Simple models of monsoon bifurcation and global warming

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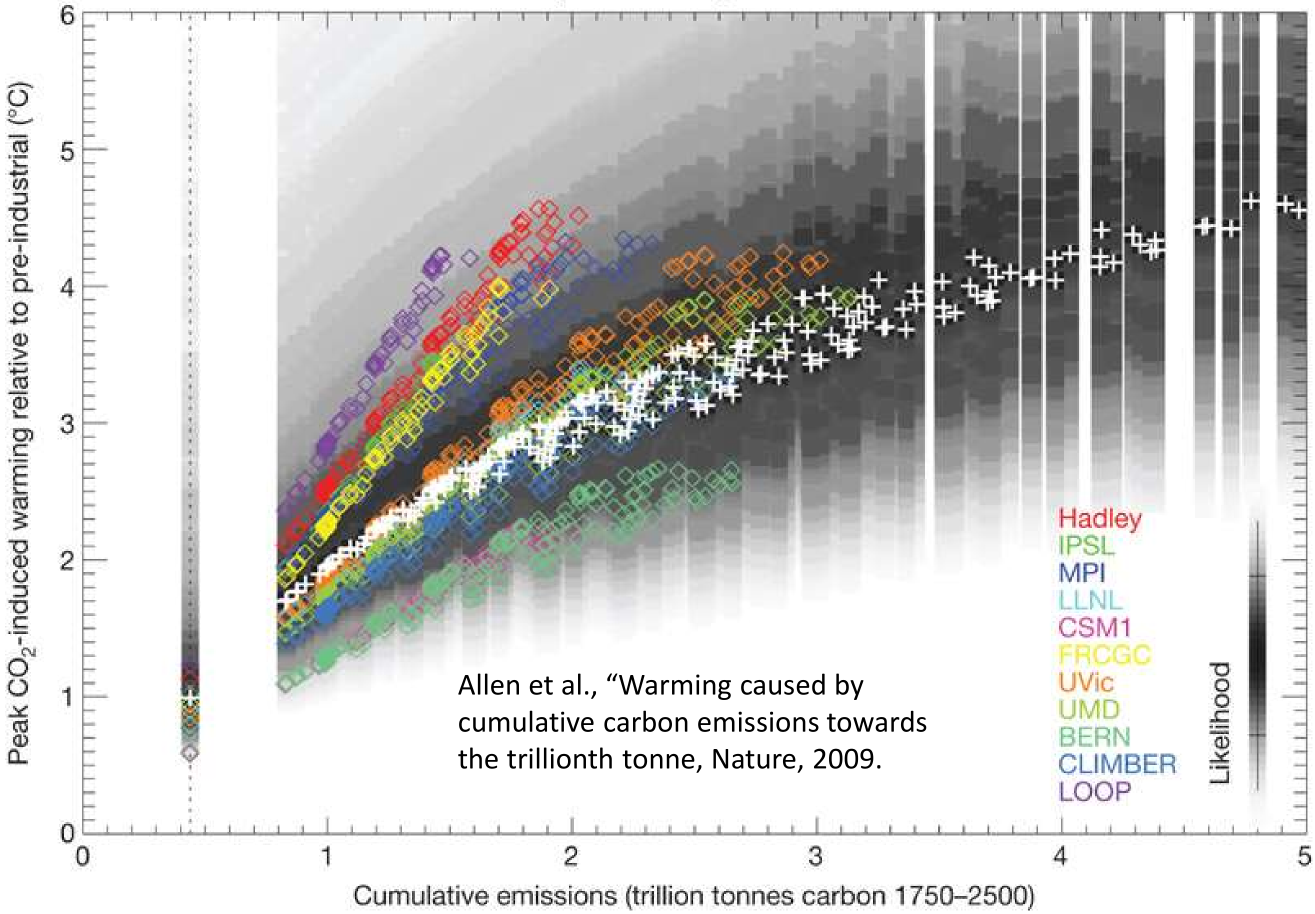
Workshop on Climate Studies
International Centre for Theoretical Sciences (ICTS-TIFR), Bangalore
02 March 2022

Proportionality between global warming and cumulative carbon dioxide emissions (CO_2)(Allen et al., 2009; Matthews et al., 2009)



Allen et al., "Warming caused by cumulative carbon emissions towards the trillionth tonne, Nature, 2009.

Also: Matthews et al., "Proportionality of global warming to cumulative carbon emissions", Nature, 2009.

Cumulative emissions accounting

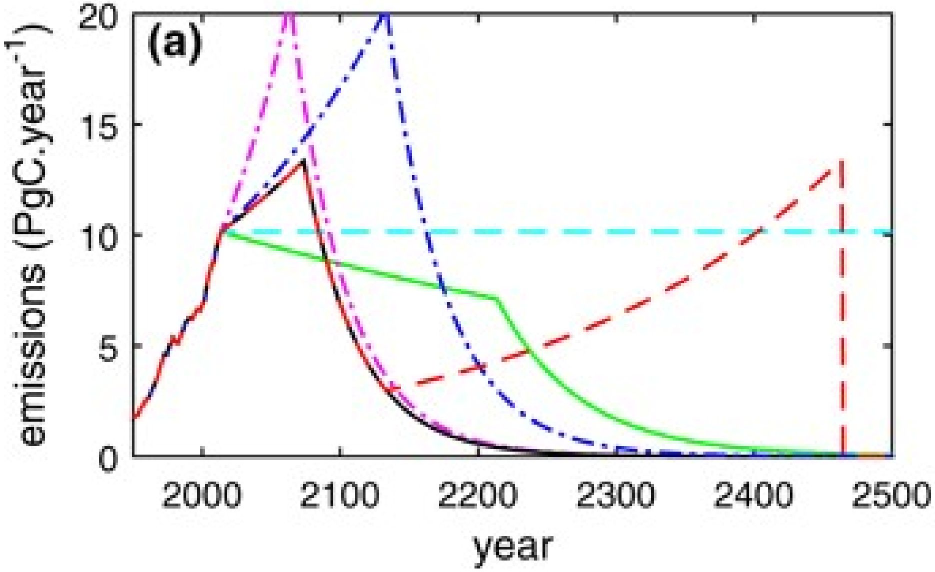
"The ratio of GMST [Global Mean Surface Temperature] change to total cumulative anthropogenic carbon emissions is relatively constant and independent of the scenario, but is model dependent, as it is a function of the model cumulative airborne fraction of carbon and the transient climate response. For any given temperature target, higher emissions in earlier decades therefore imply lower emissions by about the same amount later on." (IPCC AR5 WG1, Technical Summary, 2013)

IPCC AR5 WG1, Summary for Policymakers, 2013.

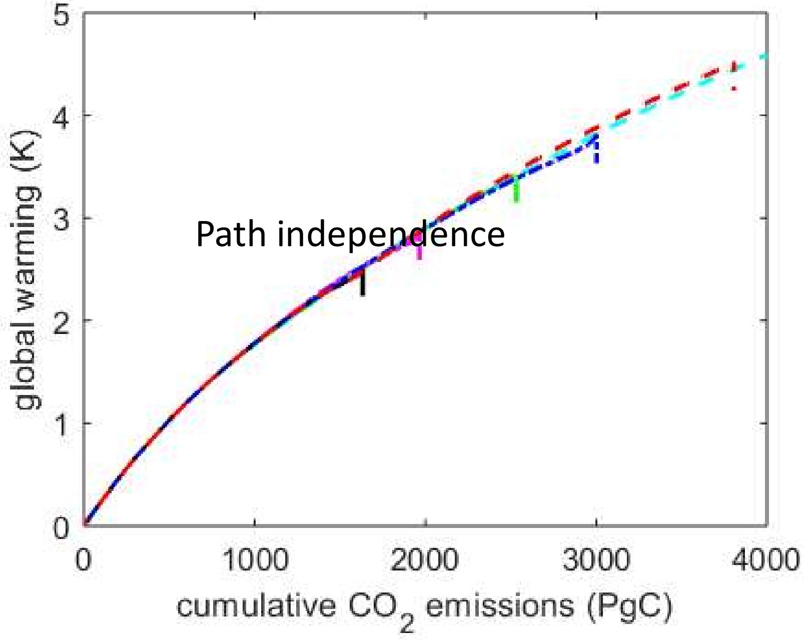
Cumulative emission accounting only requires path independence between global warming and cumulative emissions. Global warming should be a function of cumulative emissions alone, but not necessarily a linear function.

Illustration of path independence between global warming and cumulative CO_2 emissions

Different emissions pathways:



All pathways have the same relation between global warming & cumulative CO_2 emissions



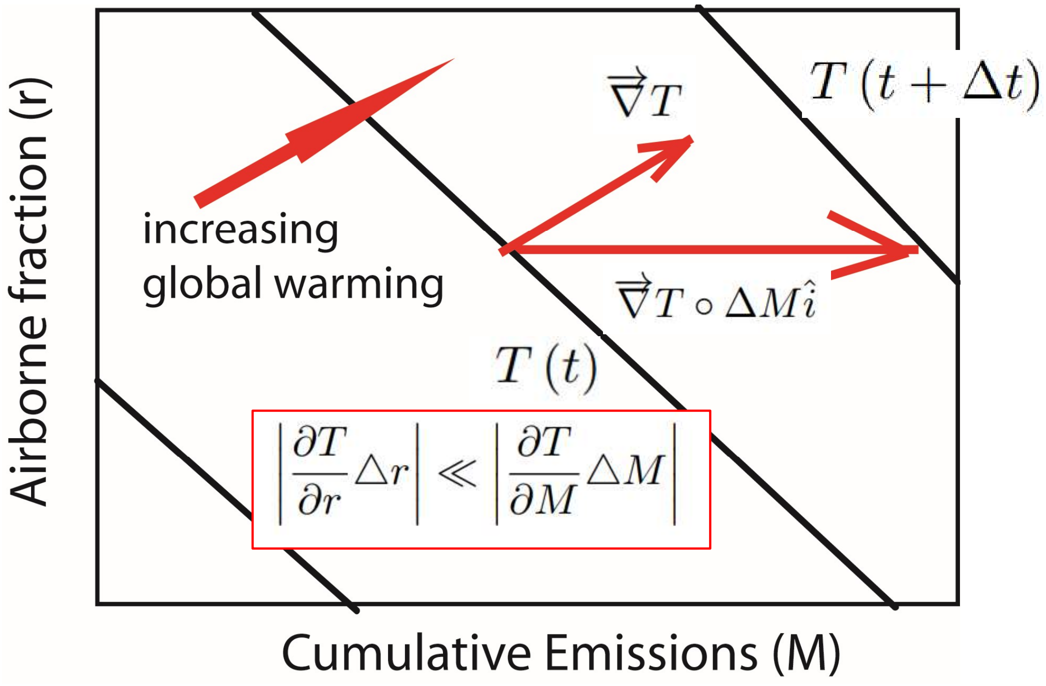
Path independence

Integration of energy balance model:

$$T(t) \approx \frac{\tau_f}{c_f}\left(\underbrace{F(t)}_{\text{Fast response}} + \frac{1}{\tau_D} e^{-t/\tau_s} \underbrace{\int_0^t e^{z/\tau_s} F(z)\,dz}_{\text{Slow response}}\right)$$

Seshadri, “Origin of path independence between cumulative CO_2 emissions and global warming”, Climate Dynamics, 2017.

Path independence occurs if the timescale for cumulative emissions change is short



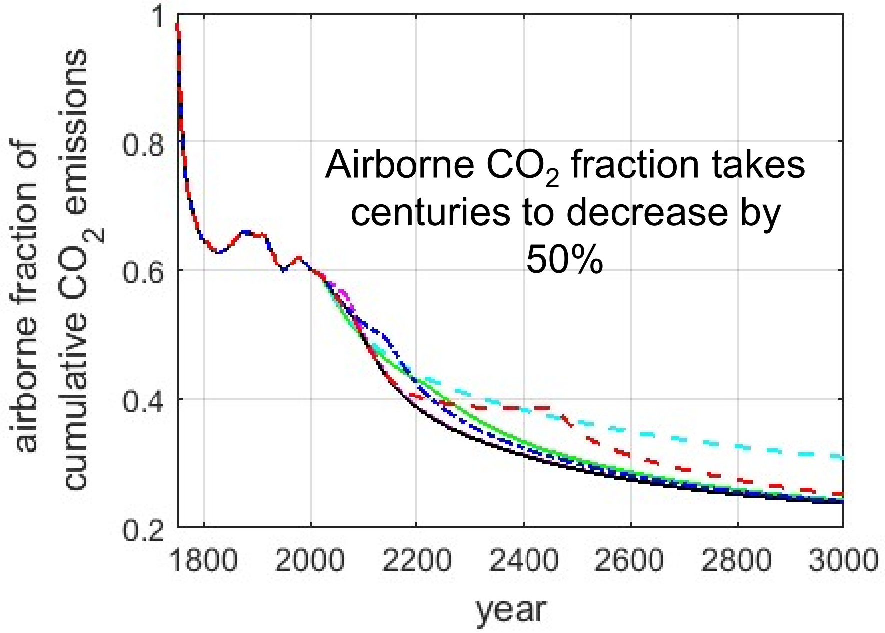
$$\overbrace{\tau_r}^{\text{Timescale for airborne fraction to change}} \gg \underbrace{\tau_M}_{\textbf{Timescale for cumulative emissions to change}} \ll \overbrace{\tau_D}^{\text{Damping timescale (slow contribution to global warming)}}$$

