Introduction to flavour experiments

In memoriam of Sheldon Stone (Feb. 14, 1946 – Oct. 6, 2021)

https://cerncourier.com/a/sheldon-stone-1946-2021/

ICTS 2022 Bengaluru (India), April 2022

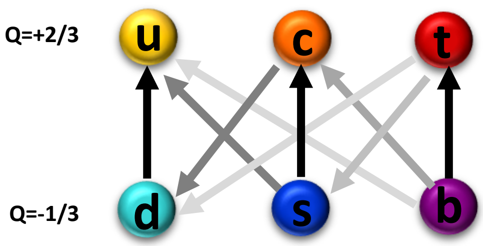
Arantza.Oyanguren@ific.uv.es

Outline

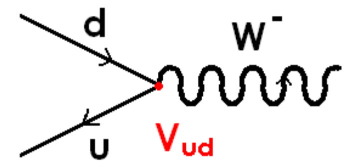
- Lesson 1: Introduction to flavour physics
- Lesson 2: The CKM matrix
- Lesson 3: Rare decays of heavy hadrons
- Lesson 4: Mixing and CP violation

The CKM matrix

- In the Standard Model of Particle Physics, transitions between different quarks are governed by the CKM mechanism:

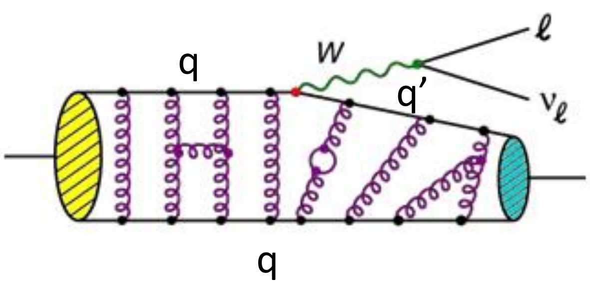


$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



- The amplitude of a hadron decay process can be described using Effective Field Theories: Operator Product Expansion (OPE)

$$A(M \to F) = \langle F|\mathcal{H}_{eff}|M\rangle = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu)\langle F|O_i(\mu)|M\rangle$$



CKM couplings

Wilson Coefficients (μ = scale)

Hadronic Matrix Elements

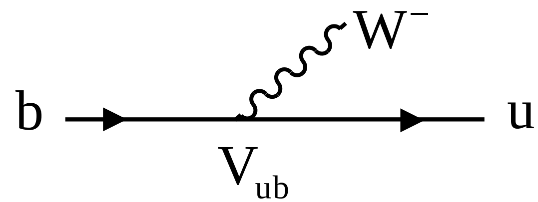
The CKM matrix

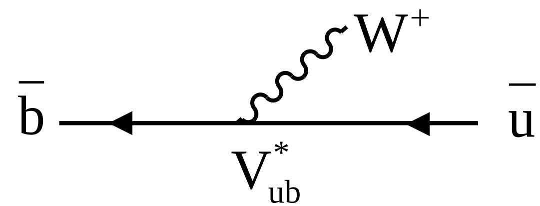
The CKM matrix V_{CKM} describes rotation for quarks between the weak eigenstates (d',s',b') and mass eigenstates (d,s,b)

Quarks $$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Antiquarks $$\begin{pmatrix} \overline{d}' \\ \overline{s}' \\ \overline{b}' \end{pmatrix} = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} \overline{d} \\ \overline{s} \\ \overline{b} \end{pmatrix}$$

V_{ij} governs the transition from quark j to quark i





CP violation due to complex phases of CKM matrix elements

The CKM matrix

- The CKM matrix is complex and unitary $\hat{V}^{+}_{CKM}\hat{V}_{CKM} = 1$
- 4 independent parameters
  - $\rightarrow$ Fundamental constants of the Standard Model
  - $\rightarrow$ Must be determined from experiment

- **Standard parametrization (PDG):**
  - 3 angles: $\theta_{12}, \theta_{23}, \theta_{13}$ and 1 phase δ

$$V_{CKM} = R_{23} \times R_{13} \times R_{12}$$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij} \quad c_{ij} = \cos\theta_{ij}$$

The CKM matrix

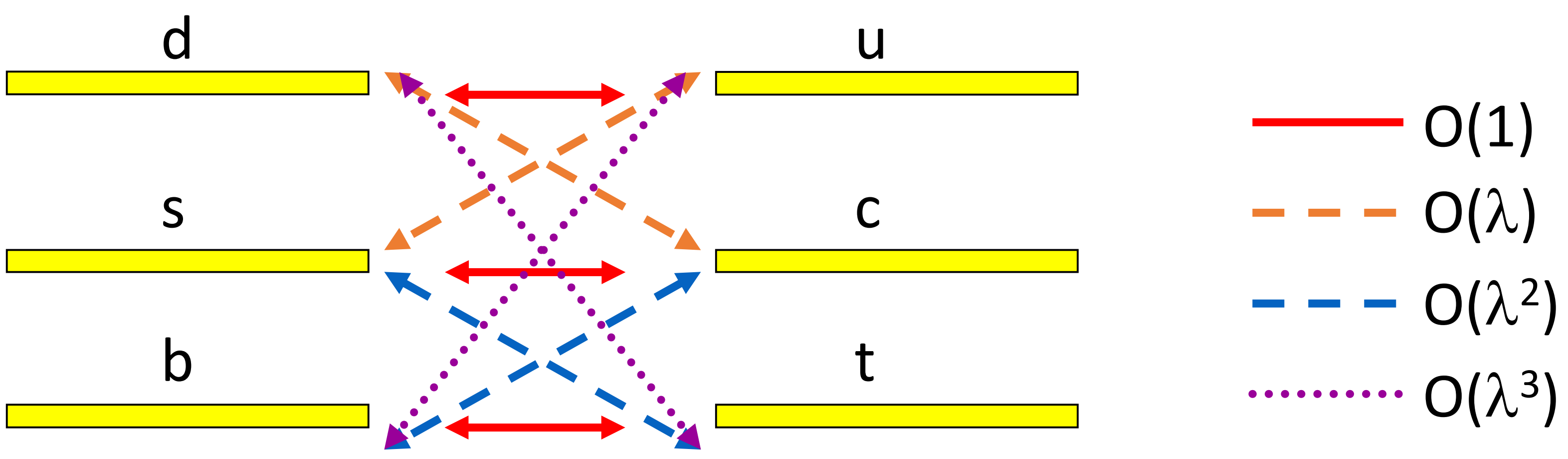
- **Wolfenstein parameterization:** $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{23} \sim 0.004$

- Perturbative, reflects the hierarchy of the matrix elements in terms of λ

$\lambda = sin\theta_{12} \approx 0.23$ (Cabibbo angle)

- The four parameters are defined as:

$$\lambda = s_{12} \quad A = \frac{s_{23}}{s_{12}^2} \quad \rho = \frac{s_{13}\cos\delta}{s_{12}s_{23}} \quad \eta = \frac{s_{13}\sin\delta}{s_{12}s_{23}}$$



The CKM matrix

Wolfenstein parameterization at O(λ^3):

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

(next-to leading order corrections in λ may be important when increasing precision)

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + A^2\lambda^5\left(1/2-\rho-i\eta\right) & 1-\lambda^2/2-\lambda^4/8\left(1+4A^2\right) & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+A\lambda^4(1/2-\rho-i\eta) & 1-A^2\lambda^4/2 \end{pmatrix} + O(\lambda^6)$$

$$(\bar{\rho},\bar{\eta}) \equiv (1-\lambda^2/2)(\rho,\eta)$$

The CKM matrix

- CP Violation in the Standard Model:

  - Requirements for CP violation

$$
\begin{aligned}
&\left(m_t^2 - m_c^2\right)\left(m_t^2 - m_u^2\right)\left(m_c^2 - m_u^2\right) \\
\times &\left(m_b^2 - m_s^2\right)\left(m_b^2 - m_d^2\right)\left(m_s^2 - m_d^2\right) \times J_{CP} \neq 0
\end{aligned}
$$

$$
J_{CP} = \left| \mathrm{Im}\left\{ V_{i\alpha} V_{j\beta} V_{i\beta}^{*} V_{j\alpha}^{*} \right\} \right| \quad (i \neq j, \alpha \neq \beta)
$$

  - *Jarlskog* invariant:

$$
J_{CP} = s_{12} s_{13} s_{23} c_{12} c_{23} c_{13} \sin\delta = \lambda^6 A^2 \eta = O\left(10^{-5}\right)
$$

→ CP violation is small in the Standard Model

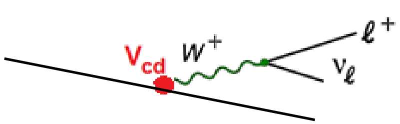
(and cannot explain the observed baryon asymmetry of the Universe)

The CKM matrix

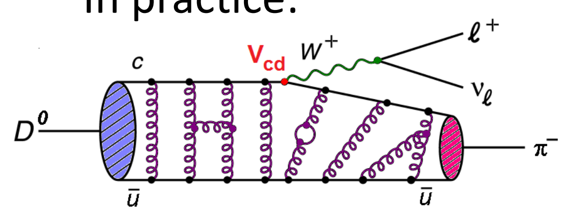
- PDG 2021: [https://pdg.lbl.gov/2021/reviews/contents_sports.html]

