Dynamical Freezing and Emergent Conservations in Interacting Systems

SQMVS 2024

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Conservation laws and Statistical Mechanics

Gibbs' Statistical Mechanics $\equiv$ Entropy Maximization Subject to ***Exact (local, additive)*** Conservation Laws

Many-body System

Quantum Mechanics

+

Hypothesis: ETH/Quantum Chaos

$$\rho_{Equi} = \frac{e^{-\sum_\kappa \lambda_\kappa \hat{\mathcal{I}}_\kappa}}{\mathcal{Z}}$$

Additive Local Conserved Quantities

Lagrange Multipliers

$[\mathcal{O}, H] = 0 \Rightarrow \mathcal{O}$ is conserved

$[\mathcal{O}, H] \neq 0 \Rightarrow \mathcal{O}$ is not conserved

What if:

$[\mathcal{O}, H] = \varepsilon(\neq 0)$ but small?

- ❑ Will there be approximate but perpetual (stable) conservation of $\mathcal{O}$ if ε is small enough?

➢ Not known for Sure: no KAM-like "Threshold Theorem" in quantum Mechanics.

General Belief: Conservations are destroyed even for an infinitesimal ε

Non-trivial Steady States in the absence of any exact conservation in "closed" systems?

Generic System

$$H = \sum_{i>j=1}^{10^{23}} \frac{J_{ij}}{r_{ij}^{\alpha}} \vec{S}_i \cdot \vec{S}_j + \dots$$

$\sim 10^{23}$ Initial Conditions

Statistical Mechanics: locally $\approx \; Z = \sum e^{-\beta H}$

One initial Condition (in ETH sense)

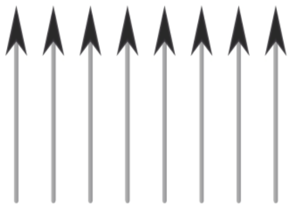
One Conservation Law

\+

Drive **(Let's Kill the Energy Conservation)**

Are there non-trivial "Stable States" in a closed many-body system?

Killing The Energy Conservation *Minimally*: The Floquet Question



Interacting Degrees of Freedom

+

(No External Bath)

(Unitary Periodic Drive)

$$H(t);\ H(t+T) = H(t)$$

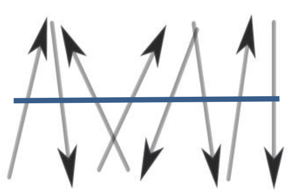
=

?

(A) Intuitive/Conventional

Unbounded "Heating" (energy absorption) from the drive

(Fermi's Golden Rule type scenario)



(B) Recent

Interesting Steady States/Non-equilibrium Ensembles stabilized by quantum effects:

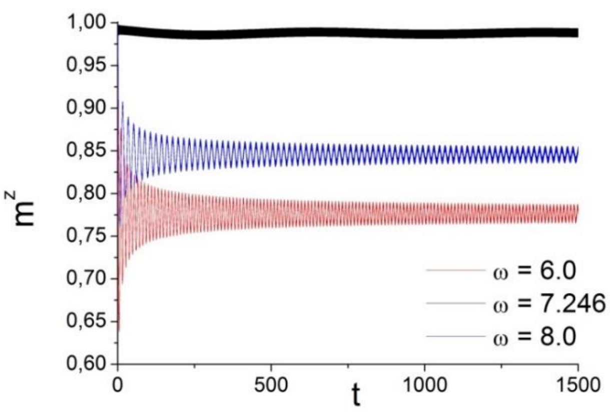
New sets of Floquet Conservation Laws/ Constraints in the presence of the drive

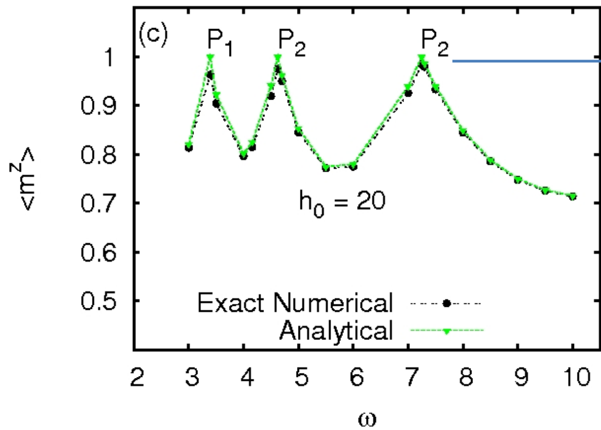
Floquet Quantum Matter

Integrable Ising Chain:
Dynamical Freezing and Emergent Conservation

$$H = -\frac{J}{2}\left[\sum_{i=1}^{L}\sigma_i^x\sigma_{i+1}^x + h_0\cos(\omega t)\sum_i \sigma_i^z\right]$$

