Simple models of climate variability and change

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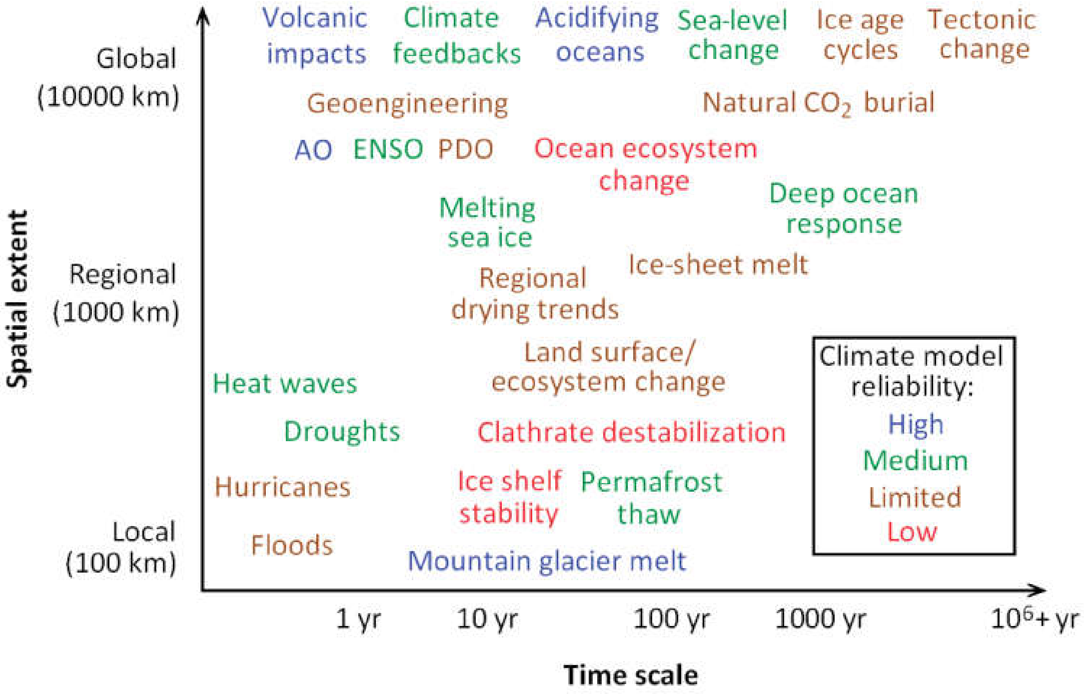
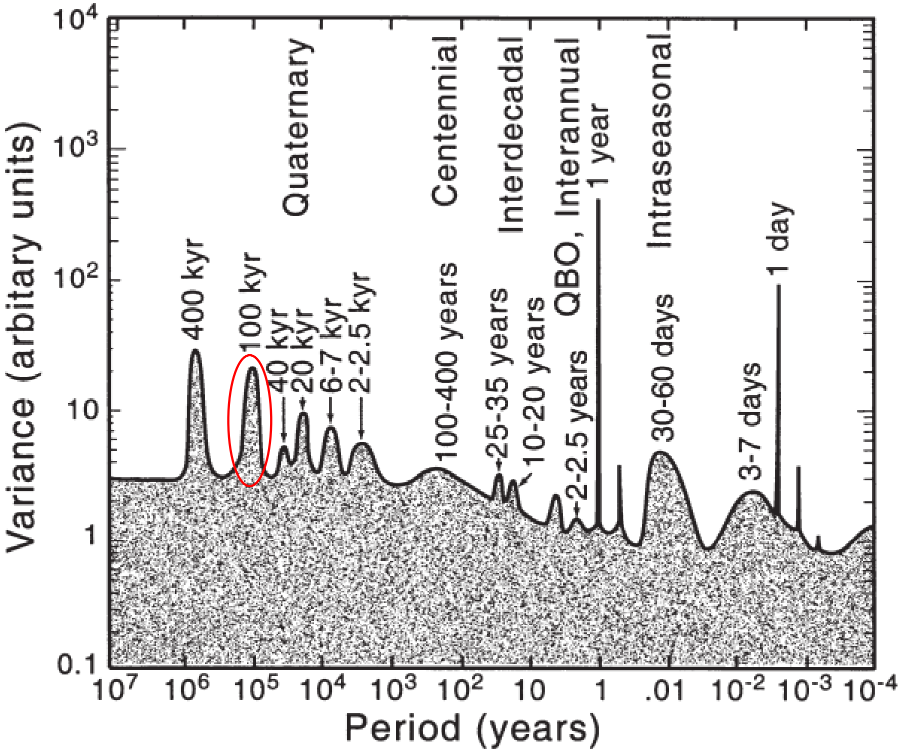
Wide range of space and timescales

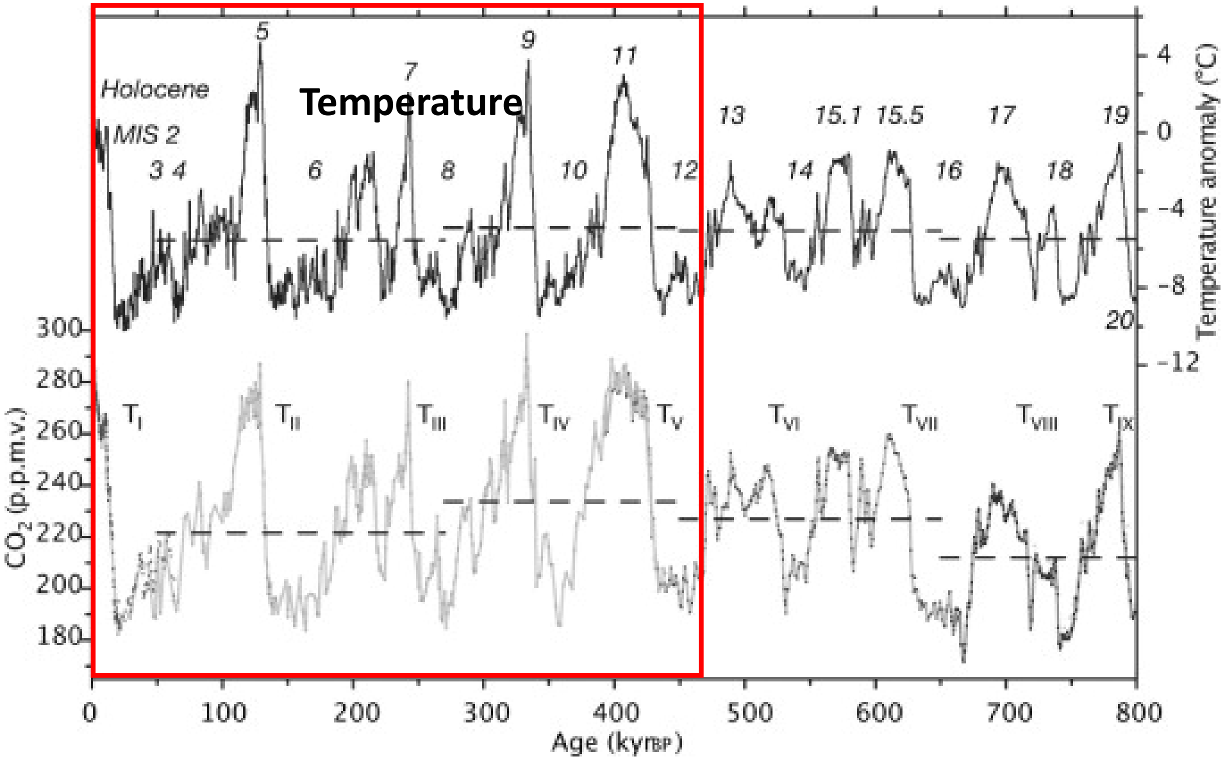


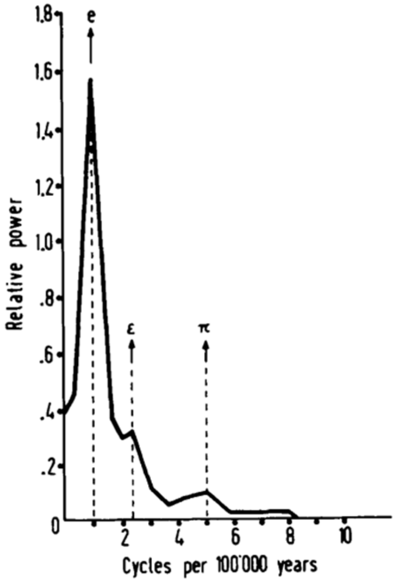
National Research Council. 2012. *A National Strategy for Advancing Climate Modeling*.

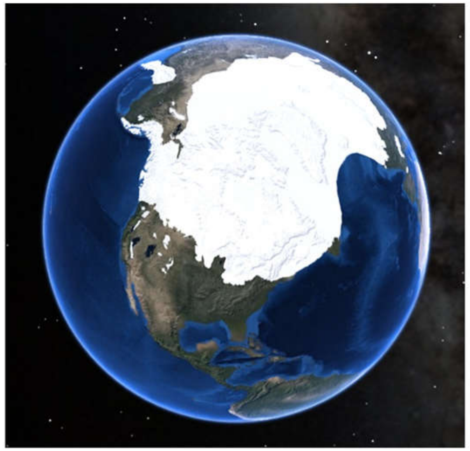


Dijkstra and Ghil, 2005

Glacial-Interglacial cycles







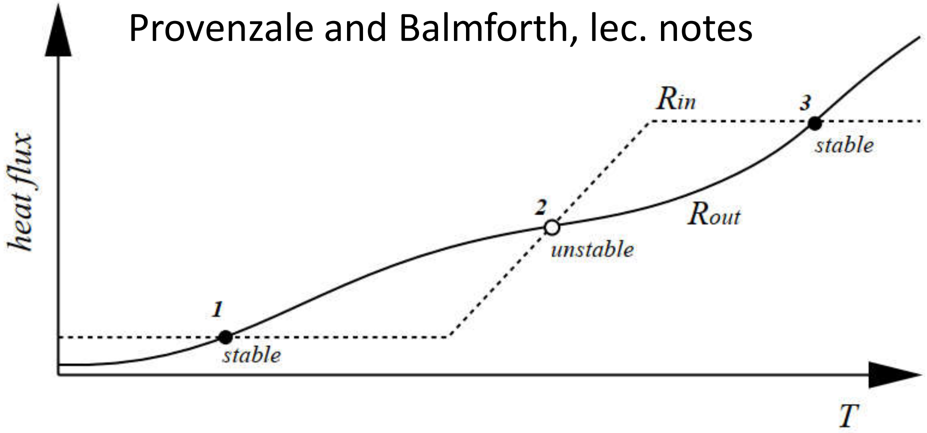
A reconstruction of the maximum extent of the Laurentide and Cordilleran Ice Sheets, which covered much of North America approximately 20,000 years ago

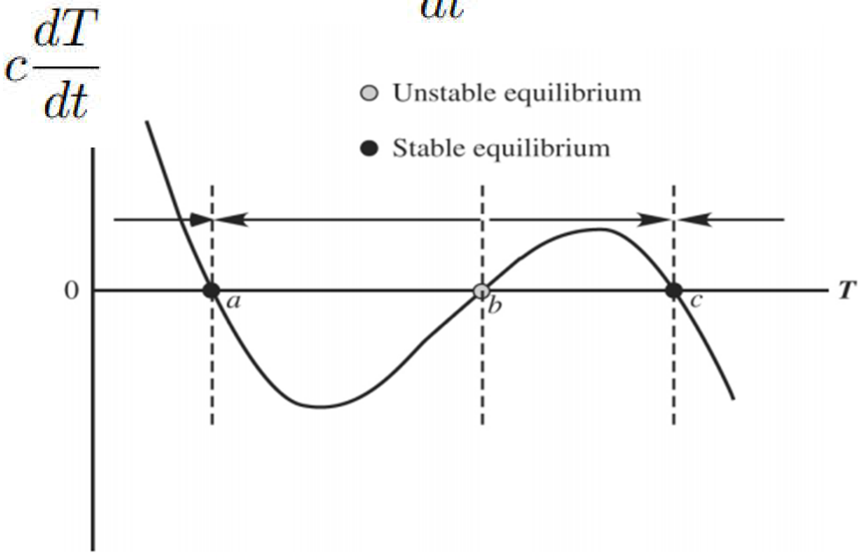
Courtesy of NOAA Science on a Sphere

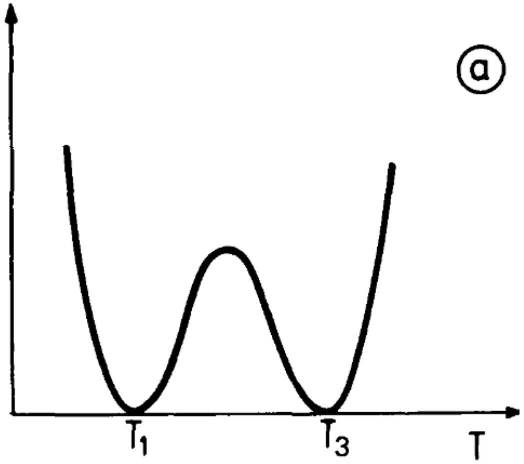
Multiple equilibria in Earth's climate system