0.97401 ± 0.00011 superallowed $0^+ \rightarrow 0^+$ β decays	0.22650 ± 0.00048 semileptonic / leptonic kaon decays hadronic tau decays	$(3.61 \pm 0.11) \times 10^{-3}$ semileptonic / leptonic B decays
0.22636 ± 0.00048 semileptonic charm decays charm production in neutrino beams	0.97320 ± 0.00011 semileptonic / leptonic charm decays	$(40.53 \pm 0.83) \times 10^{-3}$ semileptonic B decays
$(8.54 \pm 0.23) \times 10^{-3}$ B_d oscillations	$(39.78 \pm 0.82) \times 10^{-3}$ B_s oscillations	0.999172 ± 0.000035 single top production

In theory:



In practice:



→ Need theory to describe QCD effects (lattice QCD)

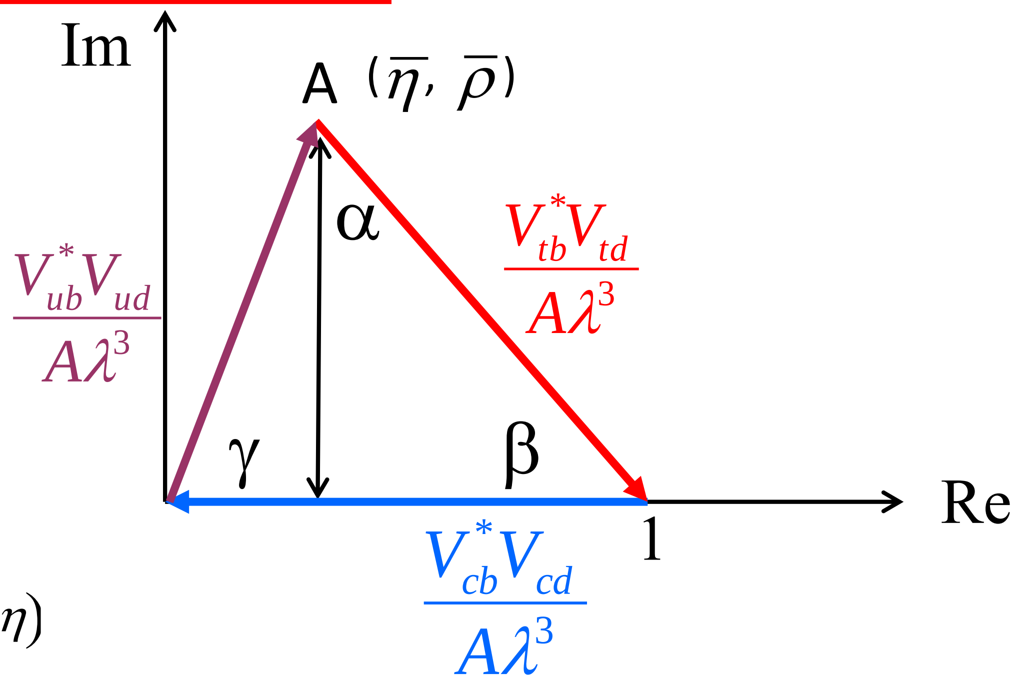
Q : How well is satisfied unitarity from the measured CKMs?

The CKM matrix

The Unitarity Triangle → CKM is unitary:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{cd} & V_{td} \\ V_{us} & V_{cs} & V_{ts} \\ V_{ub} & V_{cb} & V_{tb} \end{pmatrix}^* = I$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$$(\bar{\rho}, \bar{\eta}) \equiv (1 - \lambda^2/2)(\rho, \eta)$$

$$\gamma \equiv \arg\left[-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right] = \tan^{-1}\frac{\eta}{\rho} \sim 70^{\circ}$$

$$\beta \equiv \arg\left[-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right] = \tan^{-1}\frac{\bar{\eta}}{1-\bar{\rho}} \sim 21^{\circ}$$

$$\alpha \equiv \pi - \beta - \gamma$$

The CKM matrix

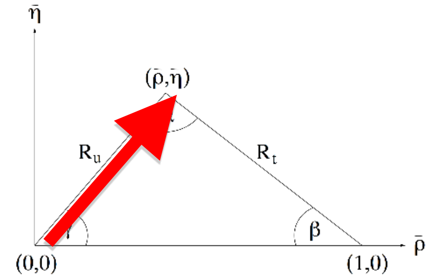
The Unitarity Triangle

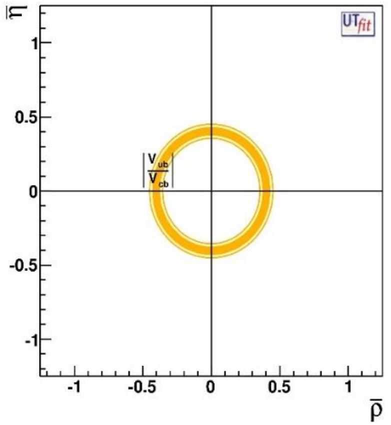
The idea: try to measure as many flavour observables as possible
overconstraint the unitarity triangle

Ex: Measuring the b→ u ℓν vs the b→ c ℓν transition

$$\frac{\Gamma(b \to u \ell \nu)}{\Gamma(b \to c \ell \nu)} \sim \left|\frac{V_{ub}}{V_{cb}}\right|^2 \sim \frac{1}{50}$$

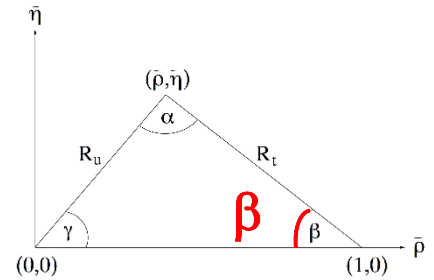
$$\left|\frac{V_{ub}}{V_{cb}}\right| = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

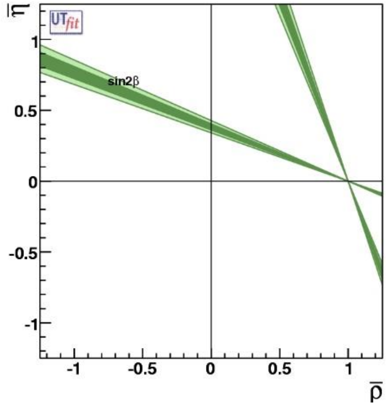




Ex: Measuring time-dependent asymmetries in b→c$\bar{c}$ s decays
(effect from interference of mixing and decay)