***AD**, PRB (2010)*





Approximately but ***Perpetually*** Conserved

$$Q = \langle m^z \rangle = 1/[1 + J_0(2h_0/\omega)]$$

❖Freezing/Conservation of m^z Happens for ***any*** Initial State!

m^z ⇒ An Emergent Conservation Law

Questions

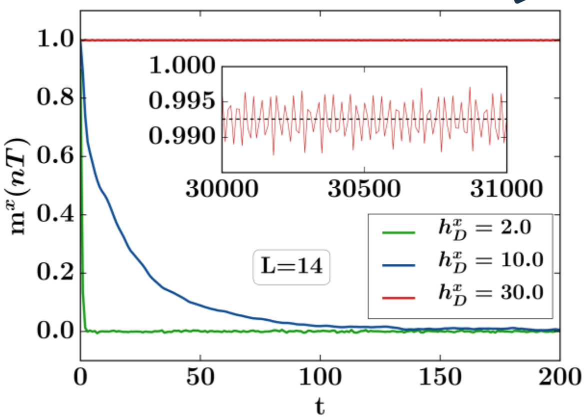
- Why no unbounded heating? What is stopping it?
- Why there is an approximate but perpetual conservation?
- What happens in interacting non-integrable systems?

A Concrete Example:
Strong Drive + Non-Integrable Static Part

(A.Haldar, R. Moessner, **AD.**, PRB 2018)

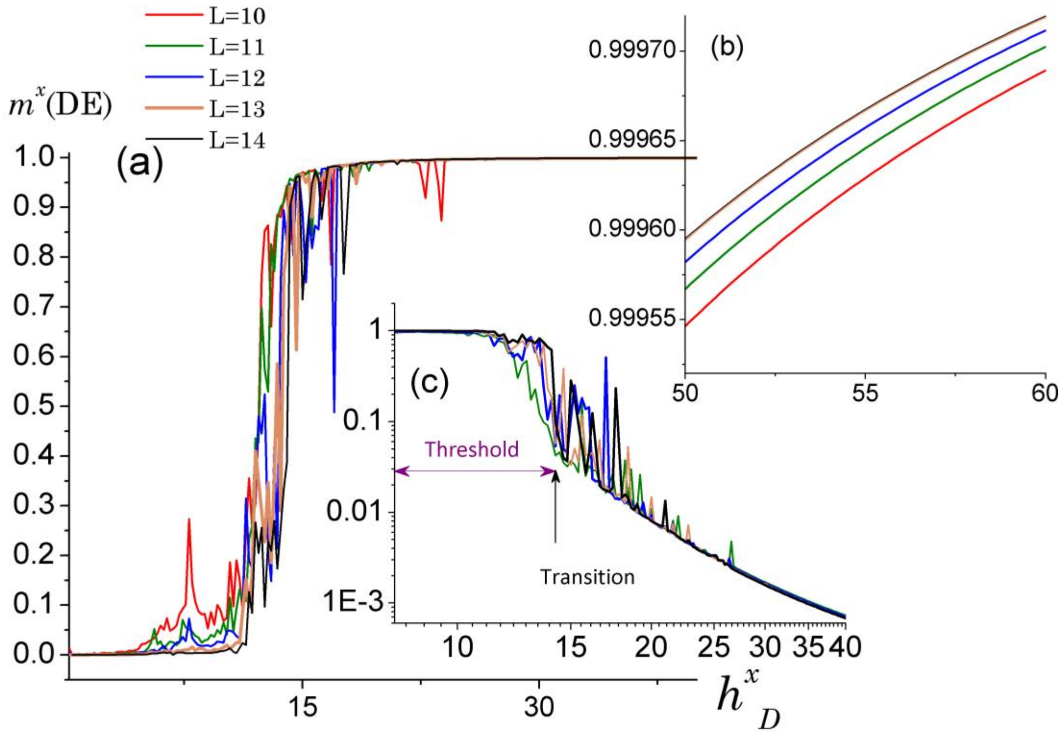
$$H(t) = H_0(t) + V, \text{where}$$
$$H_0(t) = H_0^x + \mathrm{Sgn}(\sin(\omega t))H_D, \text{with}$$
$$H_0^x = -\sum_{n=1}^{L} J\sigma_n^x\sigma_{n+1}^x + \sum_{n=1}^{L} \kappa\sigma_n^x\sigma_{n+2}^x - h_0^x\sum_{n=1}^{L}\sigma_n^x,$$
$$H_D = h_D^x \sum_{n=1}^{L}\sigma_n^x, \text{and}$$
$$V = h^z\sum_{n=1}^{L}\sigma_n^z,$$

Tested up to 10^{15} cycles recently



The Floquet ThermalizationThreshold (Reminiscence of KAM)