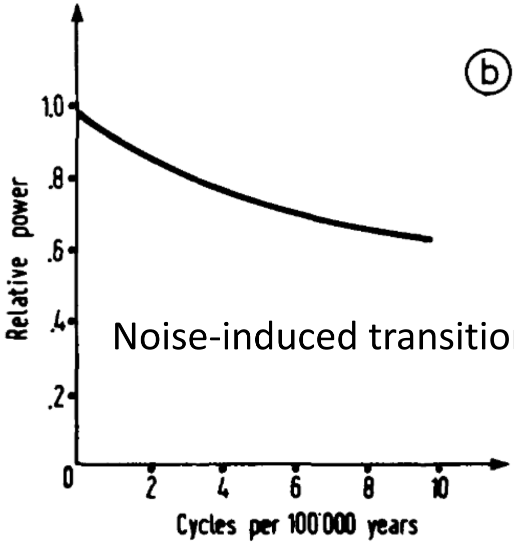
$$c\frac{dT}{dt} = R_{in} - R_{out}$$

Provenzale and Balmforth, lec. notes





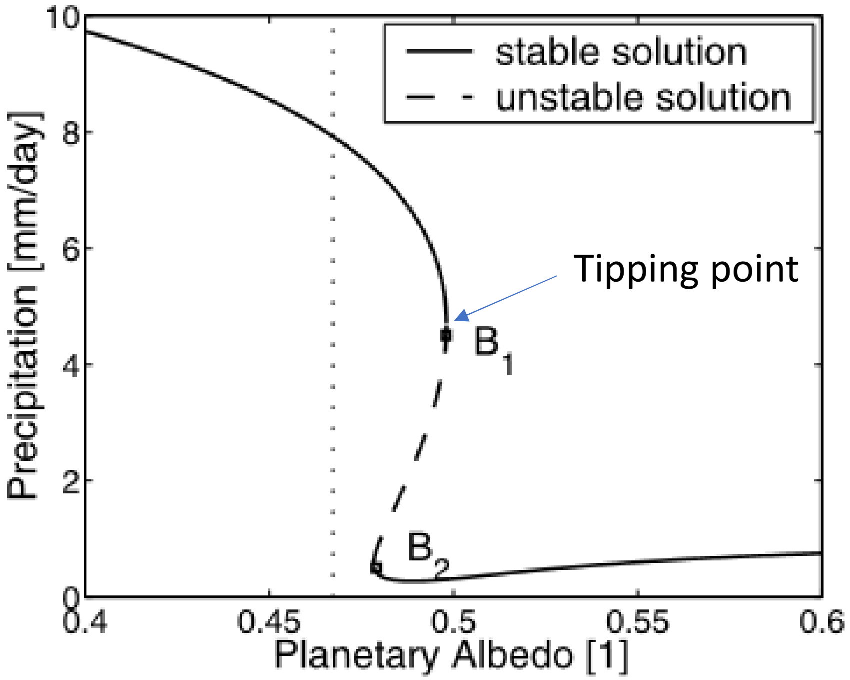




Noise-induced transition?

Benzi et al., 1982

Bifurcation in monsoon model of Zickfeld et al. (2005)



Tipping point

Figure 1. Bifurcation diagram of Indian summer precipitation against the planetary albedo. The B_1, B_2 mark the saddle-node bifurcations. The vertical dotted line indicates the present-day state.

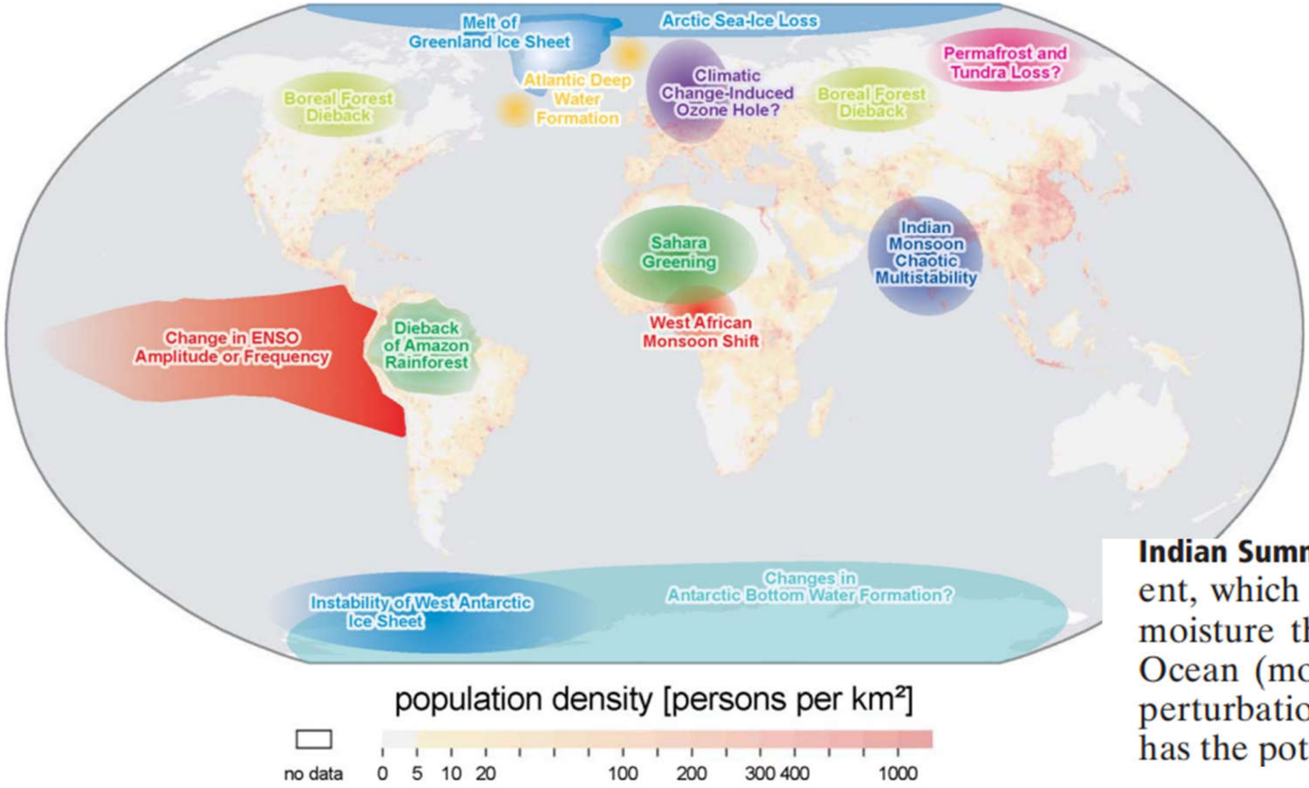
Zickfeld et al., “Is the Indian summer monsoon stable against global change?”, Geophysical Research Letters, 2005

Tipping elements in the Earth's climate system

Timothy M. Lenton[*,†], Hermann Held[‡], Elmar Kriegler[‡,§], Jim W. Hall[¶], Wolfgang Lucht[‡], Stefan Rahmstorf[‡], and Hans Joachim Schellnhuber[†,‡,‖,]**

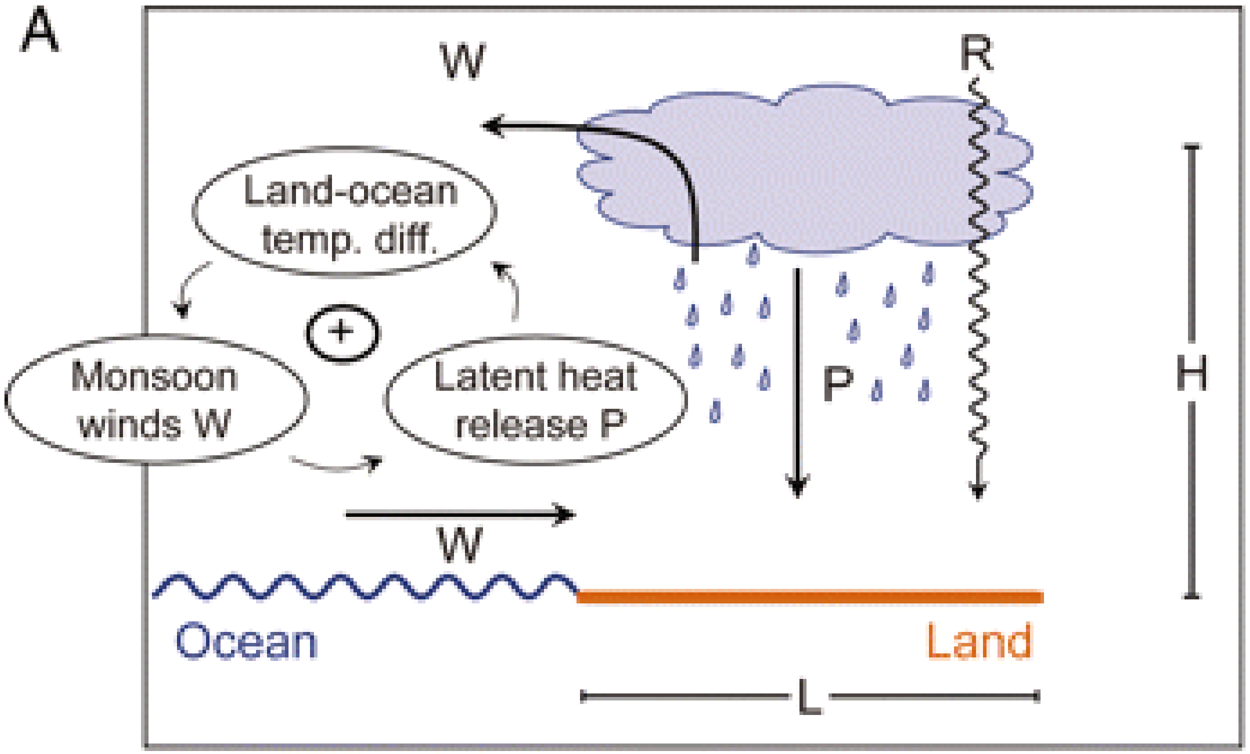
Lenton et al., Tipping elements in the Earth's climate system, PNAS, 2008

The term "tipping point" commonly refers to a critical threshold at which a tiny perturbation can qualitatively alter the state or development of a system. Here we introduce the term "tipping element" to describe large-scale components of the Earth system that may pass a tipping point. We critically evaluate potential policy-relevant tipping elements in the climate system under anthropogenic forcing, drawing on the pertinent literature and a recent international workshop to compile a short list, and we assess where their tipping points lie.



Indian Summer Monsoon (ISM). The land-to-ocean pressure gradient, which drives the monsoon circulation is reinforced by the moisture the monsoon itself carries from the adjacent Indian Ocean (moisture-advection feedback) (63). Consequently, any perturbation that tends to weaken the driving pressure gradient has the potential to destabilize the monsoon circulation. Green-

Schematic of the model



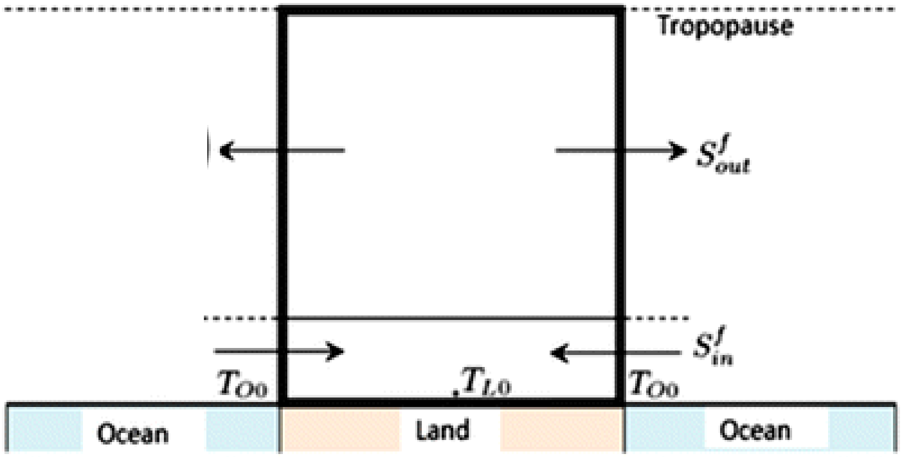
Levermann et al., “Basic mechanism for abrupt monsoon transitions”, PNAS, 2009

“The abruptness of the transition emerges through an additional stabilizing effect of the **direct heat advection which is cooling the atmospheric column [emphasis added]** and is also reduced for reduced monsoon winds. “

Moisture transported by monsoon winds further amplifies the winds. This is the positive feedback of the model. Does this make monsoons a tipping element?

Dry static energy dynamics

Stabilizing flux has linear as well as a quadratic term in horizontal temperature difference



$$s = c_p T + gz$$

$$\frac{ds}{dz} = c_p\left(\Gamma_d - \Gamma\right) > 0$$

Dry adiabatic lapse rate

Seshadri, “Energetics and monsoon bifurcations”, Climate Dynamics, 2016.

