Non-perturbative models of the Quark-Gluon Plasma

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A brief history

Alexander Soloviev

Edmond Iancu

Florian Preis
Stefan Stricker
Anton Rebhan

Aleksi Kurkela

Souvik Banerjee

Christian Ecker

Navaneeth Gaddam

Sukrut Mondkar

Toshali Mitra

See also my work on non-Fermi liquids

Giuseppe Policastro
Benoit Doucot
Sutapa Samanta

Based on

- Edmond Iancu and AM, 1410.6448 [IM]
- AM, Florian Preis, Stefan Stricker and Anton Rebhan, 1512.06445 [MPRS]
- Souvik Banerjee, Nava Gaddam and AM, 1701.01229 [BGM]
- Aleksi Kurkela, AM, Florian Preis, Anton Rebhan and Alexander Soloviev, 1805.05213 [KMPRS]
- Christian Ecker, AM, Florian Preis, Anton Rebhan and Alexander Soloviev, 1806.01850 [EMPRS]
- Toshali Mitra, Sukrut Mondkar, AM, Anton Rebhan and Alexander Soloviev, 2006.09383 [MMMRS]
- Ongoing works

Plan of the talk

- Introduction
- Principles of incorporating both weakly and strongly interacting degrees of freedom
- Consistency checks
- Applications 1: Thermodynamics
- Applications 2: Simulating HIC and the toy non-perturbative glasma
- Applications 3: Hydrodynamic attractors with hybrid degrees of freedom
- Applications 4: The Christmas tree with a cross
- Conclusions
- Outlook

Introduction

“But when the strong were too weak to hurt the weak, the weak had to be strong enough to leave”

Milan Kundera

The QGP is the probably one of the most complex and perplexing phases of matter. Understanding it may reveal the secrets of gauge theories.

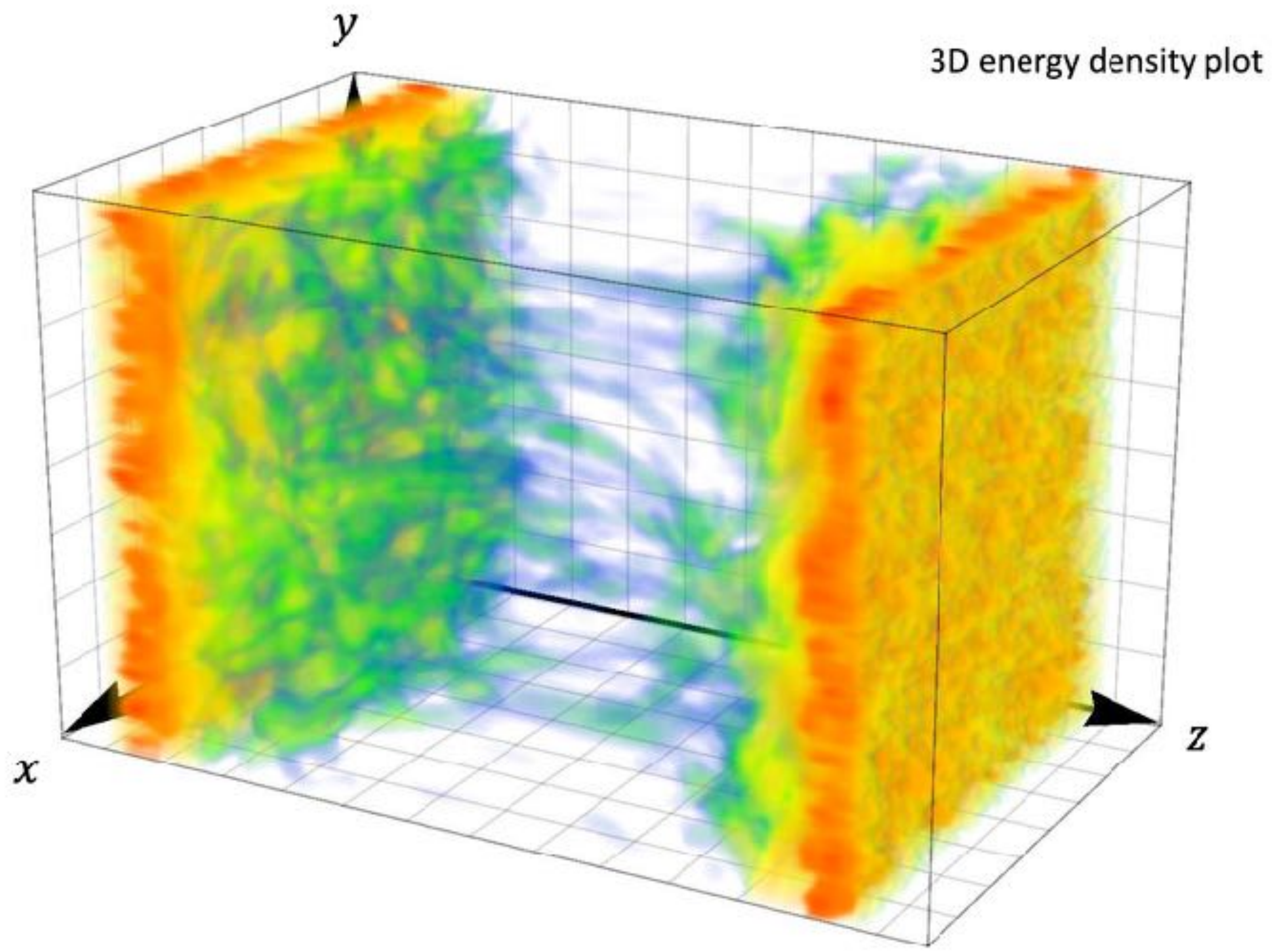
The low pT hadrons tell us that it is a strongly coupled droplet of fluid that forms incredibly rapidly (almost at the time it takes light to traverse the system). Qualitative features including the astonishingly low η/s can be understood by the holographic duality of string theory.

Lattice results do tell us that at temperatures $T_c < T < 2T_c$ the medium cannot be understood perturbatively and the degrees of freedom are complex.

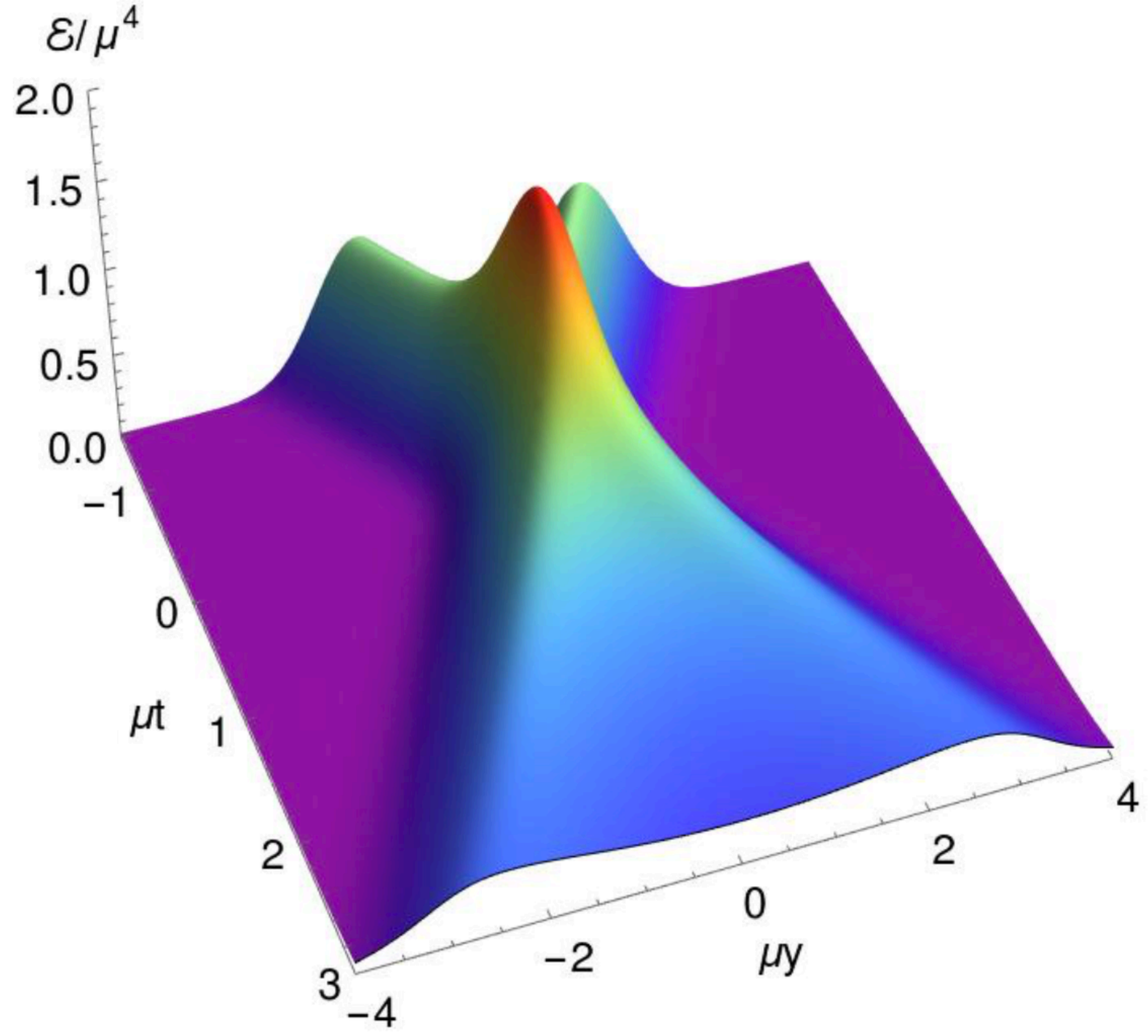
Yet very high pT jets (more collimated hadrons) can be described by perturbation theory and very badly fit holographic models. No known way to describe the intermediate pT hadrons even coarsely.

Most existing approaches either take either the exclusively strong coupling or weak coupling paradigms and focuses more on the initial/later parts of the system. However many observables like high pT jets and quarkonia can see the entire evolution. (See talks by Santosh Das, Konrad Tywonuik and Bin Wu)

Only a good theoretical framework that can combine both weakly and strongly interacting degrees of freedom can tell us how much of nonperturbative QCD we can really learn from the QGP. (See talks by Sören Schlichting, Mike Strickland and Paul Romatschke)



From David Müller’s talk in Initial Stages 2017 at Krakow



Christian Ecker’s PhD thesis

Principles

Basic Premise

The quantum effective action of QCD takes the following form at any energy scale

$$S[A_\mu] = -\frac{1}{4}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \;+ \text{perturbative corrections (Feynman diagrams)}$$

$$+ \text{non} - \text{perturbative corrections (beyond Feynman diagrams)}$$

Hypothesis: Non-perturbative corrections originate from a strongly coupled holographic theory

In the large N limit

$$\text{Non}-\text{perturbative terms} \quad = \ln Z_h[J_A = J_A[A_\mu]]$$

Partition Function of the strongly coupled theory

$$= S_{grav}[J_A = J_A[A_\mu]]$$

On-shell action of the dual classical gravity theory living in one higher dimension

J_A : **Sources for appropriate operators in the strongly coupled theory ("boundary values" of dual fields in the dual classical gravity)**

$$\Phi_A \leftrightarrow O_A \qquad \Phi_A(r,x) \approx J_A(x) r^{4-\Delta} + \cdots + O_A(x) r^{\Delta} + \cdots$$

Expectation value of the operator

Crucially, the cutoff of the holographic theory and the scale-dependent functionals $J_A[A_\mu]$ should be determined by the perturbation theory.

Major inspiration: The renormalons of the Borel summation of pQCD determine the vacuum expectation values of operators (non-perturbative data). The latter can determine the appropriate classical gravity theory and its coupling to perturbative sector. (A toy construction in BGM.)

This talk is not about such a derivation but rather about

(i) realizing such a setup and performing consistency checks

(ii) exploring phenomenological consequences for the QGP

Rules of Coupling - Issues

The primary question is how should we couple the perturbative and non-perturbative sectors?

A coupling such as

$$\gamma \int t_{\mu\nu} \tilde{t}^{\mu\nu}$$

is problematic because it spoils renormalizability of the full theory. (We claim we can construct effective non-perturbative dynamics at any scale.)

Also eg in hydrodynamic limit the full em-tensor cannot be obtained just from the hydrodynamics of the subsectors. The full nonperturbative dynamics should be obtained from just the effective perturbative dynamics at any scale.

Note the full em-tensor will involve operators which are not just the em-tensors of the subsectors but also $\mathrm{tr}(F_{\mu\alpha}F^{\alpha}_{\ \nu})$ **etc explicitly with no known way to parametrise their expectation values in terms of hydro variables without more microscopic information.**

Not a problem though if we are studying a mixture of water and alcohol but here it is inconsistent with Wilsonian RG.

Rules of Coupling - Be Democratic!

We postulate the following coupling scheme [BGM, KMPRS].

The relevant and marginal couplings and background metrics of the subsystems are promoted to algebraic functions of the operators of their complements *democratically*. Examples

$$\tilde{g}_{YM}[\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}), \mathrm{tr}(t_{\mu\nu}g^{\mu\nu}), \ldots] \qquad g_{YM}[\mathrm{tr}(\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}), \mathrm{tr}(\tilde{t}_{\mu\nu}\tilde{g}^{\mu\nu}), \ldots]$$

$$\tilde{g}_{\mu\nu}[\mathrm{tr}(t^{\alpha\beta}), \ldots] \qquad g_{\mu\nu}[\mathrm{tr}(\tilde{t}^{\alpha\beta}), \ldots]$$

It implies that although the two systems are isolated and self-consistent. Each individual sector is still renormalizable. The full dynamics is obtained by solving both systems simultaneously and self-consistently.

What determines these functions?

Simple: **There must exist a energy-momentum tensor of the full system that is a <u>polynomial</u> of the <u>relevant and marginal</u> single trace operators of the subsystems (with small anomalous dimensions) in the large N limit which should be conserved in the actual physical background metric.**

Also other conserved currents similarly.

Away from large N limit these functions also depend on the correlation functions of these marginal and relevant single operators.

One can implement this beautifully (as Alexander Soloviev showed) by promoting the couplings to auxiliary variables which can be determined from an augmented action [KMPRS].