Focus is on the t → ∞ limit: **The Diagonal Ensemble Average (DE/DEA)**



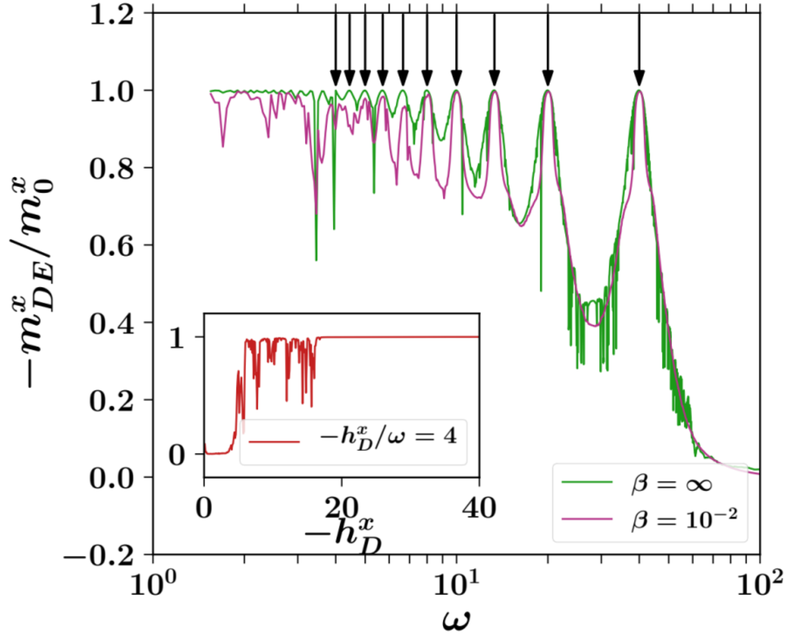
$$J = 1, \kappa = 0.7, \omega = 0.1, h_0^x = 0.01, h^z = 1.2$$

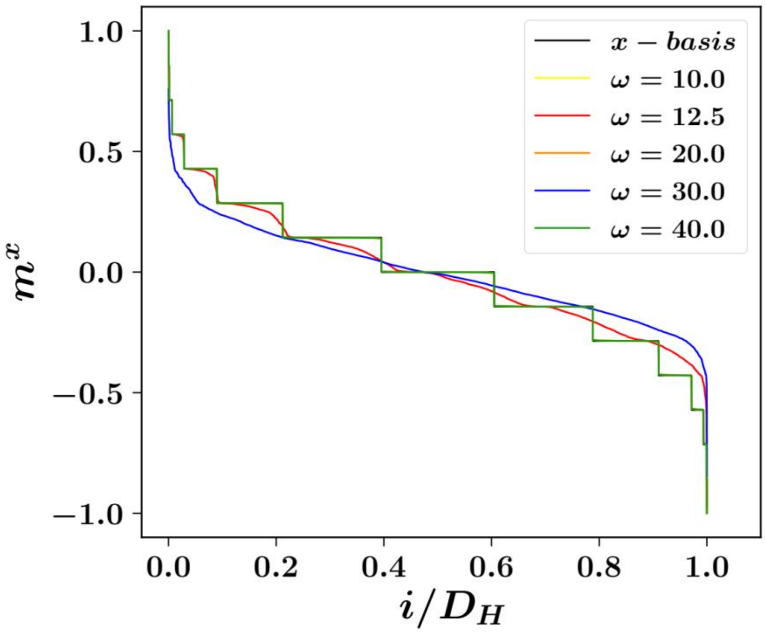
Initial State = the Ground State of *H(t=0)*

- The threshold doesn’t move with the system-size.
- Finer resolution shows, m^x is more strongly frozen for larger L above the threshold.

Beyond the Threshold: Dynamical Freezing and Emergent Conservation

$$J = 1, \kappa = 0.7\pi/3, h_0^x = e/10, h_D^x = 40, L = 14$$





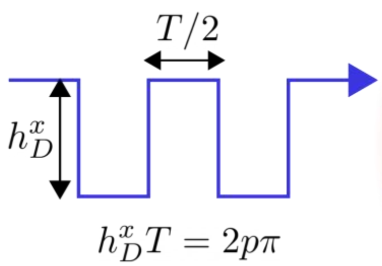
Longitudinal magnetization ***emerges*** as an approximately conserved quantity under the Drive condition:

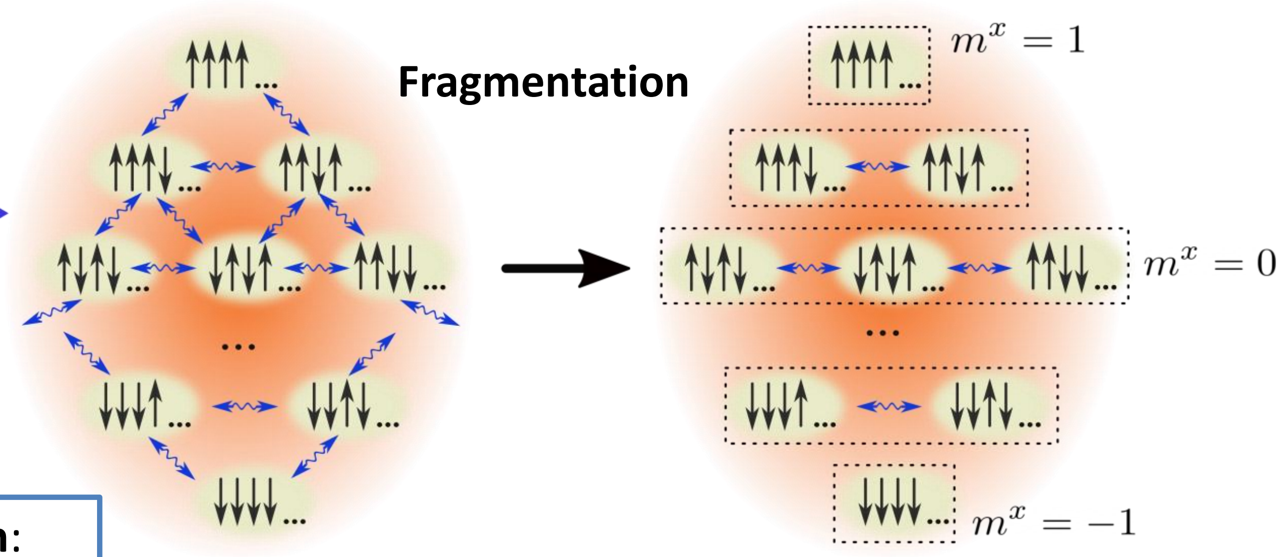
$$\boxed{h_D^x = k\omega}$$

This happens for a very broad range of ω

A. Haldar, D. Sen, R. Moessner, **AD** (PRX, 2021)

Summary of the Emergent Conservation





- **Affects Stat Mech**: Needs to take care of the Emergent Conservation to construct the correct ensemble

- **Conservations are not Planter**

$$[m^x, H_{eff}] \approx 0$$

But

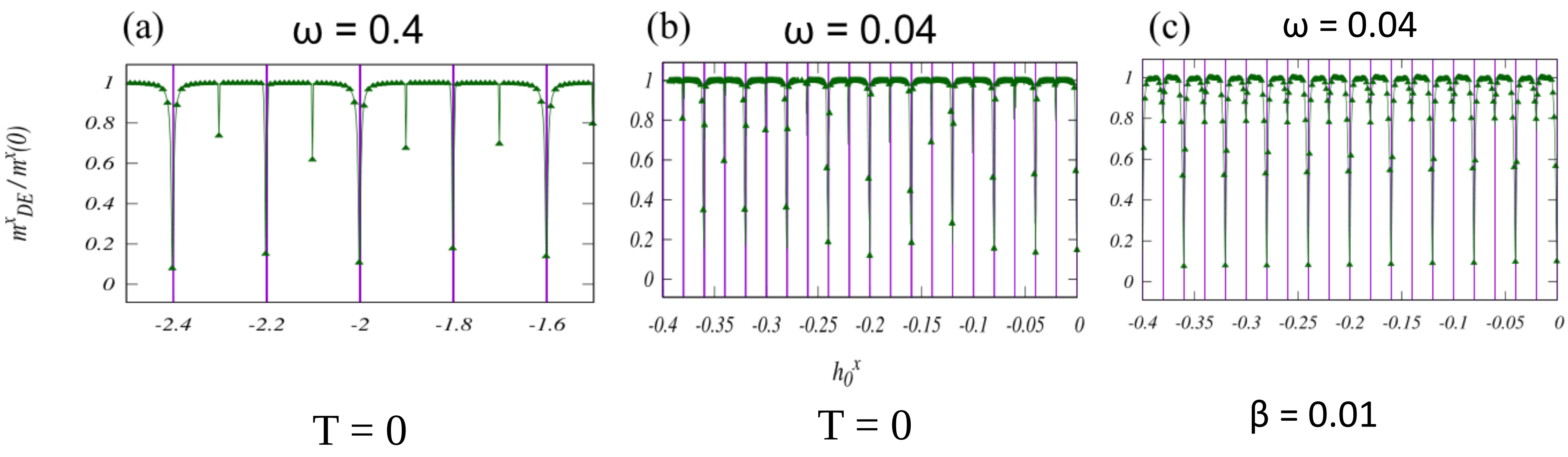
$$H_{eff} \not\approx H_D$$

Note that $m^x = -\frac{h_D^x}{L} H_D$

Conserved Quantity

Resonances

Resonances are sufficiently Isolated!



- ➢ Resonances are **Isolated**:
- ➢ Resonance-free parameter regimes: No Heating!