Conservation of dry static energy: $s = c_p T + gz$

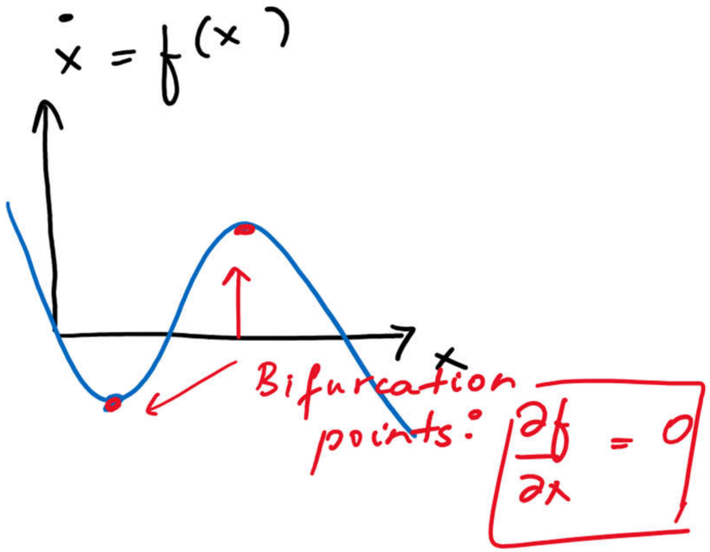
Integrate over atmospheric column

Model for column-mean temperature difference between land and that of ocean boundaries

Rate of change of temperature difference x = Radiative flux + conductive (“sensible”) heat flux + latent heat (moisture advection feedback) + horizontal flux S^f_{net} (this is the main nonlinear effect)

$$S^f_{net} \cong -\frac{Bk_w}{\Gamma_D}(p_{0O} - p_D)\left\{\underbrace{\left(\frac{R}{c_p}\frac{\Gamma_D - \Gamma_O}{\Gamma_D}\bar{T}_O\right)}_{\text{Vertical gradients in dry static energy}} x + \underbrace{\left(1 + \frac{R}{c_p}\right)}_{\text{Horizontal gradients in dry static energy}} x^2\right\}$$

To study bifurcations in this model, we look to the state dependent processes



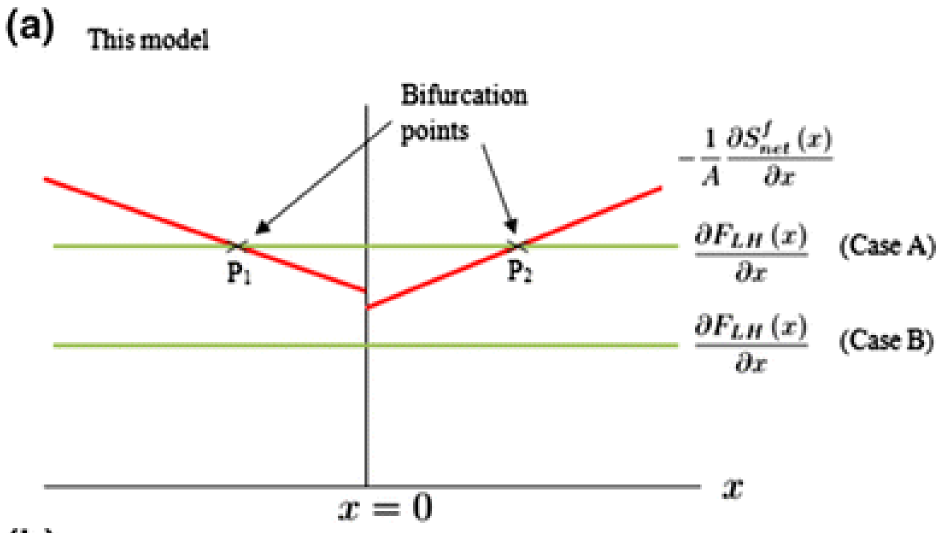
- Bifurcation occurs where the amplifying and stabilizing effects balance each other
- Where these effects are balanced, bifurcations can occur

Rate of change of temperature difference x = Radiative flux + conductive ("sensible") heat flux + latent heat (due to condensation) + horizontal flux S^{f}_{net} (this is the main nonlinear effect)

latent heat (due to condensation): Amplifying, i.e., positive & magnitude increases with x

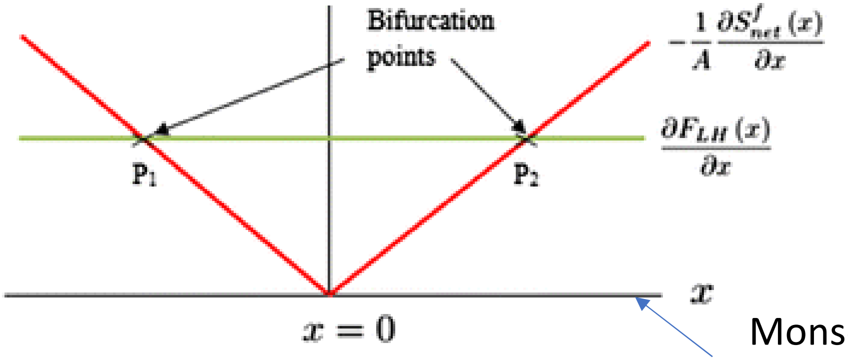
horizontal flux: Stabilizing, i.e., negative & magnitude increases strongly with x

Condition for bifurcation in dry static energy model



red: stabilizing
green: amplifying

(b) Without stratification



Monsoon steady state is somewhere here

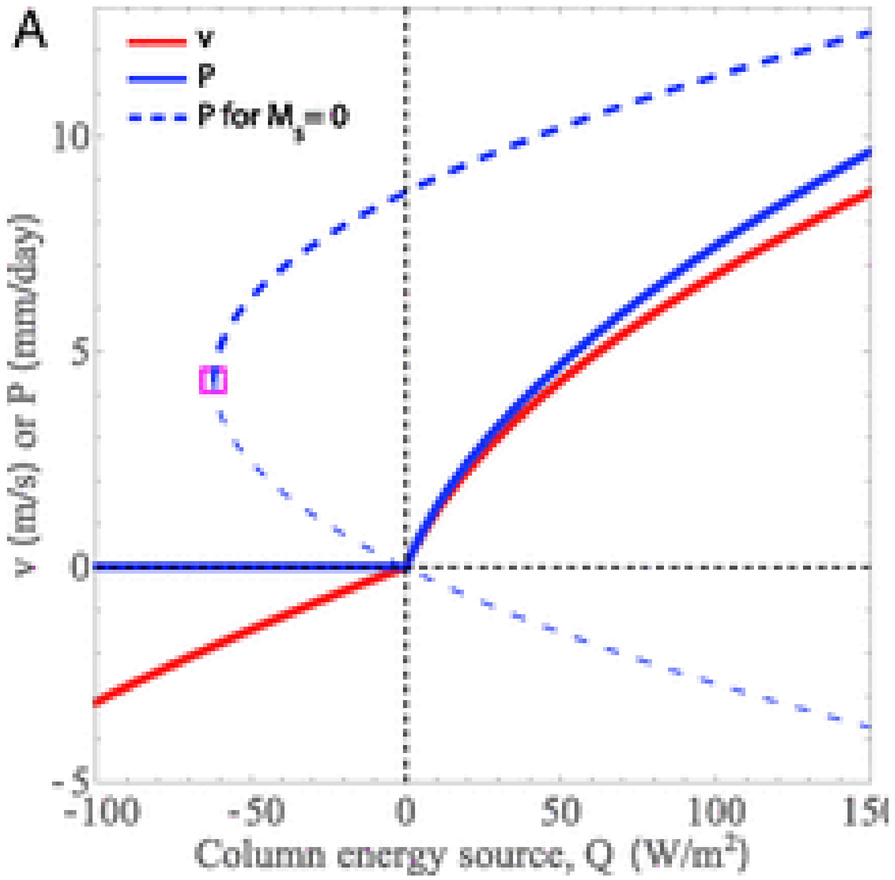
$$s = c_pT + gz$$

$$\frac{ds}{dz} = c_p\left(\Gamma_d - \Gamma\right)$$

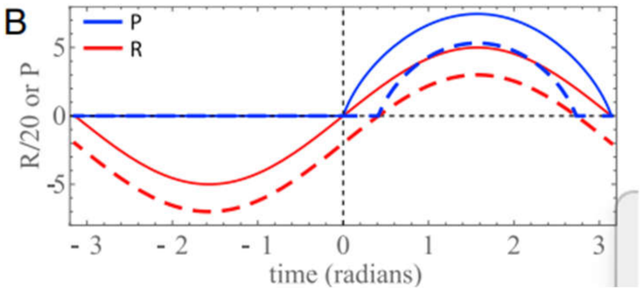
Stabilizing effect is much weaker if dry static stability is ignored

Seshadri, “Energetics and monsoon bifurcations”, Climate Dynamics, 2016.

Study based on Quasi Equilibrium Tropical Circulation Model (QTCM)



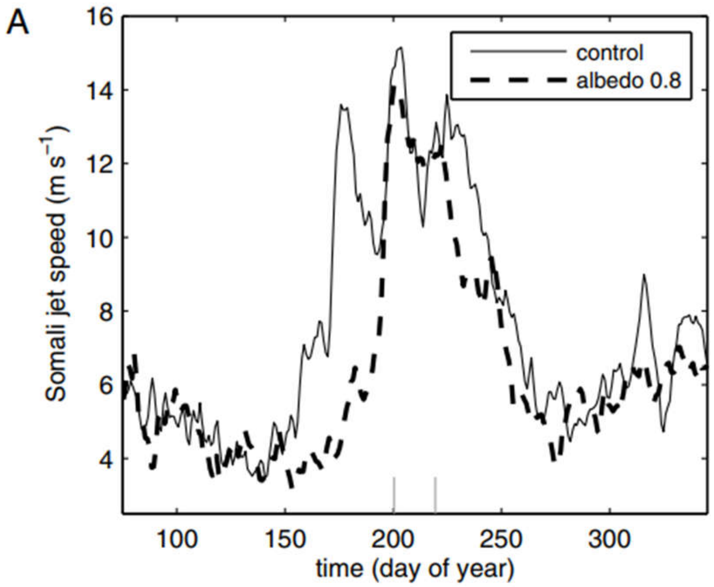
"Here it is shown that the theory used to predict such "tipping points" omits a dominant term in the equations of motion, and that both a corrected theory and an ensemble of global climate model simulations exhibit no abrupt shift in monsoon strength in response to large changes in various forcings."

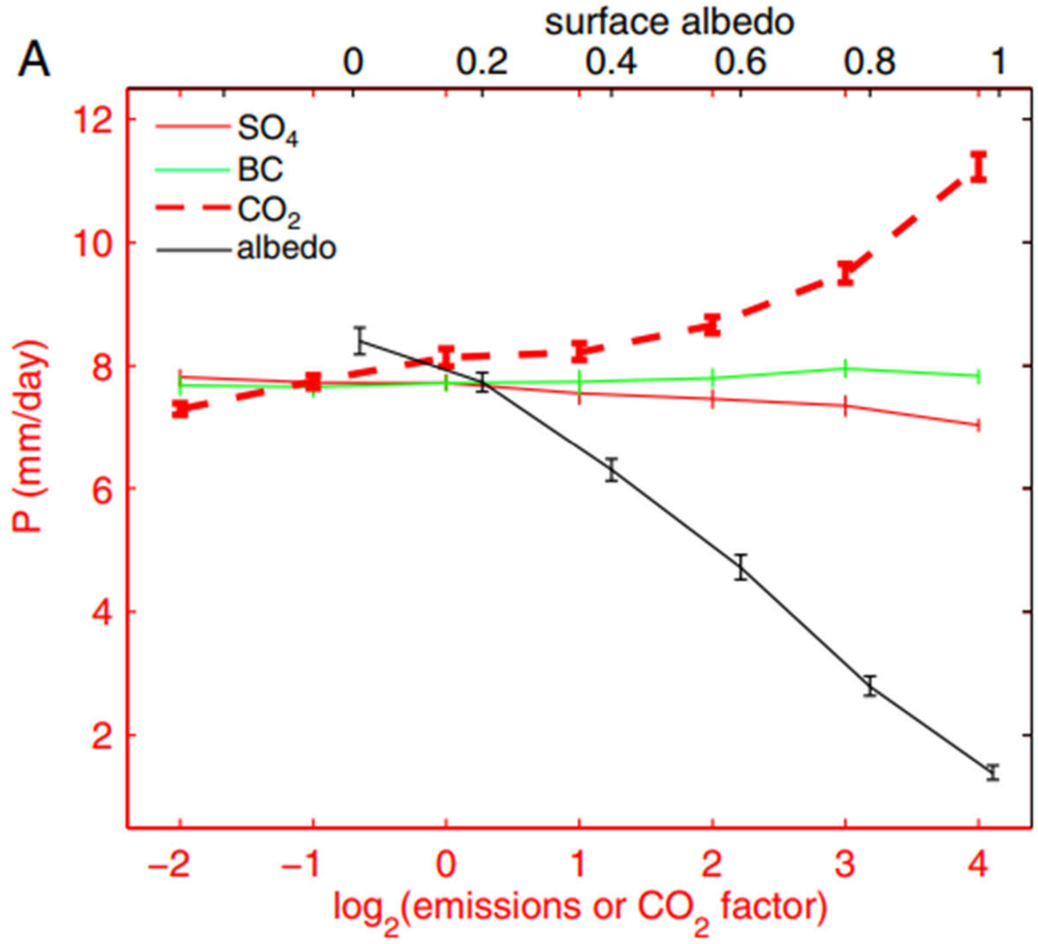


Monsoon occurs once forcing becomes positive

Boos and Storelvmo, "Near-linear response of mean monsoon strength to a broad range of radiative forcings", PNAS, 2016.

