

# **Simple models of global warming**

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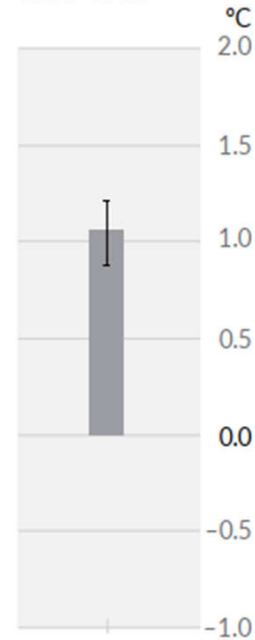
# Contributions to warming

“The **well-mixed greenhouse gases** have lifetimes long enough to be relatively homogeneously mixed in the troposphere (IPCC)”

IPCC AR6 Working Group 1

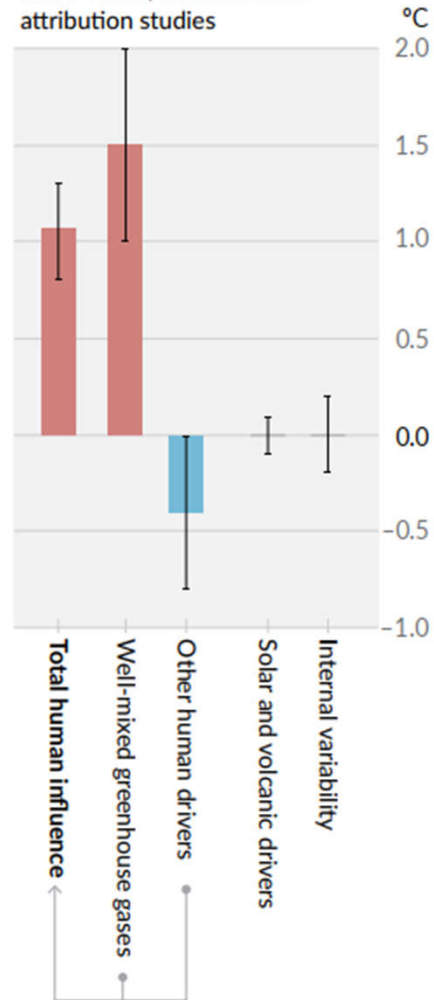
## Observed warming

(a) Observed warming 2010–2019 relative to 1850–1900

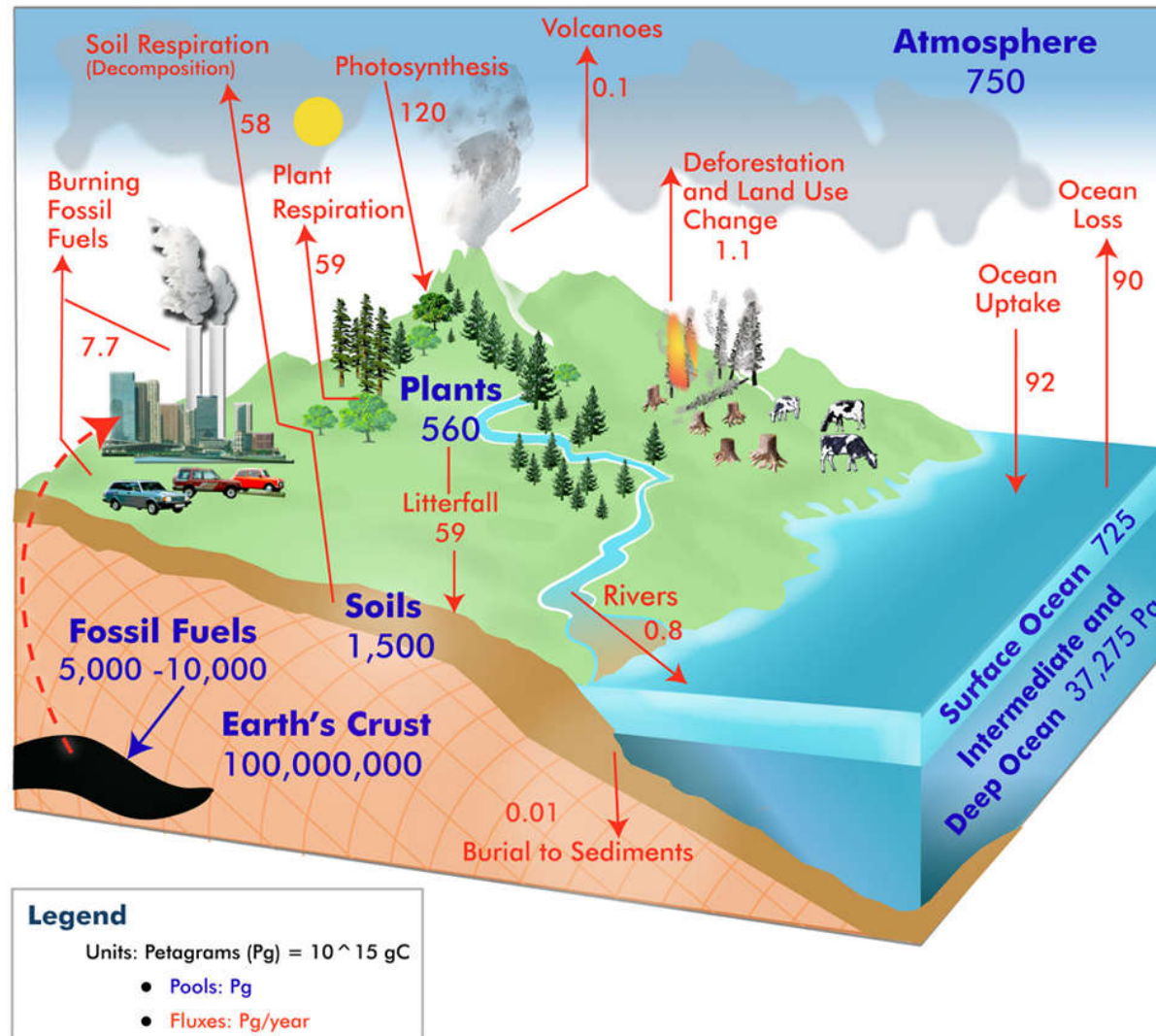


## Contributions to warming base

(b) Aggregated contributions to 2010–2019 warming relative to 1850–1900, assessed from attribution studies



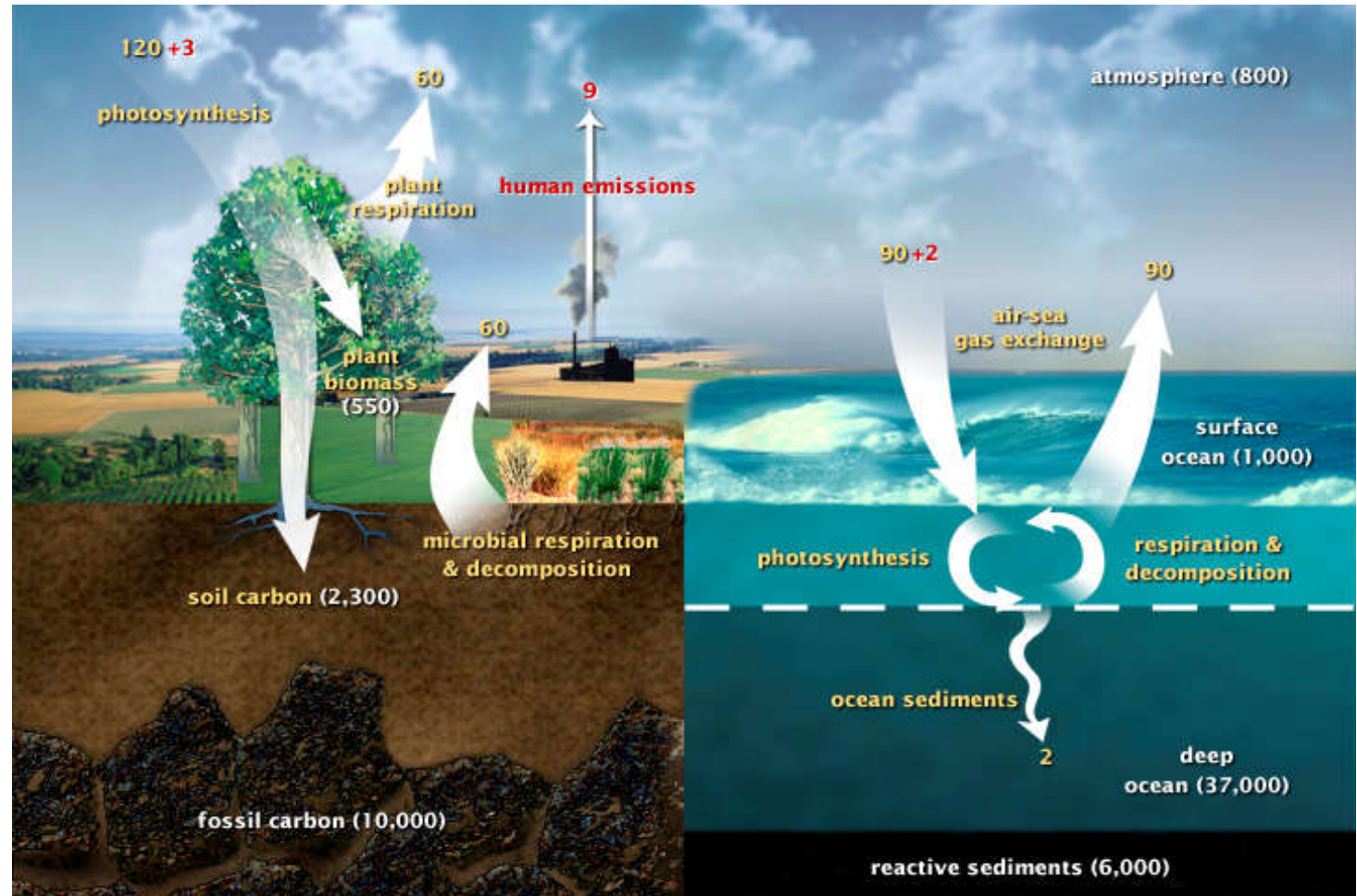
# Global Carbon Cycle



Source:  
NASA/GLOBE Program

Copyright 2010 GLOBE Carbon Cycle Project, a collaborative project between the University of New Hampshire, Charles University and the GLOBE Program Office.  
Data Sources: Adapted from Houghton, R.A. Balancing the Global Carbon Budget. Annu. Rev. Earth Planet. Sci. 007.35:313-347, updated emissions values are from the Global Carbon Project: Carbon Budget 2009.

# Fast carbon cycle

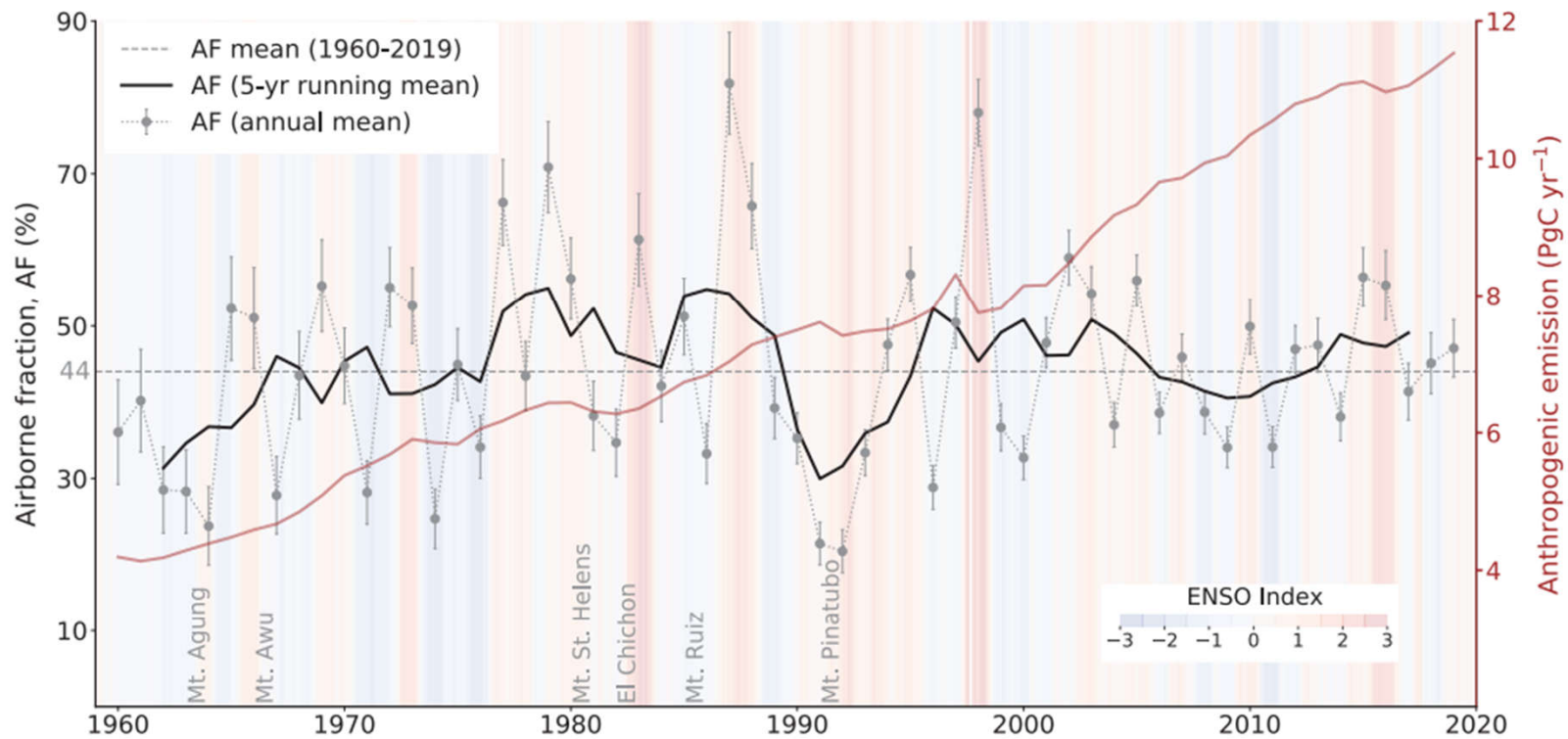


Source:  
NASA Earth Observatory

## Summary

- Fast as well as slow processes in the carbon cycle
- Natural fluxes are large
- Anthropogenic emissions are perturbing natural fluxes
- About half of carbon-dioxide emissions ends up in one of these reservoirs (ocean, biosphere)

# Airborne fraction of CO<sub>2</sub>

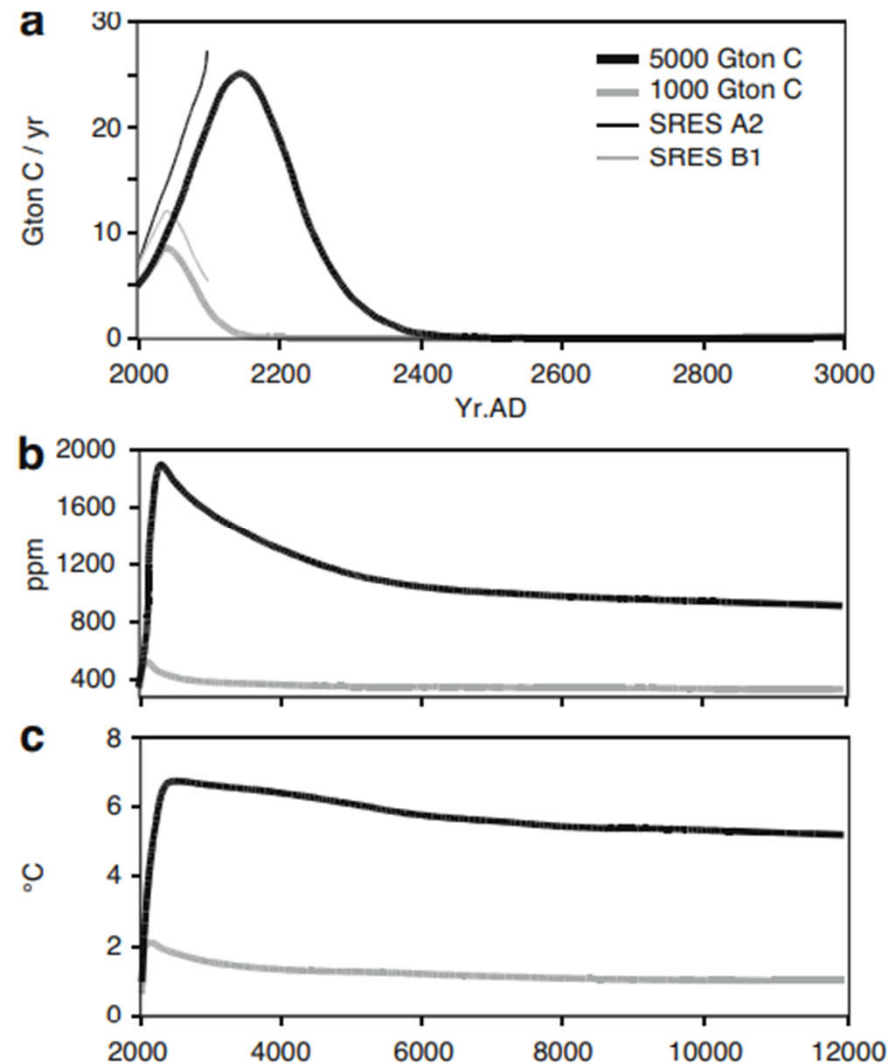


**Figure 5.7 | Airborne fraction and anthropogenic (fossil fuel and land-use change) CO<sub>2</sub> emissions.** Data as in Section 5.2.1.1. The multivariate El Niño–Southern Oscillation (ENSO) index (shaded) and the major volcanic eruptions are marked along the x-axis. Further details on data sources and processing are available in the chapter data table (Table 5.SM.6).