Analytical Approaches

- A Floquet Perturbation Theory
(Expected to explain the Resonances)

- Magnus Expansion in a Moving Frame
(Expected to explain the high ω regime)

(I) Resonances: A Floquet Perturbation Theory

$$H(t) = H_0(t) + V; \quad [H_0(t), H_0(t')] = 0 \;\; \forall t, t'$$ and $$\langle n|V|n\rangle = 0$$

$$H_0(t)|n\rangle = E_n(t)|n\rangle; \; \langle m|n\rangle = \delta_{mn}$$

- Here V is the perturbation (small) and for V = 0, |n> are the Floquet states.
- Goal = Finding the Floquet State for finite V expanding perturbatively around |n>.

TDSE:

$$i\frac{\partial|\psi_n\rangle}{\partial t} = H(t)|\psi_n(t)\rangle$$

Expansion:

$$|\psi_n(t)\rangle = \sum_m c_m(t)\, e^{-i\int_0^t dt' E_m(t')}\,|m\rangle$$

(to 1st order in V)

Coefficients:

$$c_m(0) = -\,i\,\langle m|V|n\rangle \frac{\int_0^T dt\; e^{i\int_0^t dt'[E_m(t')-E_n(t')]}}{e^{i\int_0^T dt[E_m(t)-E_n(t)]} - 1}$$

The resonance condition:

$$e^{i\int_0^T dt[E_m(t)-E_n(t)]} = 1$$

The ANNNI-Chain Case

$$H(t) = H_0(t) + V, \text{ where}$$
$$H_0(t) = H_0^x + \text{Sgn}(\sin(\omega t))H_D, \text{ with}$$
$$H_0^x = -\sum_{n=1}^{L} J\sigma_n^x\sigma_{n+1}^x + \sum_{n=1}^{L} \kappa\sigma_n^x\sigma_{n+2}^x - h_0^x\sum_{n=1}^{L}\sigma_n^x,$$
$$H_D = h_D^x\sum_{n=1}^{L}\sigma_n^x, \text{ and}$$

f (σ^x)

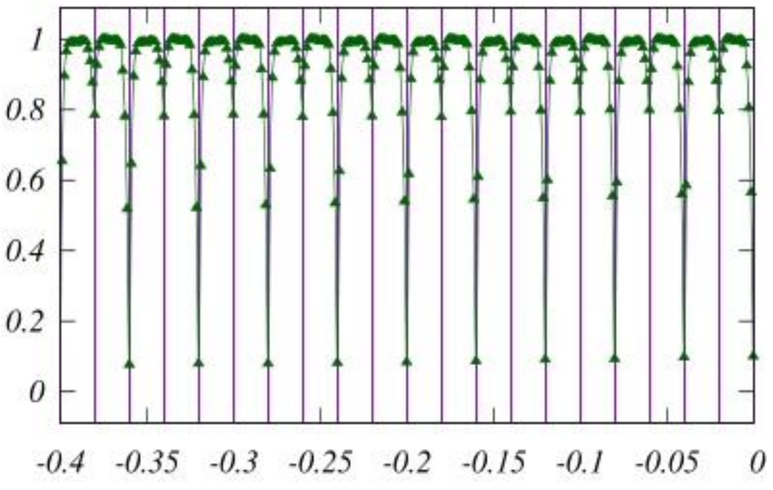
$$V = h^z\sum_{n=1}^{L}\sigma_n^z,$$

→ Single Spin-flip Perturbation

1st order Resonance Condition (isolated resonances)

$$h_0^x\sigma_0 + J\sigma_0(\sigma_{-1}+\sigma_1) - \kappa\sigma_0(\sigma_{-2}+\sigma_2) = \frac{p\omega}{2}$$

ω = 0.04, β = 0.01



Mystery: ***All*** of the Resonances are captured by the **1st order perturbation theory**!
No other divergences are observed.

(II) Strong-Drive Magnus Expansion in A Rotating Frame

Standard Magnus Expansion

$$H_{eff} = \sum_{n=0}^{\infty} H_F^{(n)} \text{where}$$

$$H_F^{(0)} = \frac{1}{T}\int_0^T dt\ H(t),$$

$$H_F^{(1)} = \frac{1}{2!(i)T}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)]$$

Contains large terms in our case

1st order is qualitatively wrong at freezing points! (No hope of even an asymptotic expansion)