Eq. from Seshadri, Climate Dynamics, 2017

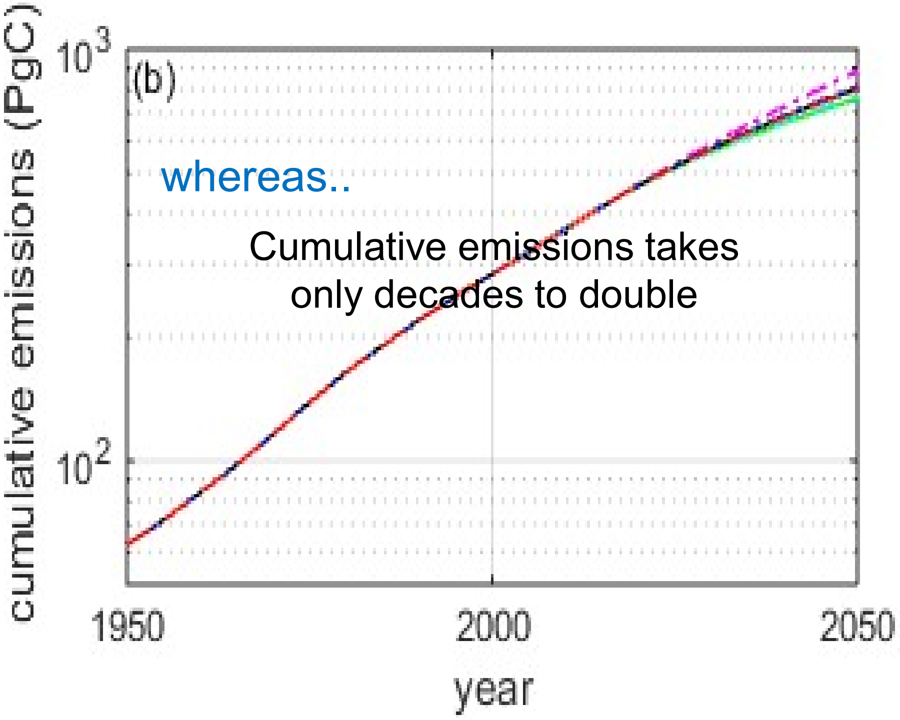
Origin of path independence for CO_2

Cumulative emissions changes more rapidly than airborne fraction

For CO_2, path independence occurs because:



Airborne CO_2 fraction takes centuries to decrease by 50%

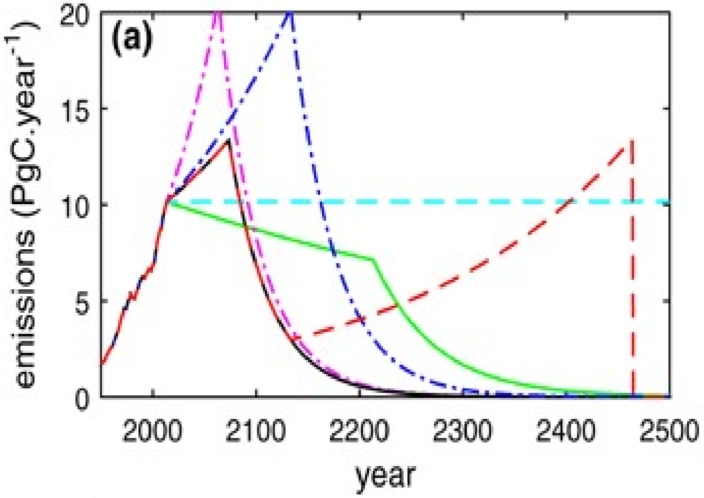


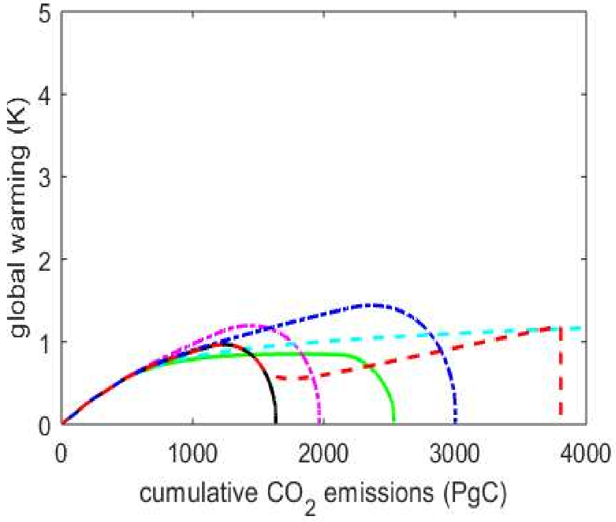
whereas..

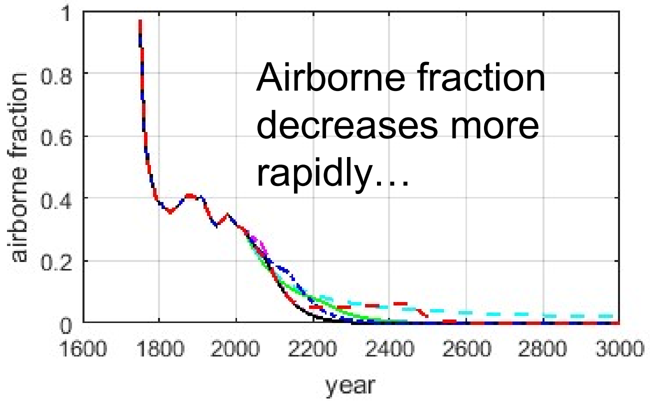
Cumulative emissions takes only decades to double

In the absence of slow processes in the carbon cycle…e.g. if CO_2 behaves like methane in the atmosphere

path independence would breakdown

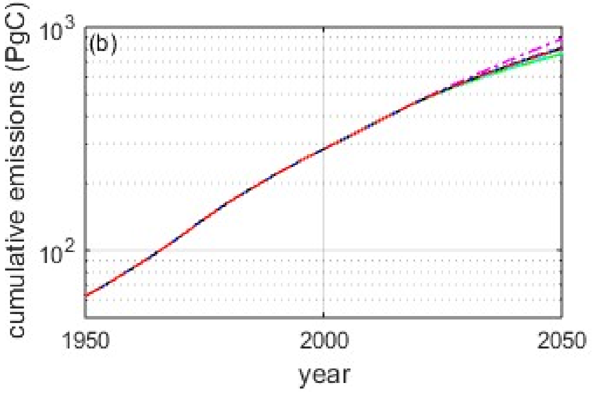






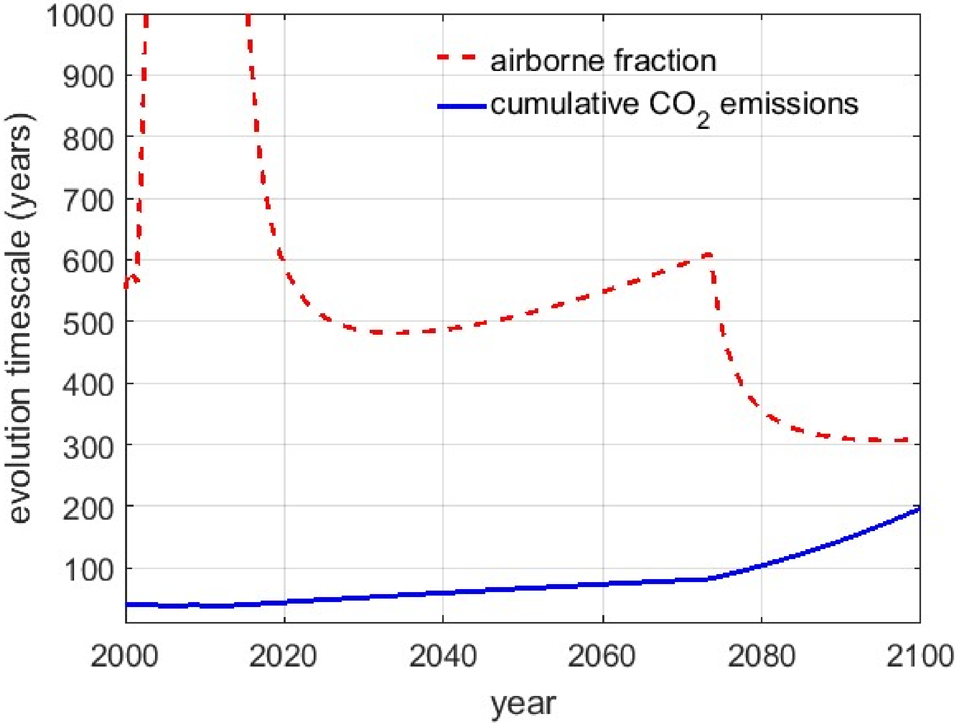
Airborne fraction decreases more rapidly…

..on timescales comparable to evolution of cumulative emissions

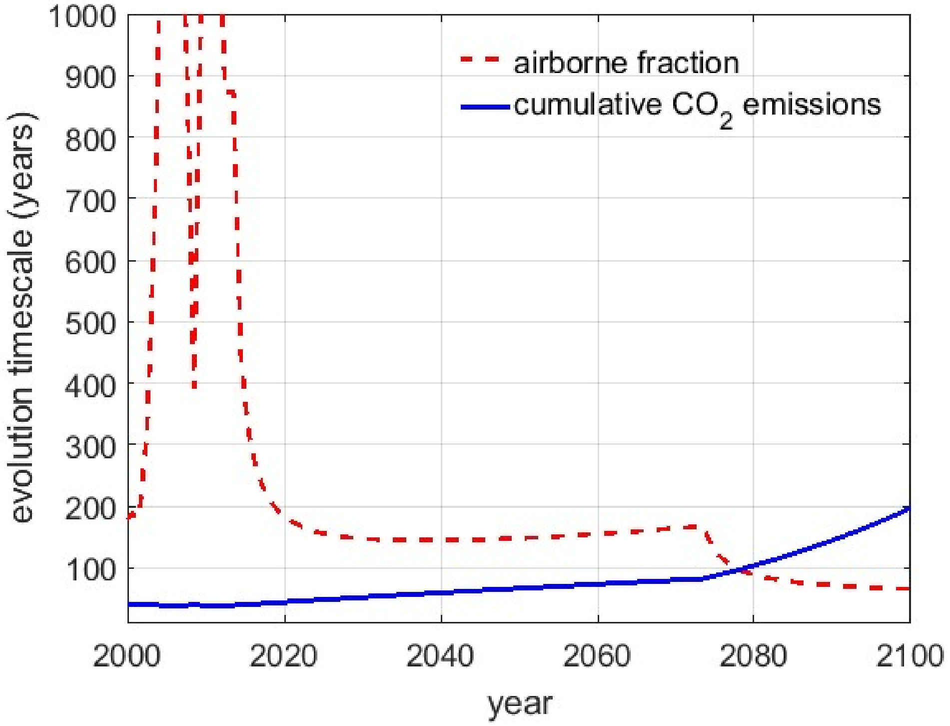


Comparison of timescales

Correct CO_2 model: with slow carbon cycle

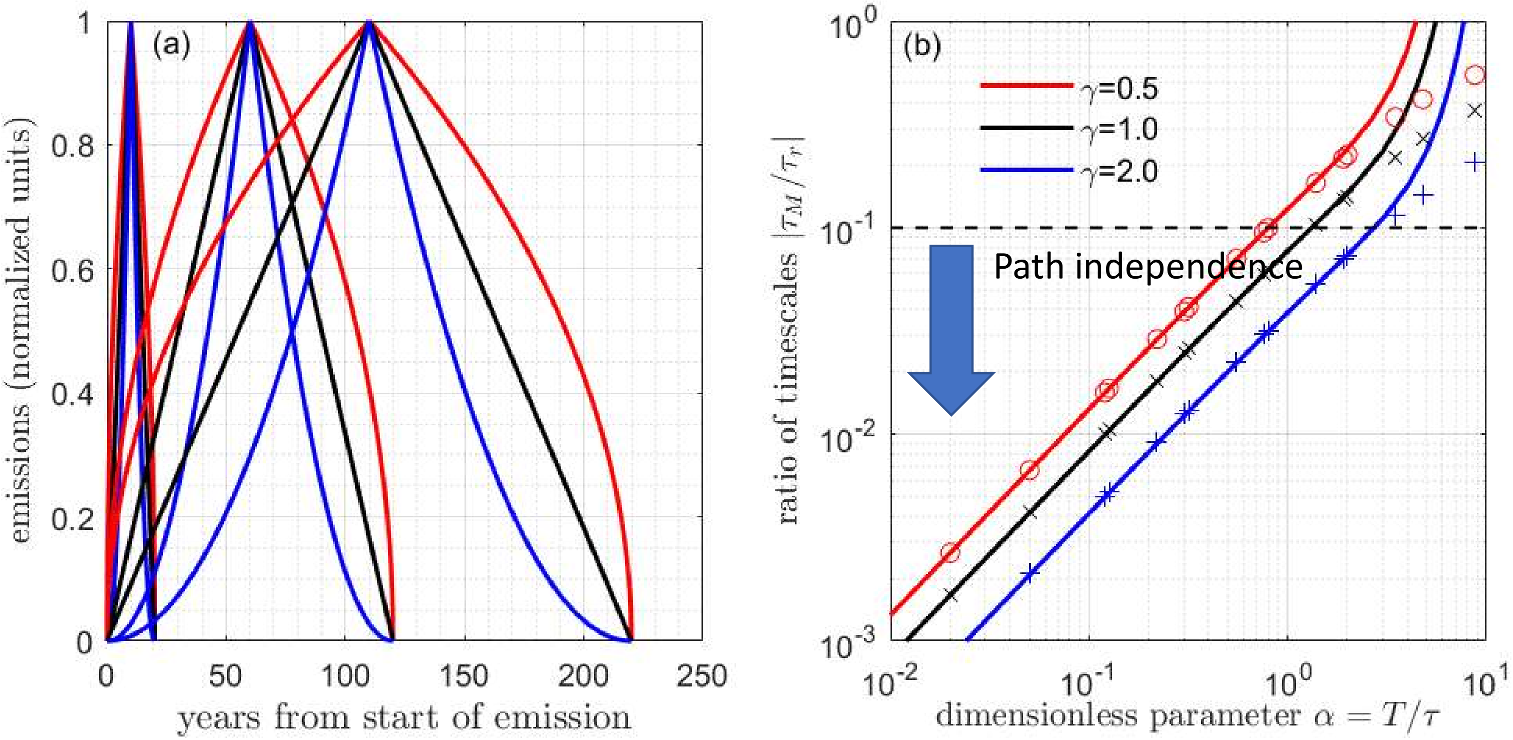


If CO_2 behaved like methane



If CO_2 behaved like methane, effect of changes in airborne fraction of CO_2 would not be negligible. Path independence would not have occurred.