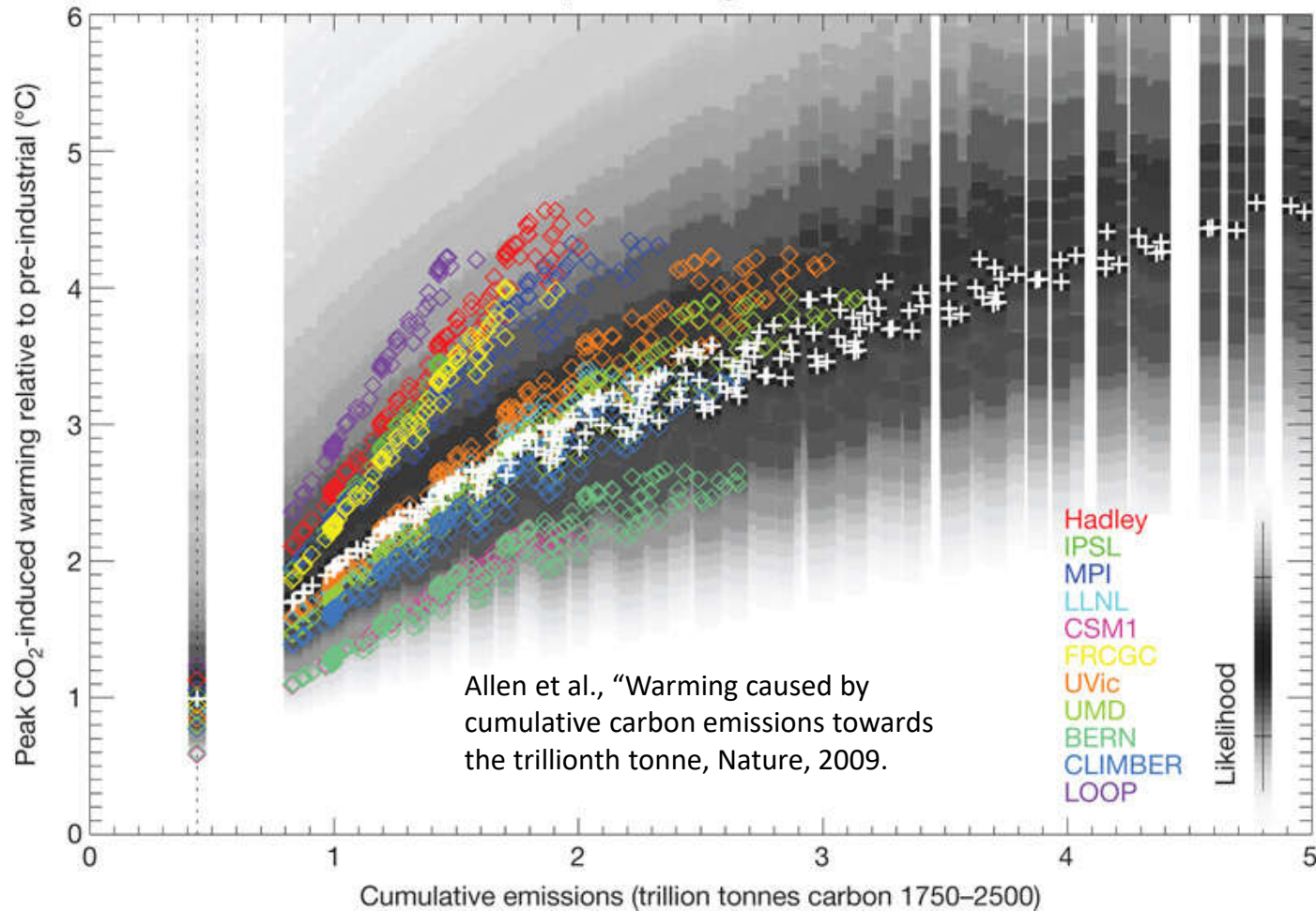
## Effects of long-lived CO<sub>2</sub>

Archer and Brovkin, **The millennial atmospheric lifetime of anthropogenic CO<sub>2</sub>**, *Climatic Change*, 2008:

“The largest fraction of the CO<sub>2</sub> recovery will take place on time scales of centuries, as CO<sub>2</sub> invades the ocean, but a significant fraction of the fossil fuel CO<sub>2</sub>, ranging in published models in the literature from 20–60%, remains airborne for a thousand years or longer. Ultimate recovery takes place on time scales of hundreds of thousands of years, a geologic longevity typically associated in public perceptions with nuclear waste.”



# Proportionality between global warming and cumulative carbon dioxide emissions (CO<sub>2</sub>) (Allen et al., 2009; Matthews et al., 2009)



Also: Matthews et al.,  
"Proportionality of global  
warming to cumulative carbon  
emissions", Nature, 2009.



## Cumulative emissions accounting

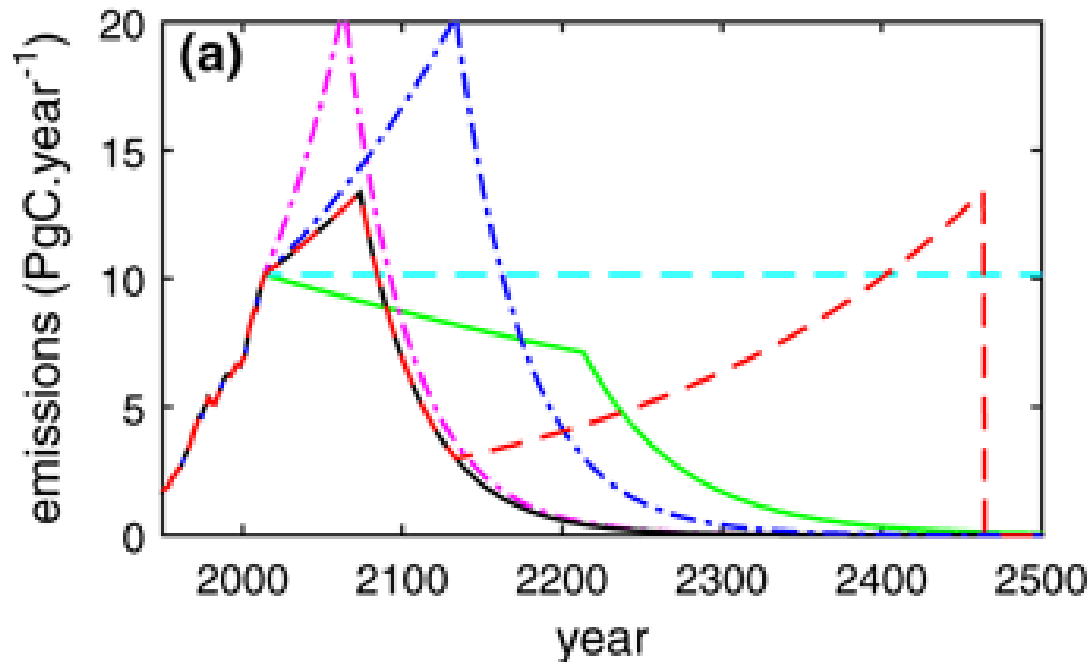
“The ratio of GMST [Global Mean Surface Temperature] change to total cumulative anthropogenic carbon emissions is relatively constant and independent of the scenario, but is model dependent, as it is a function of the model cumulative airborne fraction of carbon and the transient climate response. For any given temperature target, higher emissions in earlier decades therefore imply lower emissions by about the same amount later on.” (IPCC AR5 WG1, Technical Summary, 2013)

**IPCC AR5 WG1, Summary for Policymakers, 2013.**

Cumulative emission accounting only requires path independence between global warming and cumulative emissions. Global warming should be a function of cumulative emissions alone, but not necessarily a linear function.

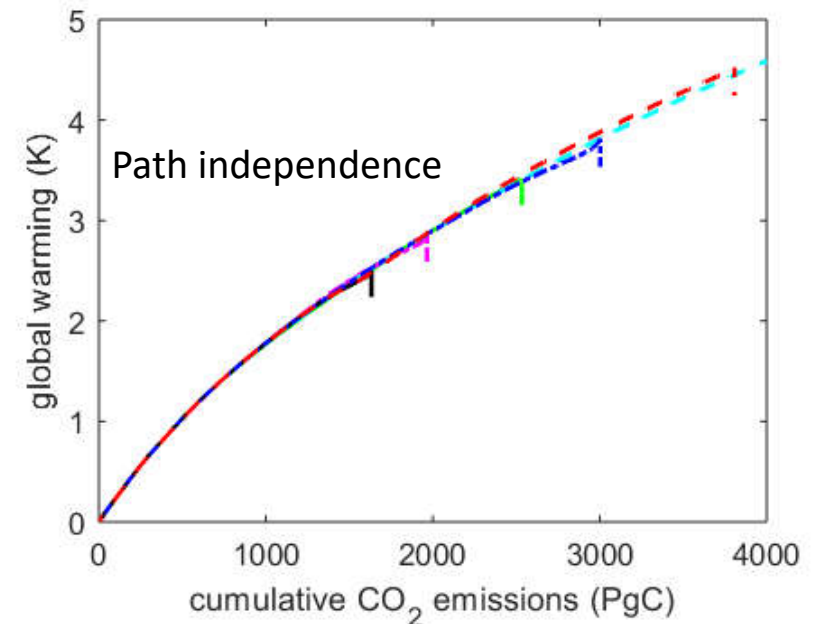
# Path independence between cumulative CO<sub>2</sub> emissions and global warming

Different emissions pathways:



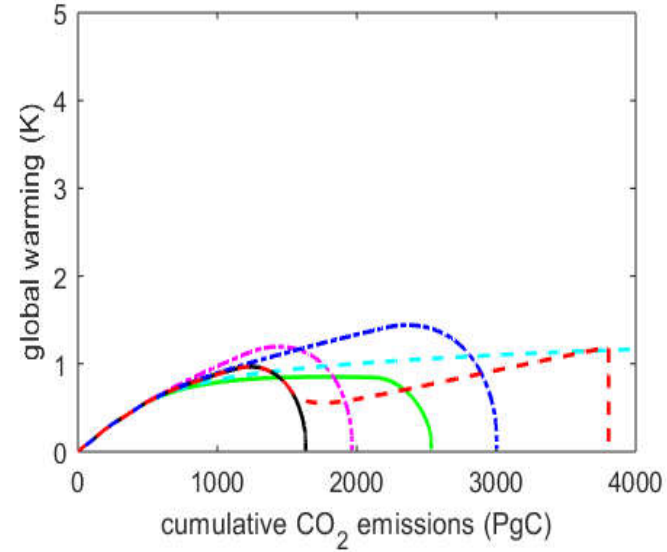
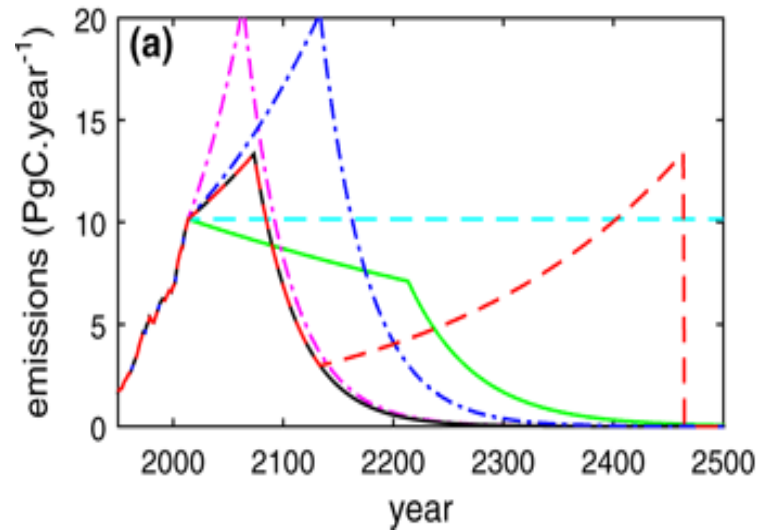
“The ratio of GMST [global mean surface temperature] change to total cumulative anthropogenic carbon emissions is relatively constant and independent of the scenario. For any given temperature target, higher emissions in earlier decades therefore imply lower emissions by about the same amount later on.”  
(IPCC AR5 WG1, Technical Summary, 2013)

All pathways have the same relation between global warming & cumulative CO<sub>2</sub> emissions