Switching to a Rotating Frame

$$|\psi_{mov}(t)\rangle = W(t)^\dagger |\psi(t)\rangle,$$

$$\hat{O}_{mov} = W(t)^\dagger \hat{O} W(t)$$

$$W(t) = \exp\left[-i\int_0^t dt' H_D \times r(t')\right]$$

This is chosen to cancel out the large term exactly

$$H_{mov} = W(t)^\dagger H(t) W(t) - iW(t)^\dagger \partial_t W$$

$$H_{mov} = H_0^x - h^z \sum_i \left[\cos(2\theta)\sigma_i^z + \sin(2\theta)\sigma_i^y\right],$$

$$\theta(t) = -h_D^x \int_0^t dt' \text{sgn}(\sin \omega t')$$

ME

The large number goes into the phase

The Effective Hamiltonian in the Moving Frame

$$H_F^{(0)} = H_0^x - \frac{h^z}{2h_D^x T}\left[\sin\left(2h_D^x T\right)\sum_i \sigma_i^z + \left(1+\cos\left(2h_D^x T\right) - 2\cos\left(h_D^x T\right)\right)\sum_i \sigma_i^y\right]$$

$$H_F^{(1)} = \frac{1}{i\ (2T)}\ \left(\Sigma_1 + \Sigma_2 + \Sigma_3\right)$$

Σ_0

$$\Sigma_1 = -\ h^z\ [H_0^x,\ \mathcal{S}_z]\left\{\frac{1}{4\ (h_D^x)^2}\left(2\cos\left(2h_D^x T\right) - \cos\left(h_D^x T\right) - 1\right) + \frac{3\ T}{4\ h_D^x}\sin\left(h_D^x T\right) + \frac{T}{4\ h_D^x}\sin\left(h_D^x T\right)\right\}.$$

$$\Sigma_2 = -\ h^z\ [H_0^x,\ \mathcal{S}_y]\left\{\left[\frac{1}{2\ (h_D^x)^2} + \frac{T^2}{4}\right]\sin\left(h_D^x\ T\right) + \frac{T}{4\ h_D^x}\left[1 - \cos h_D^x T\right]\right\}.$$

$$\Sigma_3 = 0.$$

$$\boxed{h_D^x = k\omega} \Rightarrow \Sigma_0 = \Sigma_1 = \Sigma_2 = \Sigma_3 = 0; \quad H_{eff} \approx H_0^x$$

Any H_0^x

The Generality of Dynamical Freezing: Arbitrary Two-body Heisenberg Interactions (Static)

We get A Generalized Heisenberg Model by following substitutions in the previous Hamiltonian:

$$H_0^x \rightarrow H_0^x(Heisenberg) = -\sum_{i,j} J_{ij}^x \sigma_i^x \sigma_j^x + \kappa \sum_{i,j} \sigma_i^x \sigma_{i+2}^x - h_0^x \sum_i \sigma_i^x$$

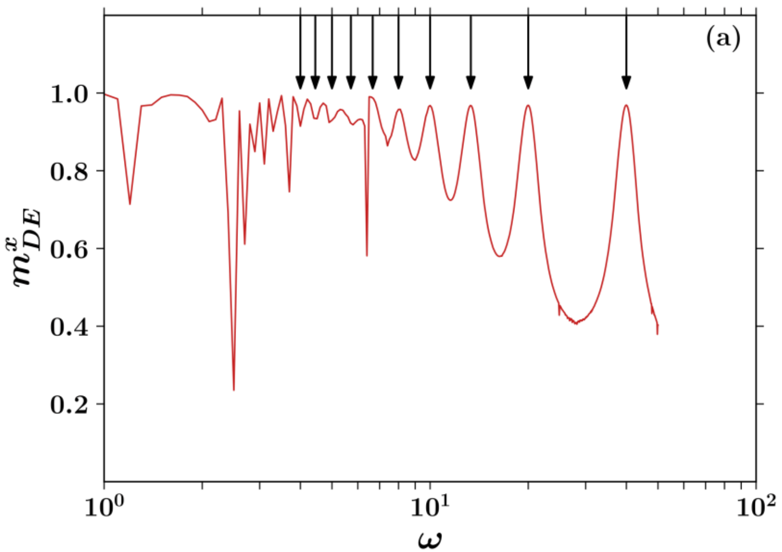
$$V \rightarrow V(Heisenberg) = -\sum_{i,j} J_{ij}^y \sigma_i^y \sigma_j^y - \sum_{i,j} J_{ij}^z \sigma_i^z \sigma_j^z - h^z \sum_i \sigma_i^z$$

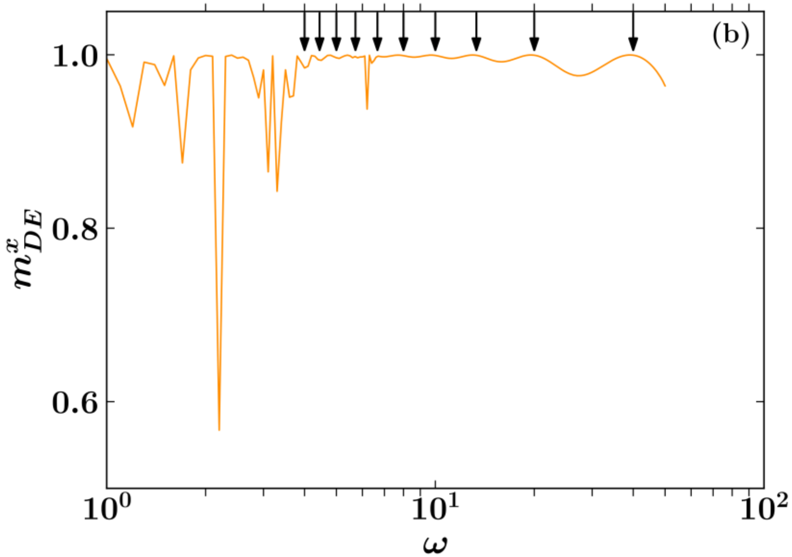
This gives (up to two initial orders of Rotating Frame ME):

