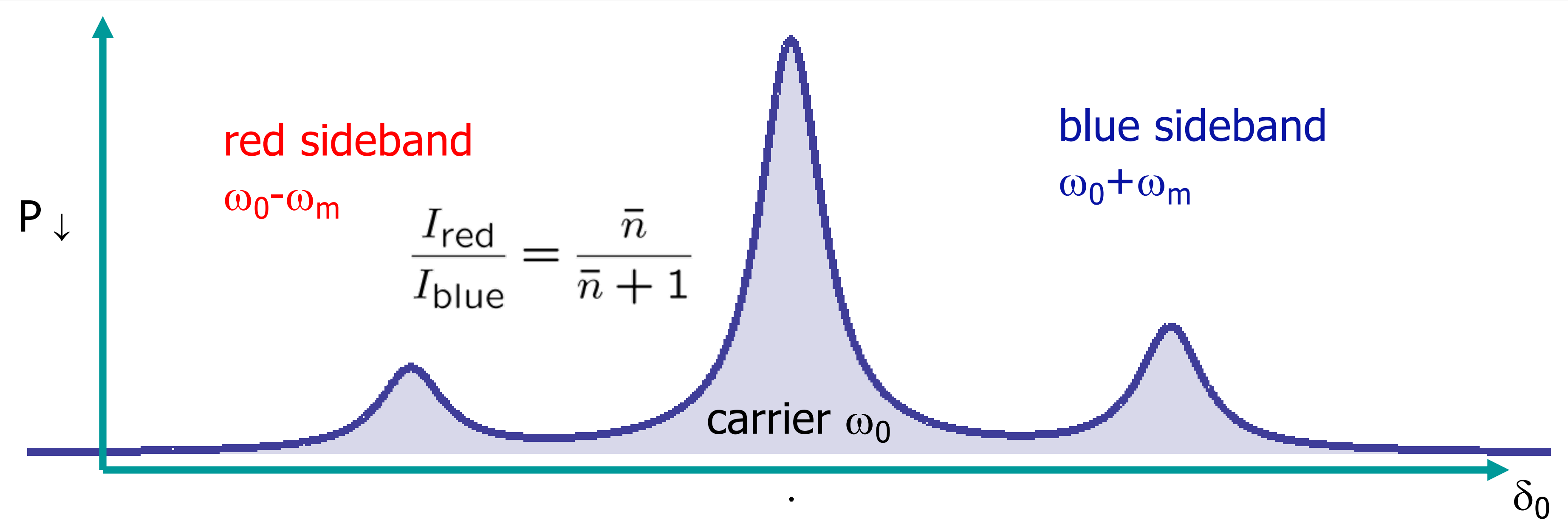
$$\dot{p}_0 = p_1 \frac{(\eta\Omega)^2}{\tilde{\Gamma}} - p_0 \left[\left(\frac{\Omega}{2\nu}\right)^2 \tilde{\eta}^2 \tilde{\Gamma} + \left(\frac{\eta\Omega}{4\nu}\right)^2 \tilde{\Gamma} \right]$$

$$\dot{p}_1 = -\dot{p}_0$$

rates vanish in steady state and ground state is nearly fully populated:

$$\bar{n} \approx p_1 \approx \left(\frac{\tilde{\Gamma}}{2\nu}\right)^2 \left[\left(\frac{\tilde{\eta}}{\eta}\right)^2 + \frac{1}{4} \right] \qquad p_0 \approx 1 - [\tilde{\Gamma}/(2\nu)]^2$$