EXAMPLE 1: Simple scalar coupling

$$S = S_1\left[\lambda^{-1} = \frac{1}{g_{YM}^2 N} + h_1\right] + S_2\left[\tilde{\lambda}^{-1} = \frac{1}{\tilde{g}_{YM}^2 N} + h_2\right] + \frac{1}{\beta}\int h_1 h_2$$

Varying we get:

$$h_1 = -\beta\frac{\delta S_1}{\delta h_2} = -\beta\frac{\delta S_1}{\delta\tilde{\lambda}^{-1}} = \frac{1}{4}\beta\mathrm{tr}(\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}) := \beta\,\widetilde{\mathscr{H}}$$

$$h_2 = -\beta\frac{\delta S_1}{\delta h_1} = -\beta\frac{\delta S_1}{\delta\lambda^{-1}} = \frac{1}{4}\beta\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) := \beta\mathscr{H}$$

Varying wrt the background metric and inserting the solutions we get that

$$T^{\mu\nu} = t^{\mu\nu} + \tilde{t}^{\mu\nu} - \beta \mathscr{H} \widetilde{\mathscr{H}} \eta^{\mu\nu}$$

Consistency check: Since each system is isolated they must satisfy individual Ward identities

$$\partial_\mu t^{\mu\nu} = \mathscr{H} \partial^\nu h_2 = \beta \mathscr{H} \partial^\nu \widetilde{\mathscr{H}}, \qquad \partial_\mu \tilde{t}^{\mu\nu} = \widetilde{\mathscr{H}} \partial^\nu h_2 = \beta \widetilde{\mathscr{H}} \partial^\nu \mathscr{H}$$

Together they do imply that

$$\partial_\mu T^{\mu\nu} = 0$$

EXAMPLE 2: **Simplest effective metric couplings**

$g^{(B)}_{\mu\nu}$:= **Physical background metric**

$$S = S_1\left[g_{\mu\nu}\right] + S_2\left[\tilde{g}_{\mu\nu}\right]$$

$$+\frac{1}{2\gamma}\int\sqrt{-g^{(B)}}\left(g_{\mu\alpha}-g^{(B)}_{\mu\alpha}\right)\left(\tilde{g}_{\nu\beta}-g^{(B)}_{\nu\beta}\right)g^{(B)\mu\nu}g^{(B)\alpha\beta}$$

$$+\frac{1}{2\gamma}\frac{\gamma'}{d\gamma'-\gamma}\int\sqrt{-g^{(B)}}\left(g_{\mu\alpha}g^{(B)\mu\alpha}-d\right)\left(\tilde{g}_{\nu\beta}g^{(B)\nu\beta}-d\right)$$

Vary the full action wrt $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ to obtain

$$g_{\mu\nu} = g^{(B)}_{\mu\nu} + \left(\gamma g^{(B)}_{\mu\alpha}\tilde{t}^{\alpha\beta}g^{(B)}_{\beta\nu} + \gamma' g^{(B)}_{\mu\nu}\tilde{t}^{\alpha\beta}g^{(B)}_{\alpha\beta}\right)\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}}$$

$$\tilde{g}_{\mu\nu} = g^{(B)}_{\mu\nu} + \left(\gamma g^{(B)}_{\mu\alpha}t^{\alpha\beta}g^{(B)}_{\beta\nu} + \gamma' g^{(B)}_{\mu\nu}t^{\alpha\beta}g^{(B)}_{\alpha\beta}\right)\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}}$$

Vary the full action wrt $g^{(B)}_{\mu\nu}$ and substitute solutions for $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ to obtain

$$T^{\mu}_{\nu} = \frac{1}{2}(t^{\mu}_{\nu} + t_{\nu}^{\mu})\sqrt{-g} + \frac{1}{2}(\tilde{t}^{\mu}_{\nu} + \tilde{t}_{\nu}^{\mu})\sqrt{-\tilde{g}} + \Delta K \delta^{\mu}_{\nu}$$

$$\Delta K = \frac{\gamma}{2}\left(t^{\mu\alpha}\sqrt{-g}\right) g^{(B)}_{\alpha\beta}\left(\tilde{t}^{\beta\nu}\sqrt{-\tilde{g}}\right) g^{(B)}_{\mu\nu} + \frac{\gamma'}{2}\left(t^{\alpha\beta}\sqrt{-g}\right) g^{(B)}_{\alpha\beta}\left(\tilde{t}^{\mu\nu}\sqrt{-\tilde{g}}\right) g^{(B)}_{\mu\nu}$$

Note indices of full system and subsystem variables are lowered/raised using $g^{(B)}_{\mu\nu}$, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively.

<u>Consistency check:</u> For each (isolated) subsystem, there are individual Ward identities

$$\nabla_\mu t^\mu_\nu = 0, \qquad \tilde{\nabla}_\mu \tilde{t}^\mu_\nu = 0$$

These together imply that

$$\nabla^{(B)}_\mu T^\mu_\nu = 0$$

When both systems are solved self-consistently, the total energy momentum tensor is conserved.

Furthermore, the full em-tensor is a polynomial of marginal/relevant subsystem operators — $t^{\mu\nu}\sqrt{-g},\ \tilde{t}^{\mu\nu}\sqrt{-\tilde{g}},\ \cdots$

Infinite number of scalar, vector and tensor (effective) couplings are possible and are most likely necessary so that the expansion in coupling of the full system is meaningful.

All scalar and tensor couplings have been classified [BGM, KMPRS].

For practical applications we advocate only lowest order couplings. We should also view this as an effective theory at a given scale (Q_s for application to HIC).

Justification: The mutual coupling between the two systems (measured with respect to the subsystem energies) must be weak. If the mutual coupling is infinite, the full system is a conformal theory even if the subsystems are not conformal (no analytical proof but our computations of thermodynamics, hydrodynamic modes, hybrid quasinormal modes, etc point out to this).

Consequences of democratic coupling

The rules of the coupling are universal and consistent. So we can proceed without knowing the actions of the subsystems explicitly.

We can use this coupling to couple:
(i) glasma (kinetic theory) to a black hole
(ii) two dissipative fluids

The full em-tensor can be computed explicitly and will be conserved automatically when the subsystems are solved self-consistently

Note that the variables of the two systems are NOT independent. However, in the context of application to HIC, we will not derive the holographic sector from pQCD but simply assume it to be 5D Einstein gravity (plus a few matter fields) with a negative cosmological constant. We can also choose a bottom-up hQCD model.

Also on the holographic side we will start with empty AdS5 (can be numerically achieved!)

So the entire evolution will be described by initial perturbative data alone. The couplings γQ_s^4, $\gamma' Q_s^4$, βQ_s^4, etc are then free parameters of the effective theory.

Self-consistent solution can be achieved iteratively. This has been confirmed numerically.

Are we making sense?

Consistency with thermodynamics and statistical mechanics

Can we describe the full system at finite temperature consistently?

If the full system is at equilibrium, so should be the subsystems at temperatures T_1 and T_2 respectively.

Since finite temperature breaks boost invariance and we must have a single thermal frame in equilibrium, we must have

$$g_{\mu\nu} = \mathrm{diag}(-a^2, b^2, b^2, b^2)$$
$$\tilde{g}_{\mu\nu} = \mathrm{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{b}^2)$$

Equations of state of subsystems $P_1(\epsilon_1)$ and $P_2(\epsilon_2)$ are given. We can obtain $T_1,\ s_1,\ T_2,\ s_2$ using the thermodynamic identities

$$\epsilon_1 + P_1 = T_1 s_1 \qquad d\epsilon_1 = T_1 ds_1 \qquad\qquad \epsilon_2 + P_2 = T_2 s_2 \qquad d\epsilon_2 = T_2 ds_2$$

Of course

$$t^{\mu\nu} = \mathrm{diag}\left(\frac{\epsilon_1(T_1)}{a^2}, \frac{P_1(T_1)}{b^2}, \frac{P_1(T_1)}{b^2}, \frac{P_1(T_1)}{b^2}\right), \qquad \tilde{t}^{\mu\nu} = \mathrm{diag}\left(\frac{\epsilon_2(T_2)}{\tilde{a}^2}, \frac{P_2(T_2)}{\tilde{b}^2}, \frac{P_2(T_2)}{\tilde{b}^2}, \frac{P_2(T_2)}{\tilde{b}^2}\right)$$

Coupling equations at leading order then reduce to:

$$1-a^2 = \left(\gamma\frac{\epsilon_2(T_2)}{\tilde{a}^2} - \gamma'\left(3\frac{P_2(T_2)}{\tilde{a}^2} - \frac{\epsilon_2(T_2)}{\tilde{a}^2}\right)\right)\tilde{a}\tilde{b}^3, \qquad b^2-1 = \left(\gamma\frac{P_2(T_2)}{\tilde{a}^2} + \gamma'\left(3\frac{P_2(T_2)}{\tilde{a}^2} - \frac{\epsilon_2(T_2)}{\tilde{a}^2}\right)\right)\tilde{a}\tilde{b}^3$$

$$1-\tilde{a}^2 = \left(\gamma\frac{\epsilon_1(T_1)}{a^2} - \gamma'\left(3\frac{P_1(T_1)}{a^2} - \frac{\epsilon_1(T_1)}{a^2}\right)\right)ab^3, \qquad \tilde{b}^2-1 = \left(\gamma\frac{P_1(T_1)}{a^2} + \gamma'\left(3\frac{P_1(T_1)}{a^2} - \frac{\epsilon_1(T_1)}{a^2}\right)\right)ab^3$$

Global equilibrium requires that the periodicity of the thermal circle is same in both subsystems, i.e.

$$T_1 a = T_2 \tilde{a} = \mathscr{T}$$

Clearly given $\mathscr{T}$ we can find a unique solution to the full system by solving a, b and $\tilde{a}, \tilde{b}$

The full em-tensor is $T^{\mu}_{\nu} = \mathrm{diag}(\mathscr{E}(\mathscr{T}), \mathscr{P}(\mathscr{T}), \mathscr{P}(\mathscr{T}), \mathscr{P}(\mathscr{T}))$

$$\mathscr{E} = \epsilon_1 a b^3 + \epsilon_2 \tilde{a}\tilde{b}^3 + \left(\frac{\gamma}{2} \left(\frac{\epsilon_1}{a^2} \frac{\epsilon_2}{\tilde{a}^2} + 3 \frac{P_1}{b^2} \frac{P_2}{\tilde{b}^2} \right) + \frac{\gamma'}{2} \left(-\frac{\epsilon_1}{a^2} + 3 \frac{P_1}{b^2} \right) \left(-\frac{\epsilon_2}{\tilde{a}^2} + 3 \frac{P_2}{\tilde{b}^2} \right) \right) a b^3 \tilde{a} \tilde{b}^3$$

$$\mathscr{P} = \epsilon_1 a b^3 + \epsilon_2 \tilde{a}\tilde{b}^3 - \left(\frac{\gamma}{2} \left(\frac{\epsilon_1}{a^2} \frac{\epsilon_2}{\tilde{a}^2} + 3 \frac{P_1}{b^2} \frac{P_2}{\tilde{b}^2} \right) + \frac{\gamma'}{2} \left(-\frac{\epsilon_1}{a^2} + 3 \frac{P_1}{b^2} \right) \left(-\frac{\epsilon_2}{\tilde{a}^2} + 3 \frac{P_2}{\tilde{b}^2} \right) \right) a b^3 \tilde{a} \tilde{b}^3$$

Where is the issue? Well given $\mathscr{E}$ and $\mathscr{P}$ we can define $\mathscr{T}$ and $\mathscr{S}$ uniquely via TIs

$$\mathscr{E} + \mathscr{P} = \mathscr{T}\,\mathscr{S}\,, \qquad d\mathscr{E} = \mathscr{T}\,d\mathscr{S}$$

But of course in eqlbm

$$\mathscr{T} = T_1 a = T_2 \tilde{a}$$

Thermodynamic consistency (TC)

Also since the two subsystems are self-consistent isolated systems, the entropy of the full system must be the sum of the two, i.e. we must have

$$\mathscr{S} = s_1 b^3 + s_2 \tilde{b}^3$$

Statistical consistency (SC)

Are TC and SC compatible with the TIs? If so the full system is consistent.

<u>Theorem:</u> For ANY democratic effective metric coupling the full system is thermodynamically and statistically consistent. [KMPRS]

<u>Proof:</u> **We note from the explicit expressions and assuming TC and SC we get**

$$\mathscr{E} + \mathscr{P} = (\epsilon_1 + P_1)ab^3 + (\epsilon_2 + P_2)\tilde{a}\tilde{b}^3 = (T_1 s_1)ab^3 + (T_2 s_2)\tilde{a}\tilde{b}^3 = \mathscr{T}(s_1 b^3 + s_2 \tilde{b}^3) = \mathscr{T}\mathscr{S}$$

Consider $g^{(B)}_{\mu\nu} = \mathrm{diag}(-e^{-\phi(\mathbf{x})},1,1,1)$. **In this case the individual system WIs reduce to Euler equations which automatically imply P = S dT given the first TI.**

Now if individual systems satisfy WIs so will the full system, therefore we must have $d\mathscr{P} = \mathscr{S}d\mathscr{T}$ **for full system since first TI holds! Therefore** $d\mathscr{E} = \mathscr{T}d\mathscr{S}$**.**

It implies all TIs are compatible with thermodynamic and statistical consistency.

Theorem: For ANY coupling of two systems for which (i) the full em-tensor is a polynomial of the subsystem em-tensors (ii) thermodynamic and statistical consistency holds, the coupling of the subsystems is a democratic effective metric coupling. [KMPRS]

Proof (Sketch): Note for the first TI it is sufficient that the full em-tensor it is of the form

$$T^{\mu}_{\nu} = \frac{1}{2}(t^{\mu}_{\nu} + t_{\nu}{}^{\mu})\sqrt{-g} + \frac{1}{2}(\tilde{t}^{\mu}_{\nu} + \tilde{t}_{\nu}{}^{\mu})\sqrt{-\tilde{g}} + \Delta K \delta^{\mu}_{\nu}$$

We make a reasonable assumption that the subsystems satisfy the TIs too. Then WIs in an effective background is necessary for the TIs to appear via Euler equations.

Then the above form is necessary so that the conservation of the subsystem em-tensors imply that of the full one.

One needs to show that all terms in ΔK can be absorbed in effective metrics (Hard part)

A combinatoric identity appears which has been proved in 2014 only.

Applications 1: Thermodynamics

Physical solutions of the equilibrium require

(i) causality i.e. $v_1 := a/b, v_2 := \tilde{a}/\tilde{b} < 1$

(ii) Lorentzian signature i.e. $ab^3, \tilde{a}\tilde{b}^3 > 0$

(iii) UV completeness: such solutions should exist at any value of $\mathscr{T}$

Causality is satisfied if $\gamma > 0$

For UV completeness we need $r := -\dfrac{\gamma'}{\gamma} > 1$

Realistic models will require $\epsilon_1(T_1)$ from HTL-resummed QCD and $\epsilon_2(T_2)$ from holographic QCD. One has to incorporate the scale appropriately.

However why not couple two conformal fluids with $\epsilon_1(T_1) = n_1 T_1^4$ and $\epsilon_2(T_2) = n_2 T_2^4$?