$$\begin{aligned} H_{eff} &= H_0^x(Heisenberg) - \frac{1}{2}\sum_{i,j}(J_{ij}^y + J_{ij}^z)\left[\sigma_i^y \sigma_j^y + \sigma_i^z \sigma_j^z\right] \\ \Rightarrow \quad & [H_{eff}, m^x] = 0 \end{aligned}$$

Emergent non-trivial U(1) Symmetric term

Generality of Dynamical Freezing and Emergent Conservation Other Ising Examples

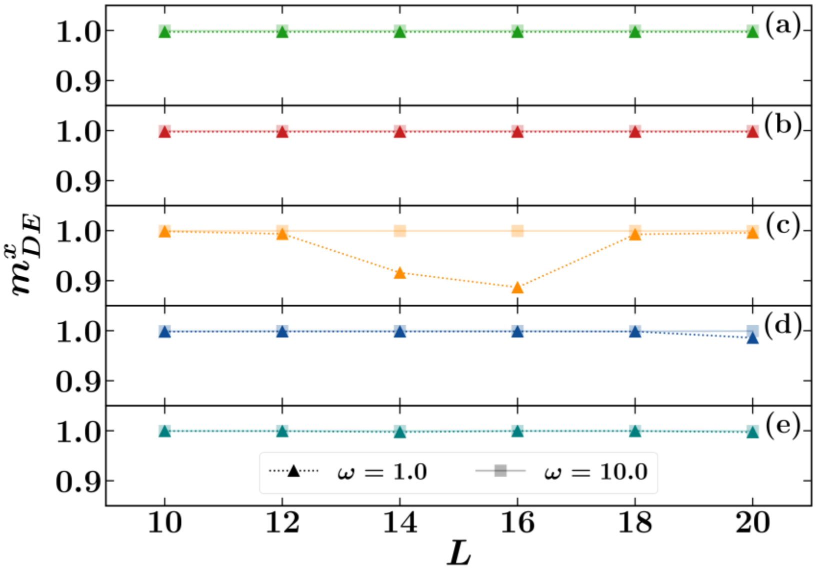




$$H_0^{x(3Spin)} = -J\sum_i \sigma_i^x\sigma_{i+1}^x + \kappa\sum_i \sigma_i^x\sigma_{i+2}^x + J_{xxx}\sum_i \sigma_i^x\sigma_{i+1}^x\sigma_{i+2}^x - h_0^x\sum_i^L \sigma_i^x.$$

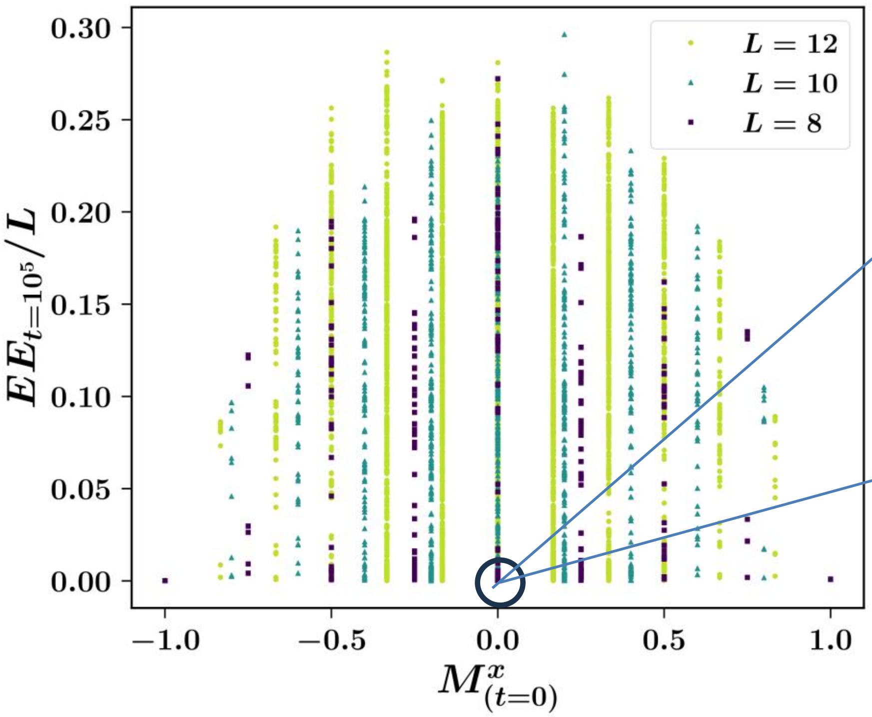
$$H_0^{x(LR)} = -J\sum_{ij}\frac{\sigma_i^x\sigma_j^x}{r_{ij}} - h_0^x\sum_i^L \sigma_i^x.$$