Sideband thermometry



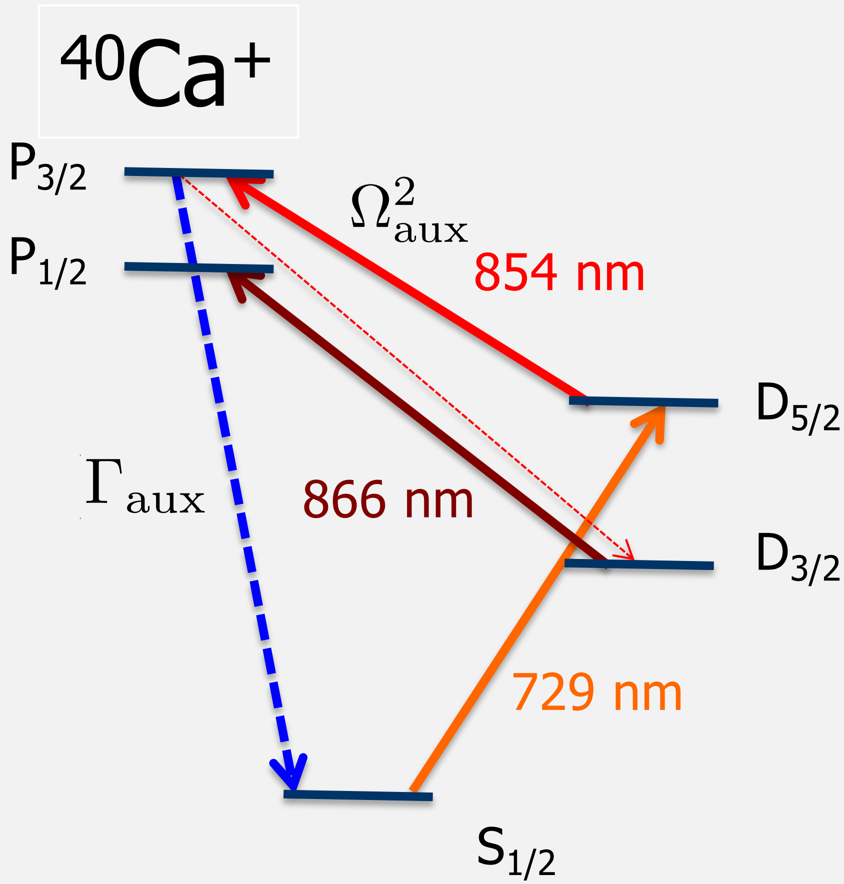
Rabi-flop resonantly on red and blue sidebands thermometry:

$$P_e^{\text{rsb}}(t) = \sum_{m=1}^{\infty} \left(\frac{\bar{n}}{\bar{n}+1} \right)^m \sin^2(\Omega_{m,m-1} t) = \frac{\bar{n}}{\bar{n}+1} \sum_{m=0}^{\infty} \left(\frac{\bar{n}}{\bar{n}+1} \right)^m \sin^2(\Omega_{m+1,m} t)$$

$$= \frac{\bar{n}}{\bar{n}+1} P_e^{\text{bsb}}(t)$$

red sideband vanishes for $\bar{n} \longrightarrow 0$ (motional ground state)

experimental example

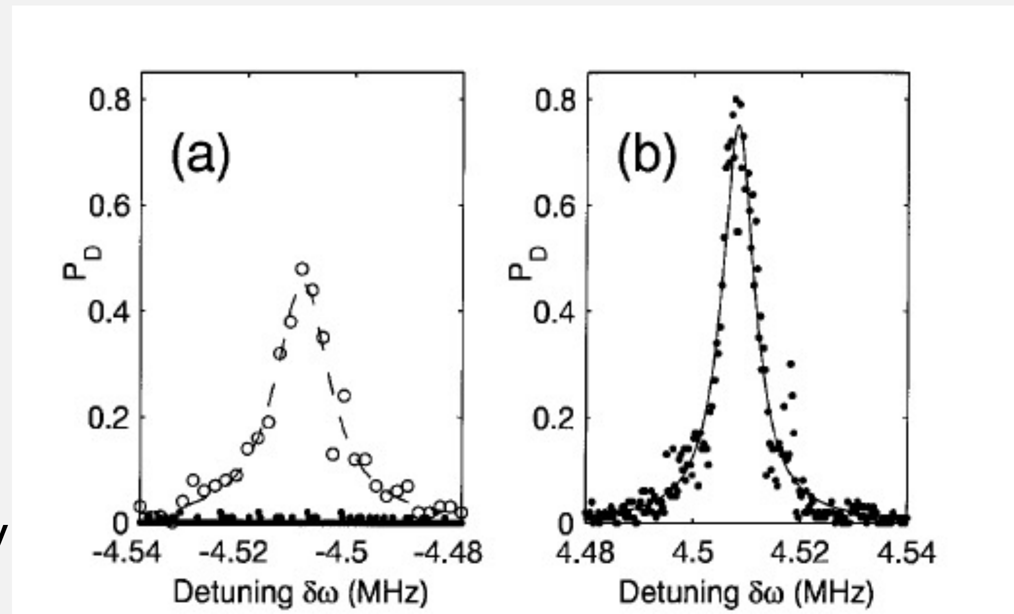


shine in red sideband drive and 854 nm repump to produce faster decay back to ground state with

$$\tilde{\Gamma} = \Gamma_{aux} \frac{\Omega^2_{aux}}{(\Gamma_{aux} + \Gamma_{nat})^2 + 4\Delta^2_{aux}}$$

plug small leak to D3/2 by pulsing on 866 nm
total cooling time 6.4 ms

Roos et al. PRL **83**, 4713(1999):