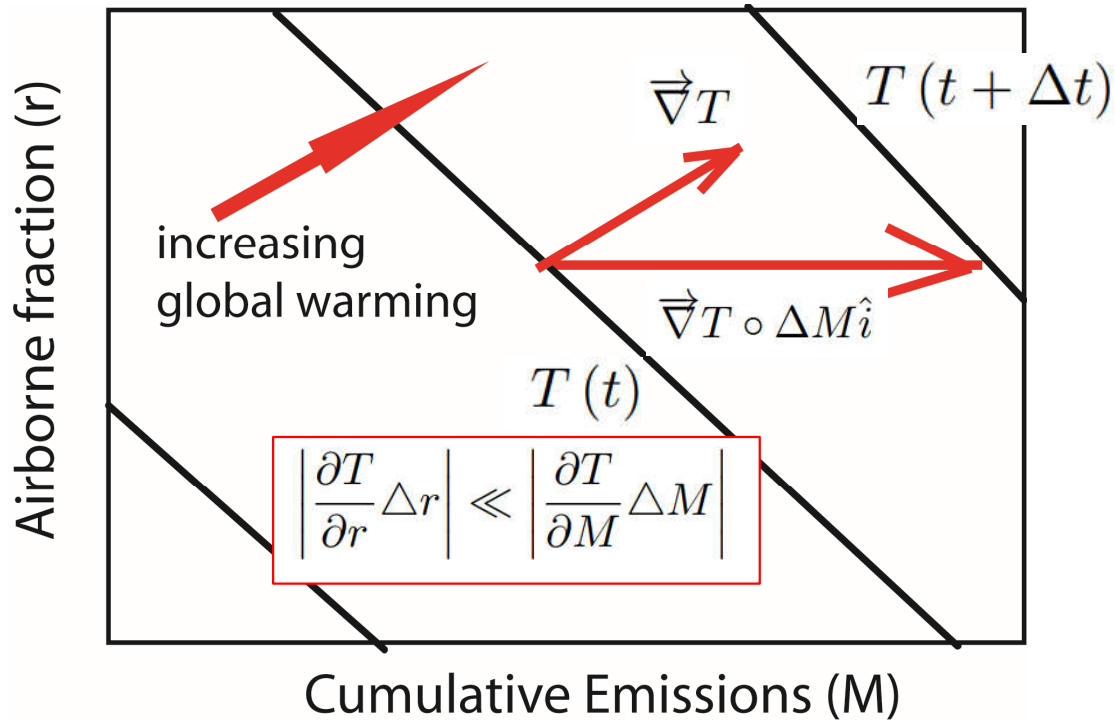


# In the absence of slow processes in the carbon cycle...e.g. if CO<sub>2</sub> behaves like methane

path independence would breakdown



# Path independence occurs if the timescale for cumulative emissions change is short



Timescale for airborne fraction to change

Damping timescale (slow contribution to global warming)

$$\tau_r \gg \tau_M \ll \tau_D$$

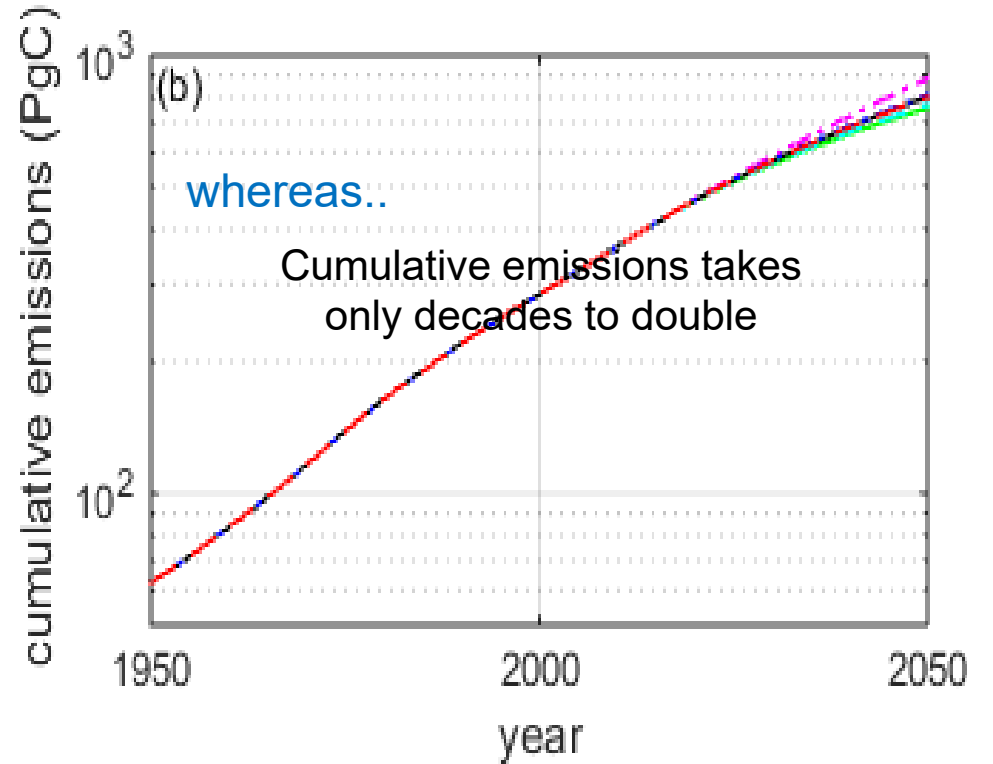
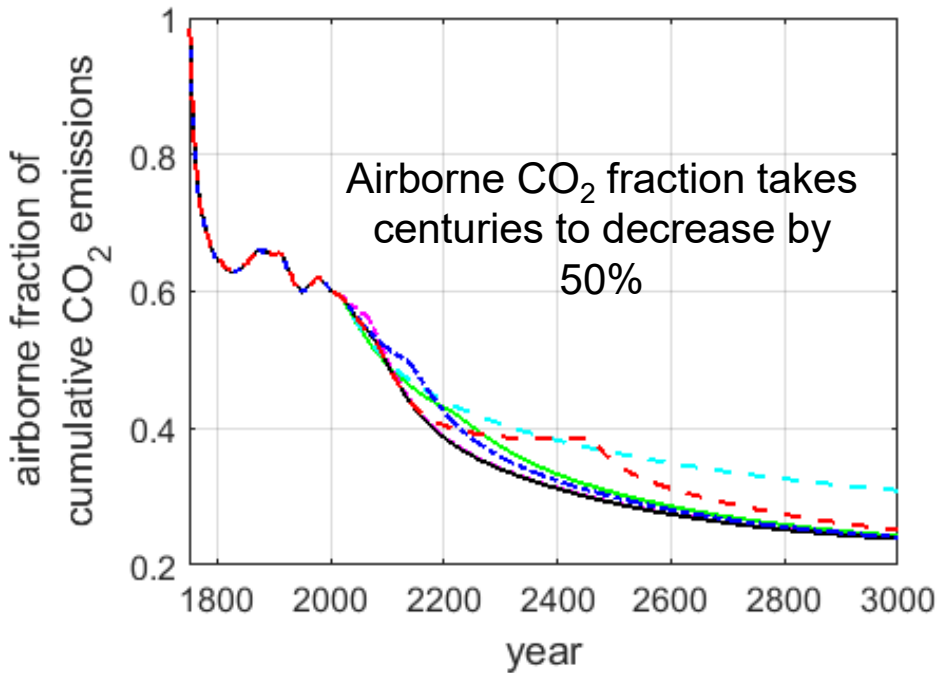
Timescale for cumulative emissions to change

Eq. from Seshadri, Climate Dynamics, 2017

# Origin of path independence for CO<sub>2</sub>

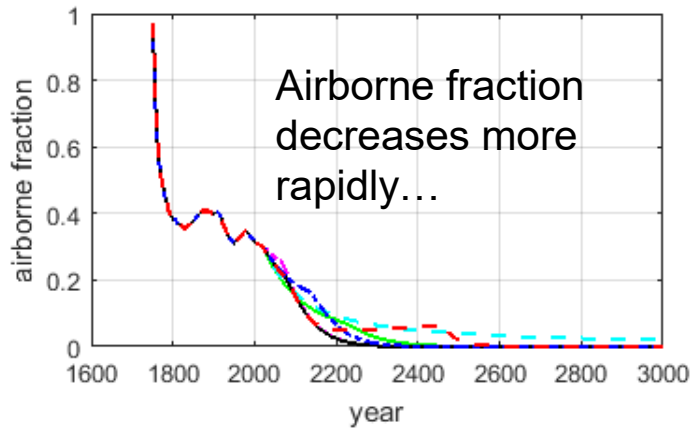
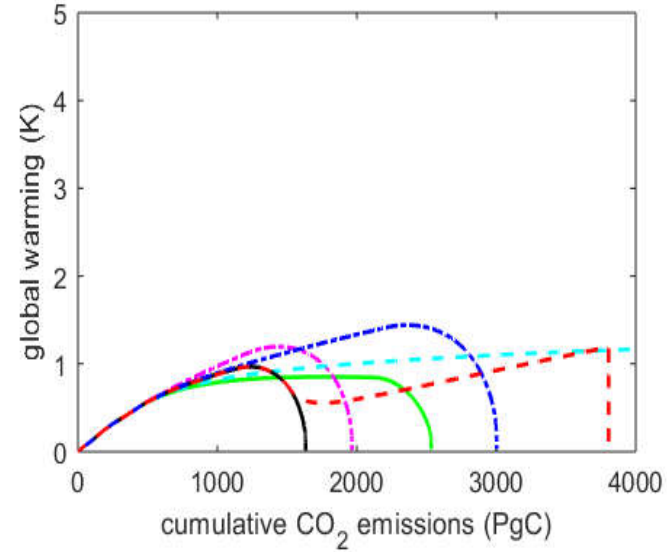
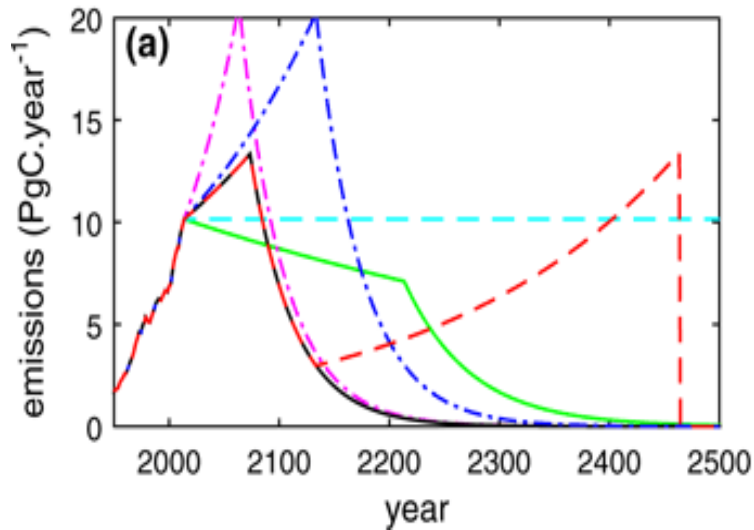
Cumulative emissions changes more rapidly than airborne fraction

For CO<sub>2</sub>, path independence occurs because:

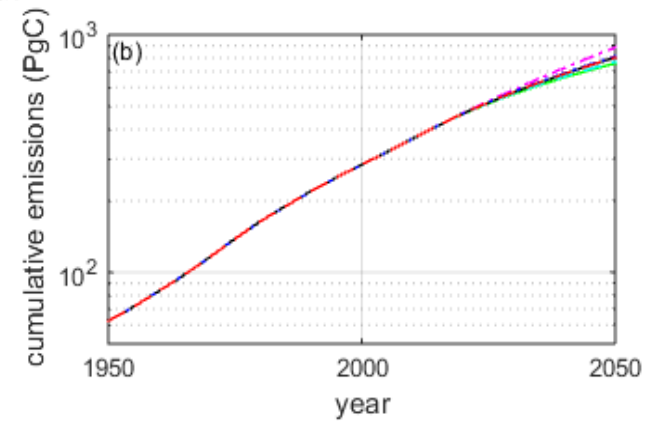


# In the absence of slow processes in the carbon cycle...e.g. if CO<sub>2</sub> behaves like methane in the atmosphere

path independence would breakdown

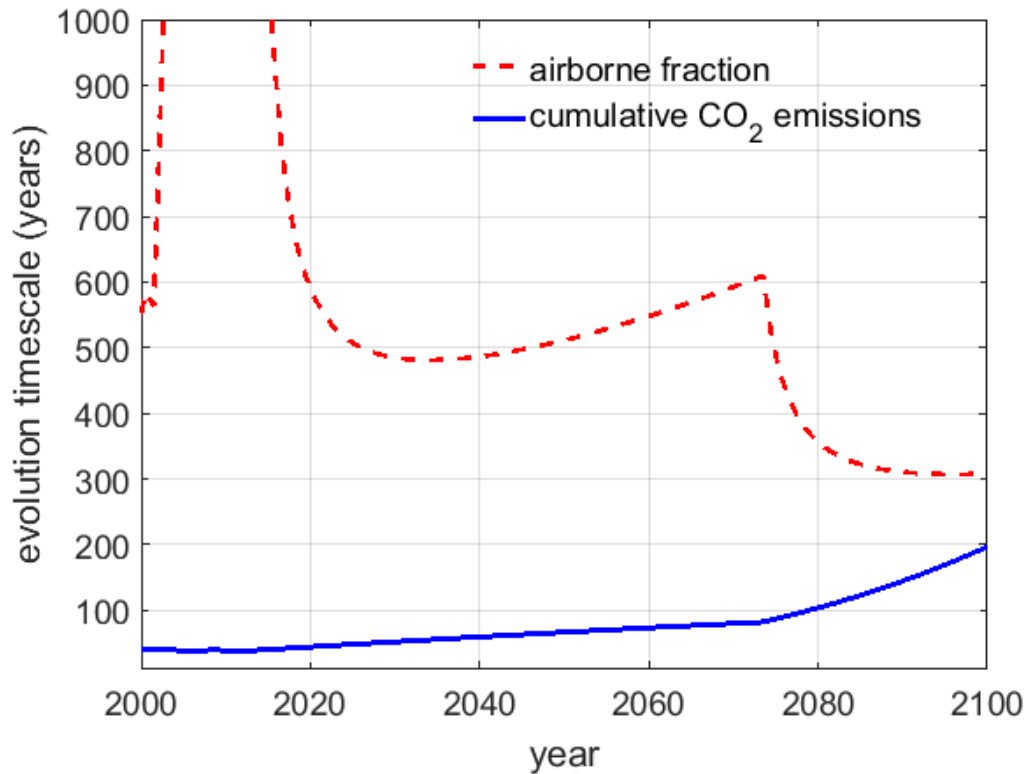


..on timescales comparable to evolution of cumulative emissions

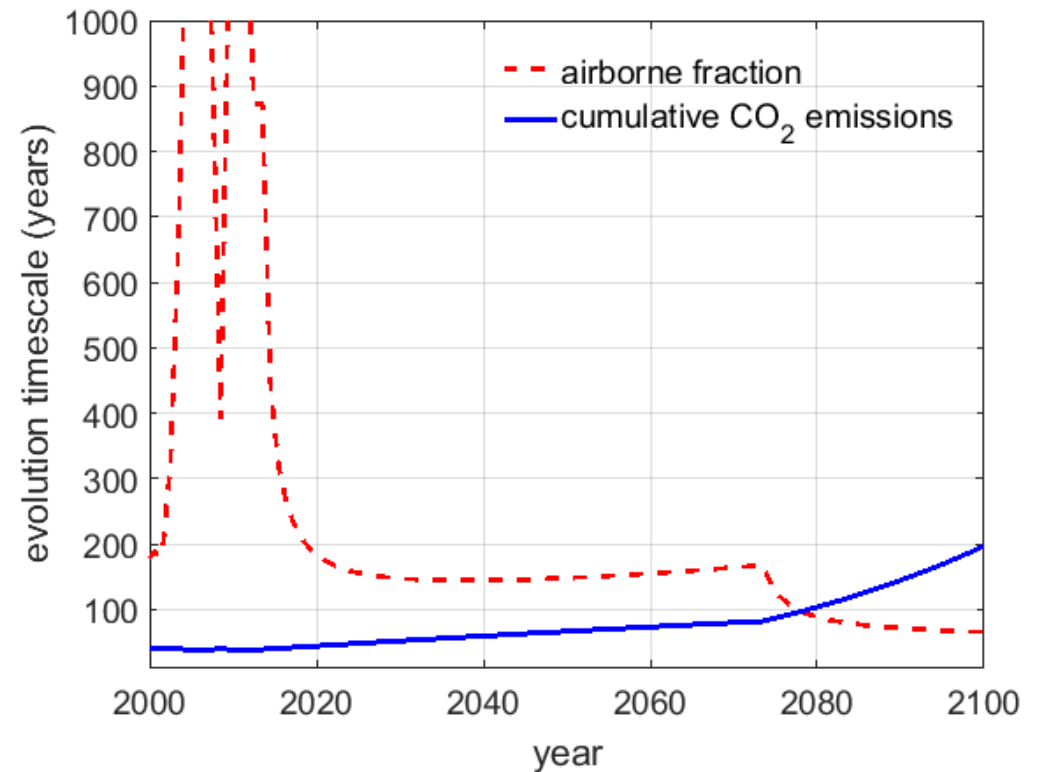


# Comparison of timescales

## Correct CO<sub>2</sub> model: with slow carbon cycle

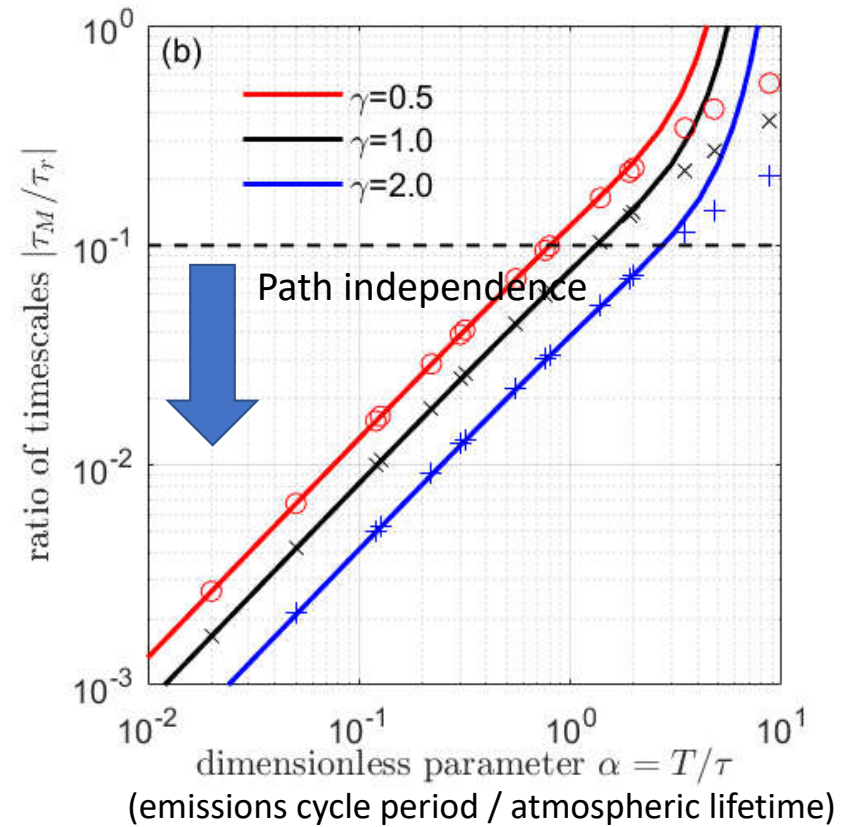
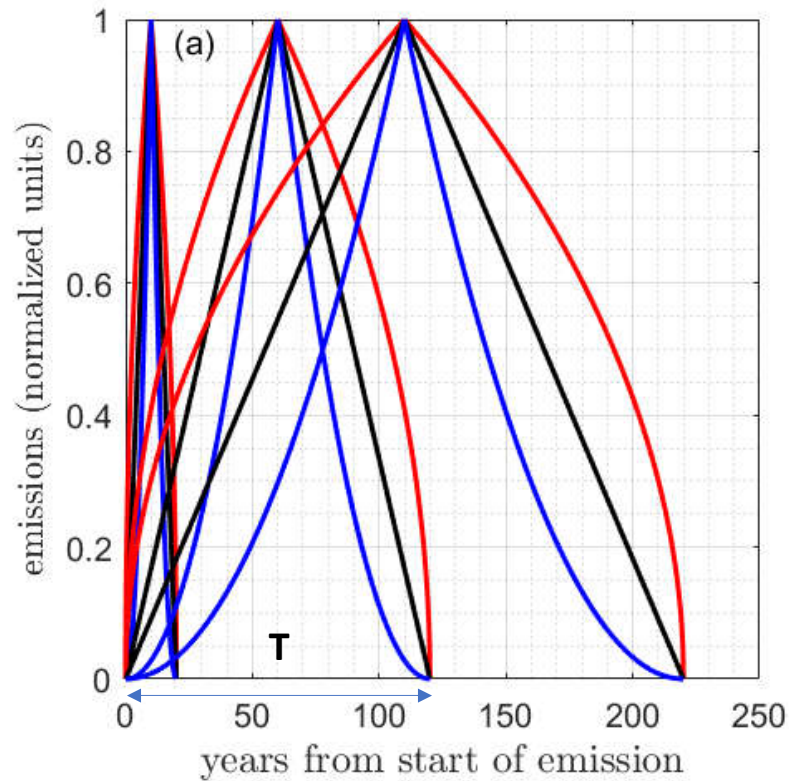


## If CO<sub>2</sub> behaved like methane



If CO<sub>2</sub> behaved like methane, effect of changes in airborne fraction of CO<sub>2</sub> would not be negligible. Path independence would not have occurred.

**For path independence, emissions cycle period should be comparable to or smaller than atmospheric lifetime**





## Derivation

airborne fraction of cumulative emissions  $r(t) = \overbrace{(C(t) - C_{eq})}^{\text{Excess concentration}} / \overbrace{M(t)}^{\text{Cumulative emissions}}$

**Airborne fraction timescale:**  $-\frac{1}{\tau_r} = \frac{1}{r} \frac{dr}{dt} = \frac{1}{C(t) - C_{eq}} \frac{d}{dt} (C(t) - C_{eq}) - \frac{1}{M(t)} \frac{d}{dt} M(t)$

**Evolution of concentration:**  $\frac{dC(t)}{dt} = m(t) - \frac{C(t) - C_{eq}}{\tau}$        $C(t) = C_{eq} + e^{-t/\tau} \int_0^t e^{s/\tau} m(s) ds$

**Rescaling time:**  $x = t/T$        $C(x) - C_{eq} = T e^{-\alpha x} \int_0^x e^{\alpha s} \hat{m}(s) ds$

where  $\hat{m}(x) = m(t/T)$  and  $\alpha = T/\tau$ .

## Derivation (contd.)

Integrating by parts:

$$e^{-\alpha x} \int_0^x e^{\alpha_i s} \hat{m}(s) ds = \hat{m}_1(x) - \alpha \hat{m}_2(x) + \alpha^2 \hat{m}_3(x) - \dots$$

where  $\hat{m}_{i+1}(x) = \int_0^x \hat{m}_i(s) ds$  is the  $i + 1^{\text{th}}$  repeated integral

$$\left| \frac{\tau_M}{\tau_r} \right| = \left| \frac{1 - \alpha \frac{\hat{m}_1(x)}{\hat{m}(x)} + \alpha^2 \frac{\hat{m}_2(x)}{\hat{m}(x)} - \dots}{1 - \alpha \frac{\hat{m}_2(x)}{\hat{m}_1(x)} + \alpha^2 \frac{\hat{m}_3(x)}{\hat{m}_1(x)} - \dots} - 1 \right|$$

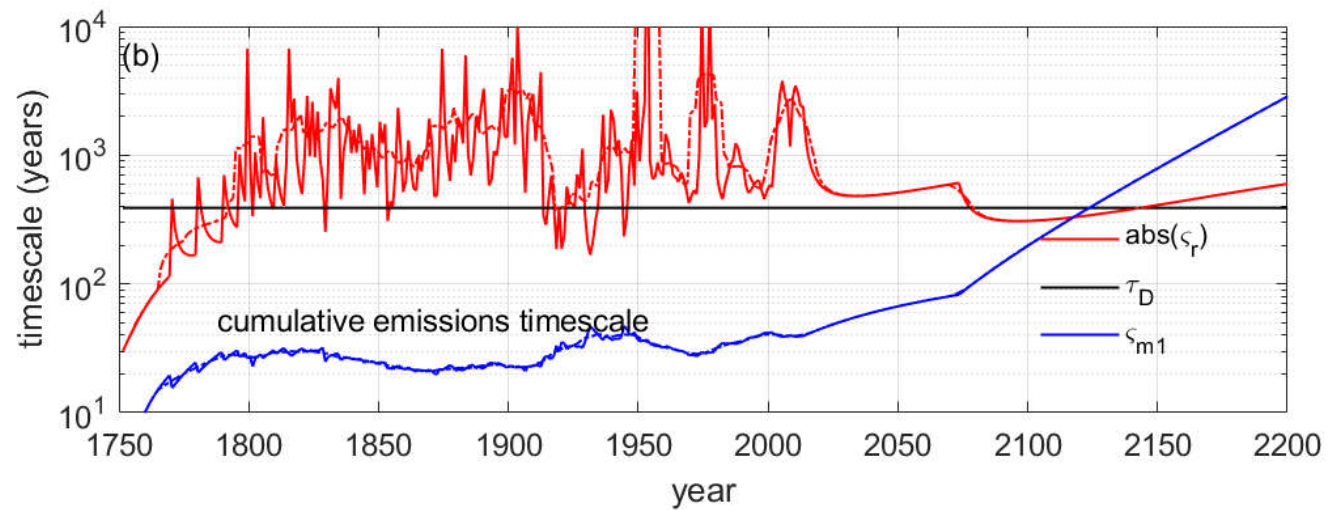
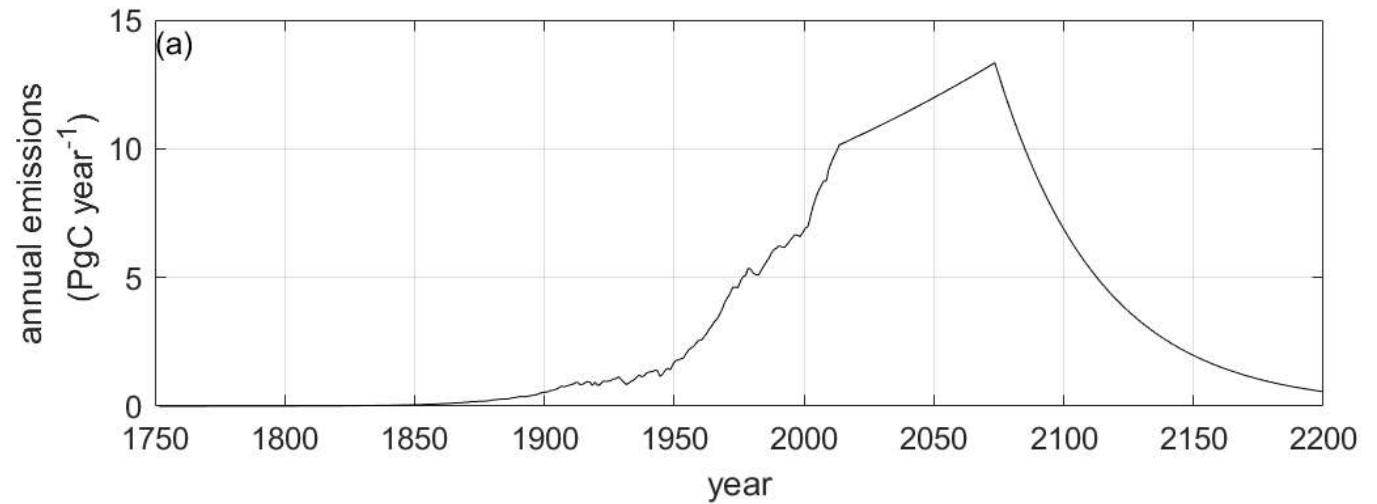
Depends on  $\alpha = T/\tau$  and  $x = t/T$

Emissions cycle period / atmospheric lifetime

Position in emissions cycle

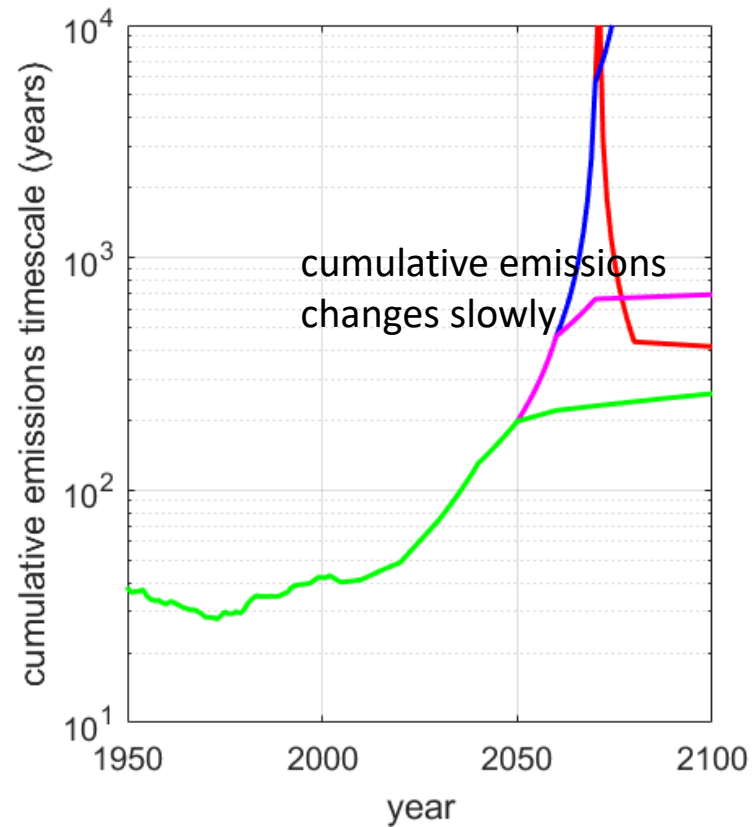
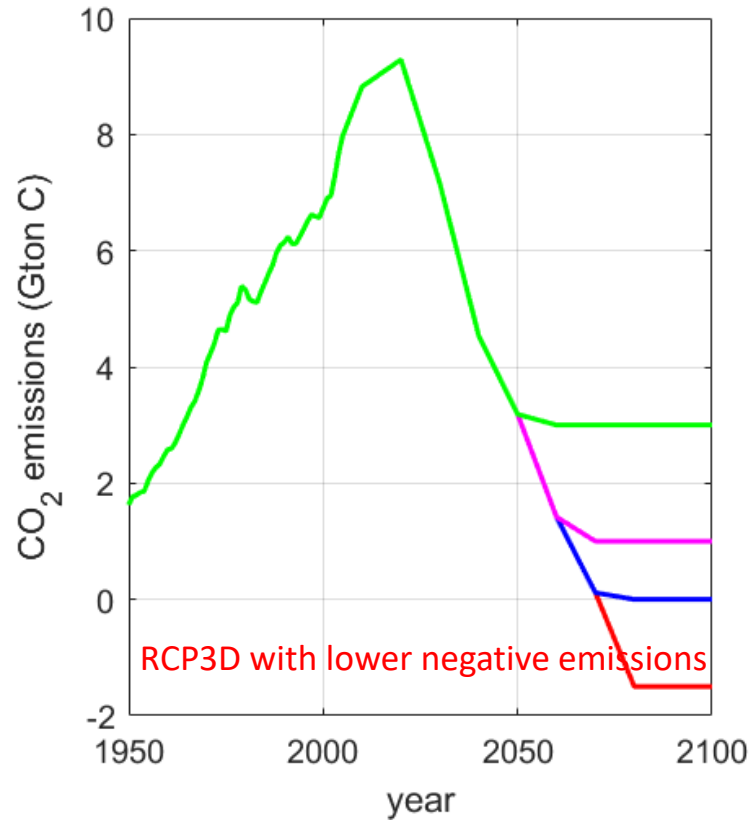
# Cumulative emissions timescale grows as emissions cycle proceeds

**cumulative emissions timescale =  
cumulative emissions / emissions**



Seshadri, "Origin of path independence between cumulative  $\text{CO}_2$  emissions and global warming", Climate Dynamics, 2017.