Various Models (L-dependence at Freezing Point)



(a) Ising: NN + NNN
(b) Ising: 3-spin Interactions
(c) Ising: 1/r Interactions (long-range)
(d) Heisenberg: Homogeneous, Isotropic
(e) Heisenberg: Homogeneous, Anisotropic

Further Structures and Conservation Laws!



$$\left|\uparrow\downarrow\uparrow\downarrow \quad \cdots \right\rangle_x \qquad \left|\downarrow\uparrow\downarrow\uparrow \quad \cdots \right\rangle_x \quad \Leftrightarrow C_1 = \sum_i \sigma_i^x \sigma_{i+1}^x$$

$$\left|\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow \quad \cdots \right\rangle_x \qquad \left|\downarrow\downarrow\uparrow\uparrow\downarrow\downarrow \quad \cdots \right\rangle_x \qquad \left|\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow \quad \cdots \right\rangle_x \qquad \left|\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow \quad \cdots \right\rangle_x \quad \Leftrightarrow C_2 = \sum_i \sigma_i^x \sigma_{i+2}^x$$

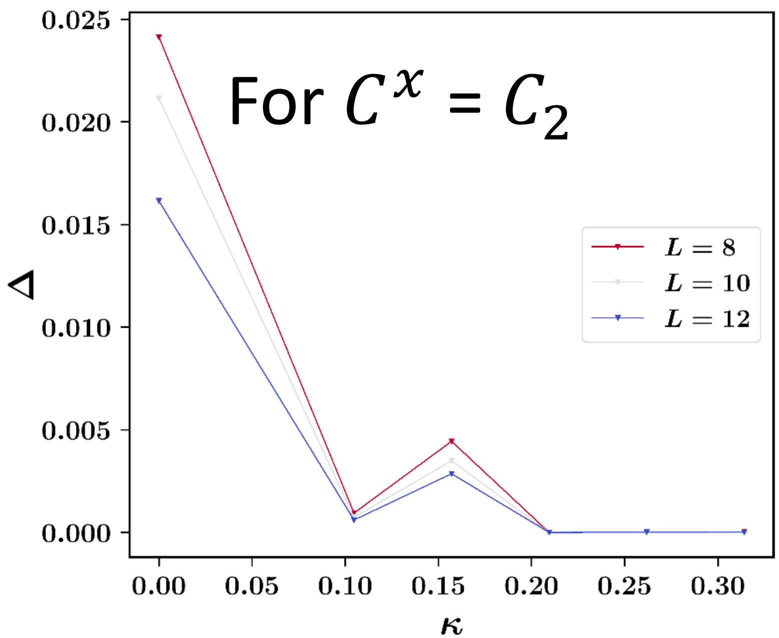
-
-
-

Strong Field:
A Surrogate Mother of Conservation Laws

$$\left.\begin{array}{rcl} |x_\alpha\rangle & = & \text{the } \alpha^{th} \text{ Eigenstate of } \{\sigma_i^x\} \\ |\mu_\alpha\rangle & = & \text{the } \alpha^{th} \text{ Floquet Eigenstate} \end{array}\right\}$$

Ordered by the expectation value of C^x over them.

$$\Delta = \frac{1}{D_H} \sum_{\alpha=1}^{D_H} \left| \langle x_\alpha | C^x | x_\alpha \rangle - \langle \mu_\alpha | C^x | \mu_\alpha \rangle \right|$$



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Probably Divergent Series Hide the key!

❖ In general, Many-Body Series (including both we discussed) have **zero radii of convergence!**
(The norm grows with the order as the number of processes explodes with the order and diverges with L)

➢ But those divergences might not have any physical significance!
The Series can be **Asymptotic to so well-behaved function** with at most (physically meaningful) isolated singularities!

Example: The Renormalized (the individual term after mass and charge renormalization) series of QED for any observable
expanded as a perturbation series in e^2 ($e =$ electron charge) after integrating the equation of motion over time.
(F. J. Dyson, Phys. Rev. **85**, 631 1952)

How to extract information from a divergent series?

Borel-Laplace summation and other **summation machines**.
Connection via Resurgence theory
(see, e.g. D. Dorigoni, Annals of Phys **409**, 167914 2019).

Thanks!

IACS **(1876)**