sidebands after Doppler cooling (open symbols) and ground state cooling (closed symbols) to 99.9% ground state probability

general rate equations for cooling

consider cooling transition with scattering rate $W(\Delta) = \Gamma\rho_{\mathrm{ex}}$
general scattering process will proceed as $|g,n\rangle \to |e,n'\rangle \to |g,n''\rangle$
could find rates for all combinations and solve coupled rate equations
(always assume coherences are negligible during cooling)

also assume LD-limit, only lowest order in η, then excitation is prop. to

carrier: Ω^2
red sideband: $\eta^2\Omega^2 n$
blue sideband: $\eta^2\Omega^2(n+1)$

terms of order η^2 or lower will limit contributions to

$$|g,n\rangle \to |e,n\pm\{0,1\}\rangle \to |g,n\pm\{0,1\}\rangle$$

total rates at detuning Δ:

$$R^n_{n+1} = \underbrace{W(\Delta)\eta^2(n+1)}_{\text{carr. / blue SB}} + \underbrace{W(\Delta-\nu)\eta^2(n+1)}_{\text{blue SB / carr.}}, \quad R^n_{n-1} = \underbrace{W(\Delta)\eta^2 n}_{\text{carr. / red SB}} + \underbrace{W(\Delta+\nu)\eta^2 n}_{\text{red SB / carr.}}.$$

carr. blue SB blue SB carr. carr. red SB red SB carr.

general rate equations for cooling II

$$R^n_{n+1} = W(\Delta)\eta^2(n+1) + \underbrace{W(\Delta-\nu)\eta^2(n+1)}, \quad R^n_{n-1} = W(\Delta)\eta^2 n + \underbrace{W(\Delta+\nu)\eta^2 n}.$$

carr. blue SB blue SB carr. carr. red SB red SB carr.

populations only arrive from/depart to adjacent states

$$\begin{aligned}\frac{d}{dt}P_n &= R_n^{n+1}P_{n+1} + R_n^{n-1}P_{n-1} - (R^n_{n-1} + R^n_{n+1})P_n \\ &= A_-[P_{n+1}(n+1) - P_n(n)] + A_+[P_{n-1}n - P_n(n+1)]\end{aligned}$$

A_+/A_- are independent of n $\quad A_\pm = \eta^2[W(\Delta) + W(\Delta \mp \nu)].$

this allows for deriving simple equation of motion for average occupation

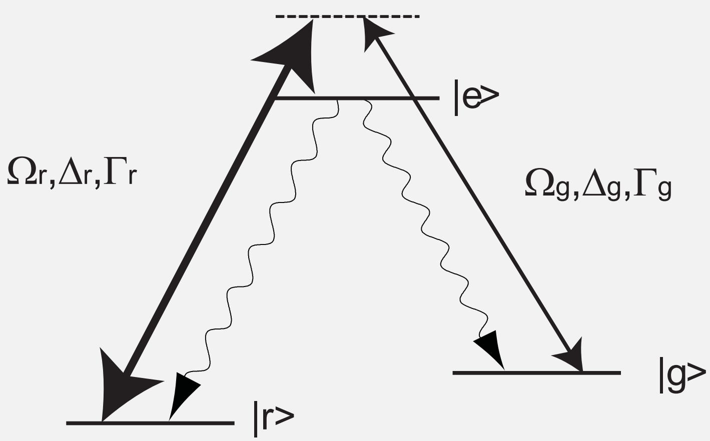
$$\frac{d}{dt}\bar{n} = \sum_{n=1}^{\infty} n \frac{d}{dt}P_n = -(A_- - A_+)\bar{n} + A_+$$

as long as $A_+ < A_-$ (red sideband scattering dominates) there exists a steady state

$$\bar{n}_f = \frac{A_+}{A_- - A_+} = \frac{W(\Delta) + W(\Delta-\nu)}{W(\Delta+\nu) - W(\Delta-\nu)}$$

application to EIT cooling

consider 3-level Λ-system



$$\frac{d\rho_{rr}}{dt} = i\frac{\Omega_r}{2}(\rho_{re} - \rho_{er}) + \Gamma_r\rho_{ee}$$

$$\frac{d\rho_{gg}}{dt} = i\frac{\Omega_g}{2}(\rho_{ge} - \rho_{eg}) + \Gamma_g\rho_{ee}$$

$$\frac{d\rho_{rg}}{dt} = i\left[(\Delta_g - \Delta_r)\rho_{rg} + \frac{\Omega_g}{2}\rho_{re} - \frac{\Omega_r}{2}\rho_{eg}\right]$$

$$\frac{d\rho_{re}}{dt} = i\left[\frac{\Omega_r}{2}(\rho_{rr} - \rho_{ee}) + \frac{\Omega_g}{2}\rho_{rg} - \Delta_r\rho_{re}\right] - \frac{\Gamma}{2}\rho_{re}$$