RESULT:

We get phase transition from two decoupled conformal systems at low temperature to a composite conformal system at high termperature. At intermediate temperature the EOS of the full system is not conformal [KMPRS]

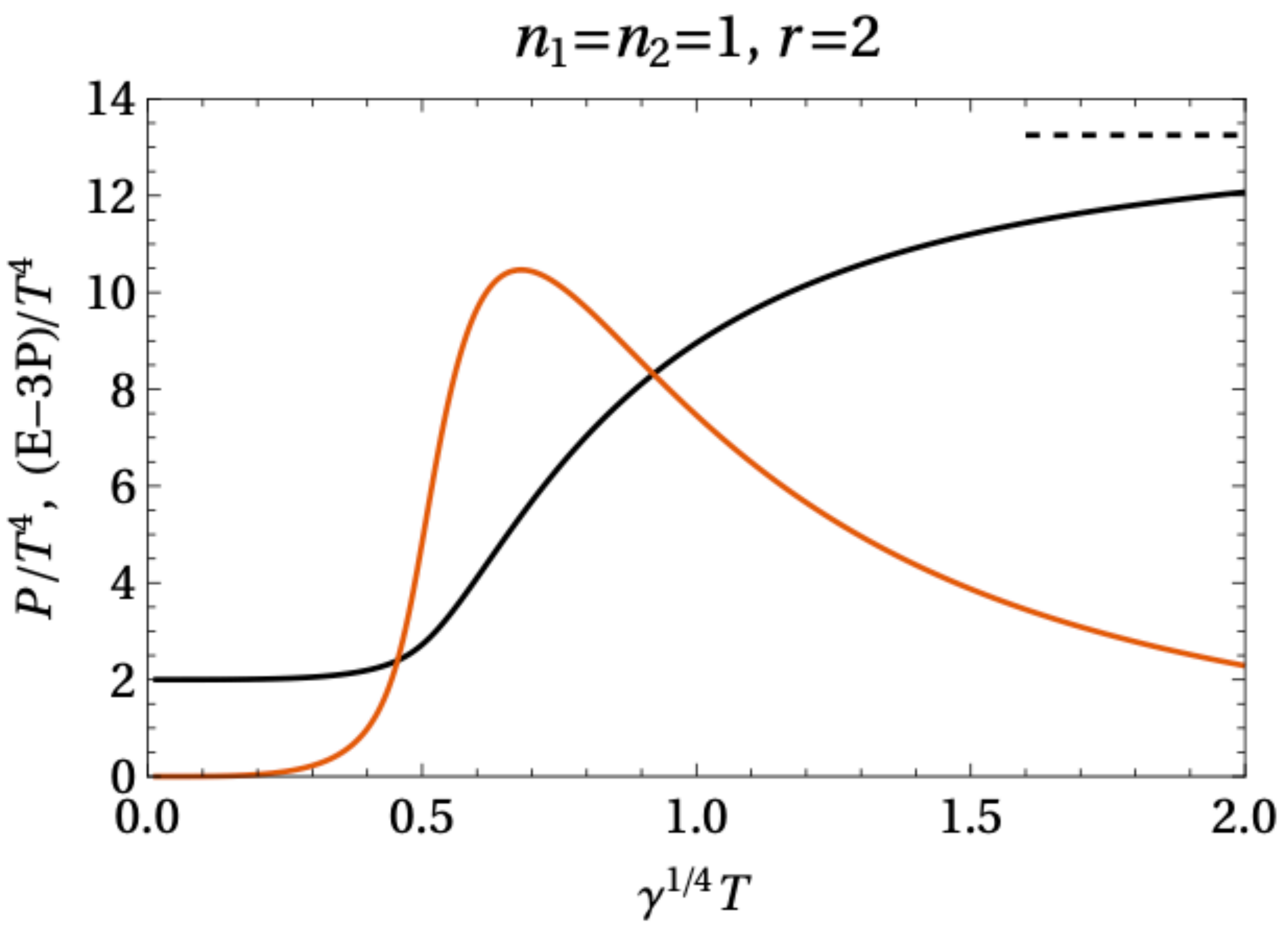
For $r > r_c$, it is crossover.

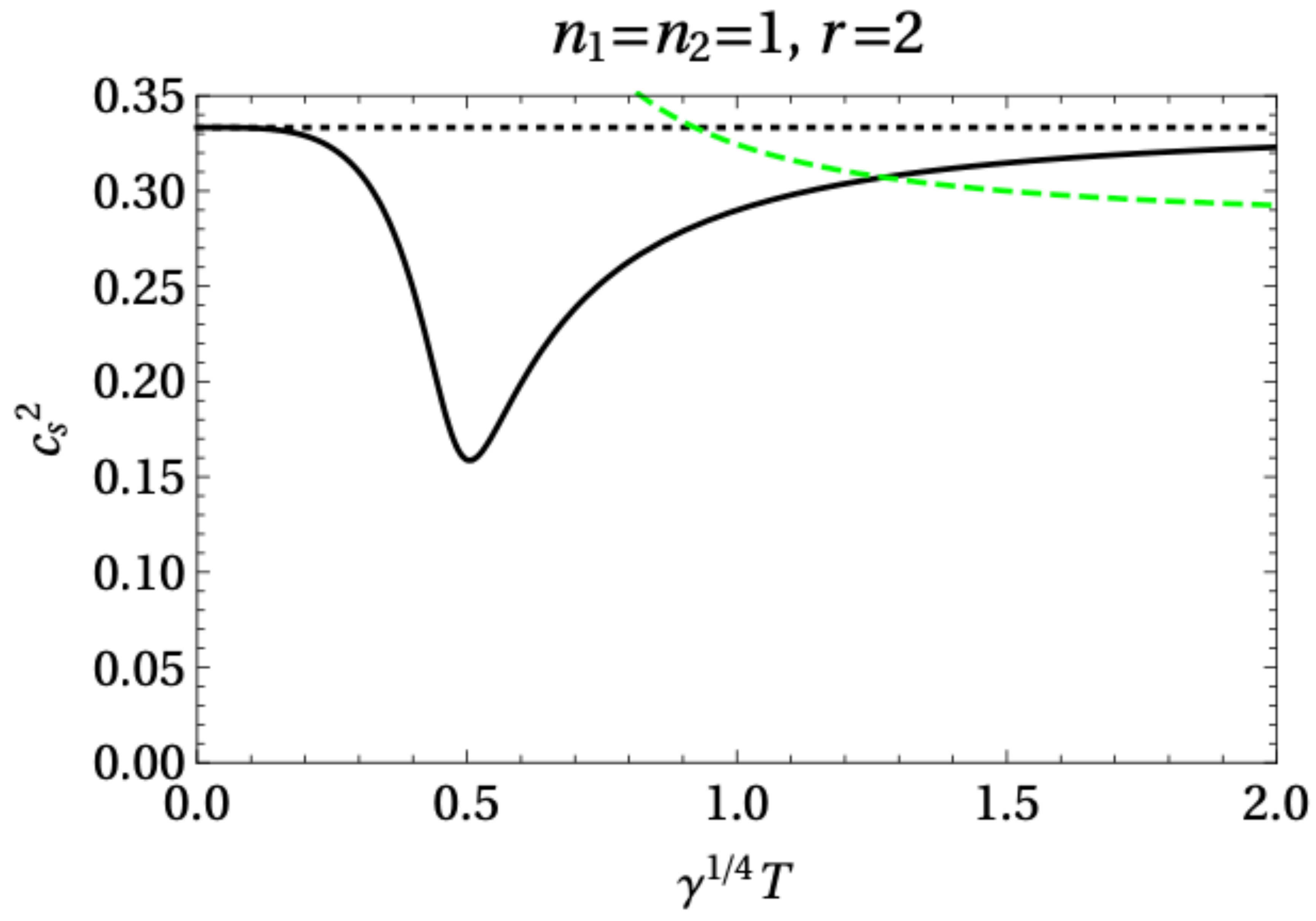
For $r = r_c$ it is a second order phase transition

For $1 < r < r_c$ it is first order phase transiton

$r_c \approx 1.1145$ when $n_1 = n_2$ and only mildly depends on n_2/n_1

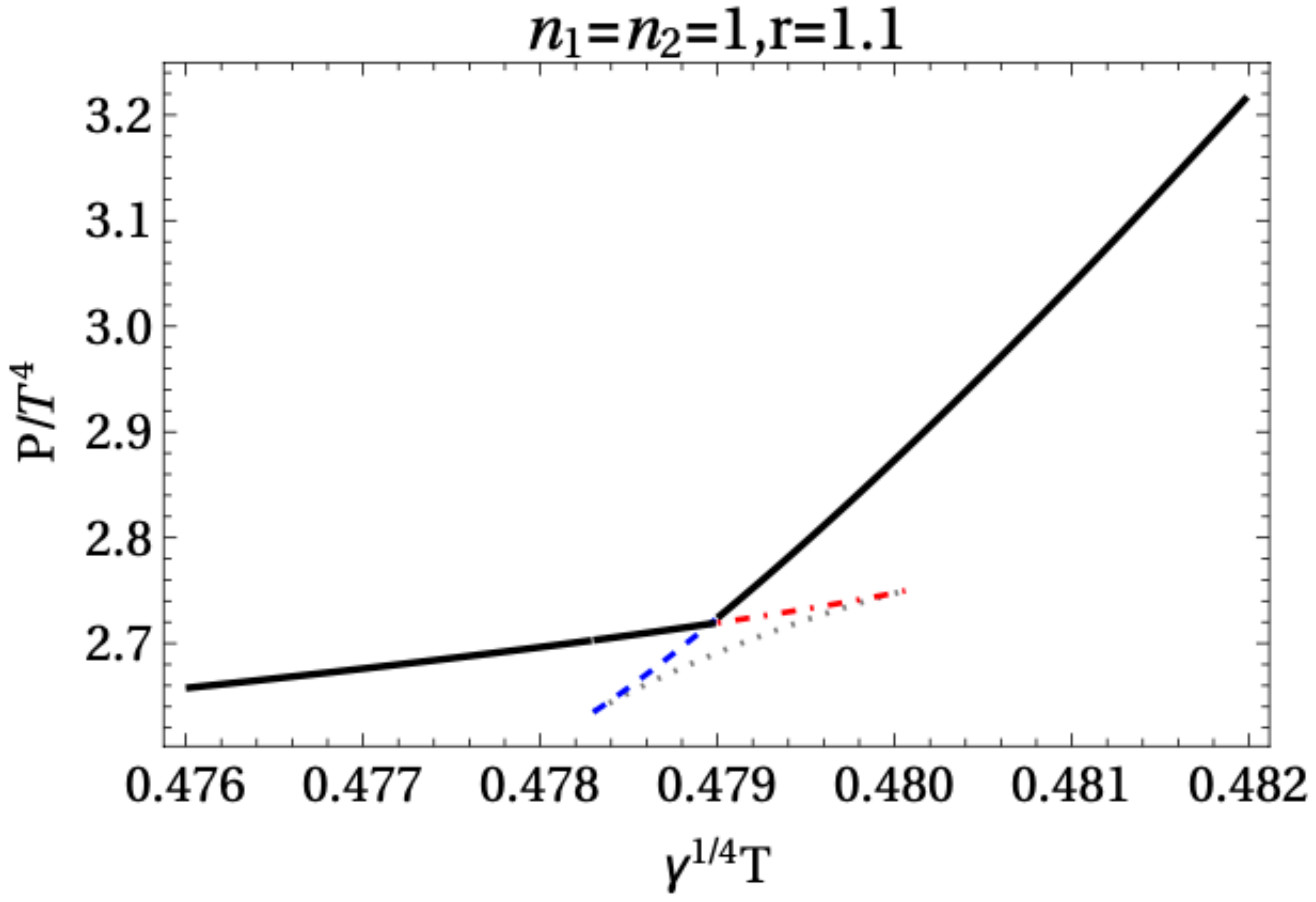
<u>Crossover:</u> **For $r > r_c$. Note conformality at higher coupling/temperature.**

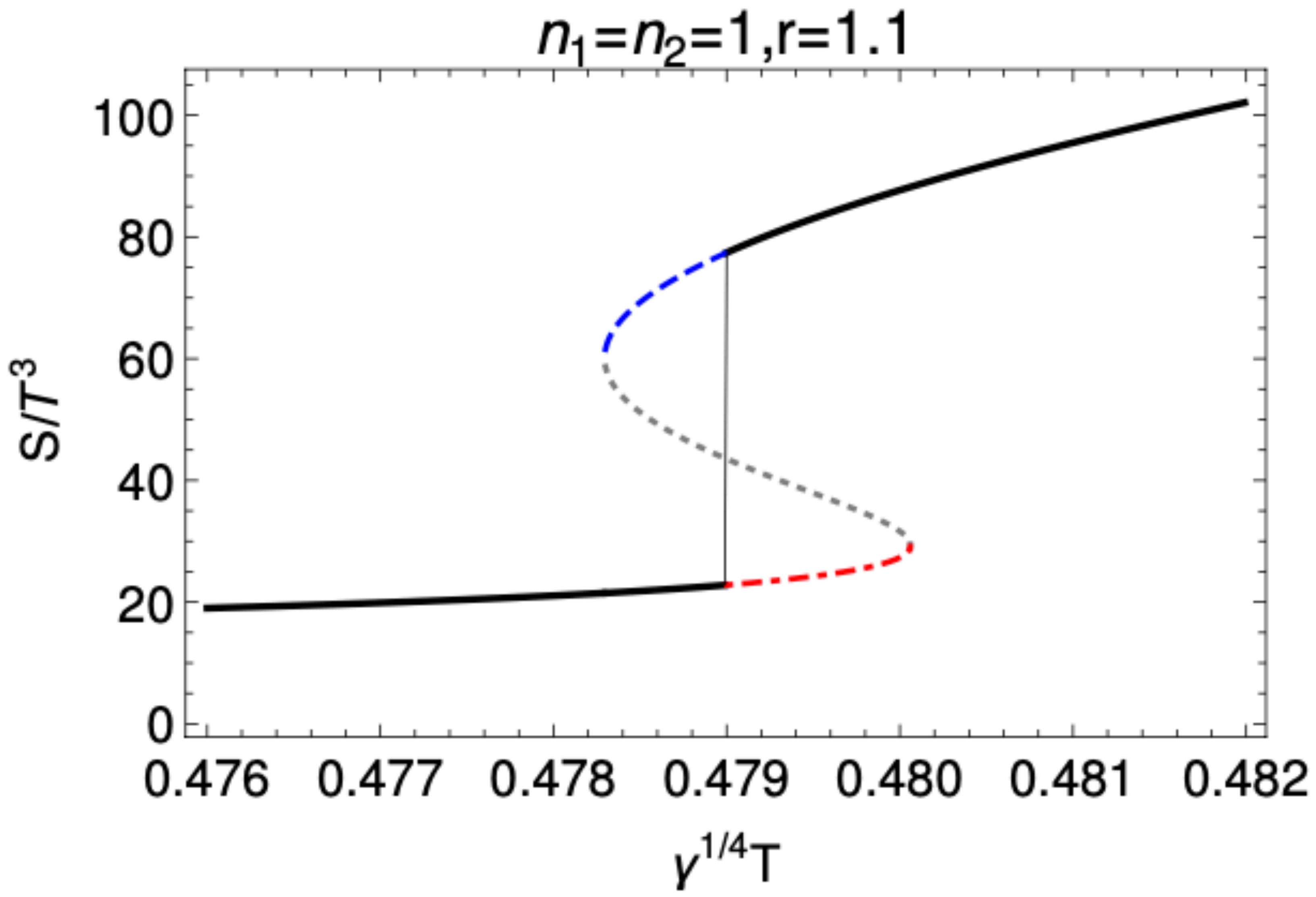




<u>First order transition:</u> **For** $1 < r < r_c$.