# Cumulative emissions timescale is persistently large in scenarios with low emissions



# Path independence does not work if emissions are small but persistent

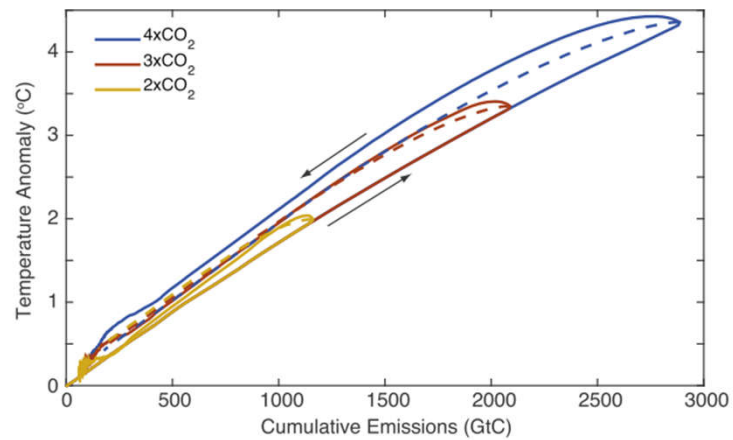
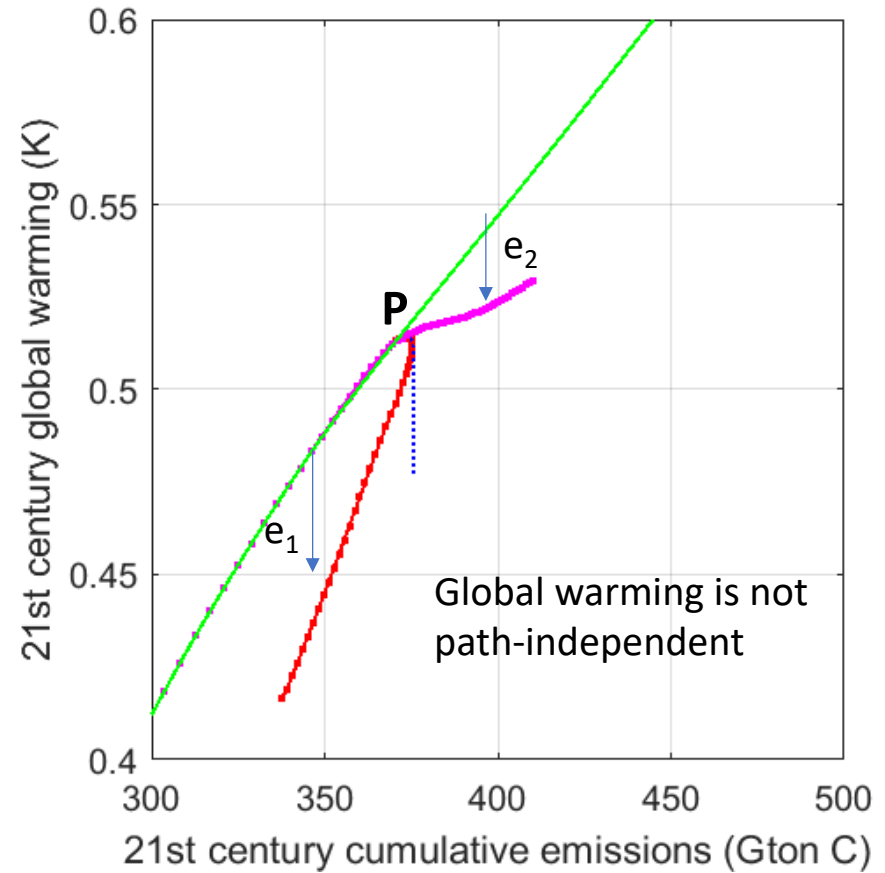


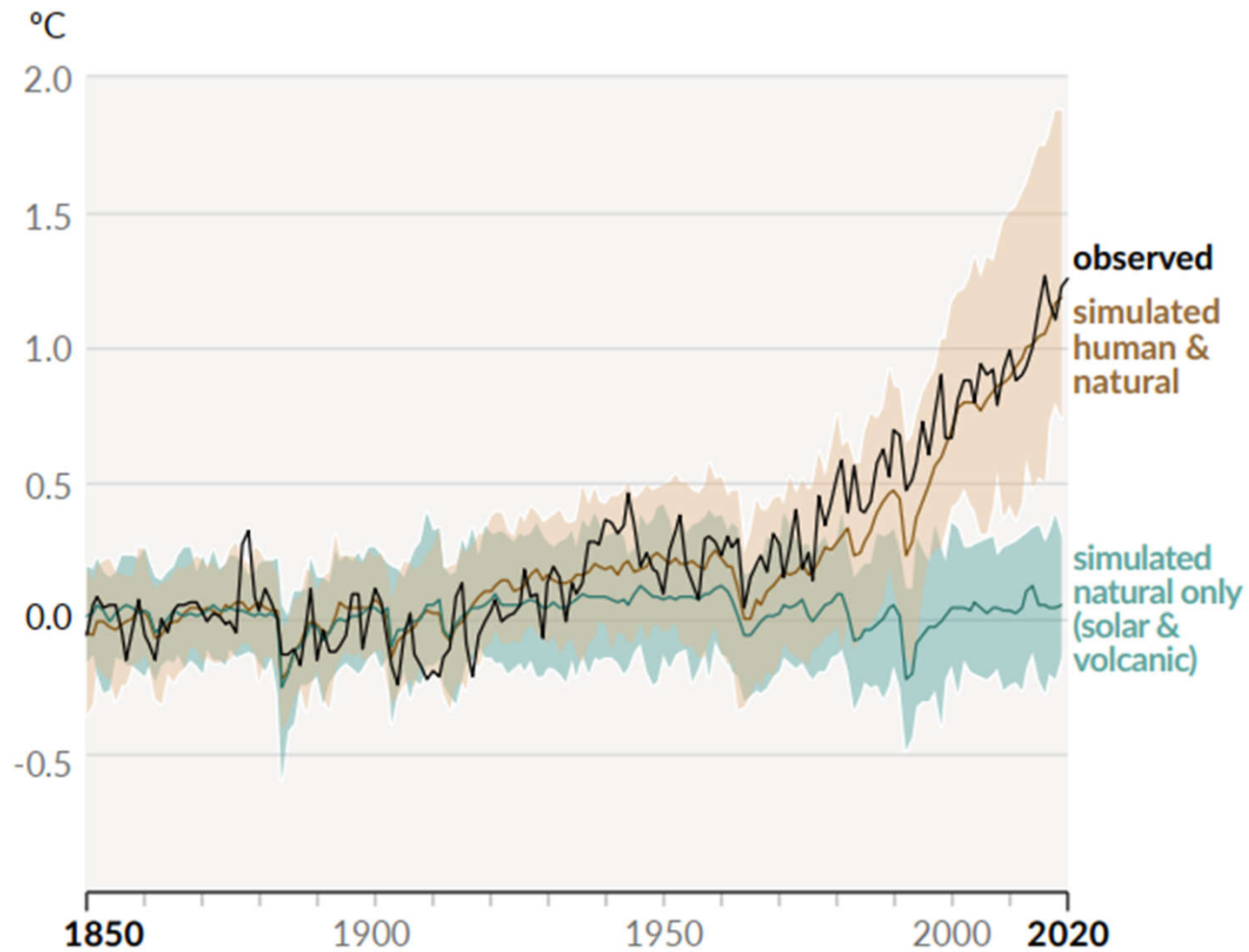
Figure 2. Surface-air temperature anomaly (relative to year 1) versus cumulative CO<sub>2</sub> emissions. Solid lines refer to the 1% CO<sub>2</sub> simulations, dashed lines refer to the 1% CO<sub>2</sub> simulations corrected for the temperature and carbon sink response in the zero CO<sub>2</sub> emissions simulations. The slope of the curves is the transient climate response to cumulative CO<sub>2</sub> emissions (TCRE).

Zickfeld et al., 2016

Average global warming in 5000-member MAGICC ensemble



# Global temperature change



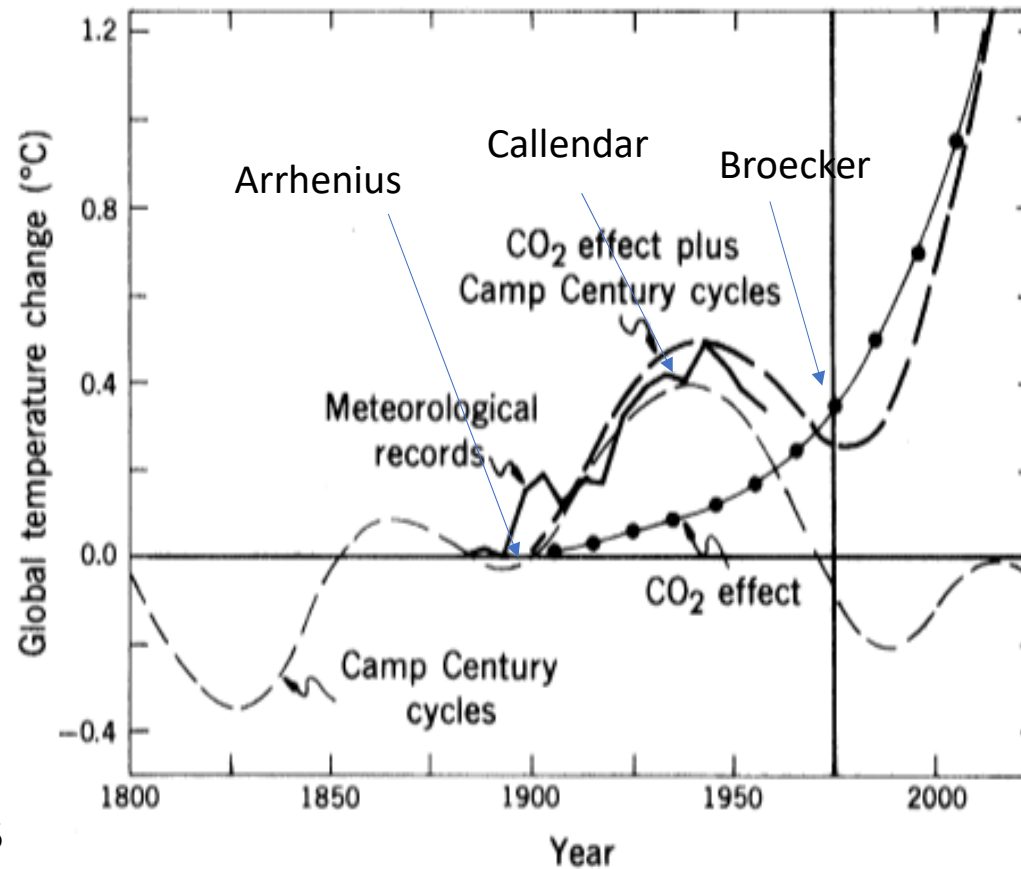
IPCC AR6 WG1 Summary for Policymakers

## Scientific milestones

- **1824:** Earth would be far colder without atmosphere (Fourier)
- **1859:** Some gases block infrared radiation, and changes in their concentration of the gases could bring climate change (Tyndall)
- **1879:** International Meteorological Organization begins to compile and standardize global weather data
- **1896:** First calculation of global warming from increased CO<sub>2</sub> (Arrhenius)
- **1938:** Claim that CO<sub>2</sub> greenhouse global warming is underway (Callendar)
- **1955:** Computer model of the global atmosphere (Phillips)
- **1956:** Adding CO<sub>2</sub> to the atmosphere will have significant effect on the radiation balance (Plass)
- **1957:** CO<sub>2</sub> produced by humans will not be readily absorbed by the oceans (Revelle)
- **1963:** Radiation calculations suggest that water vapor feedback could make the climate sensitive to changes in CO<sub>2</sub> level
- **1966:** Analysis of deep-sea cores and ancient corals show that ice ages were controlled by small orbital shifts, suggesting sensitivity to small changes (Emiliani; Broecker)
- **1968:** Studies show that Antarctic ice sheets could collapse
- **1972:** Ice cores show large shifts in climate around 11,000 years ago
- **1967-75:** First calculation using climate model of temperature effect of doubling CO<sub>2</sub> (Manabe and Wetherald)

## The CO<sub>2</sub> effect was noticeable by the mid-1970s

Fig. 1. Curves for the global temperature change due to chemical fuel CO<sub>2</sub>, natural climatic cycles, and the sum of the two effects. The measured temperature anomaly for successive 5-year means from meteorological records over the last century is given for comparison.



Wallace Broecker, *Science*, 1975



## In the news in the late 1970s...

- “The world may be inching into a prolonged warming trend that is the direct result of burning more and more fossil fuels...” *US News and World Report, 1976*
- CO<sub>2</sub> "may be the world's biggest environmental problem, threatening to raise the world's temperature", *Business Week, 1976*
- “A poll of climate scientists is evenly divided on warming or cooling. But the leading experts strongly expected future warming.” *New York Times, 1976*
- "All scientists agree that a new factor has entered the game of climate change, a 'wild card' never there before — man himself.“ *Reader’s Digest, 1977*

## Fingerprints of greenhouse gases?

the greenhouse and natural variations. No one type of change is likely to convince the scientific community of the reality of the greenhouse effect or its true magnitude until well into the next century, when the world could be condemned to dramatic changes. The best bet for early detection seems to be the identification of a number of changes—warmer weather, warmer ocean water, a cooler stratosphere, and increased precipitation, for example—that together would, in all likelihood, be caused by a greenhouse.

The approach now being pursued is more like developing a composite picture of a culprit rather than arresting the first suspect who has the same color eyes. Researchers call the technique fingerprinting, although it is hardly as conclusive as the human technique. And they are beginning to scrutinize what may be the first components, albeit fuzzy ones, of the greenhouse fingerprint.

The one component of the fingerprint that almost everyone now agrees on is the current global warming. The world is clearly

# Optimal fingerprints

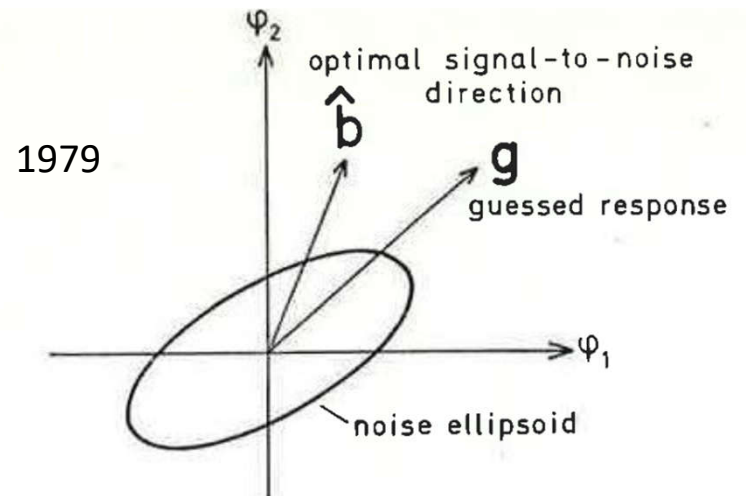
On the signal-to-noise problem in atmospheric response studies

By KLAUS HASSELMANN

*Max-Planck-Institut für Meteorologie, Hamburg*

## SUMMARY

The problem of identifying the mean atmospheric response to external forcing in the presence of the natural variability of the atmosphere is treated as a pattern-detection problem. It is shown that without application of filtering techniques to reduce the number of degrees of freedom of the response pattern the atmospheric response inferred from data or model experiments will normally fail a multi-variate significance test. A step-wise pattern construction method is proposed which avoids these difficulties. Starting from a