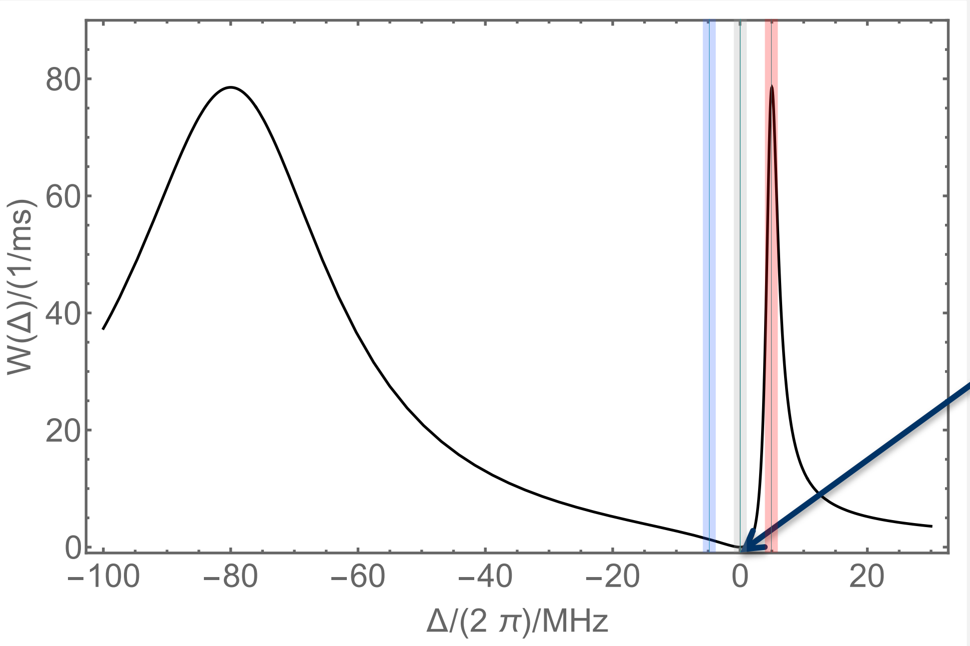
$$\frac{d\rho_{ge}}{dt} \doteq i\left[\frac{\Omega_g}{2}(\rho_{gg} - \rho_{ee}) + \frac{\Omega_r}{2}\rho_{gr} - \Delta_g\rho_{ge}\right] - \frac{\Gamma}{2}\rho_{ge}$$

assume, density matrix is going to steady state much faster than motion changes and that $\Omega_r >> \Omega_g$, $\Delta_r >> \Omega_g$ (strong pumping into g), then

$$W(\Delta) = \Gamma\rho_{ee}(\Delta) \approx \frac{\Delta^2\Omega_g^2\Gamma}{\alpha\left[\Delta^2\Gamma^2 + 4(\Omega_r^2/4 - \Delta\Delta_g)^2\right]}$$

$$\Delta = \Delta_g - \Delta_r$$
$$\Gamma = \Gamma_g + \Gamma_r$$

application to EIT cooling II



$$\Delta = \Delta_g - \Delta_r \quad \Gamma = \Gamma_g + \Gamma_r \quad \alpha = \Gamma_g/\Gamma$$

$$W(\Delta) \approx \frac{\Delta^2 \Omega_g^2 \Gamma}{\alpha\left[\Delta^2\Gamma^2 + 4(\Omega_r^2/4 - \Delta\Delta_g)^2\right]}$$

rate vanishes at Δ=0,
maxima occur at

$$\Delta_\pm = \frac{1}{2}\left(\pm\sqrt{\Delta_r^2 + \Omega_r^2} - \Delta_r\right)$$

here at 5 MHz/-80 MHz

desired:

strong scattering on red sideband $\quad \nu = 1/2(\sqrt{\Delta_r^2 + \Omega_r^2} - \Delta_r)$

no scattering on carrier $\quad \Delta = \Delta_g - \Delta_r = 0$

weak scattering on blue sideband $\quad$ wing of broad peak

$$\bar{n}_s = \frac{W(-\nu)}{W(\nu) - W(-\nu)} = \left(\frac{\Gamma}{4\Delta_r}\right)^2$$

here ca. 2 10^{-2}

set $\Delta_r = \Delta_g >> \Gamma$ for good ground state cooling, trade-off with cooling rate