ALESSANDRO BACCHETTA, PAVIA U. AND INFN

STATUS OF QUARK TMDS

A NEW ERA OF DISCOVERY

THE 2023 LONG RANGE PLAN FOR NUCLEAR SCIENCE

2023 | VERSION 1.2

https://science.osti.gov/-/media/np/nsac/pdf/202310/NSAC-LRP-2023-v12.pdf

A NEW ERA OF DISCOVERY

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RECOMMENDATION 3
We recommend the expeditious completion of the EIC as the highest priority for facility construction.

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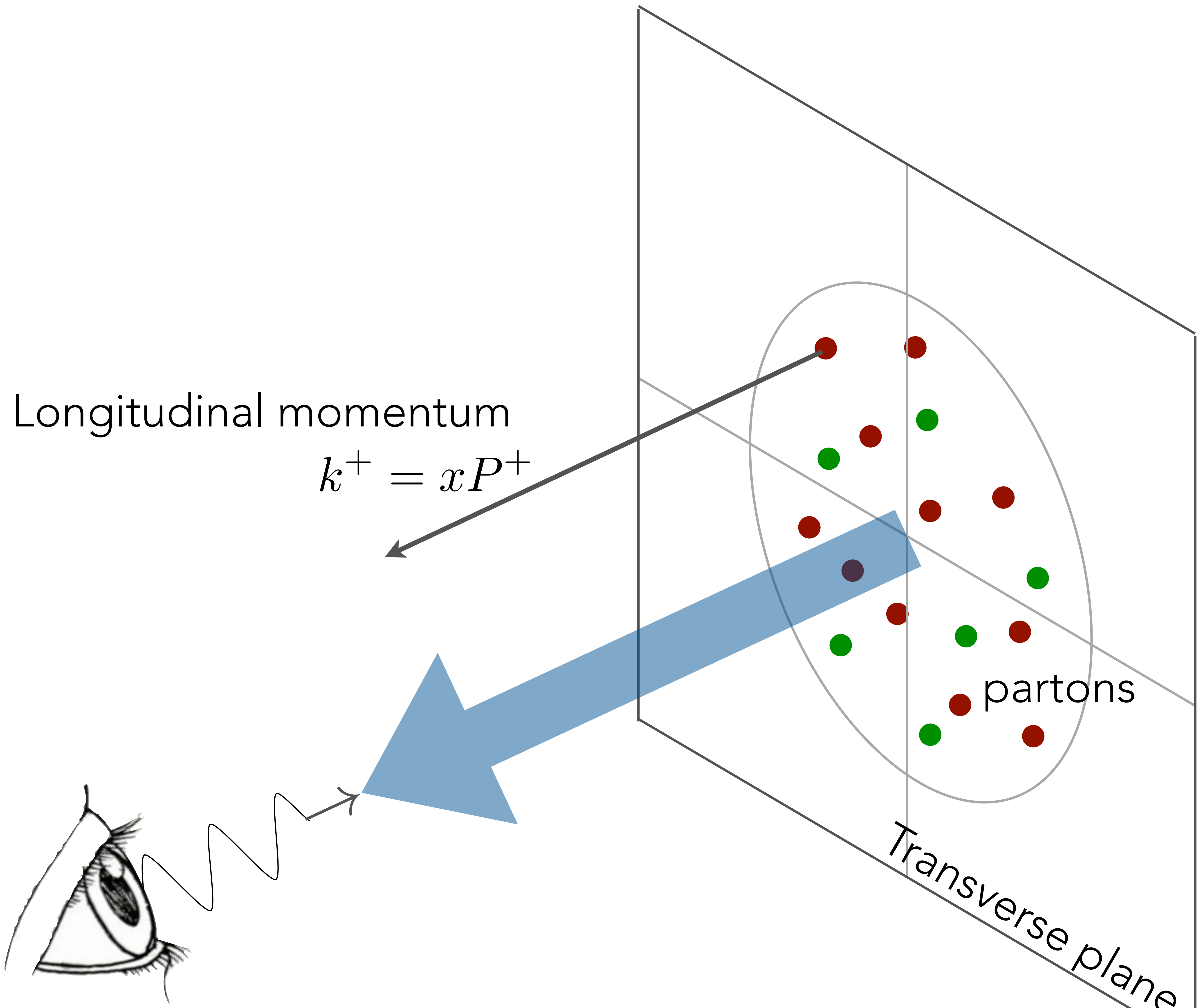
RECOMMENDATION 3
We recommend the expeditious completion of the EIC as the highest priority for facility construction.

> The EIC is a powerful discovery machine, a precision microscope capable of taking three-dimensional pictures of nuclear matter at femtometer scales.

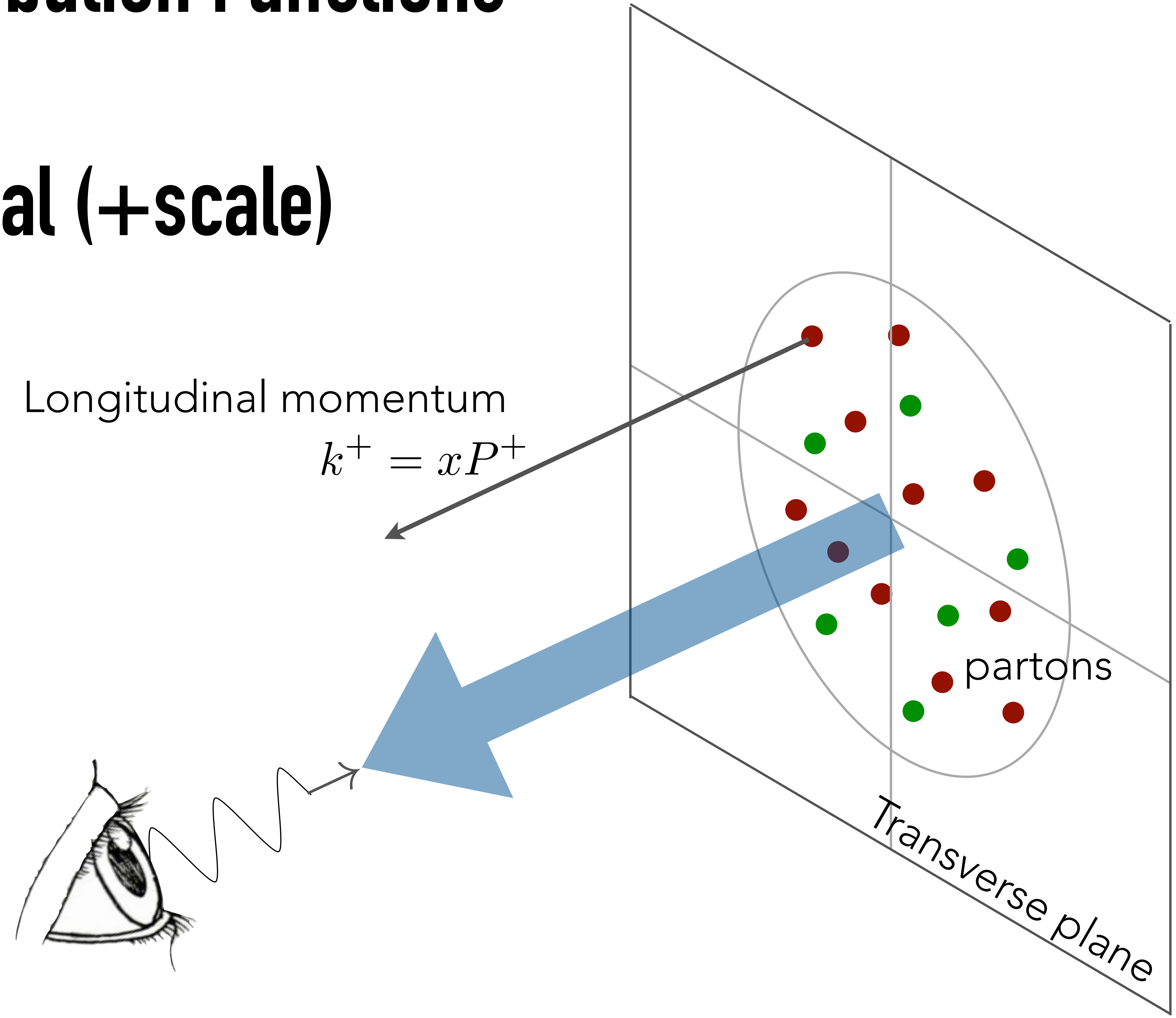
https://science.osti.gov/-/media/np/nsac/pdf/202310/NSAC-LRP-2023-v12.pdf

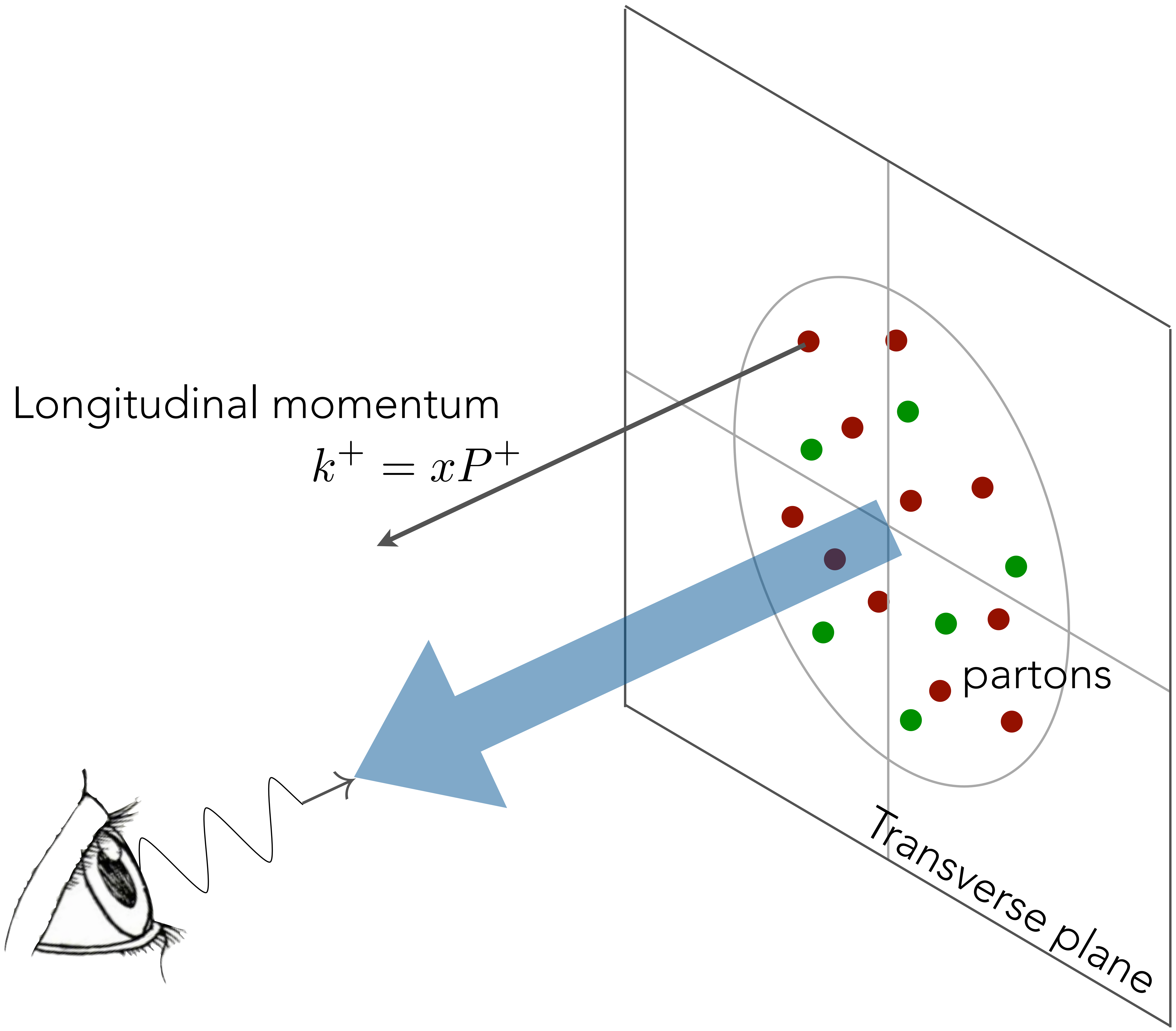

Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

Parton Distribution Functions

$f(x)$

1 dimensional (+scale)





Longitudinal momentum

$k^+ = xP^+$

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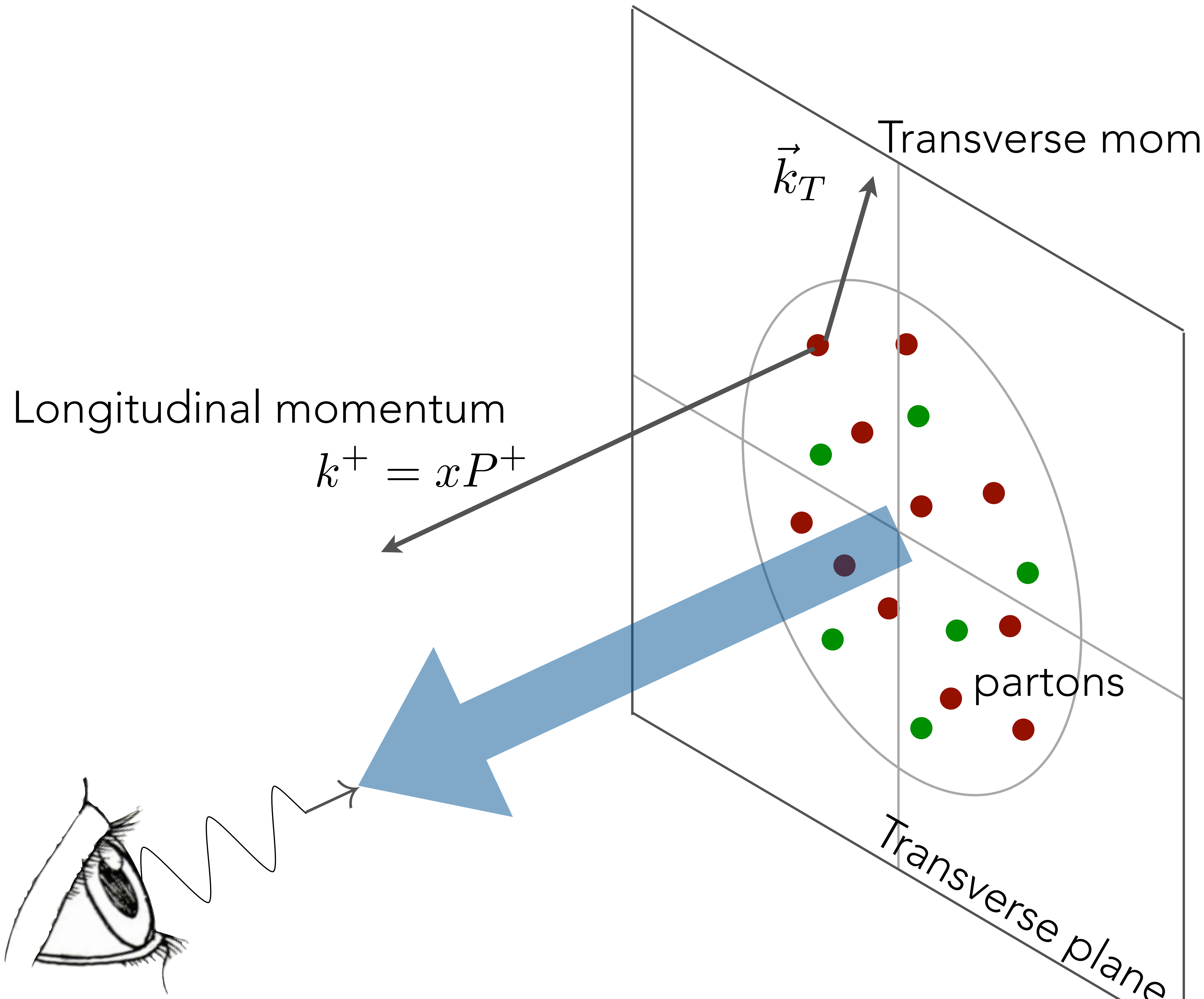
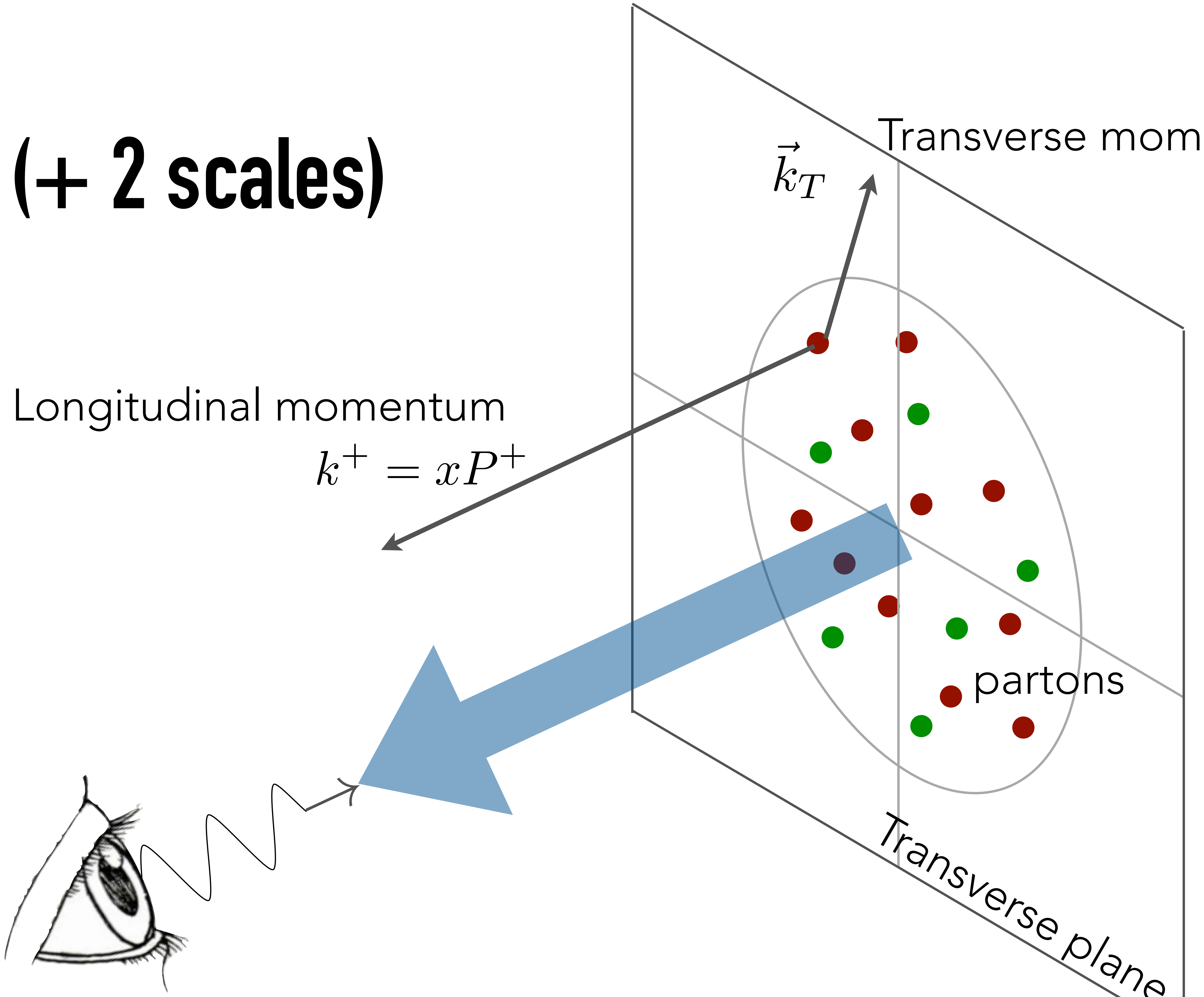
Transverse plane

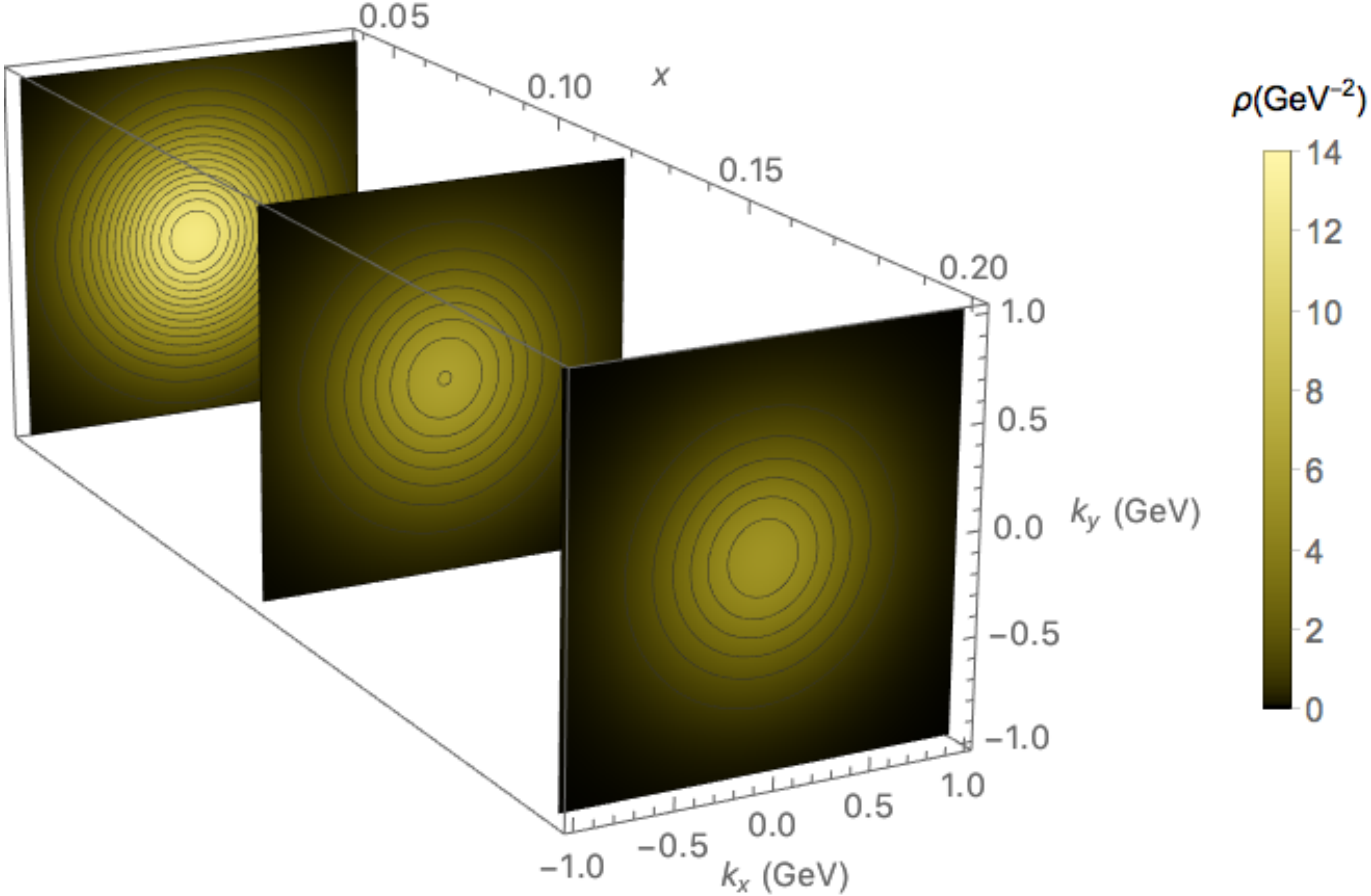

Transverse momentum
$\vec{k}_T$
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

Transverse-Momentum Distributions

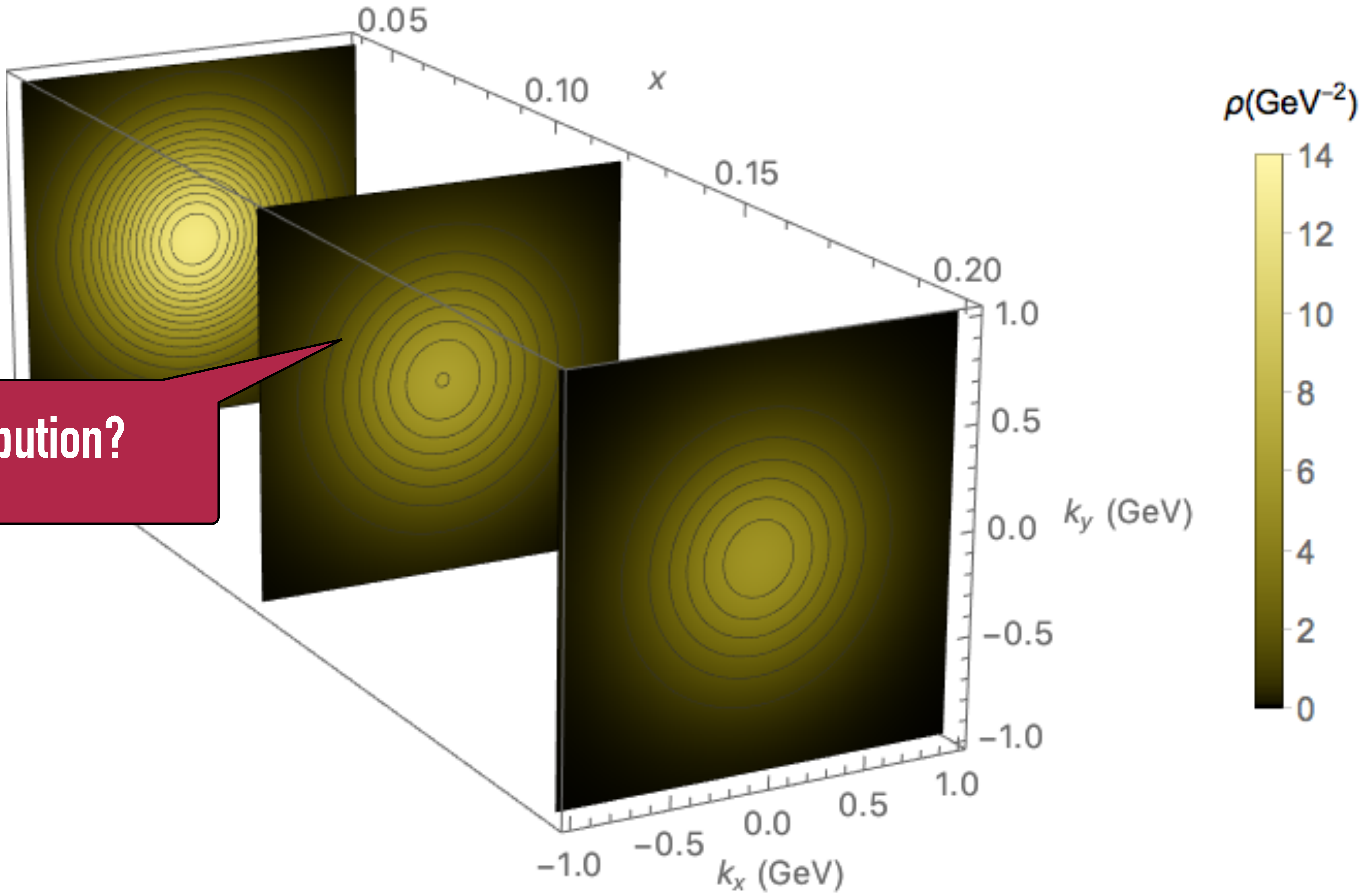
$f(x, \vec{k}_T)$

3 dimensional (+ 2 scales)





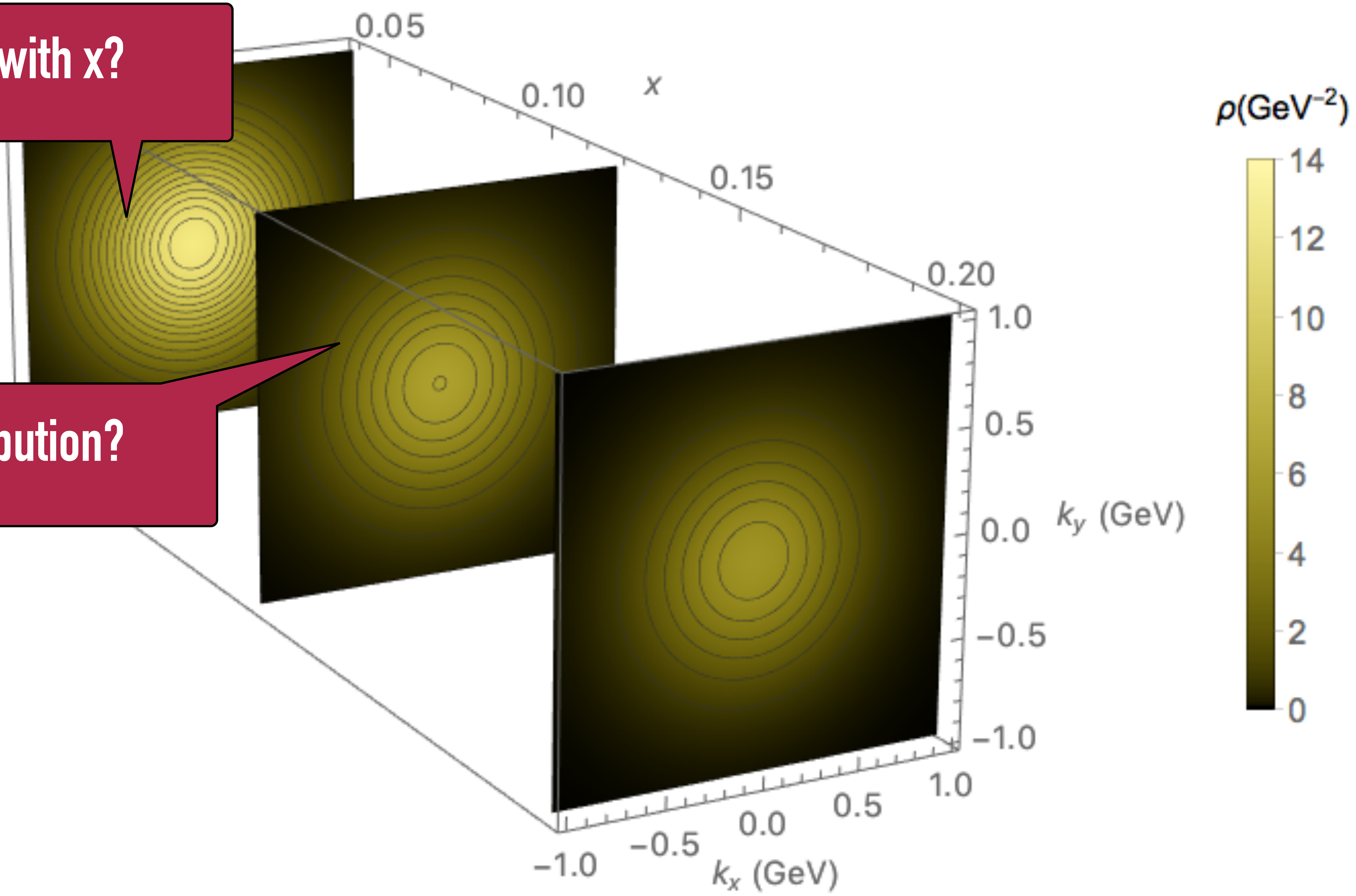
0.05
0.10
x
0.15
0.20
1.0
0.5
0.0
−0.5
−1.0
k_y (GeV)
−1.0
−0.5
0.0
0.5
1.0
k_x (GeV)
ρ(GeV^{-2})
14
12
10
8
6
4
2
0

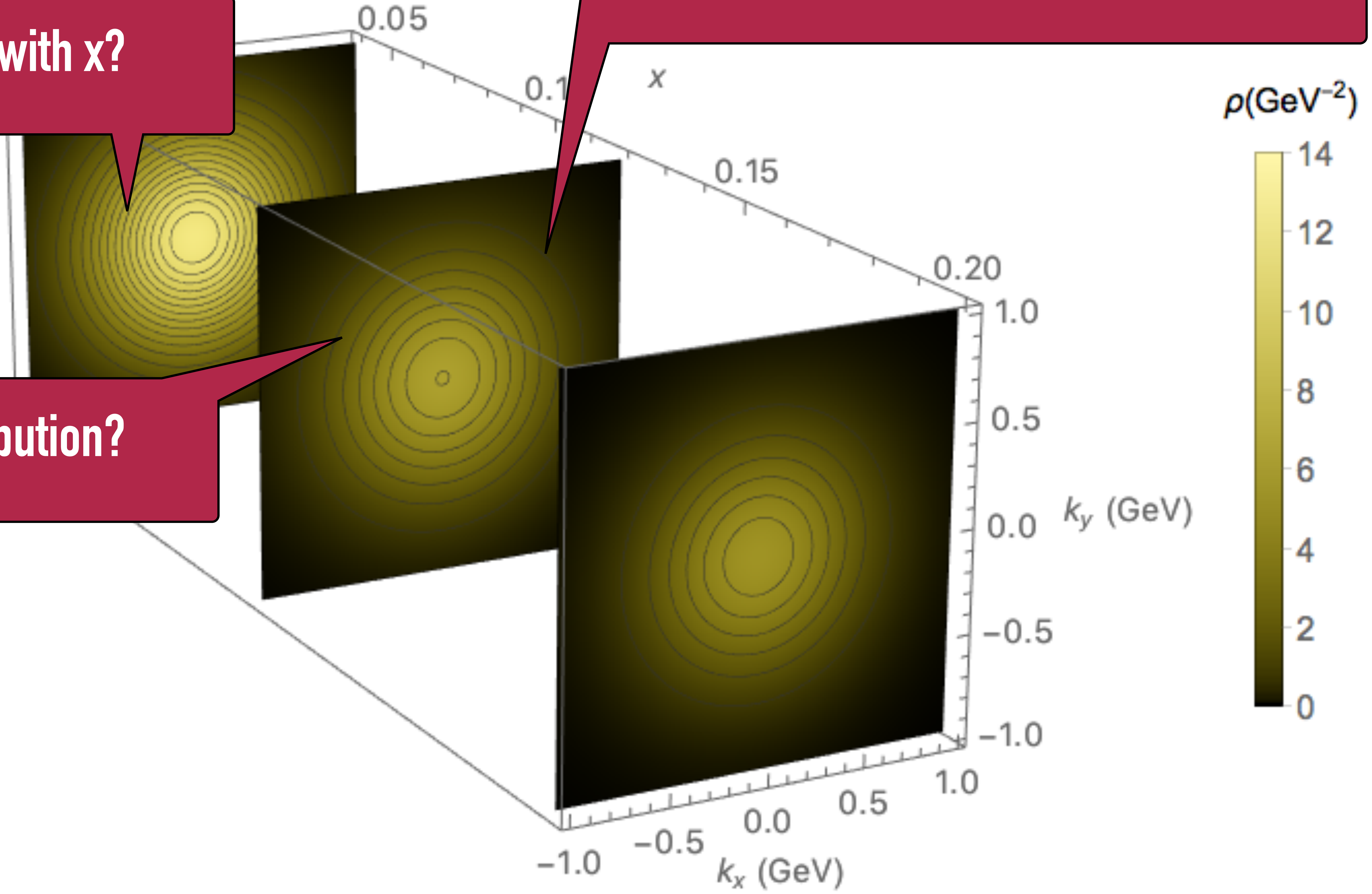


How “wide” is the distribution?

How does it change with x?

How “wide” is the distribution?

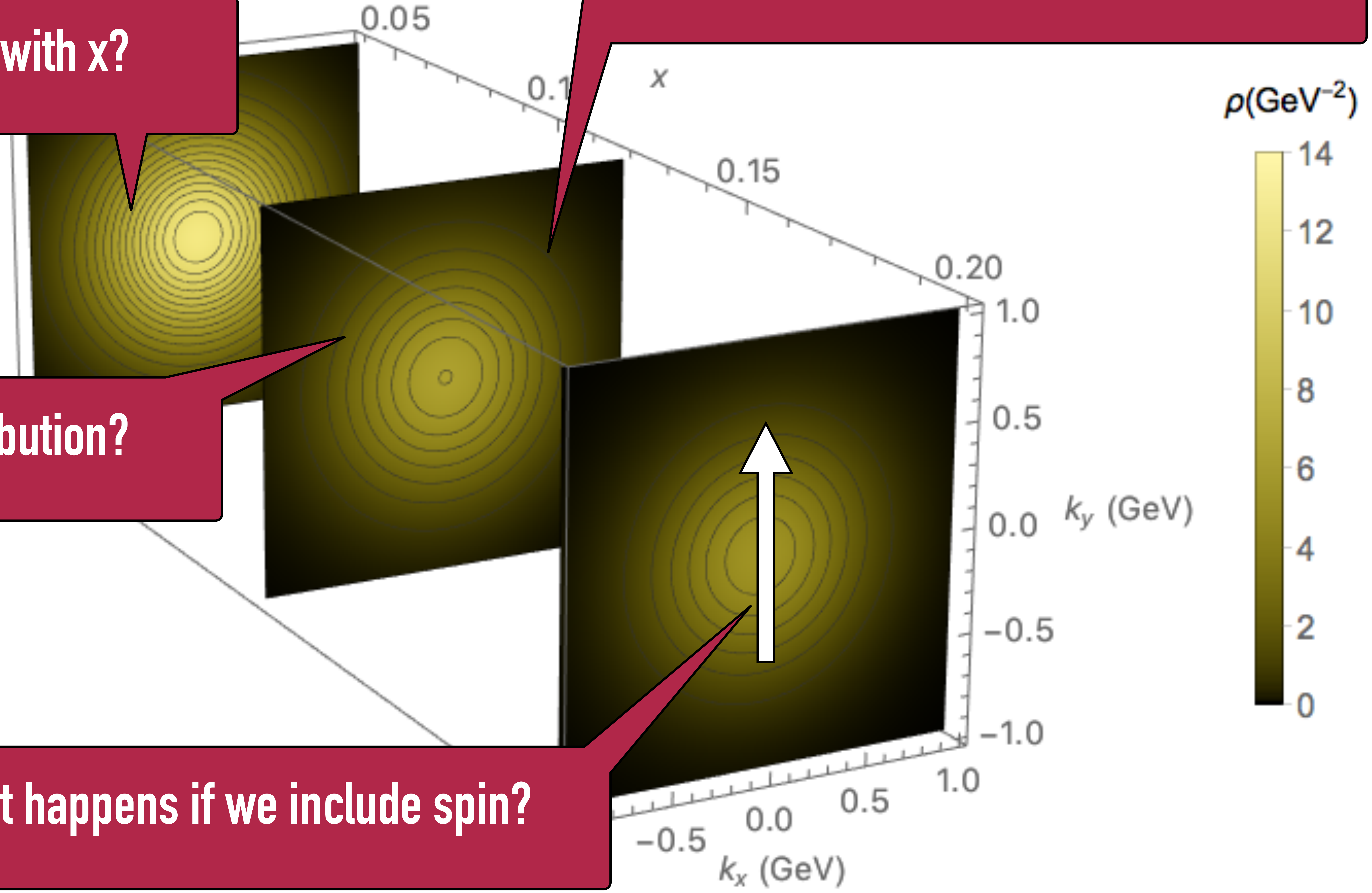




How does it change with x?

Is there a difference between flavors?

How “wide” is the distribution?



How does it change with x?

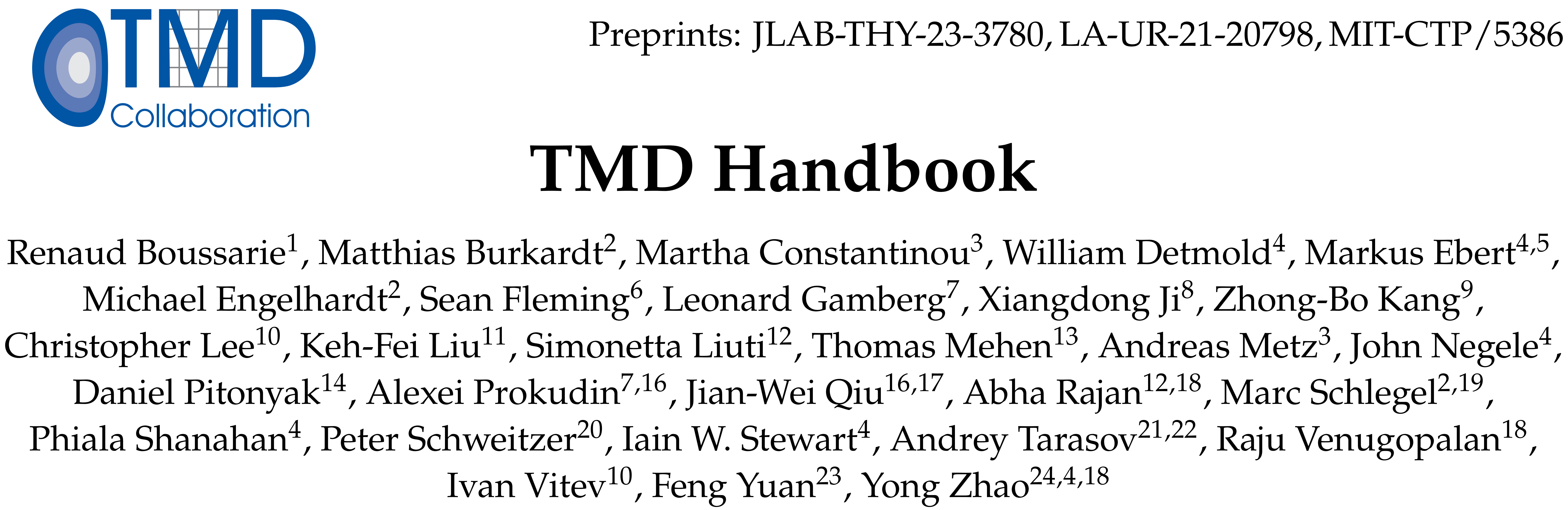
Is there a difference between flavors?

How “wide” is the distribution?

What happens if we include spin?