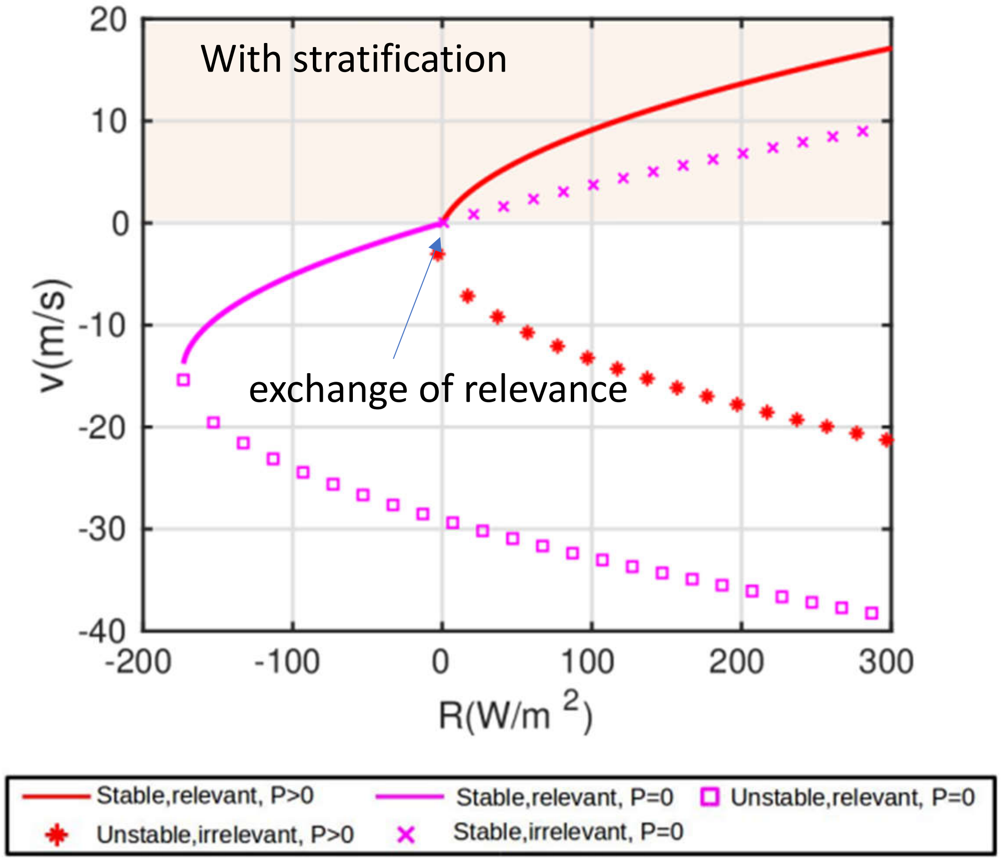
Effect of albedo in Community Earth System Model simulations





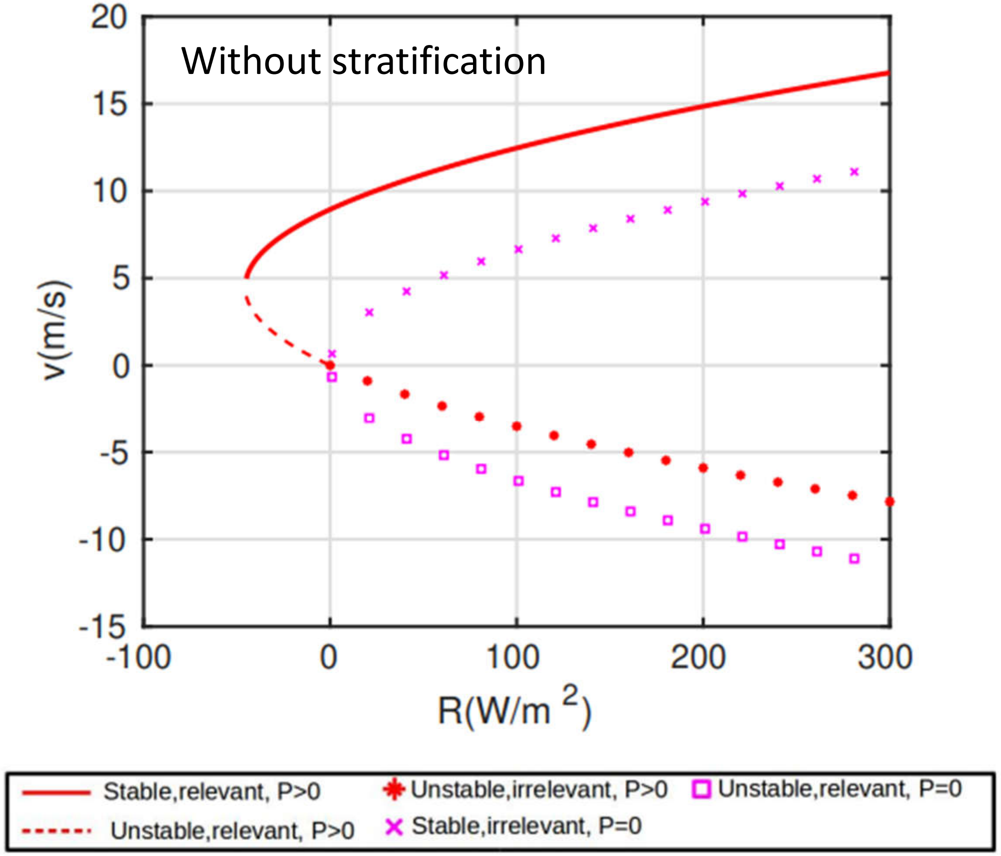
Figures from Boos and Storelvmo, 2016

Further interpretations using low-order models



With stratification

exchange of relevance

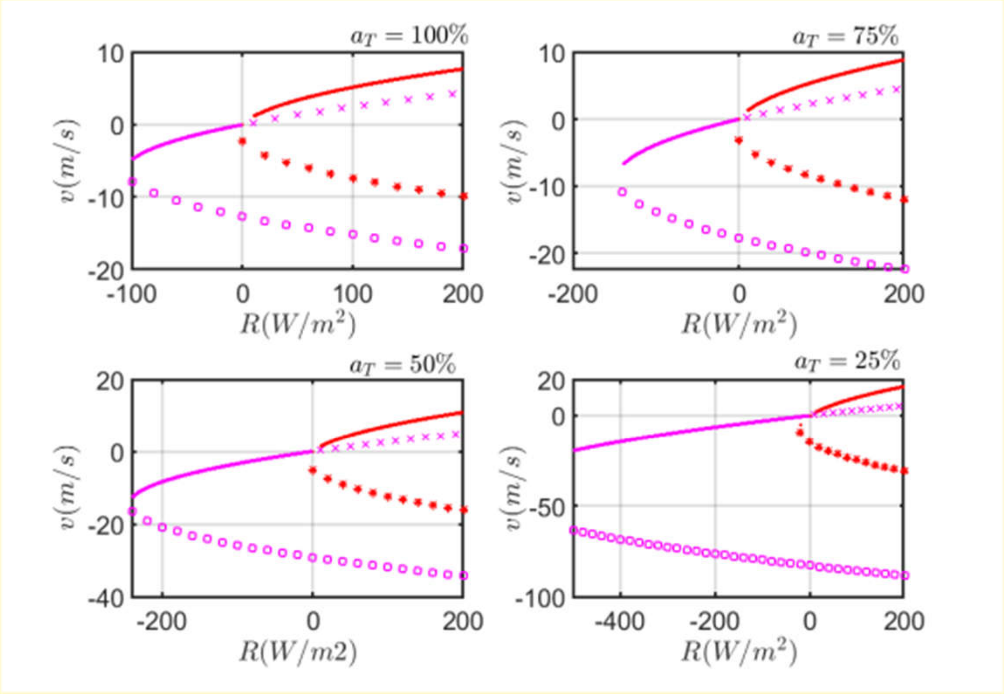


Without stratification

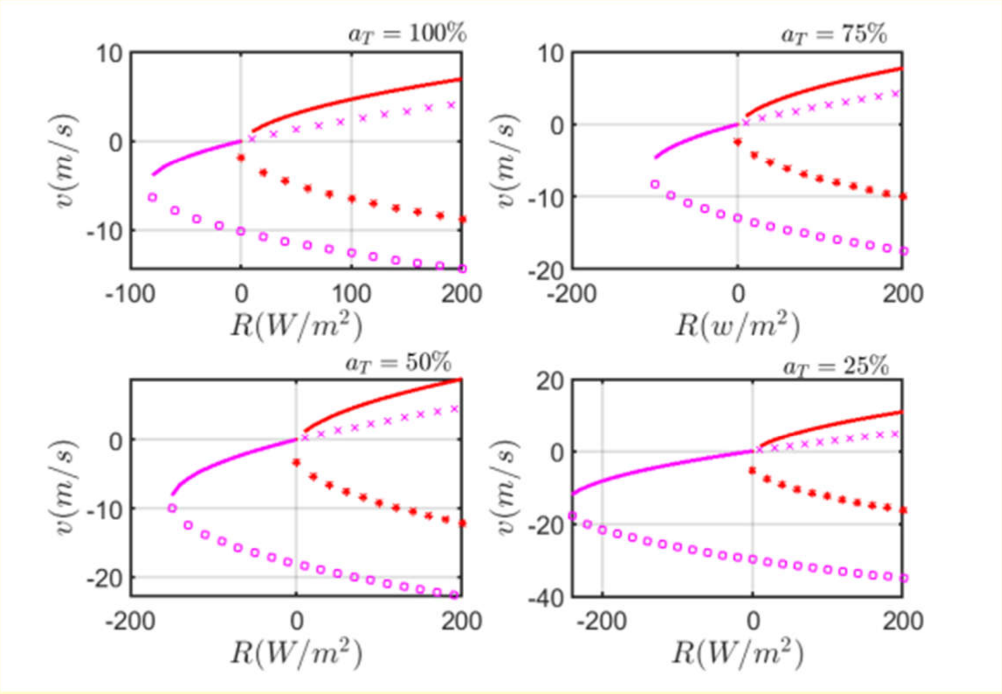
Kumar and Seshadri, Earth System Dynamics Discussions, 2022

Towards a model hierarchy

Without planetary rotation
Land-sea breeze

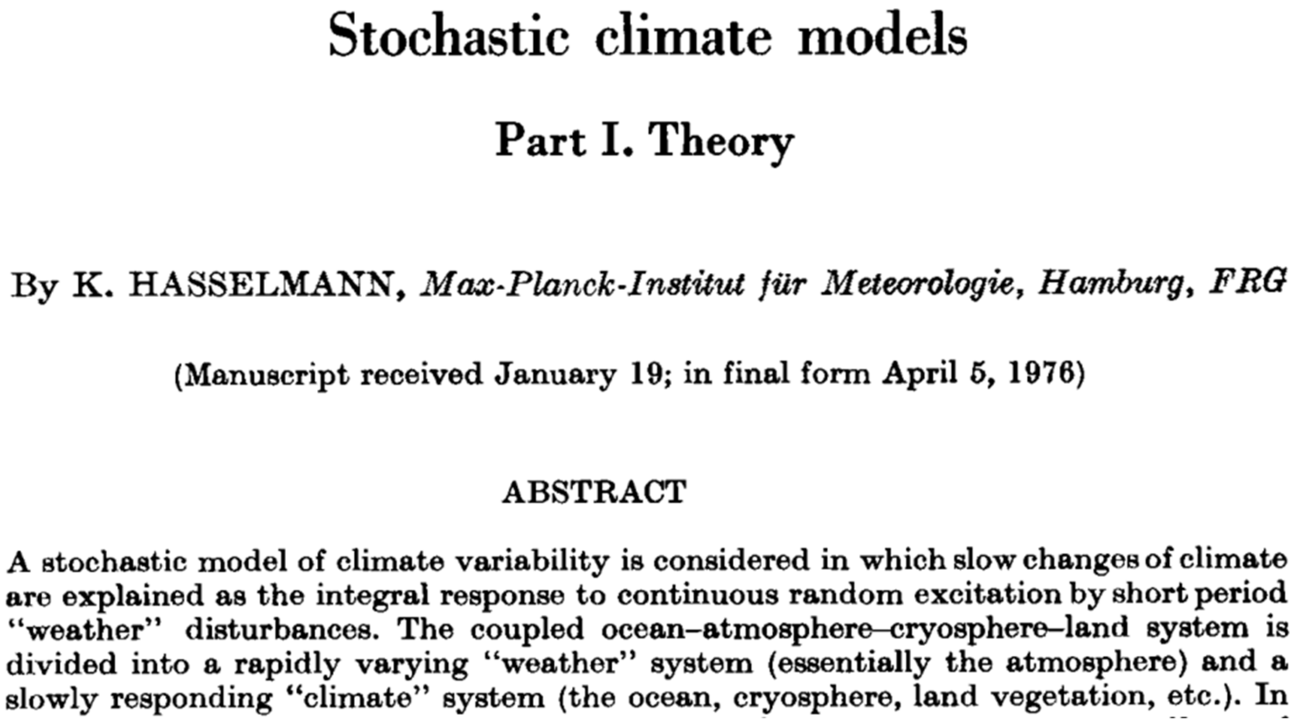


With planetary rotation
Monsoon



Kumar and Seshadri, 2023a

Hasselmann K. 1976. Stochastic Climate Models



Stochastic climate models

Part I. Theory

By K. HASSELMANN, *Max-Planck-Institut für Meteorologie, Hamburg, FRG*

(Manuscript received January 19; in final form April 5, 1976)

ABSTRACT

A stochastic model of climate variability is considered in which slow changes of climate are explained as the integral response to continuous random excitation by short period "weather" disturbances. The coupled ocean–atmosphere–cryosphere–land system is divided into a rapidly varying "weather" system (essentially the atmosphere) and a slowly responding "climate" system (the ocean, cryosphere, land vegetation, etc.). In

Fast weather and slow climate

Weather is stochastic input to climate

Climate does not feedback on the weather

Stochastic differential eq. for climate

Climate predictability in the context of chaotic weather, where weather is essentially "noise" on climate timescales

Weather and Climate

Flooding from Hurricane Iota, Nov 2020

Weather: maximum power at high frequencies

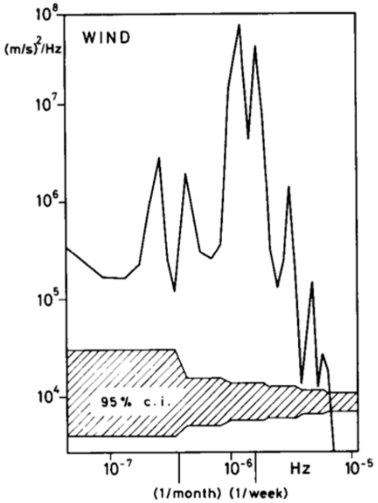


Fig. 1. Simulated spectrum of the wind velocity at $x = ($
$y = L_y/4$.