For path independence, emissions cycle period should be comparable to or smaller than atmospheric lifetime



Path independence

(emissions cycle period / atmospheric lifetime)

Seshadri, Climate Dynamics, 2021

Derivation

$$\text{airborne fraction of cumulative emissions } r(t) = \overbrace{\left(C(t) - C_{eq}\right)}^{\text{Excess concentration}} / \overbrace{M(t)}^{\text{Cumulative emissions}}$$

Airborne fraction timescale:

$$-\frac{1}{\tau_r} = \frac{1}{r}\frac{dr}{dt} = \frac{1}{C(t) - C_{eq}}\frac{d}{dt}\left(C(t) - C_{eq}\right) - \frac{1}{M(t)}\frac{d}{dt}M(t)$$

Evolution of concentration:

$$\frac{dC(t)}{dt} = m(t) - \frac{C(t) - C_{eq}}{\tau} \qquad C(t) = C_{eq} + e^{-t/\tau}\int_0^t e^{s/\tau} m(s)\, ds$$

Rescaling time: $x = t/T$

$$C(x) - C_{eq} = Te^{-\alpha x}\int_0^x e^{\alpha s}\hat{m}(s)\, ds$$

$$\text{where } \hat{m}(x) = m(t/T) \text{ and } \alpha = T/\tau.$$

Seshadri, Climate Dynamics, 2021

Derivation (contd.)

Integrating by parts:

$$e^{-\alpha x}\int_0^x e^{\alpha_i s}\hat{m}(s)\,ds = \hat{m}_1(x) - \alpha\hat{m}_2(x) + \alpha^2\hat{m}_3(x) - \dots$$

where $\hat{m}_{i+1}(x) = \int_0^x \hat{m}_i(s)\,ds$ is the $i+1^{\mathrm{th}}$ repeated integral

$$\left|\frac{\tau_M}{\tau_r}\right| = \left|\frac{1 - \alpha\frac{\hat{m}_1(x)}{\hat{m}(x)} + \alpha^2\frac{\hat{m}_2(x)}{\hat{m}(x)} - \dots}{1 - \alpha\frac{\hat{m}_2(x)}{\hat{m}_1(x)} + \alpha^2\frac{\hat{m}_3(x)}{\hat{m}_1(x)} - \dots} - 1\right|$$

Depends on $\alpha = T/\tau$ and $x = t/T$

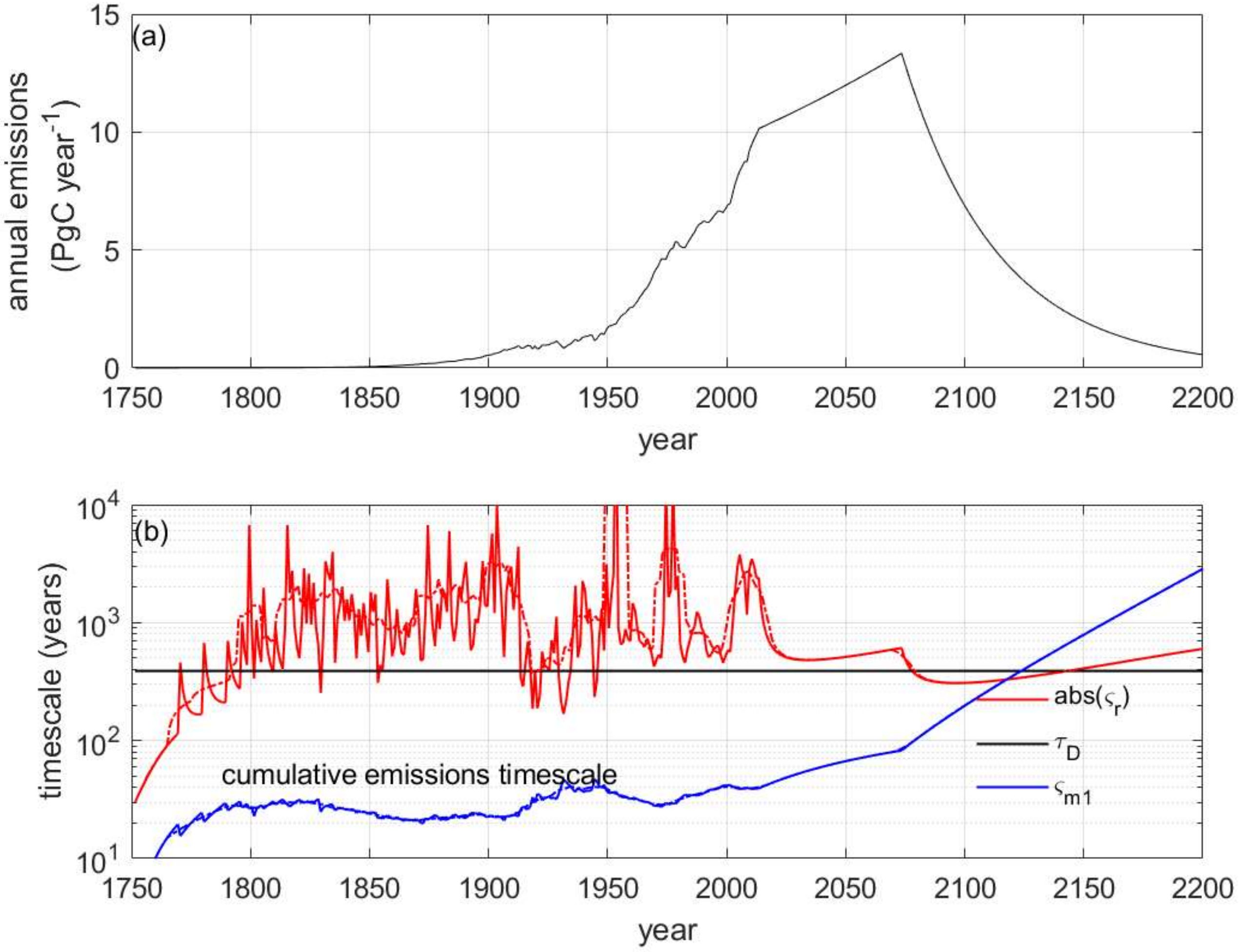
Emissions cycle period / atmospheric lifetime

Position in emissions cycle

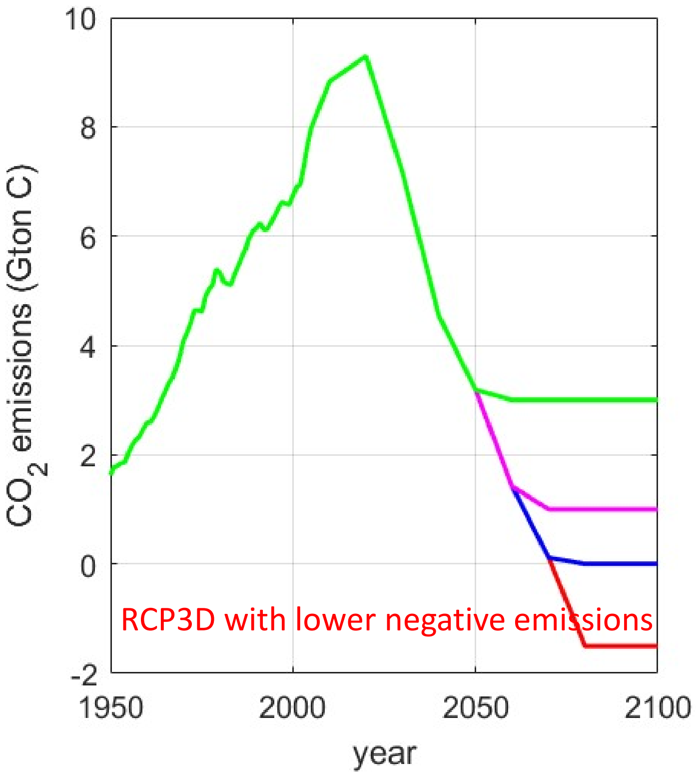
Cumulative emissions timescale grows as emissions cycle proceeds

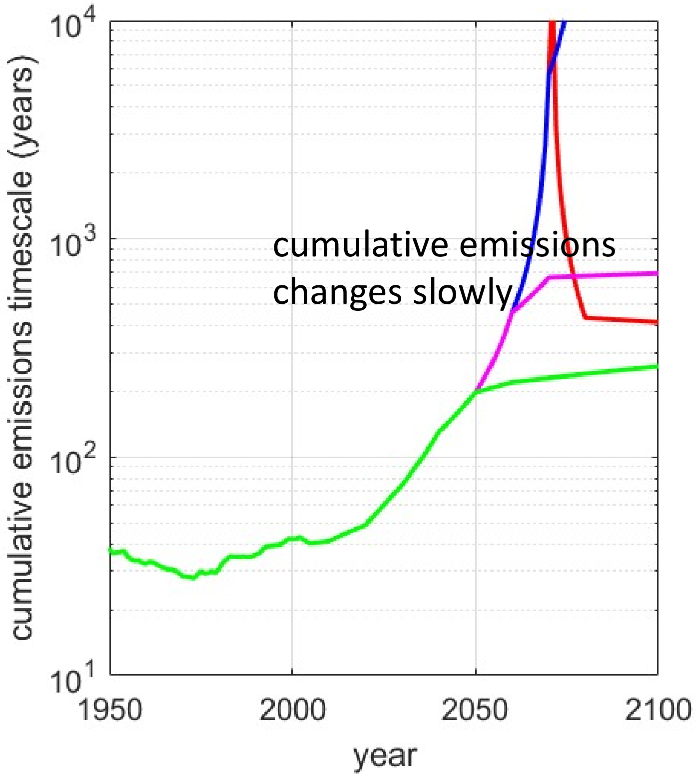
cumulative emissions timescale = cumulative emissions / emissions

Seshadri, “Origin of path independence between cumulative CO_2 emissions and global warming”, Climate Dynamics, 2017.

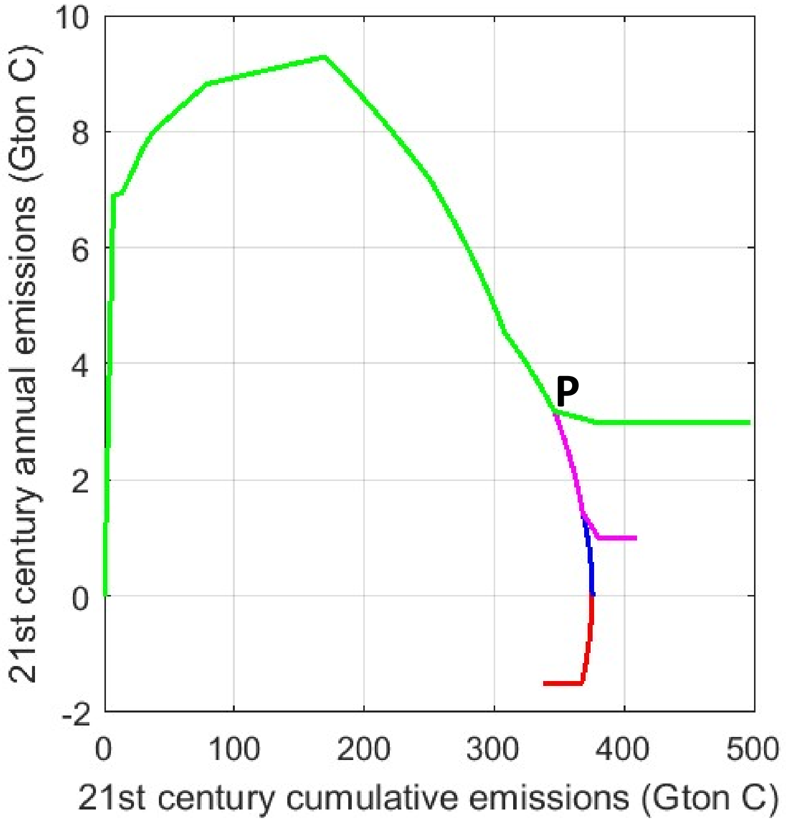


Cumulative emissions timescale is persistently large in scenarios with low emissions

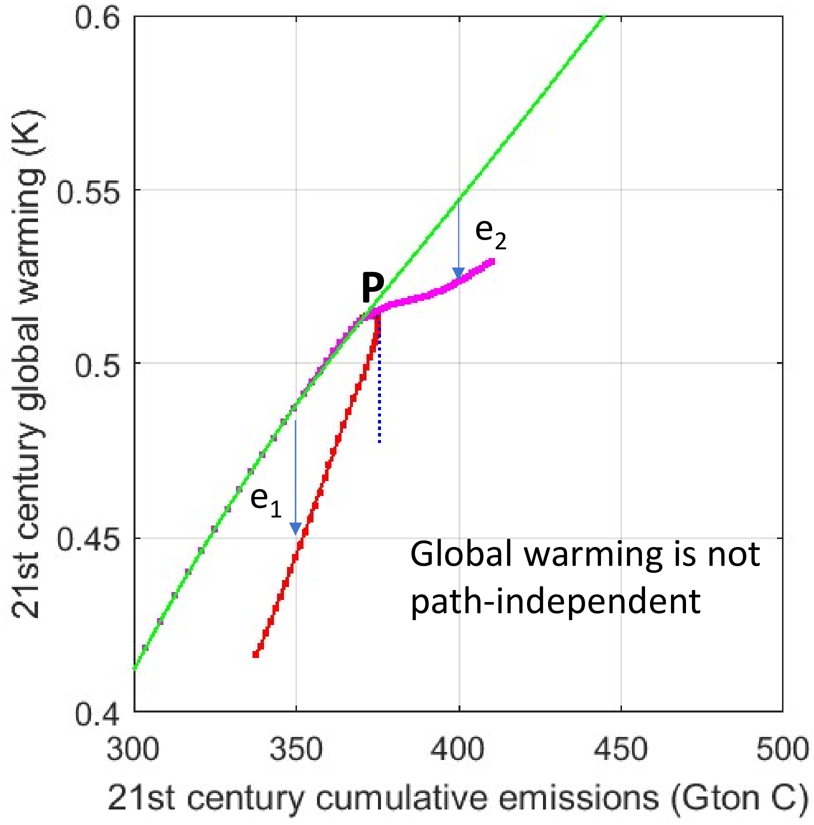

RCP3D with lower negative emissions


cumulative emissions changes slowly

Path independence does not work if emissions are small but persistent



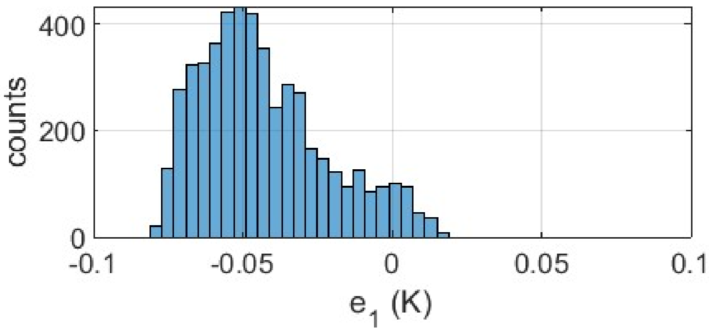
Average global warming in 5000-member MAGICC ensemble