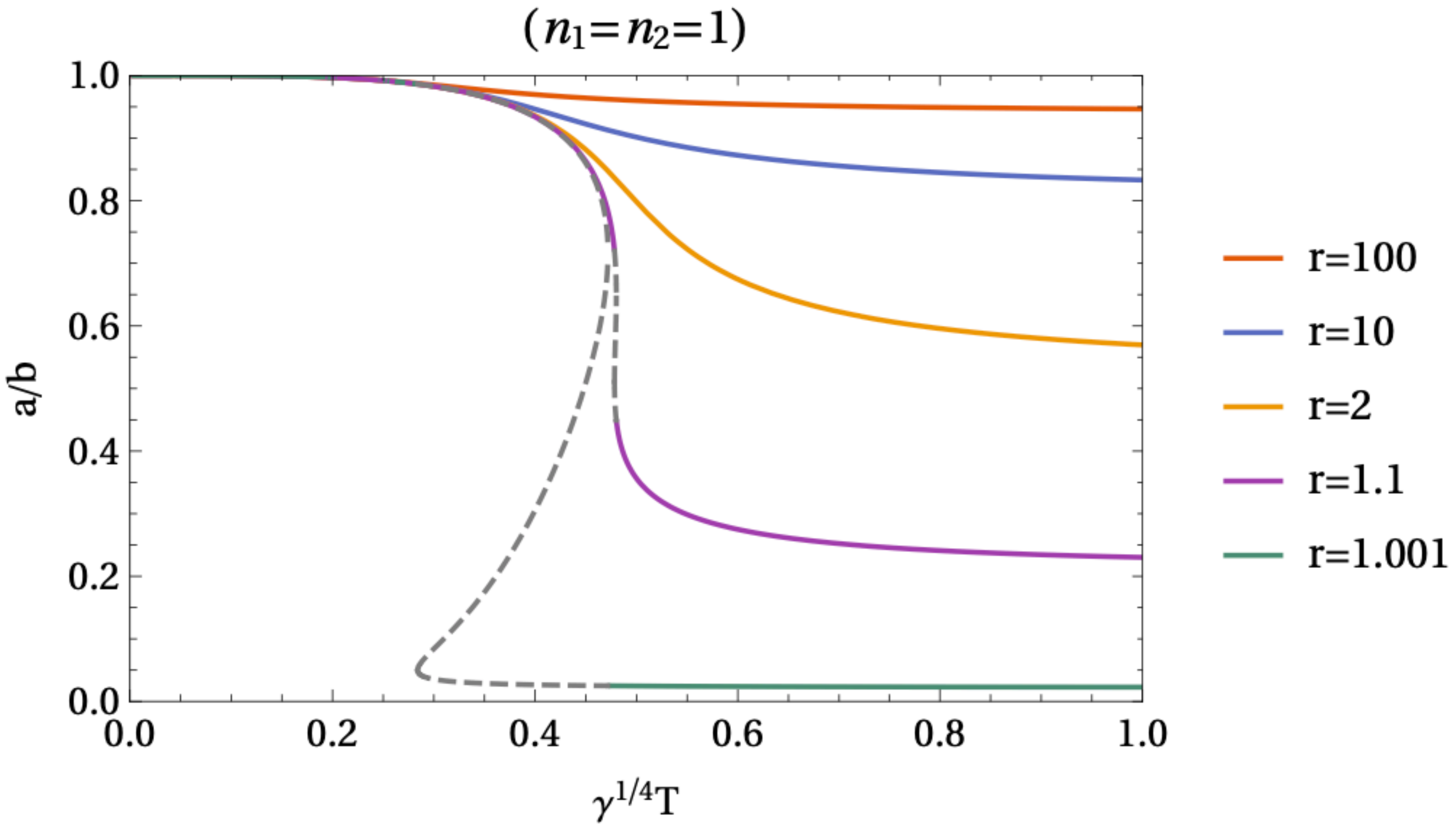
Note the superheating and supercooling lines





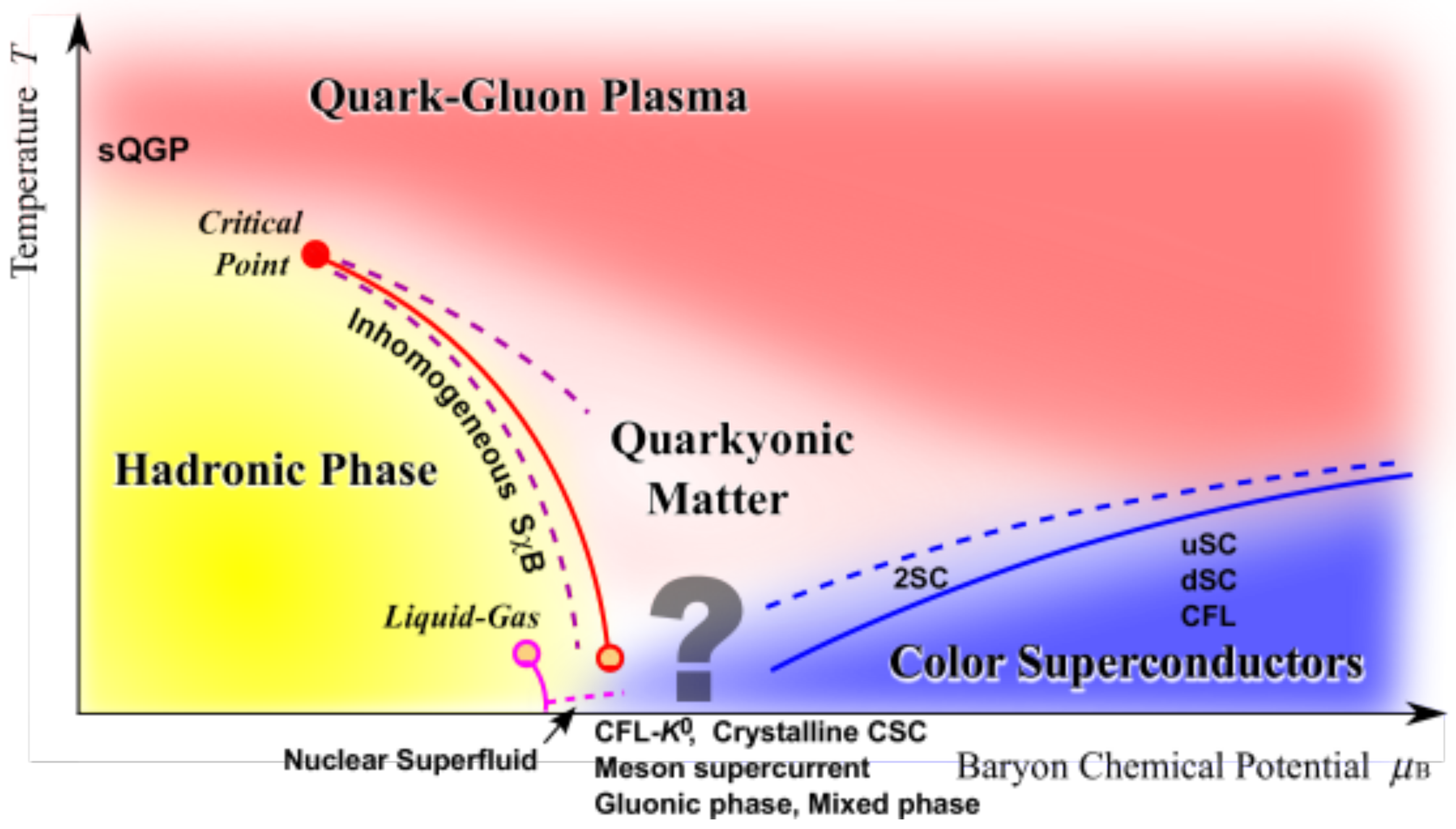
Second order transition: For $r = r_c$.

One can understand the critical exponents from the behavior of v_1, v_2



$$v_\infty^2 := \lim_{\gamma T^4 \to \infty} v^2 = \frac{3r - \sqrt{3}\sqrt{4r-1}}{3r-1}$$

We find that $C_V \approx |T - T_c|^{-\alpha}, \quad \alpha = 2/3$ **for any** n_1/n_2. **Same as in Ashkin-Teller model.**



Conjectured QCD phase diagram

Source: https://www.arxiv-vanity.com/papers/1005.4814/

Note γ' and hence r has nothing to do with the chemical potential apparently. Nevertheless γ' is a pure-trace interaction and effectively is turned on by chemical potential via a $\rho\tilde{\rho}$ type term.

Question: Can we produce characteristics of the QCD phase diagram with the right critical point behavior?

Applications 2: HIC and Toy Glasma

The first major application could be the understanding of the HIC.

In the large N limit, we expect that classical gravity implies black hole formation for generic initial conditions with sufficient energy. However we can now start with perturbative initial conditions at boundary and empty bulk AdS5 — no shock waves in the bulk.

Once the black hole is formed, it will suck up all the energy from the boundary eventually.

Away from large N, the story is different (specific comments later).

Analyzing linear fluctuations (end of talk) and also a toy non-linear example will reveal that the irreversible transfer is dominated not by the (deformed) quasinormal modes but a different pole closer to the origin. This is a slow process even in the large N limit.

What do we want to do?

Couple glasma/kinetic theory to holographic gravity — Einstein gravity coupled to an axion and a dilaton and with a negative cosmological constant.

One needs to solve classical YM eqns (kinetic theory) and gravitational equations simultaneously in a self-consistent way.

$$S = S_{YM}[g_{\mu\nu}, \lambda, \theta] + S_{grav}[\tilde{g}_{\mu\nu}, \tilde{\lambda}, \tilde{\theta}] + \text{auxiliary action for sources}$$

$\tilde{g}_{\mu\nu}$:= **Boundary metric of AdS,** $\tilde{g}_{YM}$:= **source of dilaton,** $\tilde{\theta}$:= **source of axion**

In Fefferman Graham gauge:

$$ds^2 = \frac{L^2}{r^2}(dr^2 + G_{\mu\nu}(r,x)dx^\mu dx^\nu)$$

Asymptotic expansion (schematically):

$$G_{\mu\nu} = \tilde{g}_{\mu\nu} + \cdots + r^4\left(\frac{4\pi G_5}{L^3}\tilde{t}_{\mu\nu} + \cdots\right) + \cdots$$

$$\Phi = \tilde{\lambda}^{-1} + \cdots + r^4\left(\frac{4\pi G_5}{L^3}\mathscr{H} + \cdots\right) + \cdots$$

$$a = \tilde{\theta} + \cdots + r^4\left(\frac{4\pi G_5}{L^3}\mathscr{K} + \cdots\right) + \cdots$$

Actually $\tilde{t}^{\mu\nu}, \mathscr{H} . \mathscr{K}$ depend on higher derivatives of the sources. Defined explicitly via *holographic renormalization*

YM equations of motion (with sources on the light cone not shown explicitly):

$$\mathscr{D}_\mu(\lambda^{-1}F^{\mu\nu}) - (\partial_\mu\theta)\tilde{F}^{\mu\nu} = 0$$

$\mathscr{D}_\mu :=$ **gauge-covariant derivative in the background $g_{\mu\nu}$**

YM equations of motion:

$$\mathscr{D}_\mu(\lambda^{-1}F^{\mu\nu}) - (\partial_\mu\theta)\tilde{F}^{\mu\nu} = 0$$

$\mathscr{D}_\mu :=$ **gauge-covariant derivative in the background** $g_{\mu\nu}$

Coupling equations (solutions for the sources):

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(\gamma\eta_{\mu\alpha}\tilde{t}^{\alpha\beta}\eta_{\beta\nu} + \gamma'\eta_{\mu\nu}\tilde{t}^{\alpha\beta}\eta_{\alpha\beta}\right)\sqrt{-\tilde{g}}$$

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \left(\gamma\eta_{\mu\alpha}t^{\alpha\beta}\eta_{\beta\nu} + \gamma'\eta_{\mu\nu}t^{\alpha\beta}\eta_{\alpha\beta}\right)\sqrt{-g}$$

$$\frac{1}{\lambda} = \frac{1}{g_{YM}^2 N} + \beta\mathscr{H}$$

$$\frac{1}{\tilde{\lambda}} = 0 + \frac{\beta}{4}\mathrm{tr}(\mathrm{F}^2)$$

$$\theta = \alpha\mathscr{K}$$

$$\tilde{\theta} = \frac{\alpha}{4}\mathrm{tr}(\mathscr{F}\tilde{\mathscr{F}})$$

Numerical scheme (Edmond Iancu and AM [IM]:

1. Solve the YM equations with glasma initial conditions (see talk by David Müller) in background $g_{\mu\nu} = \eta_{\mu\nu}$ with constant λ and $\theta = 0$ (Do exactly what Andi & David are doing)

2. Use these to compute $\tilde{g}_{\mu\nu}, \tilde{\lambda}, \tilde{\theta}$ — the sources in gravitational theory. Start with empty AdS5 and obtain the unique gravitational solution. Follow Chesler-Yaffe scheme but note there is NO shock wave in the bulk

3. Use the gravitational solution to compute $\tilde{t}^{\mu\nu}, \mathscr{H}, \mathscr{K}$ so that we get our new $g_{\mu\nu}, \lambda, \theta$ for the glasma. Solve the glasma equations again with the SAME initial conditions. (See IM for validity of the same initial conditions. Since higher derivative of YM-fields appear we have to set these to zero at initial time.)

4. Use the new glasma solution to compute $\tilde{g}_{\mu\nu}, \tilde{\lambda}, \tilde{\theta}$ and solve the gravitational problem again with the SAME empty AdS5 initial condition

5. Iterate until we get complete convergence. The full $T^{\mu\nu}$ will be conserved at the end of iteration.

The full em-tensor takes the form:

$$T^{\mu}_{\nu} = \frac{1}{2}(t^{\mu}_{\nu} + t_{\nu}^{\mu})\sqrt{-g} + \frac{1}{2}(\tilde{t}^{\mu}_{\nu} + \tilde{t}_{\nu}^{\mu})\sqrt{-\tilde{g}} + \left(\Delta K - \frac{\beta}{4}\mathscr{H}\sqrt{-\tilde{g}}\,\mathrm{tr}(\mathrm{F}^2)\sqrt{-g} - \frac{\alpha}{4}\mathscr{K}\sqrt{-\tilde{g}}\,\mathrm{tr}(\mathrm{F}\tilde{\mathrm{F}})\sqrt{-g}\right)\delta^{\mu}_{\nu}$$

$$\Delta K = \frac{\gamma}{2}\left(t^{\mu\alpha}\sqrt{-g}\right)\eta_{\alpha\beta}\left(\tilde{t}^{\beta\nu}\sqrt{-\tilde{g}}\right)\eta_{\mu\nu} + \frac{\gamma'}{2}\left(t^{\alpha\beta}\sqrt{-g}\right)\eta_{\alpha\beta}\left(\tilde{t}^{\mu\nu}\sqrt{-\tilde{g}}\right)\eta_{\mu\nu}$$

Self-consistent solutions of both systems will ensure $\partial_{\mu}T^{\mu\nu} = 0$