RECENT REVIEW

TMD collaboration, "TMD Handbook," arXiv:2304.03302



TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5],
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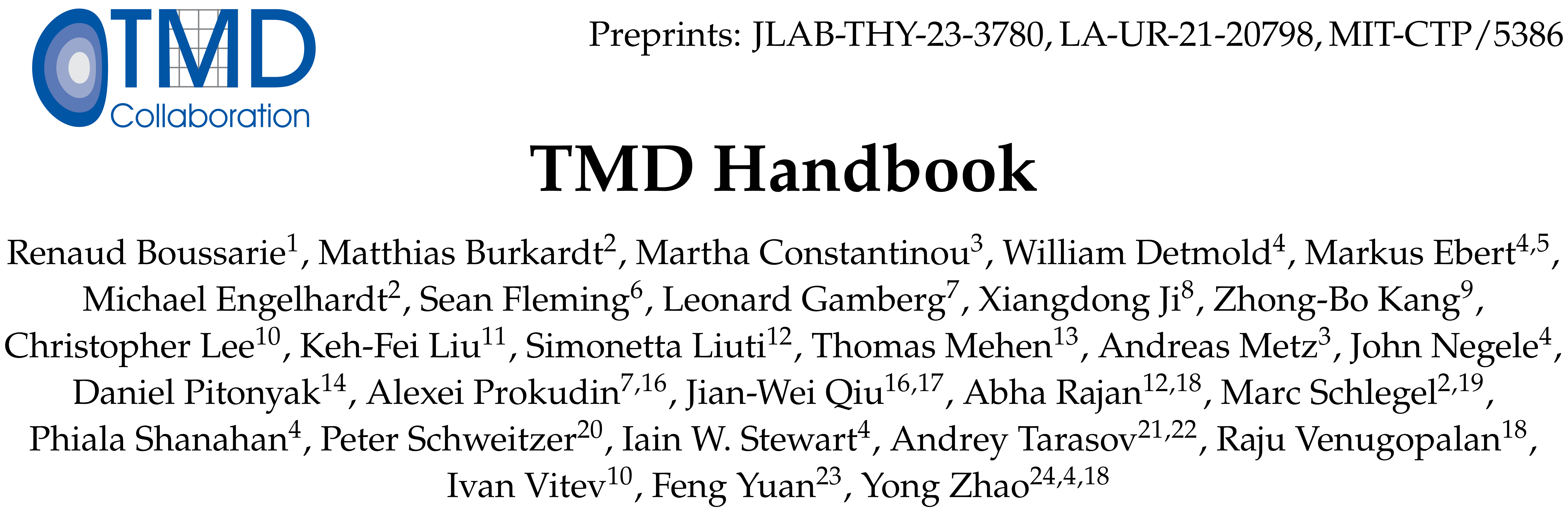

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471 PAGES

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

		quark pol.		
		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

TMD TABLES: QUARK, LEADING TWIST

[Mulders-Tangerman, NPB 461 (96)](#)
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nucleon pol.

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Sivers

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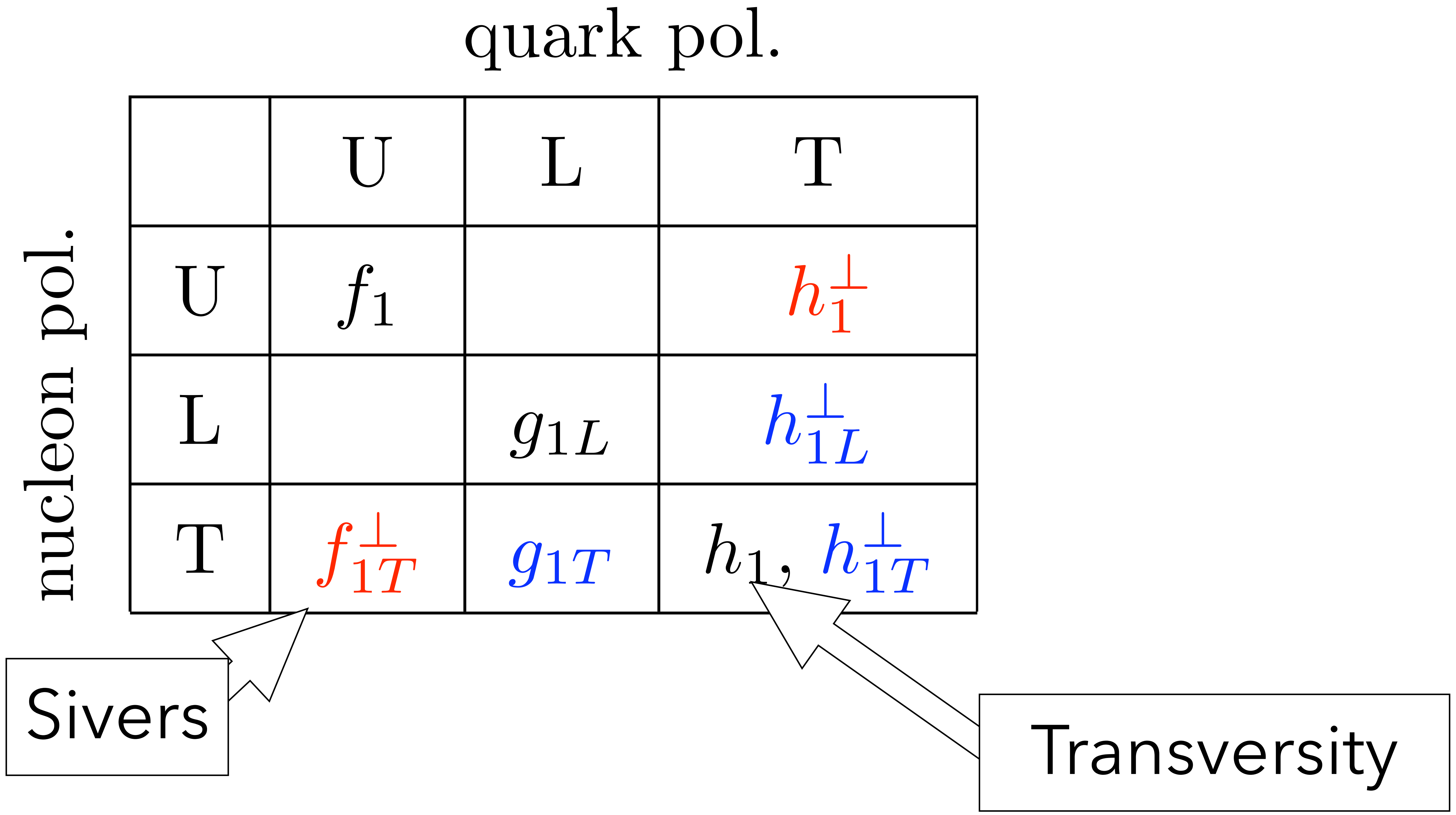

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Sivers

Transversity

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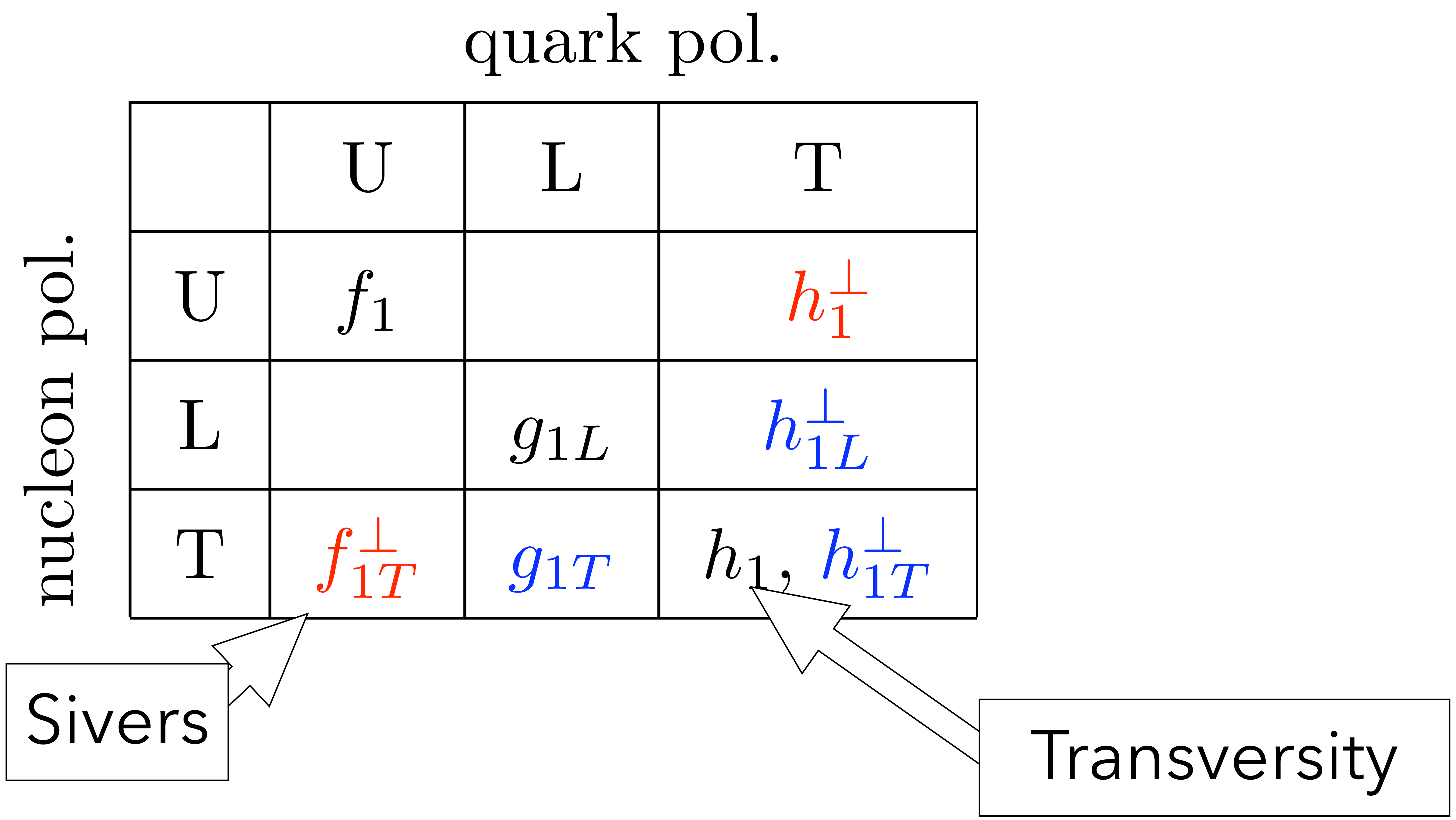

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Sivers

Transversity

- Very good knowledge of x dependence of f_1 and g_{1L}

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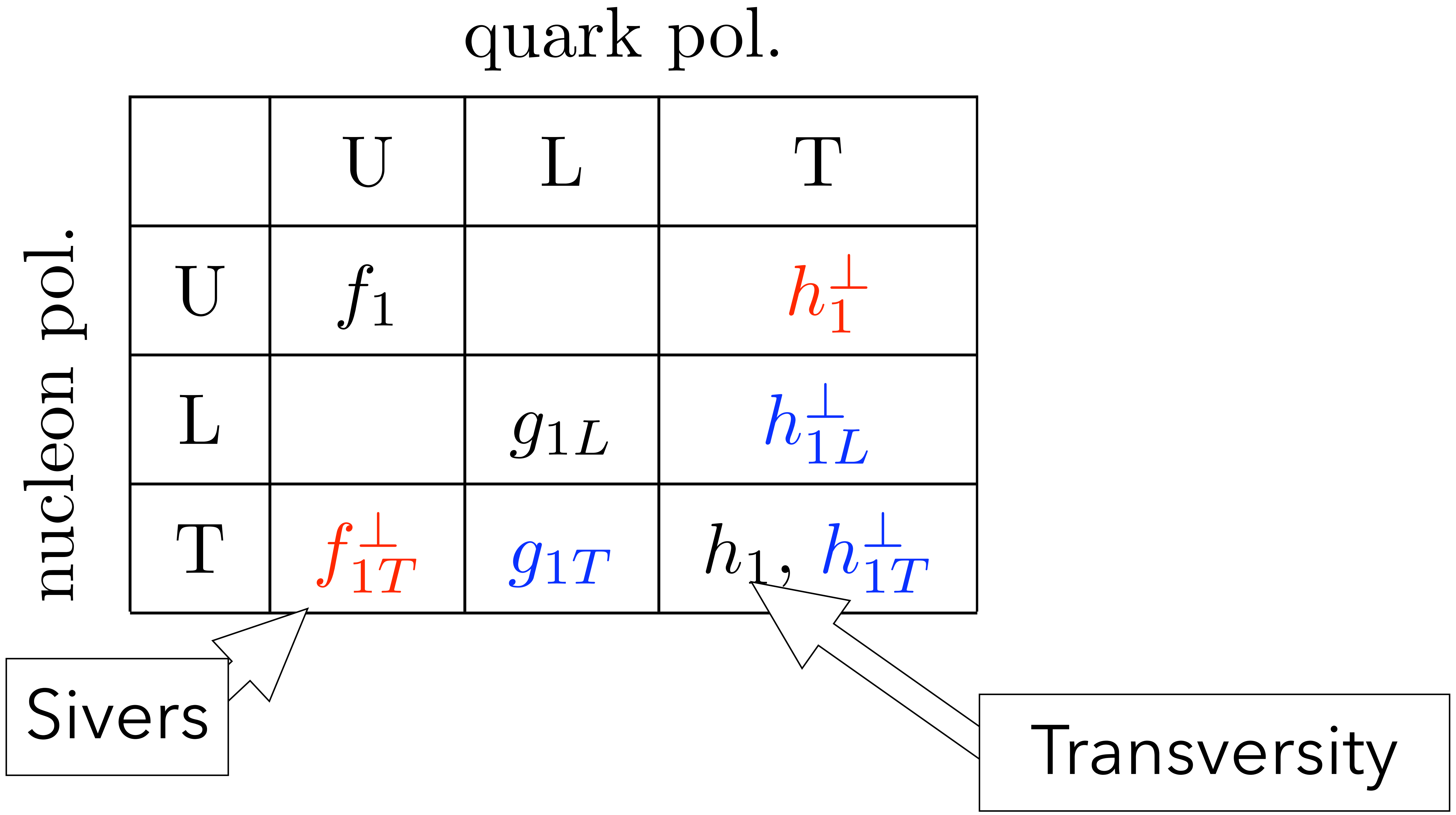

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Sivers

Transversity

- Very good knowledge of x dependence of f_1 and g_{1L}
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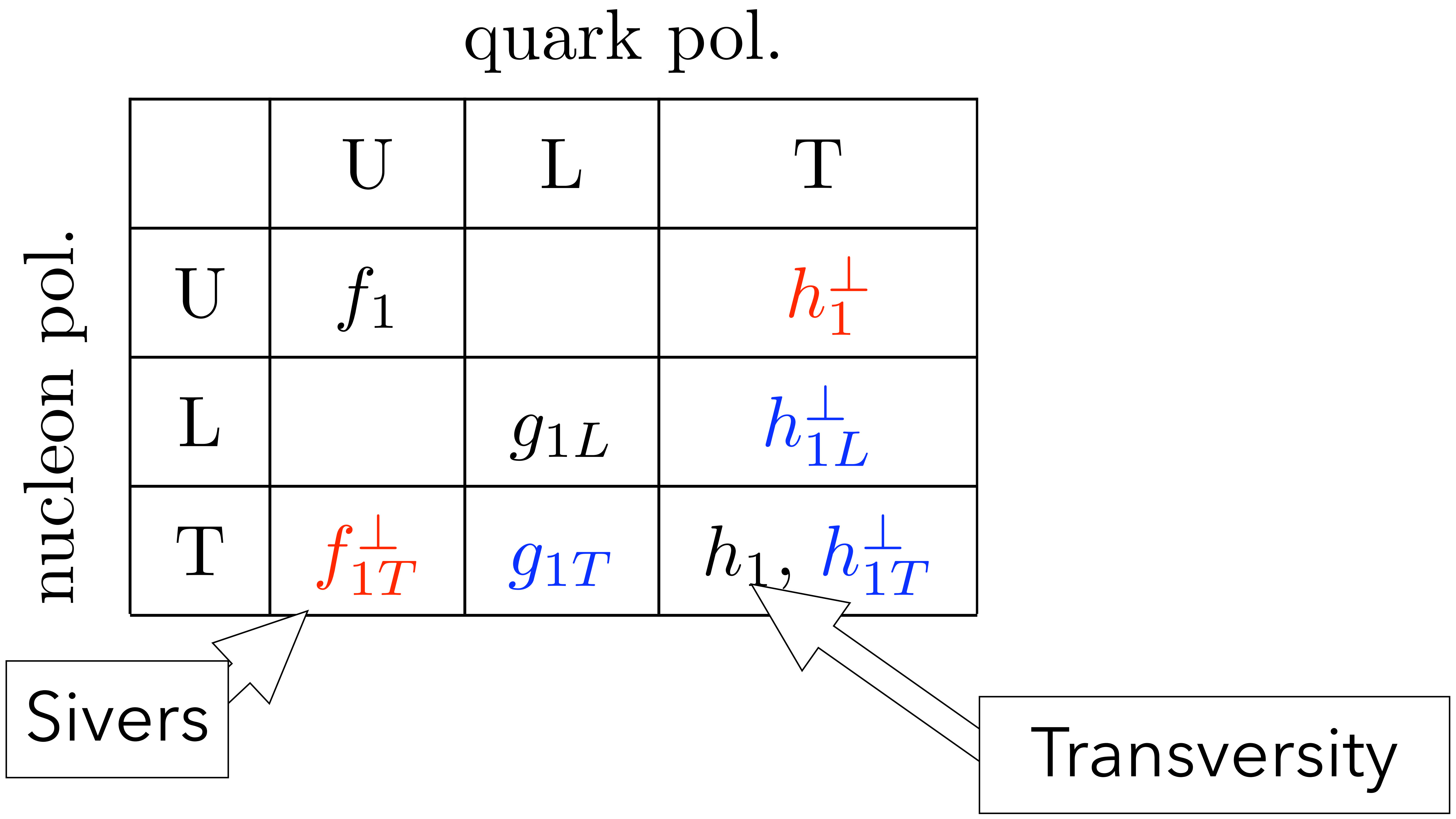

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- Fair knowledge of Sivers and transversity (mainly x dependence)

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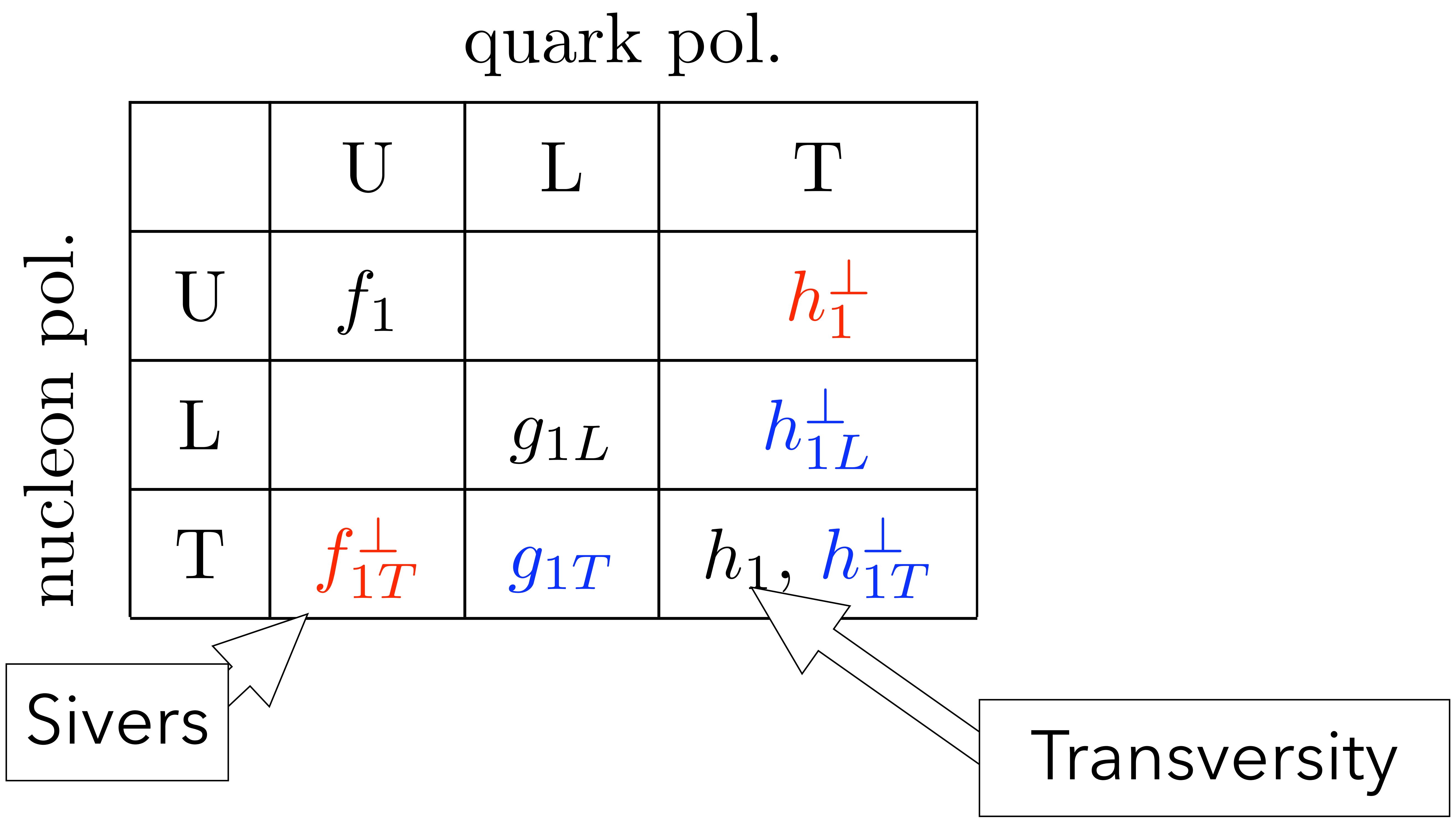

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- Very good knowledge of x dependence of f_1 and g_{1L}
- Good knowledge of the k_T dependence of f_1 (also for pions)
- Fair knowledge of Sivers and transversity (mainly x dependence)
- Some hints about all others

INTERPRETATION OF POLARIZED TMDS

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-○➡ ⊙ nucleon with transverse or longitudinal spin

-○➡ ⊙ parton with transverse or longitudinal spin

➡ parton transverse momentum

INTERPRETATION OF POLARIZED TMDS

Proton goes out of the screen/
photon goes into the screen

- ⟶ ⊙ nucleon with transverse or longitudinal spin
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- ➡ parton transverse momentum

INTERPRETATION OF POLARIZED TMDS

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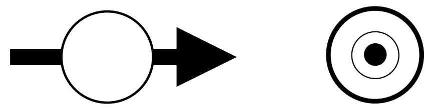
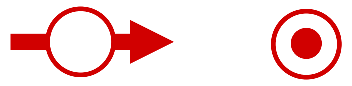
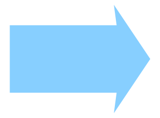
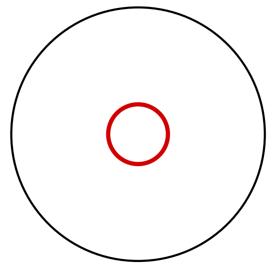
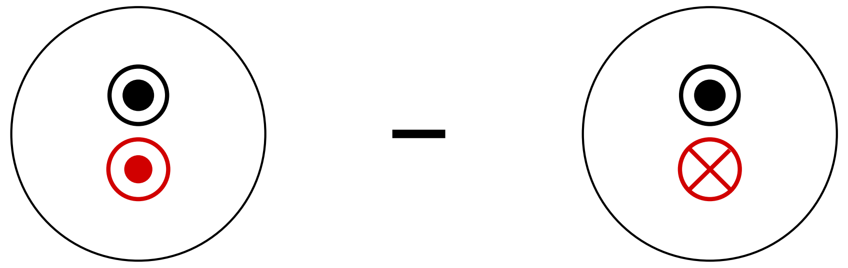

nucleon with transverse or longitudinal spin

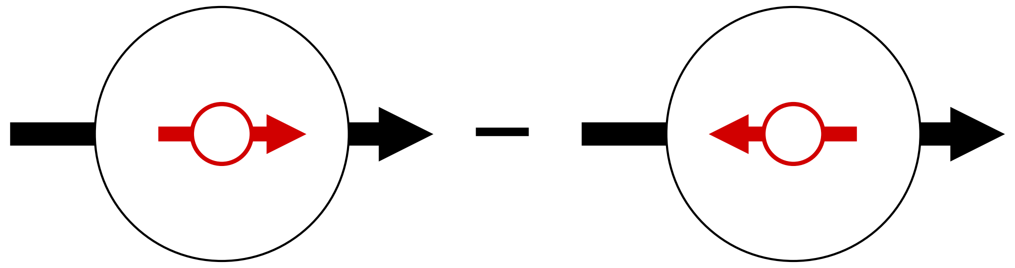

parton with transverse or longitudinal spin


parton transverse momentum

$f_1 =$ 

$g_1 =$ 

Helicity

$h_1 =$ 

Transversity

INTERPRETATION OF POLARIZED TMDS

Proton goes out of the screen/
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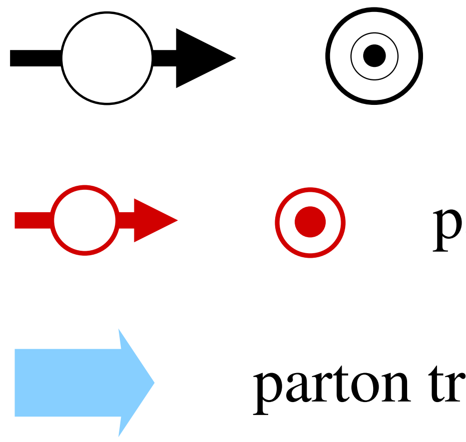
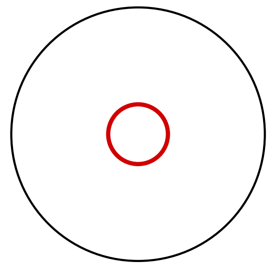
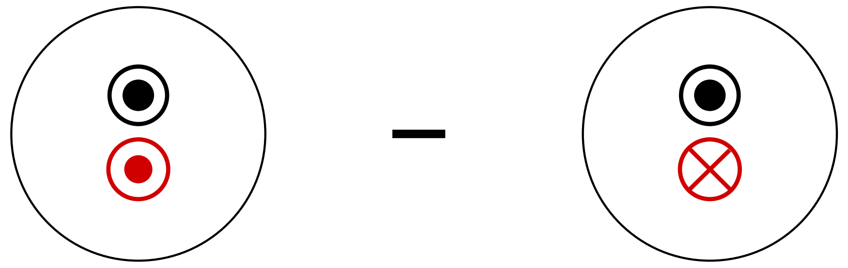


nucleon with transverse or longitudinal spin

parton with transverse or longitudinal spin

parton transverse momentum

$f_1 =$ 

$g_1 =$ 

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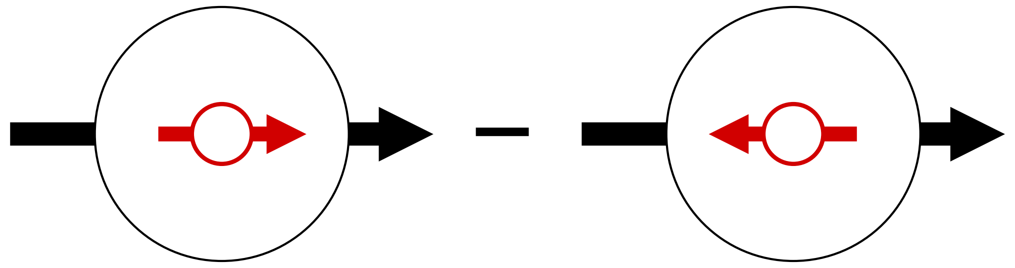
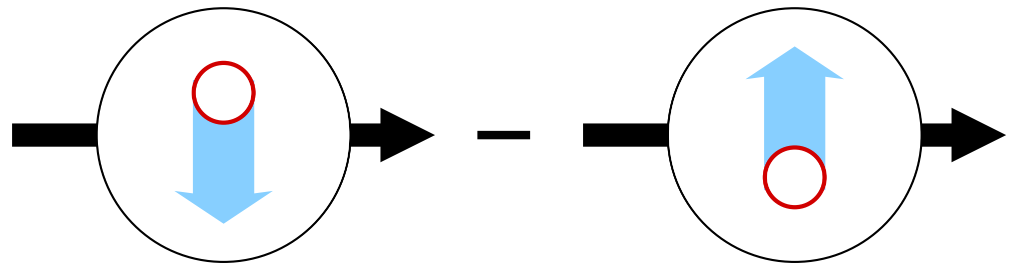
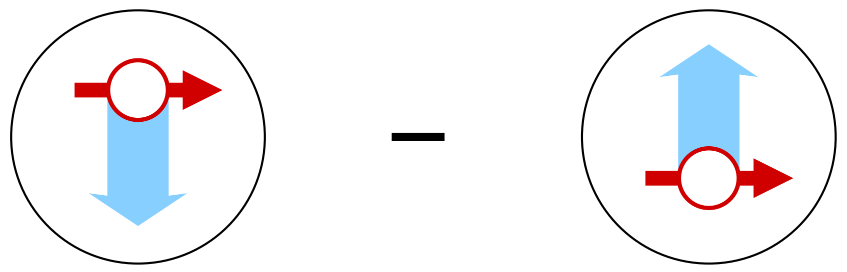
$h_1 =$ 

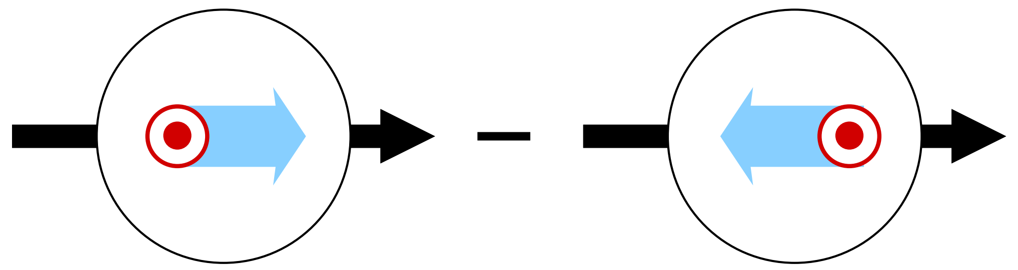
Transversity

$f_{1T}^{\perp} =$ 

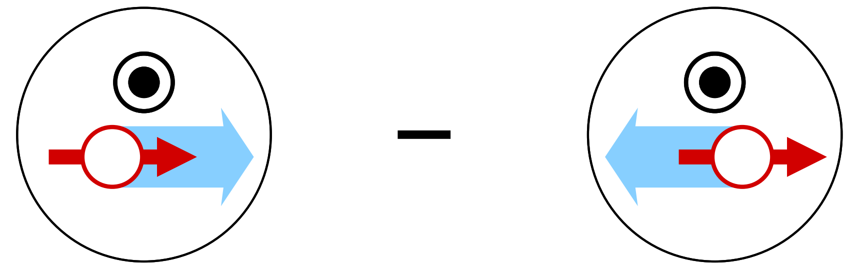
Sivers

$h_1^{\perp} =$ 

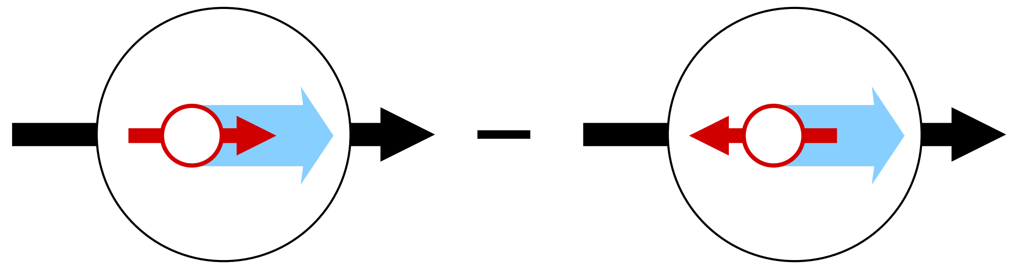
Boer-Mulders

$g_{1T} =$ 

worm-gear

$h_{1L}^{\perp} =$ 

worm-gear

$h_{1T}^{\perp} =$ 

pretzelosity

AVAILABLE EXTRACTIONS (NEWEST ONLY)

Unpol. TMD	MAP 22 arXiv:2206.07598, ART23 2305.07473
Helicity	
Transversity	arXiv:1505.05589, arXiv:1612.06413, arXiv:2205.00999
Sivers	MAP20 arXiv:2004.14278, arXiv:2009.10710, arXiv:2103.03270, arXiv:2205.00999, arXiv:2304.14328
Boer-Mulders	arXiv:0912.2031, arXiv:1502.04214, arXiv:2004.02117
Worm-gear g1T	arXiv:2110.10253
Worm-gear h1L	
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Worm-gear h1L	
Pretzelosity	arXiv:1411.0580

Not mentioned: pion TMDs, TMD fragmentation functions, nuclear TMDs

QUARK, SUBLEADING TWIST

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Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
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- Some knowledge of g_T x-dependence
- First hints about e x-dependence

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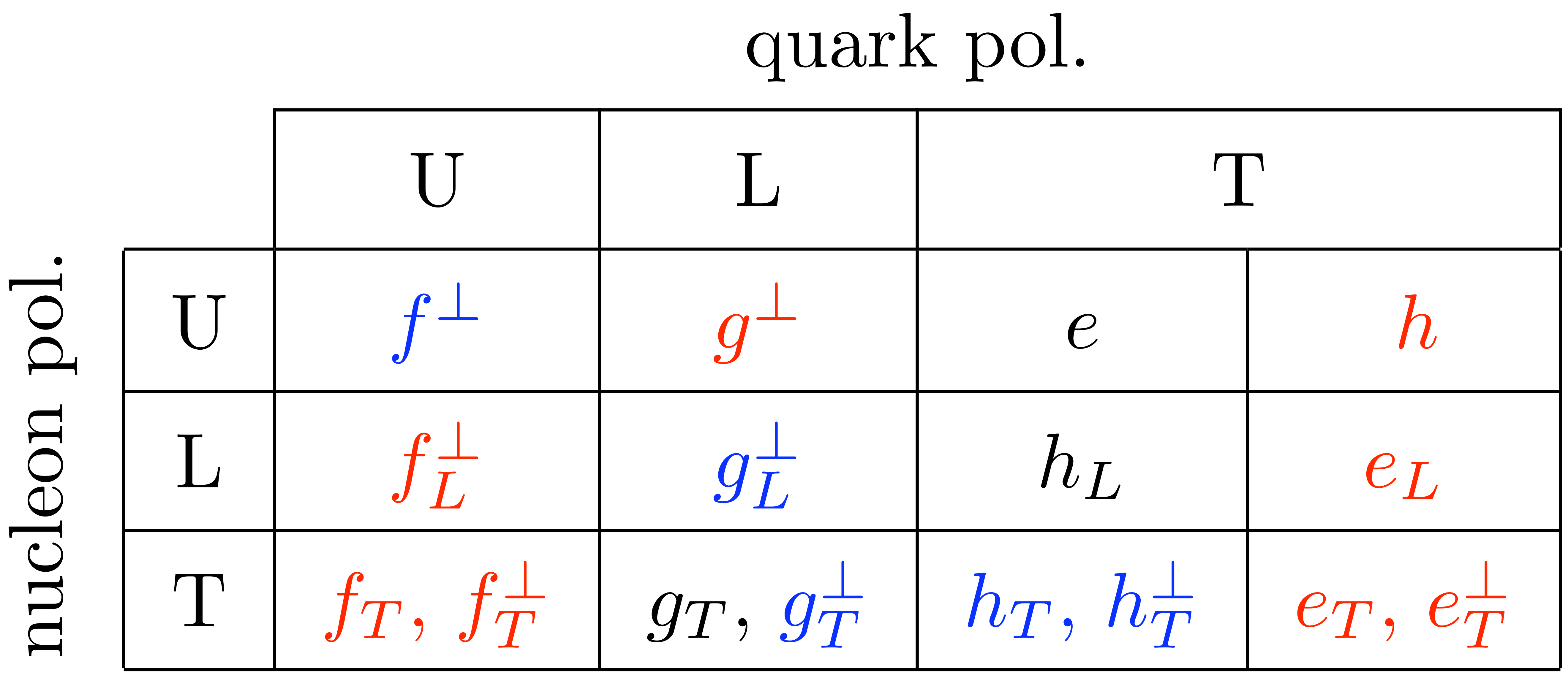

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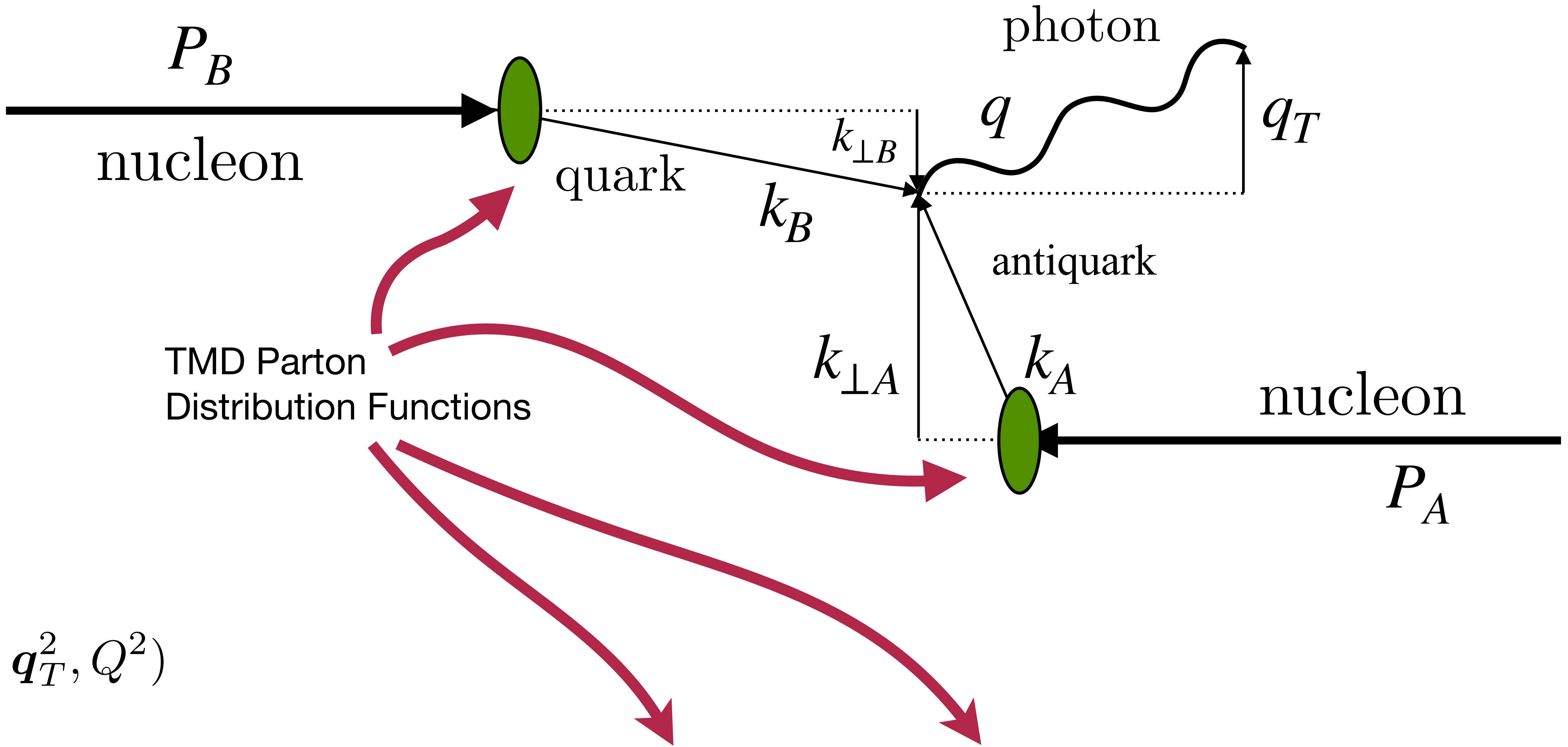
- Lots of progress from the theory side
- Some knowledge of g_T x-dependence
- First hints about e x-dependence
- All others unknown

UNPOLARIZED TMD

TMDS IN DRELL-YAN PROCESSES

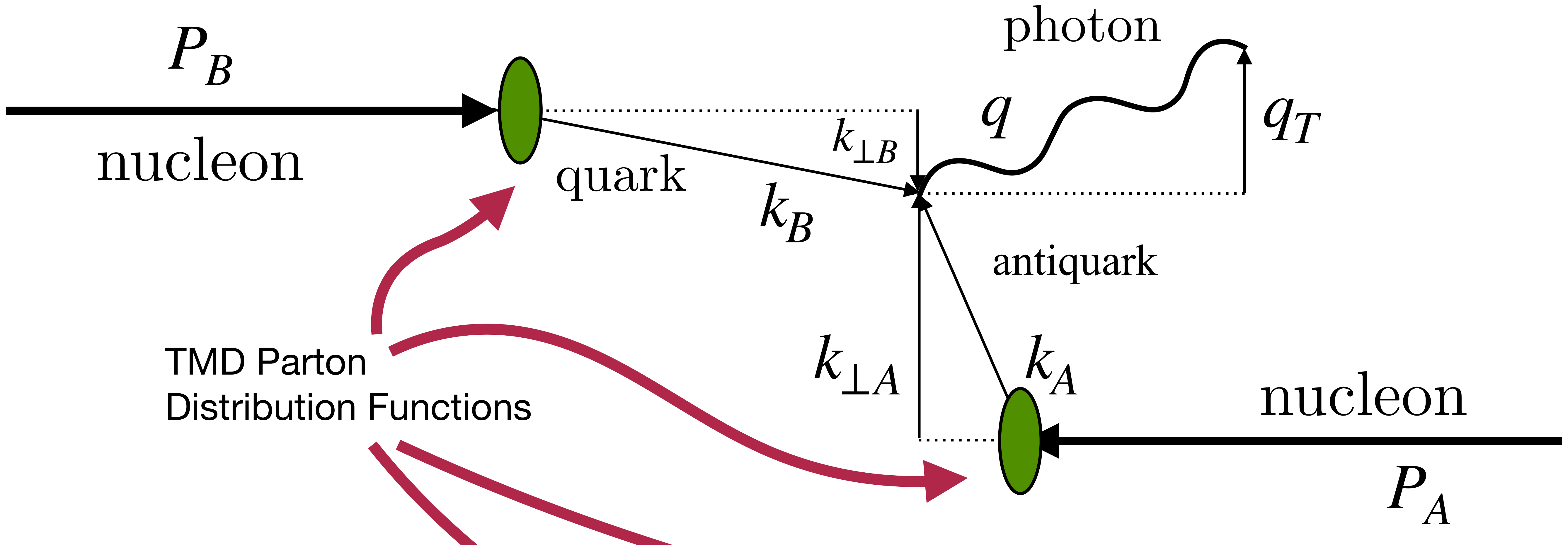


TMD Parton
Distribution Functions

$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}^{1q}_{UU}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q\big(x_A, \boldsymbol{k}^2_{\perp A}; \mu^2\big)\, f_1^{\bar{q}}\big(x_B, \boldsymbol{k}^2_{\perp B}; \mu^2\big)\, \delta^{(2)}\big(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\big)$$