Climate: maximum power at low frequencies

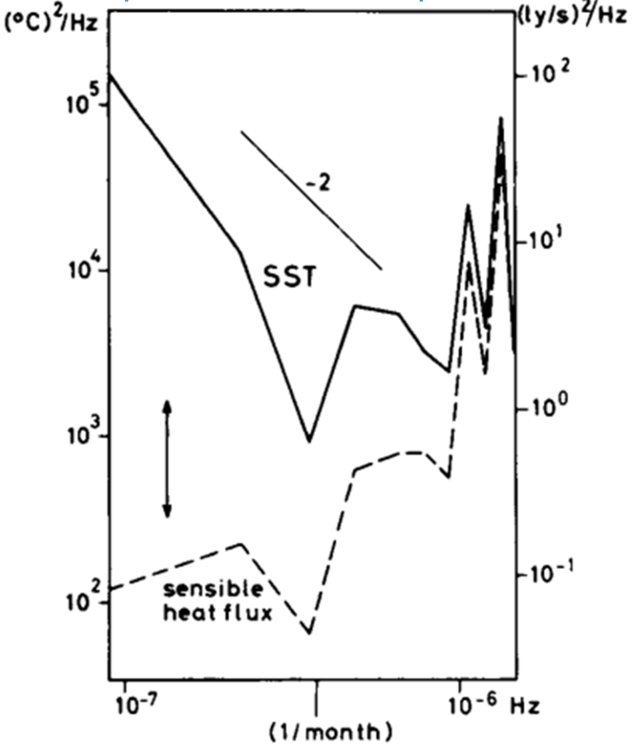


Fig. 4. Simulated spectrum of sensible heat flux (dashed lines) and SST anomaly (continuous lines) at $x = 0$, $y = L_y/4$. The arrows indicate the 95% confidence interval.

Diagrams from: Frankignoul and Hasselmann, Stochastic climate models, Part II, Application to sea-surface temperature anomalies and thermocline variability, *Tellus*, 1977

Chaos

Deterministic Nonperiodic Flow[1]

Edward N. Lorenz

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

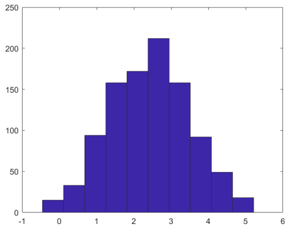
Convection, Lorenz (1963)

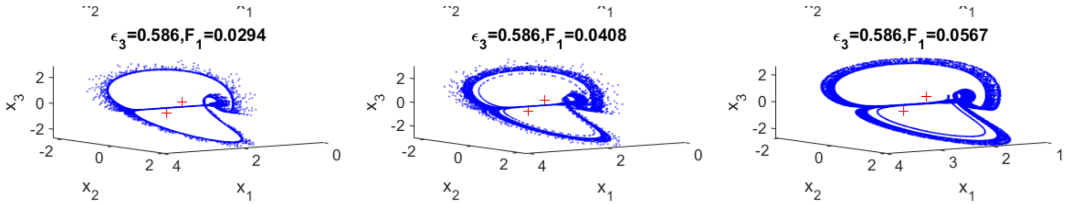
$$\dot{X} = -\sigma X + \sigma Y,$$

momentum

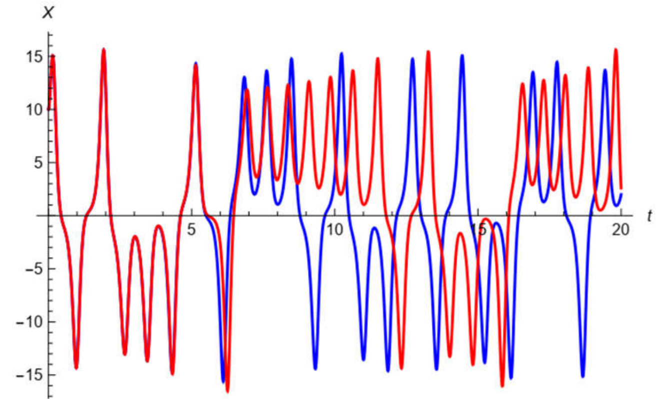
$$\dot{Y} = -XZ + rX - Y,$$
$$\dot{Z} = XY - bZ.$$

thermodynamics





Abraham and Shaw, Dynamics: The Geometry of Behavior



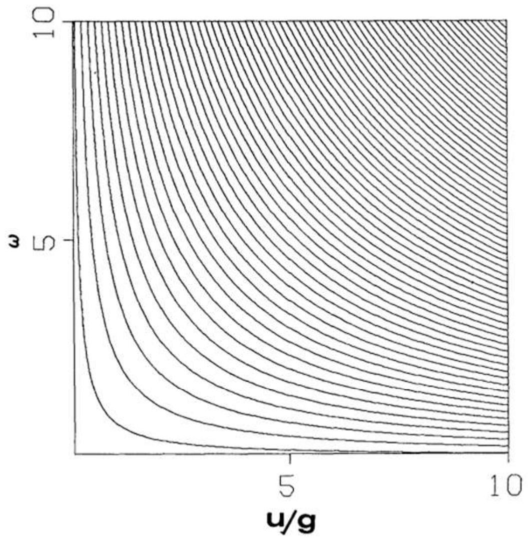
Stable : a nearby trajectory remains nearby

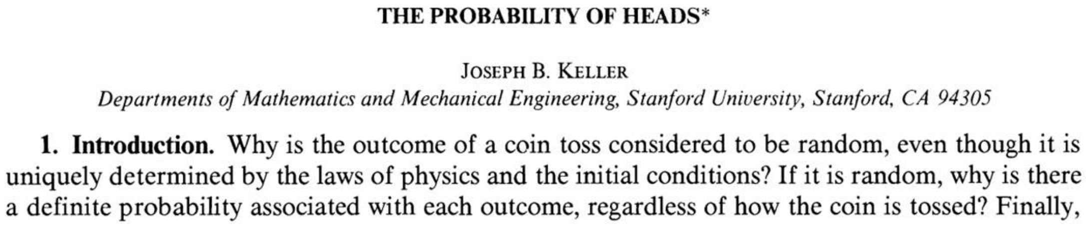
Nonperiodic: neither periodic nor quasiperiodic

Nonperiodic flow is unstable

“ A central trajectory…is unstable if it is nonperiodic **[so distance between initial conditions must grow].** In view of the impossibility of measuring initial conditions precisely, and distinguishing between a central trajectory and nearby noncentral trajectory, all nonperiodic trajectories are effectively unstable from the point of view of practical prediction” (Lorenz, 63)

Randomness via deterministic laws





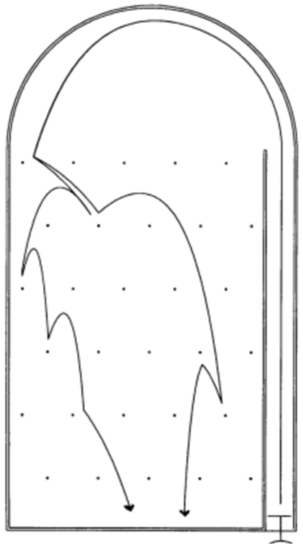
THE PROBABILITY OF HEADS*

JOSEPH B. KELLER

Departments of Mathematics and Mechanical Engineering, Stanford University, Stanford, CA 94305

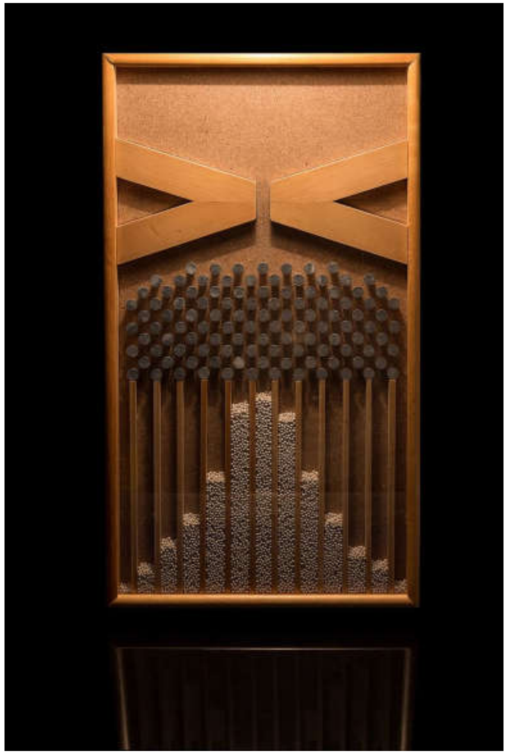
1. Introduction. Why is the outcome of a coin toss considered to be random, even though it is uniquely determined by the laws of physics and the initial conditions? If it is random, why is there a definite probability associated with each outcome, regardless of how the coin is tossed? Finally,

Pinball machine



Lorenz, Essence of Chaos

Galton Board



Stochastic climate model

$$\frac{dy}{dt} = -\Lambda y + \bar{\sigma}\xi(t)$$ Stochastic climate model

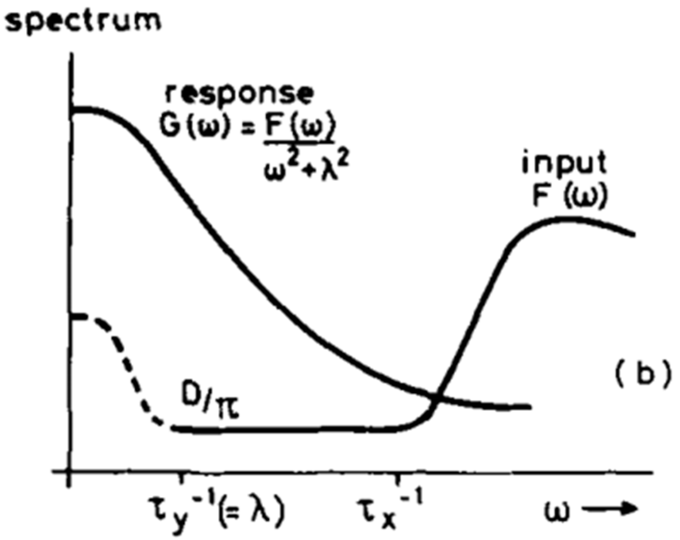
Weather + Climate Model: $\underbrace{\dot{x}_i}_{\text{weather}} = f_i(x,y)$ and $\underbrace{\dot{y}_j}_{\text{climate}} = g_j(x,y),$

Weather: $x = \underbrace{\langle x|y\rangle}_{\text{mean weather, given climate}} + \underbrace{x^*}_{\text{weather anomaly}}$

Climate Evolution:

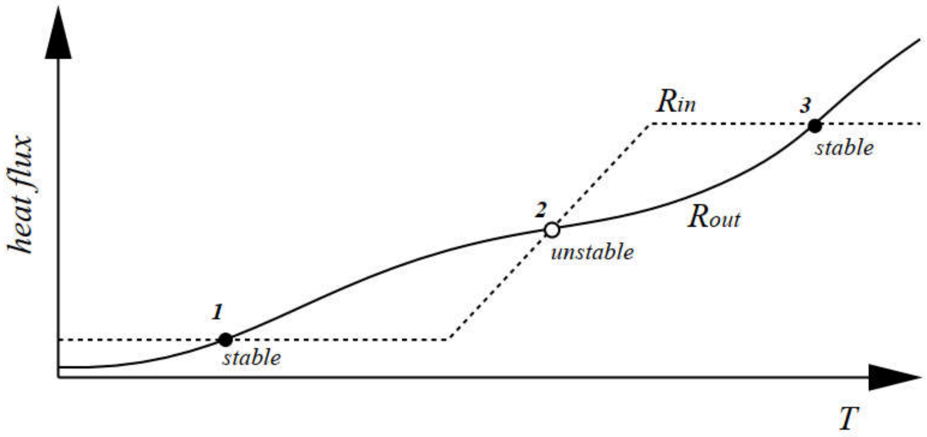
$$\underbrace{g(x,y)}_{\text{rate of change of climate}} \equiv g\left(\langle x|y\rangle + x^*, y\right) \approx g(\langle x|y\rangle, y) + \partial_x g(\langle x|y\rangle, y)x^*$$
$$\equiv -\frac{dU(y)}{dy} + \bar{\sigma}(y)\underbrace{\xi(t)}_{\text{Stochastic term}},$$

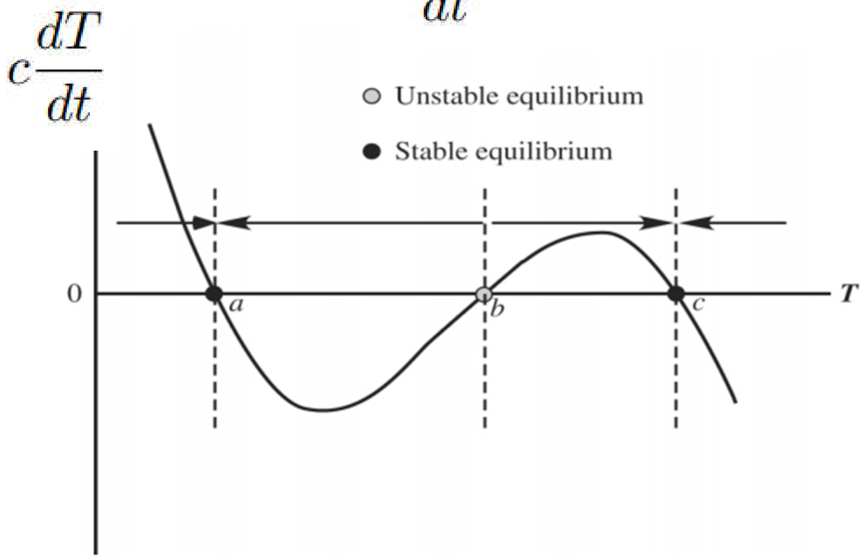
Can study its:
- Probability distribution of y(t)
- Stationary probability distribution
- Power spectrum