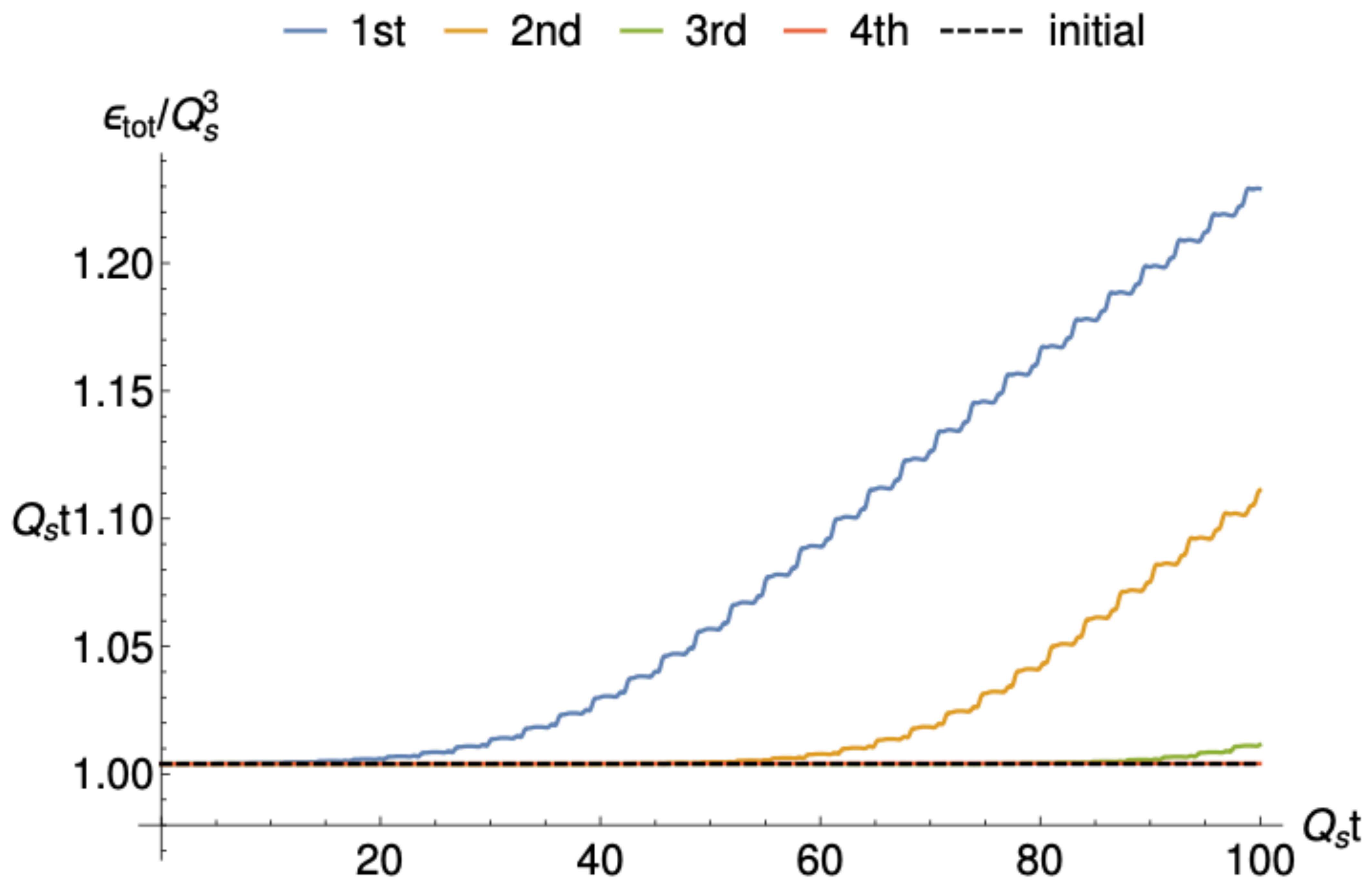
The iterative scheme indeed works! Simplest non-trivial way to test in the full non-linear theory is to go to 2+1D in homogeneous non-expanding scenario. The bulk geometry is then AdS4. [EMPRS]

In the SU(2) YM sector use temporal gauge $A^t = 0$ and $A_i^a = f(t)\delta_i^a$. To satisfy Gauss constraint we may have $f(t_{in}) = 0, \dot{f}(t_{in}) = a_0$

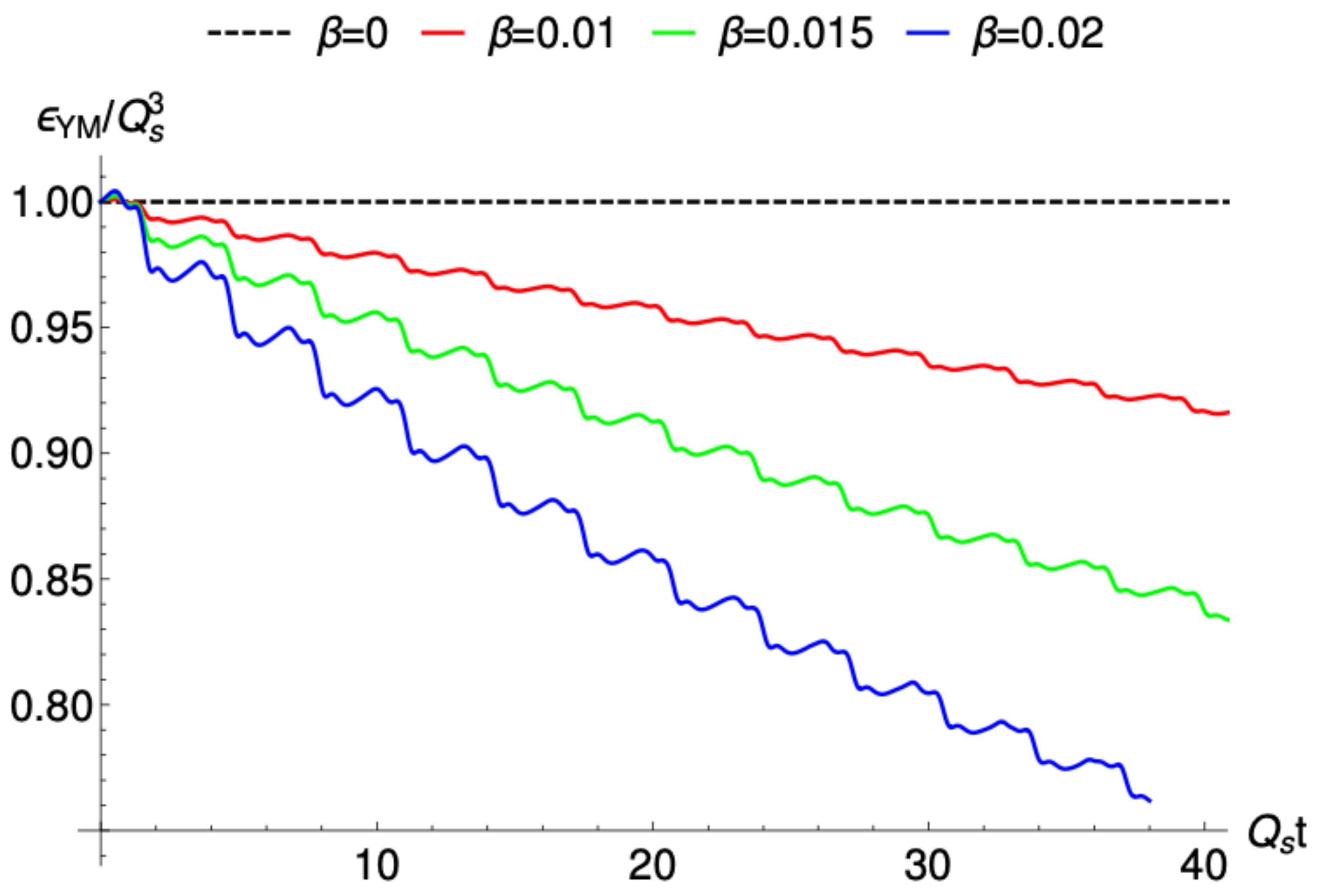
Set $\alpha = \gamma = \gamma' = 0$ but $\beta \neq 0$

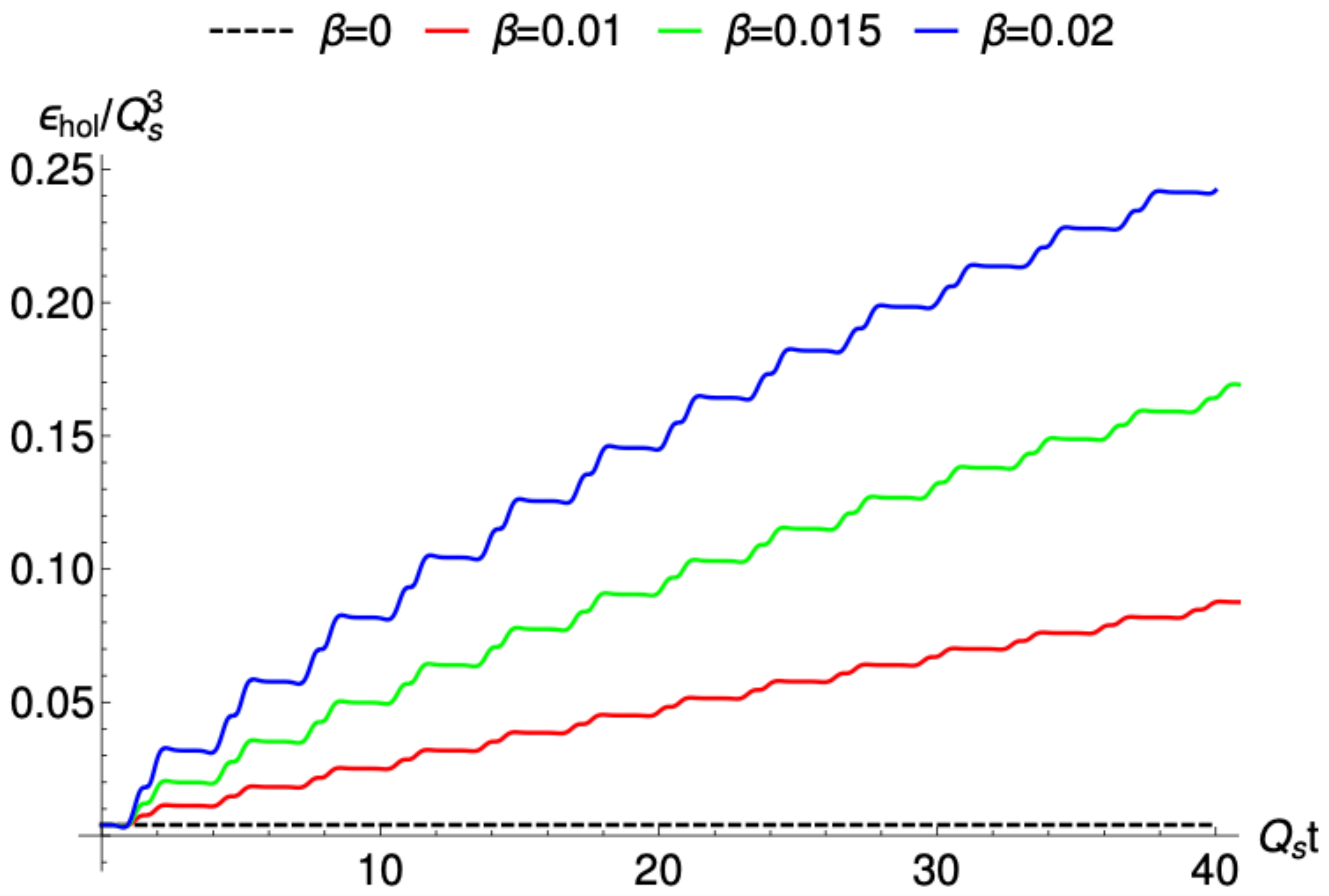
Converges in about 4 iterations! [MPRS, EMPRS]

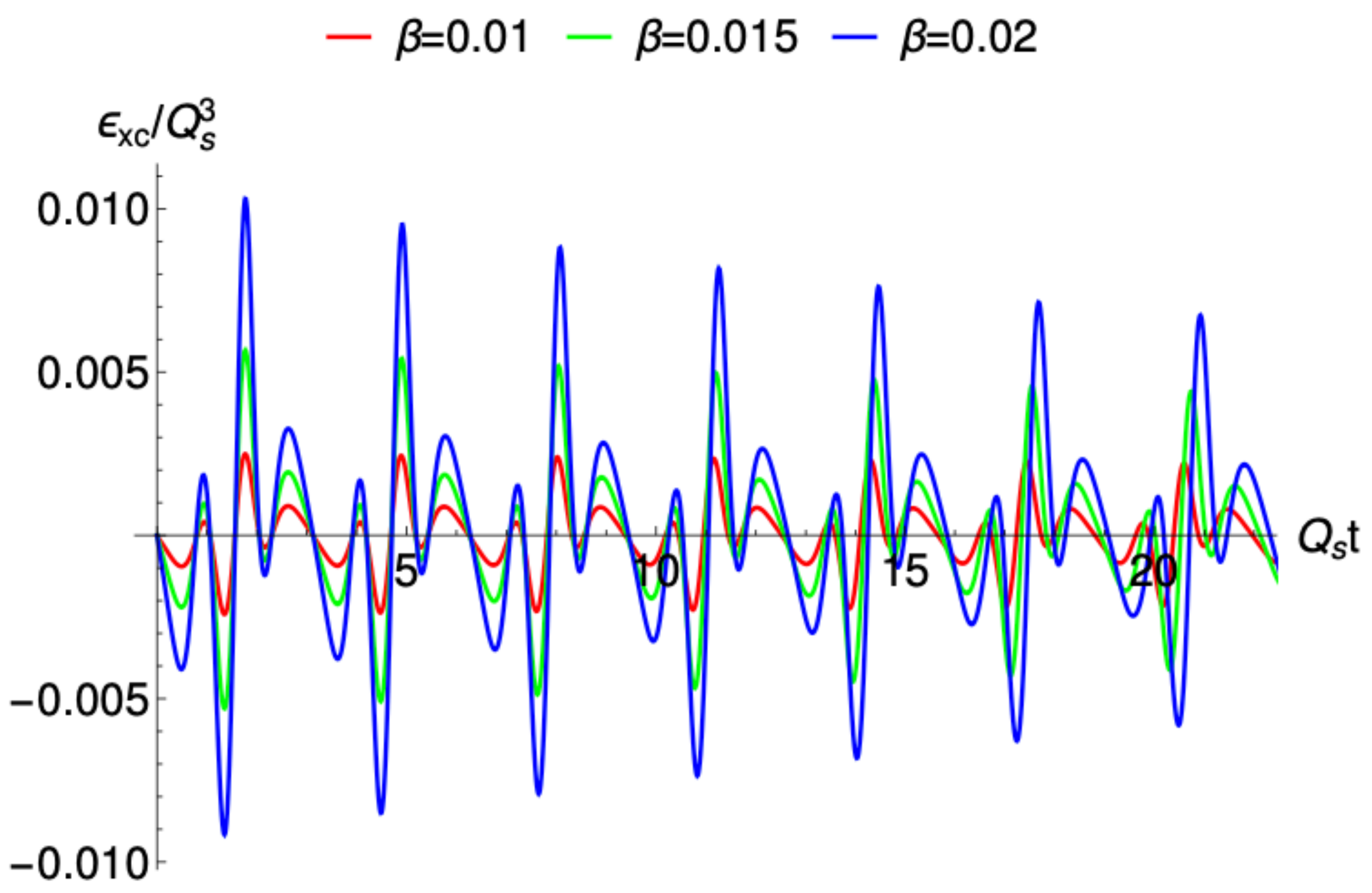
It is possible to take the limit of initial empty AdS4 numerically! [EMPRS]



Irreversible transfer of energy to the black hole! It is slow for small mutual coupling.