TMDS IN DRELL-YAN PROCESSES



TMD Parton
Distribution Functions

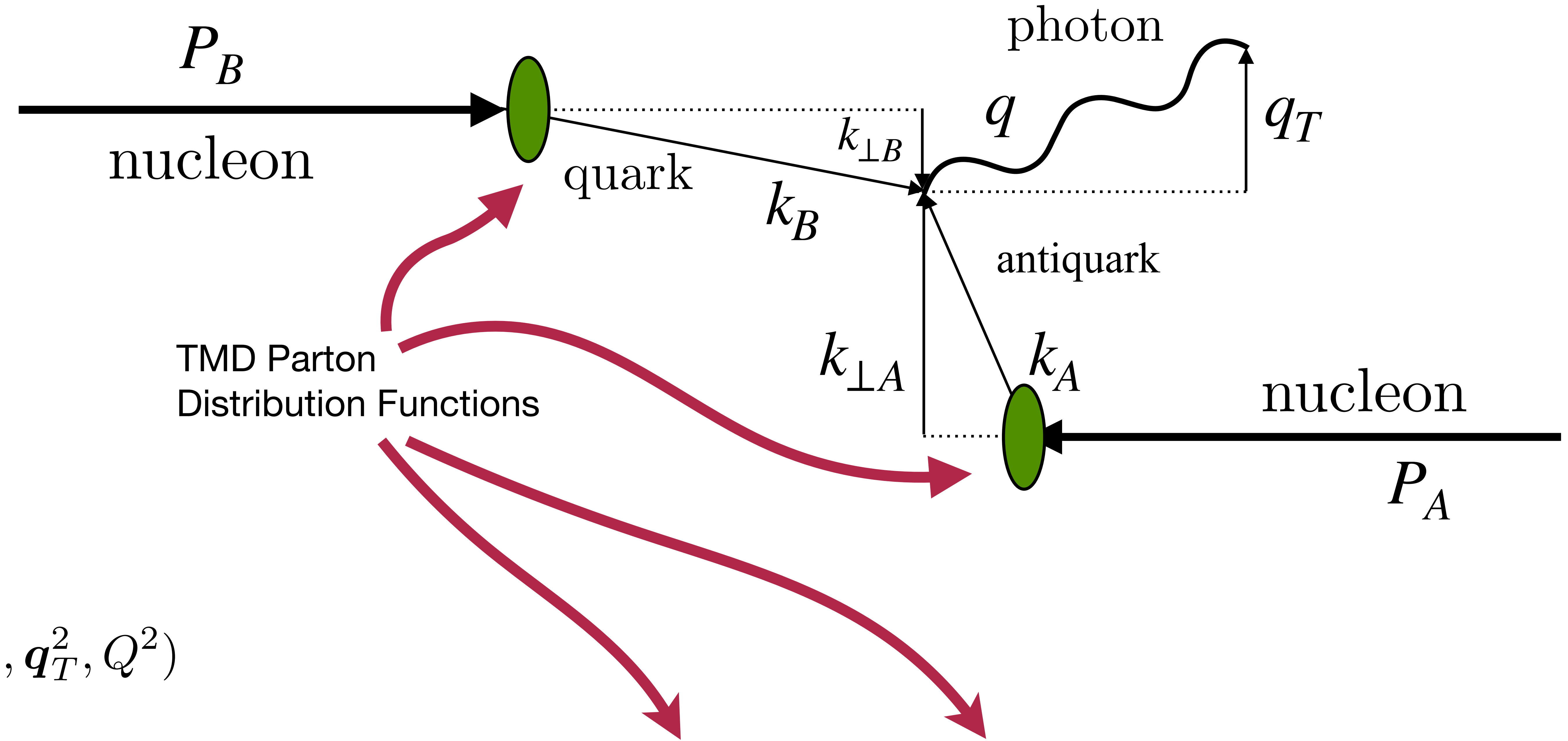
$$F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2)\, f_1^{\bar{q}}(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2)\, \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

The analysis is usually done in Fourier-transformed space

TMDS IN DRELL-YAN PROCESSES



TMD Parton
Distribution Functions

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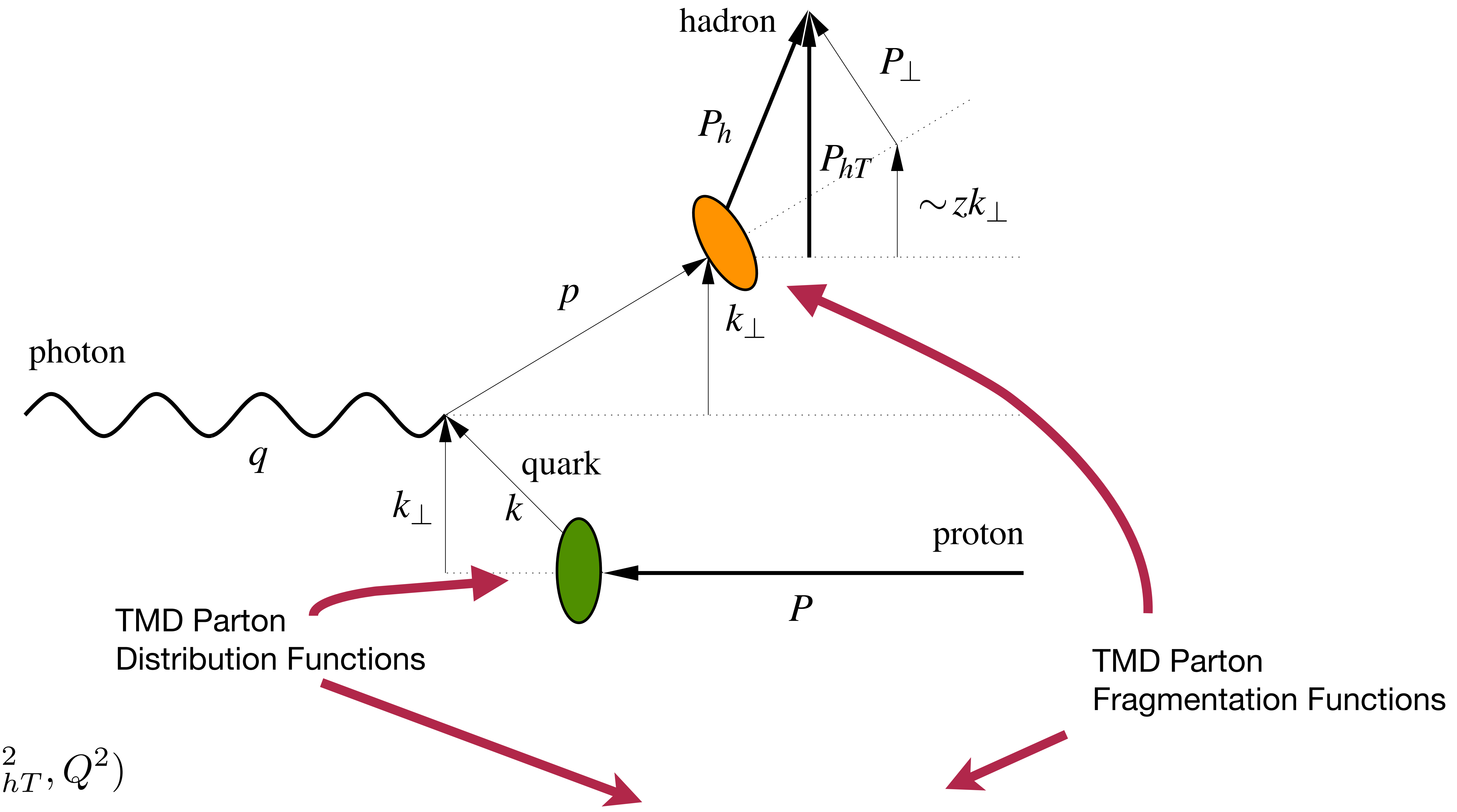
The analysis is usually done in Fourier-transformed space

TMDs formally depend on two scales, but we set them equal.

TMDS IN SEMI-INCLUSIVE DIS (SIDIS)



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2)\, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2)\, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

$$= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|)\hat{f}_1^q(x, z^2 b_\perp^2; \mu^2)\, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

TMD STRUCTURE

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\text{resum}} + g_K}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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TMD STRUCTURE

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

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$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

perturbative Sudakov
form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\mathrm{resum}} + g_K}$$

collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

matching coefficients
(perturbative)

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

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$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

perturbative Sudakov
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$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C \otimes f_1](x,\mu_{b_*})\; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu}\left(\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\mathrm{resum}}+g_K}$$

collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

$$\Downarrow$$

$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

matching coefficients
(perturbative)

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C \otimes f_1](x,\mu_{b_*})\; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu}\left(\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\text{resum}}+g_K} f_{1\,NP}(x,b_T^2;\zeta_f,Q_0)$$

collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

⬇

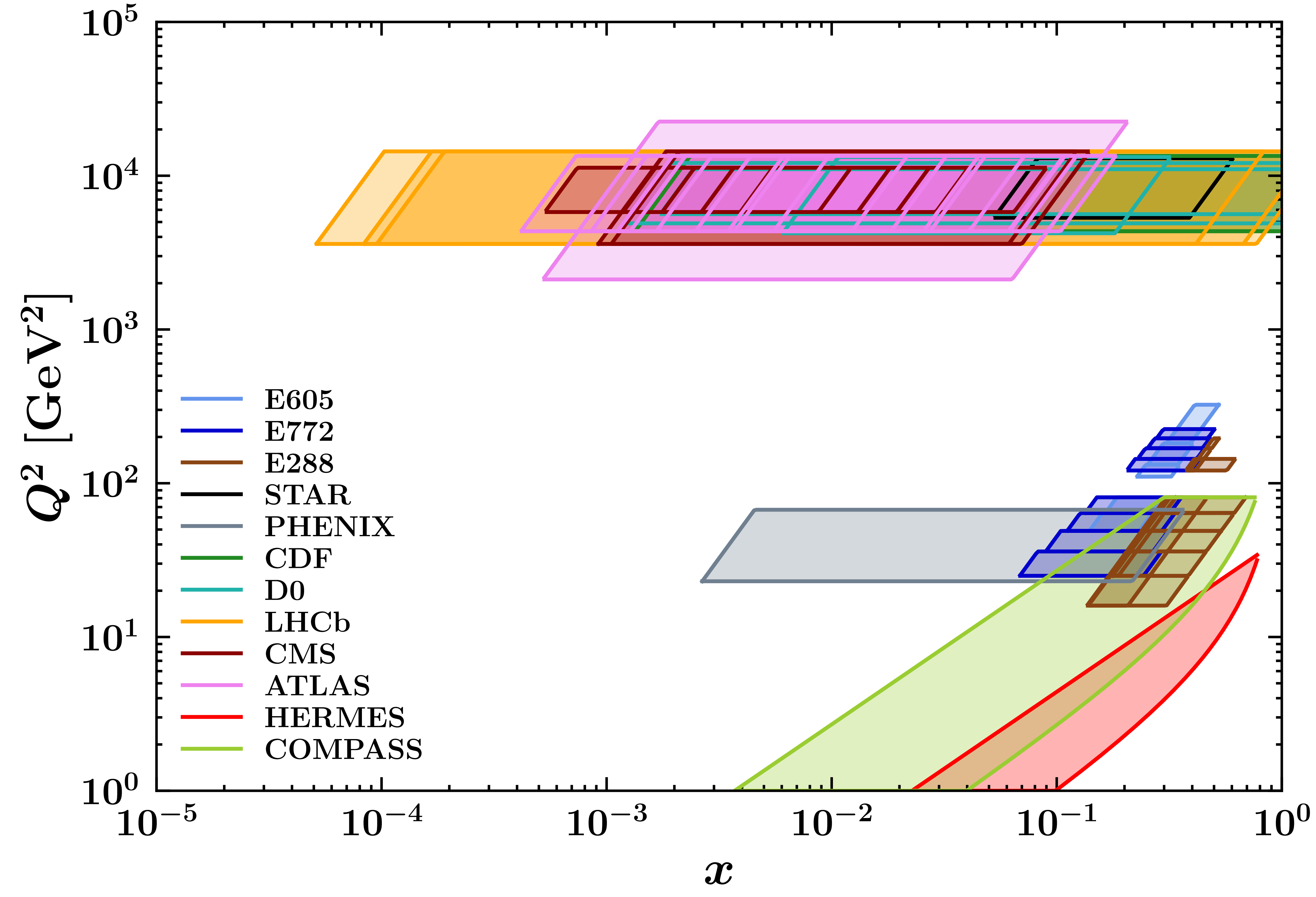
$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

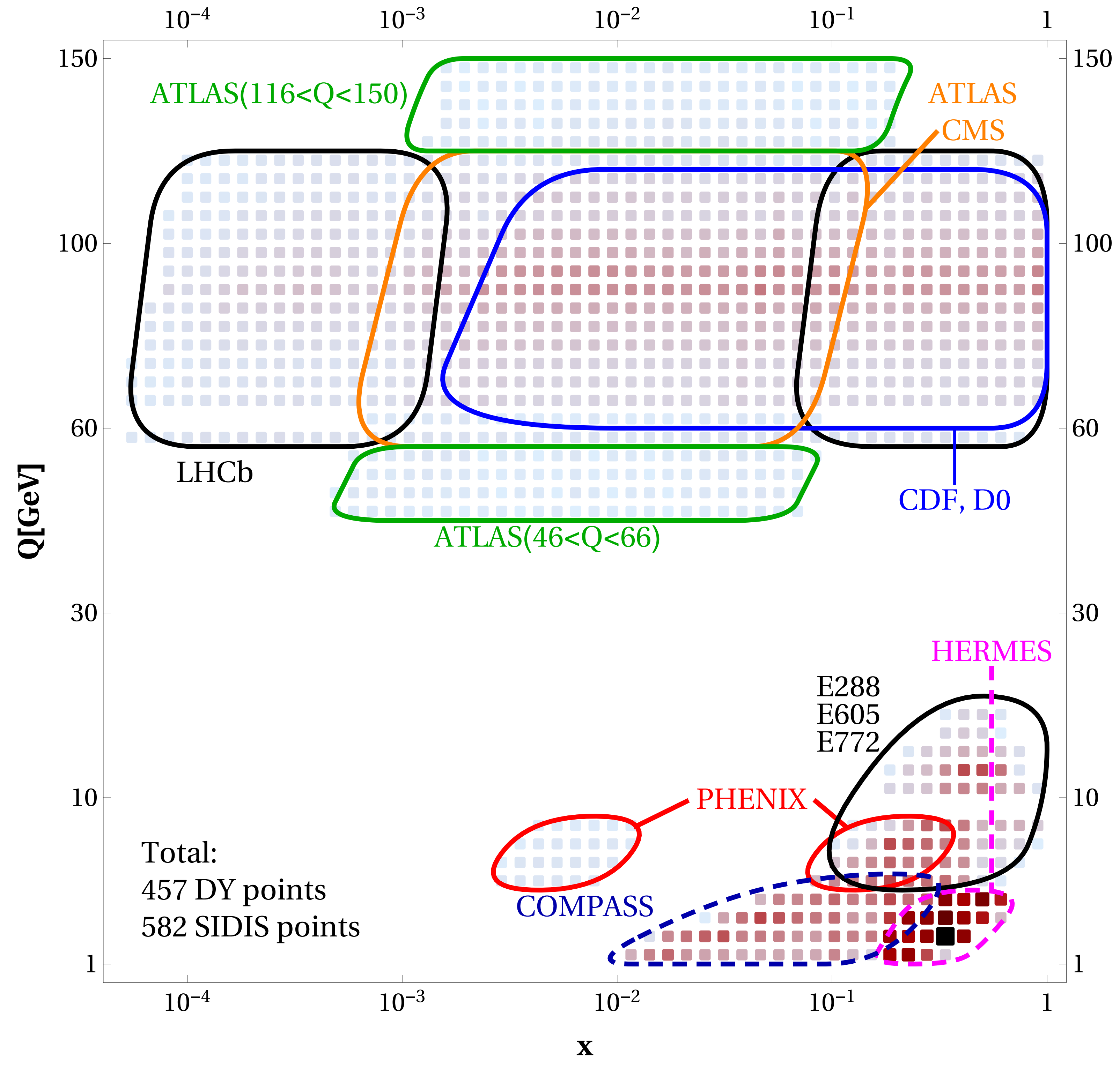
TMD GLOBAL FITS

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✓	✓	✓	✓	2031	1.06

x–Q² COVERAGE

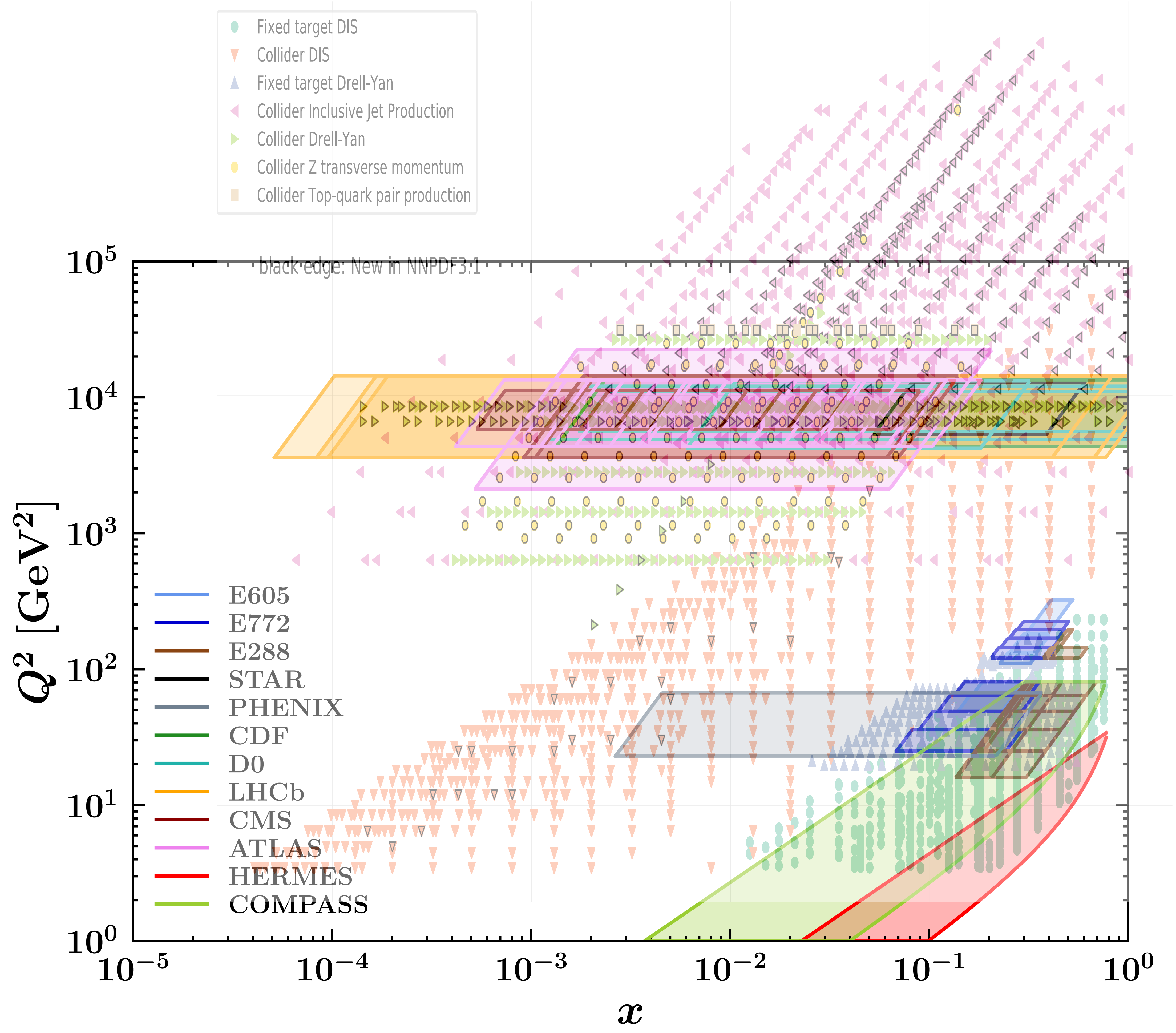


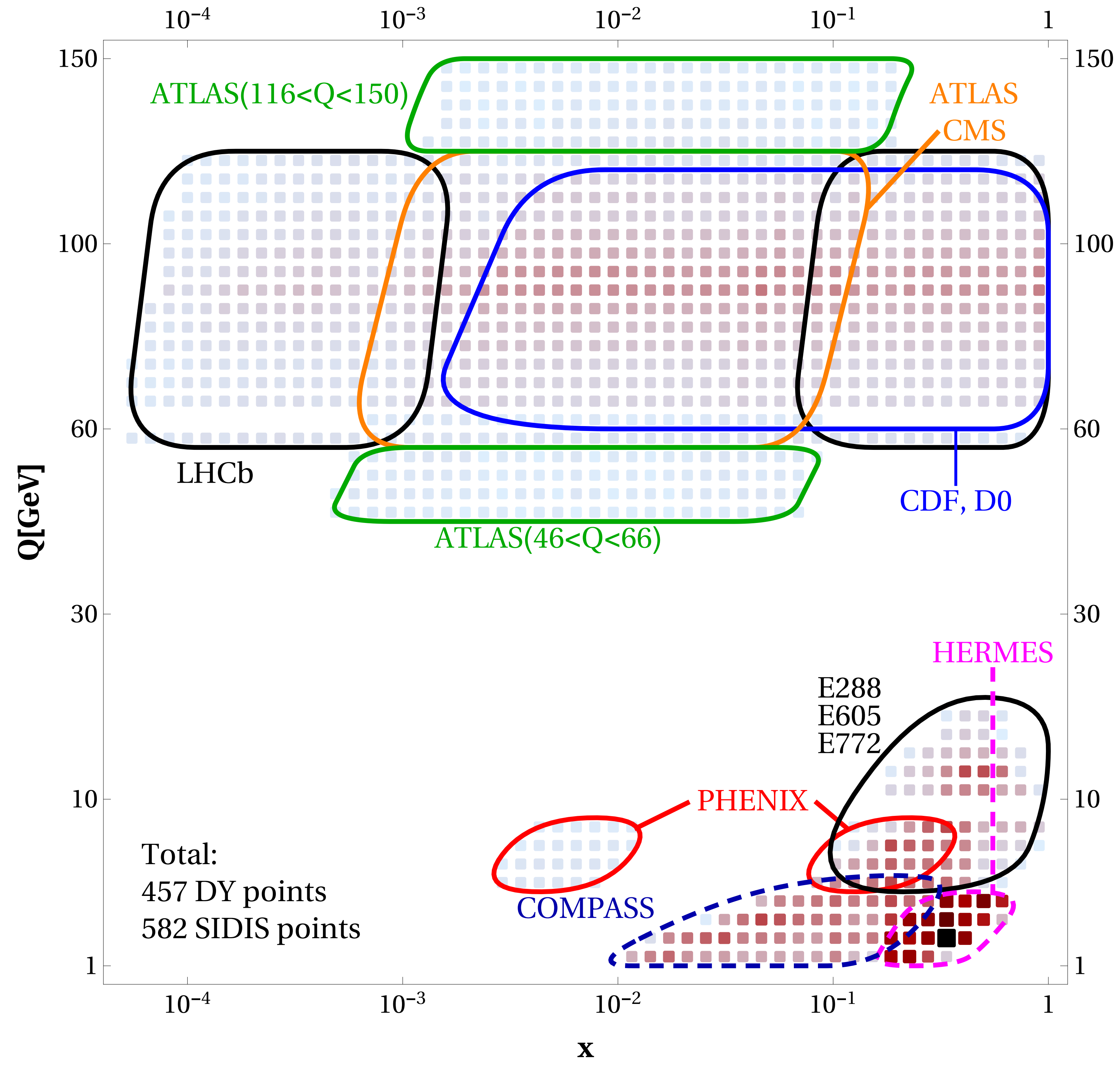
MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, arXiv:2206.07598



Scimemi, Vladimirov, arXiv:1912.06532

x–Q^2 COVERAGE





MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, arXiv:2206.07598

Scimemi, Vladimirov, arXiv:1912.06532

Data set	N_{dat}	χ^2_D/N_{dat}	$\chi^2_\lambda/N_{\text{dat}}$	χ^2_0/N_{dat}
Tevatron total	71	0.87	0.06	0.93
LHCb total	21	1.15	0.3	1.45
ATLAS total	72	4.56	0.48	5.05
CMS total	78	0.53	0.02	0.55
PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
DY collider total	251	1.86	0.2	2.06
DY fixed-target total	233	0.85	0.4	1.24
HERMES total	344	0.48	0.23	0.71
COMPASS total	1203	0.62	0.3	0.92
SIDIS total	1547	0.59	0.28	0.87
Total	**2031**	**0.77**	**0.29**	**1.06**

MAP Collaboration, arXiv:2206.07598

EXAMPLE OF RESULTING TMDS

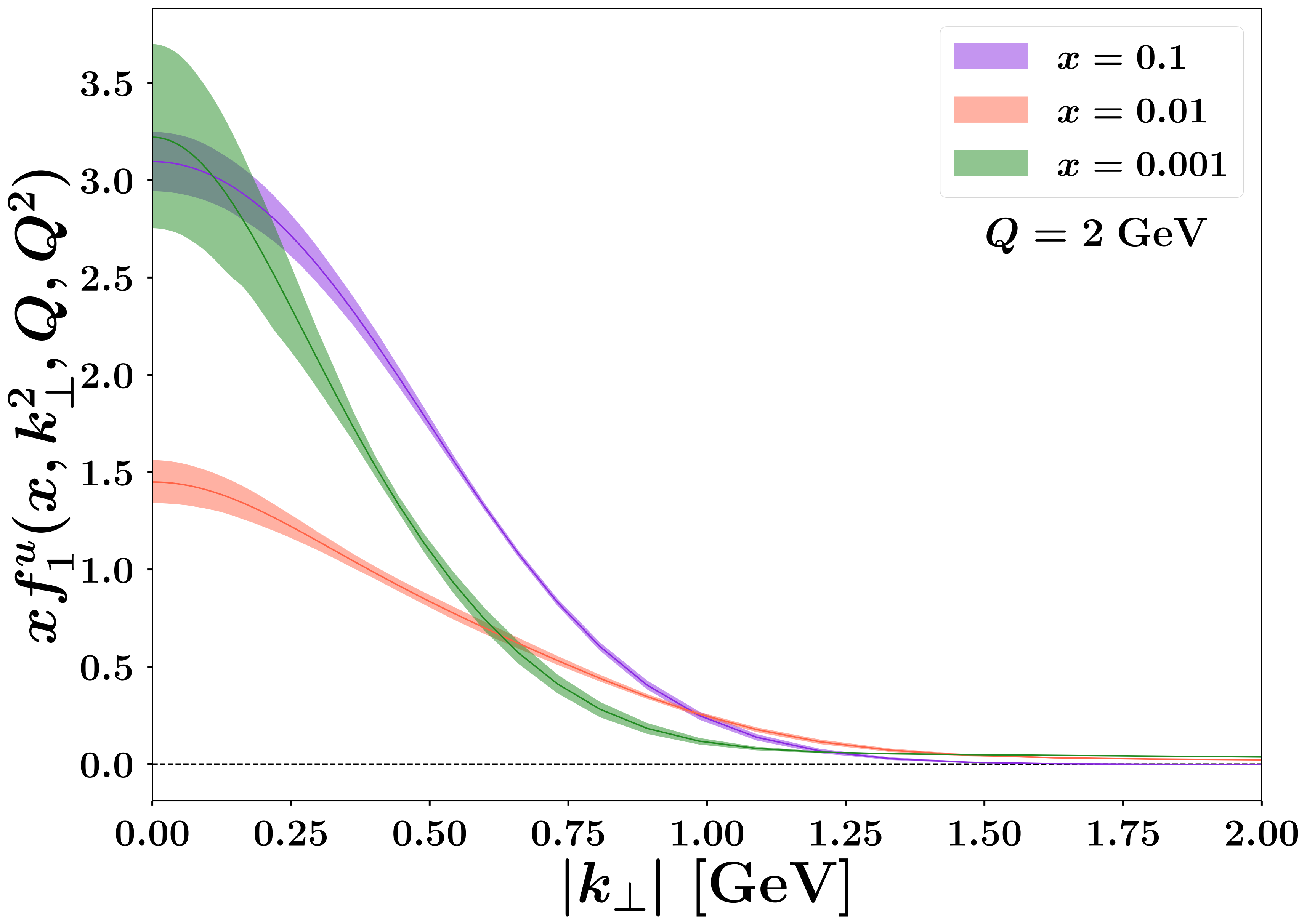
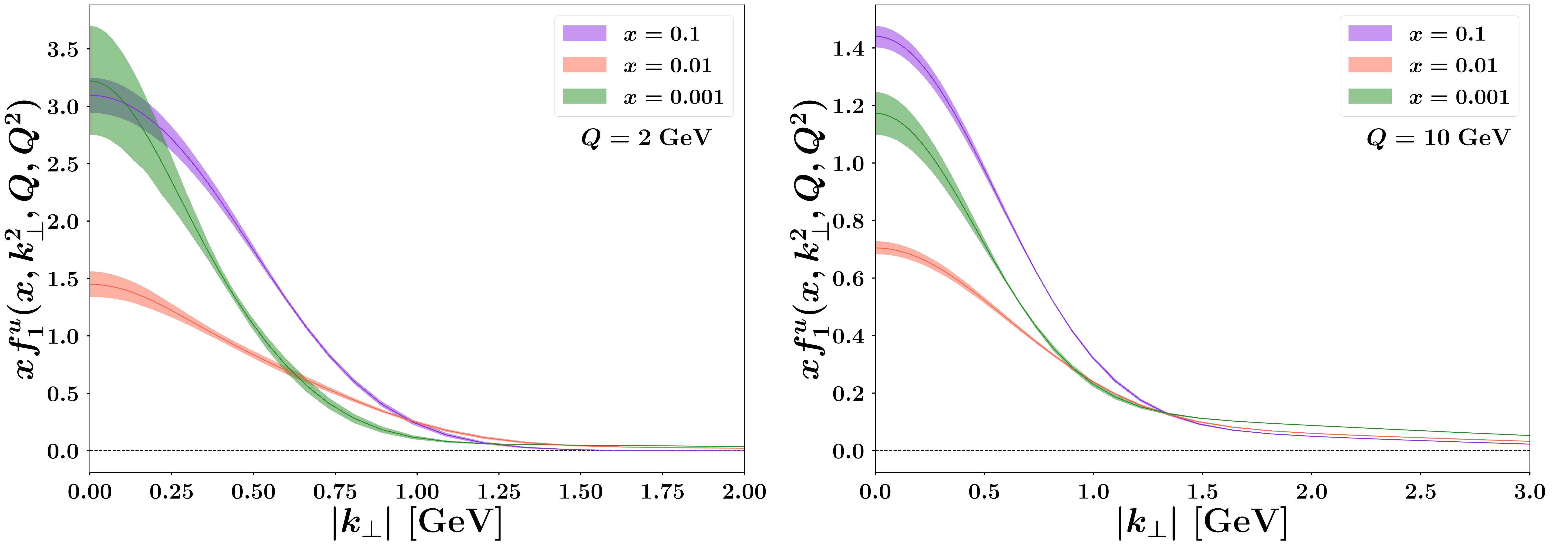


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for $x = 0.001$, 0.01 and 0.1. The uncertainty bands represent the 68% CL.

EXAMPLE OF RESULTING TMDS



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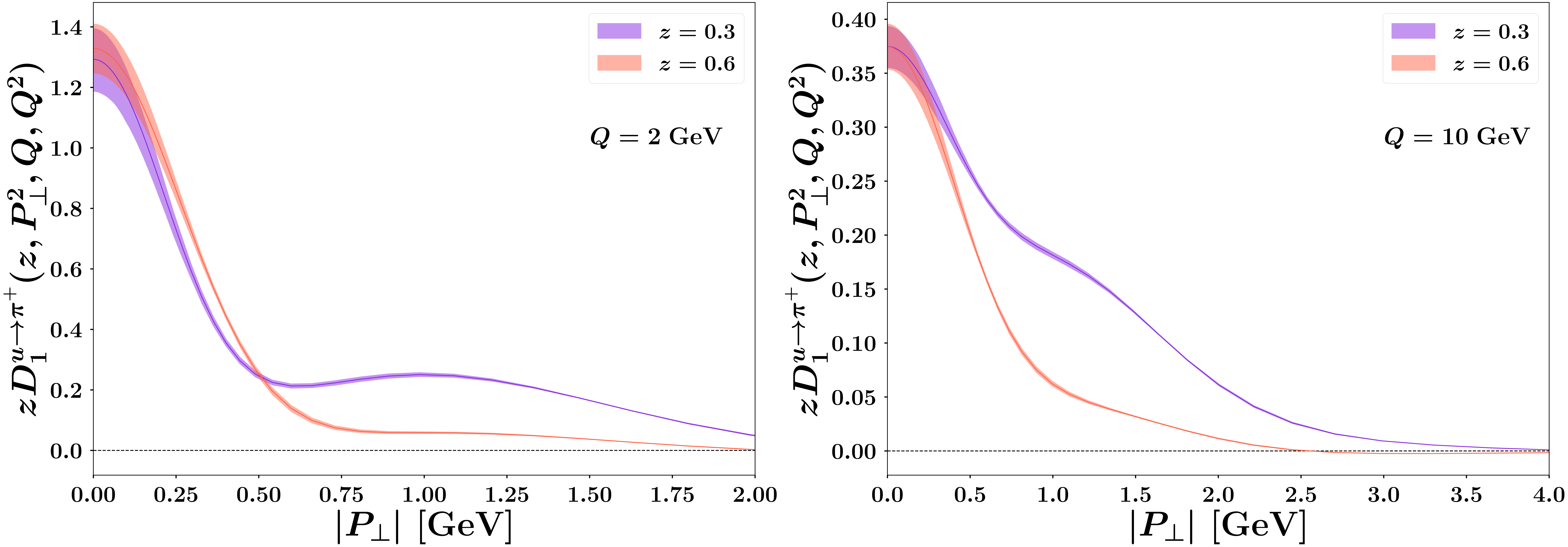
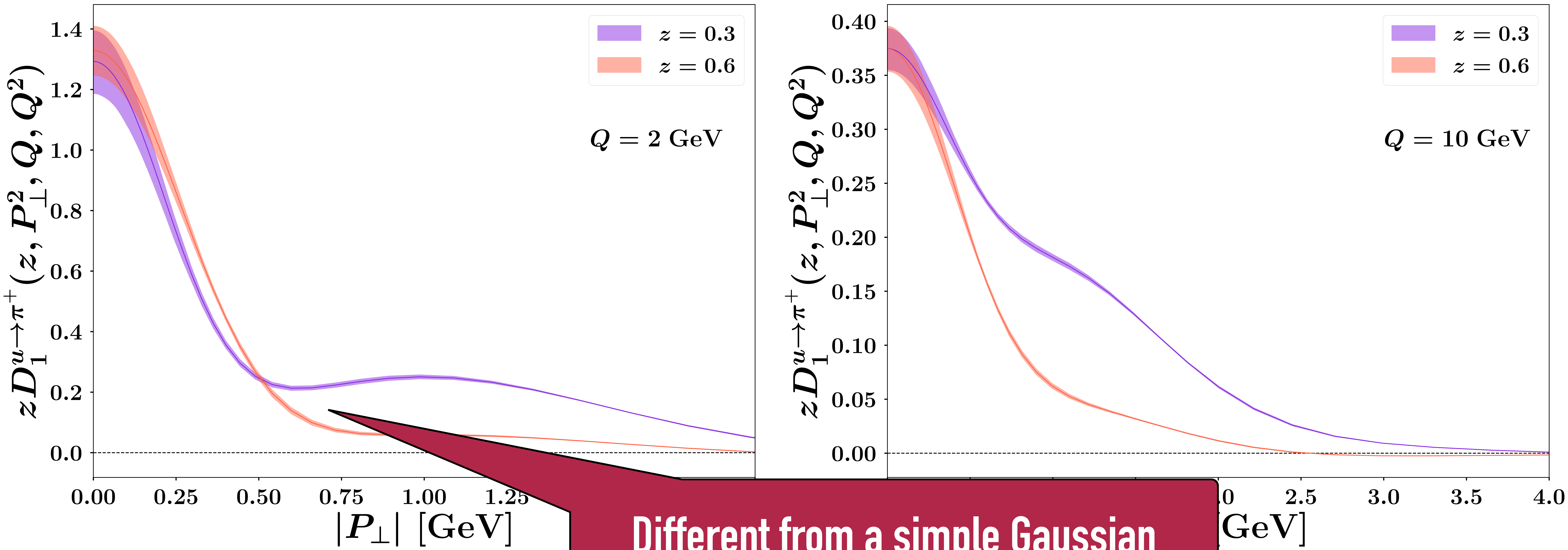
RESULTING TMD FRAGMENTATION FUNCTIONS

MAP Collaboration, arXiv:2206.07598



RESULTING TMD FRAGMENTATION FUNCTIONS

MAP Collaboration, arXiv:2206.07598

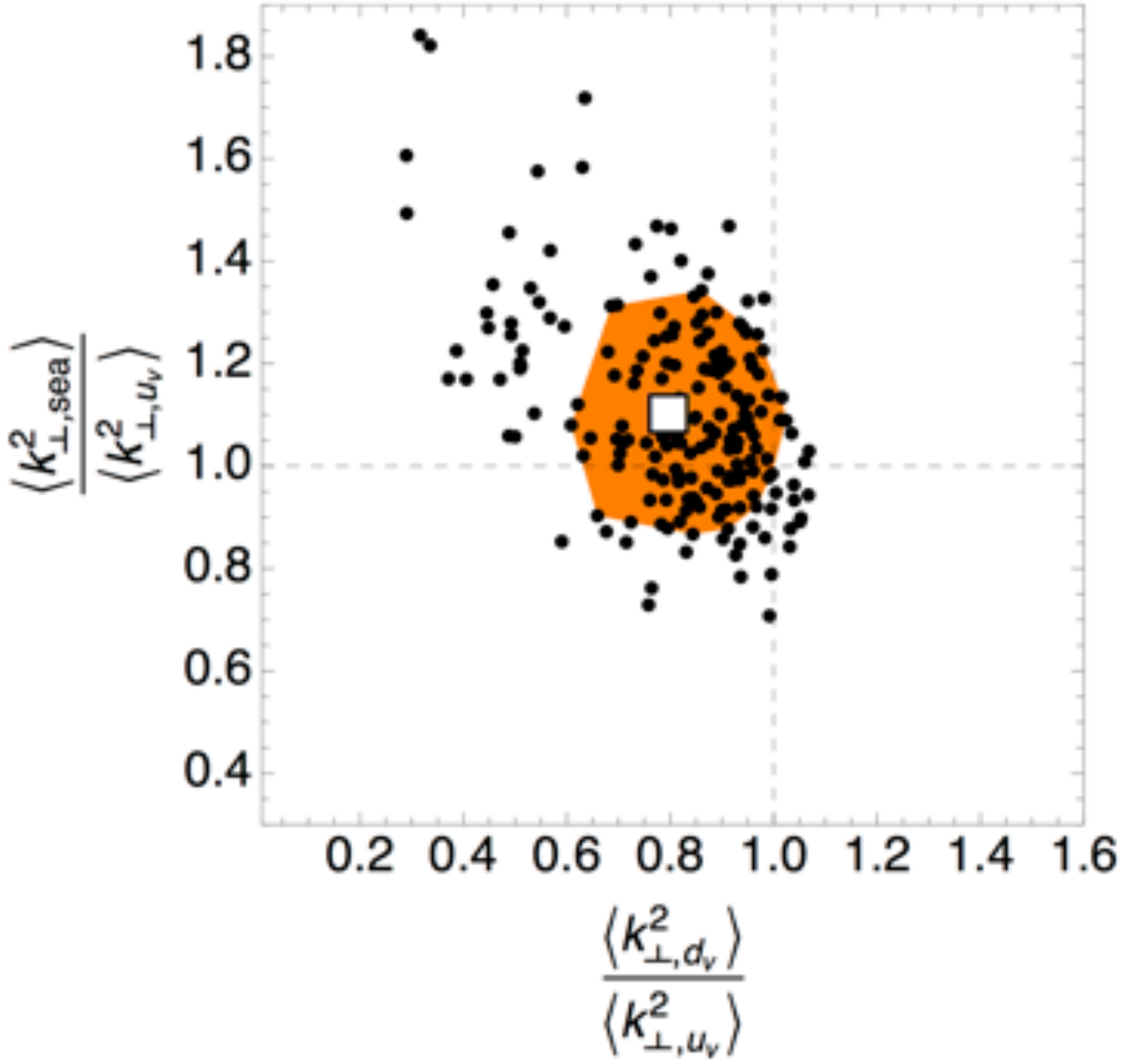


Different from a simple Gaussian

FLAVOR DEPENDENCE OF TMDS

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)