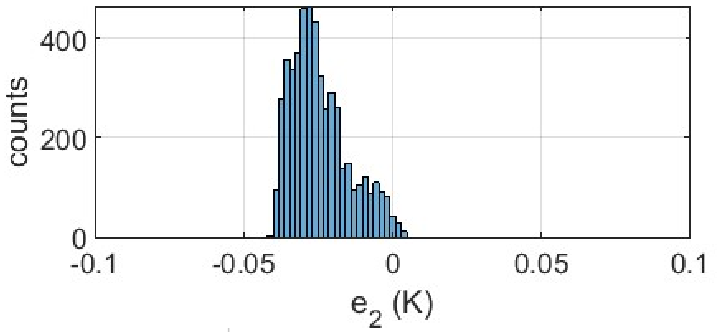


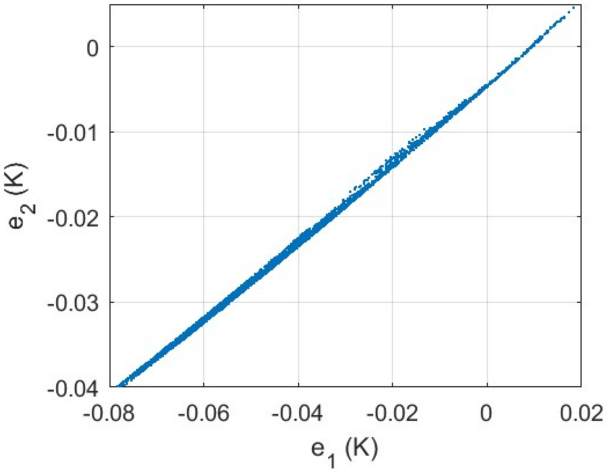
Global warming is not path-independent

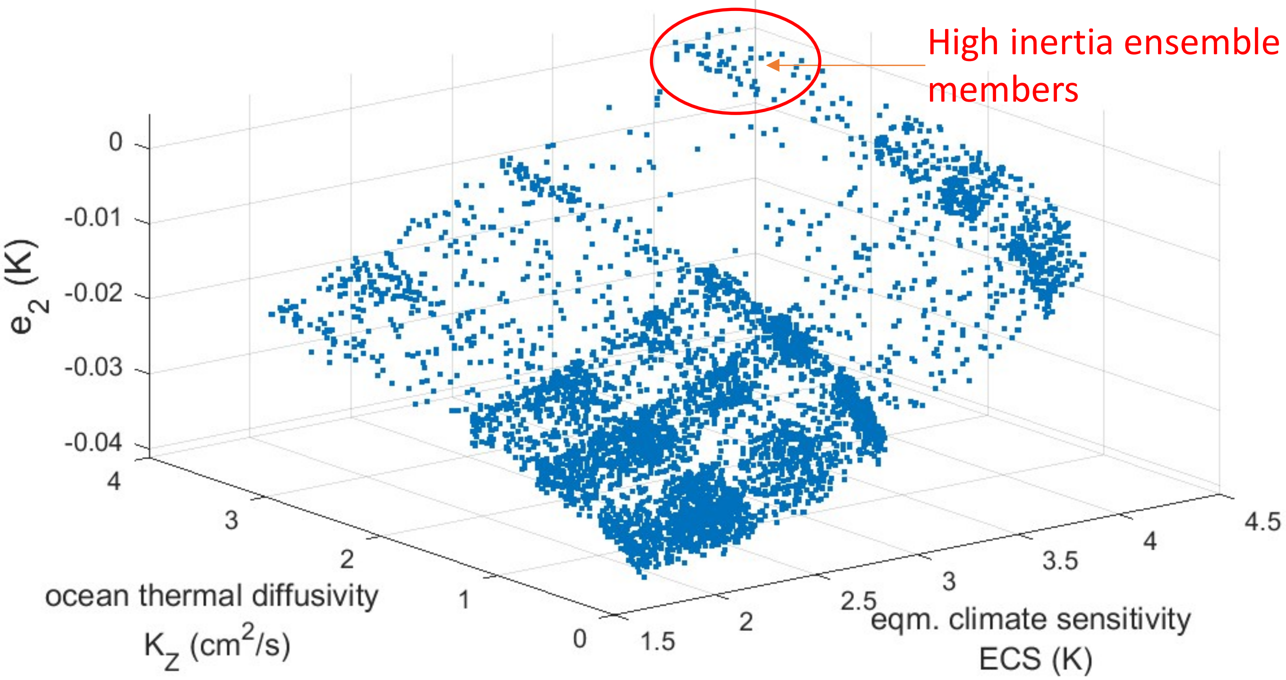
Path independence error depends on inertia in climate system

Results of analysis of 5000-member MAGICC ensemble



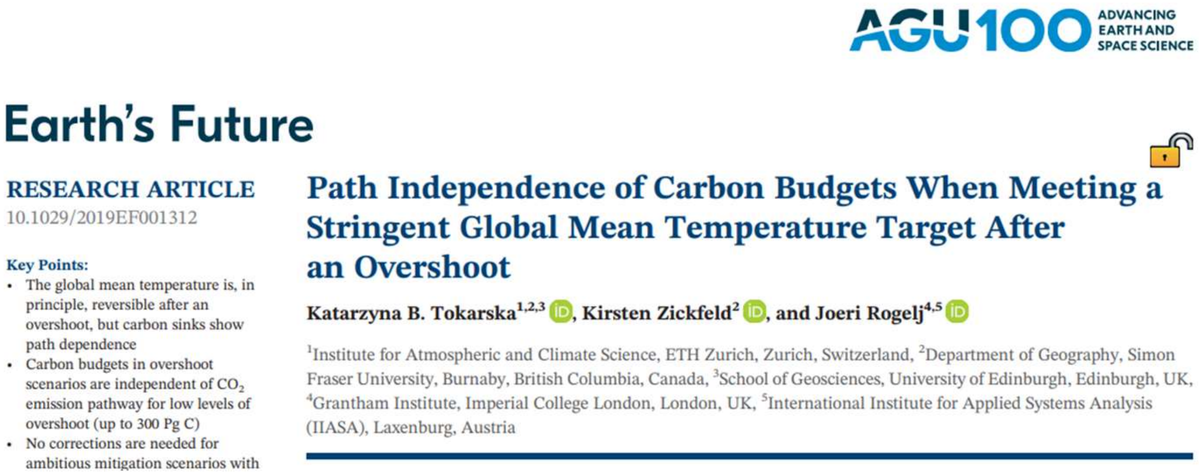






High inertia ensemble members

Earth system models are required to understand terrestrial and ocean feedbacks and resulting hysteresis



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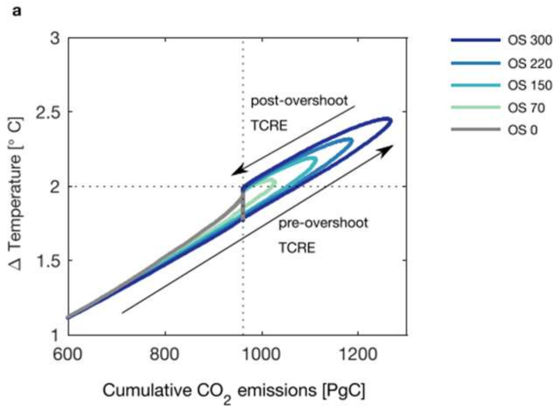
Key Points:

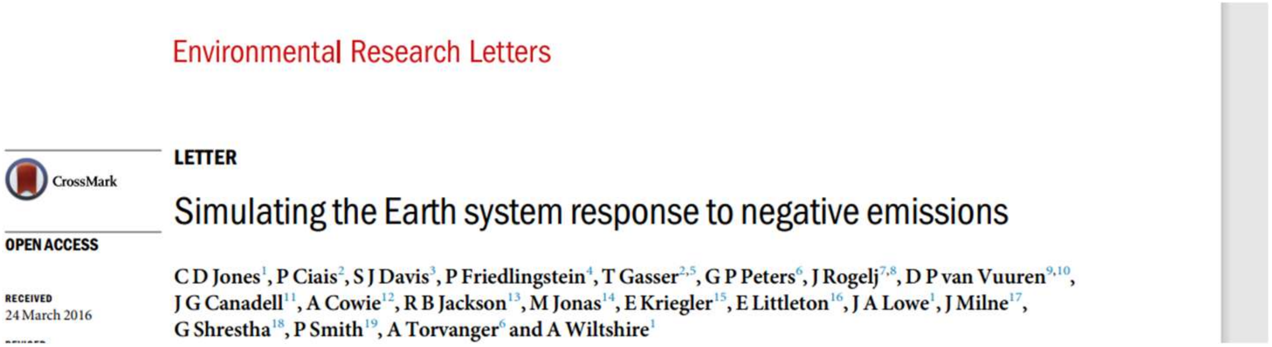
- The global mean temperature is, in principle, reversible after an overshoot, but carbon sinks show path dependence
- Carbon budgets in overshoot scenarios are independent of CO_2 emission pathway for low levels of overshoot (up to 300 Pg C)
- No corrections are needed for ambitious mitigation scenarios with

Path Independence of Carbon Budgets When Meeting a Stringent Global Mean Temperature Target After an Overshoot

Katarzyna B. Tokarska[1,2,3], Kirsten Zickfeld[2], and Joeri Rogelj[4,5]

[1]Institute for Atmospheric and Climate Science, ETH Zurich, Zurich, Switzerland, [2]Department of Geography, Simon Fraser University, Burnaby, British Columbia, Canada, [3]School of Geosciences, University of Edinburgh, Edinburgh, UK, [4]Grantham Institute, Imperial College London, London, UK, [5]International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria





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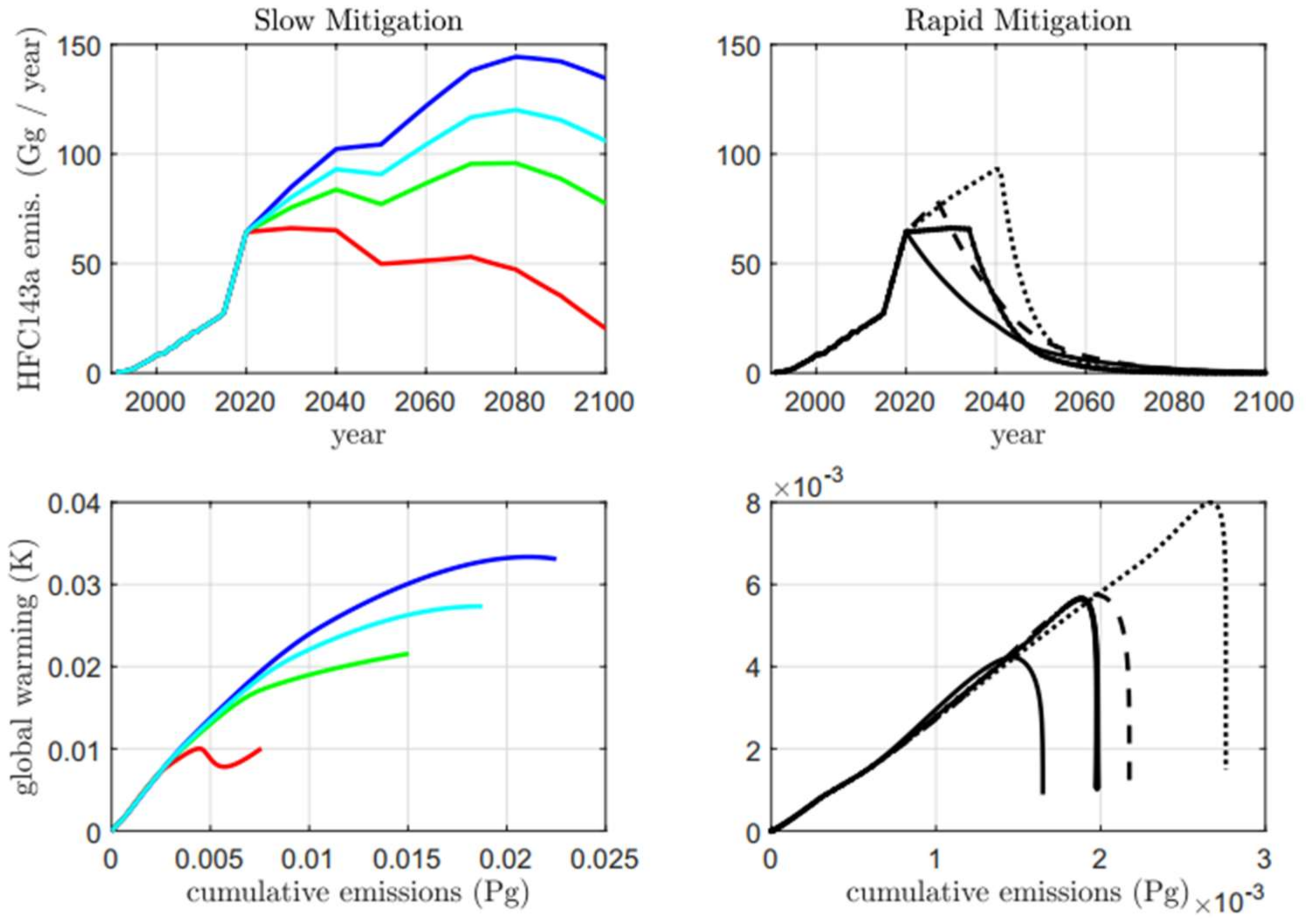
LETTER