$$\beta = \tan^{-1}\frac{\bar{\eta}}{1-\bar{\rho}}$$



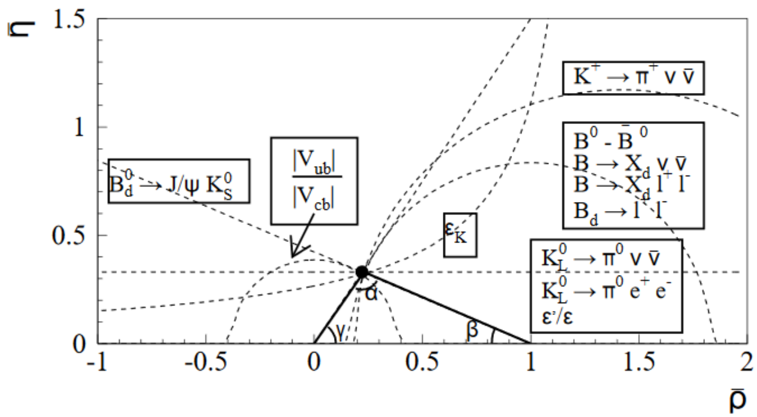


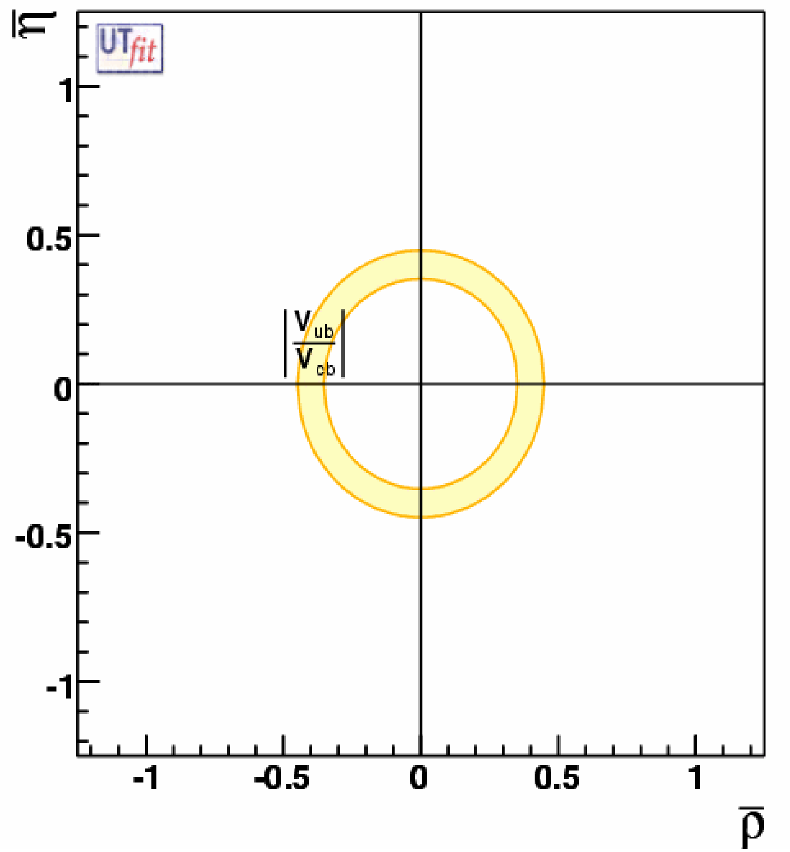
The CKM matrix

The Unitarity Triangle

The idea: try to measure as many flavour observables as possible

overconstrain the unitarity triangle



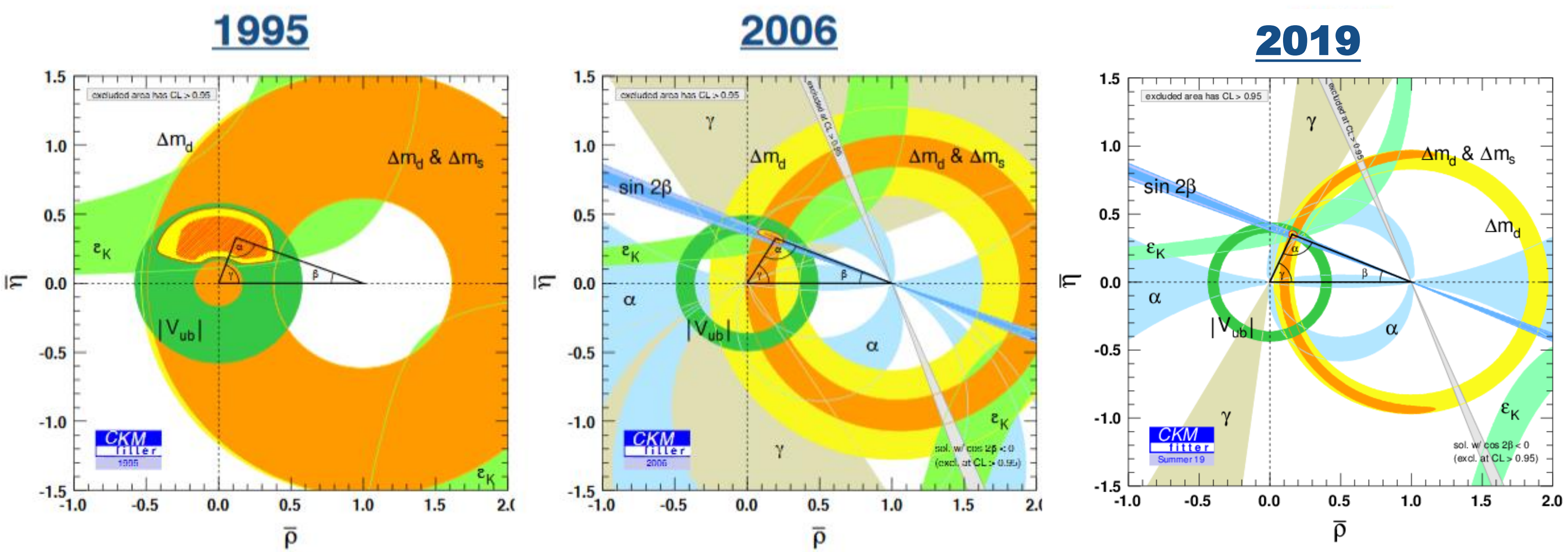


- If all measurements meet in the same apex→ good understanding of the flavour structure of the SM

- If not → New Physics !

The CKM matrix

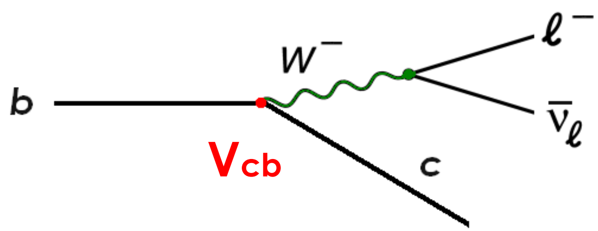
The Unitarity Triangle



- Very precise measurements of CKM elements (fundamental parameters!)
- Angles and sides have to be measured in many different ways to look for inconsistencies $\rightarrow$ quantum effects from new particles
- Tree level processes (new physics is less expected) should be compared to loop processes sensitive to new particles
- If everything is consistent with increasing precision the new physics scale has to be higher

The CKM matrix

Measuring CKM elements from semileptonic decays (an example)
(sides of the Unitarity Triangle)



$$V_{\rm CKM} = \begin{pmatrix} V_{\rm ud} & V_{\rm us} & V_{\rm ub} \\ V_{\rm cd} & V_{\rm cs} & V_{\rm cb} \\ V_{\rm td} & V_{\rm ts} & V_{\rm tb} \end{pmatrix}$$

Decay width, partial width and lifetime:

$$\Gamma = \sum_f \Gamma_f = \frac{1}{\tau}$$

Decay length:

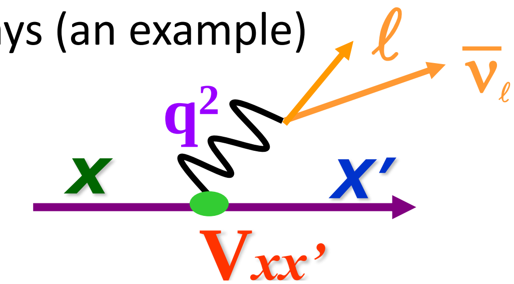
$$\langle \Delta \ell \rangle = \beta \gamma c \tau$$

Branching fraction:

$$1 = \sum_{f=1}^{m} \frac{\Gamma_f}{\Gamma} = \sum_{f=1}^{m} B_f$$

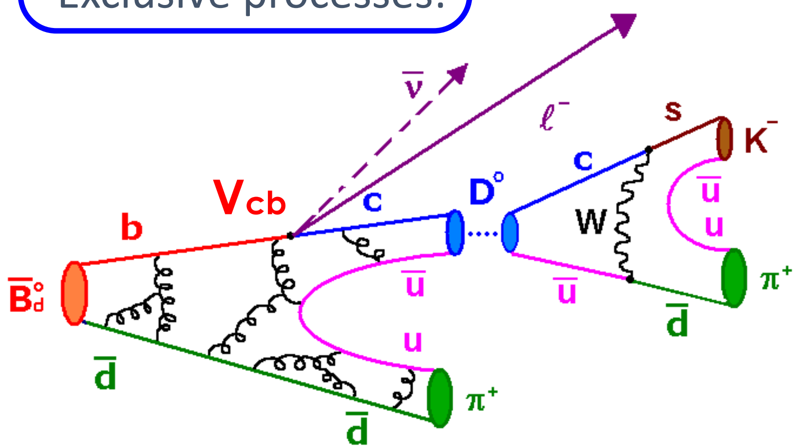
The CKM matrix

Measuring CKM elements from semileptonic decays (an example)
(sides of the Unitarity Triangle)



$$\Gamma_{sl} = |V_{xx'}|^2 \, f(\text{theory})$$

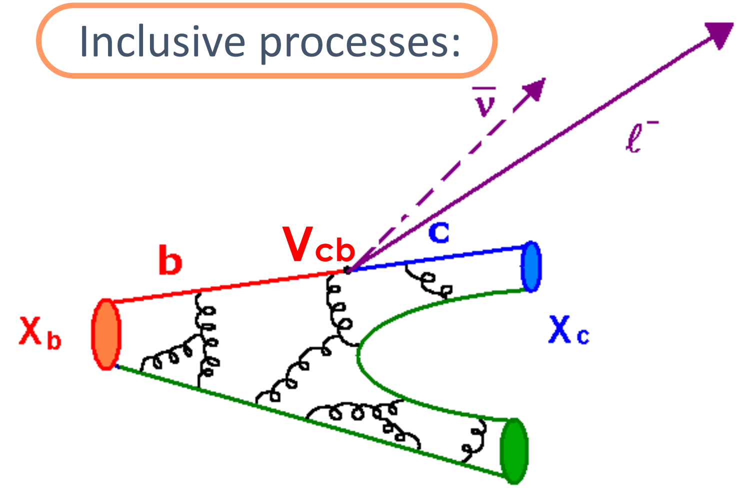
Exclusive processes:



$$\Gamma_{sl}^{D^*} = \frac{\mathcal{B}(\overline{B} \to D^* \ell^- \overline{\nu}_\ell)}{\tau_{\overline{B}}}$$

Theory: HQET, LQCD, LCSR
Experiment: low backgrounds (mainly D**)

Inclusive processes:

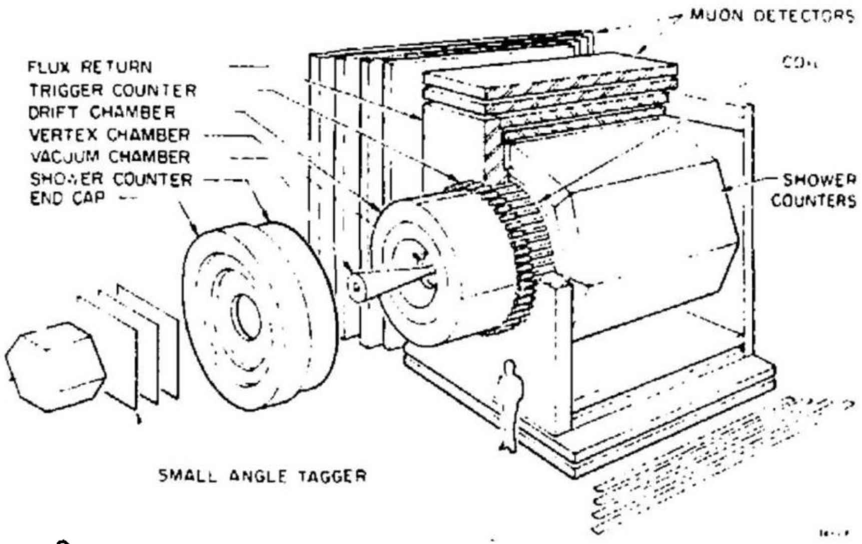


$$\Gamma_{sl}^{tot} = \frac{\mathcal{B}(b \to c \ell^- \overline{\nu}_\ell)}{\tau_b}$$