# Optimal Fingerprints for the Detection of Time-dependent Climate Change

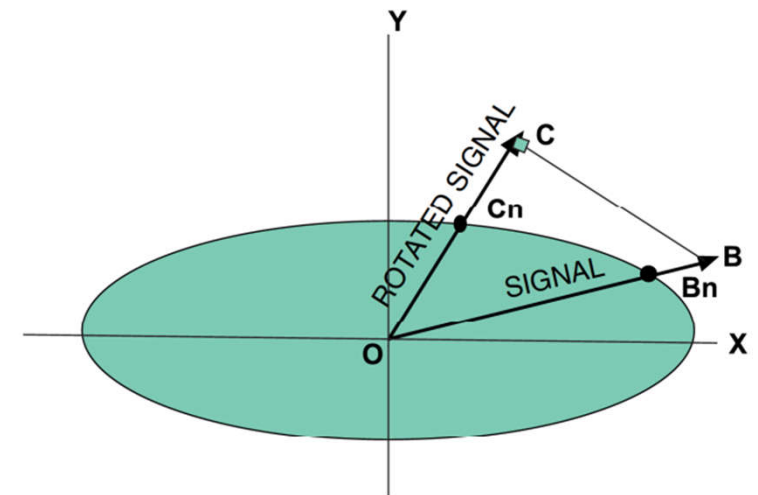
K. HASSELMANN

*Max-Planck-Institut für Meteorologie, Hamburg, Germany*

(Manuscript received 24 August 1992, in final form 17 March 1993)

## ABSTRACT

An optimal linear filter (fingerprint) is derived for the detection of a given time-dependent, multivariate climate change signal in the presence of natural climate variability noise. Application of the fingerprint to the observed (or model simulated) climate data yields a climate change detection variable (detector) with maximal signal-to-noise ratio. The optimal fingerprint is given by the product of the assumed signal pattern and the inverse of the climate variability covariance matrix. The data can consist of any, not necessarily dynamically



# Fingerprints of global warming

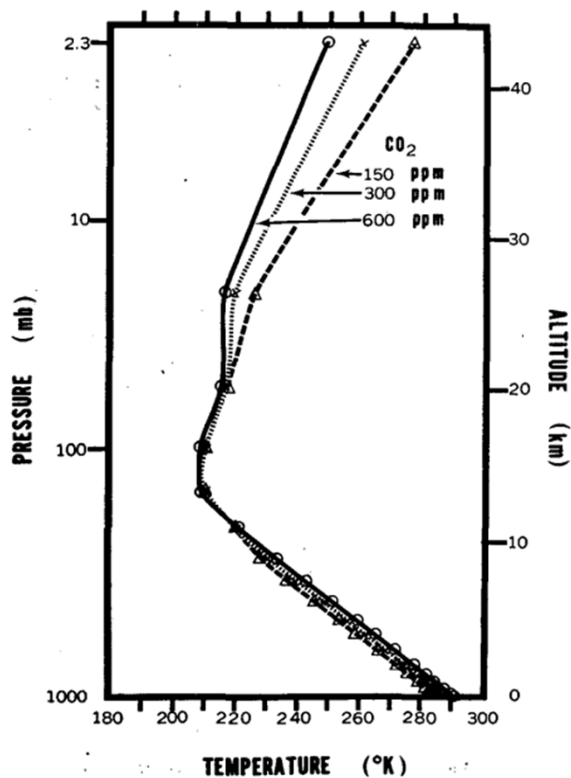
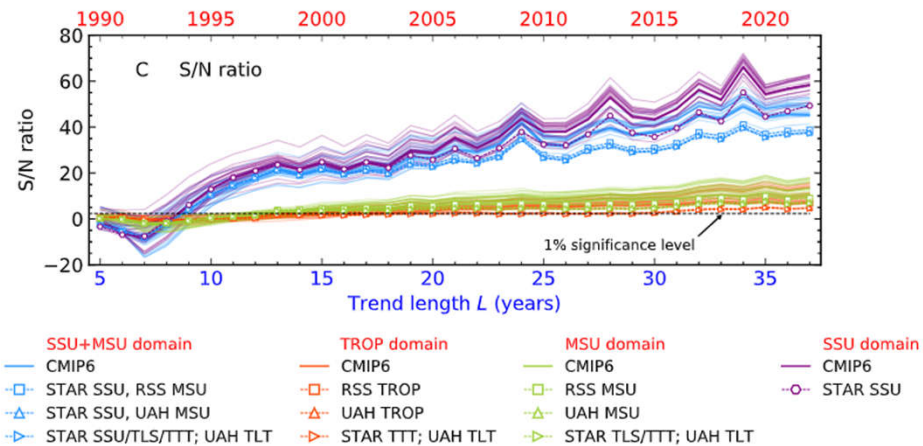
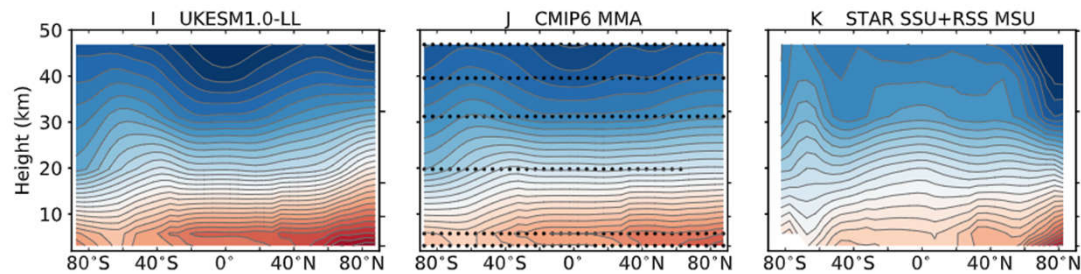


FIG. 16. Vertical distributions of temperature in radiative convective equilibrium for various values of CO<sub>2</sub> content.

Manabe and Wetherald, 1967



Santer et al., 2023

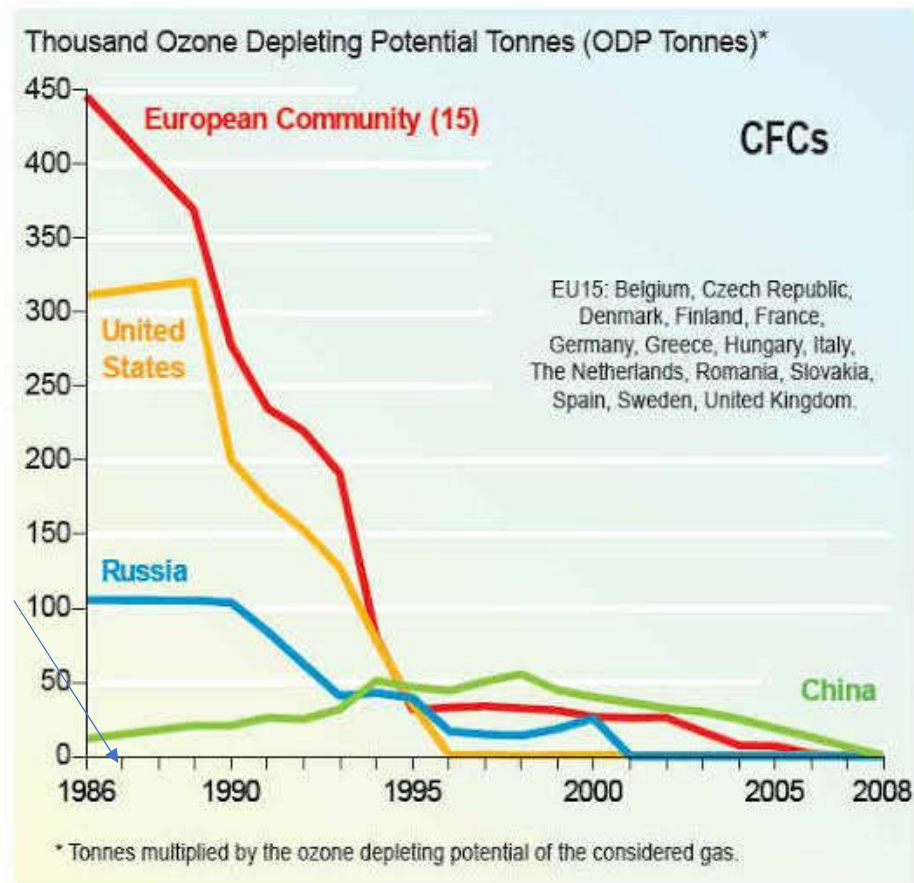
Article V. Ramanathan

## Trace-Gas Greenhouse Effect and Global Warming

Underlying Principles and Outstanding Issues  
Volvo Environmental Prize Lecture-1997

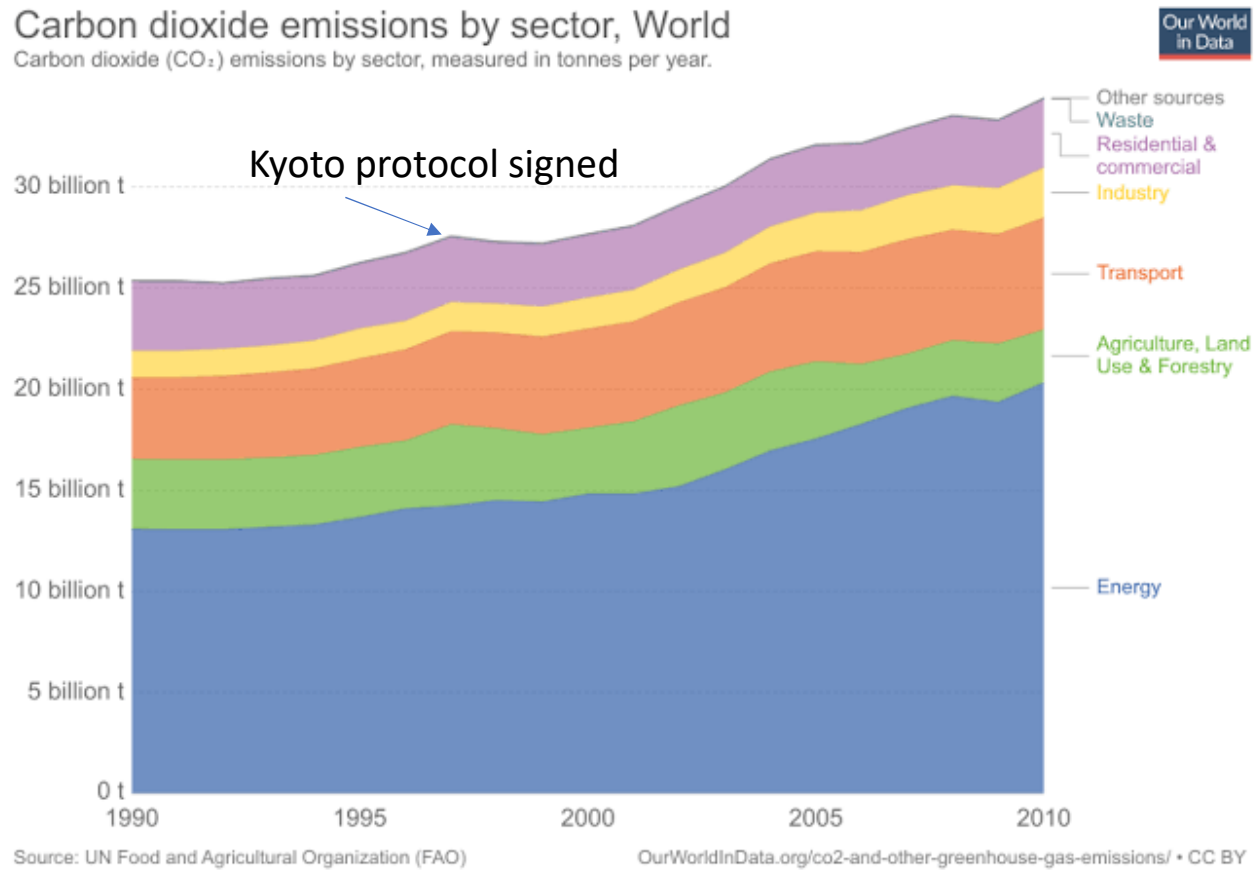
# Effect of Montreal Protocol to control ozone depleting substances

## PRODUCTION OF MAIN ODS GASES



Montreal Protocol  
signed

# Effect of Kyoto protocol to limit greenhouse gases



## Kyoto vs Montreal: what are the differences?

Kyoto	Montreal
Treaty (“...an international agreement concluded between States in written form and governed by international law)	Treaty
Legally binding	Legally binding
Founded on scientific evidence	Founded on scientific evidence
Countries saw it as too expensive to act alone on greenhouse gases	Countries saw it in their interest to act alone on ozone depleting substances
Believed that substitutes for fossil-fuel activities too expensive	Substitutes for CFCs had been invented
“GHG affects others” in the future and far away	“UV radiation affects us”
Domestic pressure only reaching a tipping point at the present time	Domestic pressure for action
Limited scope	Broad scope

I am pleased to sign the instrument of ratification for the Montreal protocol [governing] substances that deplete the ozone layer. The protocol marks an important milestone for the future quality of the global environment and for the health and well-being of all peoples of the world. Unanimous approval of the protocol by the Senate on March 14<sup>th</sup> [sic] demonstrated to the world community this country's willingness to act promptly and decisively in carrying out its commitments to protect the stratospheric ozone layer . . . .

—Ronald Reagan<sup>1</sup>

I oppose the Kyoto Protocol because it . . . would cause serious harm to the U.S. economy. The Senate's vote, 95-0, shows that there is a clear consensus that the Kyoto Protocol is an unfair and ineffective means of addressing global climate change concerns.