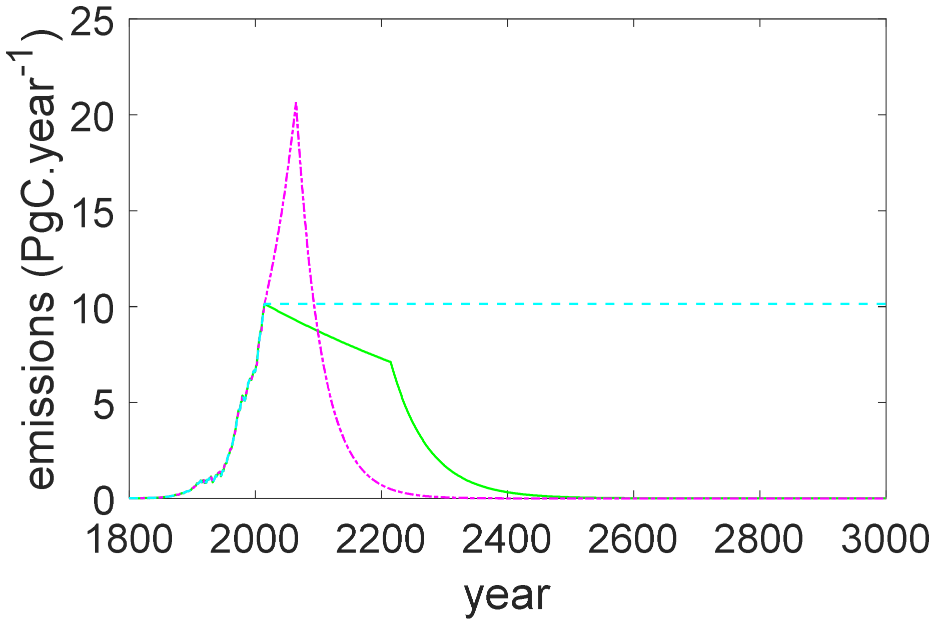
Simulating the Earth system response to negative emissions

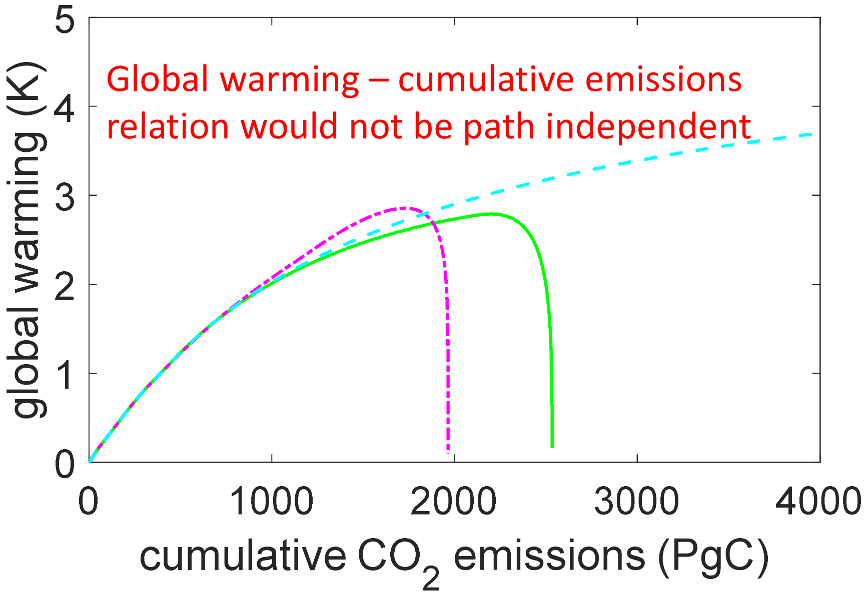
C D Jones[1], P Ciais[2], S J Davis[3], P Friedlingstein[4], T Gasser[2,5], G P Peters[6], J Rogelj[7,8], D P van Vuuren[9,10], J G Canadell[11], A Cowie[12], R B Jackson[13], M Jonas[14], E Kriegler[15], E Littleton[16], J A Lowe[1], J Milne[17], G Shrestha[18], P Smith[19], A Torvanger[6] and A Wiltshire[1]

Cumulative emissions accounting works if the emissions cycle plays out over a period shorter than the atmospheric lifetime (an example)



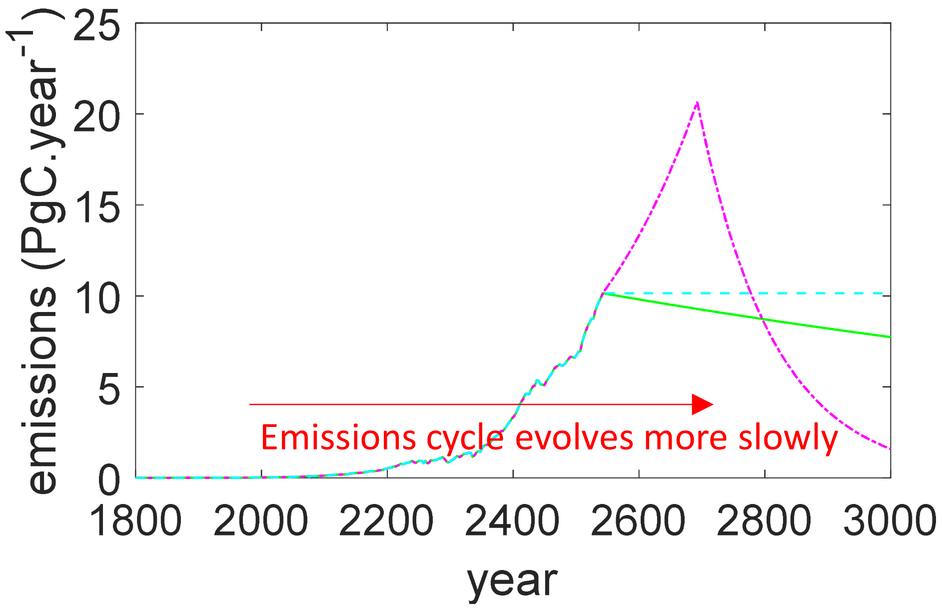
Why is there cumulative emissions accounting for CO_2?
For illustration, imagine that CO_2 had atmospheric lifetime of 100 years:



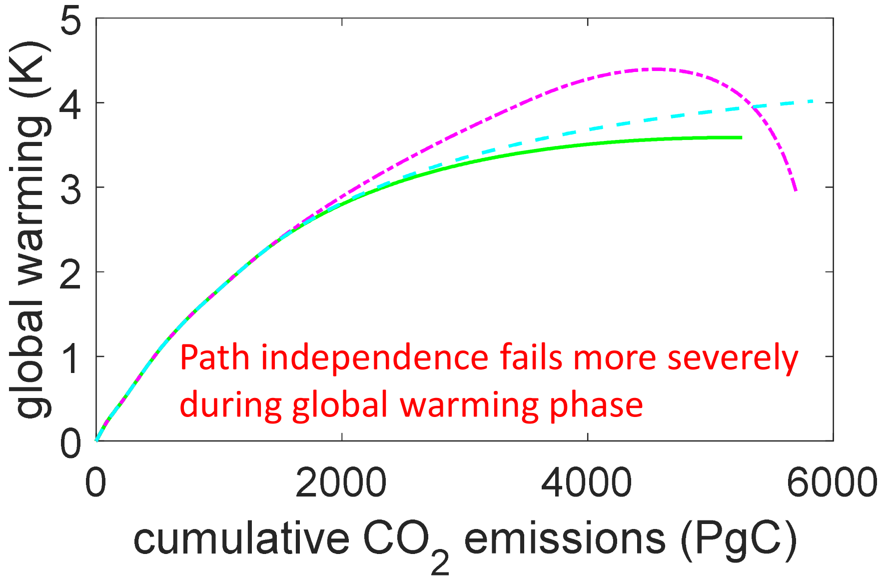


Global warming – cumulative emissions relation would not be path independent

If emissions evolves more slowly, the path independence approximation is poorer

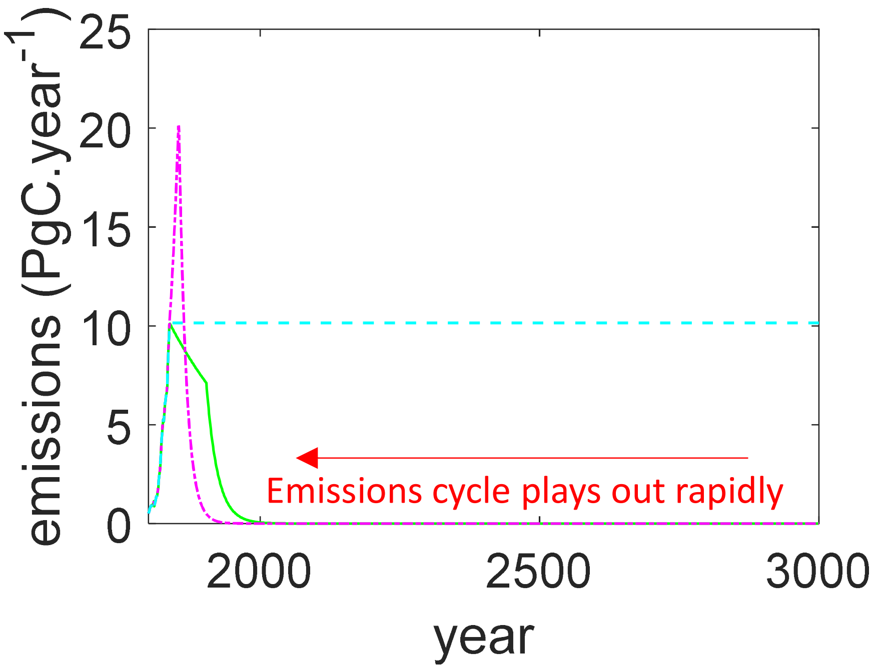


Emissions cycle evolves more slowly

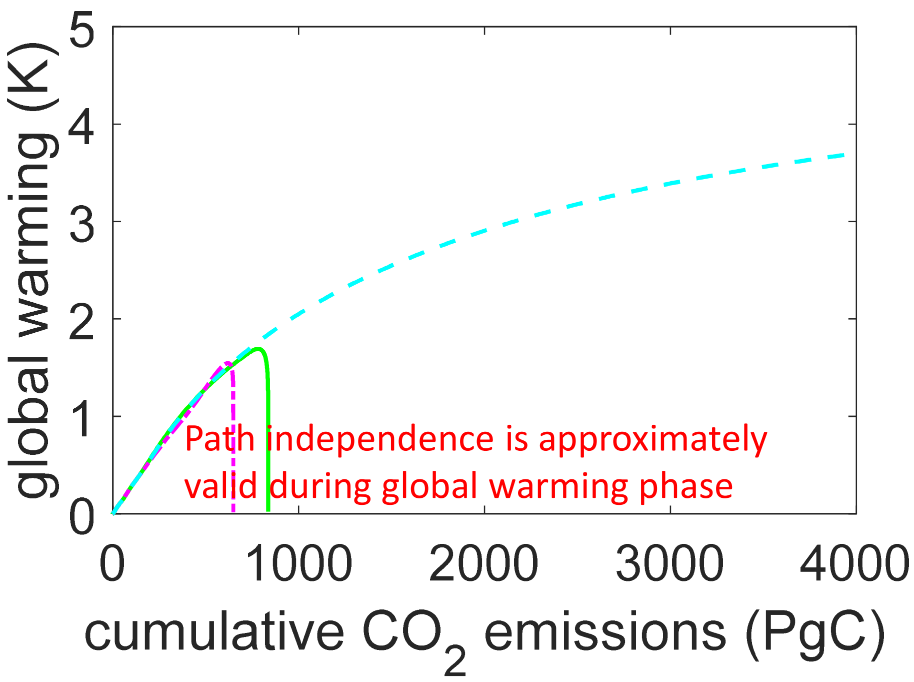


Path independence fails more severely during global warming phase

If the emissions cycle had played out more quickly, path independence would have been accurate, even with an atmospheric lifetime of 100 years



Emissions cycle plays out rapidly



Path independence is approximately valid during global warming phase

Summary

- Path independence arises from weak constraints, some timescales must be much smaller than others: hence provides a firmer basis than proportionality
- Also gives rise to counterfactual accounts about its origins (can ask questions not only about the "how" of path independence but also the "why")
- For CO_2, owing to slow processes in the carbon cycle, airborne fraction changes on multi-century timescales (i.e. slowly)
- Cumulative emissions timescale is comparatively short (decades), when considering emissions scenarios that play out over a couple of centuries (20th & 21st century)
- Changes in cumulative emissions drive global warming, and new temperature contours are picked by changes in cumulative emissions
- However, near the end of the emissions cycle, cumulative emissions varies slowly. It is no longer necessarily the dominant effect. Moreover, this is where path independence can fail.
- For scenarios with persistent nonzero but small emissions, global warming from CO_2 is no longer independent of emissions pathway ("path independence"). Earth system models are necessary to estimate the resulting effects.