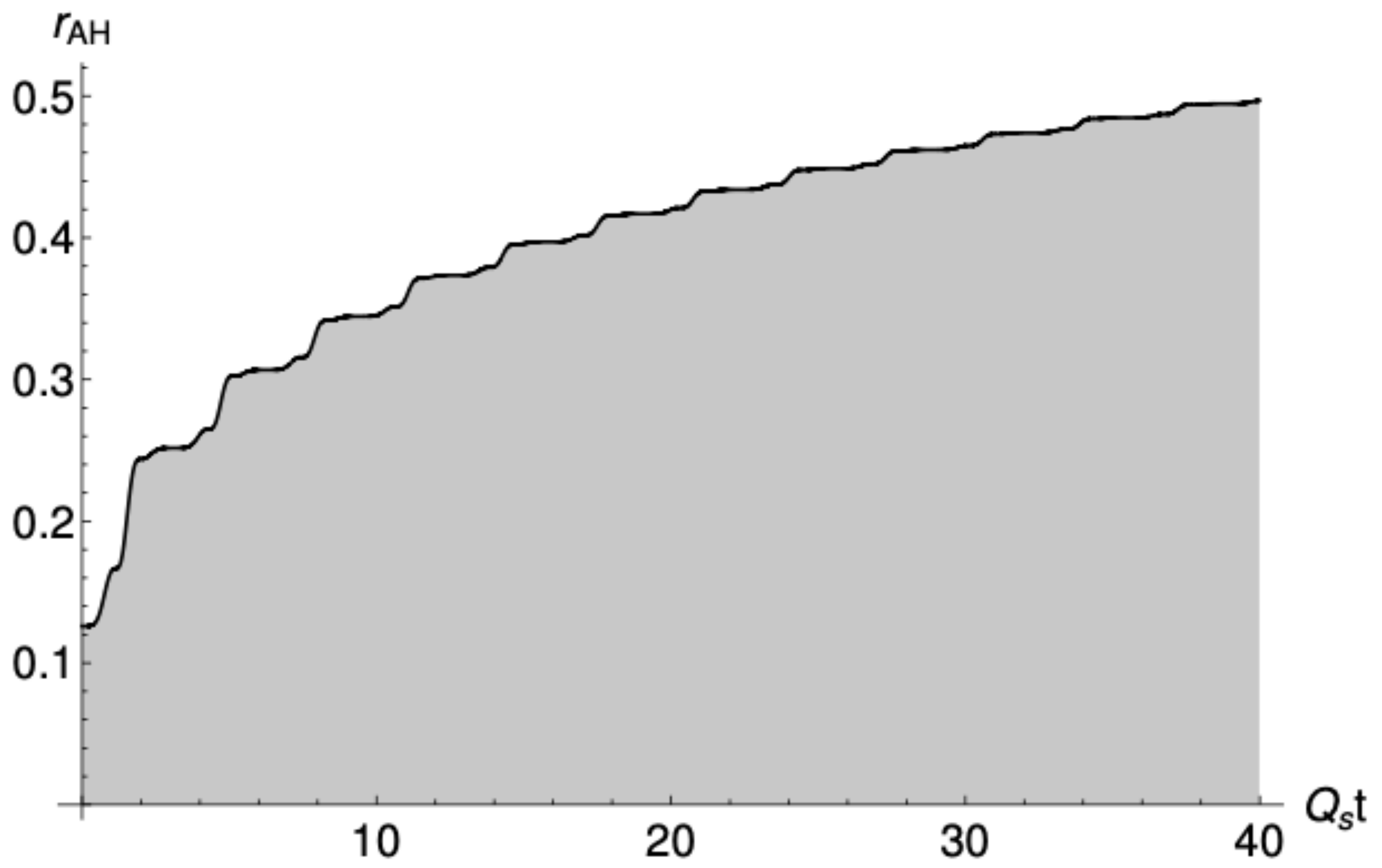


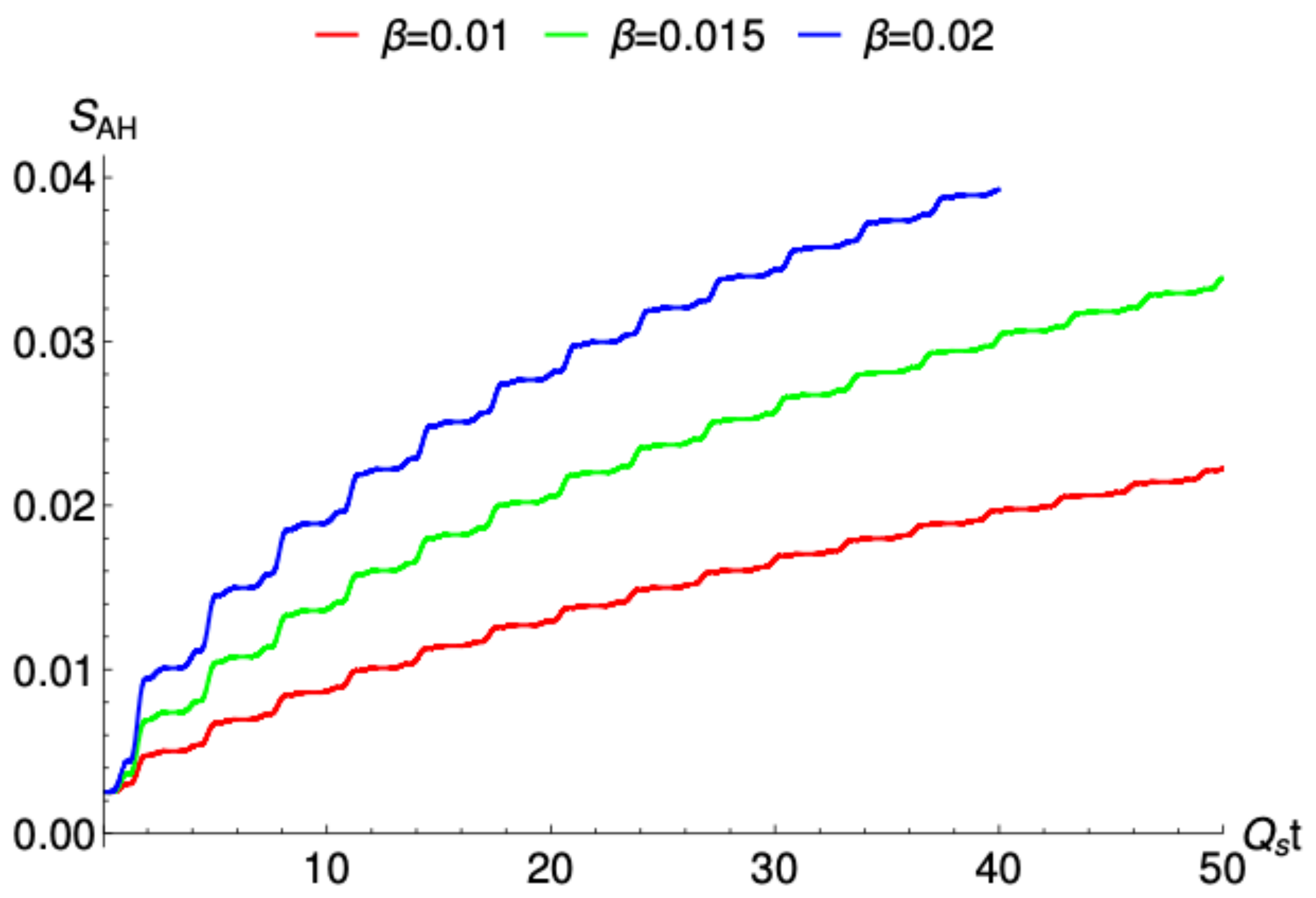




$$E = \epsilon_{YM} + \epsilon_{hol} + \epsilon_{xc}$$

Apparent horizon grows and grows and so does the entropy (area of apparent horizon)





Our toy glasma works exactly as expected!

Applications 3: Bi-hydrodynamic attractor

Since both the weak and strong sectors can be described by fluids at long time, we naively expect that the long time behavior can be described by a two-fluid model coupled by the democratic effective metric couplings.

This is actually wrong because there is no mechanism of irreversible energy transfer to the soft sector.

Nevertheless, at small mutual coupling the transfer of energy to soft sector takes long time. Therefore bi-hydro model may be a good model for a significant fraction of time and capture some qualitative features.

However, the two fluids do not locally equilibrate — there are two sound modes and two shear modes about thermal equilibrium [KMPRS].

This happens because there are two independent entropy currents in the large N limit (two self-consistent isolated systems after all — so something irreversible needed at fundamental level for global equilibration!)

The total entropy current is just $S^\mu = s^\mu \sqrt{-g} + \tilde{s}^\mu \sqrt{-\tilde{g}}$. The total entropy production ceases if the two subsystems equilibrate at different temperatures not related by global equilibrium condition.

Of course replacing the strongly coupled fluid by black hole leads to global equilibration. Our framework makes sense in the large N limit only if one of the sectors is holographic and strongly coupled (something that is borne out while analyzing renormalons of pQCD).

Away from large N limit, global equilibration will happen via stochastic fluctuations and there will be only one (total) entropy current. Currently Toshali is demonstrating this explicitly in the bi-hydro limit.

The hybrid attractor (talk by Toshali Mitra)

Couple two conformal Muller-Israrel-Stewart systems with different amounts of viscosities undergoing Bjorken flow and find the attractor of the full system.

Choose $r > r_c$ for simplicity.

The degrees of freedom are $\epsilon, \tilde{\epsilon}, \phi, \tilde{\phi}$ (energy densities and pressure anisotropies).

$$t^{\mu}_{\nu} = \mathrm{diag}\left(\epsilon, P+\frac{\phi}{2}, P+\frac{\phi}{2}, P-\phi\right), \quad \tilde{t}^{\mu}_{\nu} = \mathrm{diag}\left(\tilde{\epsilon}, \tilde{P}+\frac{\tilde{\phi}}{2}, \tilde{P}+\frac{\tilde{\phi}}{2}, \tilde{P}-\tilde{\phi}\right)$$

$$\chi := \frac{\phi}{\epsilon + P}, \quad \tilde{\chi} := \frac{\tilde{\phi}}{\tilde{\epsilon} + \tilde{P}} \qquad C_\eta = \frac{\eta}{s}, \quad \tilde{C}_\eta = \frac{\tilde{\eta}}{\tilde{s}} \qquad \sigma = \frac{C_\eta}{\tau_\pi T}, \quad \tilde{\sigma} = \frac{\tilde{C}_\eta}{\tilde{\tau}_\pi \tilde{T}}$$

There exists a 2D attractor surface ruled by curves. The two constants $\alpha := \lim_{\tau\to\infty} \epsilon\tau^{4/3}$ and $\beta := \lim_{\tau\to\infty} \tilde{\epsilon}\tau^{4/3}$ label these curves. Any initial condition evolves to one of these curves on the attractor surface. [MMMRS]

Example of an attractor curve is in the plot below (from Toshali's talk)

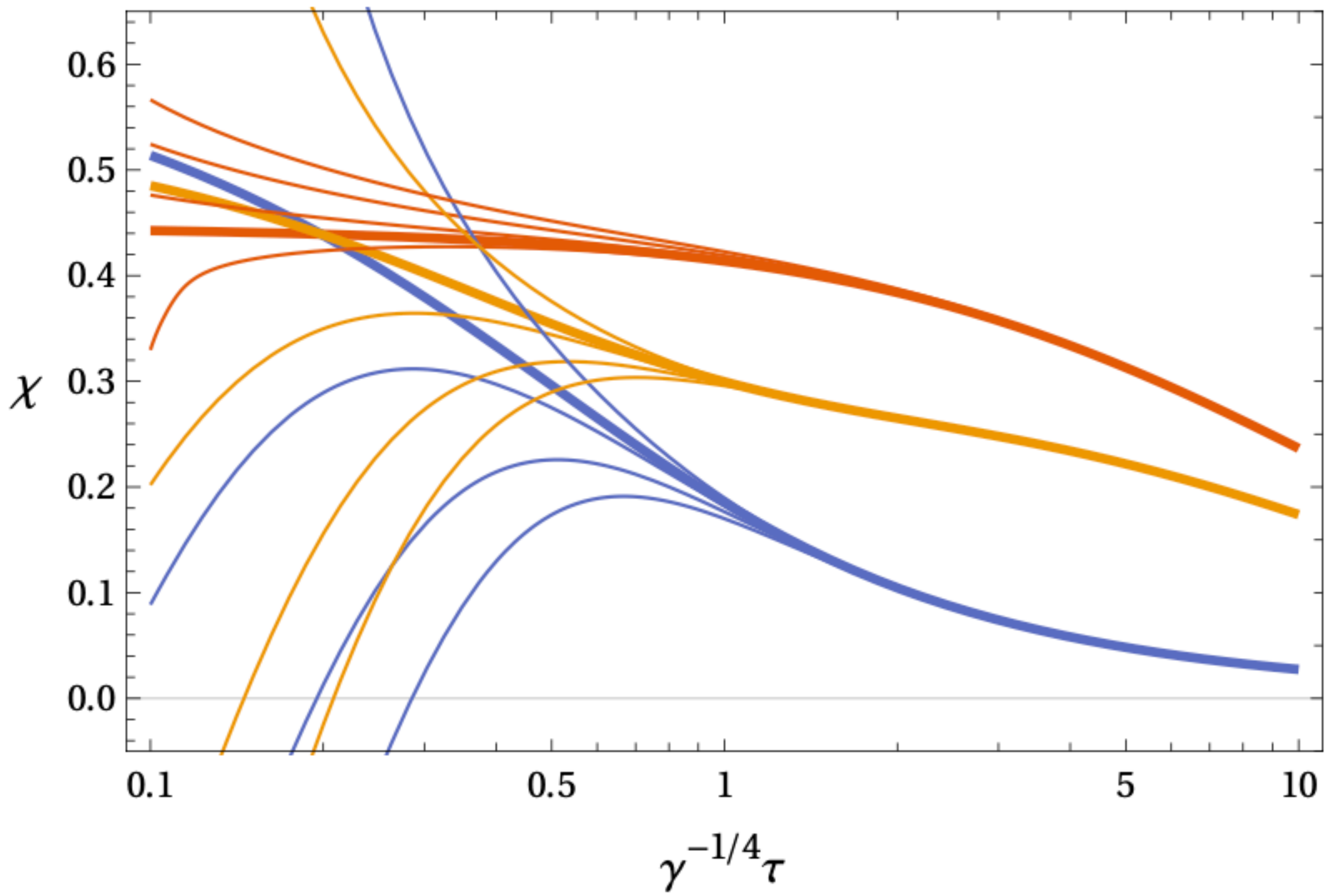


Figure: **Attractor solutions** , thin lines are neighbouring trajectories.
Less viscous system, *More viscous system* and *Full system*

Bottom-up thermalization is universal as long as one of the systems is weakly coupled and another is strongly coupled i.e. $\tilde{\sigma} > \sigma$!