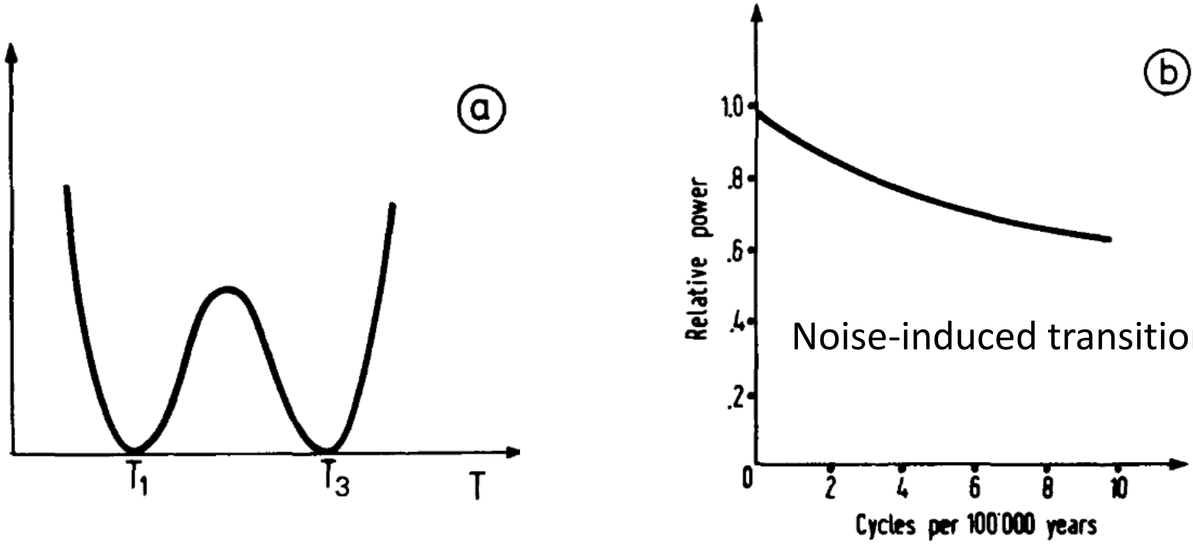


Multiple equilibria in Earth's climate system

$$c\frac{dT}{dt} = R_{in} - R_{out}$$



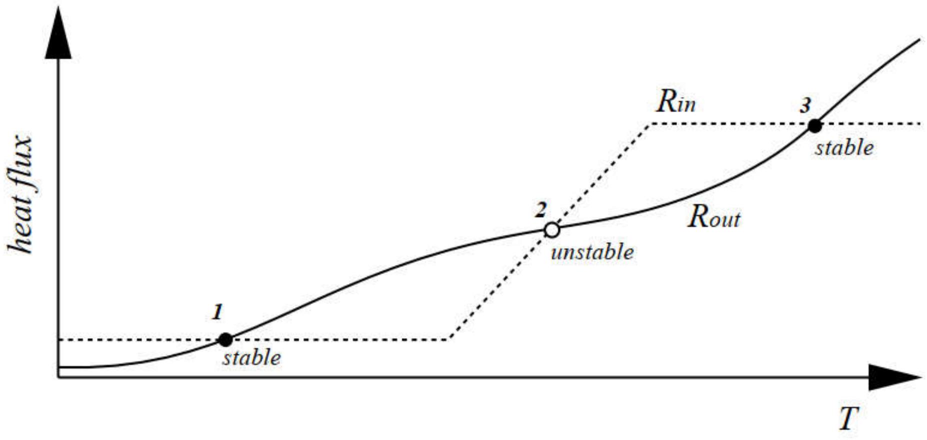


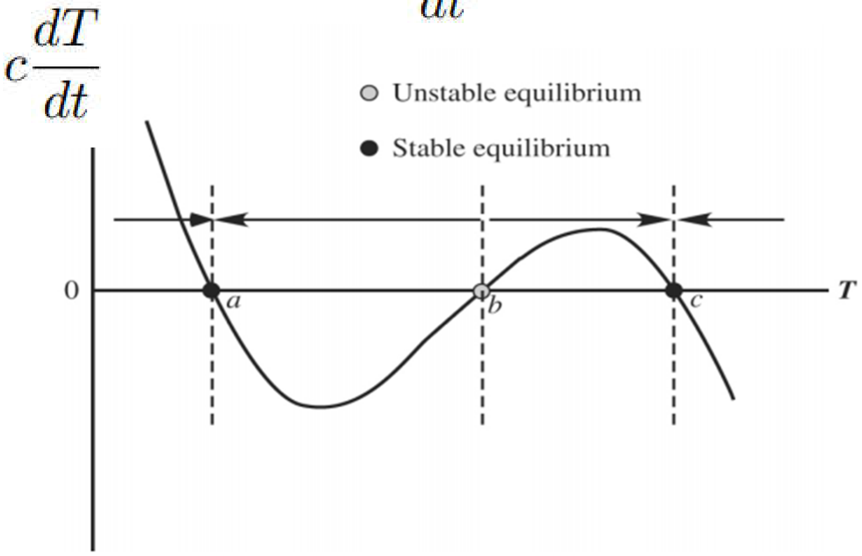


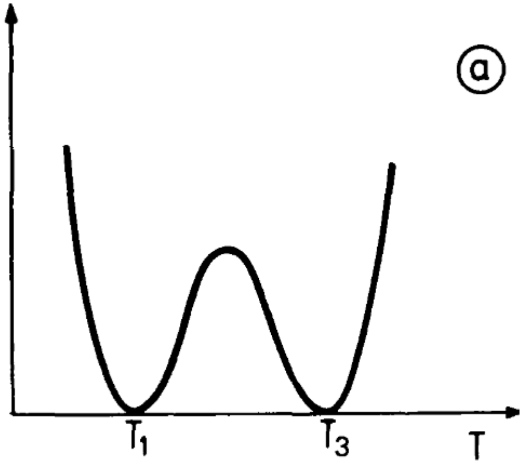
Noise-induced transition?

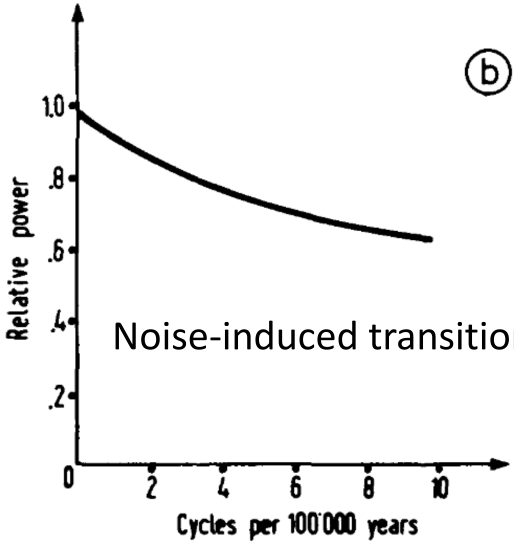
Multiple equilibria in Earth’s climate system

$$c\frac{dT}{dt} = R_{in} - R_{out}$$

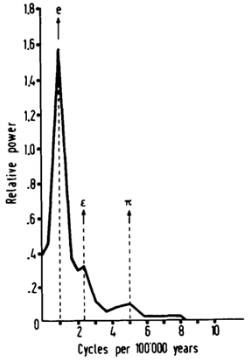








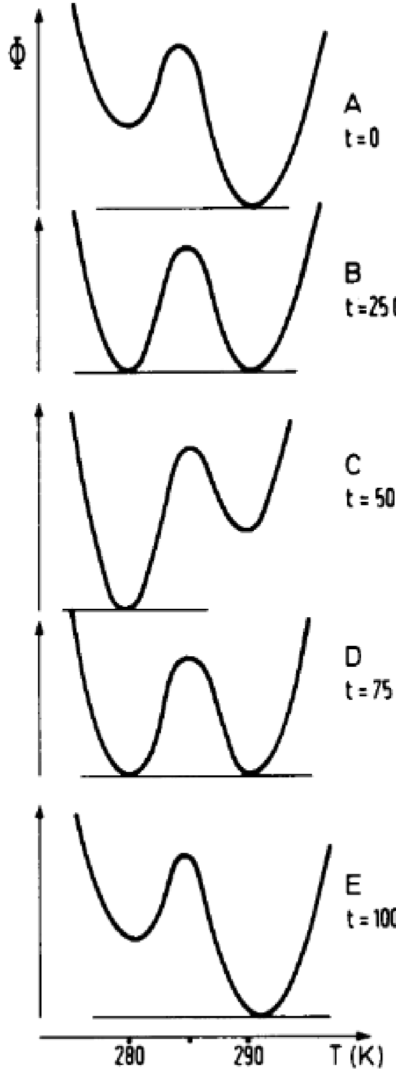
Noise-induced transition?

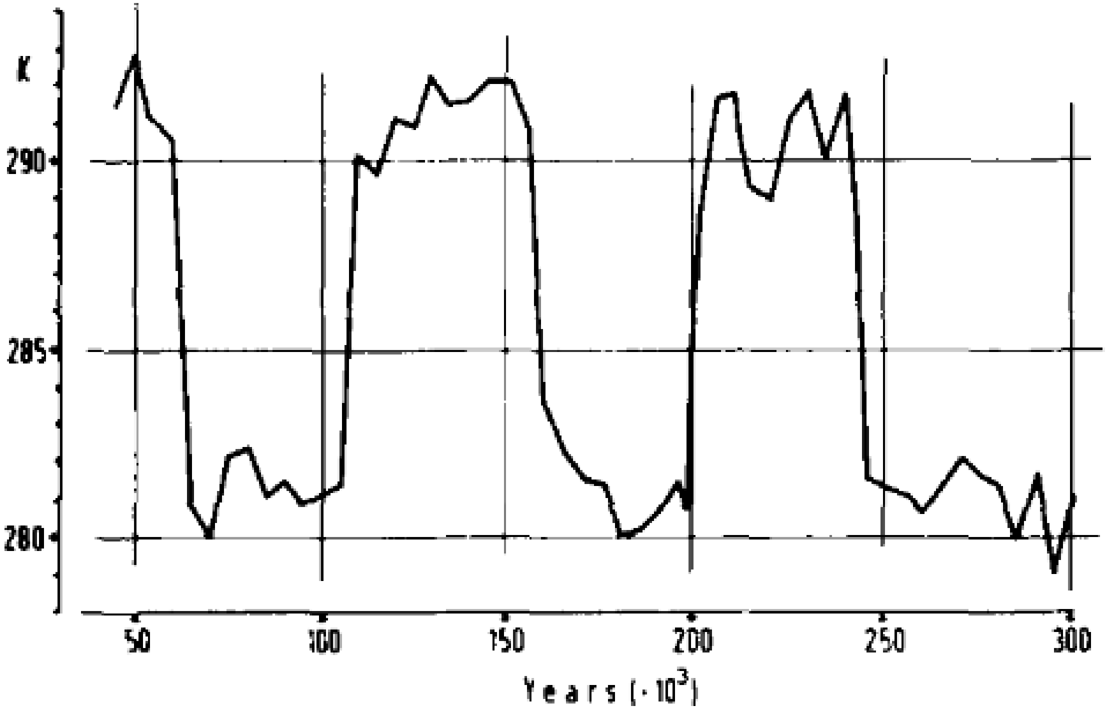


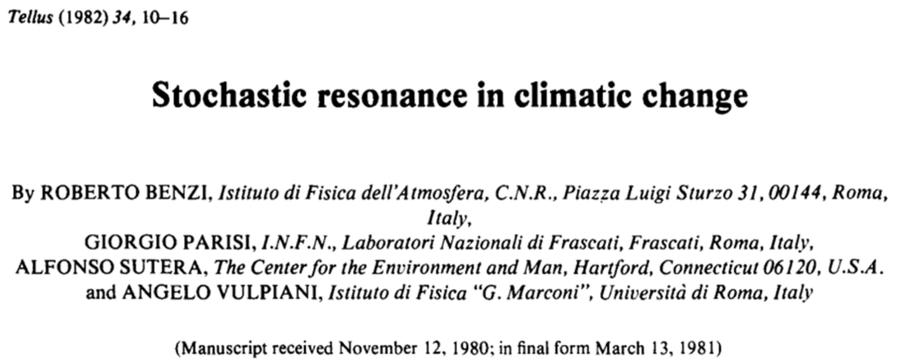
Benzi et al., 1982

Stochastic resonance

Slow internal dynamics







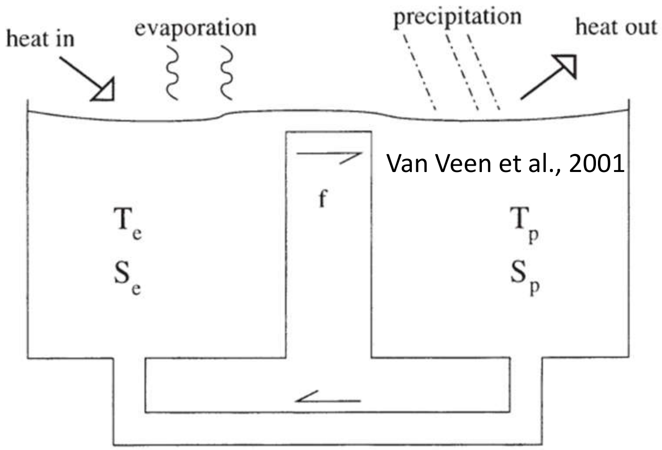
Tellus (1982) *34*, 10–16

Stochastic resonance in climatic change

By ROBERTO BENZI, *Istituto di Fisica dell'Atmosfera, C.N.R., Piazza Luigi Sturzo 31, 00144, Roma, Italy,*
GIORGIO PARISI, *I.N.F.N., Laboratori Nazionali di Frascati, Frascati, Roma, Italy,*
ALFONSO SUTERA, *The Center for the Environment and Man, Hartford, Connecticut 06120, U.S.A.*
and ANGELO VULPIANI, *Istituto di Fisica "G. Marconi", Università di Roma, Italy*