Ratio of width of sea / width of up valence

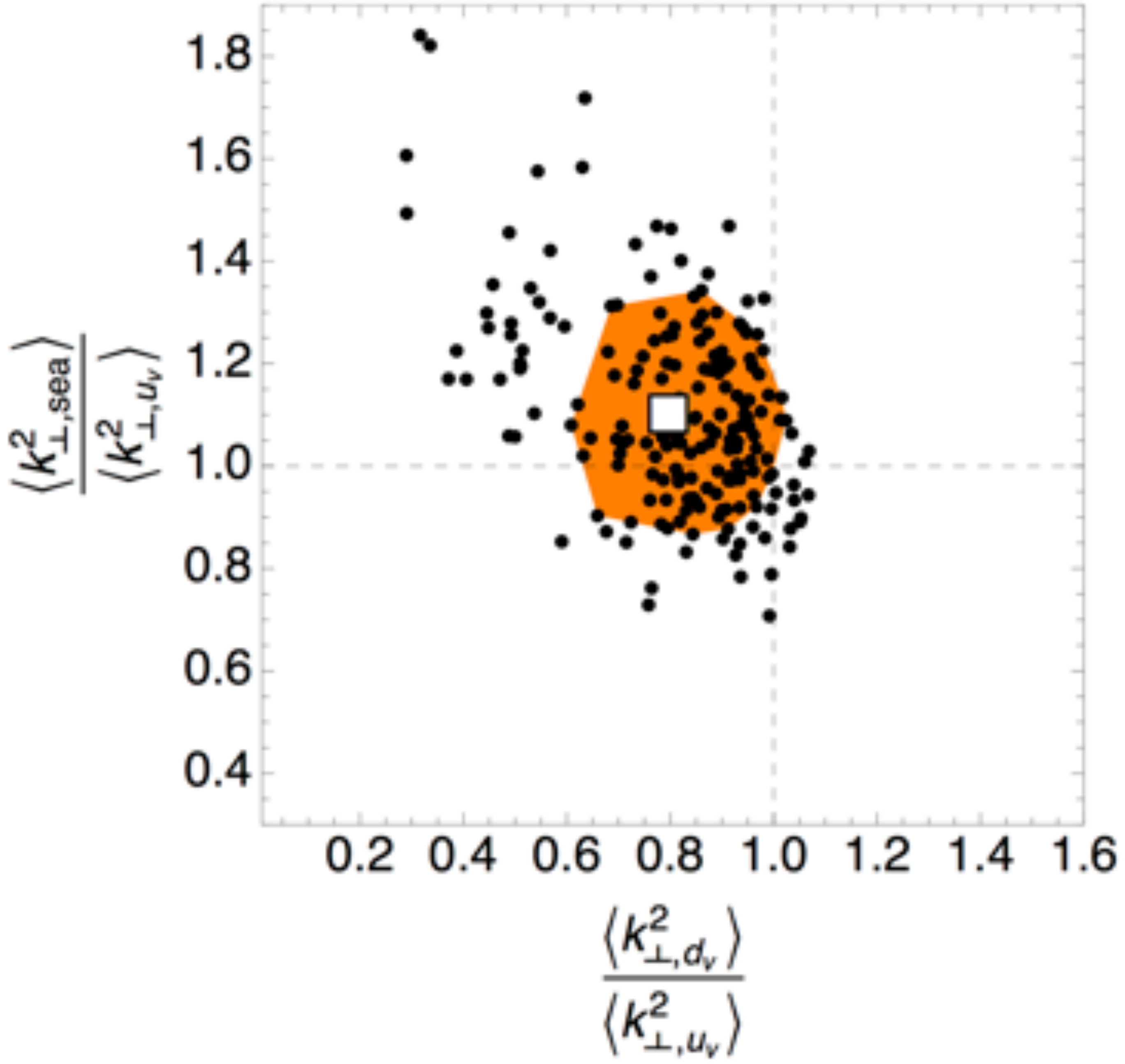


Ratio width of down valence/
width of up valence

FLAVOR DEPENDENCE OF TMDS

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)

Ratio of width of sea / width of up valence



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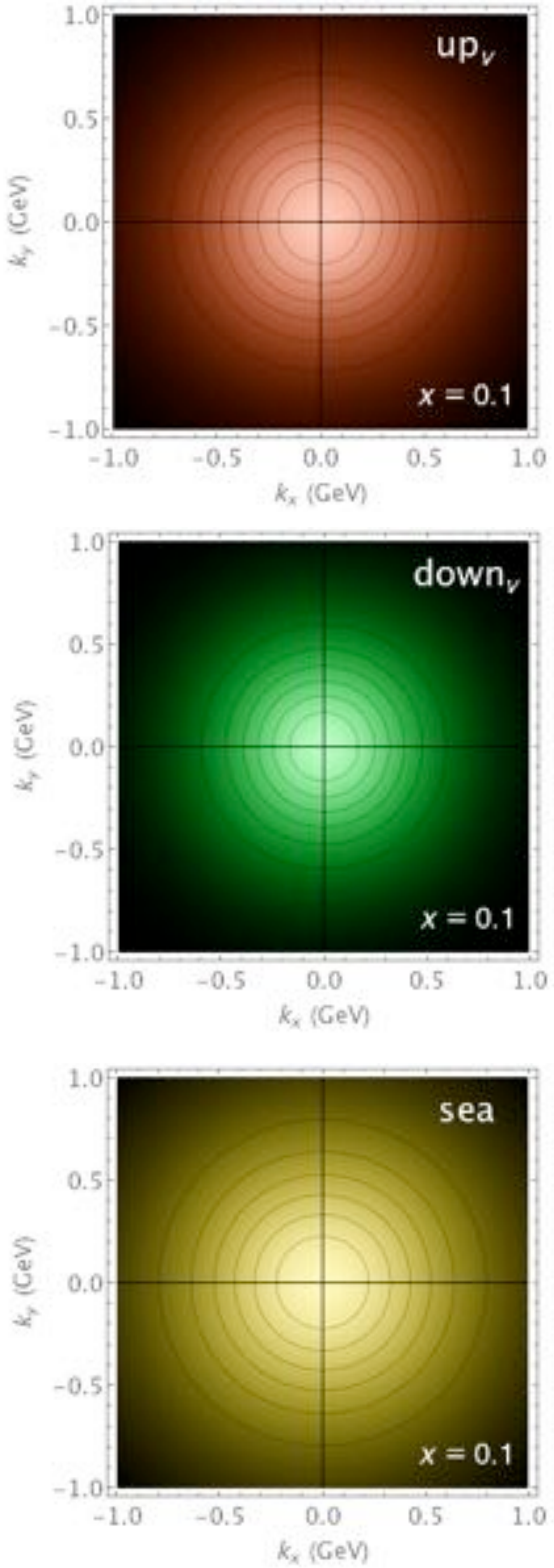

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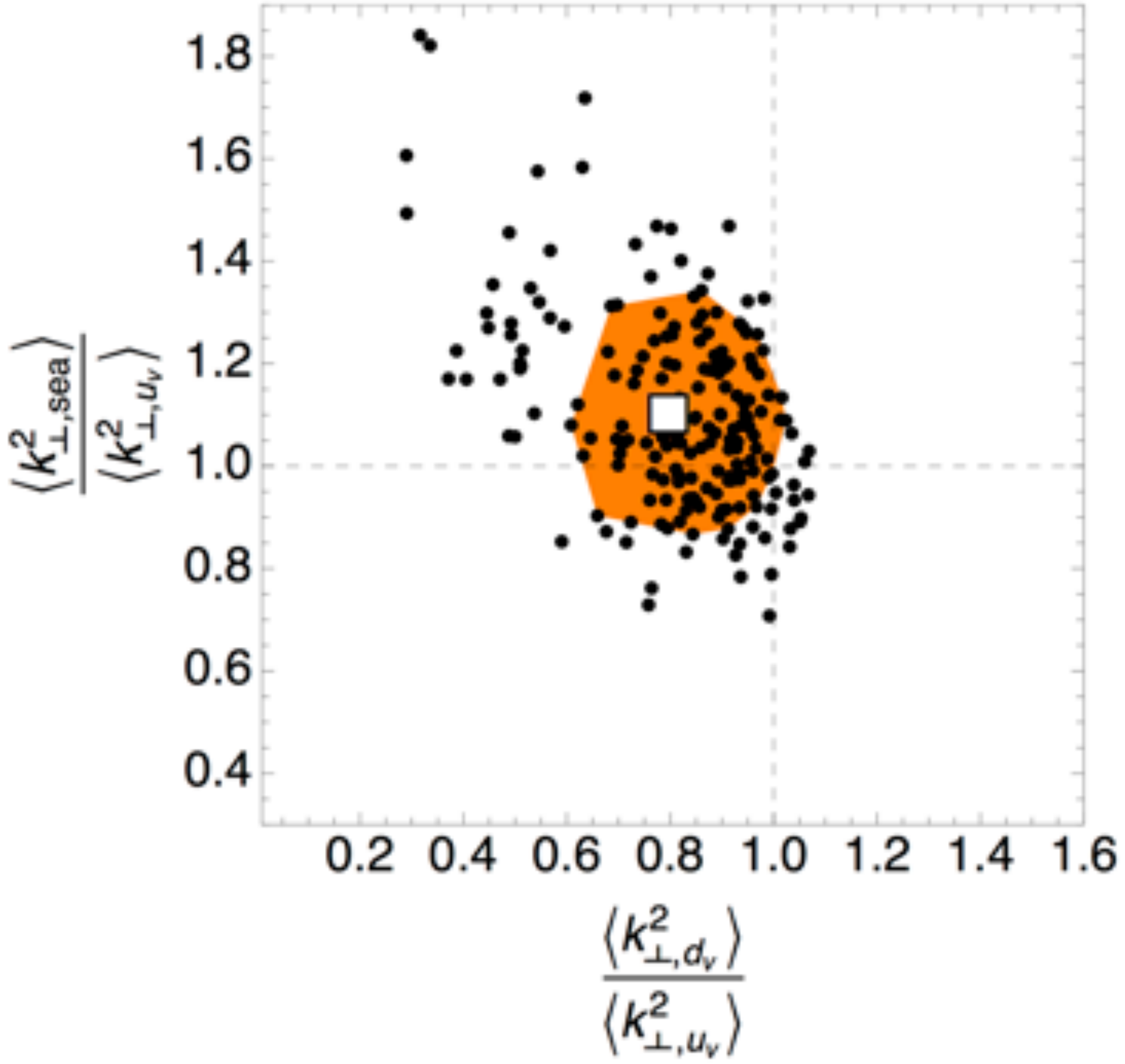

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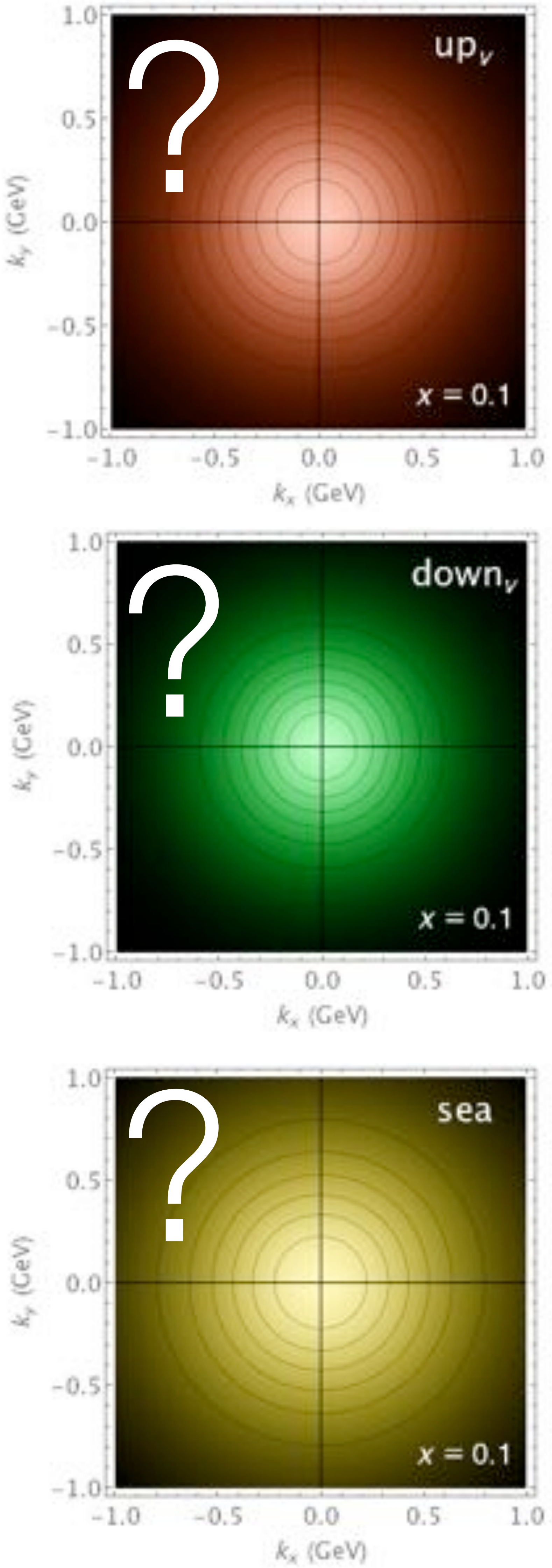

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Ratio of width of sea /
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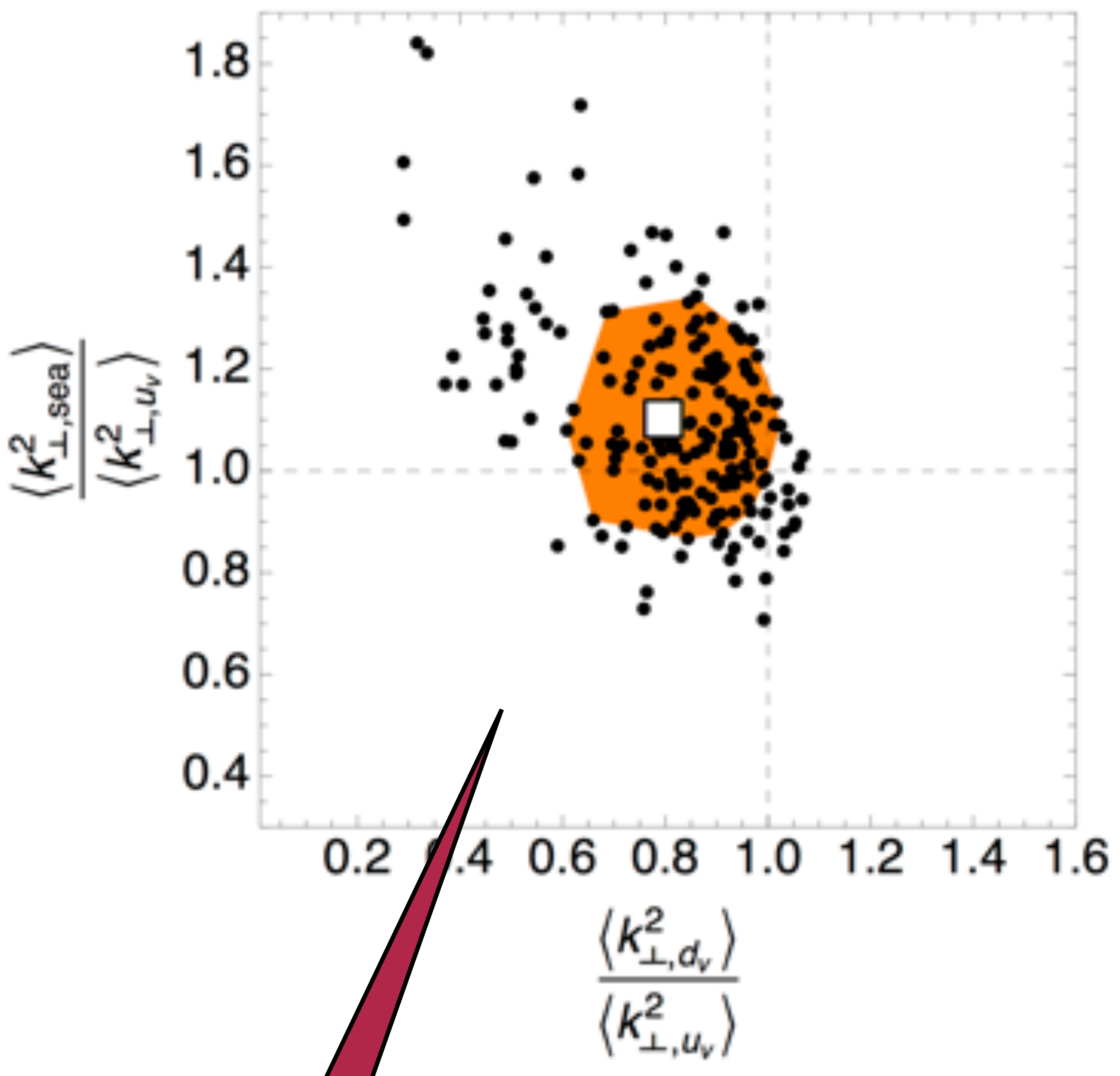

Ratio width of down valence/
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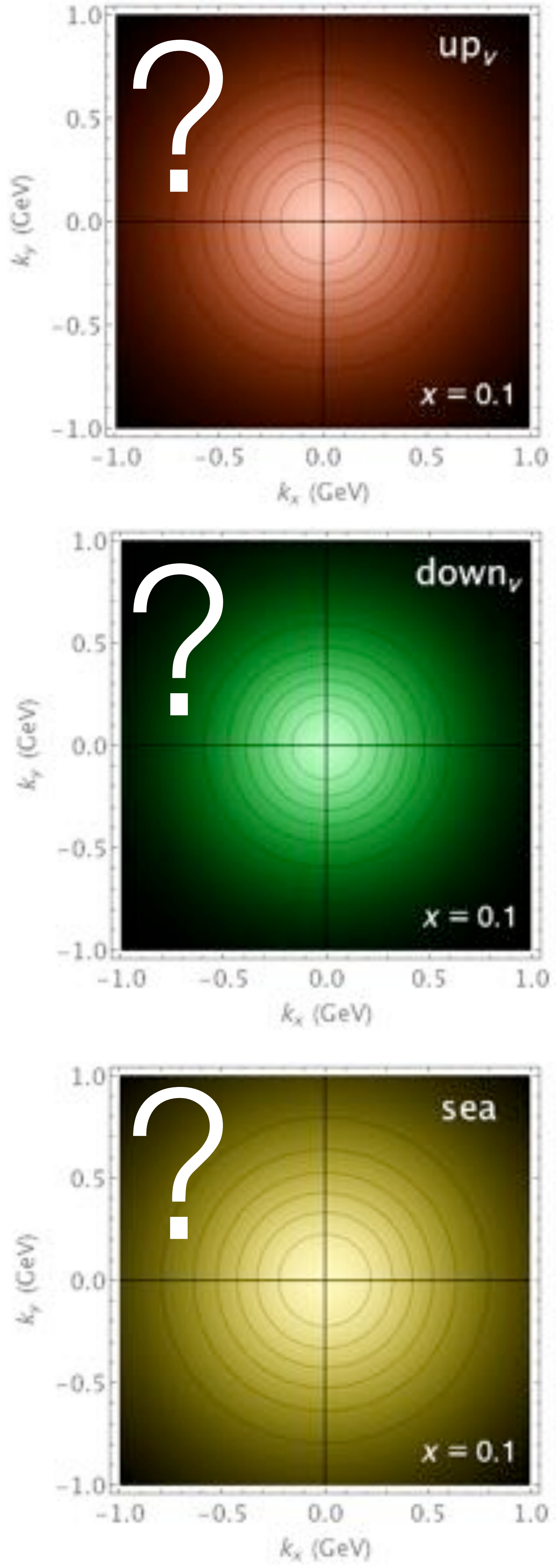

First investigation 10 years ago:
there is room for flavor dependence,
but we don't control it well

MOST RECENT EXTRACTION

ART23

Moos, Scimemi, Vladimirov, Zurita, 2305.07473

N^4LL^- accuracy

Drell-Yan only

MOST RECENT EXTRACTION

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Moos, Scimemi, Vladimirov, Zurita, 2305.07473

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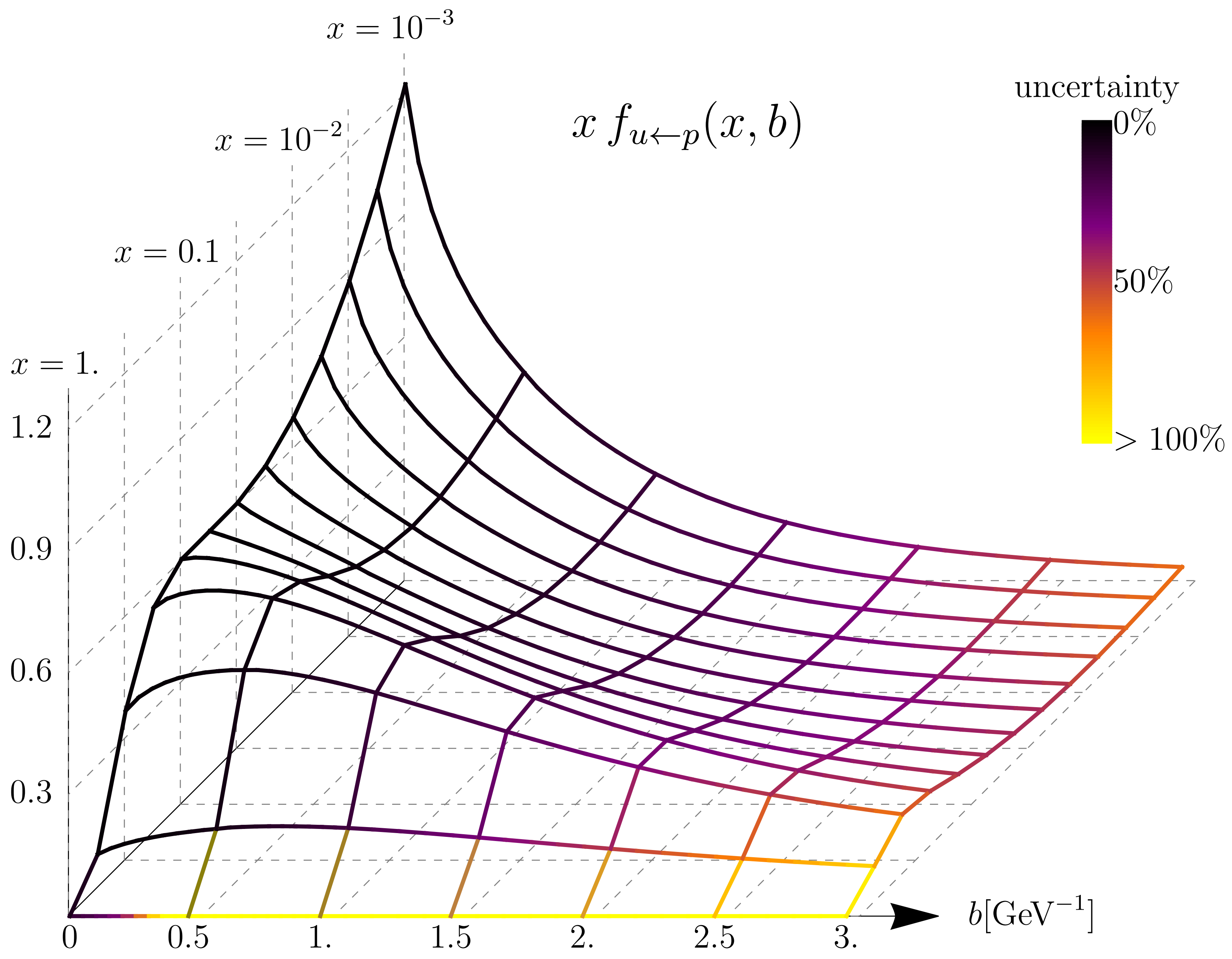
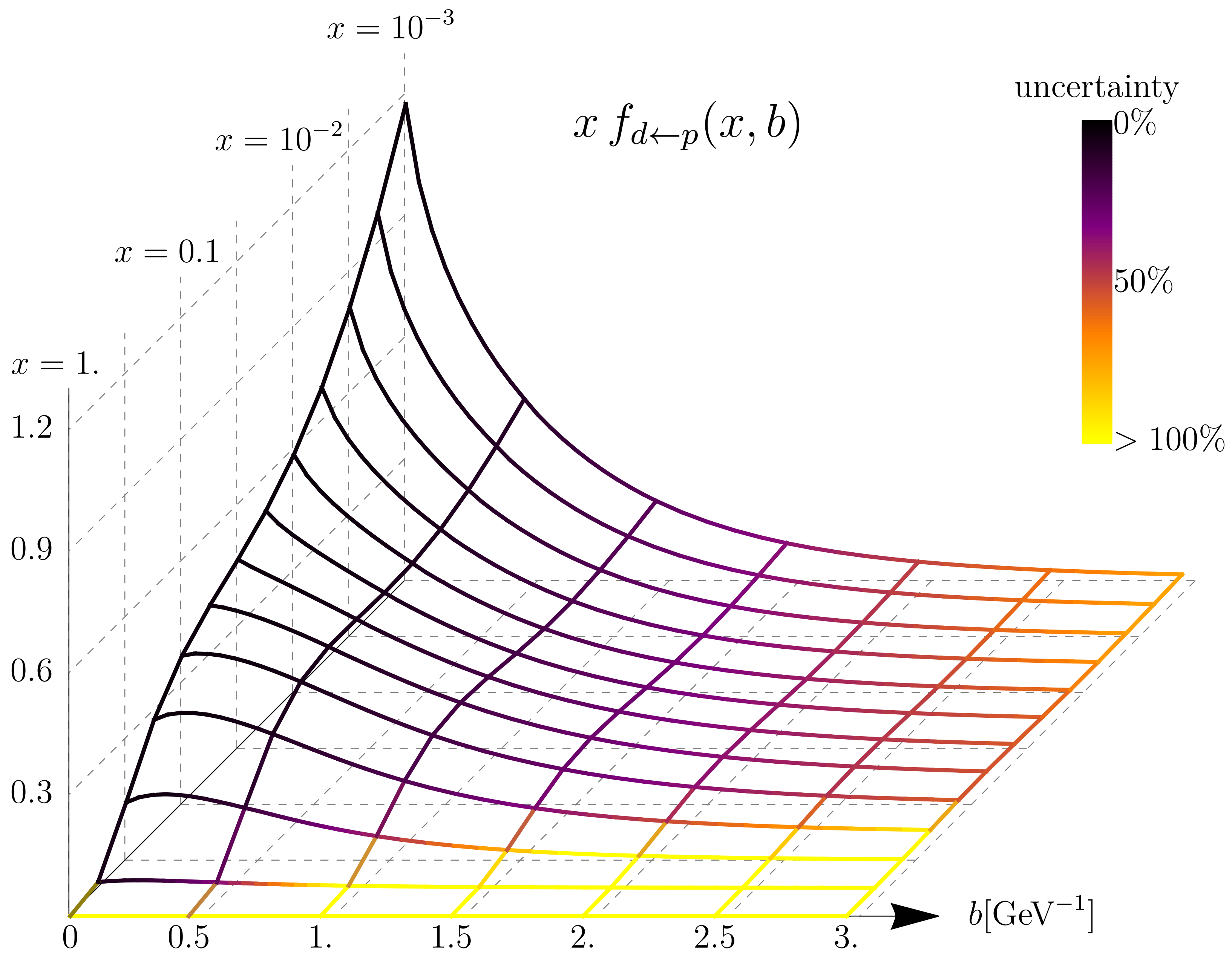




MOST RECENT EXTRACTION

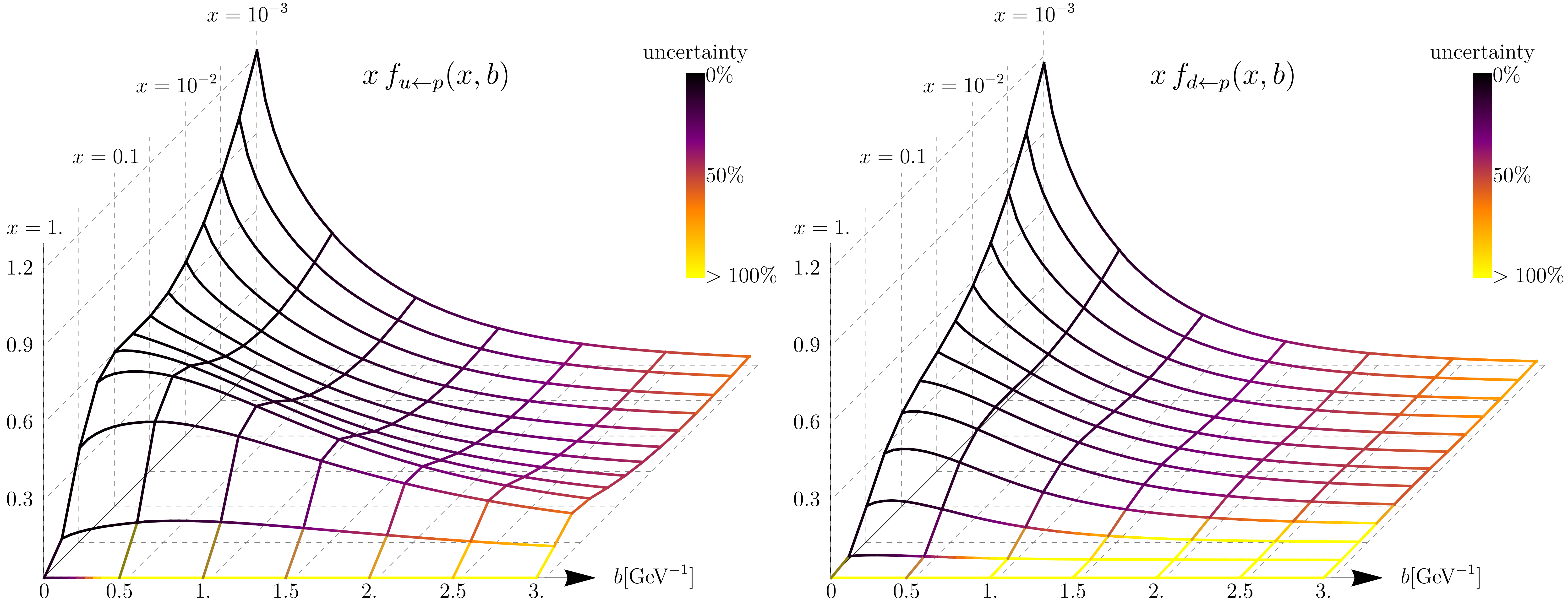
ART23

Moos, Scimemi, Vladimirov, Zurita, 2305.07473

N^{4}LL$^-$ accuracy

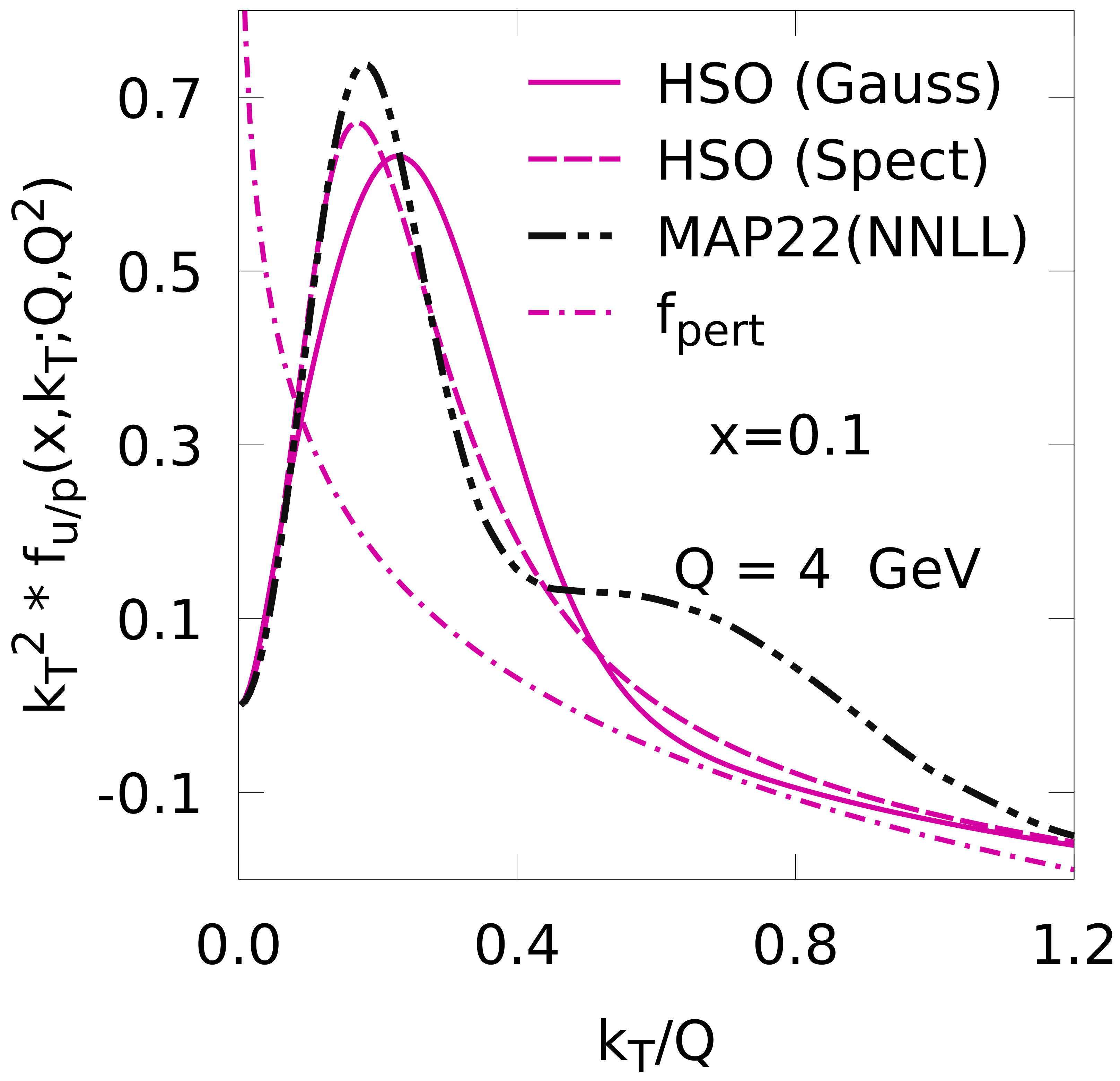
Drell-Yan only

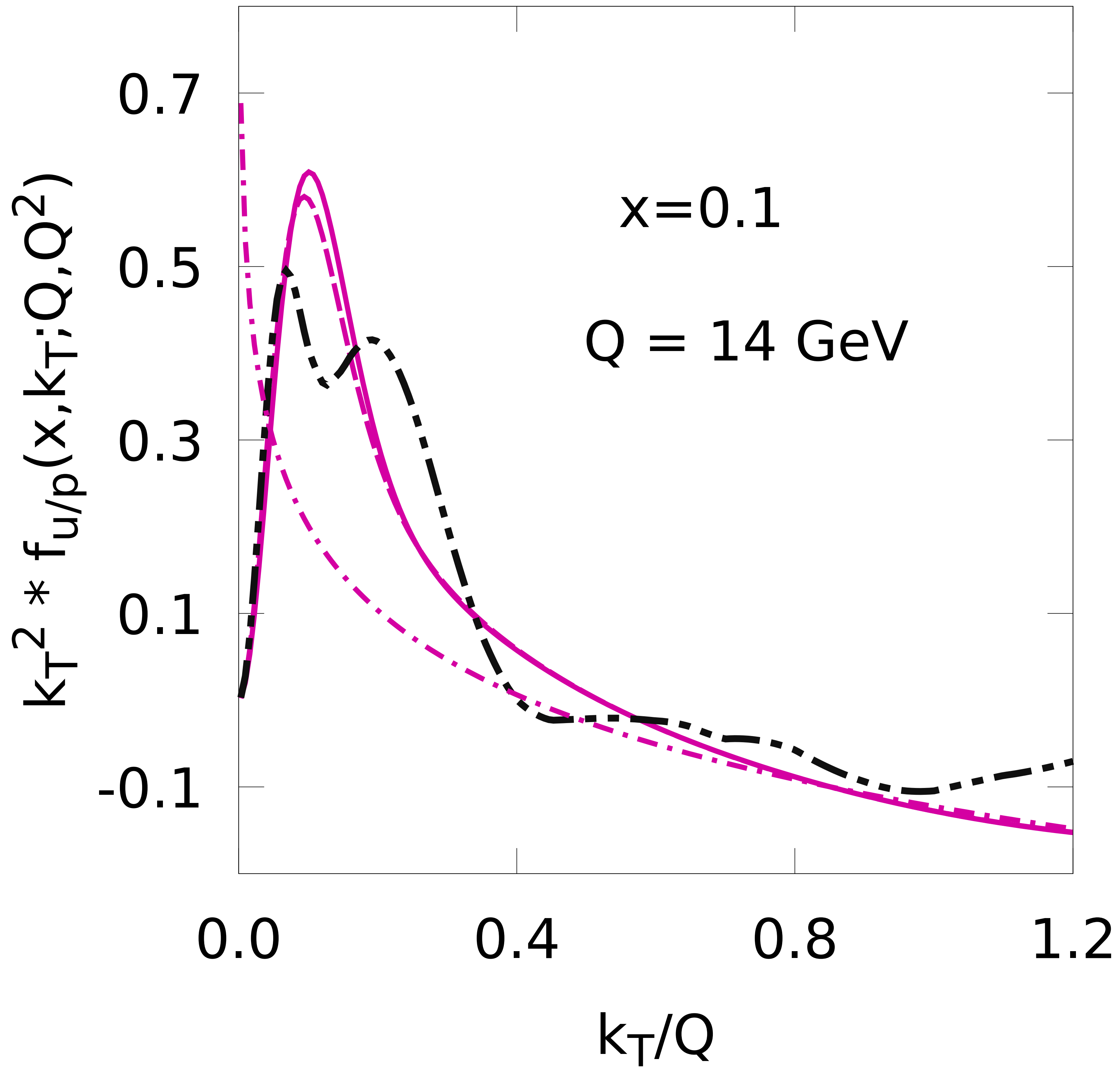
Different up and down TMDs



RECENT DISCUSSION IN “HSO” APPROACH

Aslan, Boglione, Gonzalez-Hernandez, Rainaldi, Rogers, Simonelli, 2401.14266





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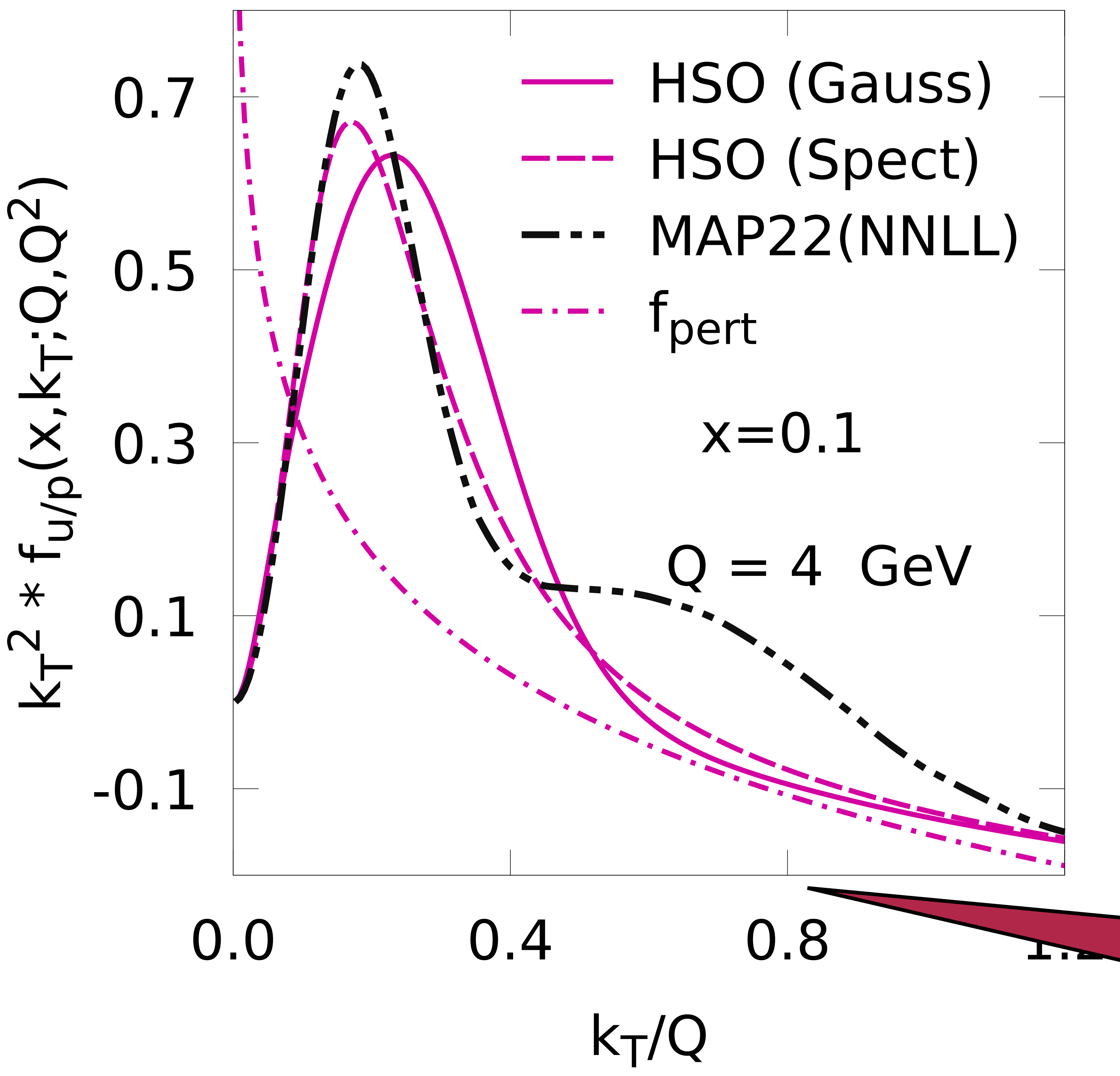