As $\tau \to 0$, $\dfrac{\epsilon\sqrt{-g}}{\tilde{\epsilon}\sqrt{-\tilde{g}}} \to \tau^{(8/3)(\tilde{\sigma}-\sigma)}$ on the attractor. However later the weak system dominates again as if in the hadron gas crossover

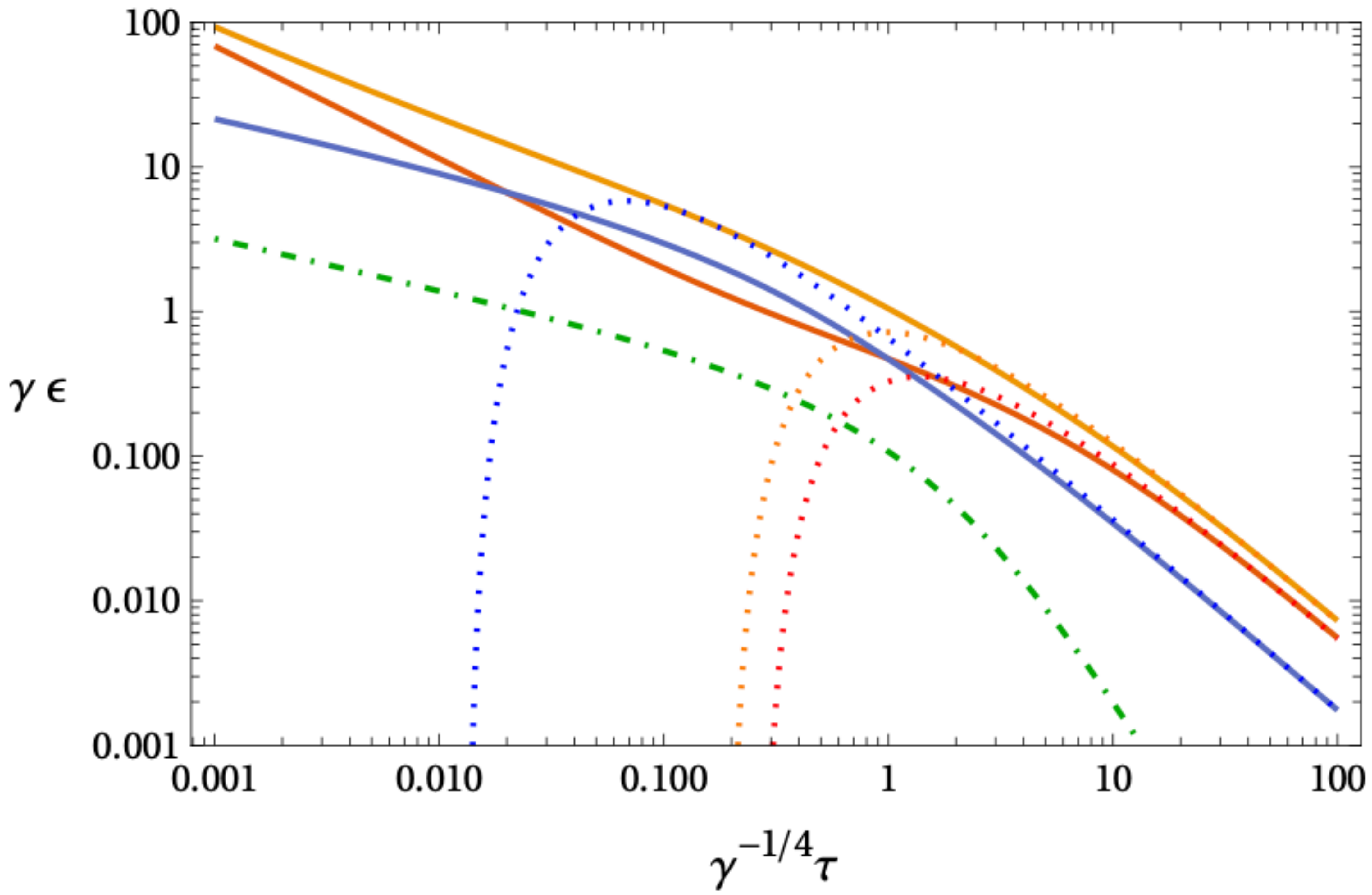


Figure: *Less viscous system*, *More viscous system* and *Full system*

The two systems do not equilibrate but still the full system em-tensor can be described as a SINGLE FLUID at late time.

The EOS and shear viscosity determined by the curve on the attractor surface to which the system evolves at late time (amount of energy sharing between the two systems)

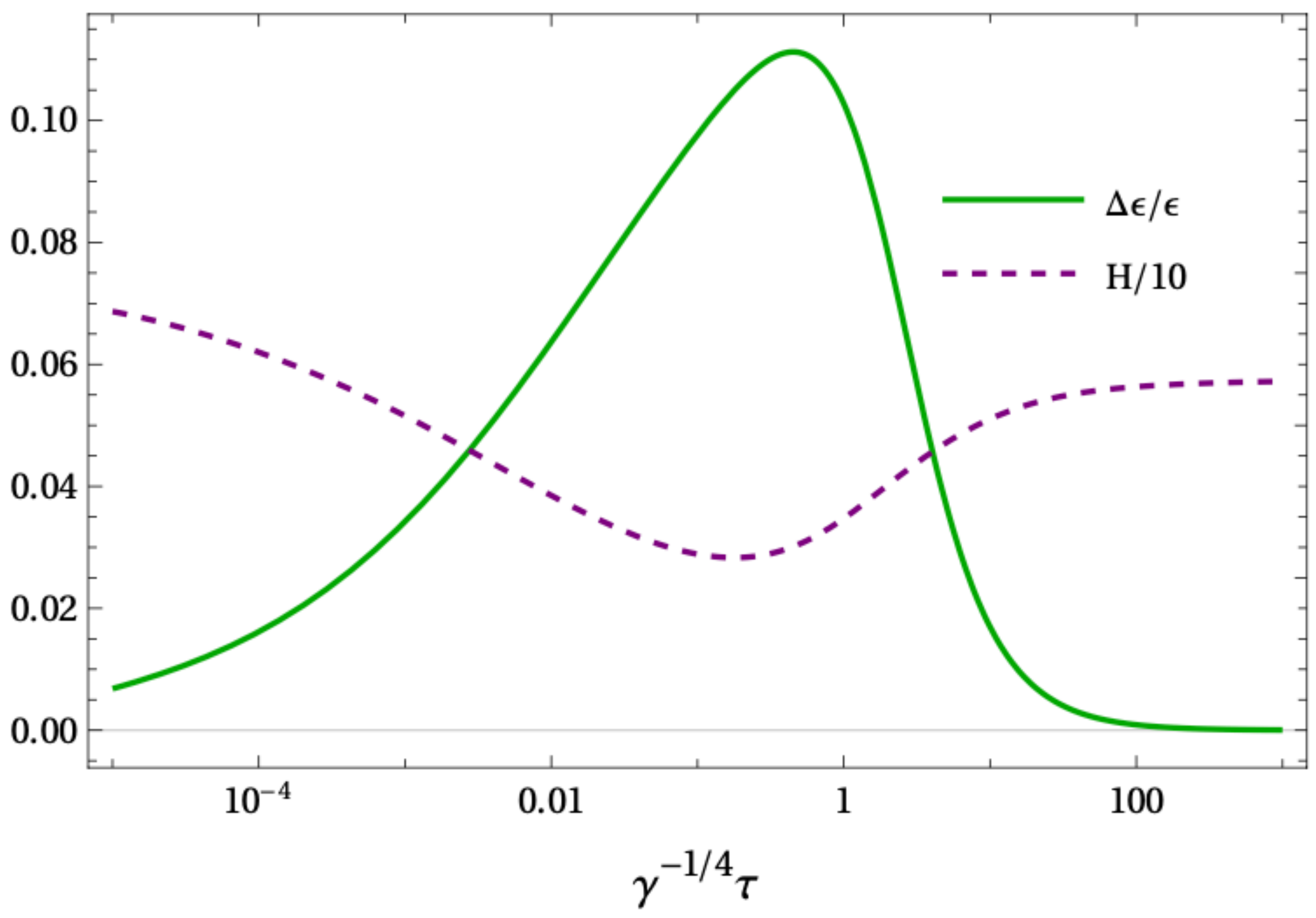


Figure: *Interaction energy of the subsystems over total energy*
Averaged effective shear viscosity

$$\left(\frac{\eta}{s}\right)^{\text{full}} = C_\eta^{\text{eff}} := \lim_{\tau\to\infty} H(\tau)$$

$$H(\tau) = \frac{C_\eta \epsilon^{4/3}(\tau) + \tilde{C}_\eta \tilde{\epsilon}^{4/3}(\tau)}{\epsilon^{4/3}(\tau) + \tilde{\epsilon}^{4/3}(\tau)}$$

$$C_\eta^{\text{eff}} = \frac{C_\eta \alpha^{4/3} + \tilde{C}_\eta \tilde{\beta}^{4/3}}{\alpha^{4/3} + \tilde{\beta}^{4/3}}$$

Bottom-up hydrodynamiczation: The weak system hydrodynamizes later than the strong system unless the strong system has extremely tiny fraction of energy at a reference time $\tau_0 = \gamma^{1/4} \approx Q_s^{-1}$

However the ratio of hadrodynamization times become extreme as the total energy of the system at reference time is decreased. Gives insights into small vs large system collision.

$\mathcal{E}(1)$	τ_{hd}	$w_{\mathrm{hd}}/10$	χ_{hd}	$\tilde{\tau}_{\mathrm{hd}}$	$\tilde{w}_{\mathrm{hd}}$	$\tilde{\chi}_{\mathrm{hd}}$	R_{hd}
0.26	12.0	0.609	0.215	2.08	1.42	0.101	5.76
0.32	10.2	0.705	0.203	3.90	2.82	0.0525	2.62
0.052	25.5	0.608	0.210	1.39	0.613	0.211	18.4

Anisotropy, $A = \frac{P_\perp - P_L}{P} \sim 6\chi$

w scales as $\frac{4\pi\eta}{s}$

$$\frac{|\Delta P_L|}{P} := \frac{|\phi - \phi_{1st}|}{P} < 0.1, \quad \tau > \tau_{hd}$$

Applications 4: Hybrid QNM modes — Christmas tree with a cross

ongoing work with Sukrut Mondkar, Anton Rebhan and Alexander Soloviev

What about relaxation?

Coupling kinetic theory (instead of fluid) to strongly coupled MIS reveals that although the hydrodynamics of both systems are affected strongly, the damping modes of the kinetic sector $\omega/T = -i\Gamma/T$ are hardly affected by the mutual coupling.

Furthermore the cut merely shrinks due to the narrowing of the effective lightcone.

Half of the damping sector is still described very well perturbation theory. Many observables may still be well described by perturbative approaches alone.

Why so slow?

Why is the transfer of energy to soft sector so slow at weak mutual coupling?

We can find out by coupling kinetic theory with a black hole.

If we couple classical YM to black hole then the linearized perturbations about the final equilibrium state (black hole + an empty YM sector) do not see the coupling at all, hence they remain unaffected.

Try this: In 2+1D case couple a massless scalar to 4D BH linearly. In the homogeneous case (note β is the mutual coupling)

$$S = \frac{1}{2}\int \dot{\chi}^2 + S_{grav}[\lambda^{-1} = \beta\chi]$$

$\lambda :=$ Source of massless dilaton bulk field Φ $\quad \Phi = \beta\chi(t) + \cdots + r^3\Phi_3(t) + \cdots$

The total conserved energy of the full non-linear system is

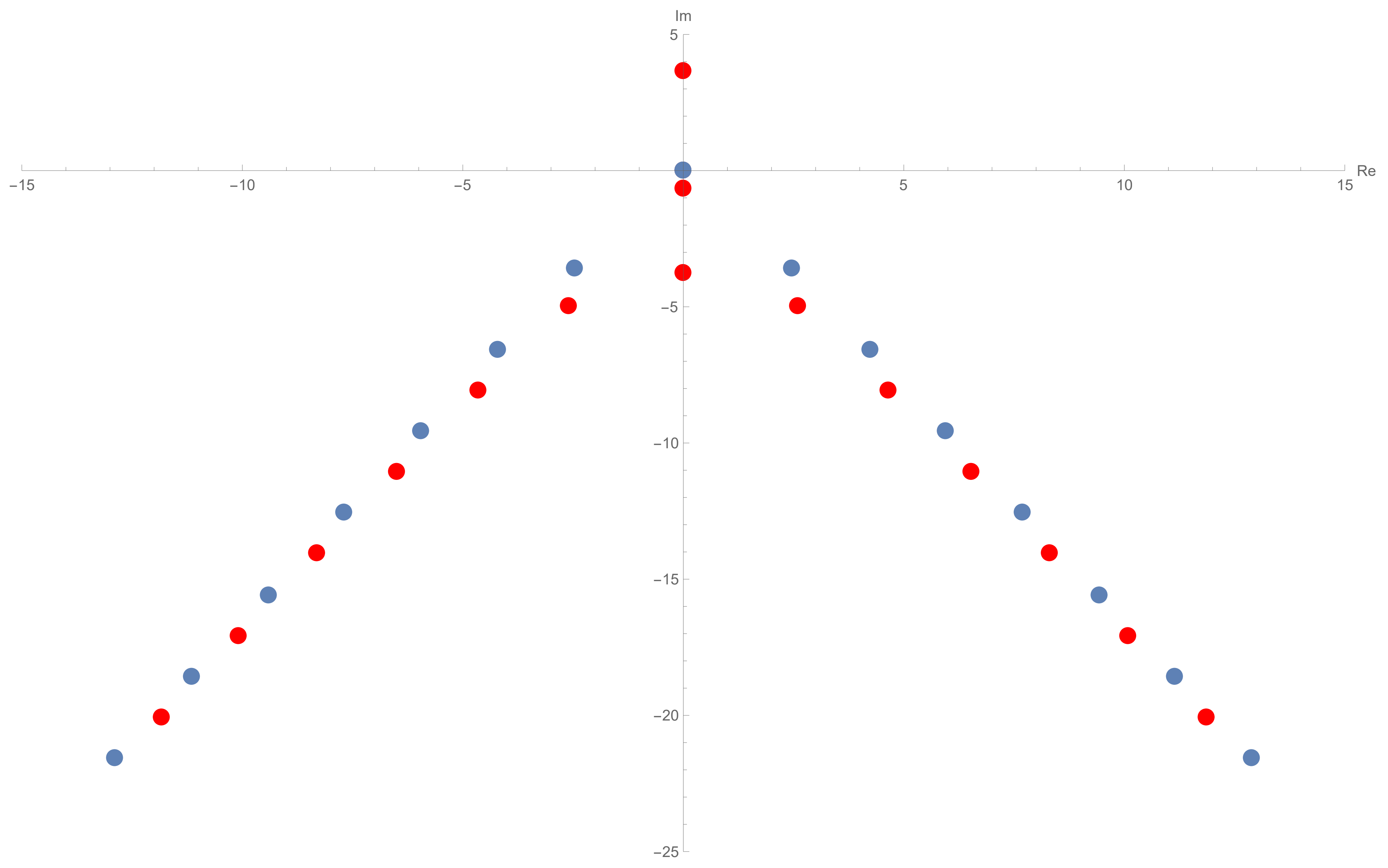
$$E = \frac{1}{2}\dot{\chi}^2 + M_{ADM}$$

Eom of χ

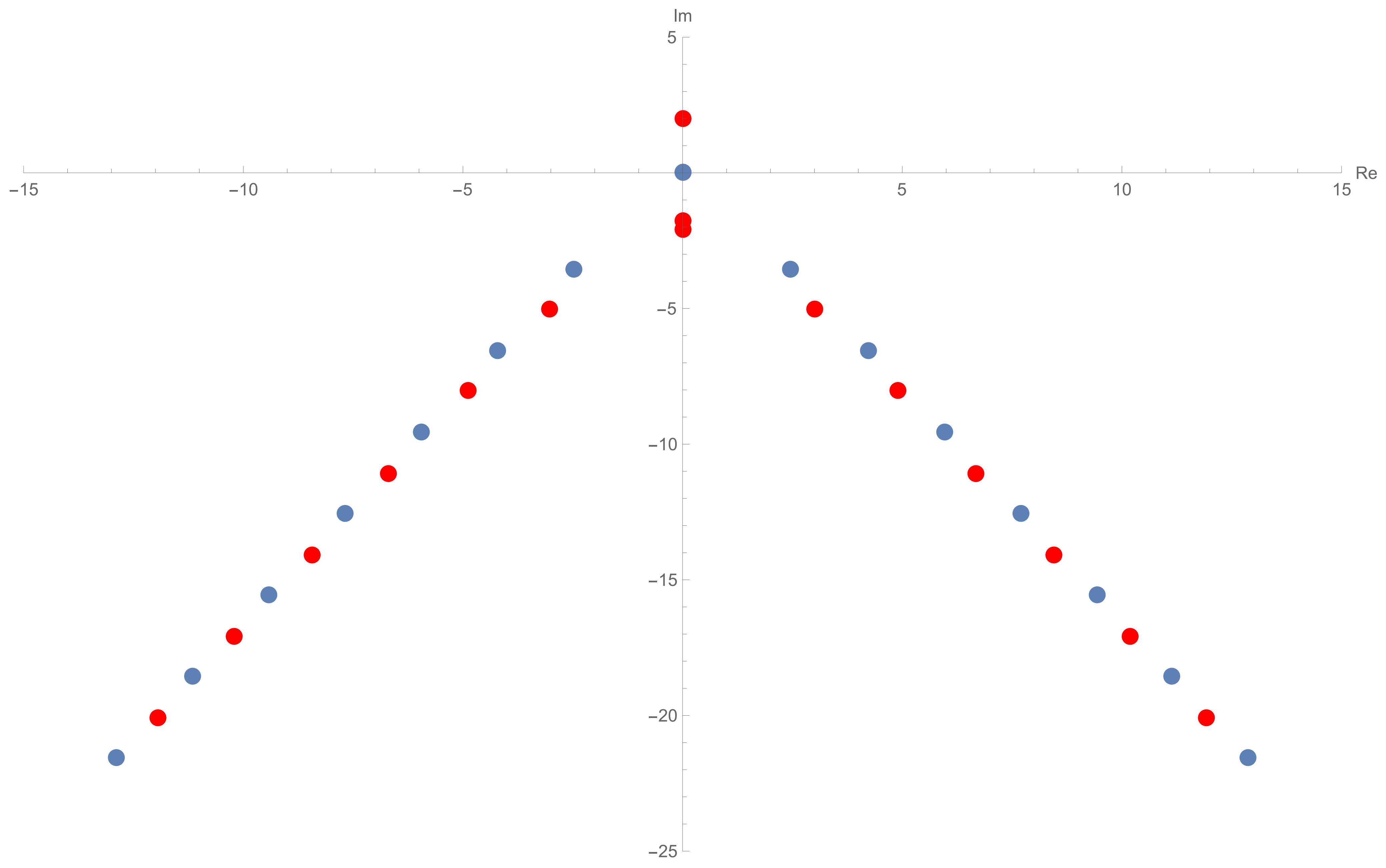
$$\ddot{\chi} = \beta O, \quad O = \beta(3\phi_3 + \dddot{\chi})$$

Blue dots: Decoupled case $\beta = 0$ (note β is the mutual coupling)

Note $\omega = 0$ mode is doubly degenerate in decoupled limit. But degeneracy is lifted by coupling.