Theory: OPE
Experiment: higher backgrounds

The CKM matrix

Measurement of the $|V_{cb}|$ element by MARK II at PEP e+e- collider at 29 GeV (California, 1983)



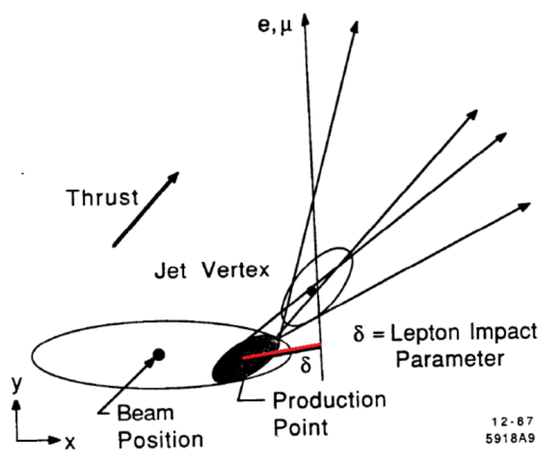
Lifetime of b-hadrons

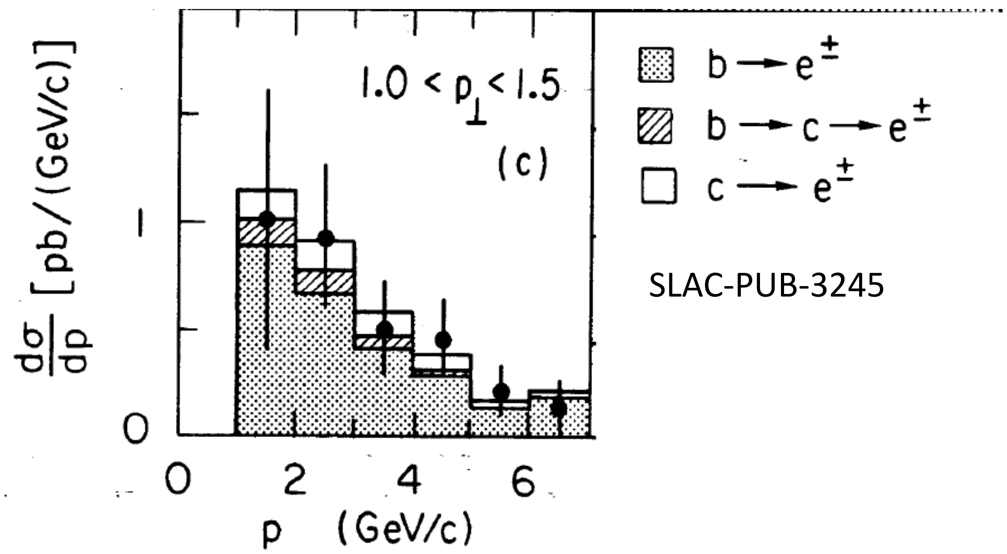
Phys. Rev. Lett. 51 (1983) 1316

$$\frac{1}{\tau_b} \equiv \Gamma_{tot} = \frac{\Gamma(b \to xl\nu)}{BR(b \to xl\nu)}$$

$$\Gamma(b \to xl\nu) = \frac{G_F^2\, m_b^5}{192\,\pi^3} \left(|V_{ub}|^2 + 0.49\,|V_{cb}|^2 \right) (\mathrm{GeV})$$

Impact parameter:

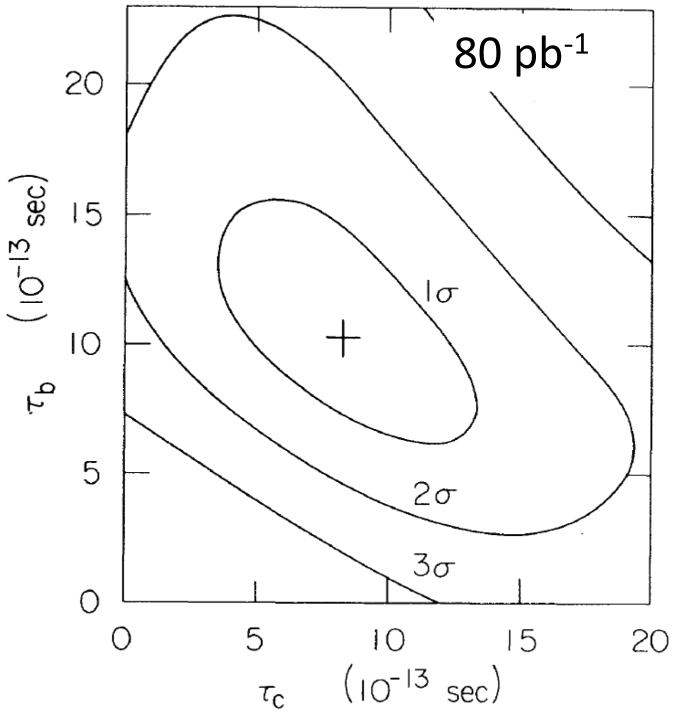




SLAC-PUB-3245

The CKM matrix

Phys. Rev. Lett. 51 (1983) 1316



$$\tau_b = (12.0^{+4.5}_{-3.6} \pm 3.0) \times 10^{-13}\ \text{sec}$$

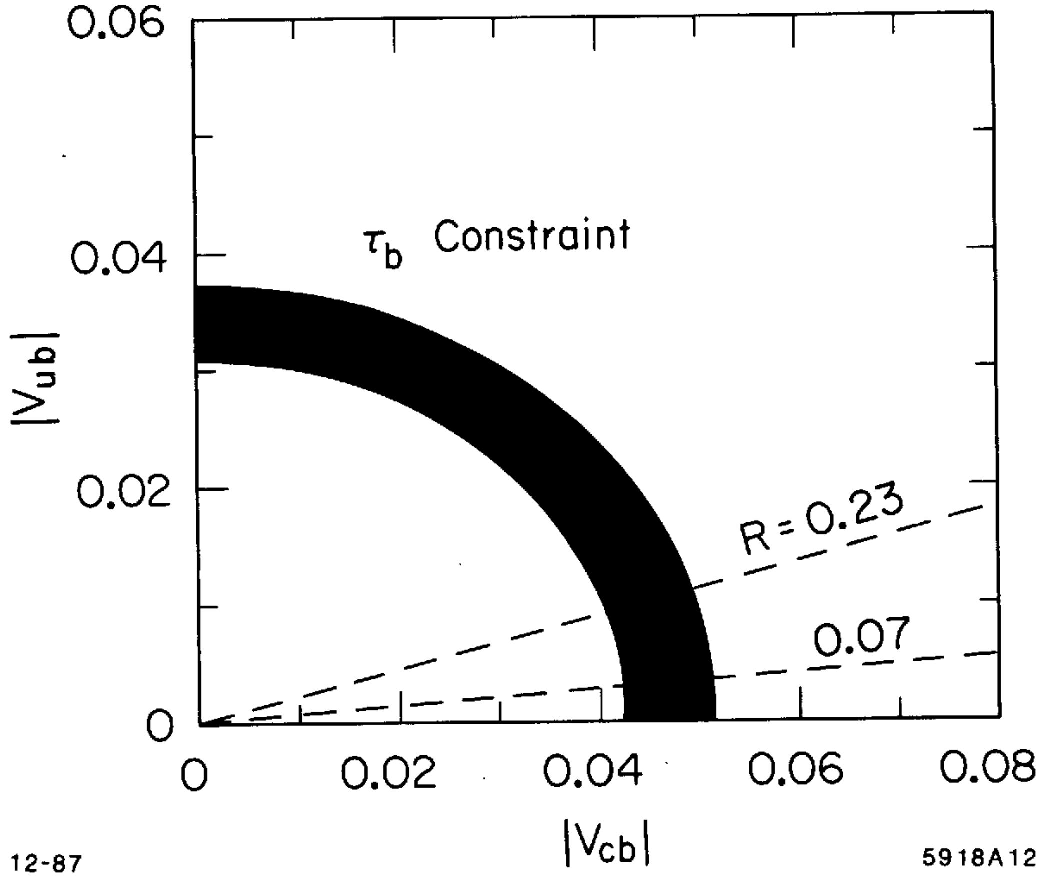
The B lifetime is long → the mixing between the third generation of quarks and the lighter quarks is much weaker that the mixing between the two first generations.

the U_{bu} term in the expression for the lifetime is negligible. If we put $m_b = 5\ \text{GeV}/c^2$, we find $|U_{bc}| = 0.053^{+0.010}_{-0.009}$, where the error is statistical only. This value is appreciably smaller than that of the analogous matrix element which describes strange-particle decay, $U_{su} = 0.22$, the sine of the Cabibbo angle.