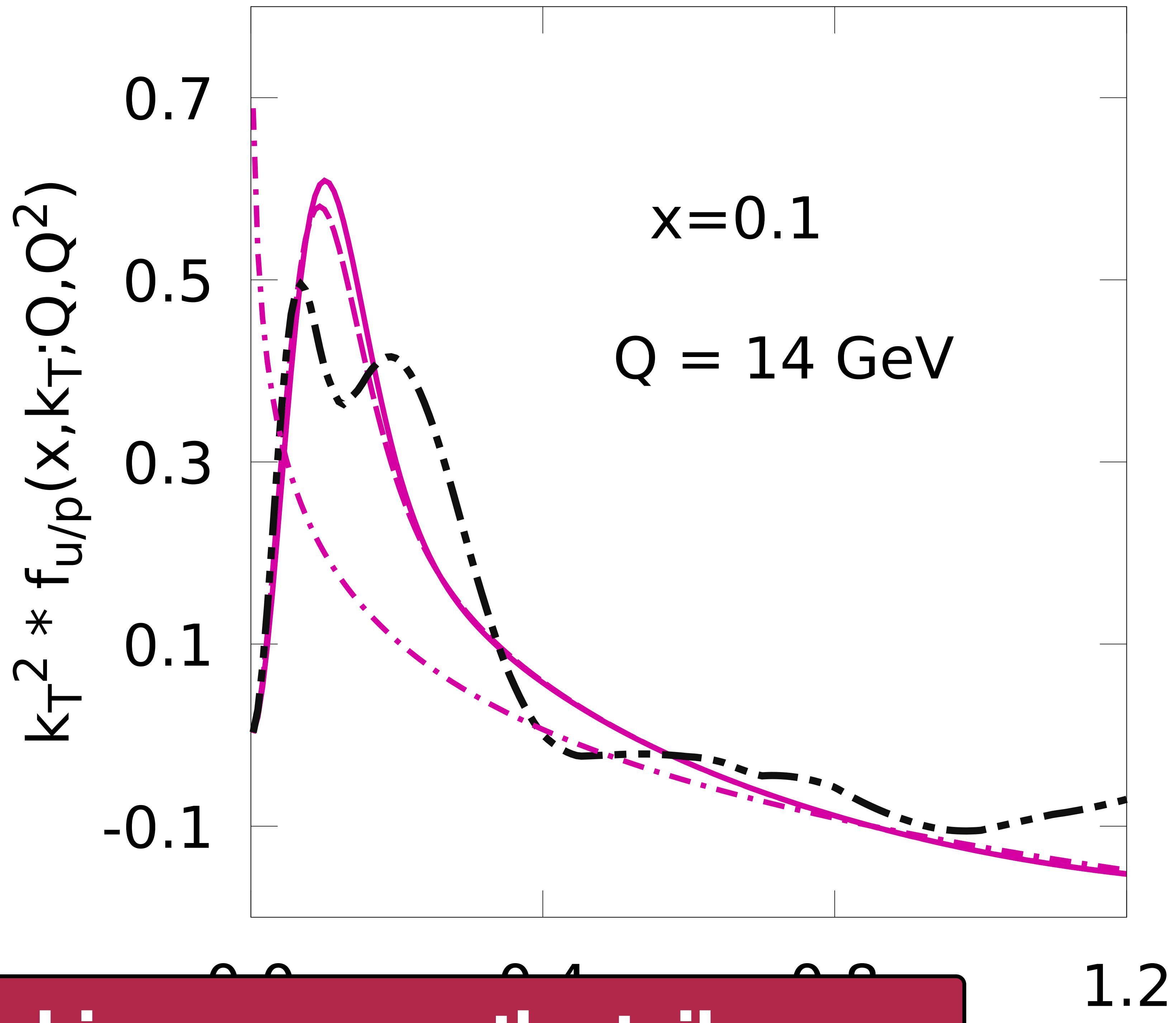

The k_T^2 weighing exposes the tails (multiplied by a factor of 10 in this case)

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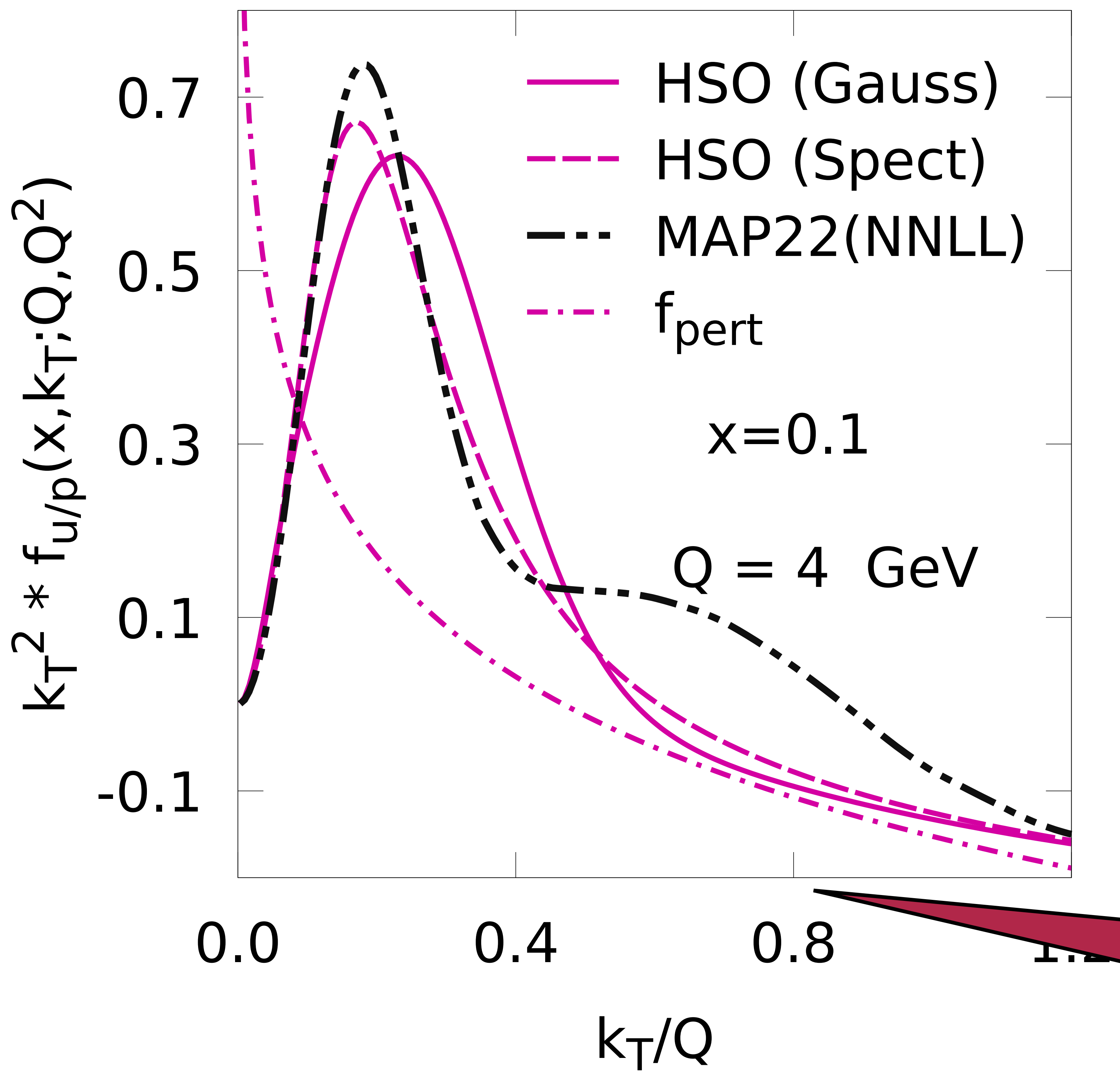
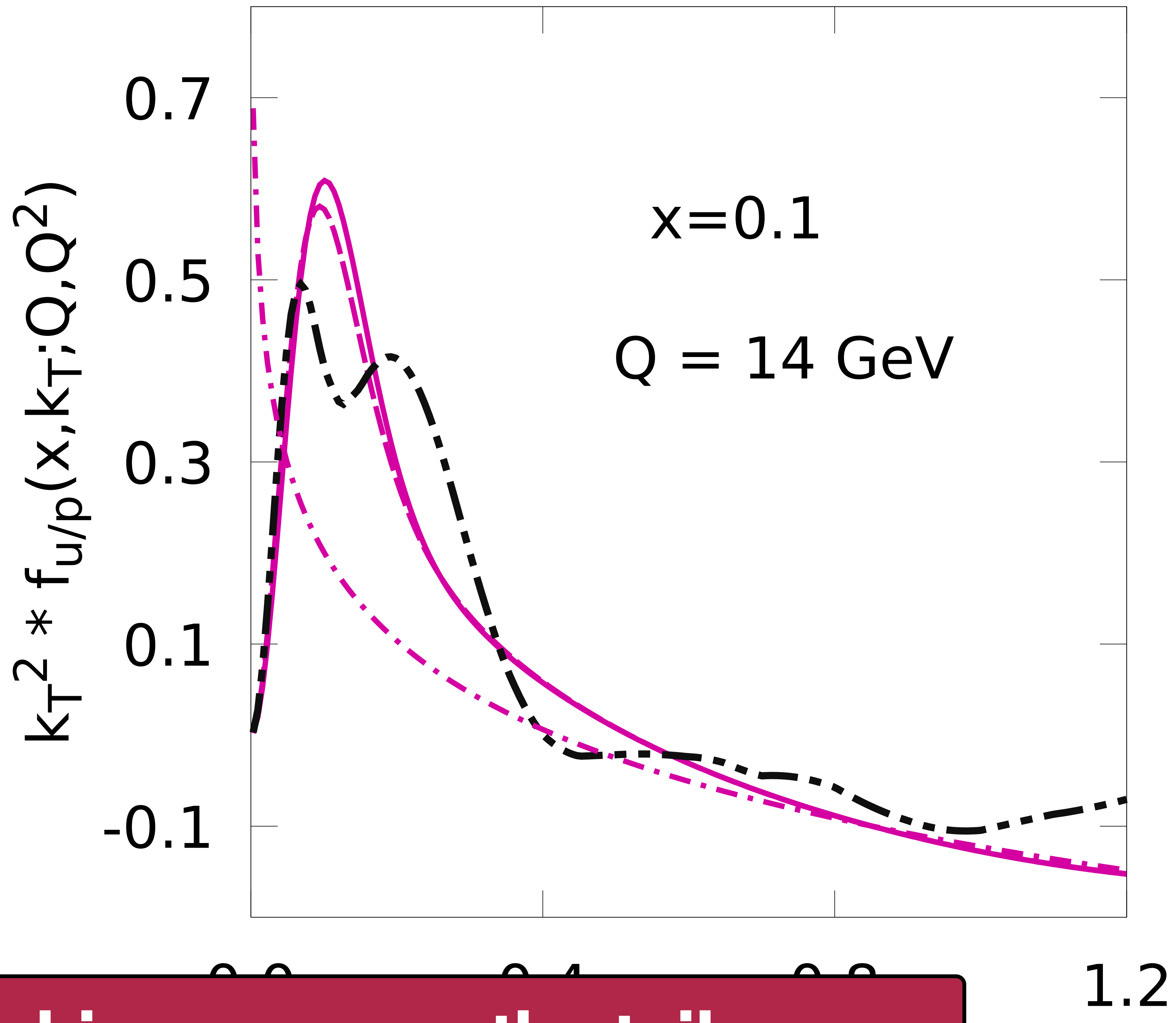




The k_T^2 weighing exposes the tails (multiplied by a factor of 10 in this case)

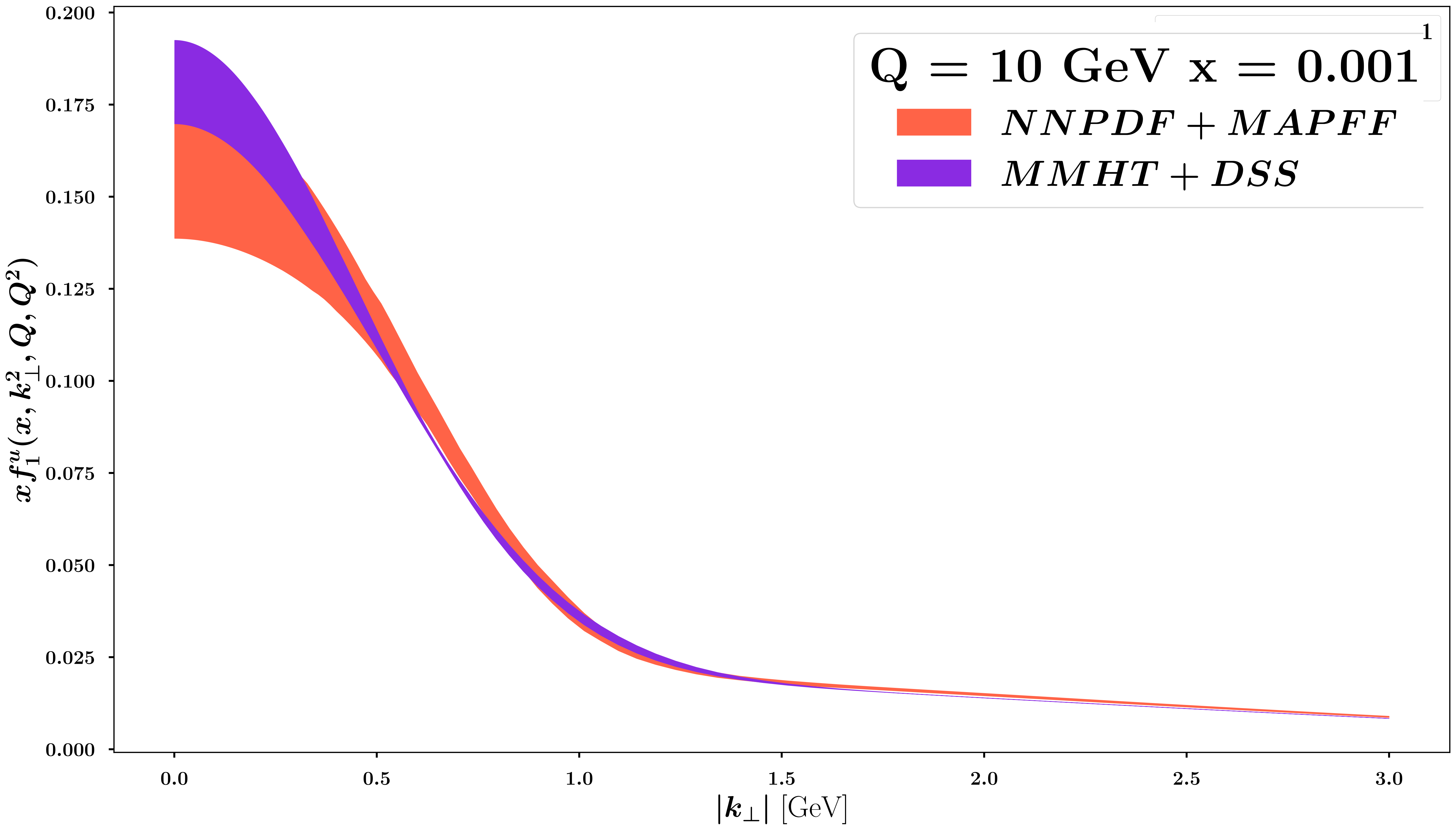
The paper emphasizes the relevance of prescription choices and simultaneous TMD-PDF fit, but does not provide a fit to extended data sets.

INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

MMHT2014 → NNPDF3.1

Hessian set → Monte Carlo set



INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

MMHT2014

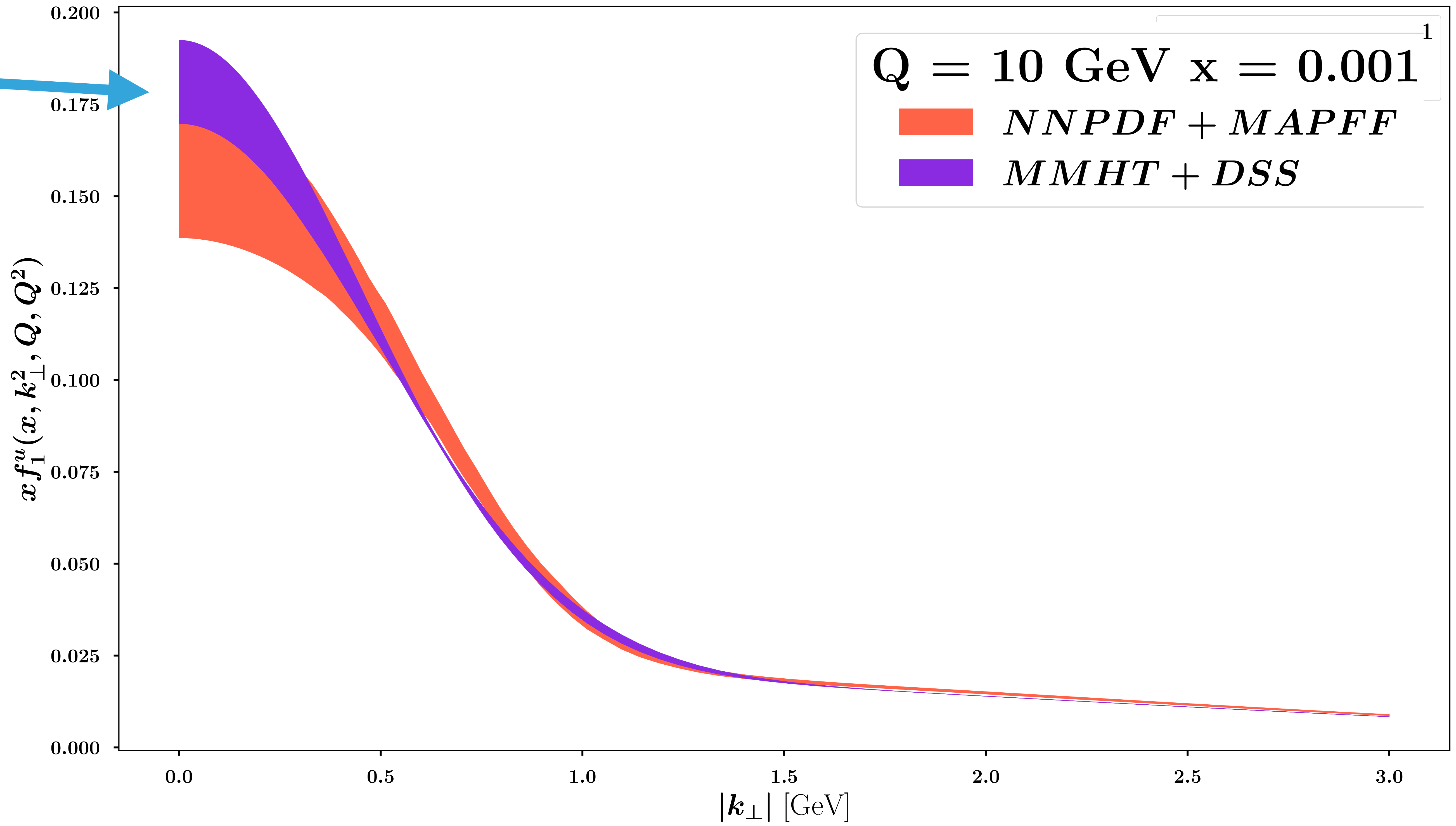
Hessian set

→

NNPDF3.1

Monte Carlo set

MAP22 fit



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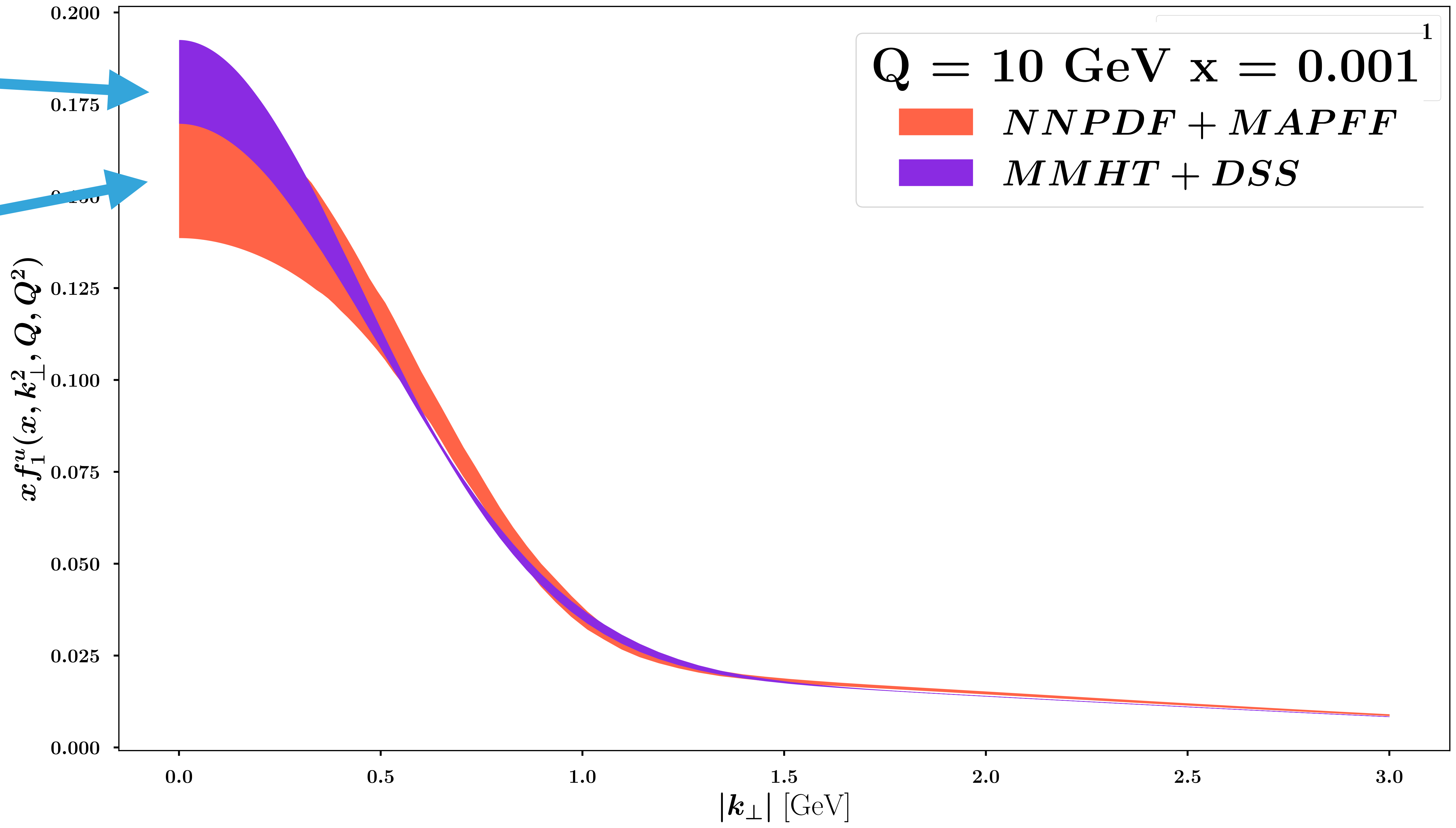

MAP22 fit

Fit with NNPDF set

INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

MMHT2014

Hessian set

→

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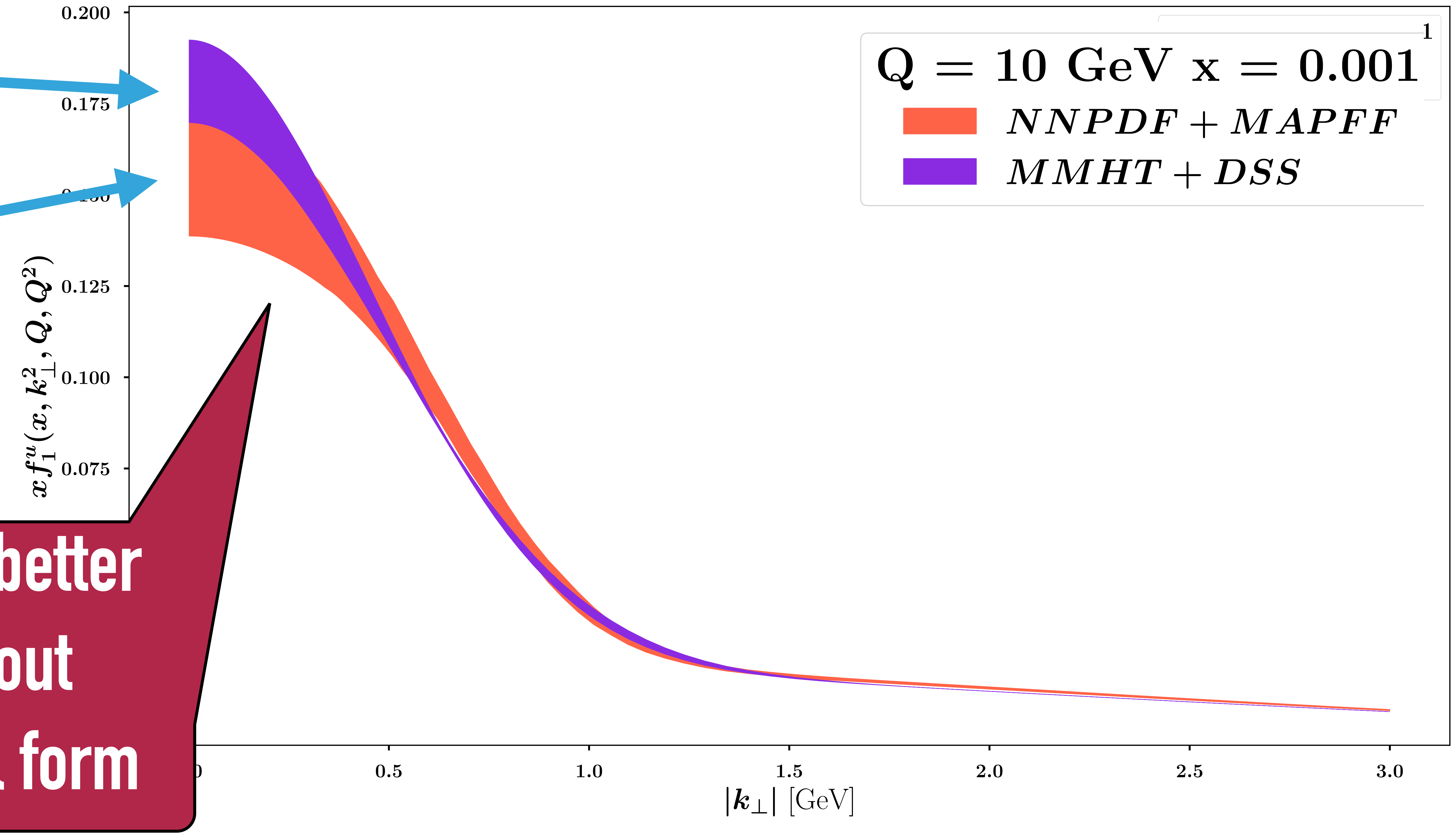


MAP22 fit

Fit with NNPDF set

The use of a Monte Carlo PDF set gives a better idea of the full TMD uncertainties, without dramatically changing the TMD functional form

INCLUSION OF HADRON DEPENDENCE IN TMD FF

Nonpert. TMD components of FF
equal for pions and kaons

INCLUSION OF HADRON DEPENDENCE IN TMD FF

Nonpert. TMD components of FF equal for pions and kaons

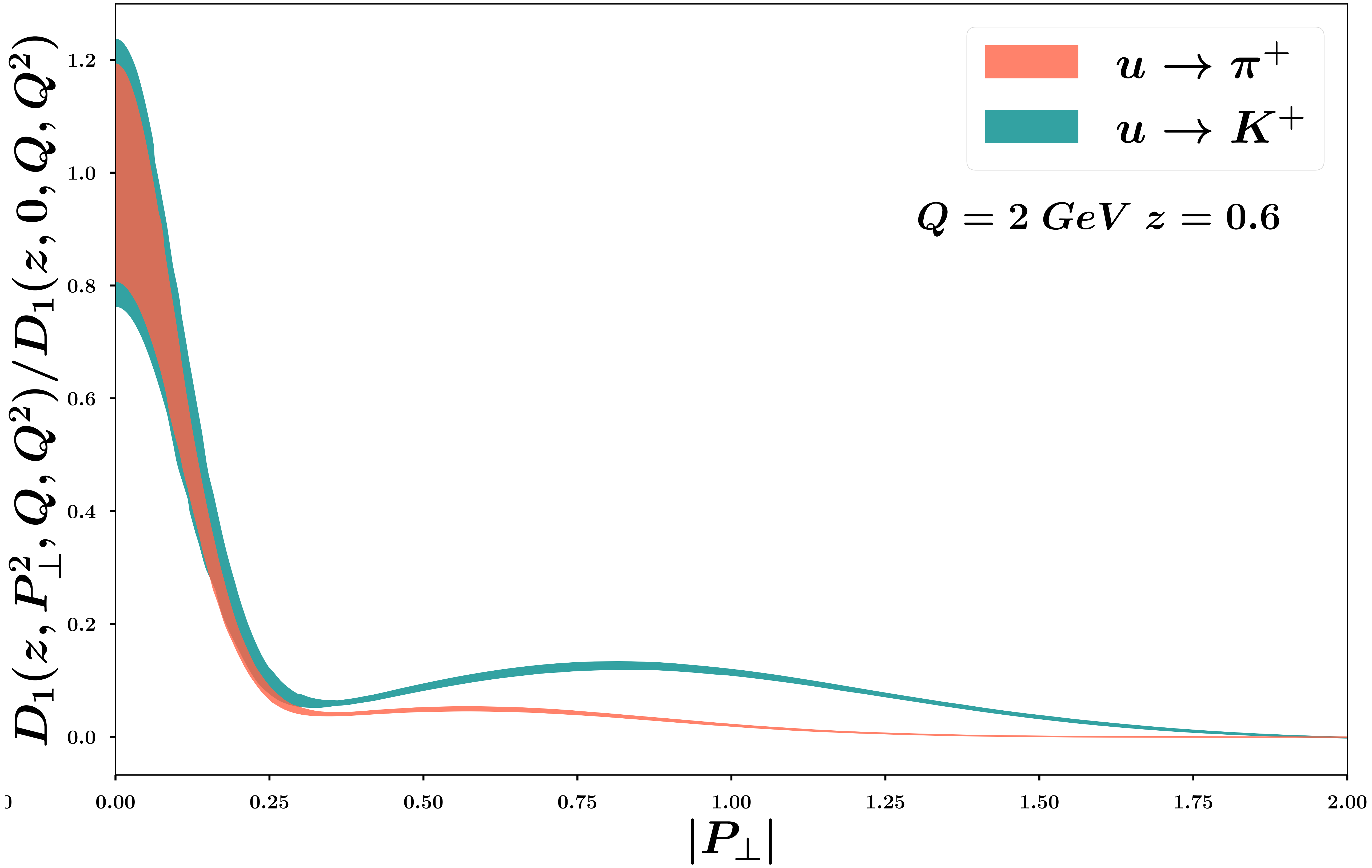
→

Distinction between pions and kaons

INCLUSION OF HADRON DEPENDENCE IN TMD FF

Nonpert. TMD components of FF equal for pions and kaons

Distinction between pions and kaons



LESSONS LEARNED

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- Simple Guassians or bell-like shapes are not sufficient to describe data

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- The TMD shape must be x-dependent

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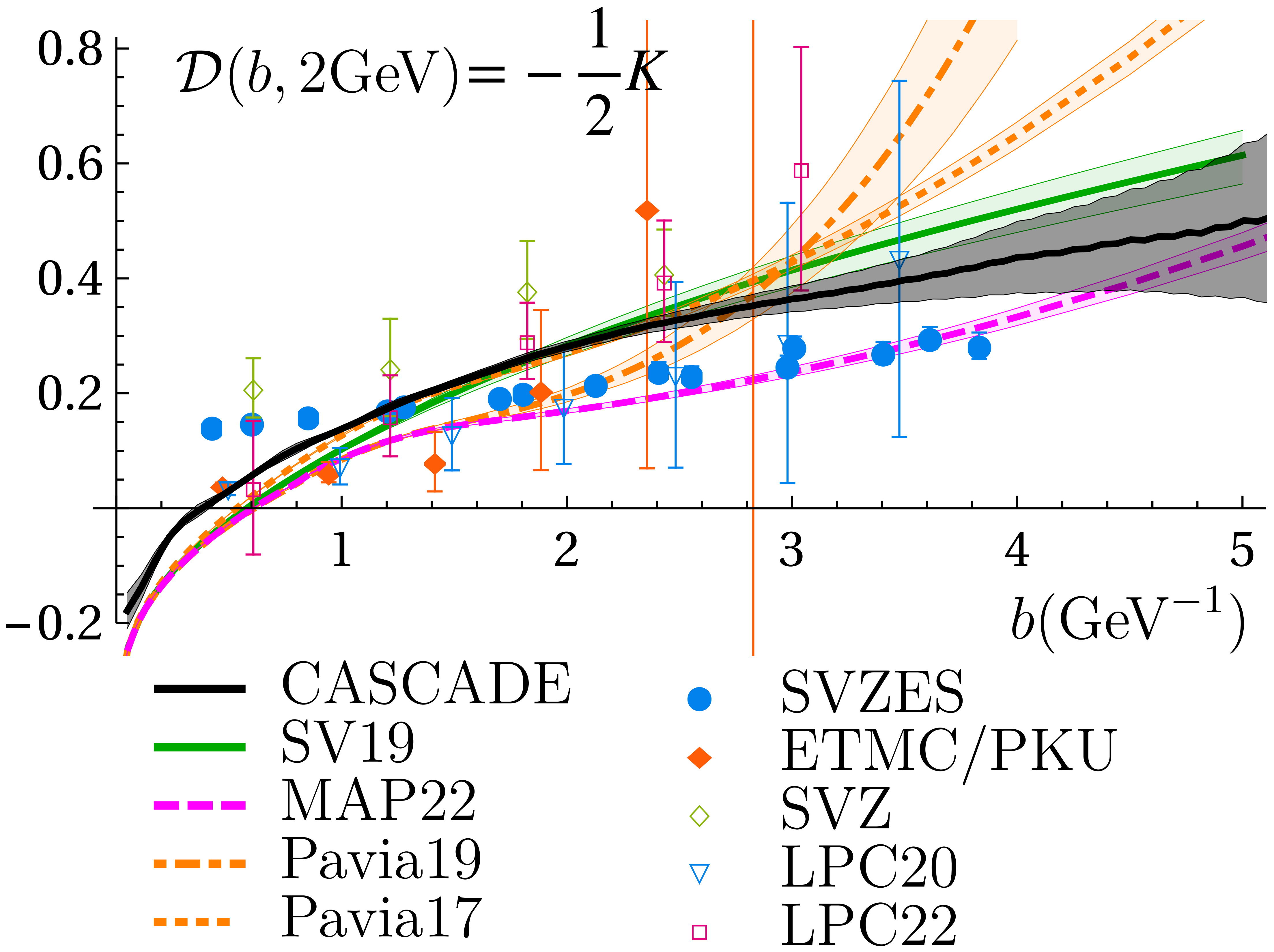
CONNECTIONS WITH LATTICE QCD: COLLINS-SOPER KERNEL

Bermudez Martinez, Vladimirov, arXiv:2206.01105



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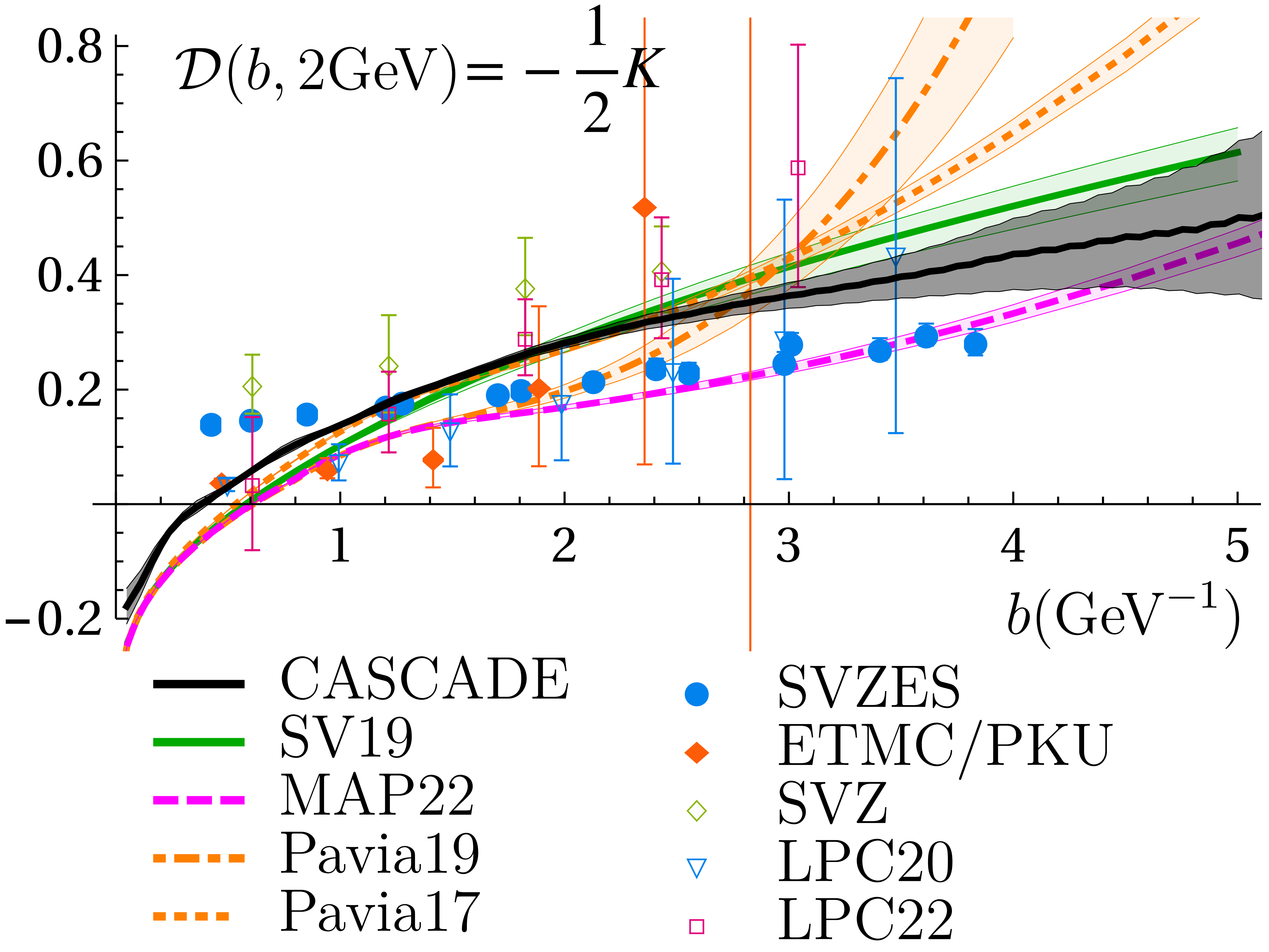

TMD phenomenology

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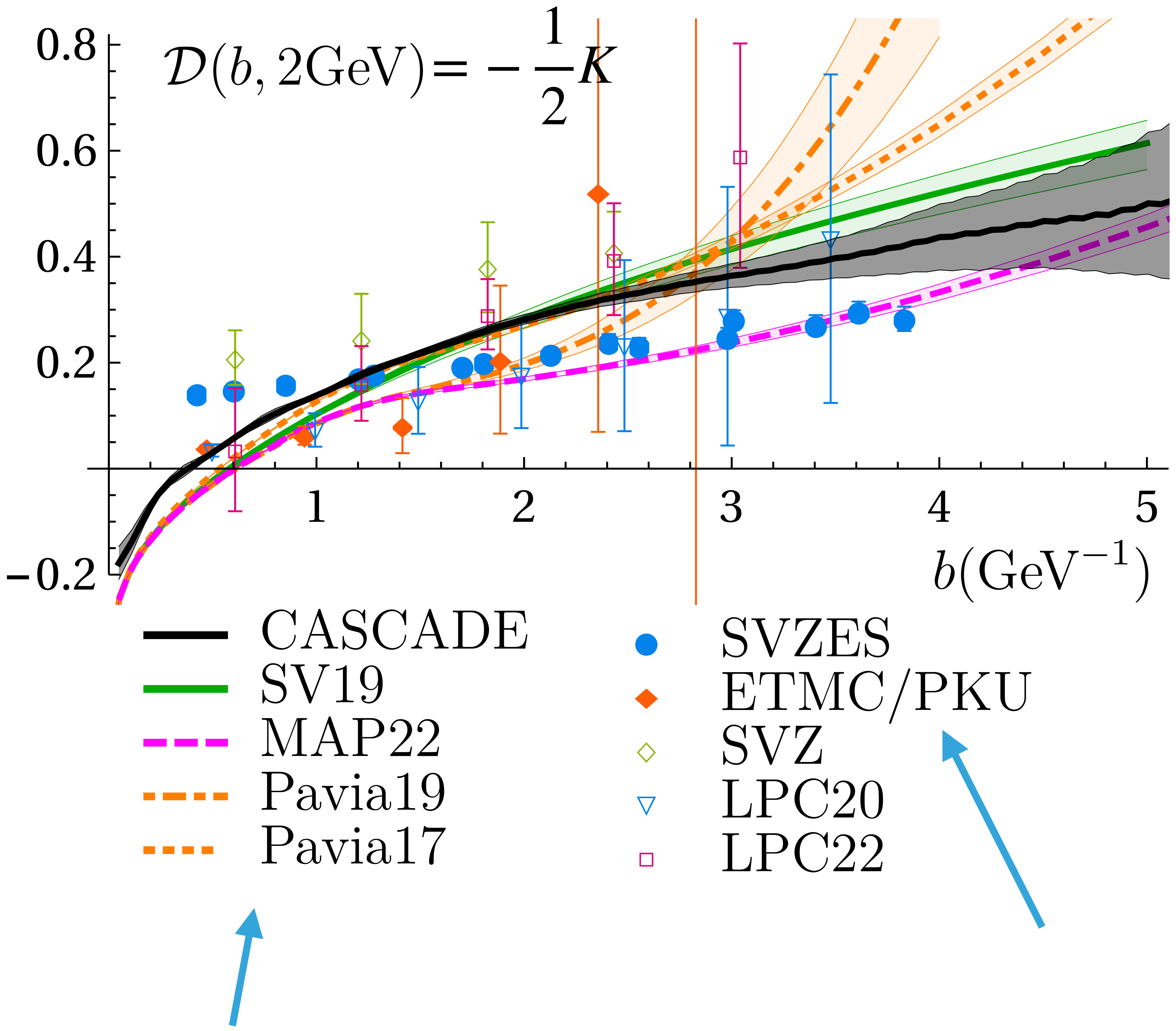