(Manuscript received November 12, 1980; in final form March 13, 1981)

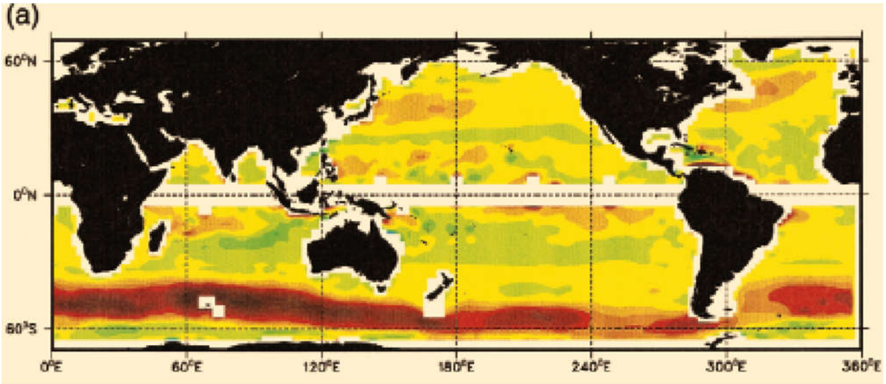
Atlantic Meridional Overturning Circulation



Van Veen et al., 2001

The Work Done by the Wind on the Oceanic General Circulation

CARL WUNSCH



Wunsch, 1998

Active and passive ocean regimes in a low-order climate model

By LENNAERT VAN VEEN[1]*, THEO OPSTEEGH[2] and FERDINAND VERHULST[1], [1]*Mathematical Institute, University of Utrecht, PO Box 80010, 3508 TA Utrecht, The Netherlands;* [2]*Royal Netherlands Meteorological Institute (KNMI), PO Box 201, 3730 AE de Bilt, The Netherlands*

Thermohaline Convection with Two Stable Regimes of Flow

By HENRY STOMMEL, Pierce Hall, Harvard University, Massachusetts

Irregularity: a fundamental property of the atmosphere*

By EDWARD N. LORENZ, ***Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A.***

On the interaction of gravitational and dynamic forcing in simple circulation models*

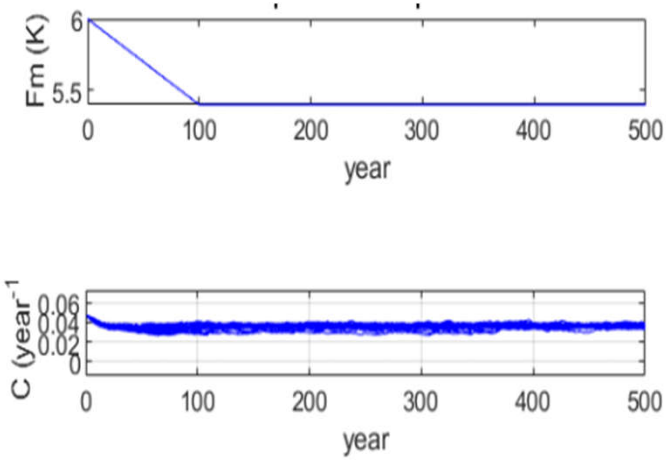
HENRY STOMMEL† and CLAES ROOTH‡

Risk of tipping the overturning circulation due to increasing rates of ice melt

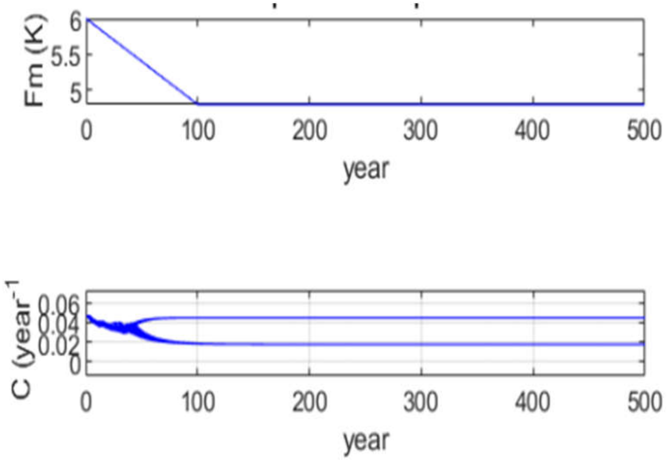
Johannes Lohmann[a,1] and Peter D. Ditlevsen[a]