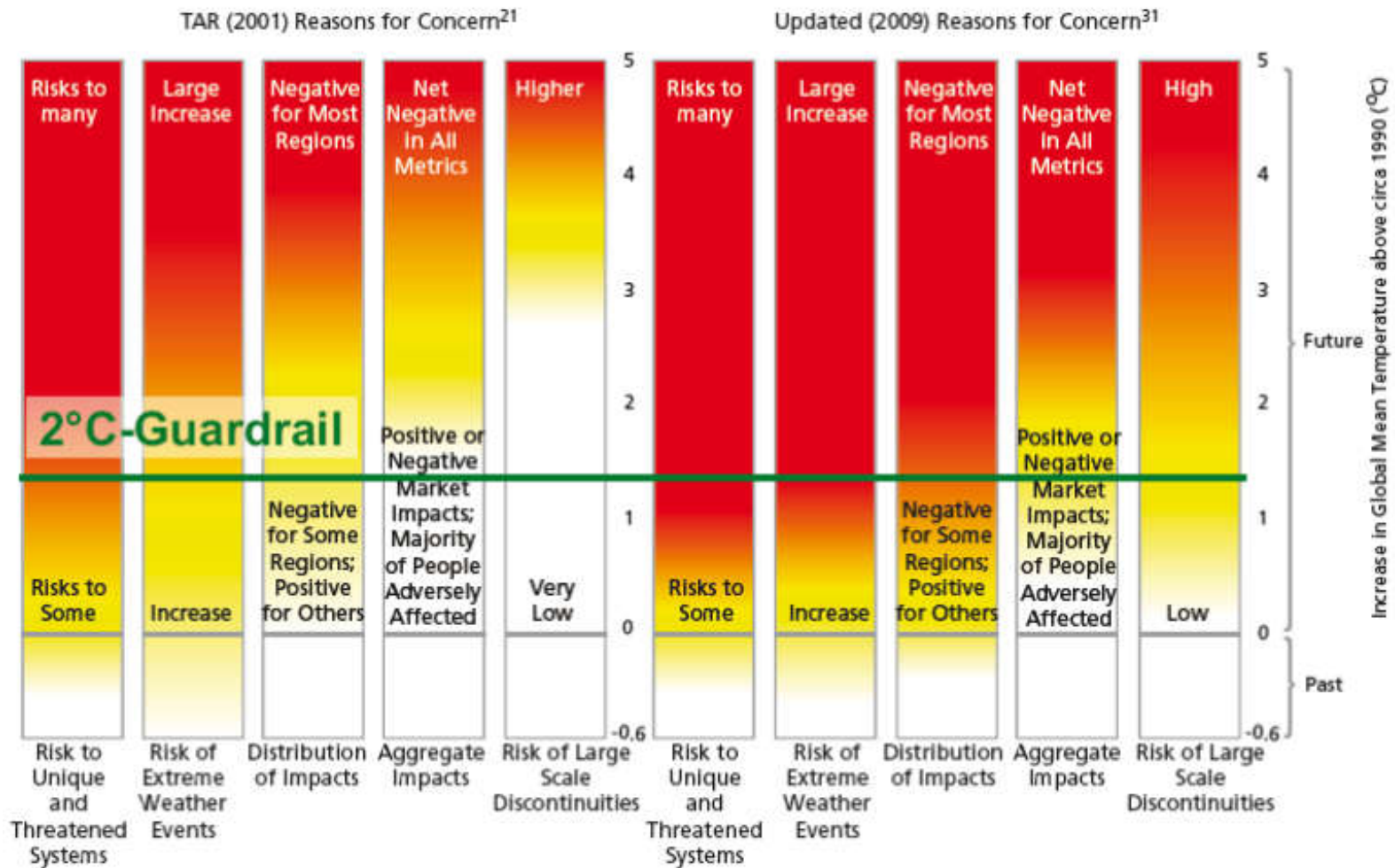
—George W. Bush<sup>2</sup>



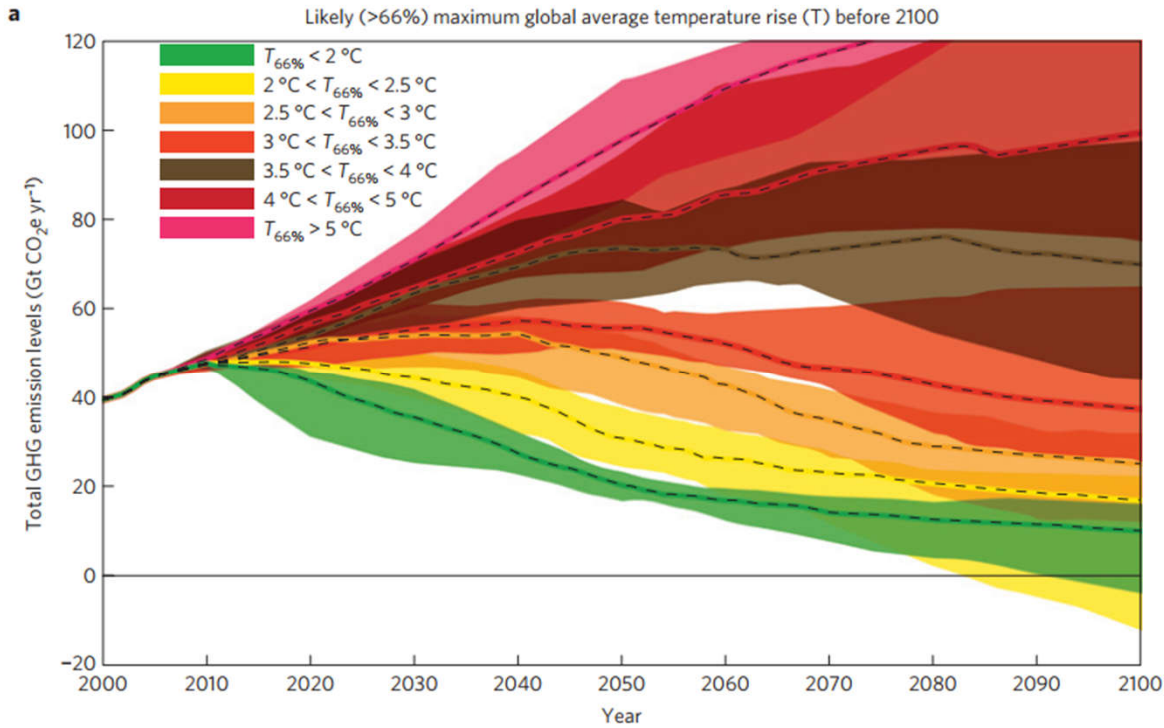
## Some highlights from the science (1997-2015)

- 2000: Variability and importance of biological feedbacks in global warming: studies found that warming may make it harder for soils and forests to take up carbon
- 2001: 3<sup>rd</sup> IPCC report, reporting that damages from climate change are higher than previously thought
- 2002: pollution has retarded global warming, but that effect is decreasing
- 2003: collapse of ice sheets over Greenland and West Antarctica can raise sea-level faster than previously thought
- 2005: tropical storms spur debate about role of global warming on storm intensity
- 2007: Greenland and Antarctic ice sheets and Arctic sea ice are shrinking faster than expected
- 2008: Even if all greenhouse gas emissions could be halted immediately, global warming will continue for millennia because of the long atmospheric lifetime of CO<sub>2</sub>.
- 2012: Studies suggest that recent disastrous heat waves, droughts, extremes of precipitation, and floods were made worse by global warming
- 2015: Collapse of West Antarctic ice sheet is irreversible, will bring meters of sea-level rise over future centuries.

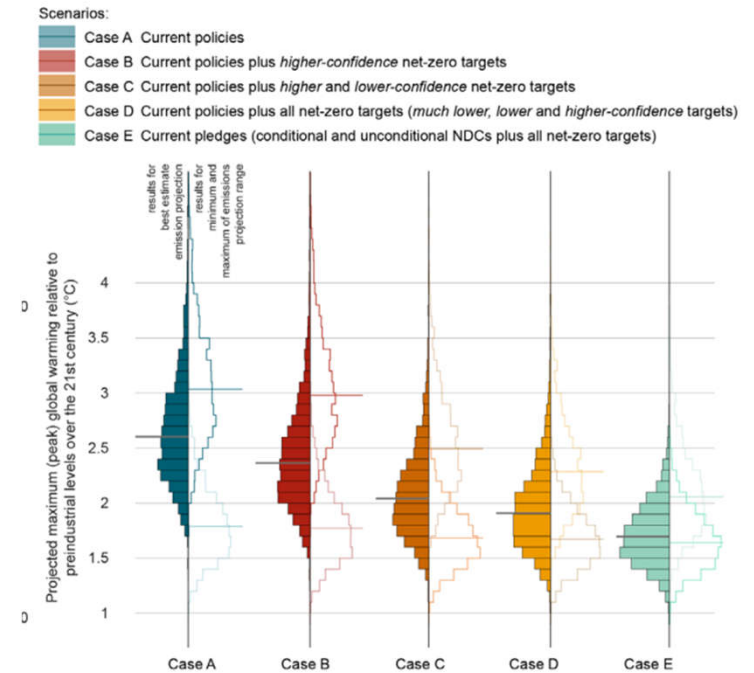
# Recent science has identified higher impacts from warming



# Mitigation involves uncertainty



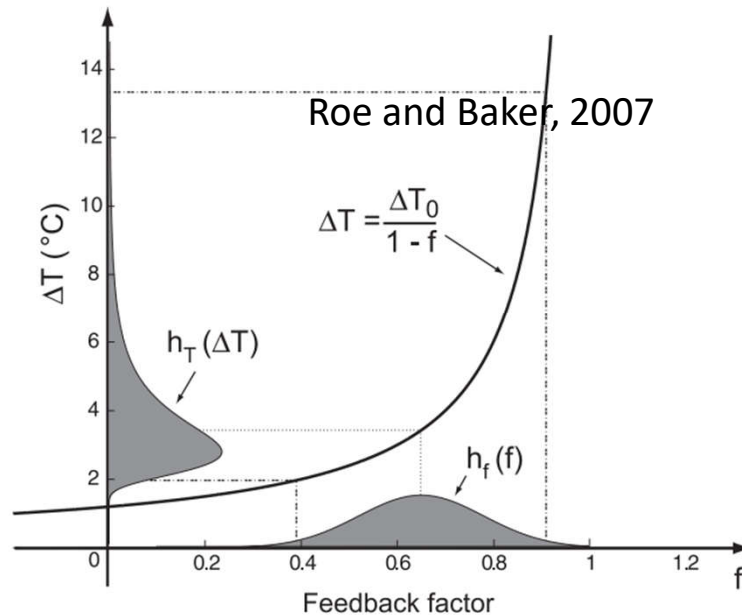
Rogelj et al., 2011



Rogelj et al., 2023

# Origins and implications of fat-tailed uncertainty in climate sensitivity

**Fig. 1.** Demonstration of the relationships linking  $h_T(\Delta T)$  to  $h_f(f)$ .  $\Delta T_0$  is the sensitivity in the absence of feedbacks. If the mean estimate of the total feedbacks is substantially positive, any distribution in  $h_f(f)$  will lead to a highly skewed distribution in  $\Delta T$ . For the purposes of illustration, a normal distribution in  $h_f(f)$  is shown with a mean of 0.65 and a SD of 0.13, typical to that obtained from feedback studies of GCMs (17, 18). The dot-dashed lines represent 95% confidence intervals on the distributions. Note that values of  $f \geq 1$  imply an unphysical, catastrophic runaway feedback.

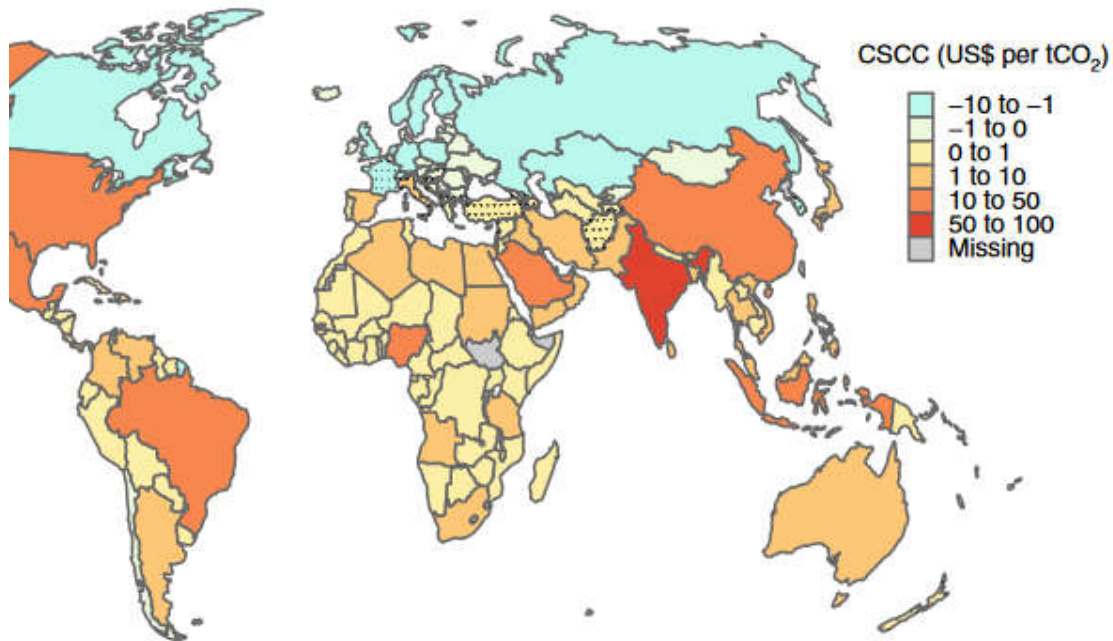


## Fat Tails and the Social Cost of Carbon<sup>†</sup>

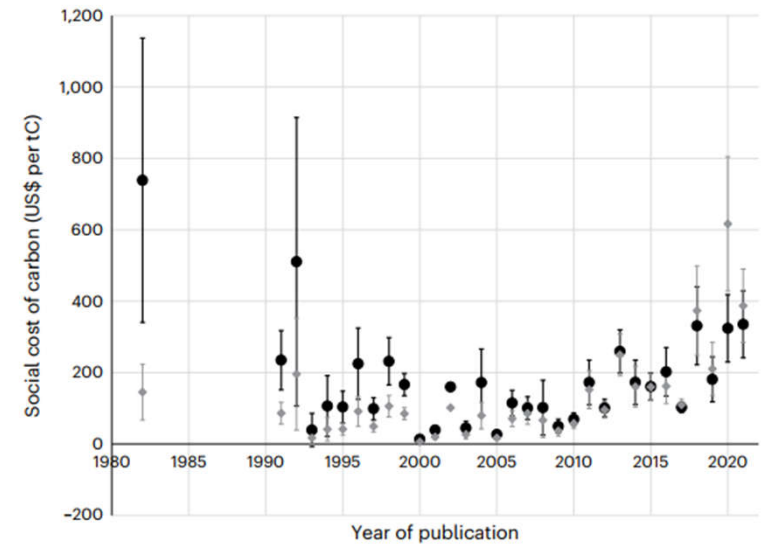
By MARTIN L. WEITZMAN\*

The “dismal theorem” is best understood as a cautionary tale. A fat tail for rare disasters has the *potential* to dominate economic calculations like the SCC. Therefore, analysis of a situation that might potentially be catastrophic cannot afford to ignore tail behavior. It is not enough in such situations to look just at measures of central tendency or even just at thin-tailed probability distributions. Ignorance of the potential fatness of an extreme bad tail is not an excuse for ignoring the potential fatness of an extreme bad tail. This warning is the main message of the “dismal theorem.”

# Social cost of carbon is estimated to be large, and estimates are increasing



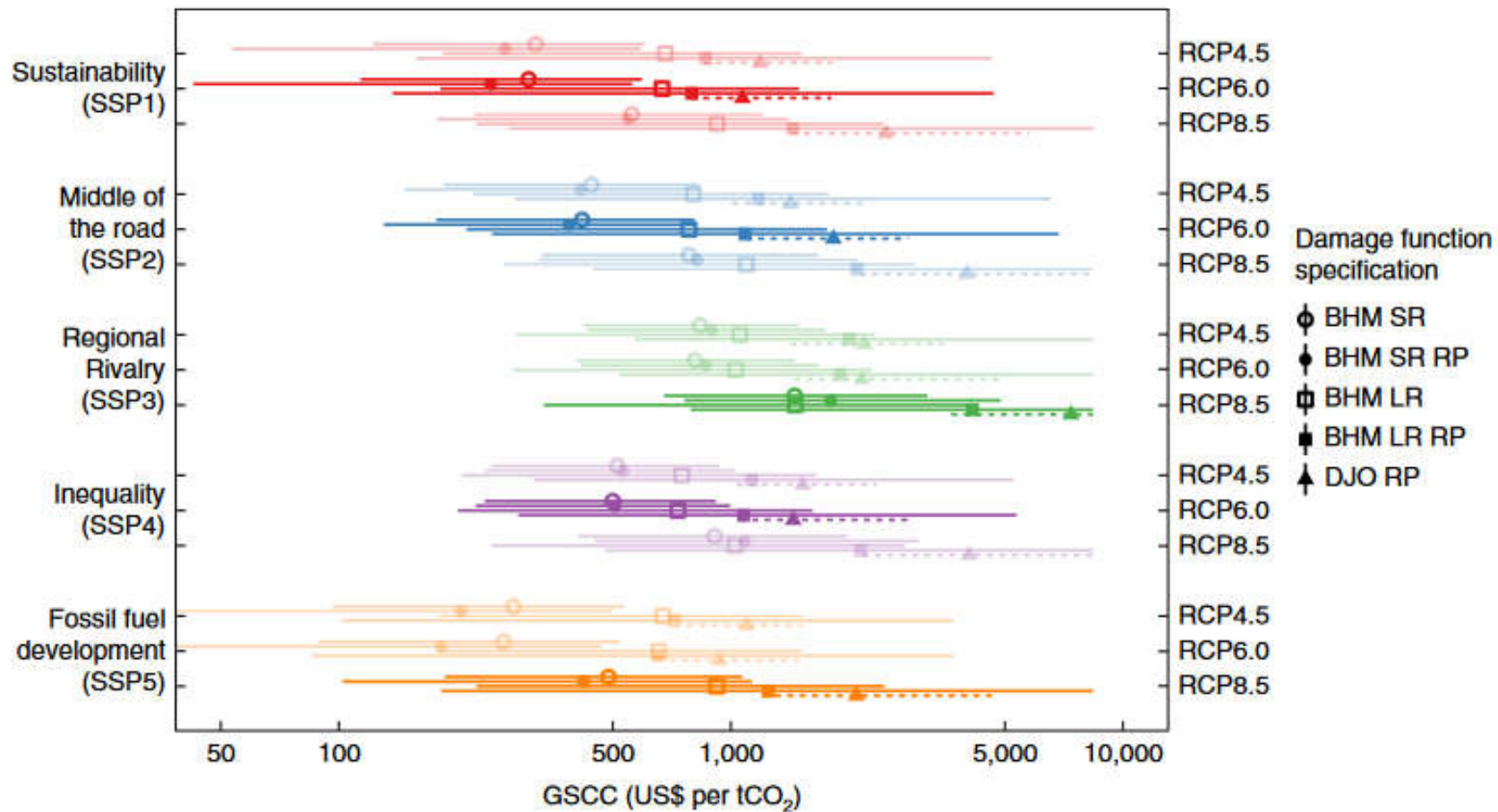
Ricke et al., 2018



**Fig. 1 | Average social cost of carbon by publication year.** Grey diamonds are as reported, black dots are corrected for inflation and year of emission. Error bars are  $\pm$  s.d. of the published estimates. Estimates are quality weighted and censored.