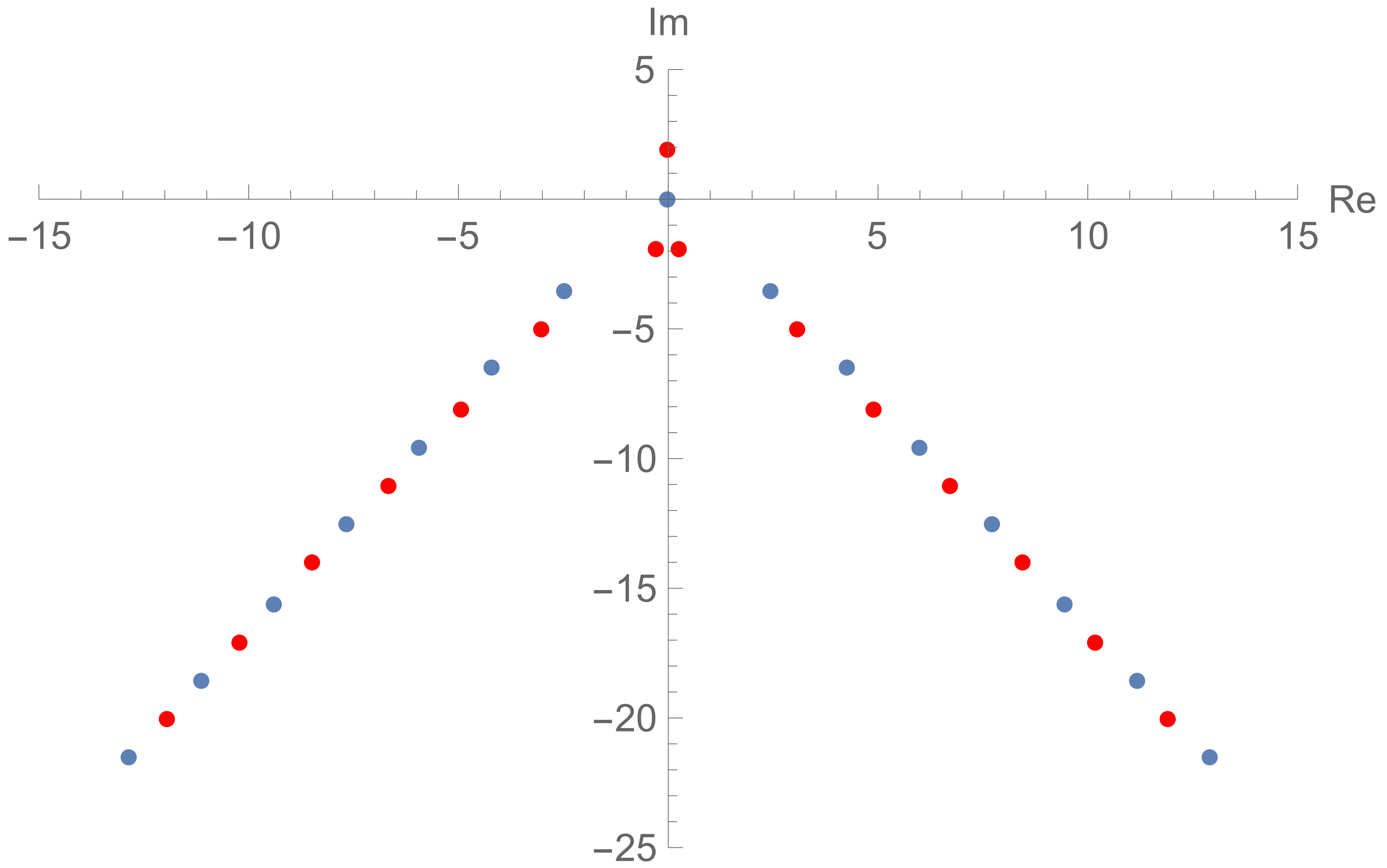
$\beta = 0.3$



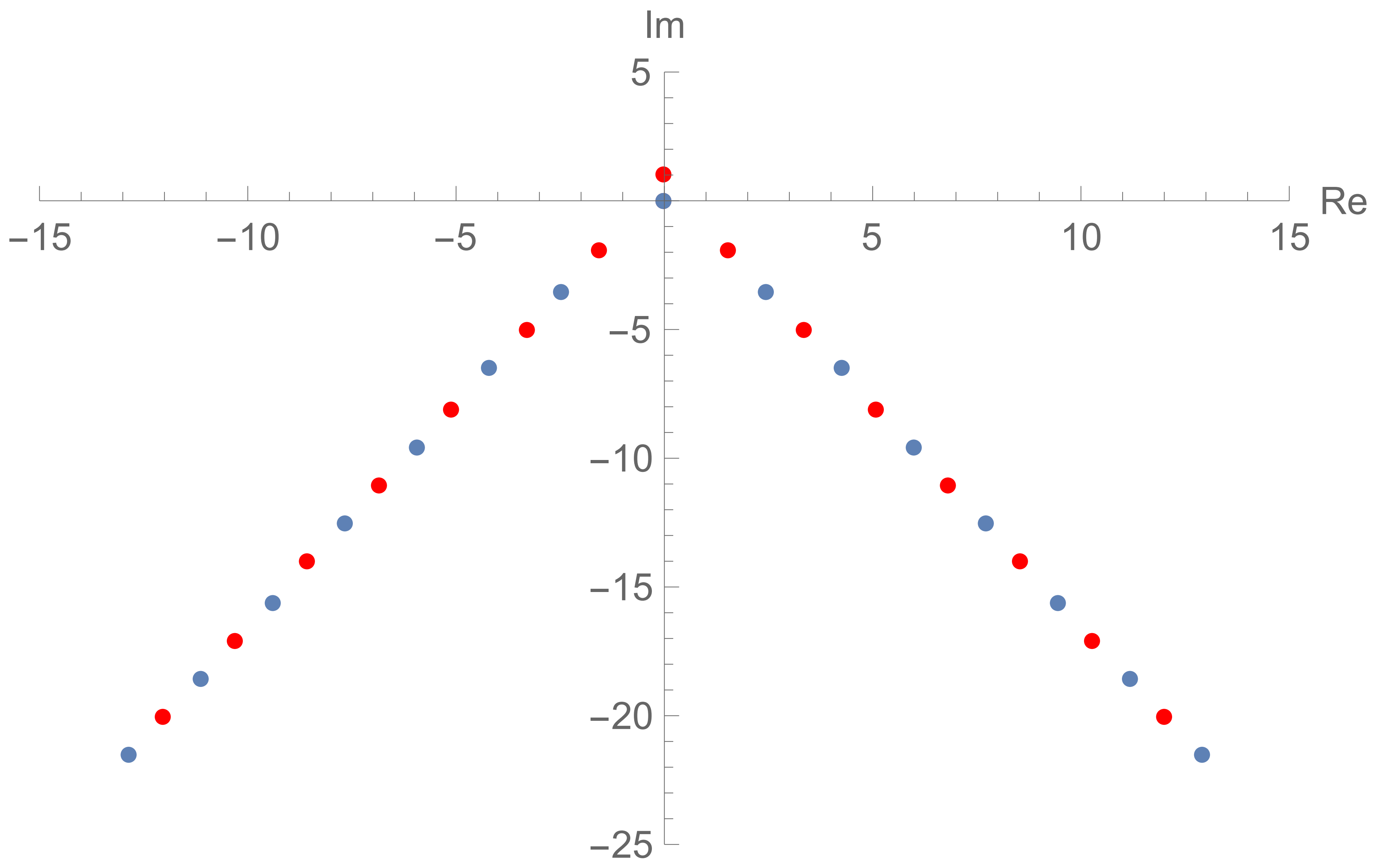
$\beta = 0.44$

Emergent conformality at infinite β as ω/T become T i.e. β independent.

The poles realign on the Christmas tree.

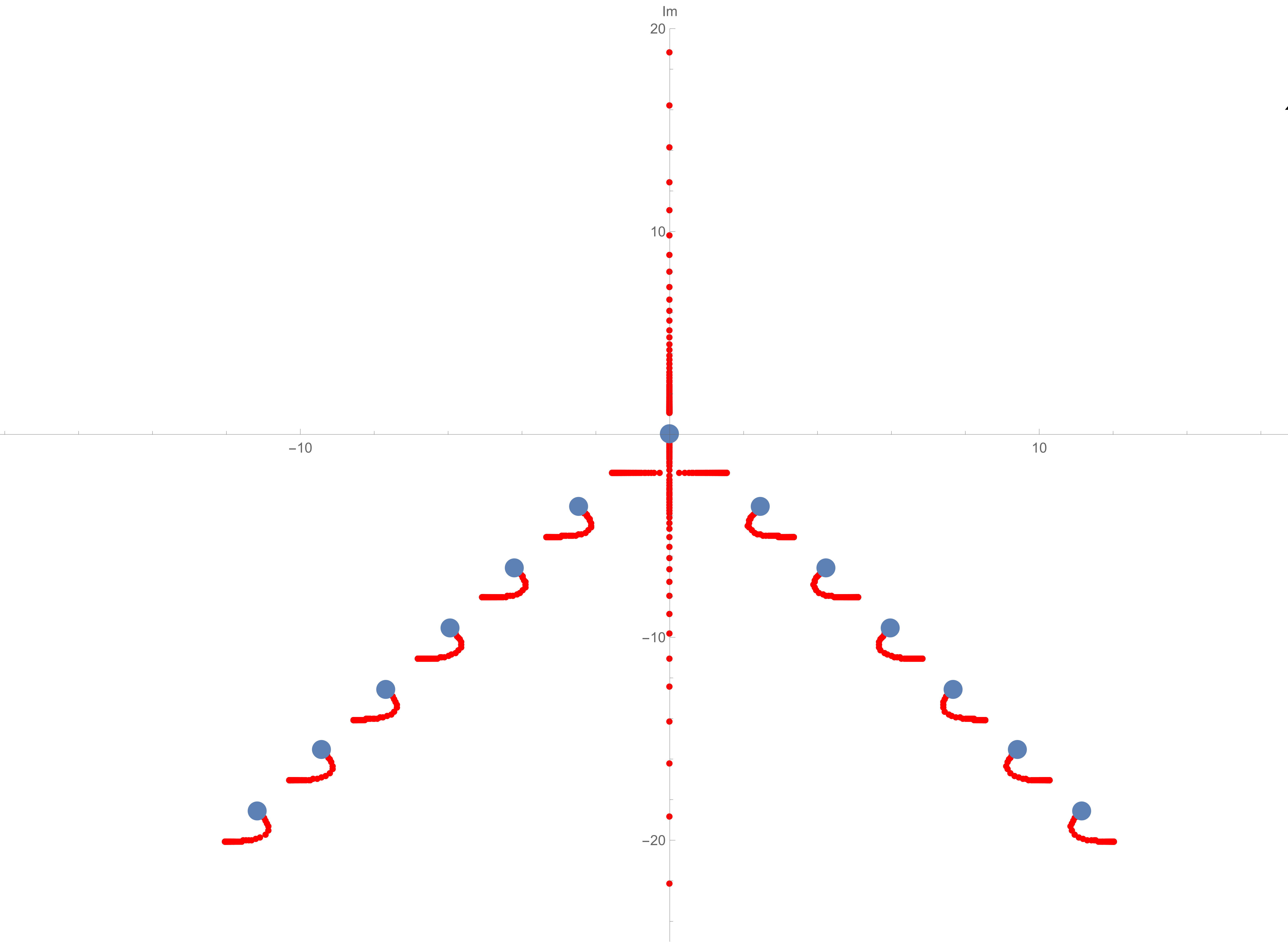


$\beta = 0.45$



$\beta = 10$

The dance of the poles with varying β



Instability is likely cured by setting $\ddot{\chi} = 0$ in the far past. Non-linear simulations indicate so.

At small β the damping mode driving transfer of energy from boundary to bulk is very close to the origin.

How to incorporate it in EFT?

Conclusions

Semi-holography can be a consistent way to understand non-perturbative dynamics of an asymptotically free theory like QCD in the large N limit.

The key proposal is to view the perturbative and holographic sectors as self-consistent isolated systems where the marginal couplings and effective metrics are deformed by the operators of the other sector.

Survives key consistency tests with thermodynamics and statistical mechanics.

Emergent conformality at infinite mutual coupling but useful at scales where mutual coupling is small.

A proposal to simulate HIC has been made. The numerical iterative scheme in which the limit of empty AdS initial condition must be taken works — checked for a non-trivial prototype.

In the large N limit and small mutual coupling, the transfer of energy to black hole occurs slowly.

Bi-hydrodynamic model reveals universality of bottom-up thermalization and emergence of the attractor surface where the full system can be described as a single fluid with EOS and transport coefficients determined by which curve on the surface the initial conditions evolve to.

“But when the strong were too weak to hurt the weak, the weak had to be strong enough to leave”
Milan Kundera

The irreversible transfer of energy to strong sector happens most likely after re-dominance of the weak sector

New insights on small vs large system collisions: Enormous hierarchy in hydrodynamization times in small systems?

Also many observables can be described well by perturbative physics.

Outlook

1. Better modeling of thermodynamics incorporating the chemical potential

2. Simulate HIC

3. Study jets and quarkonia

4. Understand how thermalization works at finite N

5. Incorporate fragmentation of the horizon and study the instanton liquid of QCD

6. Get new insights into hadronic structure

7. Understand the link between hydrodynamics, QNM modes and quantum chaos beyond holography

THANK YOU