Bifurcation in monsoon model of Zickfeld et al. (2005)

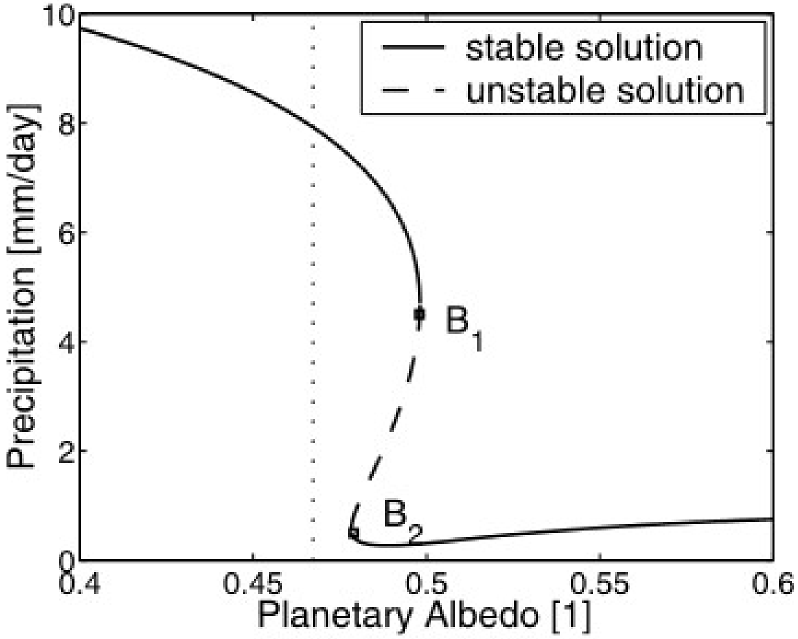
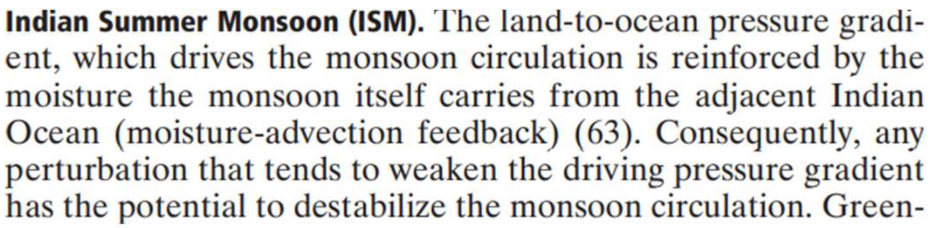


Figure 1. Bifurcation diagram of Indian summer precipitation against the planetary albedo. The B_1, B_2 mark the saddle-node bifurcations. The vertical dotted line indicates the present-day state.

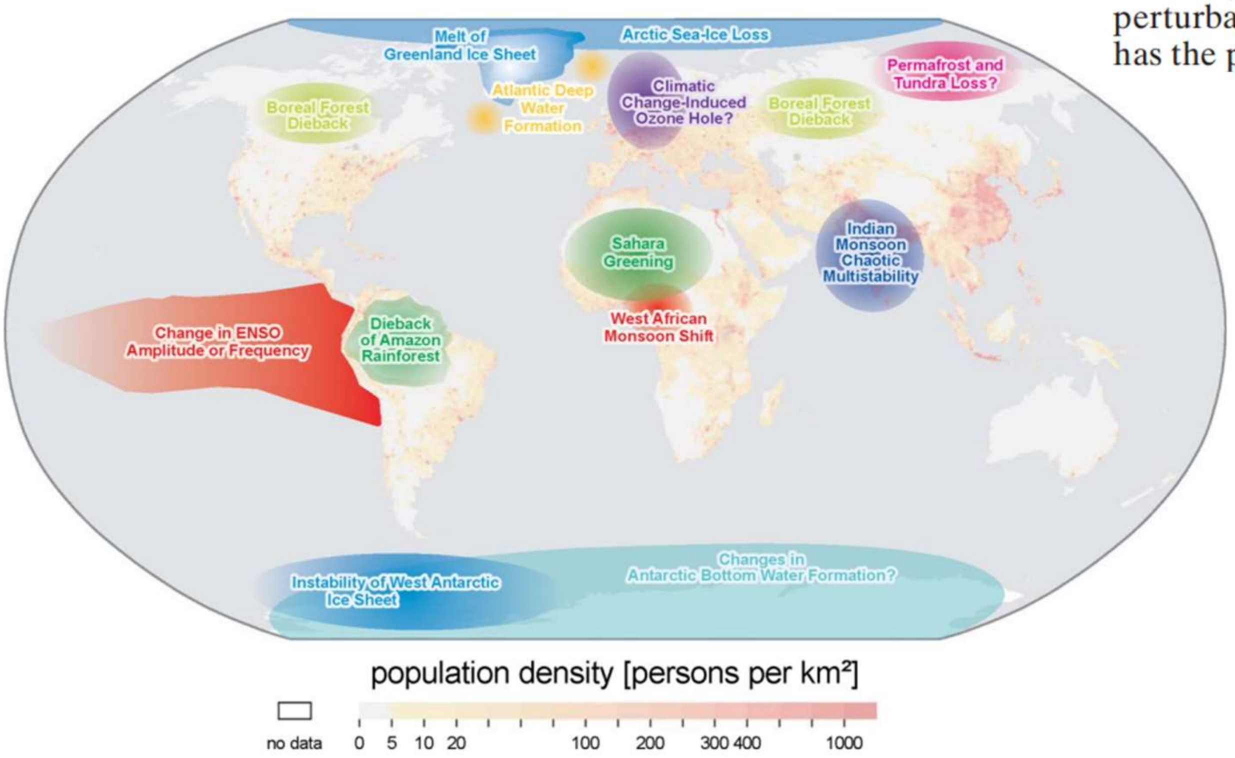
Zickfeld et al., “Is the Indian summer monsoon stable against global change?”, Geophysical Research Letters, 2005

Tipping elements in the Earth's climate system

Timothy M. Lenton*†, Hermann Held‡, Elmar Kriegler‡§, Jim W. Hall¶, Wolfgang Lucht‡, Stefan Rahmstorf‡, and Hans Joachim Schellnhuber†‡‖**



Indian Summer Monsoon (ISM). The land-to-ocean pressure gradient, which drives the monsoon circulation is reinforced by the moisture the monsoon itself carries from the adjacent Indian Ocean (moisture-advection feedback) (63). Consequently, any perturbation that tends to weaken the driving pressure gradient has the potential to destabilize the monsoon circulation. Green-



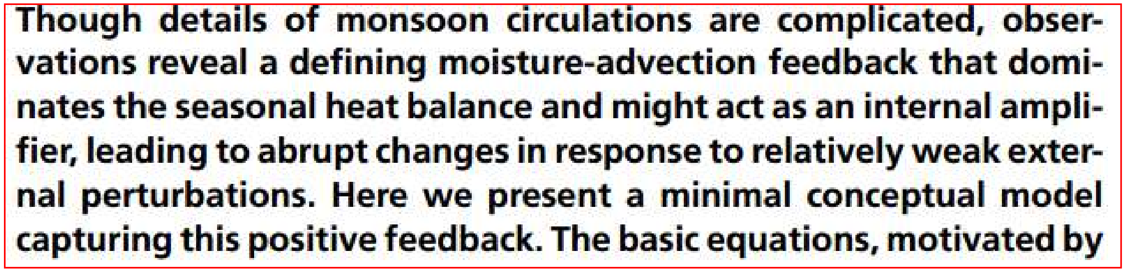
Lenton et al., Tipping elements in the Earth's climate system, PNAS, 2008

Basic mechanism for abrupt monsoon transitions

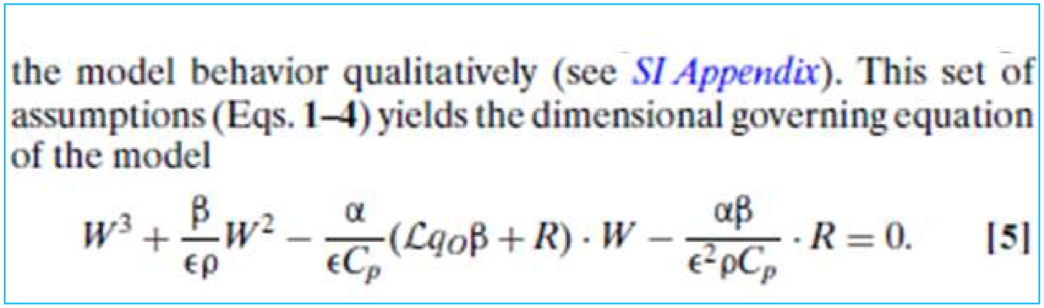
Anders Levermann[a,b,1], Jacob Schewe[a,b], Vladimir Petoukhov[a], and Hermann Held[a]

[a]Earth System Analysis, Potsdam Institute for Climate Impact Research, 14473 Potsdam, Germany; and [b]Institute of Physics, Potsdam University, 14473 Potsdam, Germany

Edited by Hans Joachim Schellnhuber, Potsdam Institute for Climate Impact Research, Potsdam, Germany and approved August 18, 2009 (received for review February 11, 2009)



Though details of monsoon circulations are complicated, observations reveal a defining moisture-advection feedback that dominates the seasonal heat balance and might act as an internal amplifier, leading to abrupt changes in response to relatively weak external perturbations. Here we present a minimal conceptual model capturing this positive feedback. The basic equations, motivated by

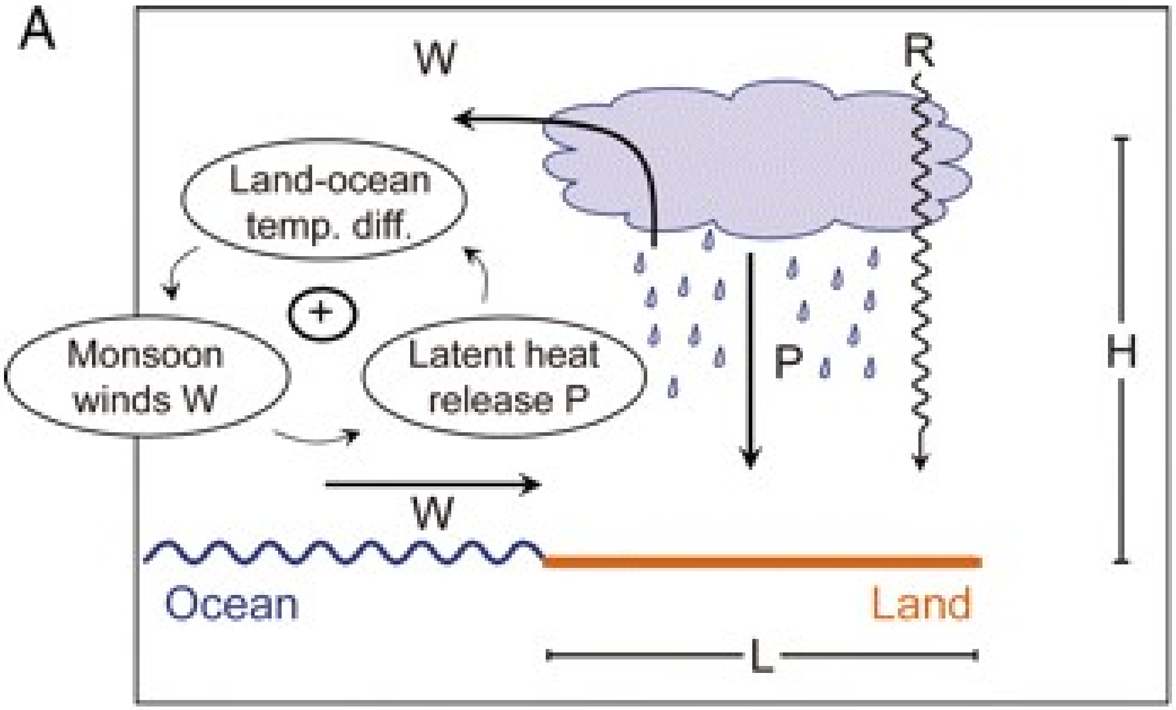


the model behavior qualitatively (see *SI Appendix*). This set of assumptions (Eqs. 1–4) yields the dimensional governing equation of the model

$$W^3 + \frac{\beta}{\epsilon\rho}W^2 - \frac{\alpha}{\epsilon C_p}(\mathcal{L}q_0\beta + R)\cdot W - \frac{\alpha\beta}{\epsilon^2\rho C_p}\cdot R = 0. \qquad [5]$$

Levermann et al., “Basic mechanism for abrupt monsoon transitions”, PNAS, 2009

Schematic of the model



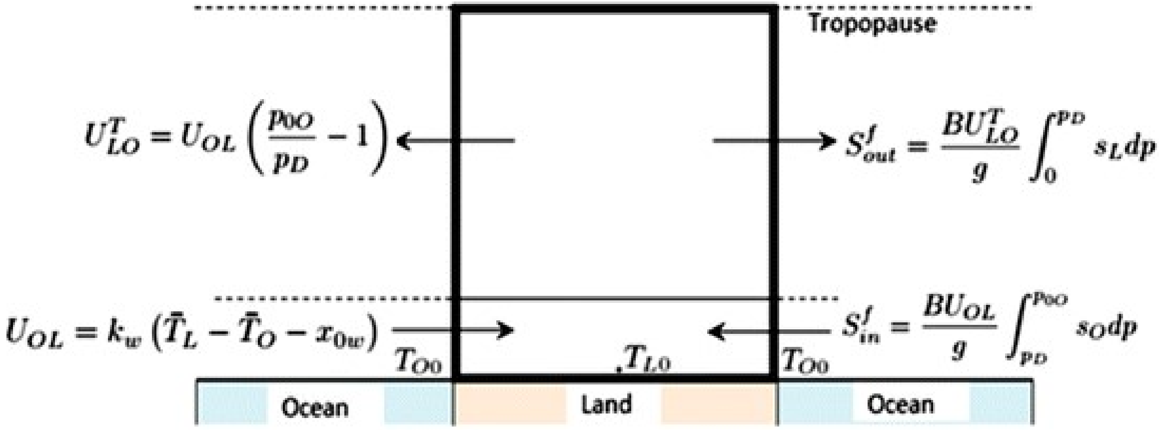
Levermann et al., “Basic mechanism for abrupt monsoon transitions”, PNAS, 2009

“The abruptness of the transition emerges through an additional stabilizing effect of the **direct heat advection which is cooling the atmospheric column [emphasis added]** and is also reduced for reduced monsoon winds. “

Bifurcation in this model is caused by interaction between destabilizing effect of moisture advection feedback and the nonlinearity of the temperature advection (which cools the column).