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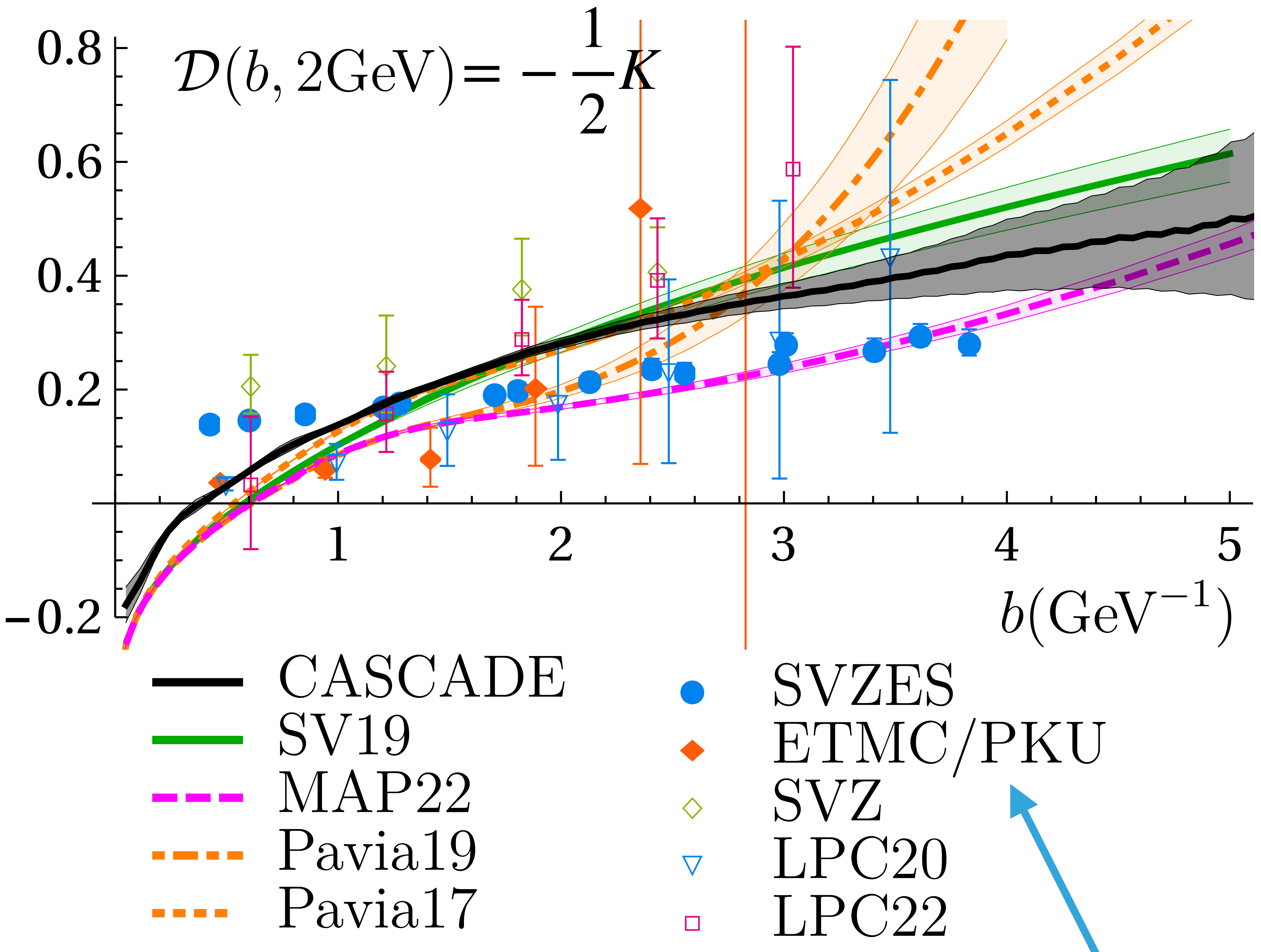
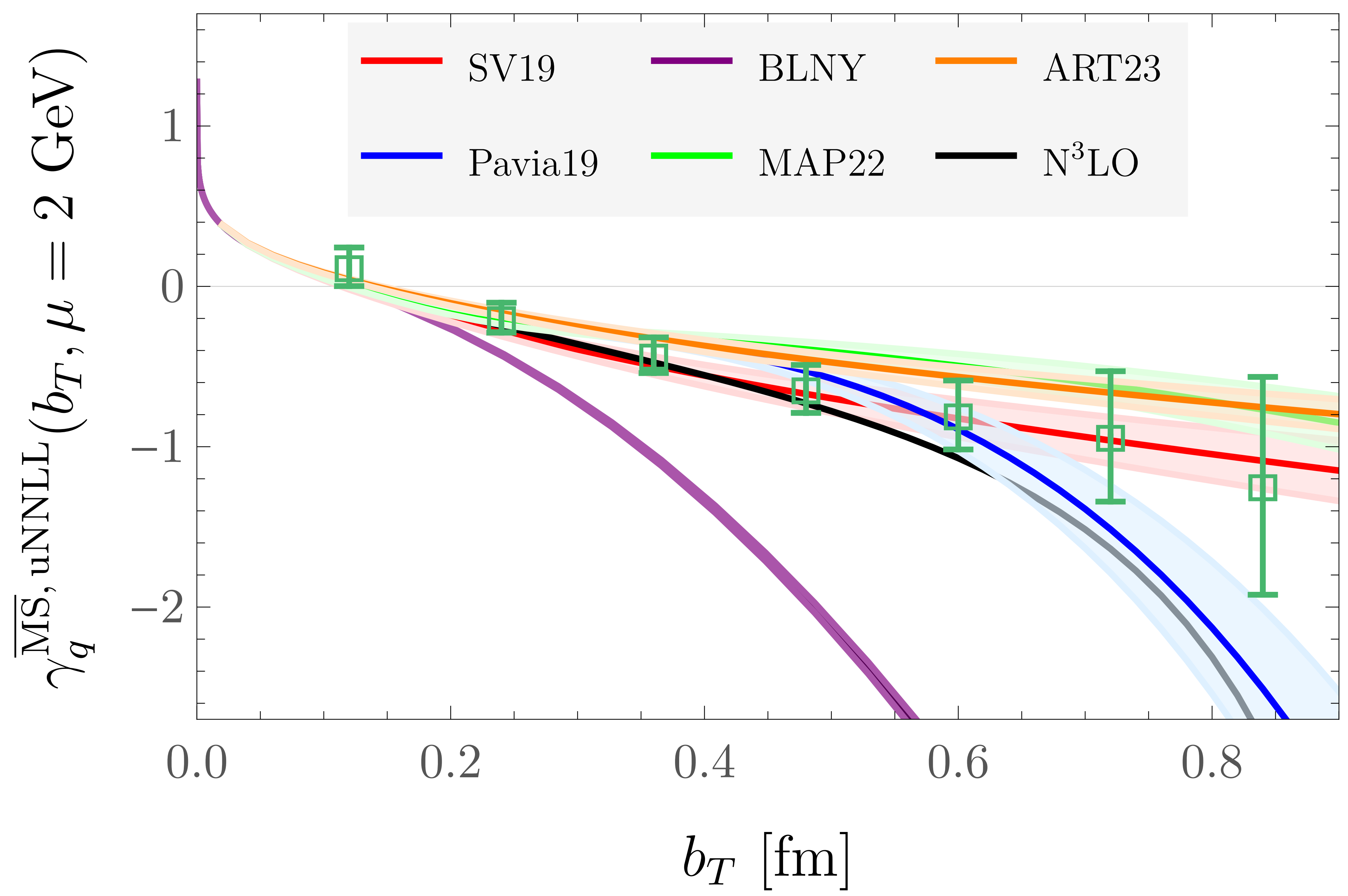


TMD phenomenology

Lattice QCD

Avkhadiev, Shanahan, Wagman, Zhao, arXiv:2307.12359



CONNECTION WITH LATTICE QCD: TMDS

LPC collaboration, arxiv:2211.02340

Lattice QCD

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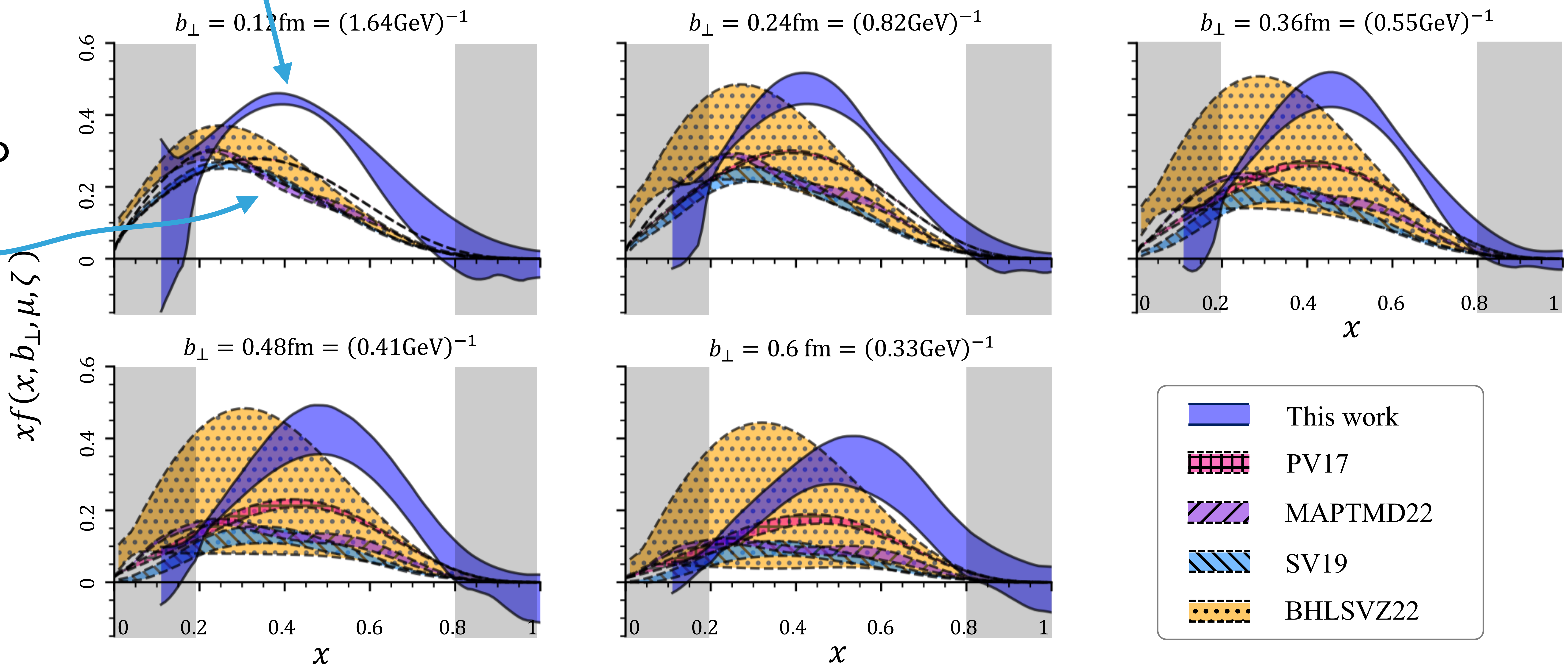
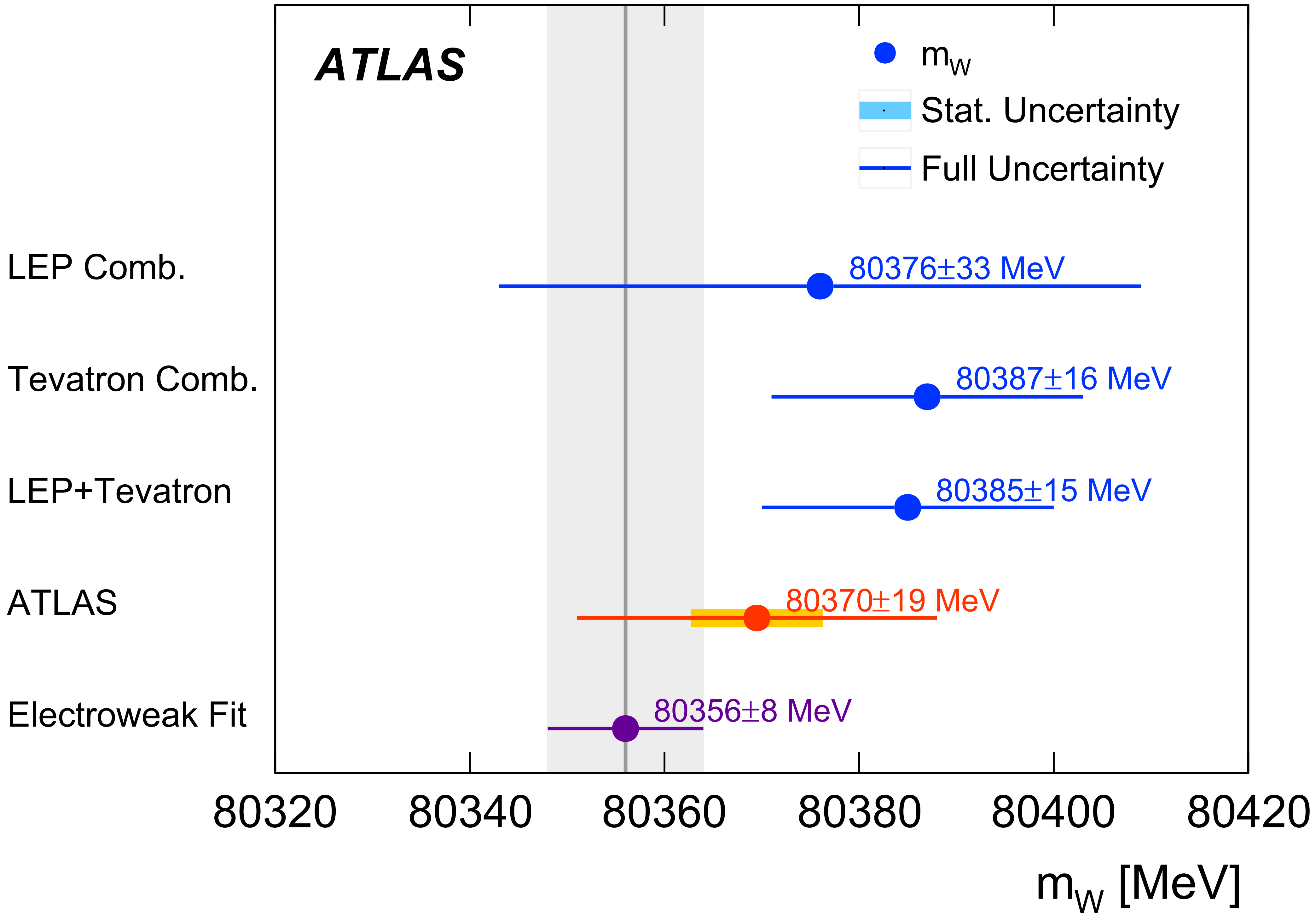


CONNECTION WITH LHC PHYSICS: M_W

ATLAS Collab. arXiv:1701.07240



$$\begin{aligned} m_W &= 80370 \pm 7\,(\text{stat.}) \pm 11\,(\text{exp. syst.}) \pm 14\,(\text{mod. syst.})\ \text{MeV} \\ &= 80370 \pm 19\ \text{MeV}, \end{aligned}$$

$$m_{W^+} - m_{W^-} = -29 \pm 28\ \text{MeV}.$$

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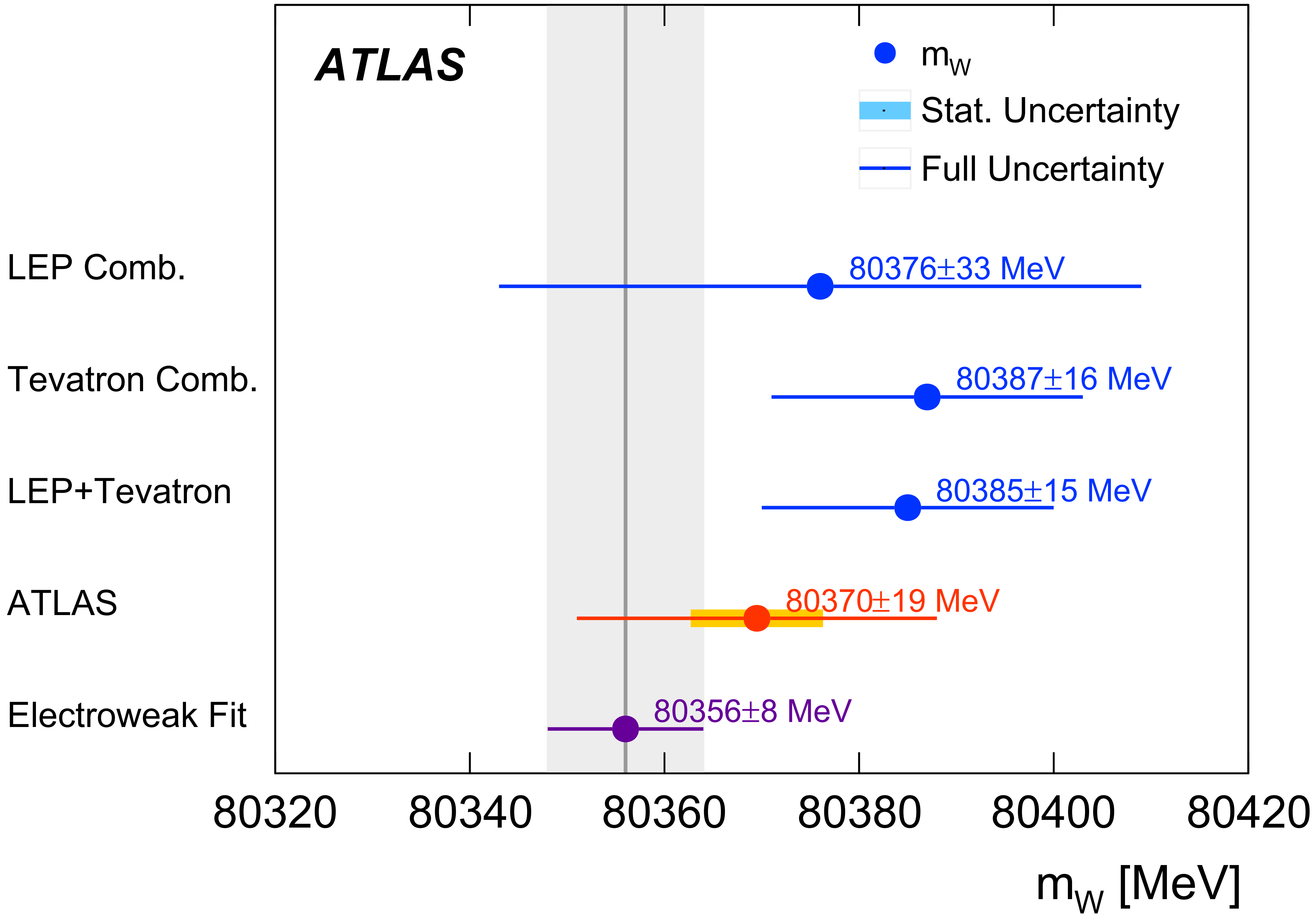
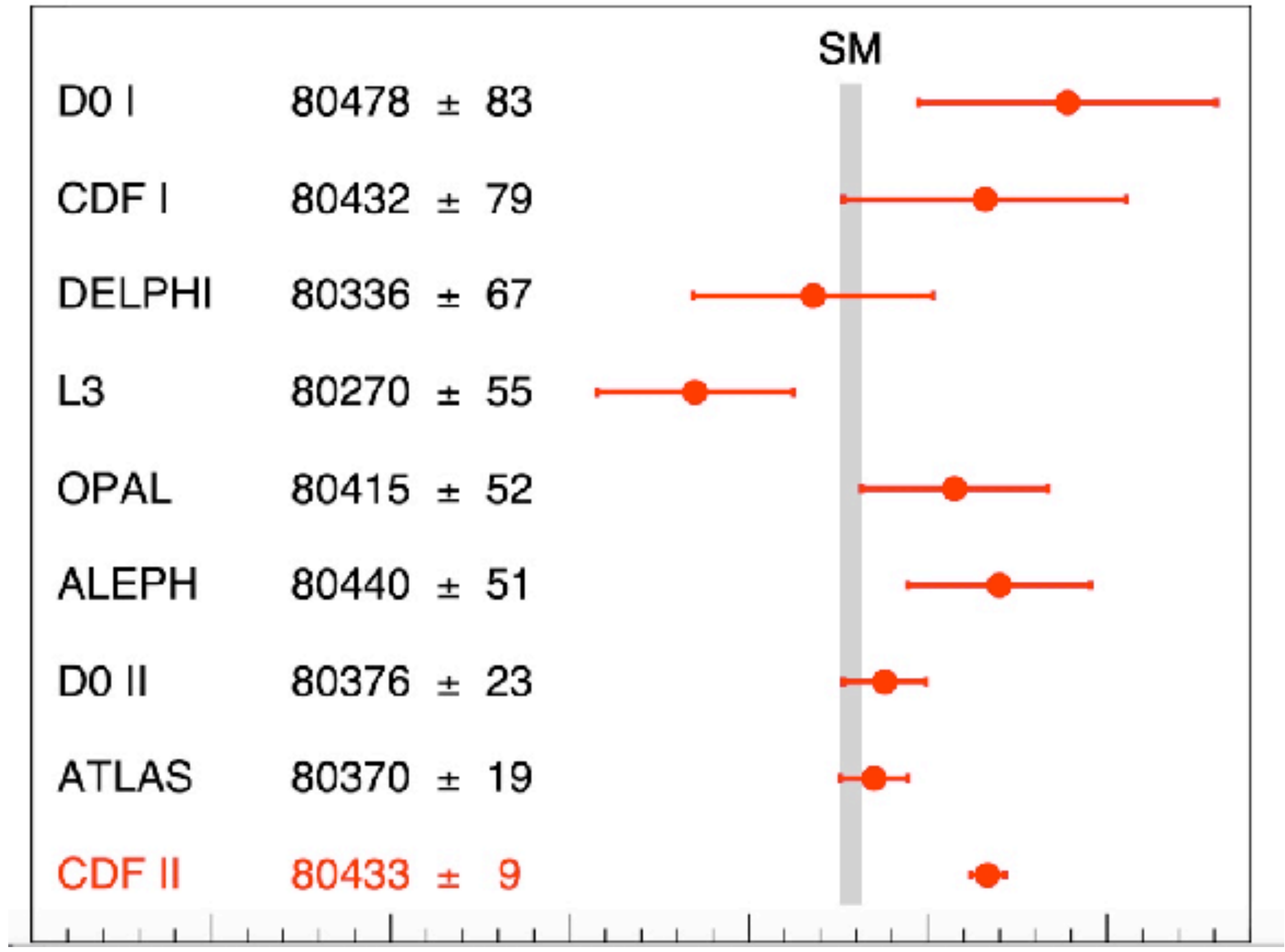
CDF Collab.. Science 2022





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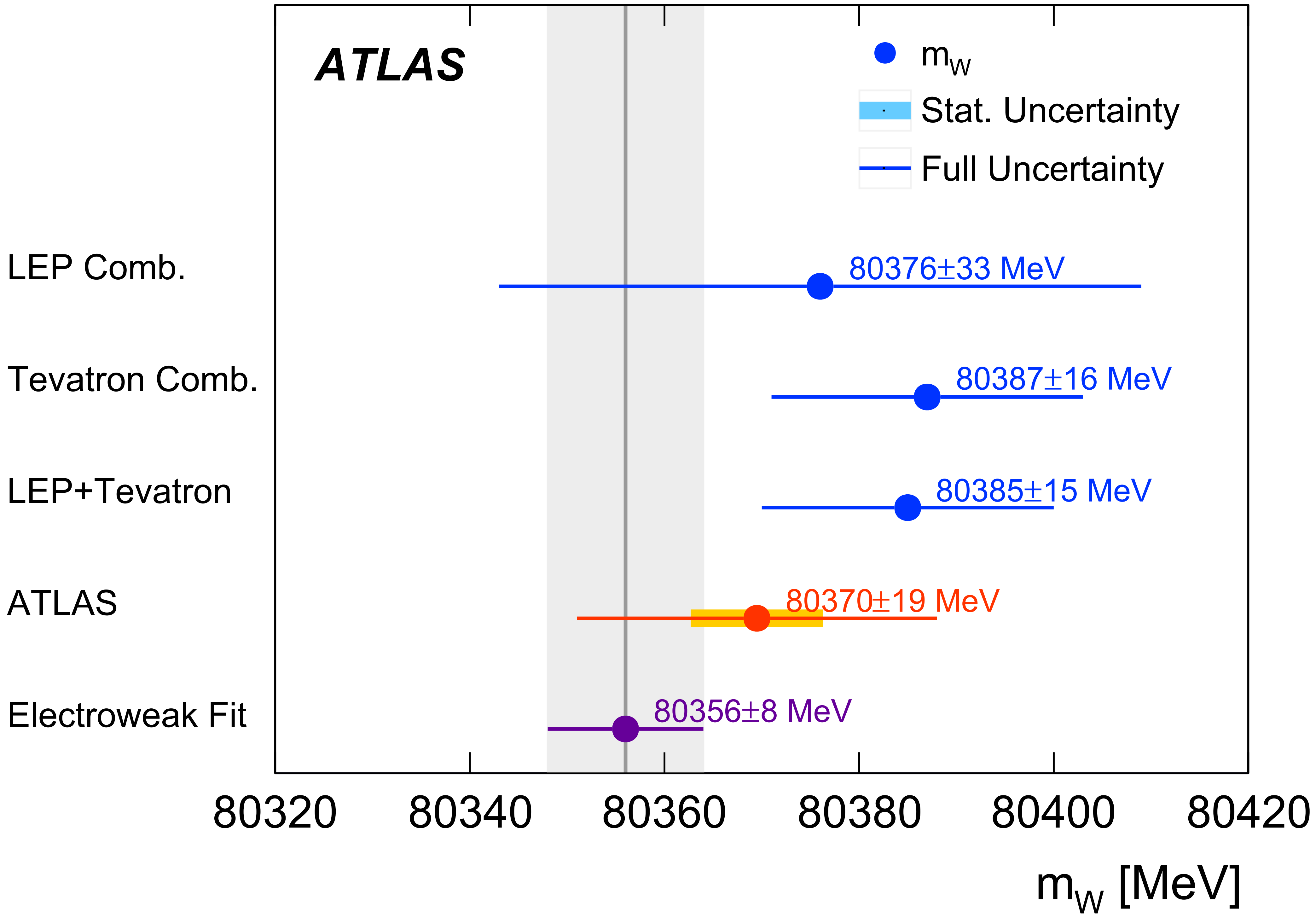

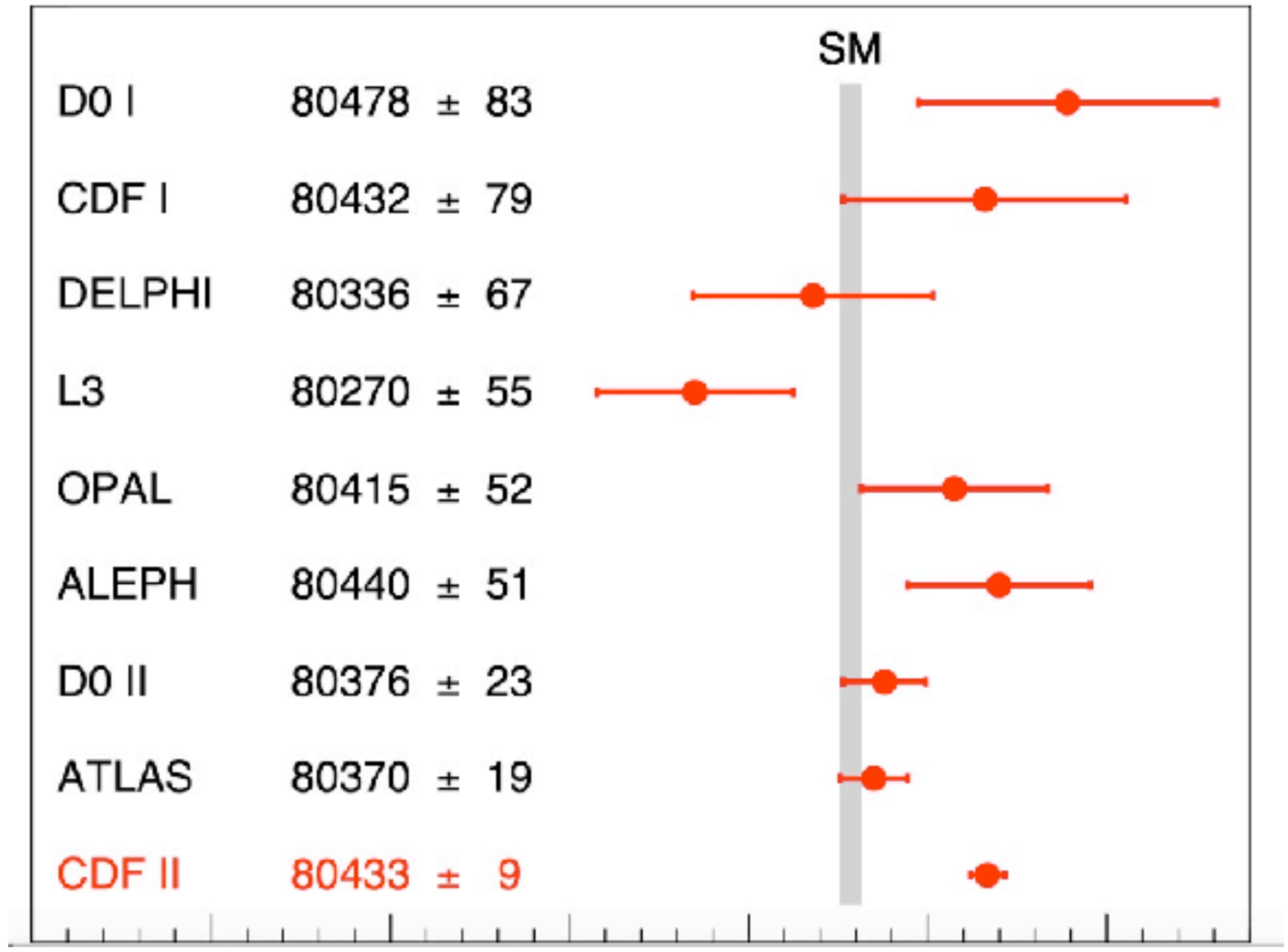

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All analyses assume that TMDs are not flavor dependent.
What happens if they are?

CONNECTION WITH LHC PHYSICS: M_W

Bacchetta, Bozzi, Radici, Ritzmann, Signori, arXiv:1807.02101

Try some judicious choices of flavour dependent widths and check

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Try some judicious choices of flavour dependent widths and check

Set	u_v	d_v	u_s	d_s	s
1	0.34	0.26	0.46	0.59	0.32
2	0.34	0.46	0.56	0.32	0.51
3	0.55	0.34	0.33	0.55	0.30
4	0.53	0.49	0.37	0.22	0.52
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narrow, medium, large
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	ΔM_{W+}		ΔM_{W-}	
Set	m_T	$p_{T\ell}$	m_T	$p_{T\ell}$
1	0	-1	-2	3
2	0	-6	-2	0
3	-1	9	-2	-4
4	0	0	-2	-4
5	0	4	-1	-3

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They all describe the Z spectrum very well

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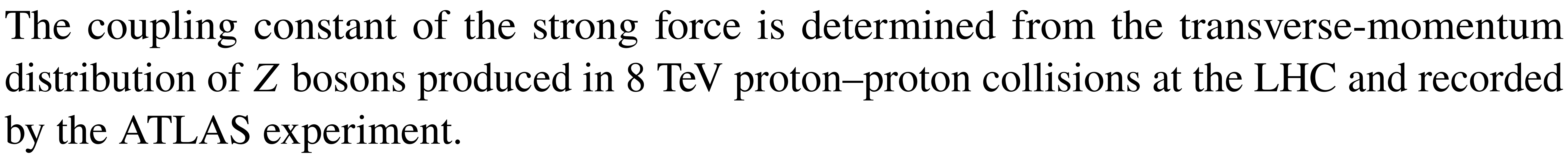
CONNECTION WITH LHC PHYSICS: α_S

ATLAS coll., arXiv:2309.12986



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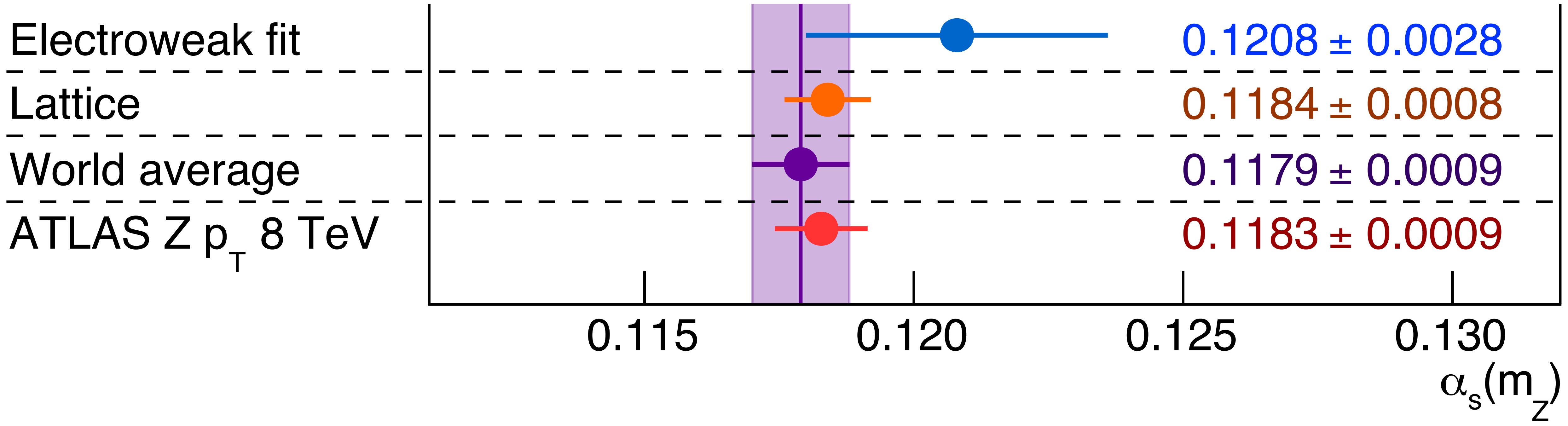

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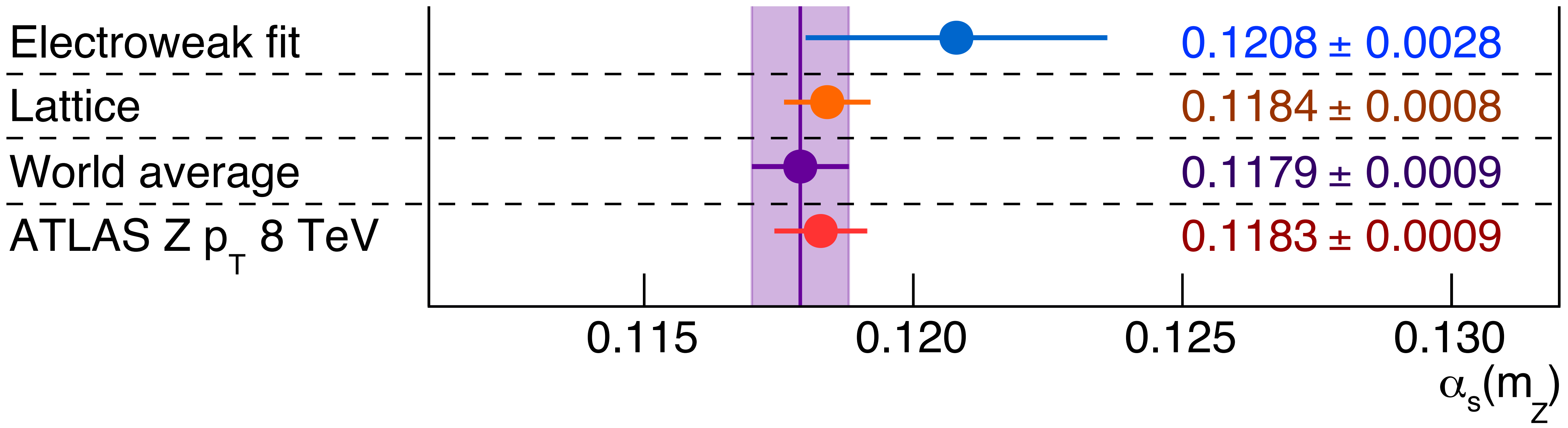

Table 1: Summary of the uncertainties in the determination of $\alpha_s(m_Z)$, in units of 10^{-3}.

Experimental uncertainty	±0.44	
PDF uncertainty	±0.51	
Scale variation uncertainties	±0.42	
Matching to fixed order	0	−0.08
Non-perturbative model	+0.12	−0.20
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N^4LL approximation	±0.04	
Total	+0.91	−0.88

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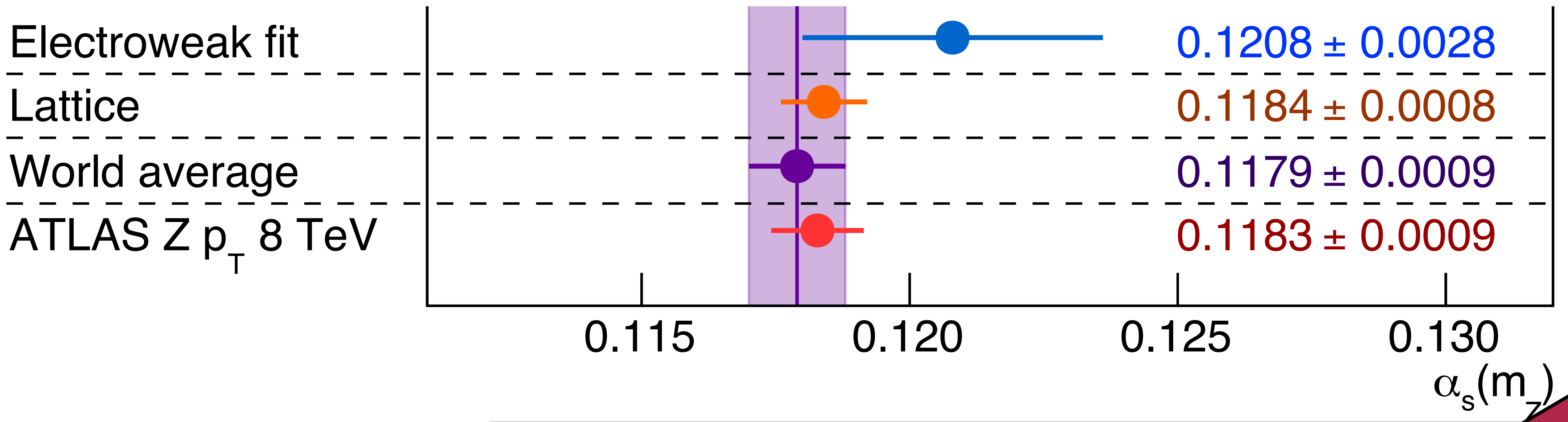

Estimate that does not take into account recent TMD results

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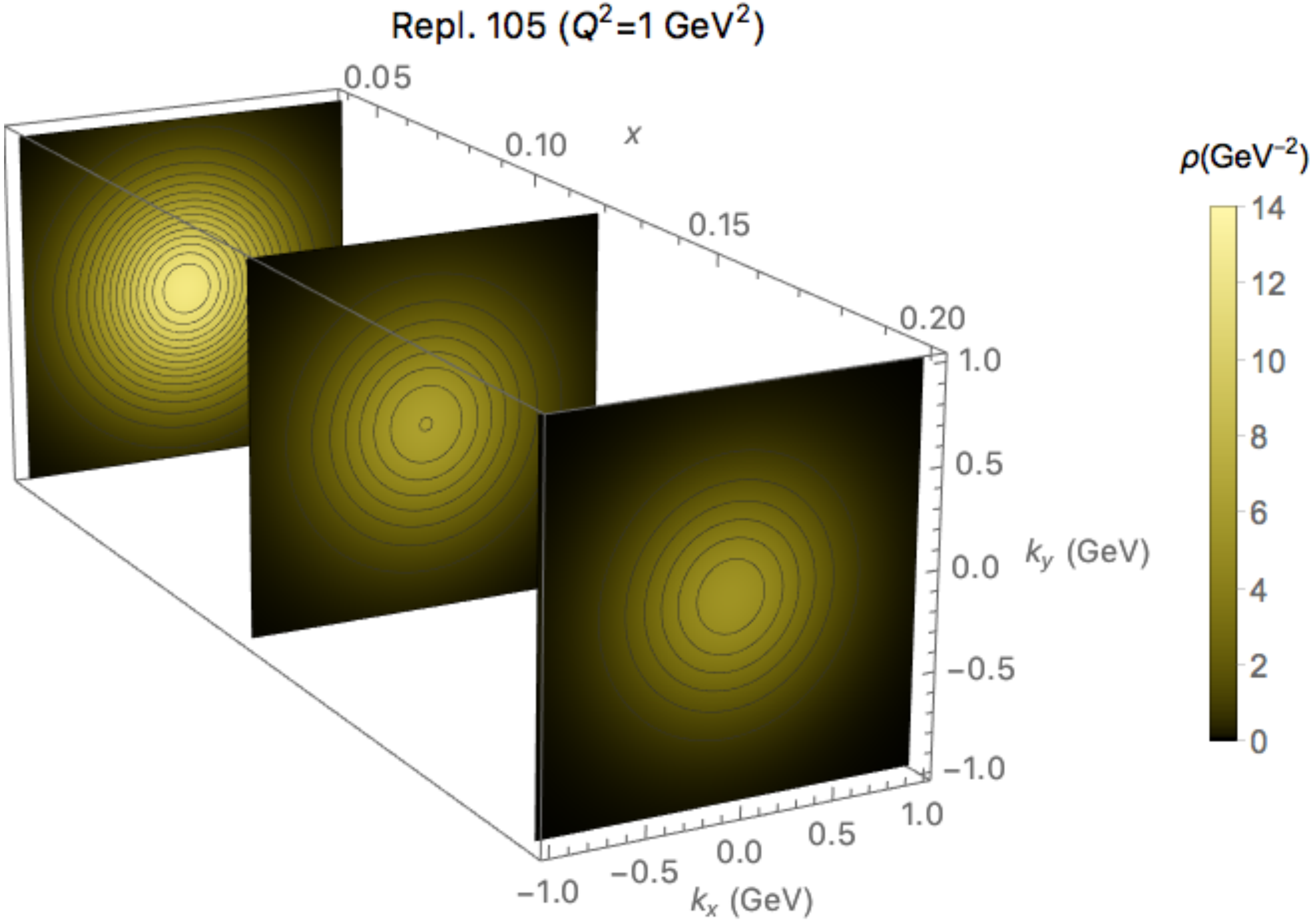
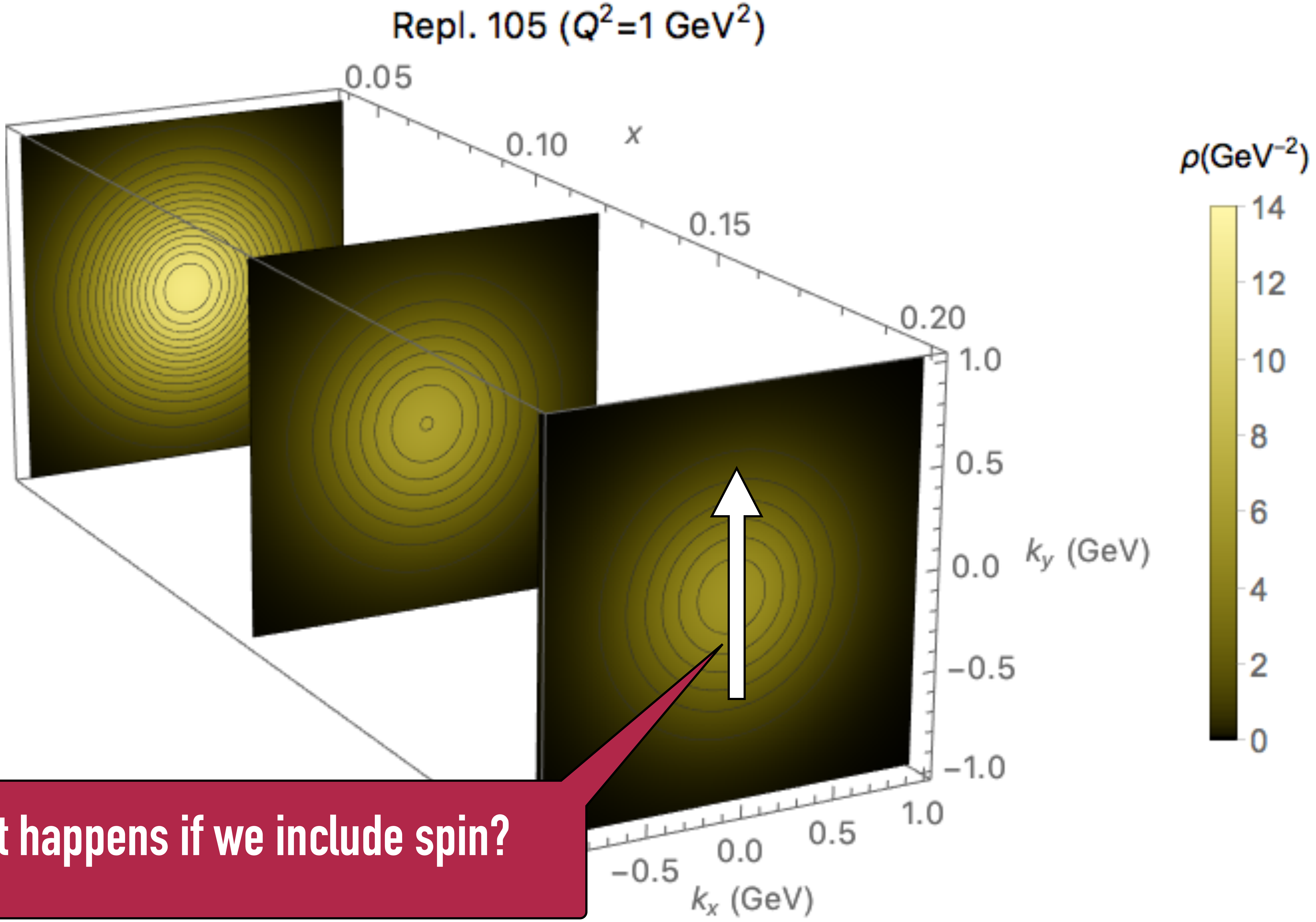
SIVERS TMD


Repl. 105 (Q^2=1 GeV^2)
0.05
0.10
x
0.15
0.20
1.0
0.5
0.0
k_y (GeV)
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
k_x (GeV)
ρ(GeV^{-2})
14
12
10
8
6
4
2
0



What happens if we include spin?