~ 20% error

The CKM matrix

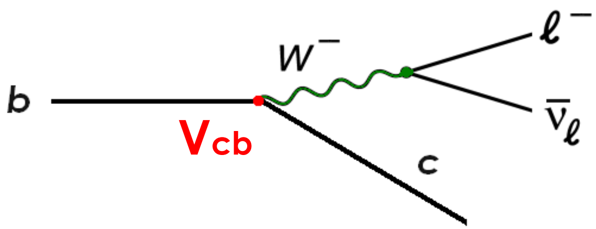
Status by December 1987…



Constraints on the KM matrix elements for B decay. The dark band corresponds to the constraint imposed by the B lifetime measurements. The dotted lines reflect the constraints imposed by the ratio R = IV_{ub}/V_{cb}]. SLAC-PUB-4503

The CKM matrix

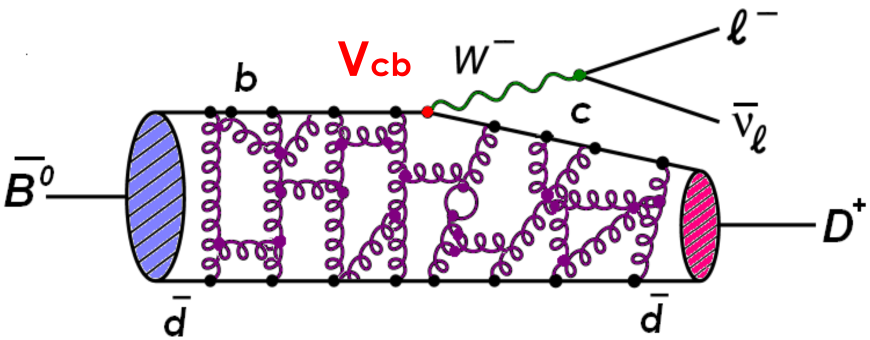
Measuring CKM elements from semileptonic decays (an example)
(sides of the Unitarity Triangle)

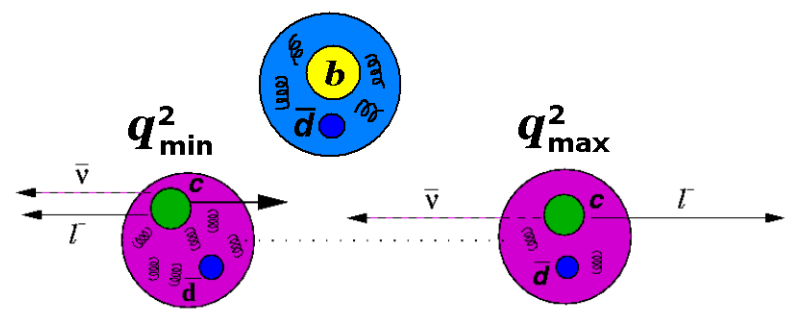


$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Quarks are confined inside the hadrons

$$q^2 = (p_{\ell^-} + p_{\overline{\nu}_\ell})^2 = (p_{\overline{B}} - p_{D^*})^2$$





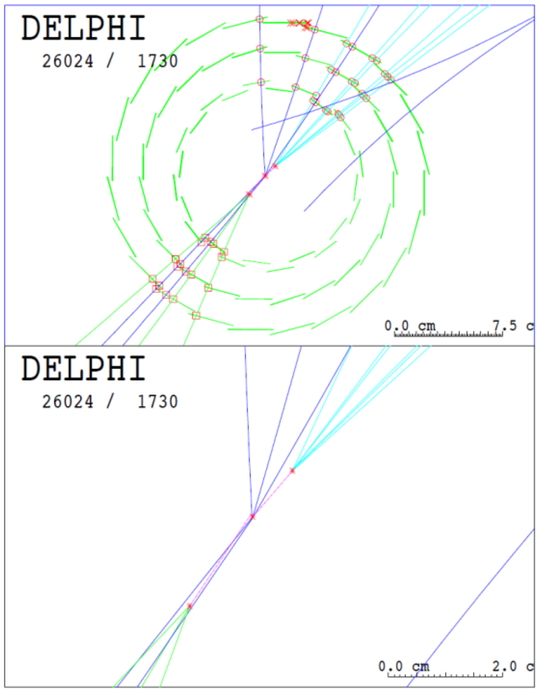
$$\frac{d\Gamma(B \to D^* \ell \nu)}{dq^2} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 \mathcal{F}(q^2)^2 \mathcal{K}(q^2)$$

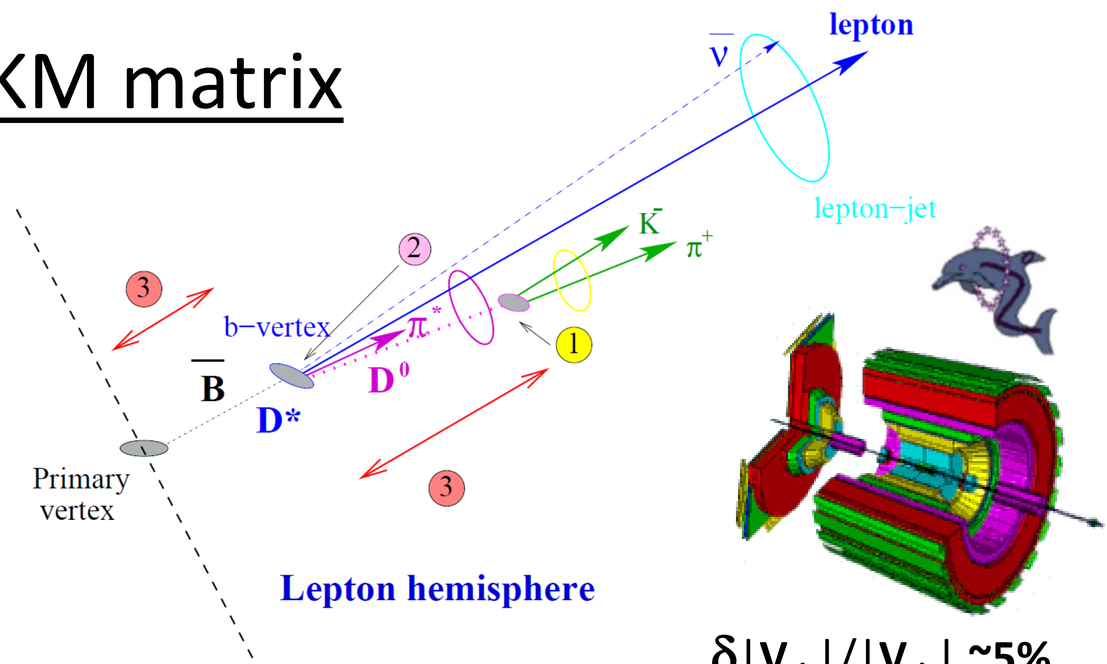
The CKM matrix

20 years after MARK II,

and before the B factories:

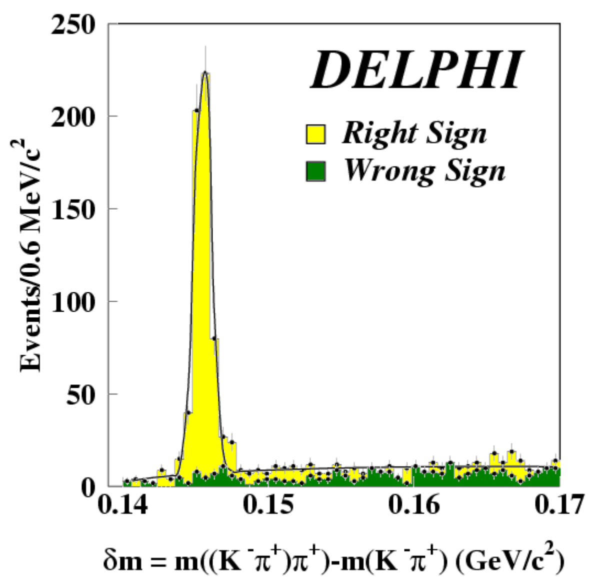
V_{cb} from Z decays at LEP

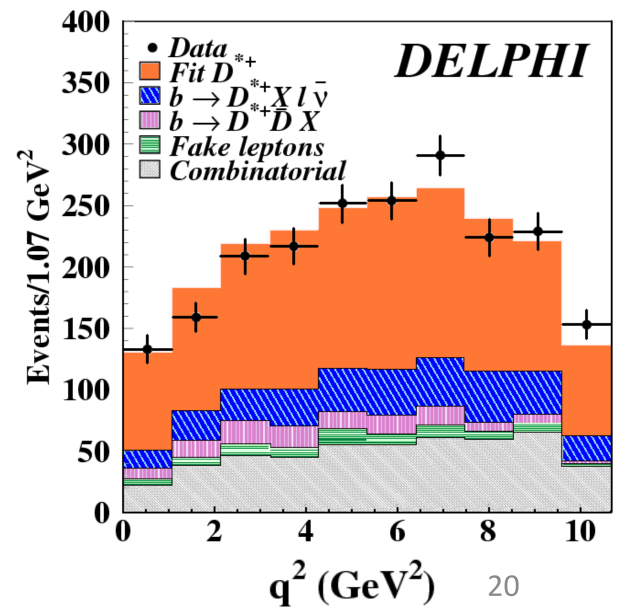




$\delta|V_{cb}|/|V_{cb}|$ ~5%

Eur. Phys. J. C33 (2004) 213





D*+ - ℓ⁻ candidates

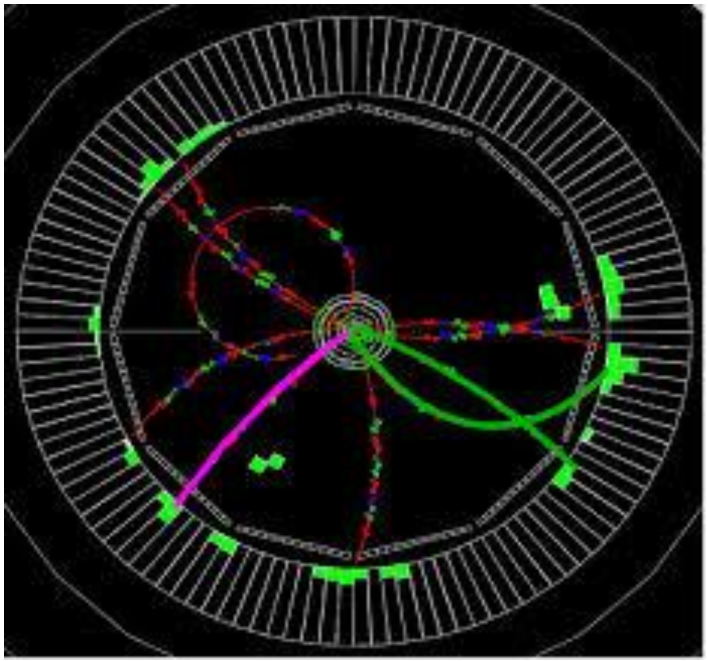
$B \to D^* \ell^- \nu$

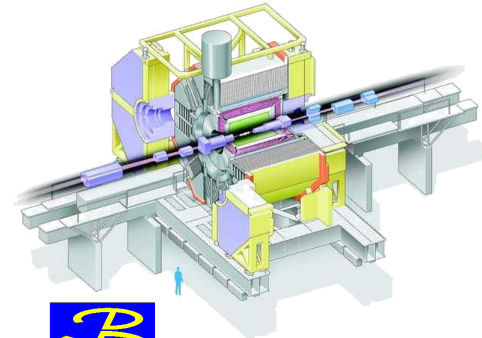
$\hookrightarrow D^0 \pi_{soft}$

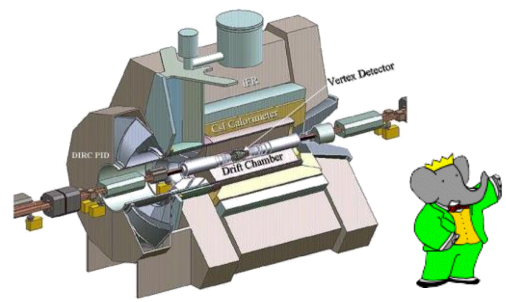
$$q^2_{\max} = (m_B - m_{D^*})^2 \approx 10.69\ \mathrm{GeV}^2$$

The CKM matrix

Measurements from BELLE and BaBar:

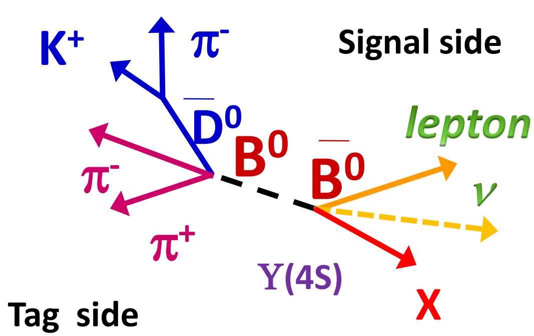


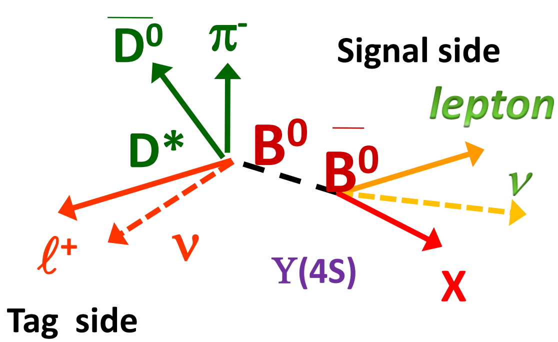


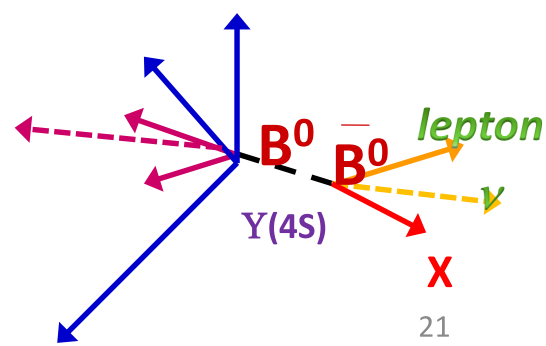


Hadronic or Semileptonic Tag (High purity)

Untagged (High efficiency)







The CKM matrix

$$\frac{d\Gamma(B \to D^{(*)}\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3\hbar}\mathcal{F}^2(w)\,\mathcal{K}(w)|V_{cb}|^2$$

$$w = V_B \cdot V_{D^{(*)}} = \frac{(M_B^2 + M_{D^{(*)}}^2 - q^2)}{(2M_B M_{D^{(*)}})}$$

(w ranges from 1 to 1.5)

Form factor
(shape & normalization)

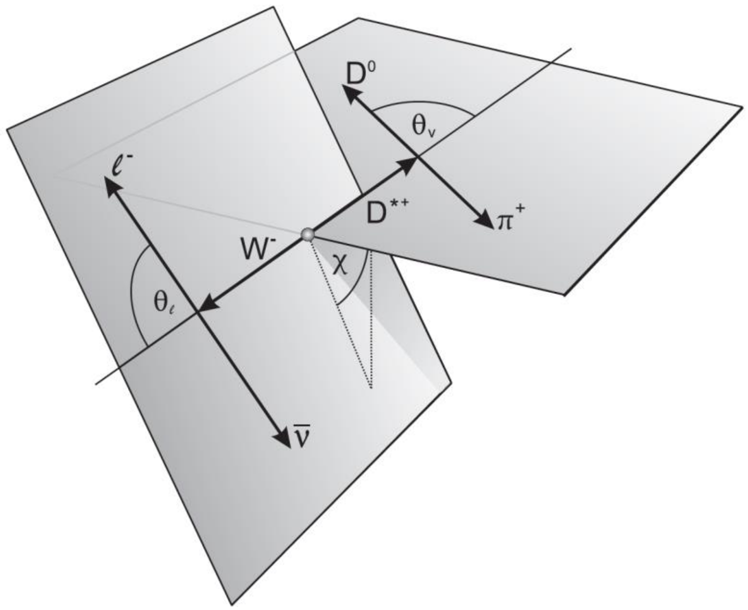
Kinematic factor

Normalized by HQET ($m_Q \to \infty$) at q^2_{max} (w= 1) $\equiv$ $F_{D*}(1)=1$

LQCD $\to$ $F_{D*}(1) = 0.921 \pm 0.024$

Belle and BaBar have a lot of data! (~ 700 fb^{-1})
Possible to fit also the angular distribution of the decay particles:

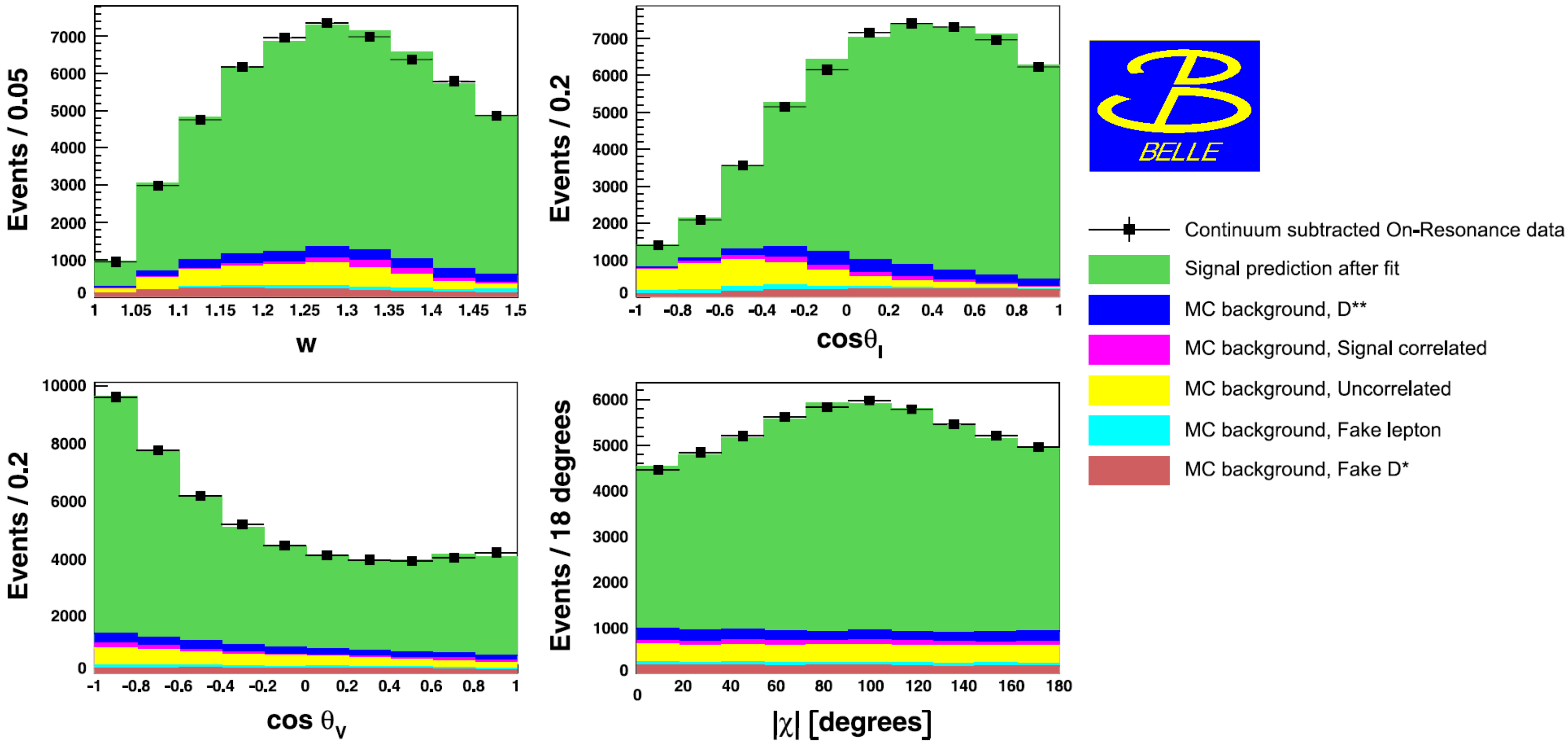
$$\frac{d^4\Gamma(B^0 \to D^{*-}\ell^+\nu_\ell)}{dw\,d(\cos\theta_\ell)\,d(\cos\theta_V)\,d\chi}$$