Tol, 2023

# Global social cost of carbon is much larger than mitigation costs



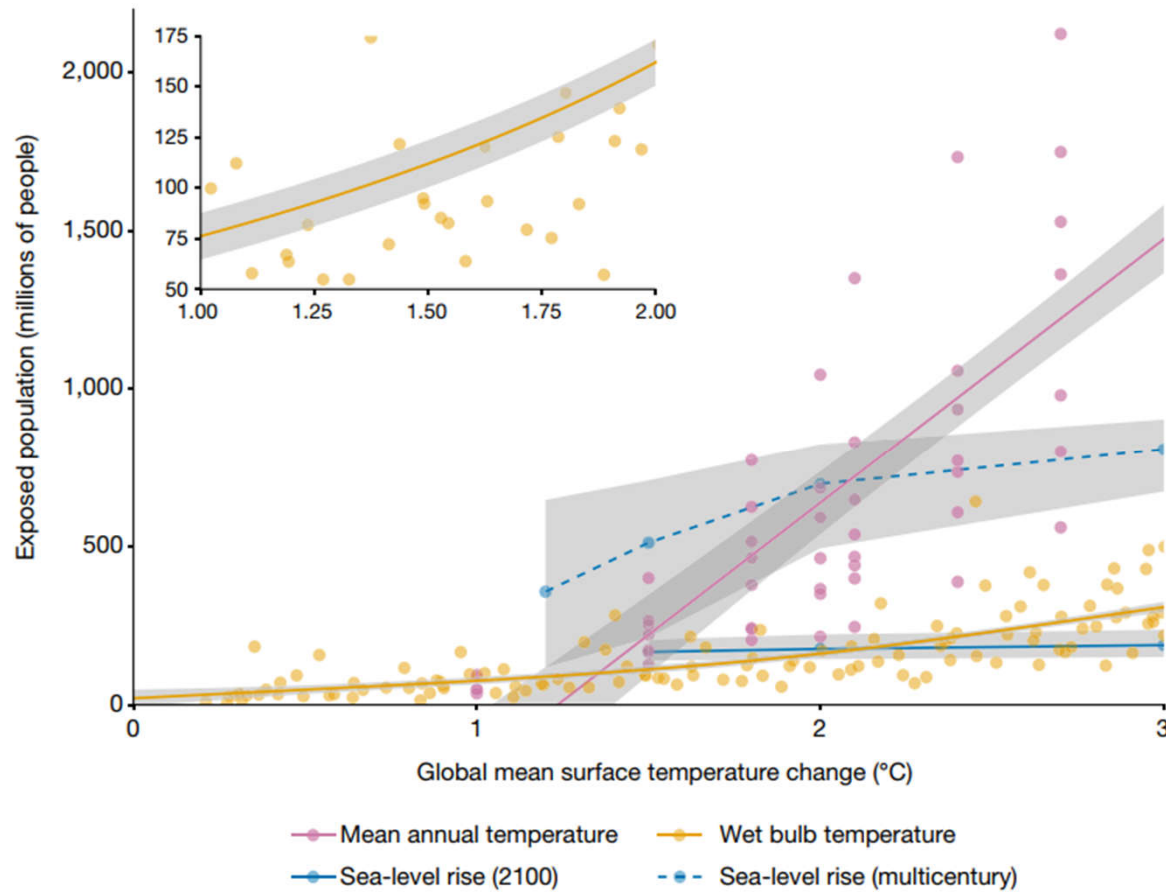
Ricke et al., Nature Climate Change, 2018

# Exposure

# Safe and just Earth system boundaries

<https://doi.org/10.1038/s41586-023-06083-8>  
Received: 23 June 2022

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Lauren S. Andersen<sup>1</sup>, David I. Armstrong McKay<sup>3,11,12</sup>, Xuemei Bai<sup>10</sup>, Govindasamy Bala<sup>13</sup>,



Rockström et al., 2023

A TIME FOR ACTION ON CLIMATE CHANGE AND A TIME  
FOR CHANGE IN ECONOMICS\*

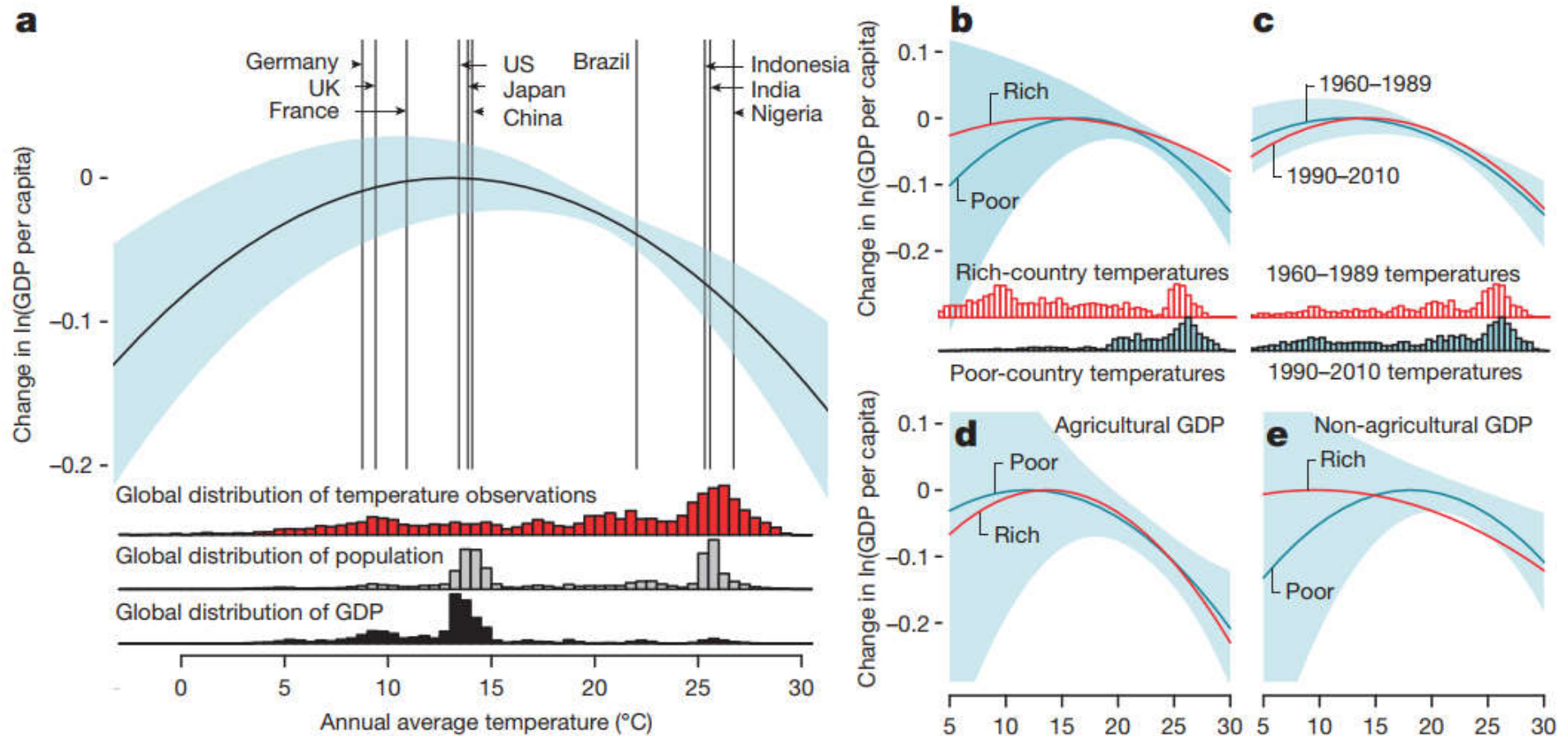
*Nicholas Stern*

**“Nordhaus’s question, recognising that there could be potential dangers from climate change and that emissions arose from activities around producing and consuming, was ‘should we grow a little less fast than we might have envisaged before we thought about climate change?’.** He proceeded in a sensible way, taking an emerging problem and applying the standard tools of economics: first the Pigouvian story of marginal social costs, marginal private costs, and taxing for the externality; second on growth, he used the framework of a standard exogenous growth model and considered the impact of climate change largely in terms of small perturbations around the underlying growth path(s). That was a sensible and valuable early contribution for the economics of climate change. **Over the following 10–15 years, it became more and more clear that climate change is not a marginal problem. We are dealing with a challenge involving huge potential disruptions. Further, rising to that challenge requires very radical changes in our production systems and ways of consuming. ....**

Their scope has been expanded, but the basic underlying features of optimisation of explicit, calibrated social welfare functions, of underlying exogenous growth and of aggregation (usually to one good) impose severe limitation on their ability to illuminate *two basic questions*. **The first is how to approach analytically the challenge of managing immense risk, which could involve loss of life on a massive scale. The second is how to chart and guide a response to this challenge which will involve fundamental structural change across a whole complex economy.** “ (Quoted from Stern, 2023)

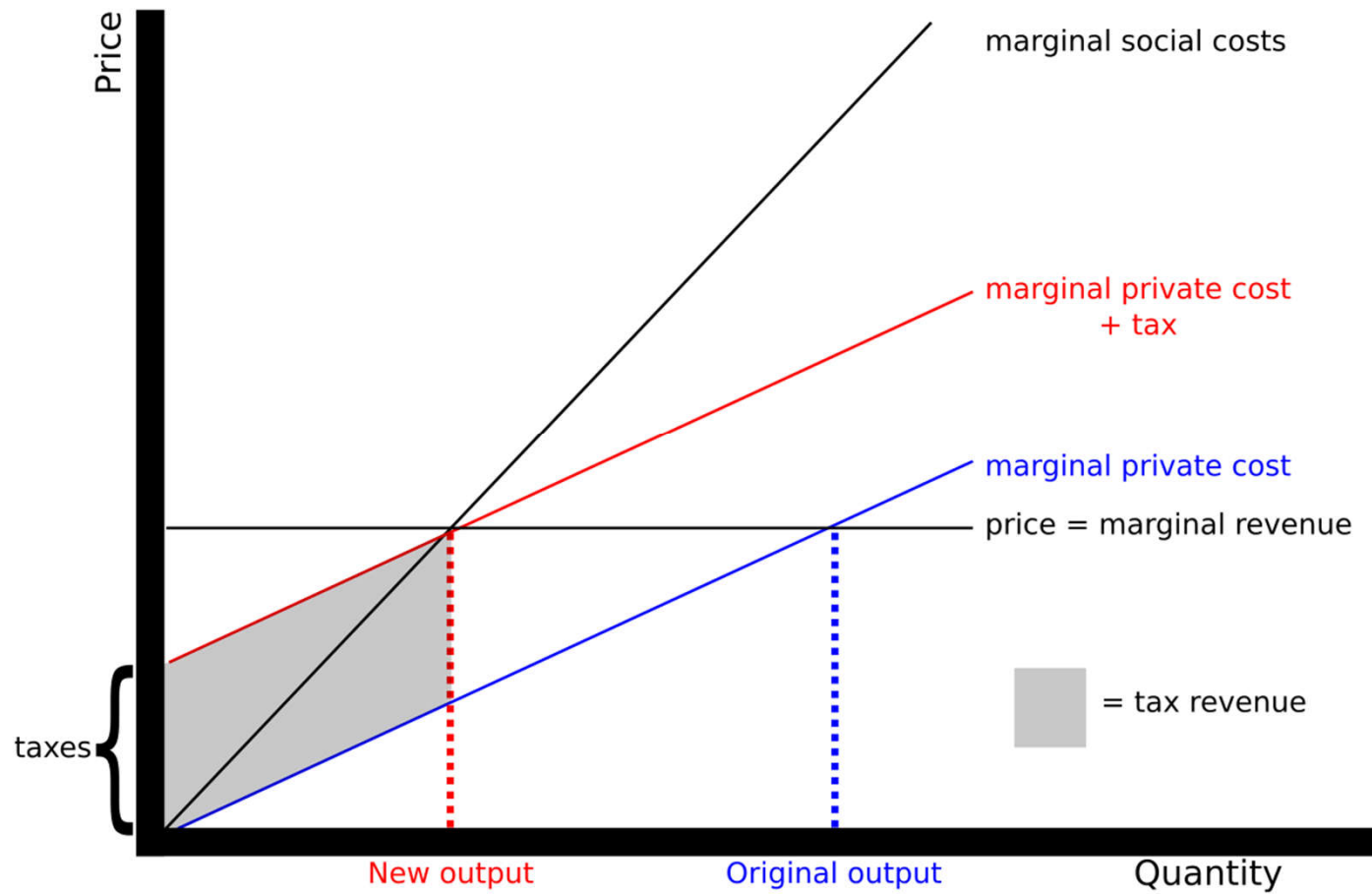


# Economic productivity peaks at around 13°C

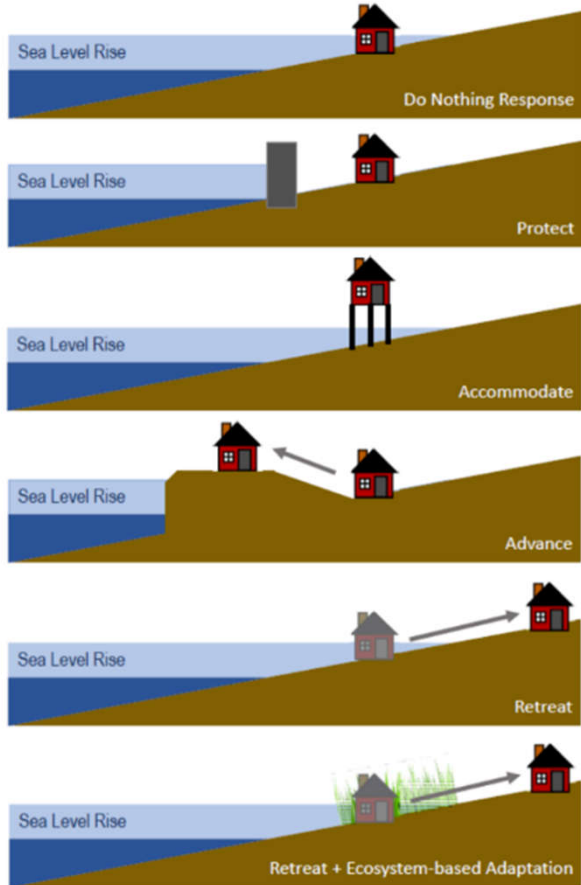


Burke et al., Nature, 2015

# Pigouvian Tax



Arthur Pigou



## Limits to adaptation to climate change: a risk approach

Kirstin Dow<sup>1</sup>, Frans Berkhout<sup>2</sup> and Benjamin L Preston<sup>3</sup>