SIVERS FUNCTION

$$\rho^q_{N\uparrow}(x, k_x, k_y; Q^2) = f^q_1(x, k_T^2; Q^2) - \frac{k_x}{M} f^{\perp q}_{1T}(x, k_T^2; Q^2)$$

In a nucleon polarized in the +y direction,
the distribution of quarks can be distorted in the x direction

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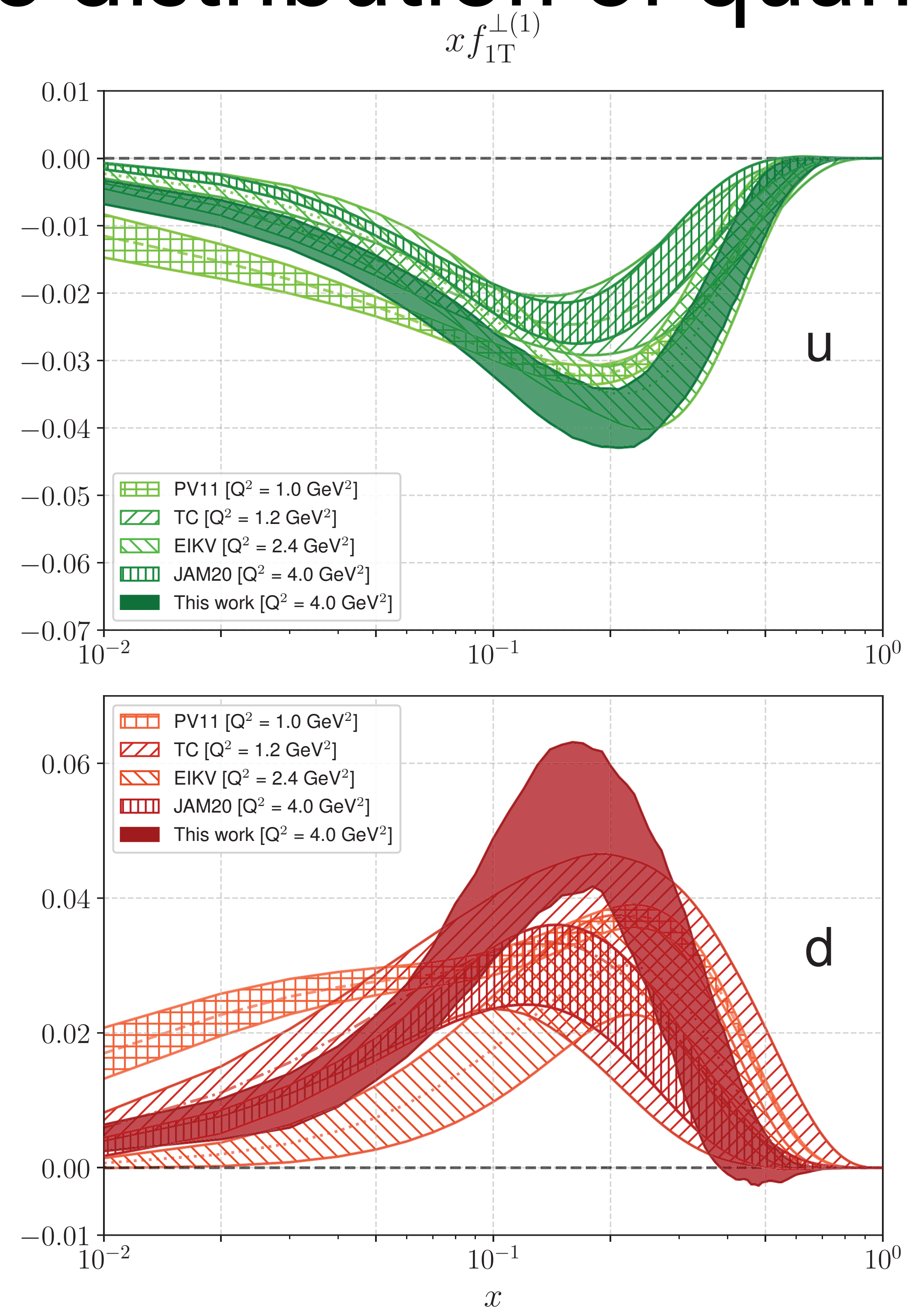
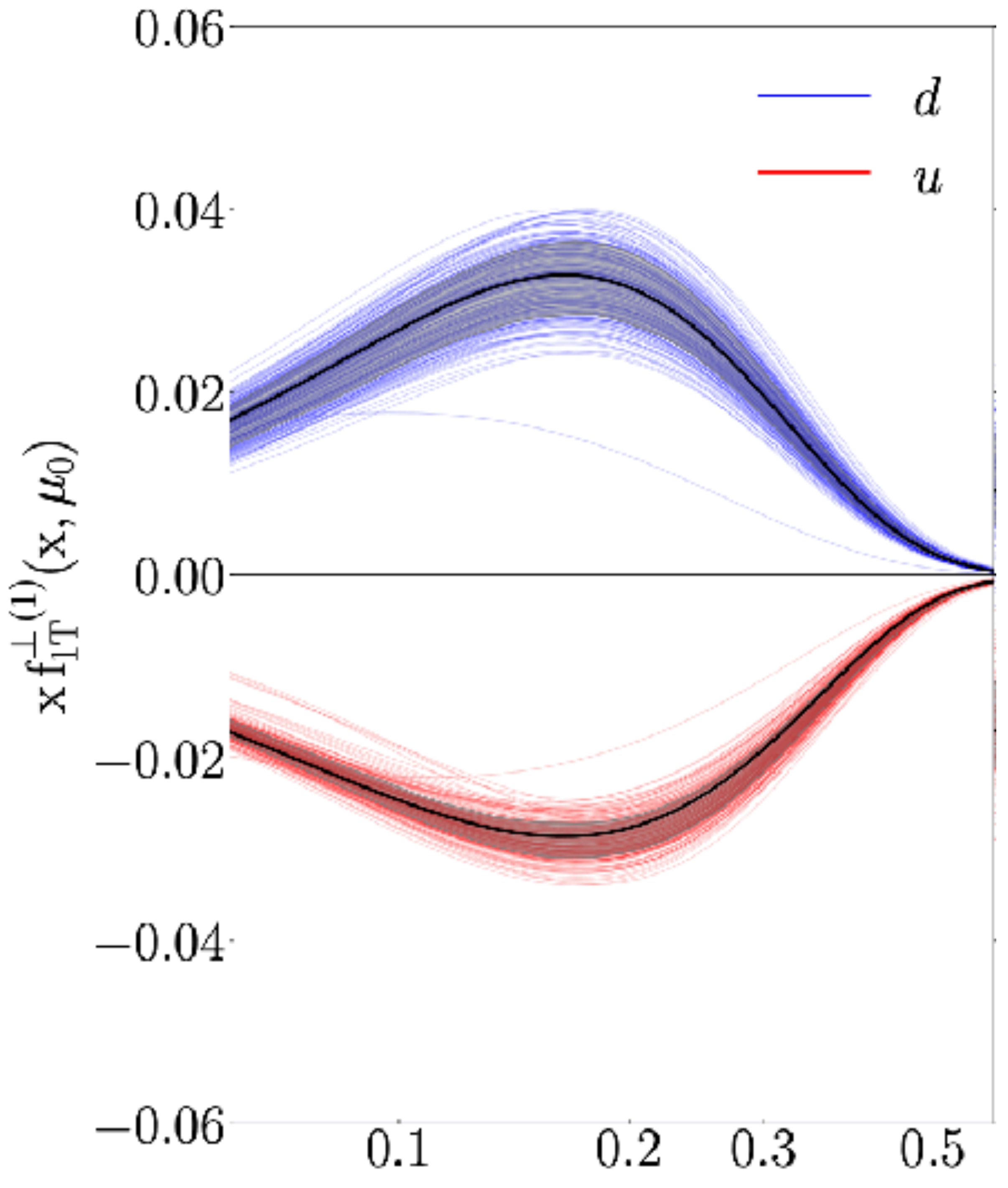
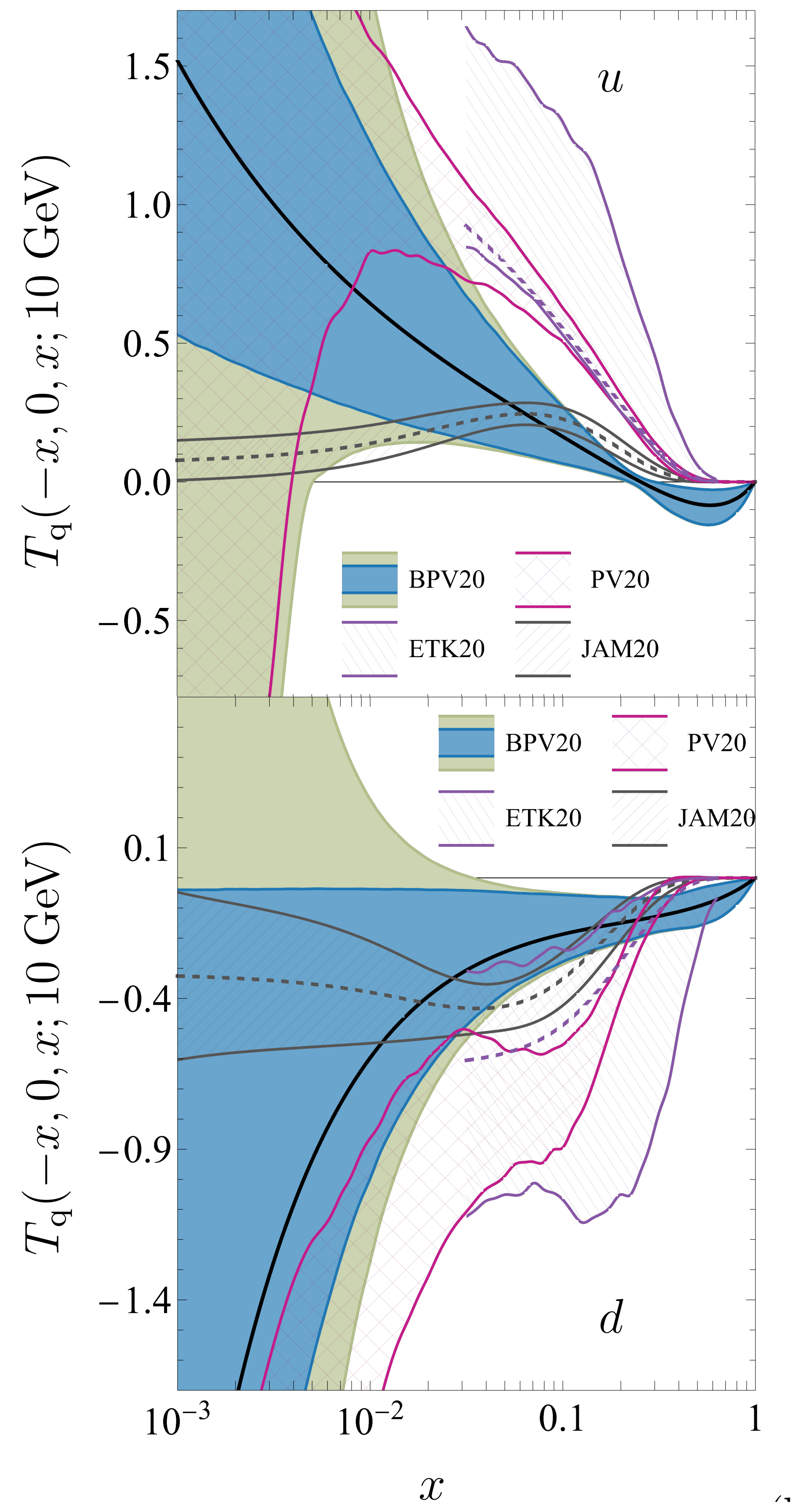


Bacchetta, Delcarro,
Pisano, Radici, arXiv:2004.14278

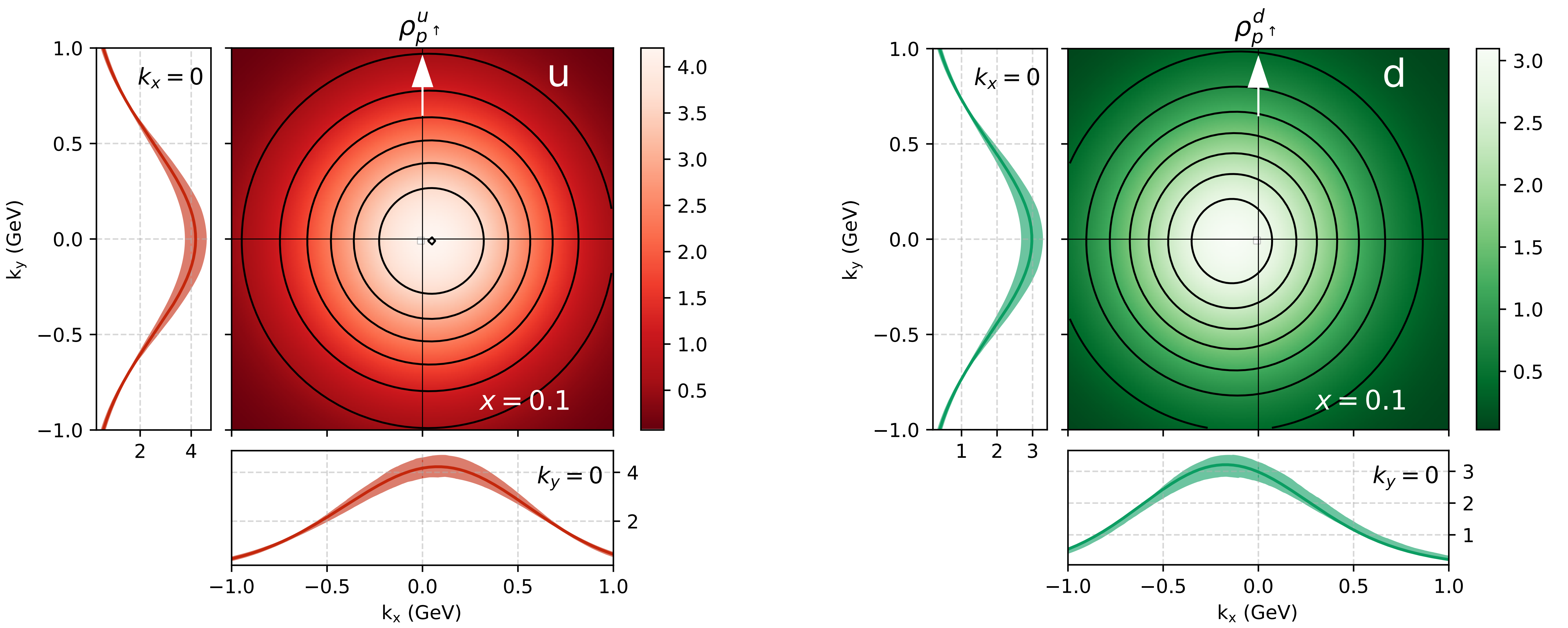


Echevarria, Kang, Terry,
arXiv:2009.10710



Bury, Prokudin, Vladimirov,
arXiv:2103.03270

3D STRUCTURE IN MOMENTUM SPACE



Q= 2GeV

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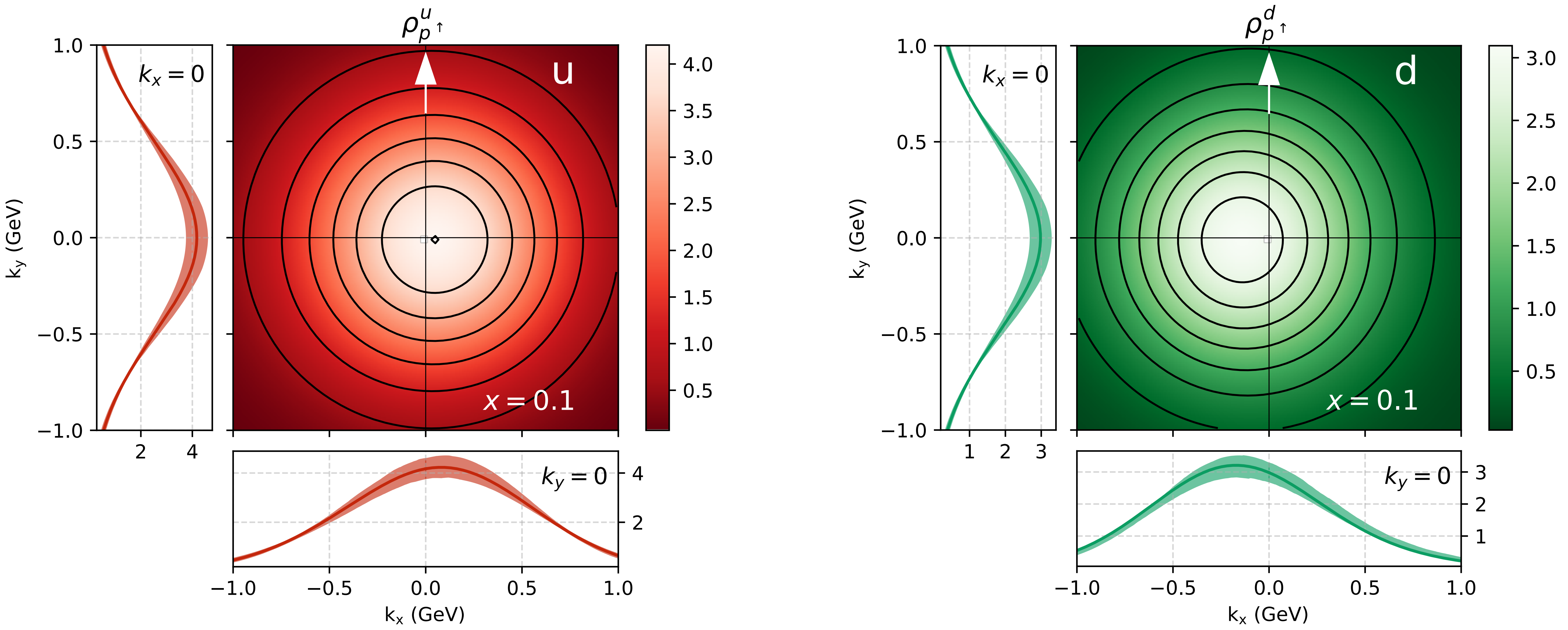

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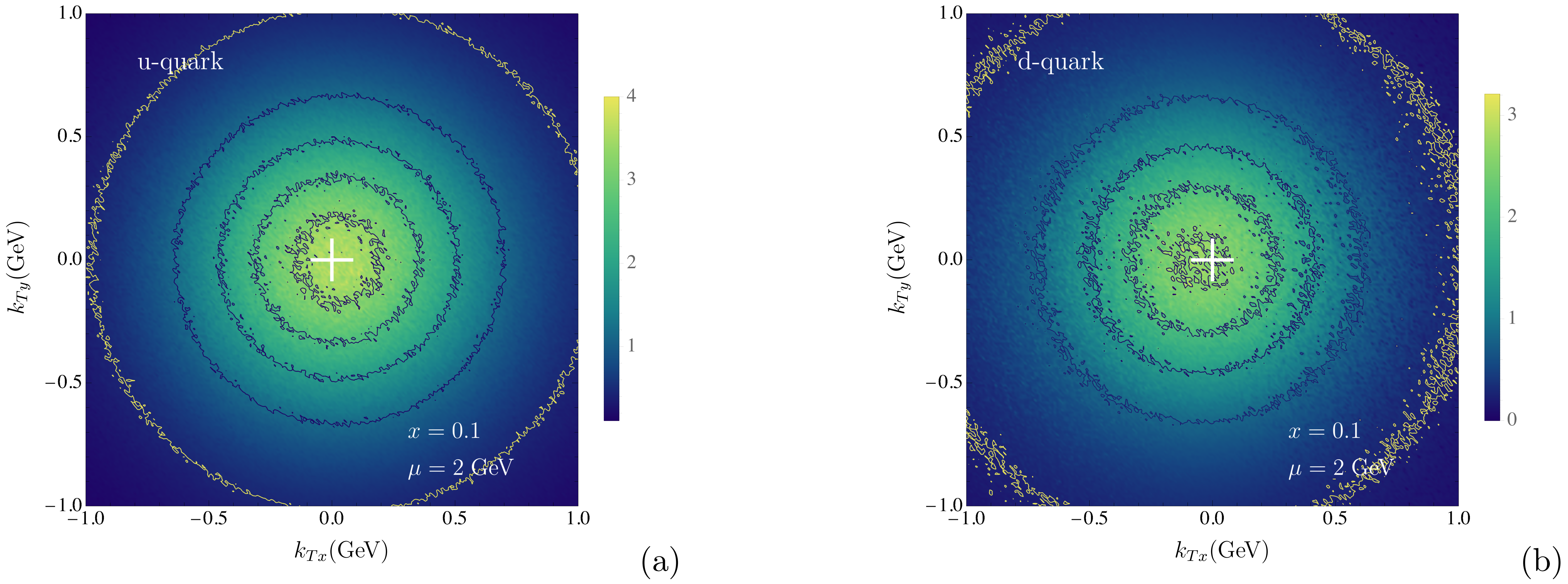
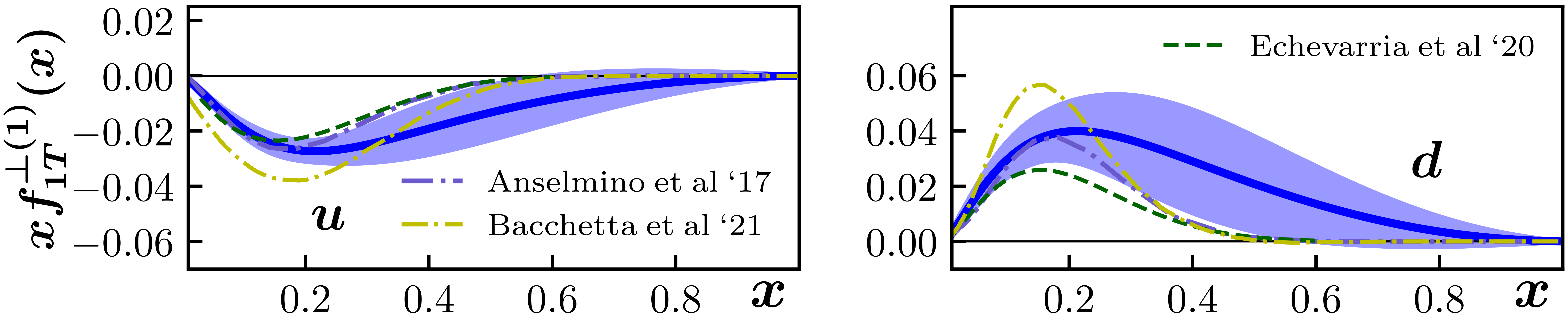


[Bury, Prokudin, Vladimirov, arXiv:2103.03270]

SIMULTANEOUS FIT OF SINGLE TRANSVERSE-SPIN ASY

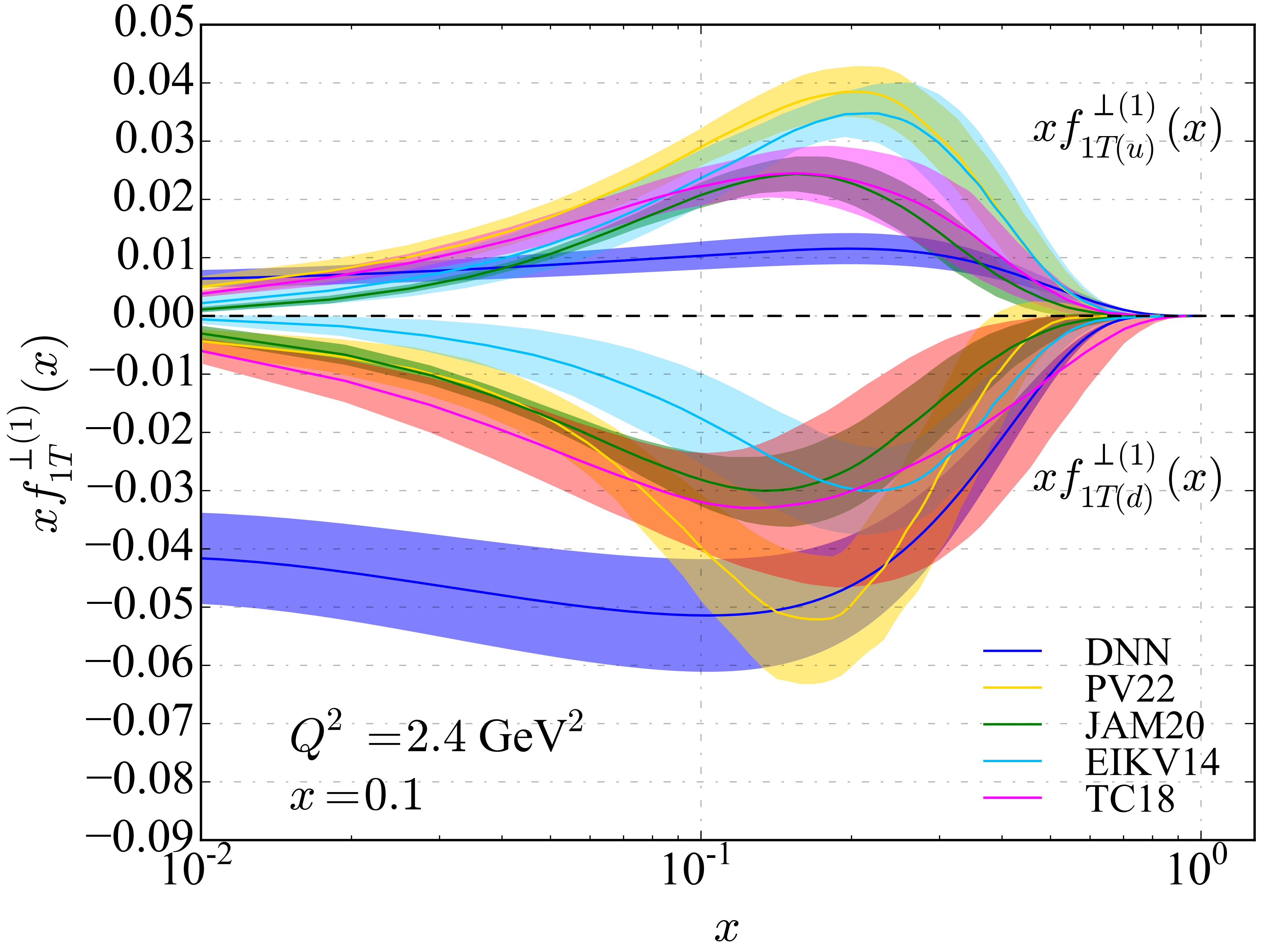
JAM, arXiv:2205.00999

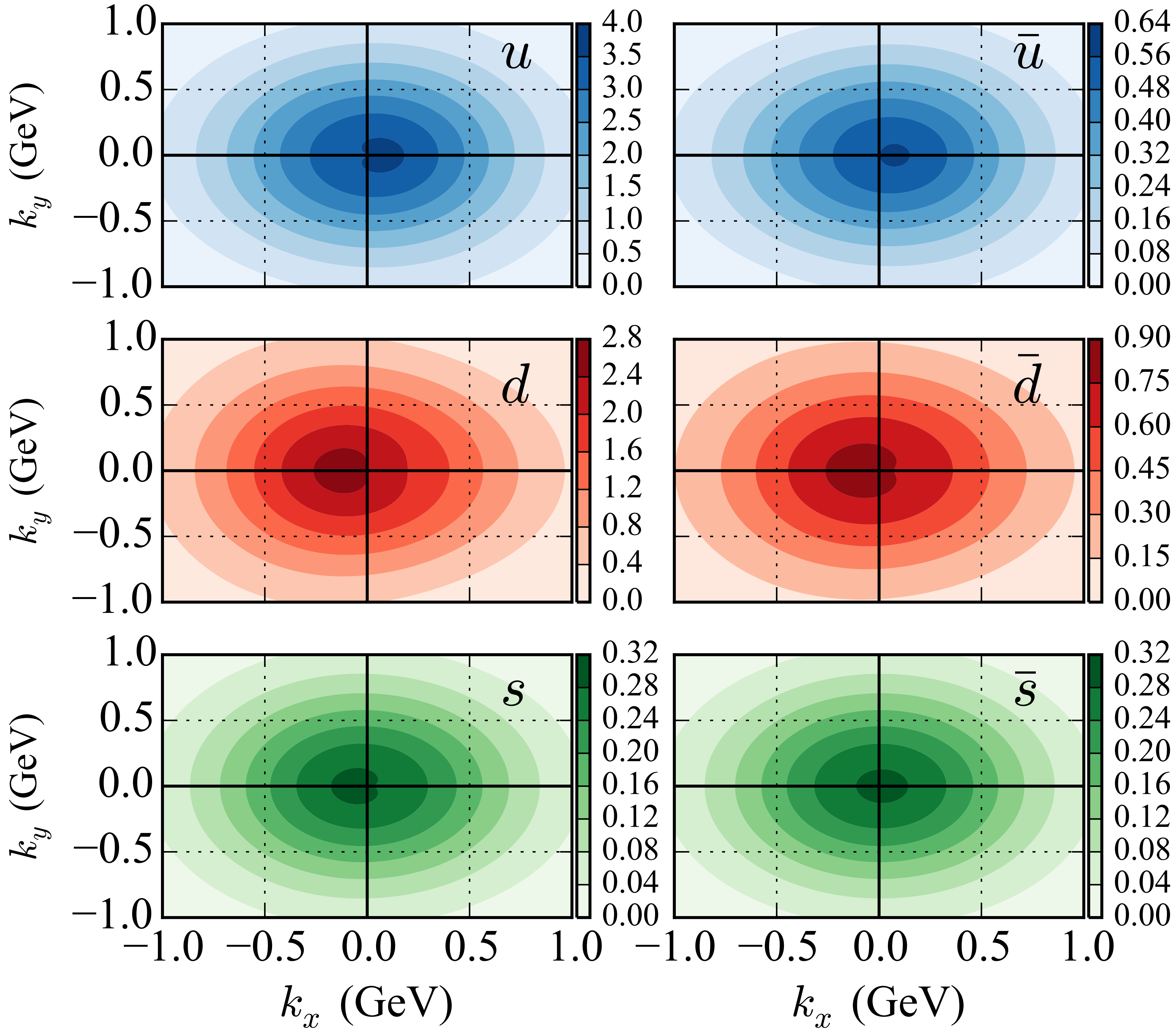


Interesting work from the point of view of simultaneous use of several measurements, but still limited from other perspectives (lack of TMD evolution and knowledge of the unpolarized function)

SIVERS FUNCTION WITH NEURAL NETWORKS

Fernando, Keller, arXiv:2304.14328





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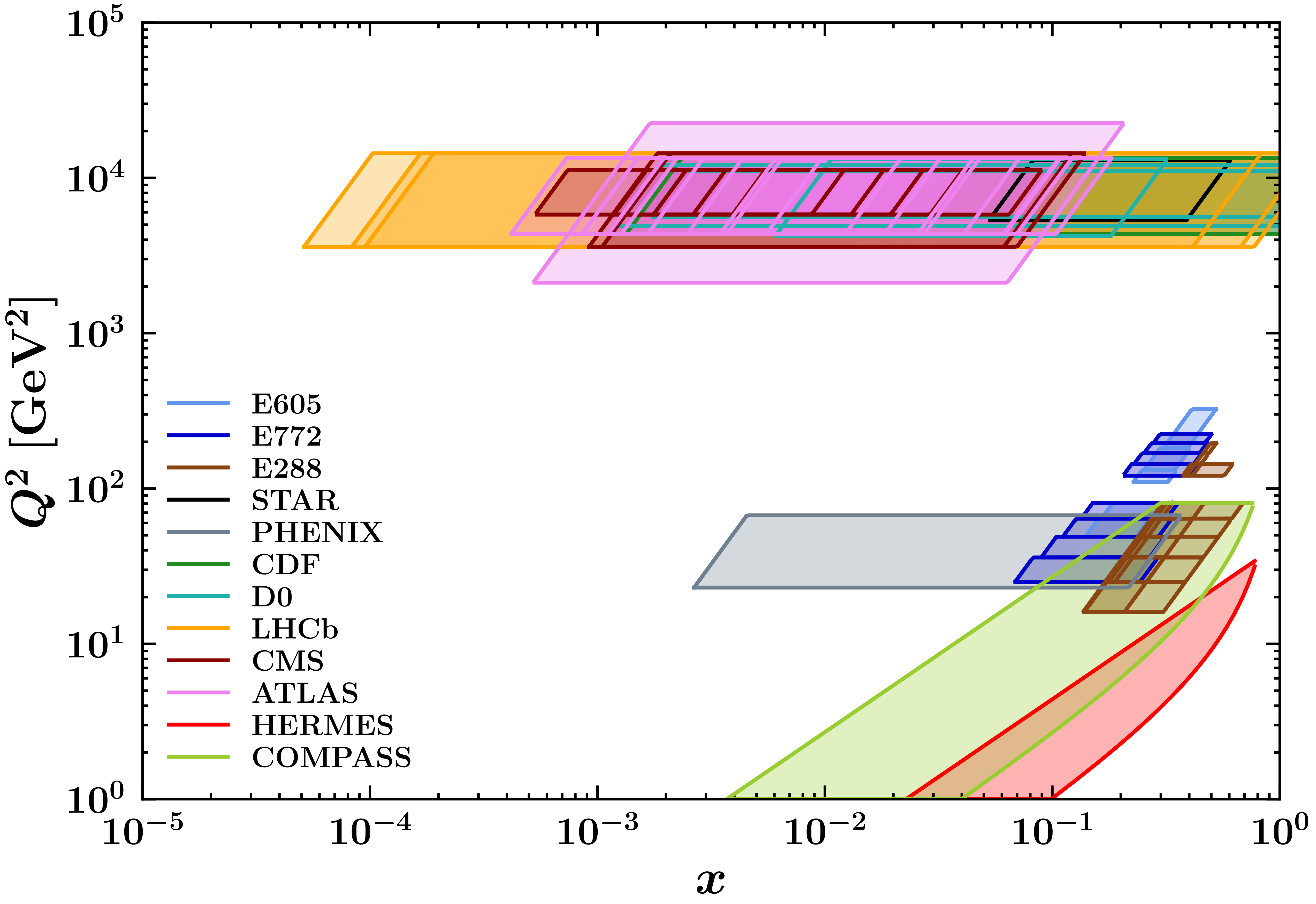
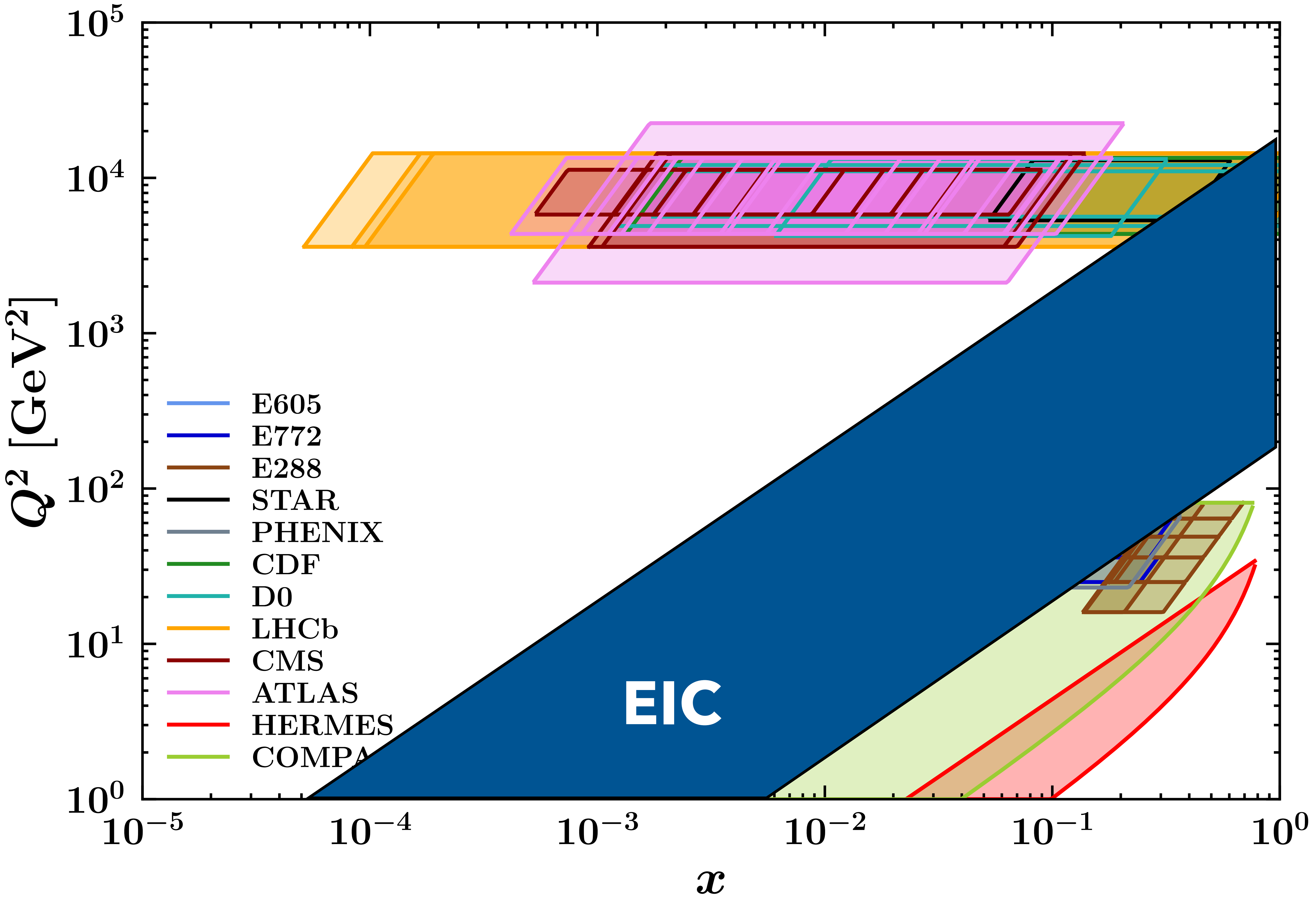
EIC IMPACT



Q^2 [GeV^2]
10^0
10^1
10^2
10^3
10^4
10^5
x
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
E605
E772
E288
STAR
PHENIX
CDF
D0
LHCb
CMS
ATLAS
HERMES
COMPASS