Dry static energy dynamics

Stabilizing flux has linear as well as a quadratic term in horizontal temperature difference



Dry static energy:

$$s = c_pT + gz$$

$$\frac{\partial \rho s}{\partial t} = \rho g \frac{\partial\left(F^R + F^S\right)}{\partial p} + \rho Q_{LH} - \nabla_H \circ \rho s\vec{V} - \frac{\partial \rho s \omega}{\partial p}$$

$$G_T(x)\frac{dx}{dt} = F(x) \equiv \left(F_{net}^{SW}(x) + F_{Net}^{LW}(x) + F_B^S(x)\right) + F_{LH}(x) + \frac{1}{A}S_{net}^{f}(x)$$

$$S_{net}^{f} \cong -\frac{Bk_w}{\Gamma_D}\left(p_{0O} - p_D\right)\left\{\left(\frac{R}{c_p}\boxed{\frac{\Gamma_D - \Gamma_O}{\Gamma_D}}\bar{T}_O\right)x + \left(1 + \frac{R}{c_p}\right)x^2\right\}$$

Dry static stability: lapse rate is smaller than dry adiabatic

Column-mean temperature gradient x is important for monsoon dynamics

Seshadri, “Energetics and monsoon bifurcations”, Climate Dynamics, 2016.

Condition for bifurcation

$$\dot{x} = f(x, \alpha)$$

$$f(x_0, \alpha_0) + \frac{\partial f}{\partial x}\triangle x + \frac{\partial f}{\partial \alpha}\triangle \alpha = 0$$

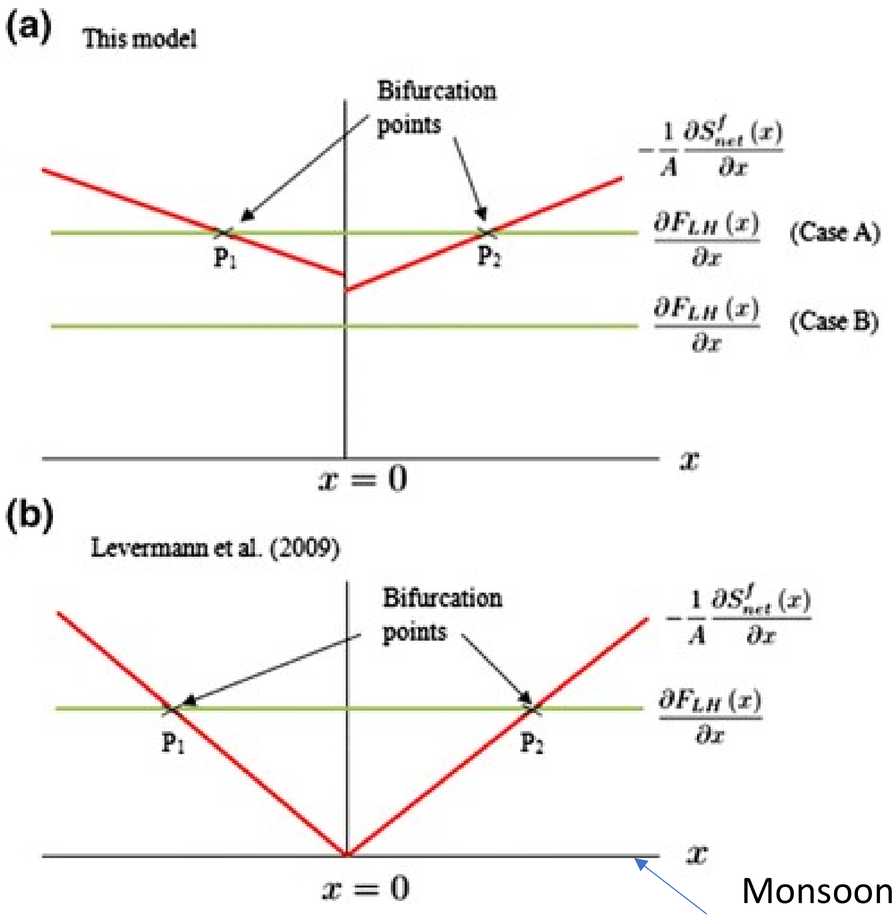
$$\triangle x = -\frac{\partial f / \partial \alpha}{\partial f / \partial x}\triangle \alpha$$

Bifurcation occurs if: $\frac{\partial f}{\partial x} = 0$ i.e. eigenvalue at steady state is zero

Governing eq. for Dry Static Energy:

$$G_T(x)\frac{dx}{dt} = F(x) \equiv \overbrace{\left(F^{SW}_{net}(x) + F^{LW}_{Net}(x) + F^{S}_{B}(x)\right)}^{\text{depends weakly on x}} + \overbrace{F_{LH}(x)}^{\text{increases strongly with x}} + \underbrace{\frac{1}{A}S^{f}_{net}(x)}_{\text{negative; increases strongly in magnitude with x}}$$

Condition for bifurcation in dry static energy model



red: negative contribution to eigenvalue
green: positive contribution to eigenvalue

$$s = c_p T + gz$$

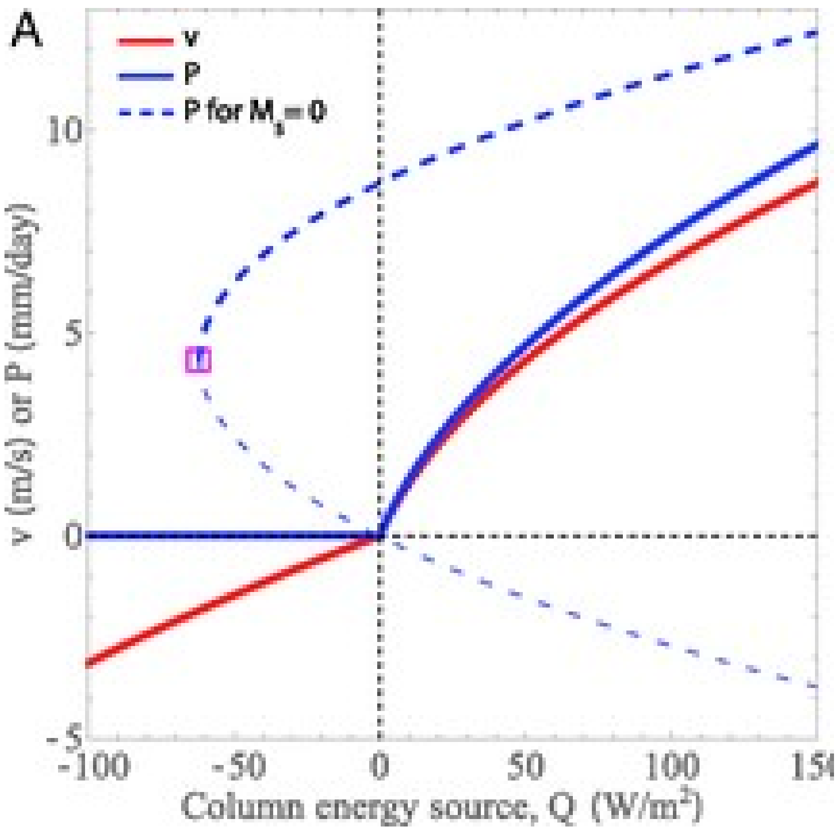
$$\frac{ds}{dz} = c_p\left(\Gamma_d - \Gamma\right)$$

Stabilizing effect is much weaker if dry static stability is ignored

Monsoon steady state is somewhere here

Seshadri, “Energetics and monsoon bifurcations”, Climate Dynamics, 2016.

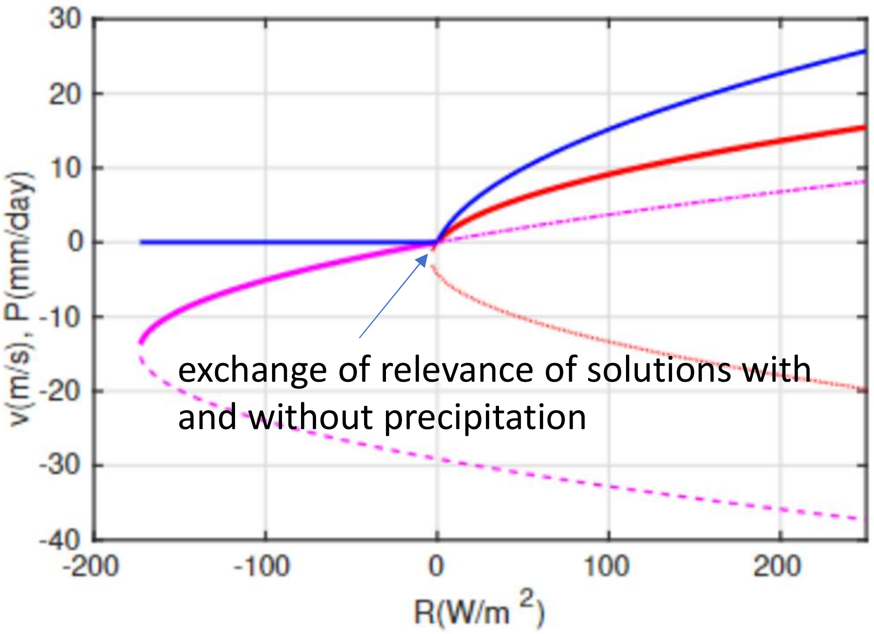
Study based on Quasi Equilibrium Tropical Circulation Model (QTCM)



“Here it is shown that the theory used to predict such “tipping points” omits a dominant term in the equations of motion, and that both a corrected theory and an ensemble of global climate model simulations exhibit no abrupt shift in monsoon strength in response to large changes in various forcings.”

Boos and Storelvmo, “Near-linear response of mean monsoon strength to a broad range of radiative forcings”, PNAS, 2016.

Bifurcation is present but not in the monsoon regime, so not physically manifest

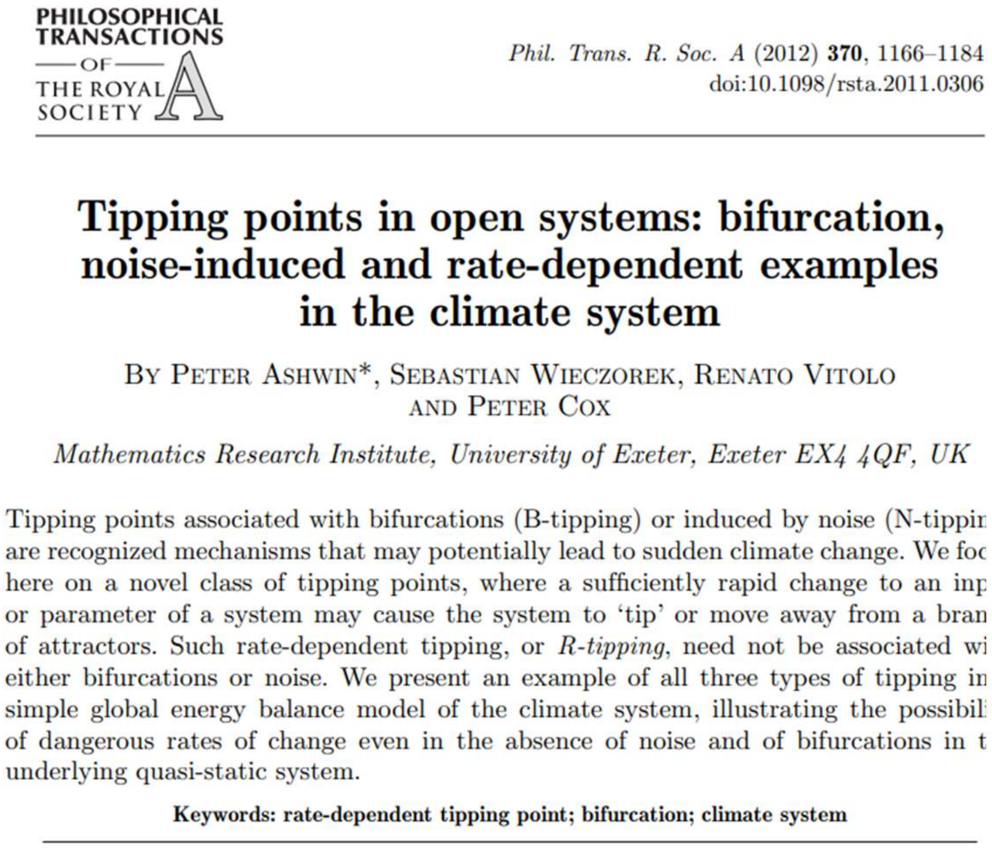


exchange of relevance of solutions with and without precipitation

Figure 4: Standard Case of Boos & Storelvmo (2016) with stratification being present- Enlarged domain to show the presence of two saddle-node bifurcation curves.

Kumar and Seshadri, in progress

Bifurcation, noise-induced, and rate-induced tipping



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

Phil. Trans. R. Soc. A (2012) **370**, 1166–1184
doi:10.1098/rsta.2011.0306

Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system

BY PETER ASHWIN*, SEBASTIAN WIECZOREK, RENATO VITOLO AND PETER COX

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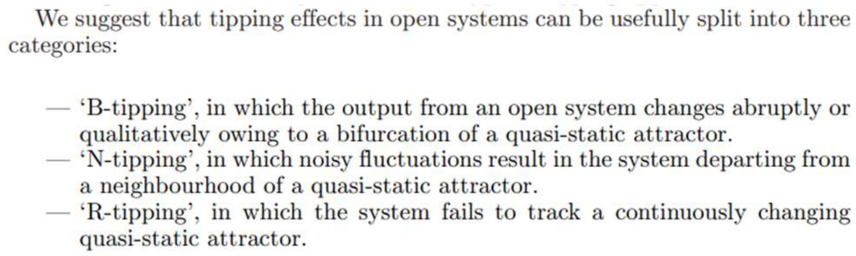
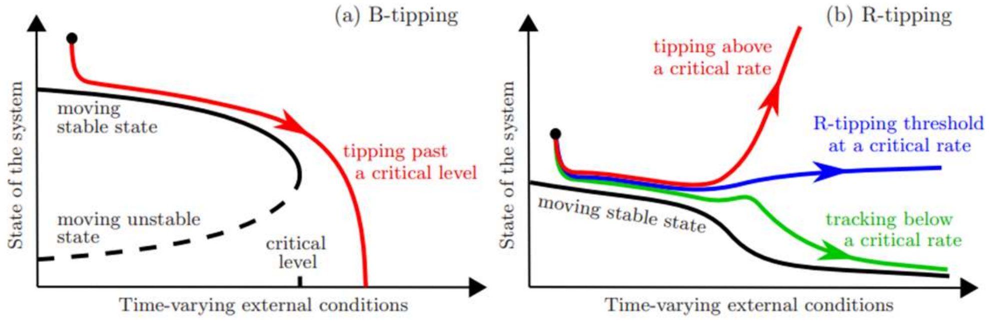
Tipping points associated with bifurcations (B-tipping) or induced by noise (N-tippir are recognized mechanisms that may potentially lead to sudden climate change. We foc here on a novel class of tipping points, where a sufficiently rapid change to an inp or parameter of a system may cause the system to 'tip' or move away from a bran of attractors. Such rate-dependent tipping, or *R-tipping*, need not be associated wi either bifurcations or noise. We present an example of all three types of tipping in simple global energy balance model of the climate system, illustrating the possibil of dangerous rates of change even in the absence of noise and of bifurcations in t underlying quasi-static system.