Three types of responses

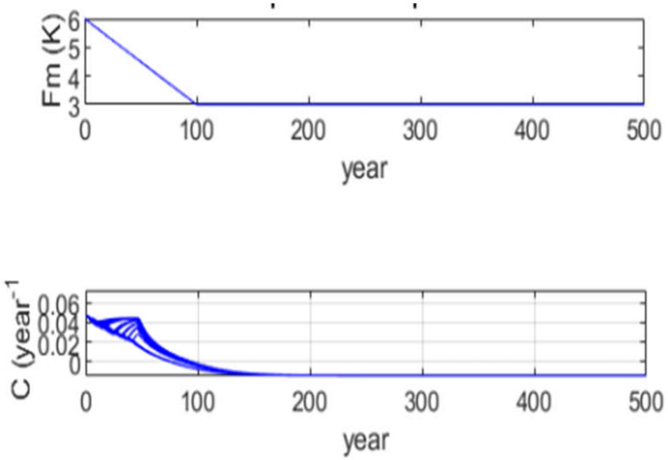
Small forcing

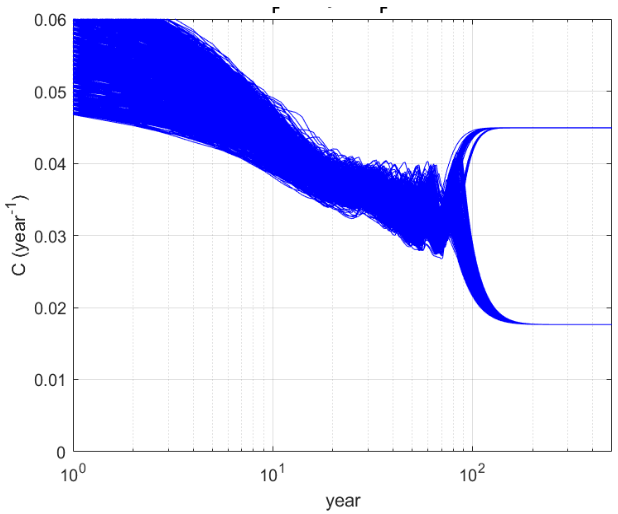


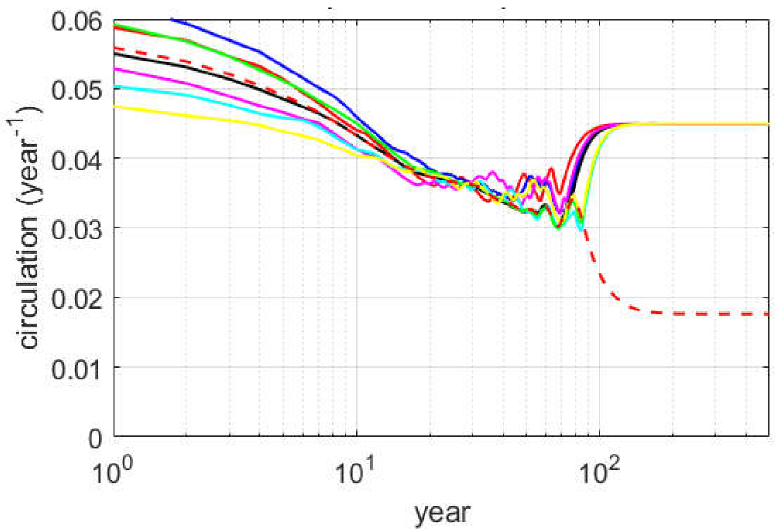
Intermediate forcing



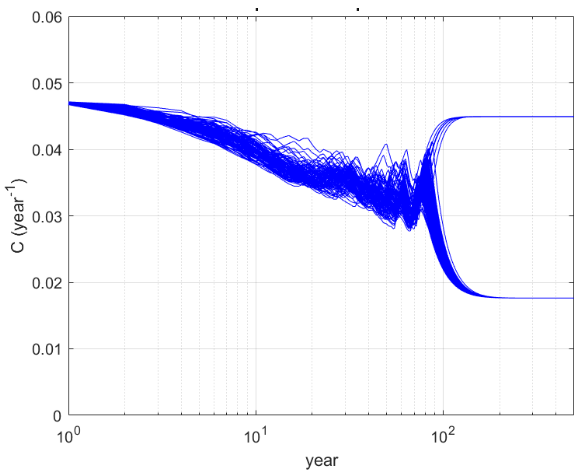
Large forcing

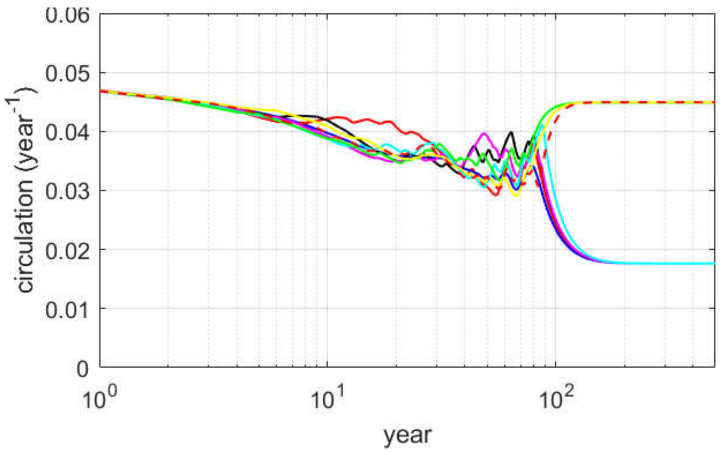


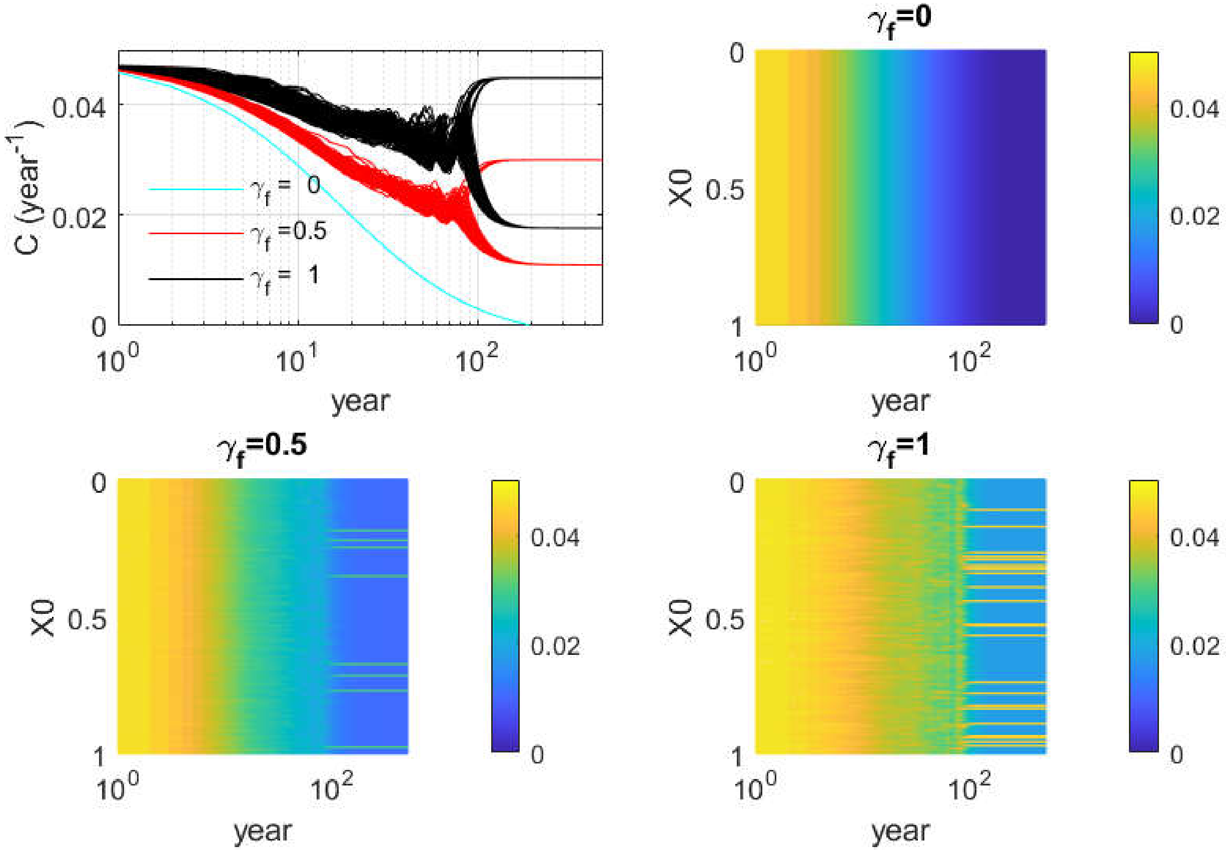




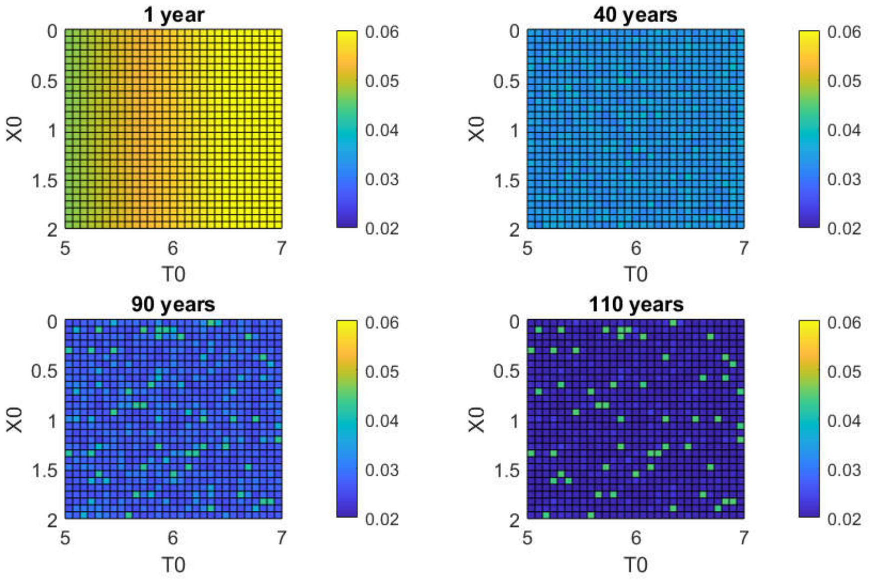
Initial condition sensitivity & role of atmosphere-ocean feedback

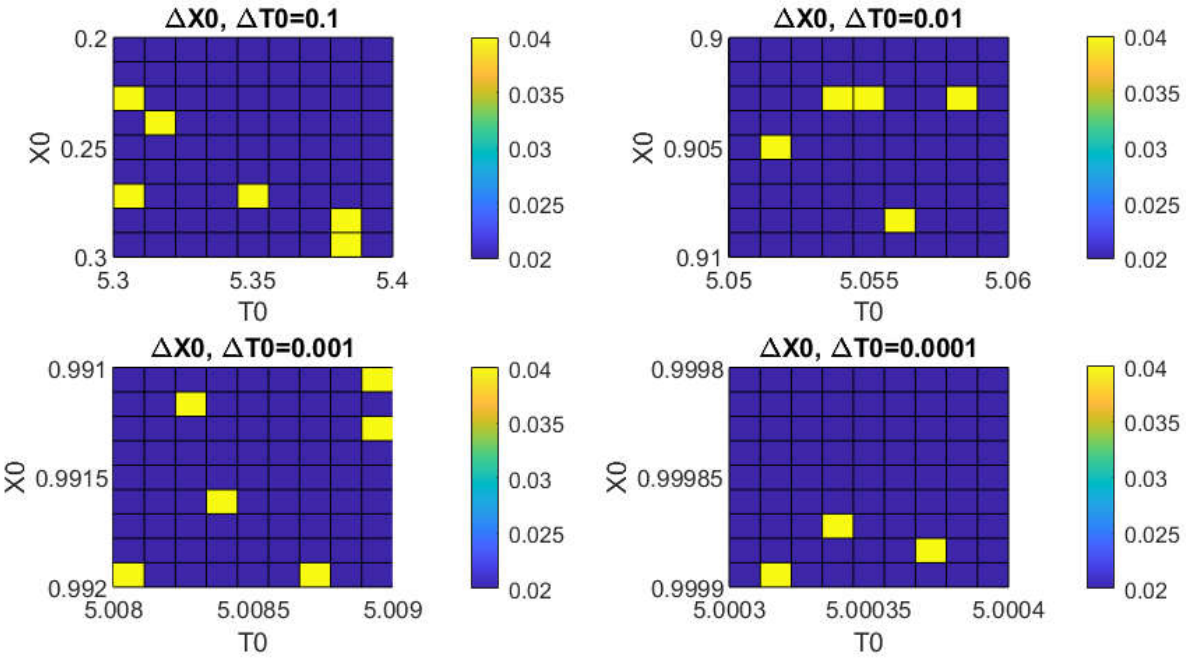




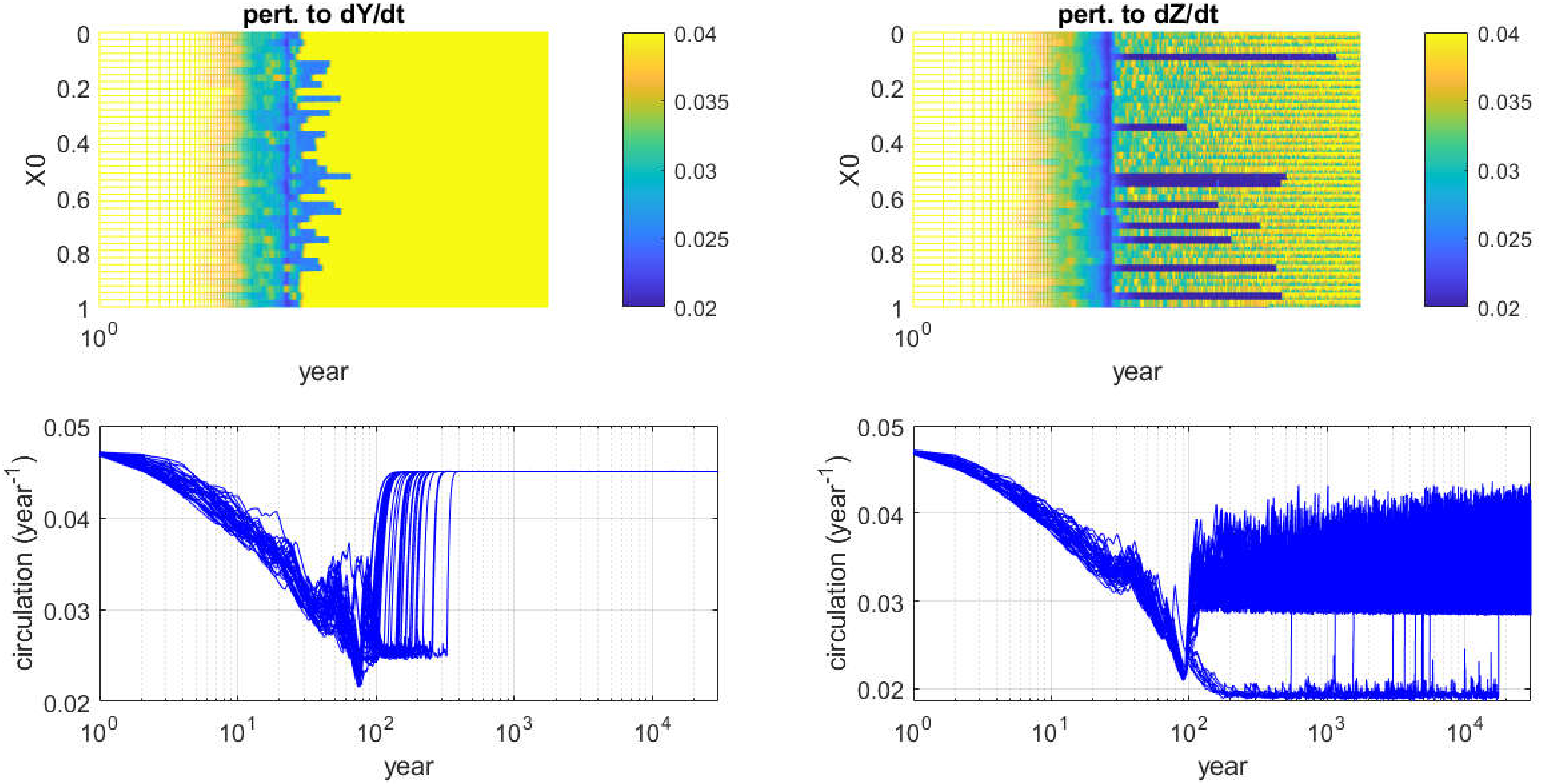


Riddled (actually, intermingled) basins

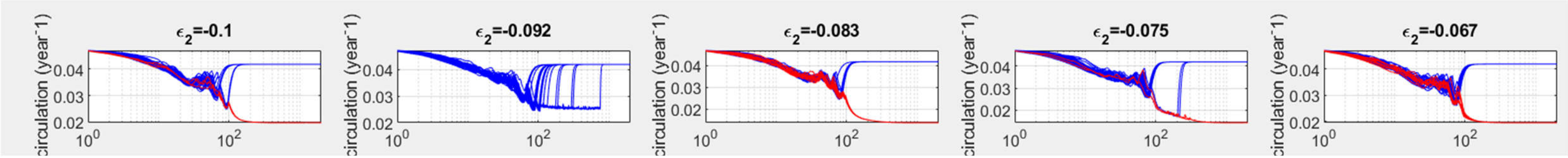


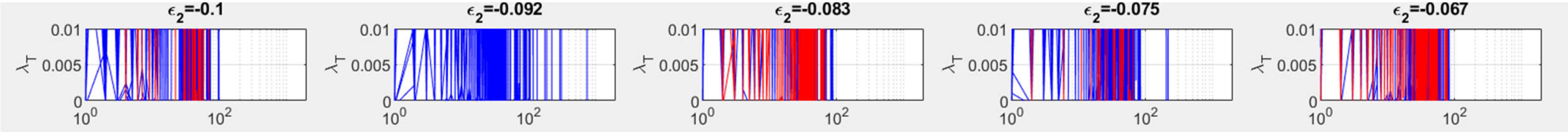


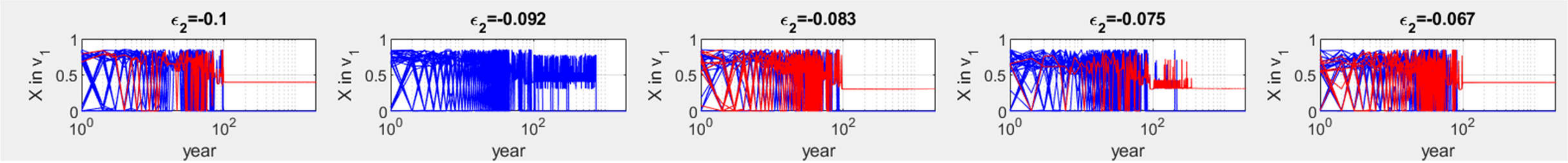
Long-term uncertainty in prediction



Effect of weather on climate







The role of resonance

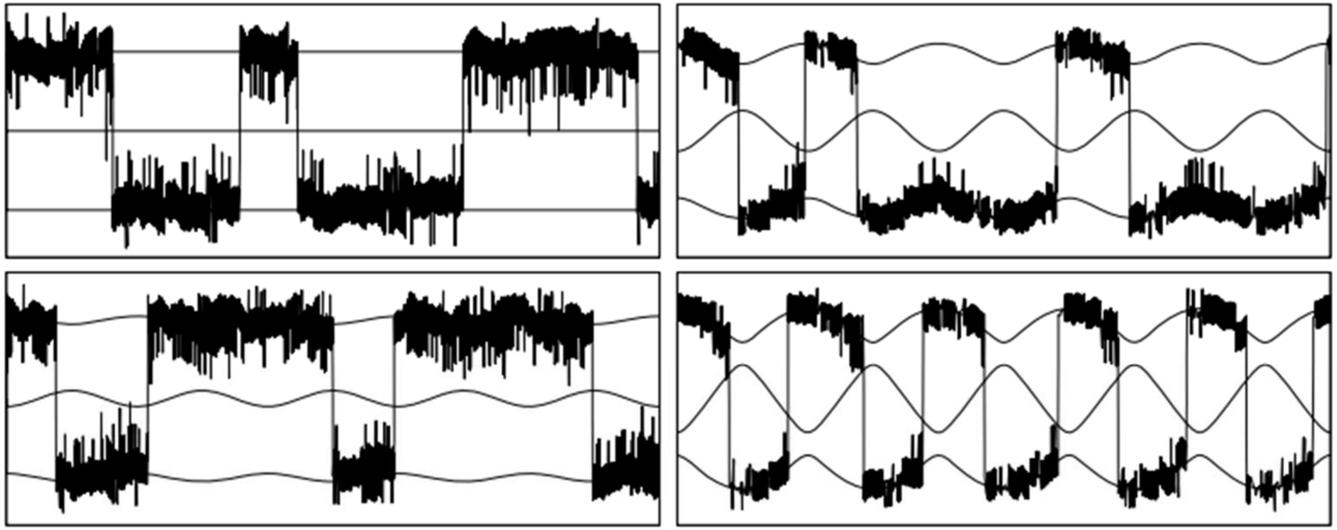


Fig. 4.2. Sample paths of Equation (4.1.11) for $\varepsilon = 0.003$ and different forcing amplitudes A and noise intensities σ. Solid lines indicate the location of the potential minima and of the saddle. From top to bottom and left to right: $A = 0$ and $\sigma = 0.3$, $A = 0.1$ and $\sigma = 0.27$, $A = 0.24$ and $\sigma = 0.2$, $A = 0.35$ and $\sigma = 0.2$.

From Berglund and Gentz, 2006