EIC IMPACT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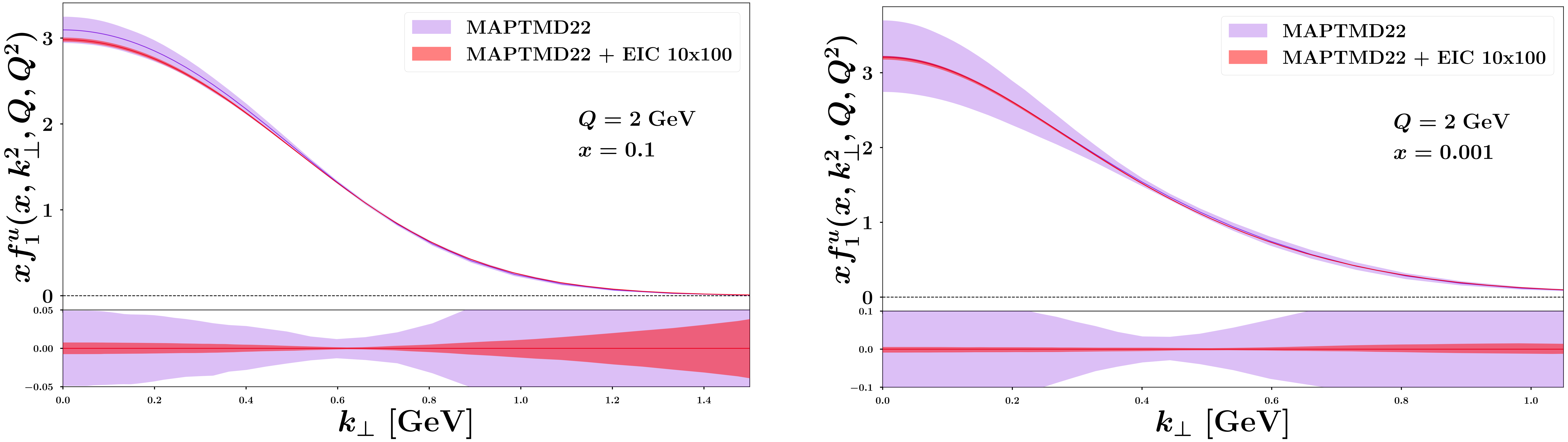

EIC

CONCLUSIONS

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CONCLUSIONS

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- Progress is ongoing concerning higher-twist and gluon TMDs (see Cristian Pisano's talk)
- Extractions of unpolarized TMDs are reaching a good level of sophistication, but there are still several open questions
- For other TMDs, the study has barely started

BACKUP

CHOICES OF FUNCTIONAL FORMS

MAP22

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_T^2}{g_1}} + \lambda^2 k_T^2 e^{-\frac{k_T^2}{g_{1B}}} + \lambda_2^2 e^{-\frac{k_T^2}{g_{1C}}} \right)$$

$$g_K(b_T^2) = -\frac{g_2^2}{2} b_T^2$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \ x^\sigma}{(1-\hat{x})^\alpha \ \hat{x}^\sigma}$$

11 parameters for TMD PDF
+ 1 for NP evolution +9 for FF
= 21 free parameters

SV19

$$f_{NP}(x, b) = \exp\left(-\frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4} \boldsymbol{b}^2}} \boldsymbol{b}^2 \right)$$

$$g_K(b_T^2) = -2c_0 \frac{b_T^2}{1 + b_T^2/B_{\mathrm{NP}}^2}$$

5 parameters for TMD PDF
+ 2 for NP evolution +5 for FF
= 11 free parameters

ART23

$$f^f_{NP}(x, b) = \frac{1}{\cosh\left(\left(\lambda_1^f(1-x) + \lambda_2^f x\right) b\right)}$$

$$\mathcal{D}_{\mathrm{NP}}(b) = b b^* \left[c_0 + c_1 \ln\left(\frac{b^*}{B_{\mathrm{NP}}} \right) \right]$$

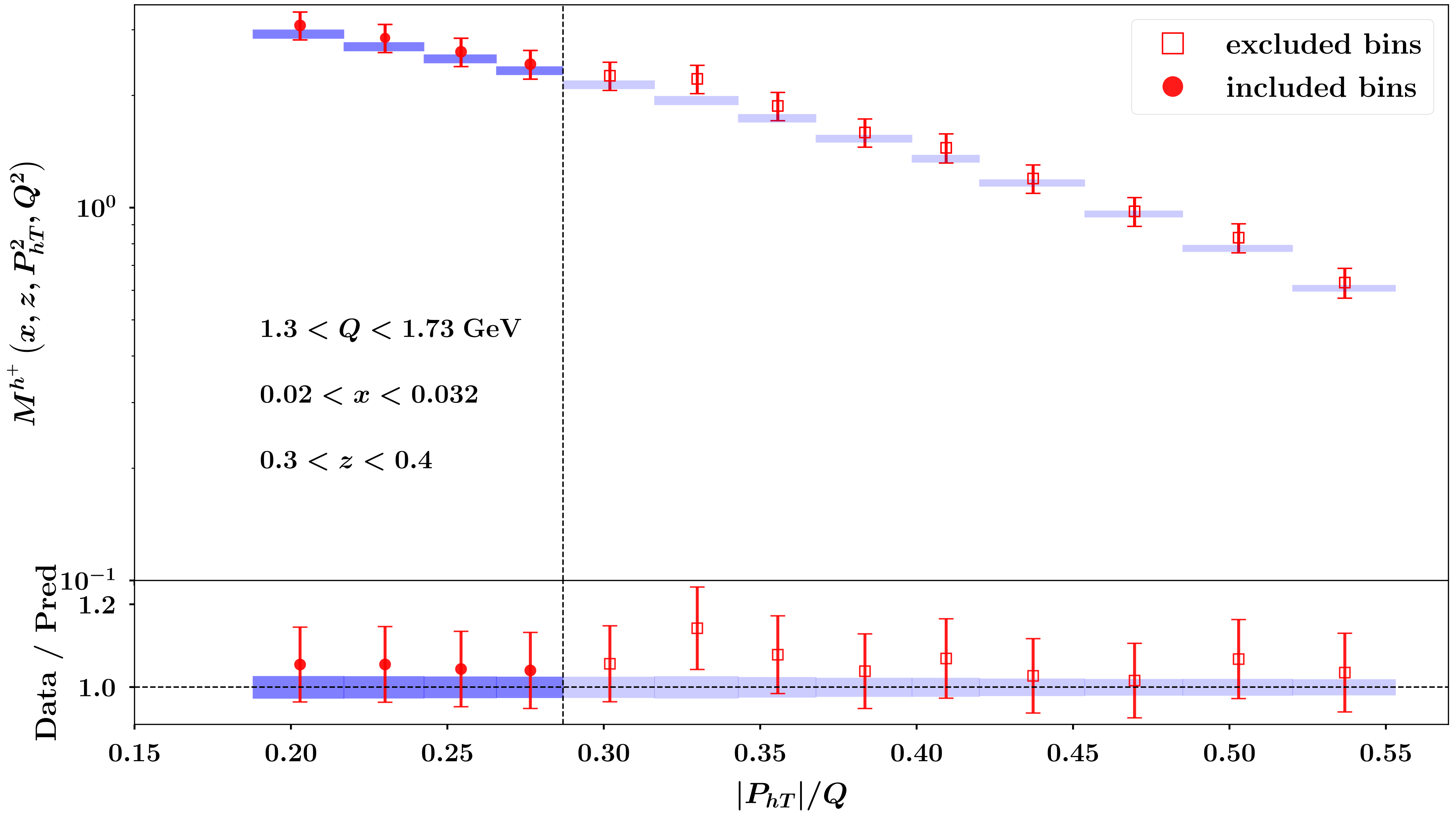
2401.14266

$$f^{\mathrm{Spect}}_{\mathrm{core},i/p}(x, \boldsymbol{k}_\mathrm{T}; Q_0^2) = \frac{1}{\pi} \frac{6\,L^6}{L^2 + 2(m_q + x\,M_p)^2} \frac{k_\mathrm{T}^2 + (m_q + x\,M_p)^2}{(k_\mathrm{T}^2 + L^2)^4}$$

10 parameters for TMD PDF
+ 3 for NP evolution
= 13 free parameters

REGION OF VALIDITY OF TMD FORMALISM

$$|q_T| = |P_{hT}|/z \ll Q$$



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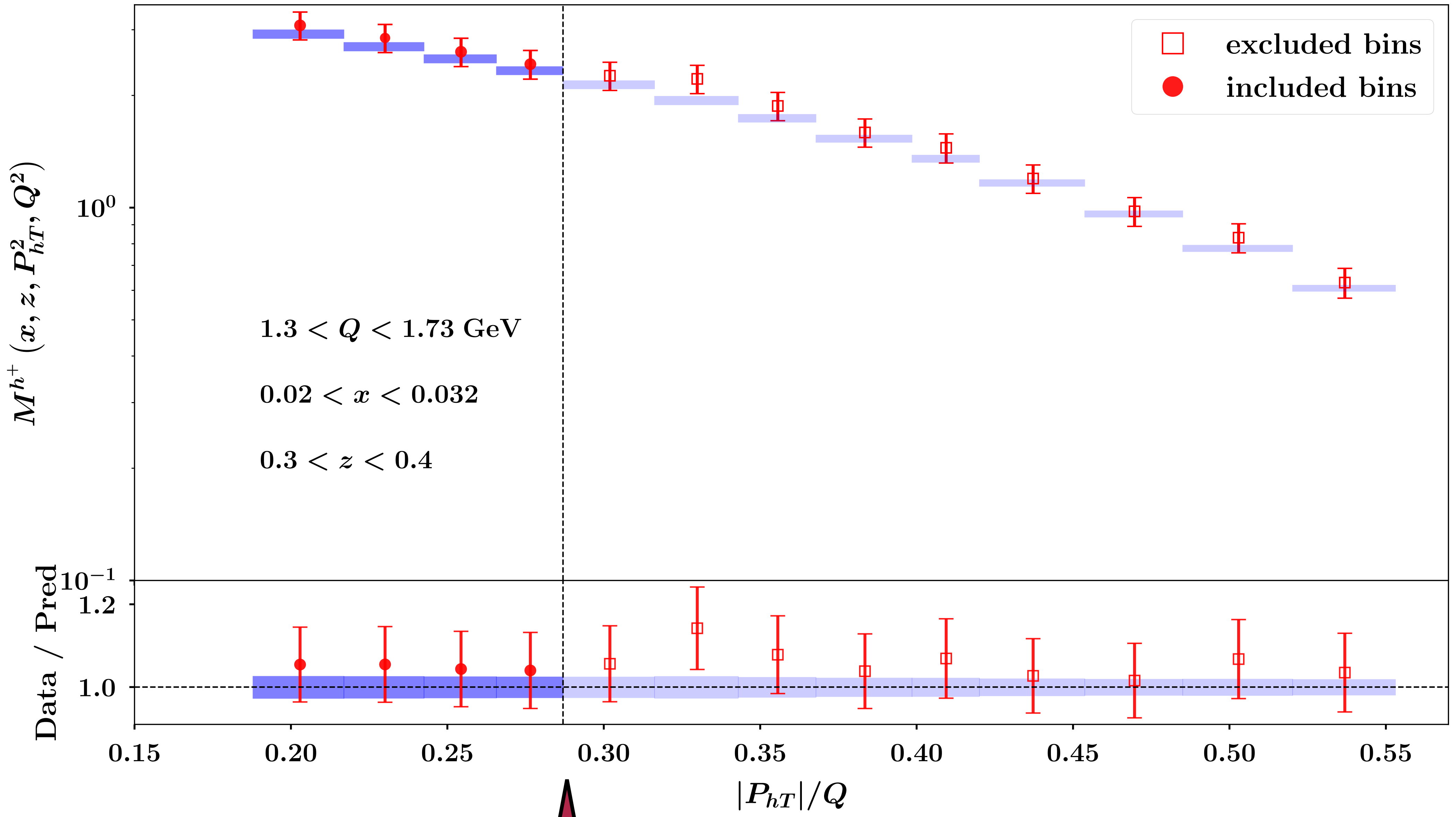

MAP22 cut

REGION OF VALIDITY OF TMD FORMALISM

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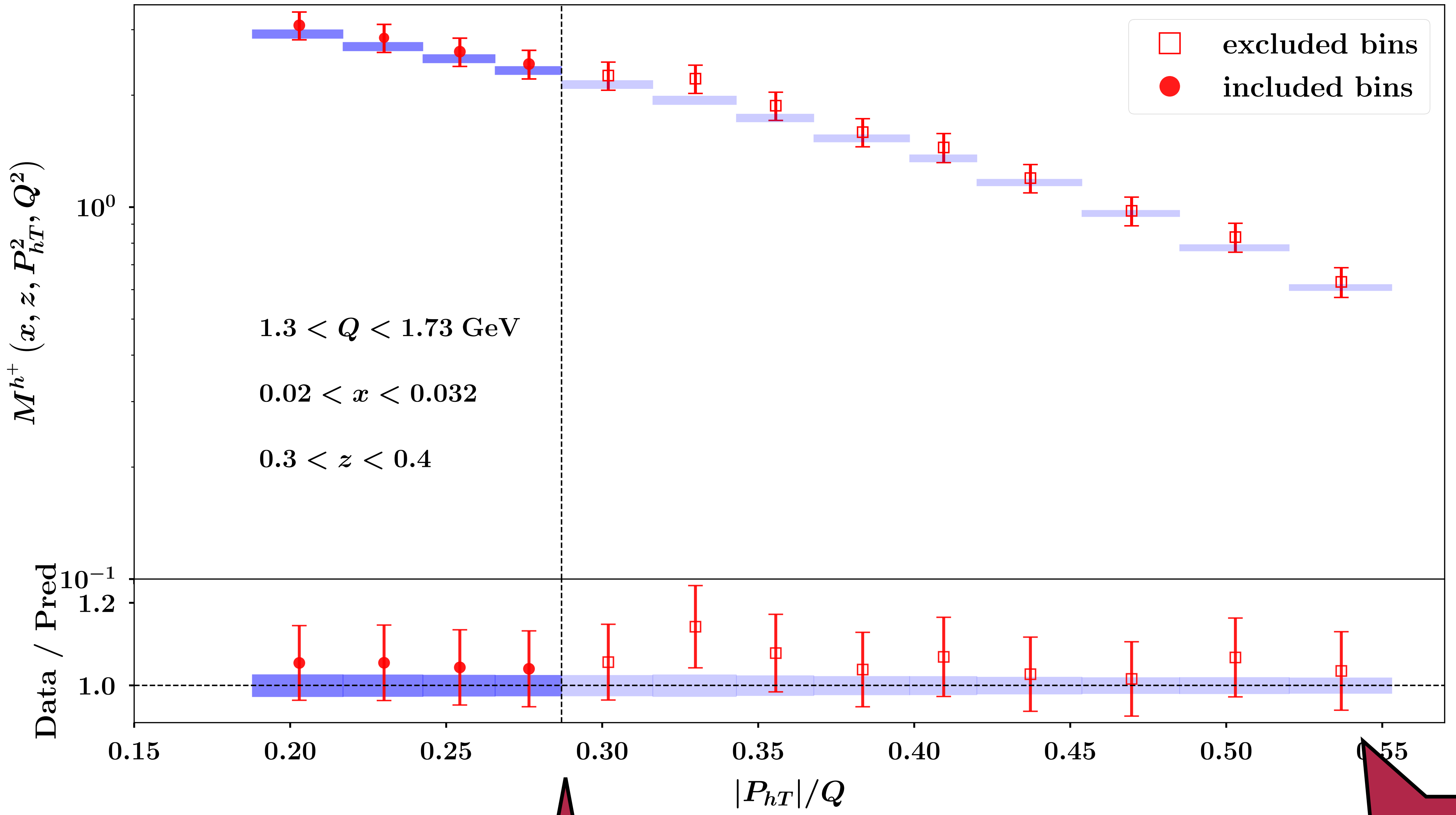

MAP22 cut

MAP22 extrapolation

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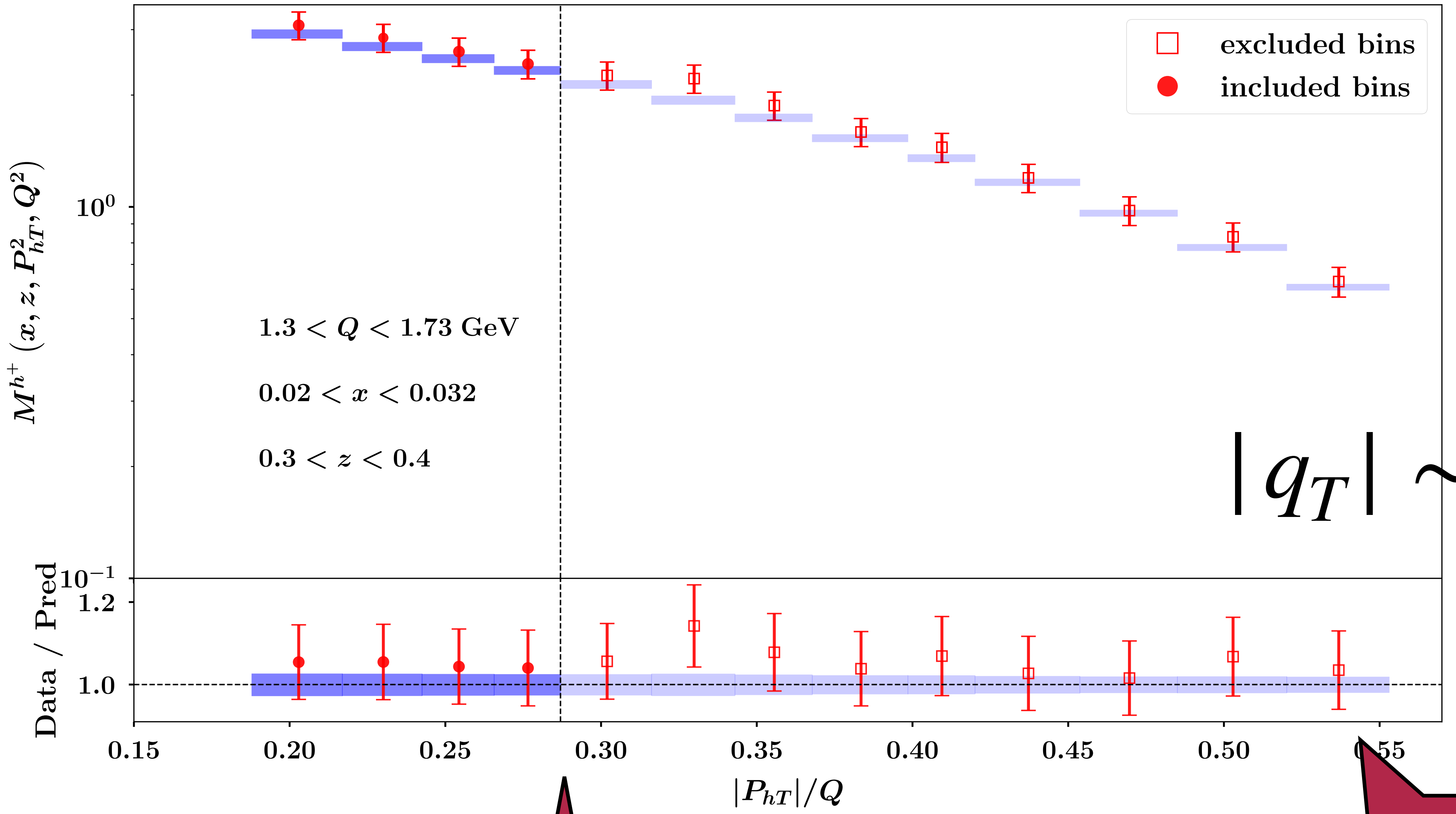

$$|q_T| \sim 1.5\ Q$$!

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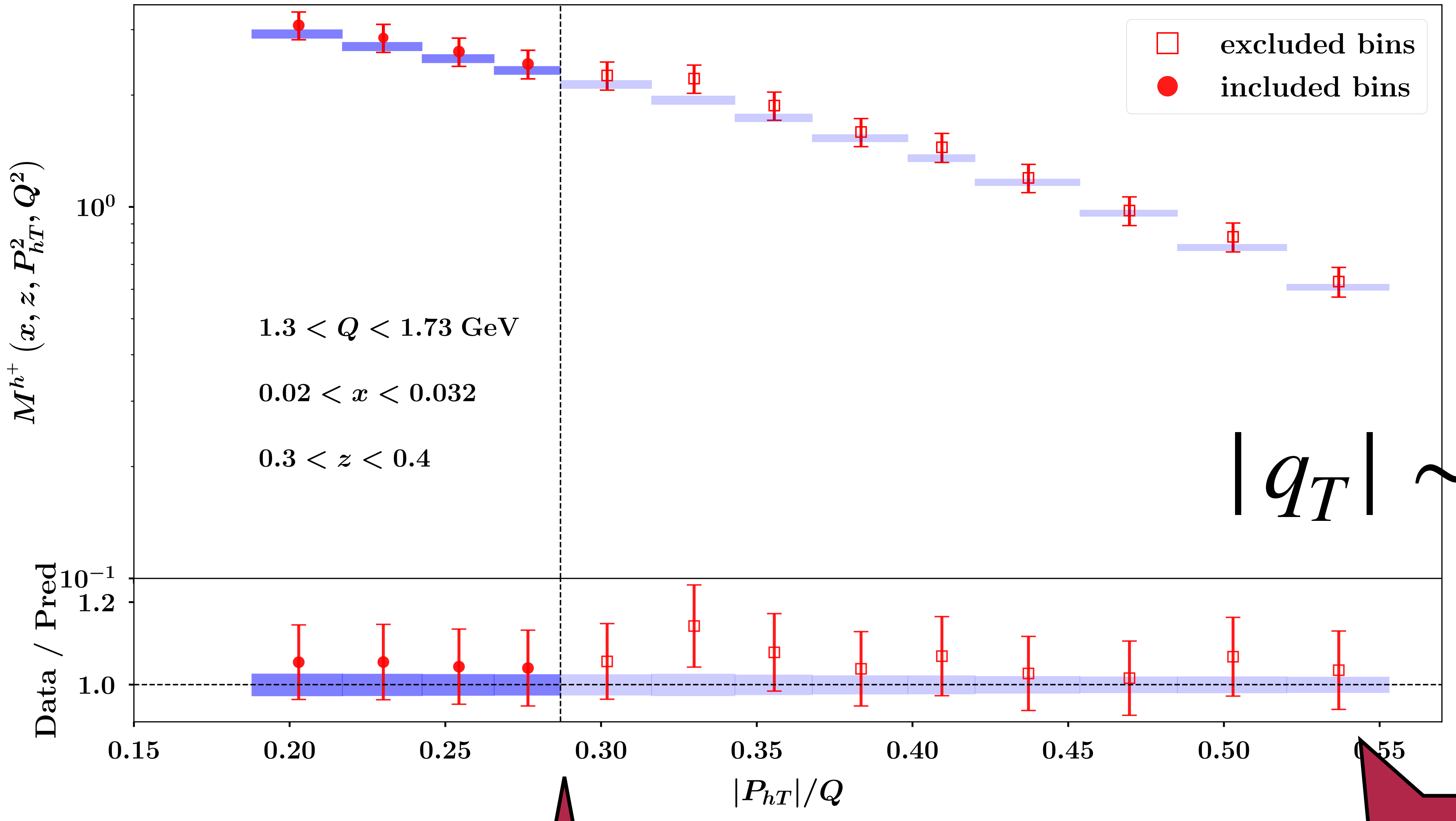

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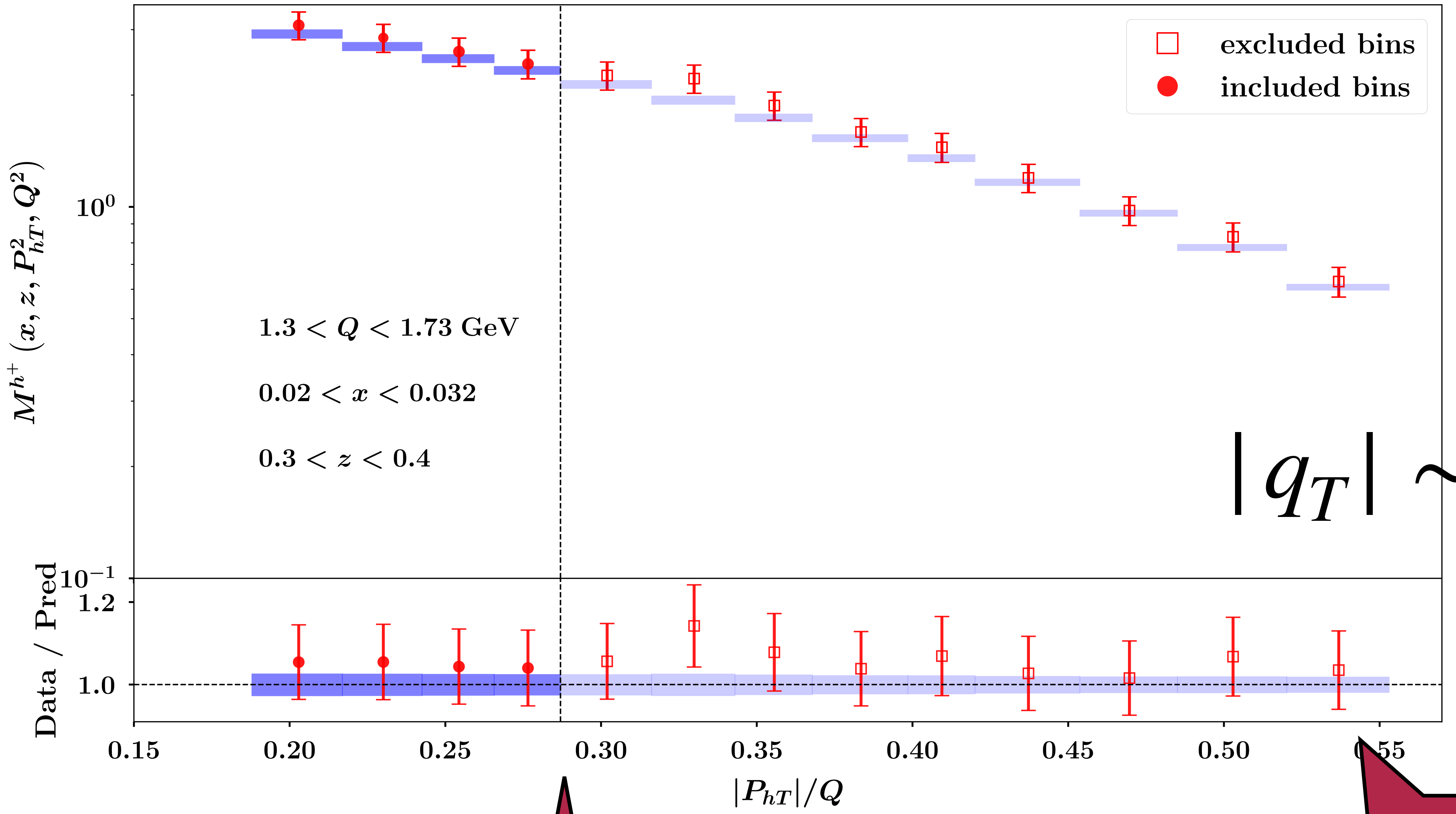

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LOW-b_T MODIFICATIONS

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

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$$b_*(b_c(b_\mathrm{T})) = \sqrt{\frac{b_\mathrm{T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

$$b_\mathrm{min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

Collins et al.
arXiv:1605.00671

PAVIA 2019 b_* PRESCRIPTION

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

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These are all choices that should be at some point checked/challenged

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$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i \left(C_{qi} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^q(x, b_T)$$

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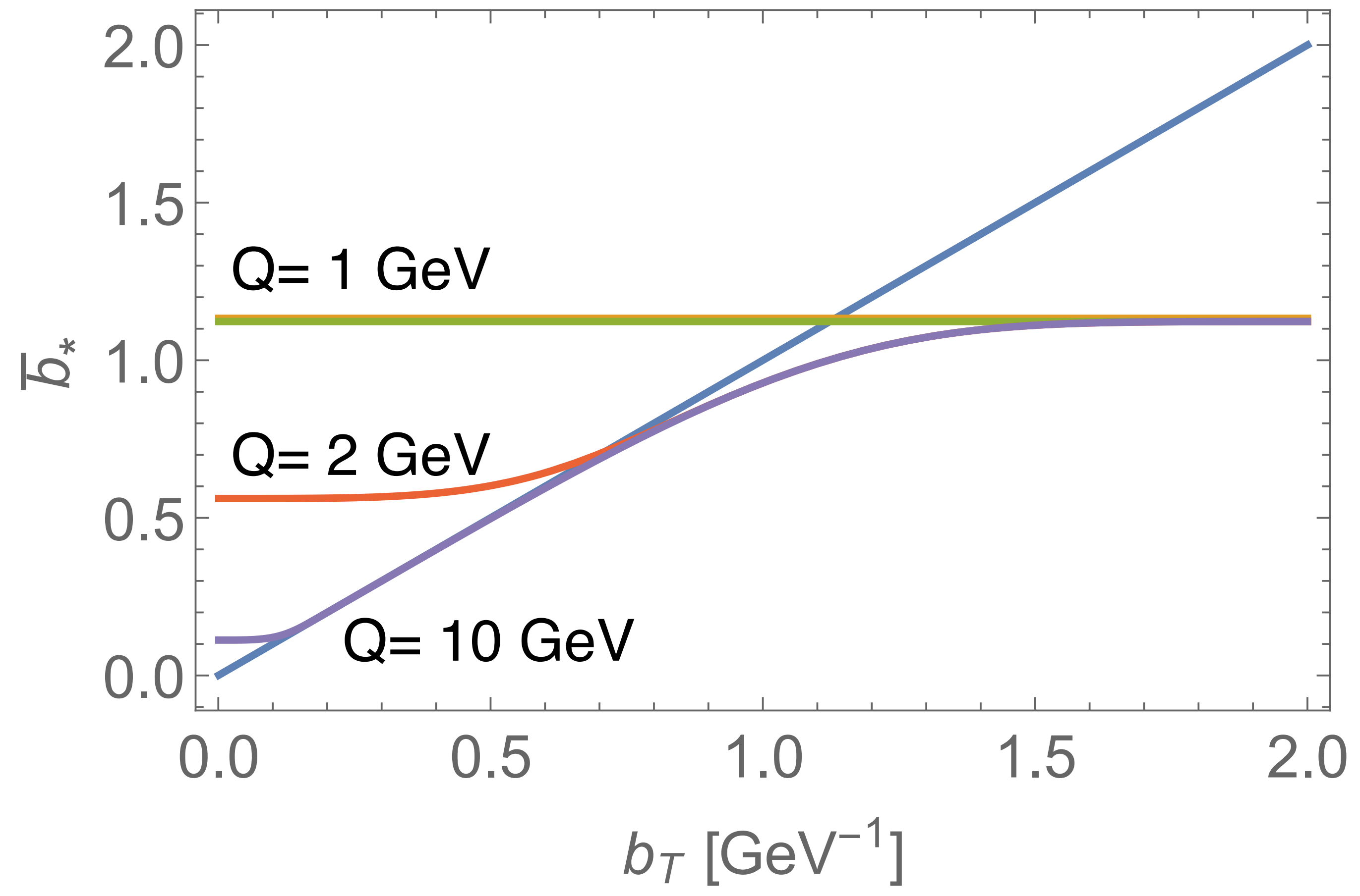

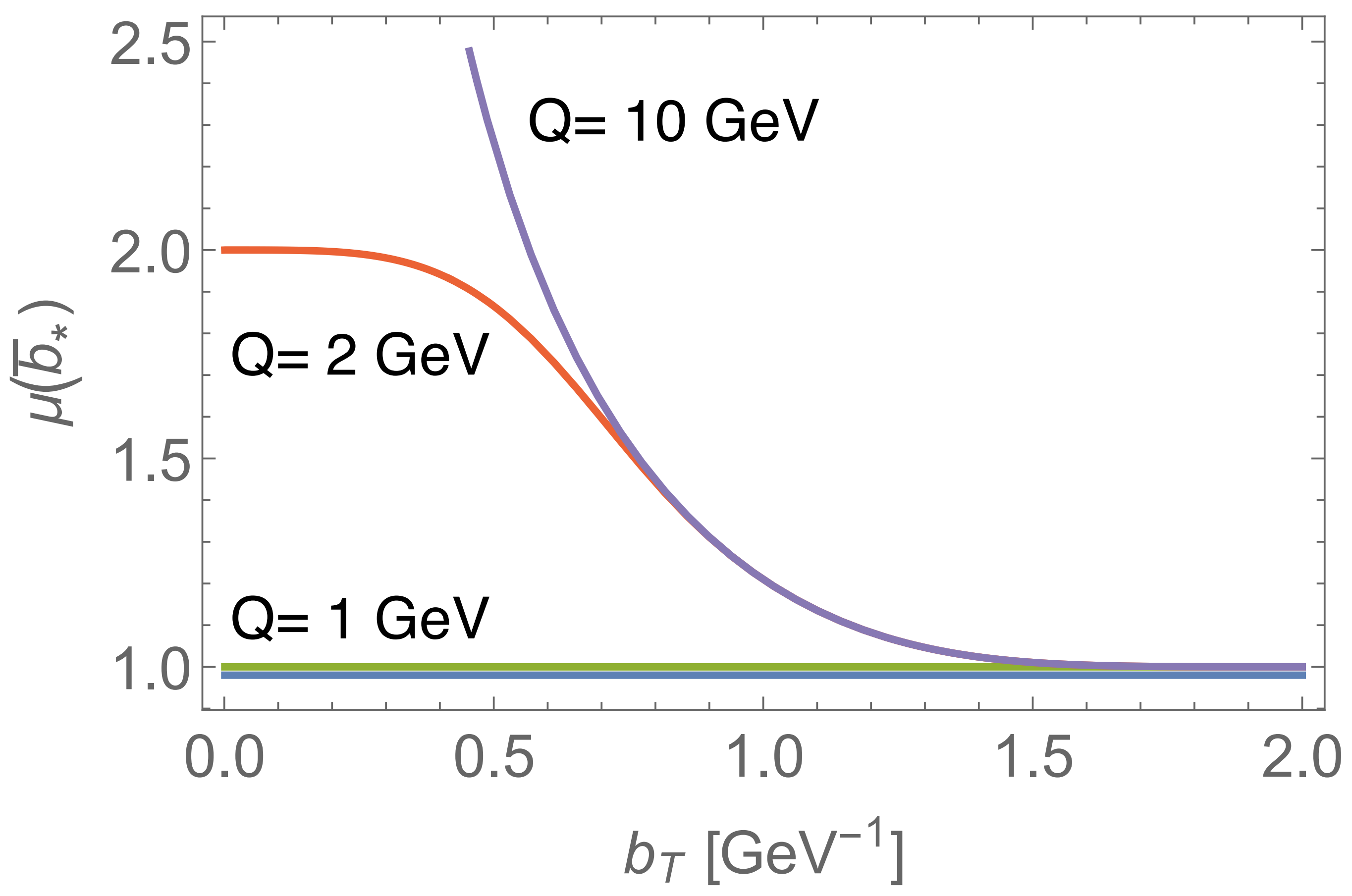

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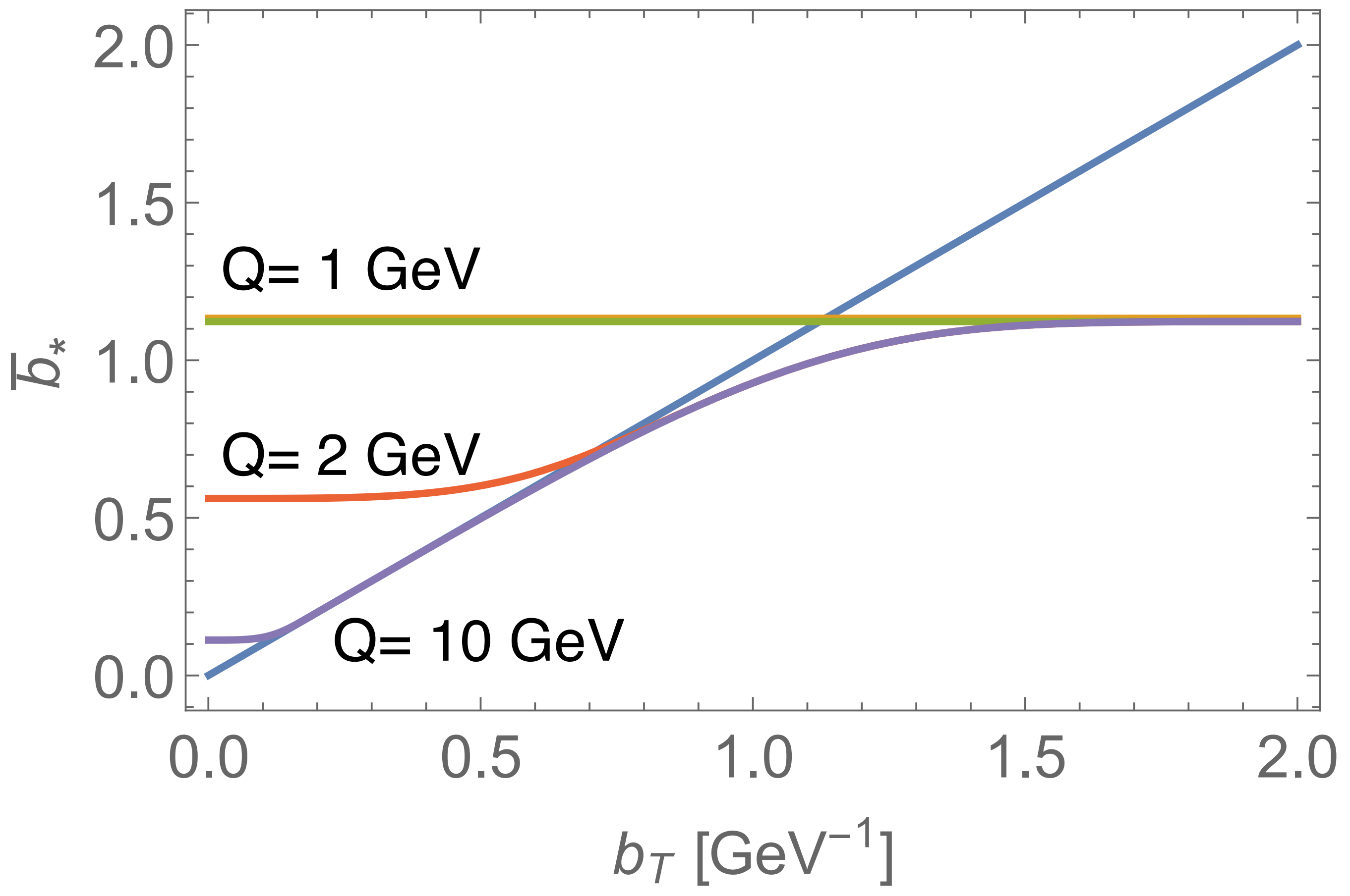

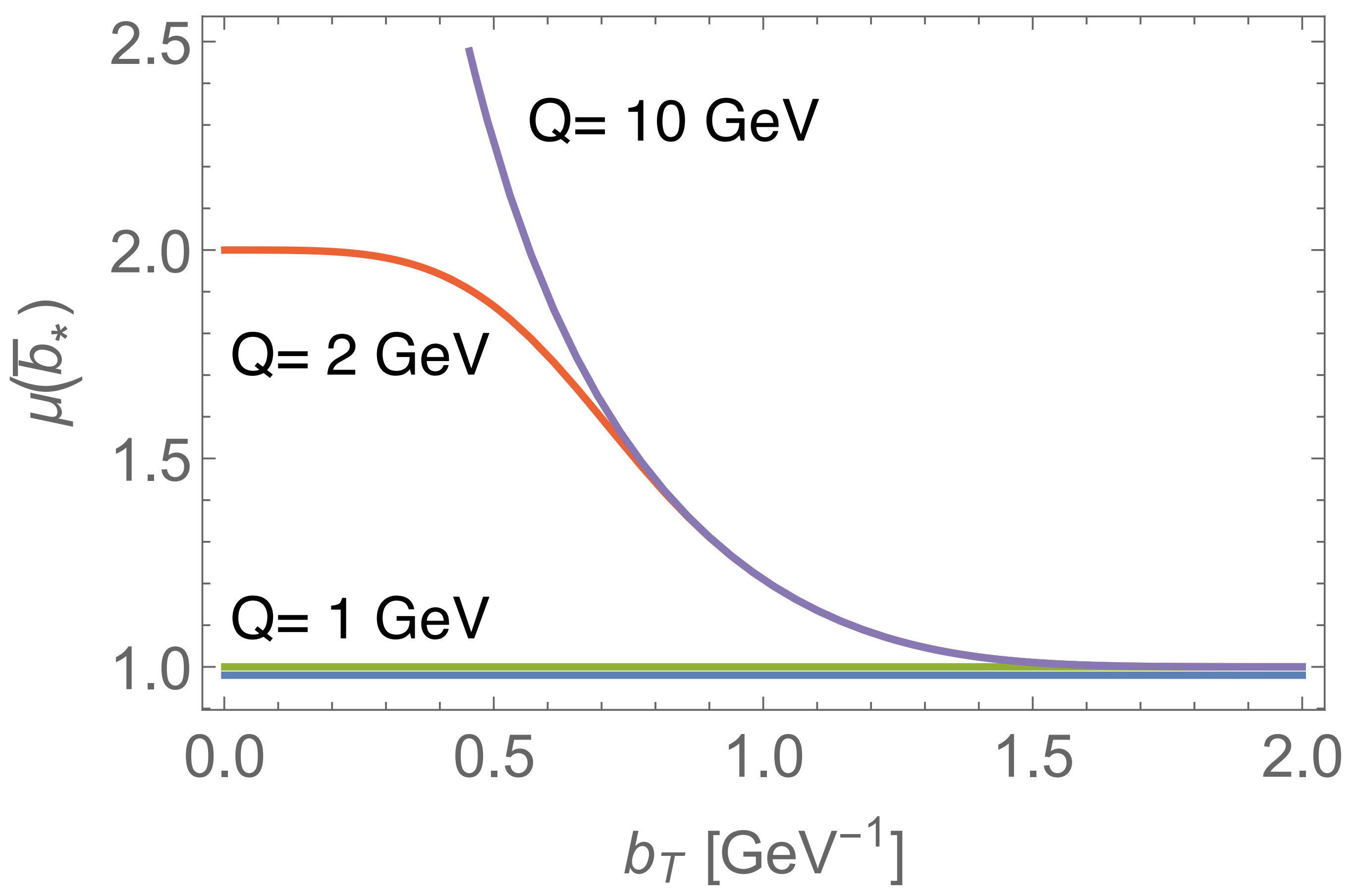

No significant effect at high Q, but large effect at low Q (inhibits perturbative contribution)