Keywords: rate-dependent tipping point; bifurcation; climate system



We suggest that tipping effects in open systems can be usefully split into three categories:

- 'B-tipping', in which the output from an open system changes abruptly or qualitatively owing to a bifurcation of a quasi-static attractor.
- 'N-tipping', in which noisy fluctuations result in the system departing from a neighbourhood of a quasi-static attractor.
- 'R-tipping', in which the system fails to track a continuously changing quasi-static attractor.



Wieczorek et al., 2021

Comment

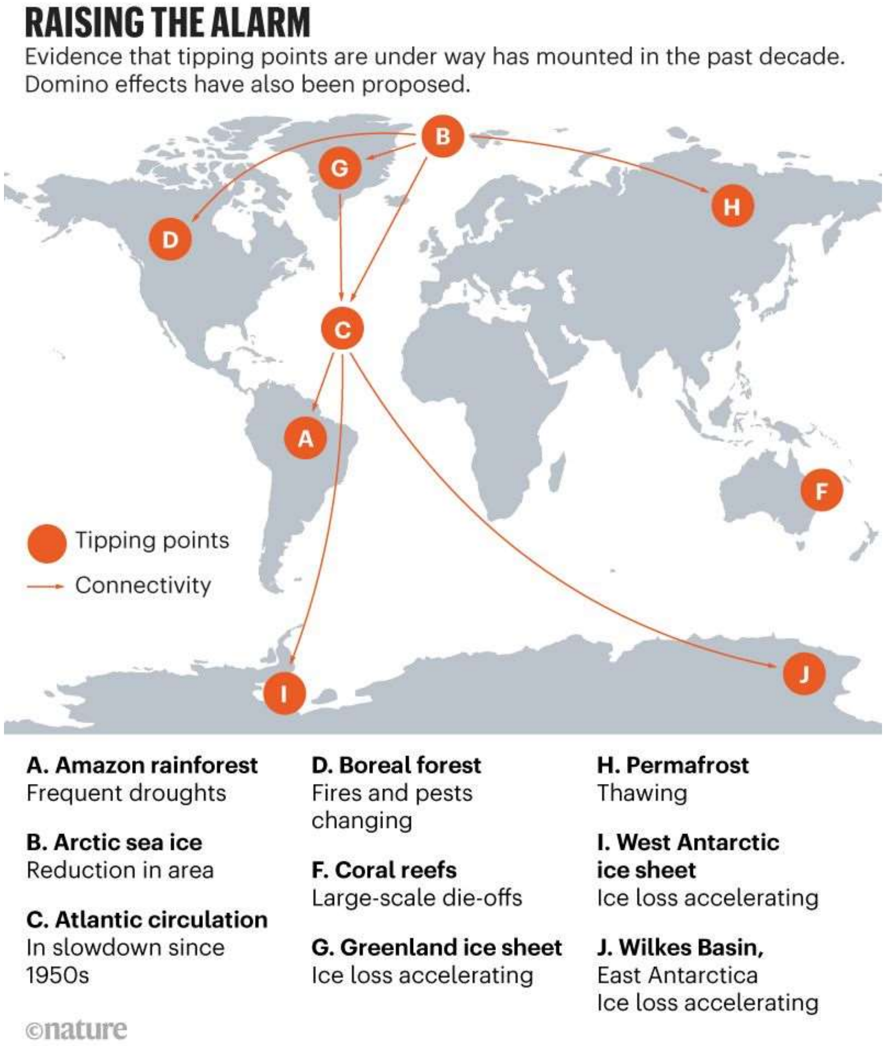
Climate tipping points — too risky to bet against

Timothy M. Lenton, Johan Rockström, Owen Gaffney, Stefan Rahmstorf, Katherine Richardson, Will Steffen & Hans Joachim Schellnhuber

The growing threat of abrupt and irreversible climate changes must compel political and economic action on emissions.

assuming that climate tipping points are of very low probability (even if they would be catastrophic), have suggested that 3 °C warming is optimal from a cost–benefit perspective. However, if tipping points are looking more likely, then the 'optimal policy' recommendation of simple cost–benefit climate-economy models[4] aligns with those of the recent IPCC report[2]. In other words, warming must be

Lenton et al., 2019



RAISING THE ALARM

Evidence that tipping points are under way has mounted in the past decade. Domino effects have also been proposed.

A. Amazon rainforest
Frequent droughts

B. Arctic sea ice
Reduction in area

C. Atlantic circulation
In slowdown since 1950s

D. Boreal forest
Fires and pests changing

F. Coral reefs
Large-scale die-offs

G. Greenland ice sheet
Ice loss accelerating

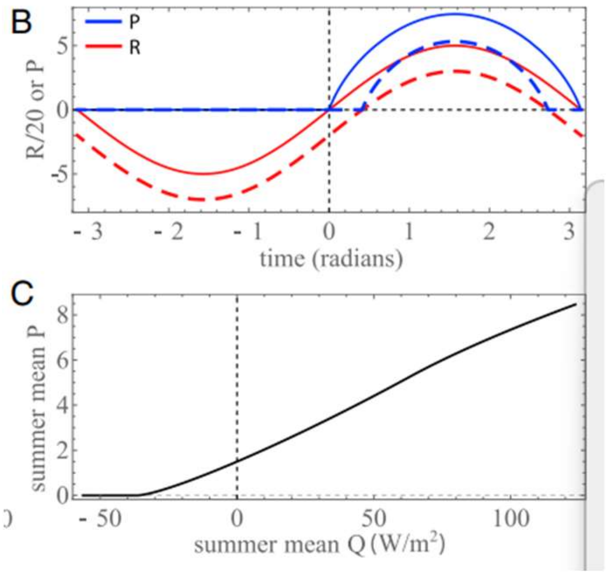
H. Permafrost
Thawing

I. West Antarctic ice sheet
Ice loss accelerating

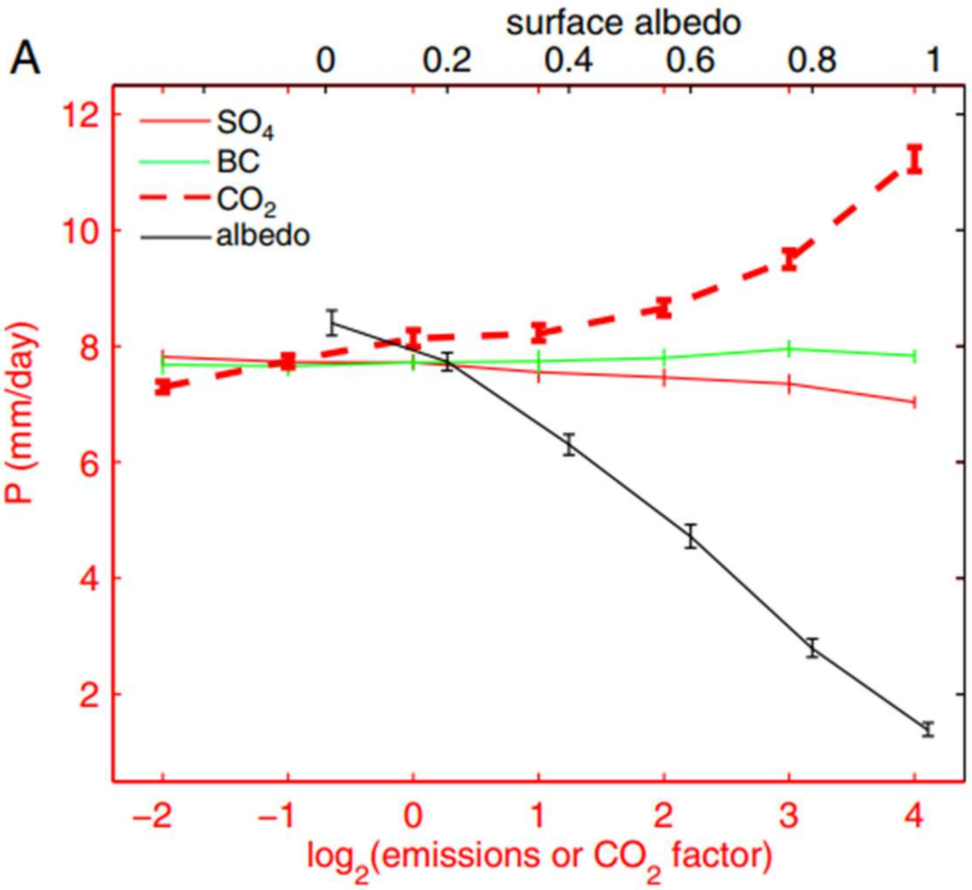
J. Wilkes Basin,
East Antarctica
Ice loss accelerating

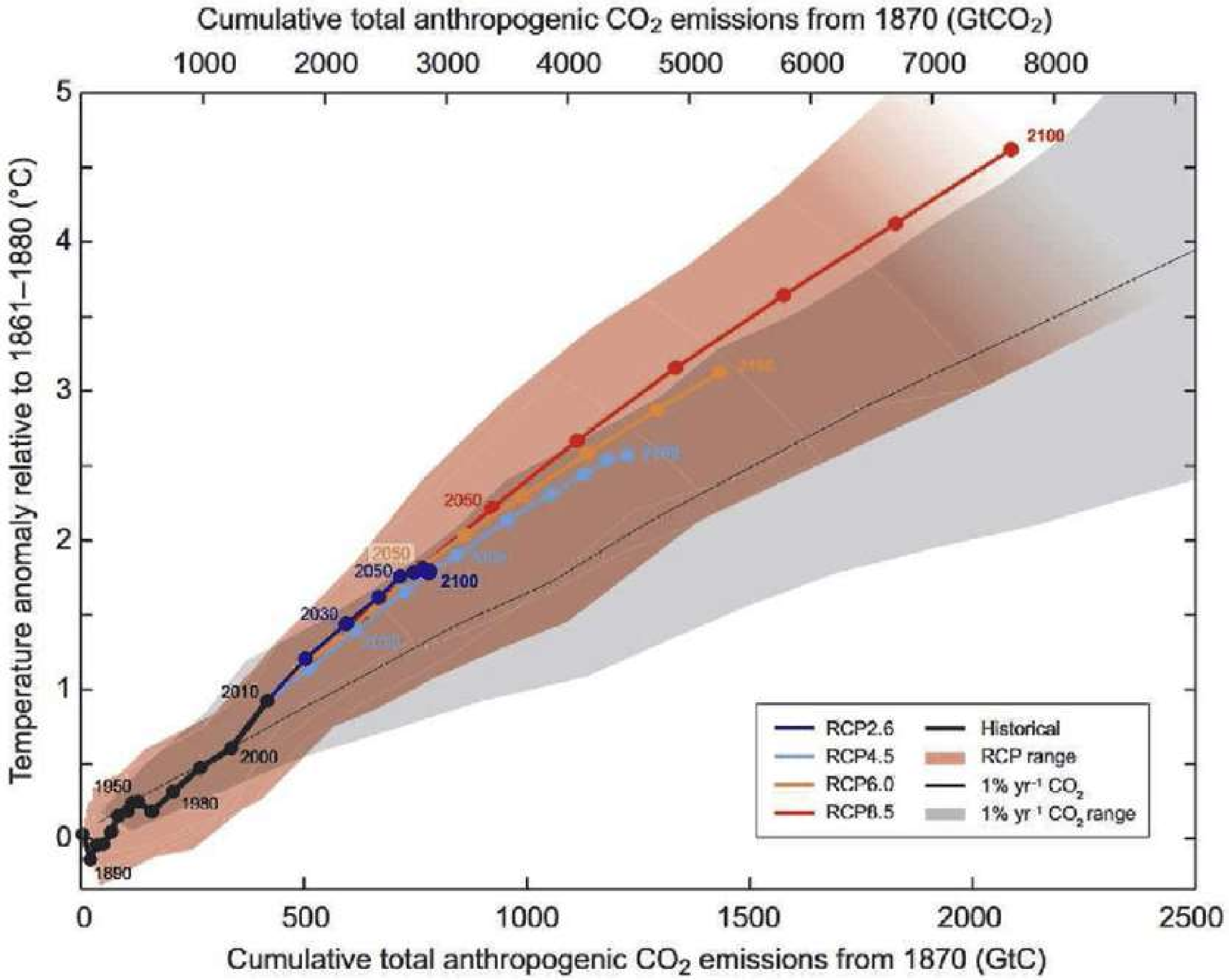
©nature

Effect of albedo in CESM runs



Boos and Storelvmo, 2016




Cumulative total anthropogenic CO₂ emissions from 1870 (GtCO₂)
1000
2000
3000
4000
5000
6000
7000
8000
Temperature anomaly relative to 1861–1880 (°C)
5
4
3
2
1
0
2100
2050
2050
2050
2100
2030
2010
2000
1950
1980
1890
RCP2.6
RCP4.5
RCP6.0
RCP8.5
Historical
RCP range
1% yr⁻¹ CO₂
1% yr⁻¹ CO₂ range
0
500
1000
1500
2000
2500
Cumulative total anthropogenic CO₂ emissions from 1870 (GtC)