J (B^0) = 0, J (D^*) = 1, J (W) = 1

The CKM matrix

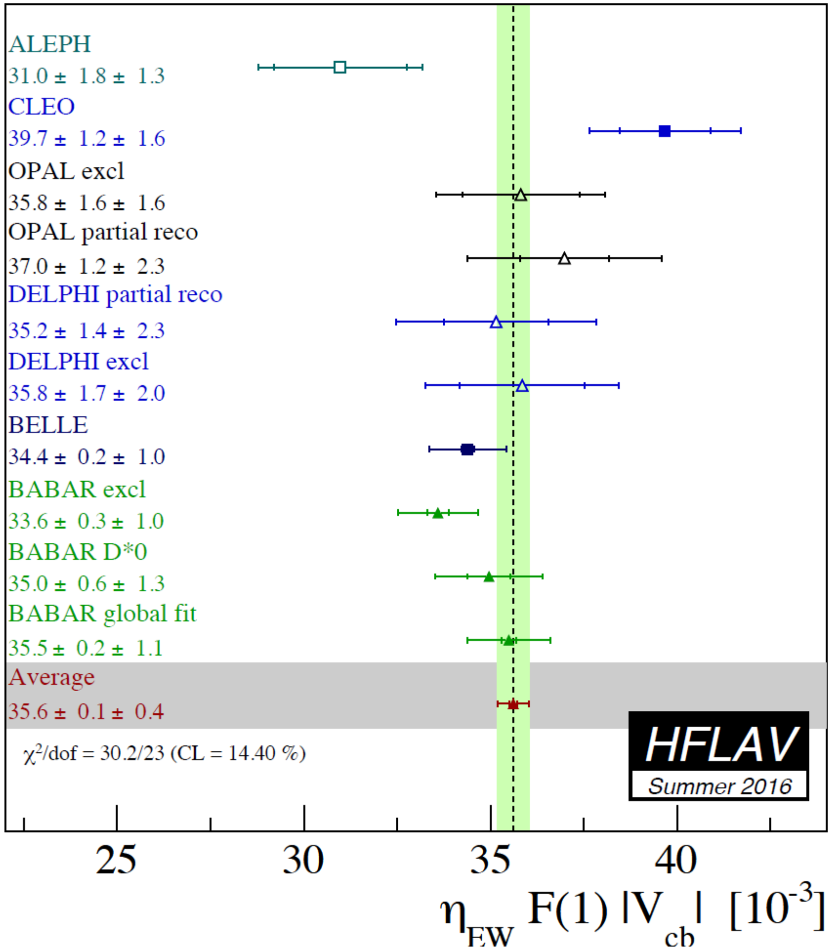
120000 $B \to D^* \ell \nu$! (muons and electrons), very few backgrounds

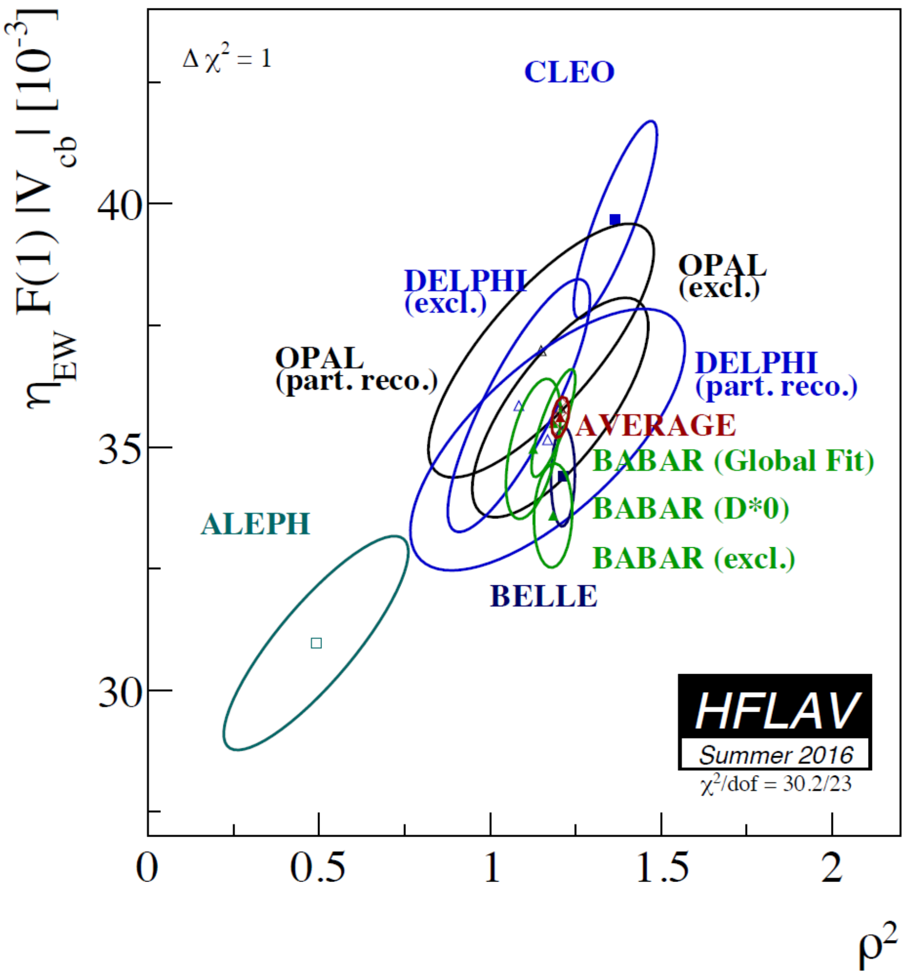


A 4 dimensional fit allows the determination of the form factor shape and $|V_{cb}|$

The CKM matrix

After BELLE and BaBar:





$\delta|V_{cb}|/|V_{cb}| \sim 1\%$!

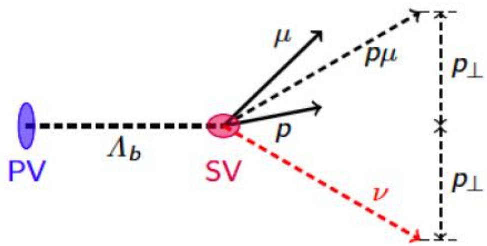
The CKM matrix

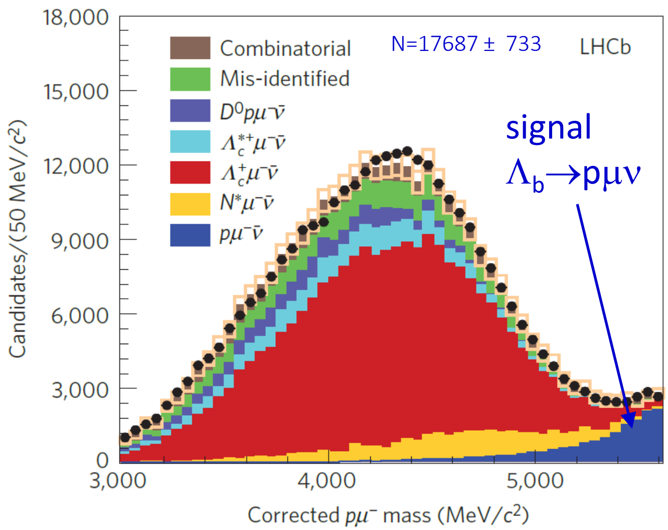
At LHCb semileptonic decays are very challenging due to the missing neutrino:

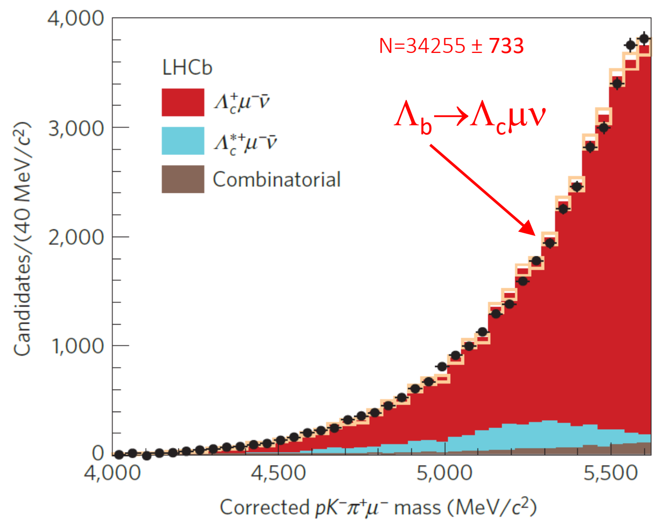
Using semileptonic decays of b-baryons:

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu^-\overline{\nu}_\mu)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\overline{\nu}_\mu)} = \frac{|V_{ub}|^2}{|V_{cb}|^2}$$ x Ratio of form factors ← (5% accuracy from LQCD)

- Use information from displaced vertex
- Select high q^2 region (theory more precise)
- Corrected mass: $m_{\text{corr}} = \sqrt{m_{h\mu}^2 + p_\perp^2} + p_\perp$







[Nature Physics 10 (2015) 1038]

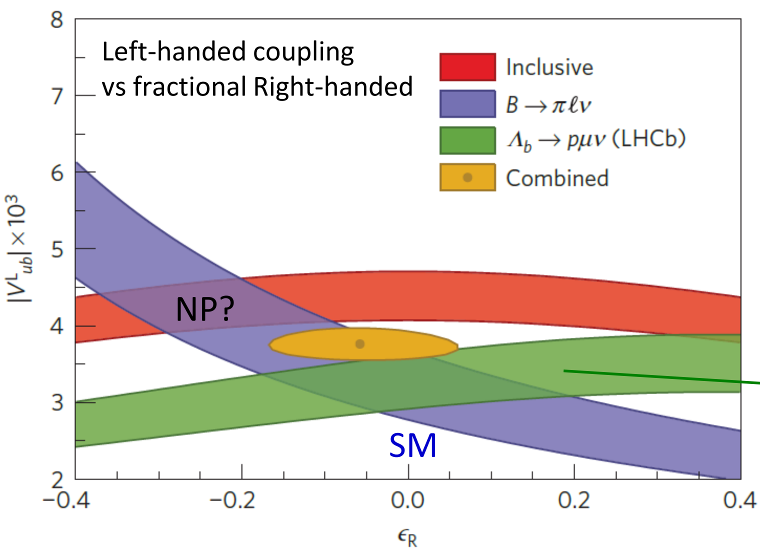
The CKM matrix

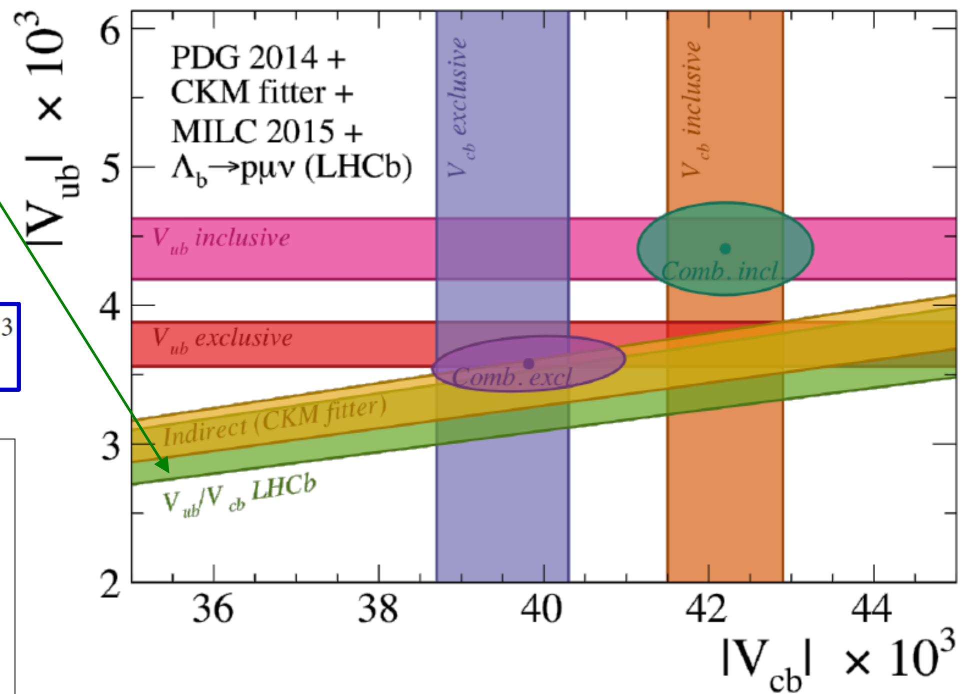
[Nature Physics 10 (2015) 1038]

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004 \pm 0.004$$

Using the world average from exclusive V_{cb}:

$$|V_{ub}| = (3.27 \pm 0.15 \pm 0.16 \pm 0.06) \times 10^{-3}$$

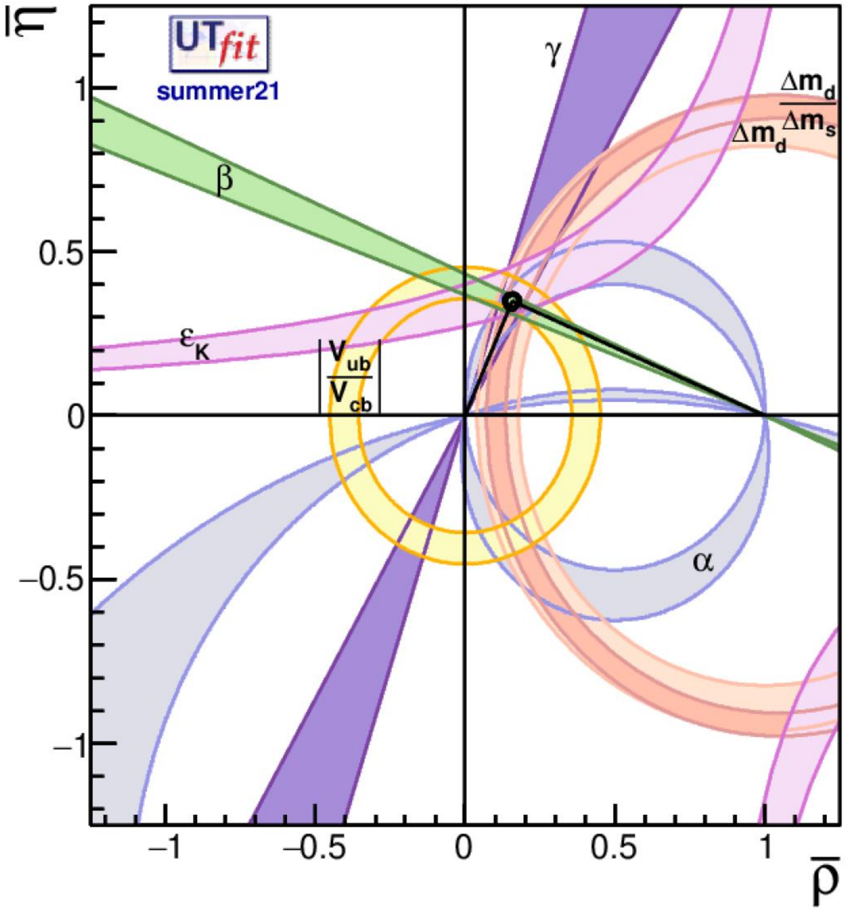




Disfavours New Phyiscs models with Right-handed currents

The CKM matrix

Present status:



- ➜ Very high level of precision (few %)
- ➜ No inconsistencies: validation of Standard Model in the flavour sector
- ➜ Understanding from QCD is crucial

$\bar\rho = 0.157 \pm 0.012$
$\bar\eta = 0.350 \pm 0.010$

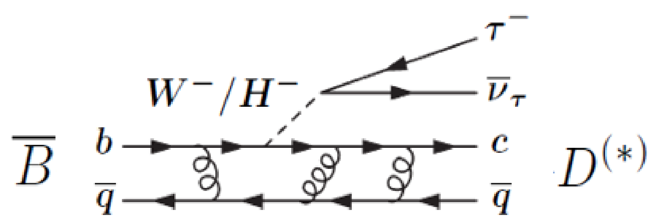
http://ckmfitter.in2p3.fr/

http://www.utfit.org/UTfit/

The CKM matrix

- Nevertheless, some anomalies are found at tree level in B semileptonic decays
  $\rightarrow$ Ratio of semi-tauonic and semi-muonic branching fractions:

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu)}$$

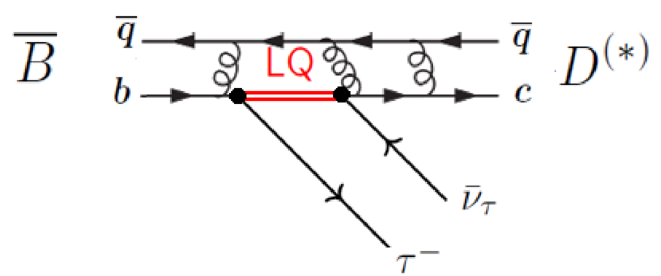


SM predictions very precise :

(V_{cb} and form factors (partially) cancel)

$R(D)_{SM}=0.299 \pm 0.003$

$R(D^*)_{SM}=0.252 \pm 0.003$



- ▶ Test of lepton universality
- ▶ Sensitive to charged Higgs bosons and leptoquarks

The CKM matrix

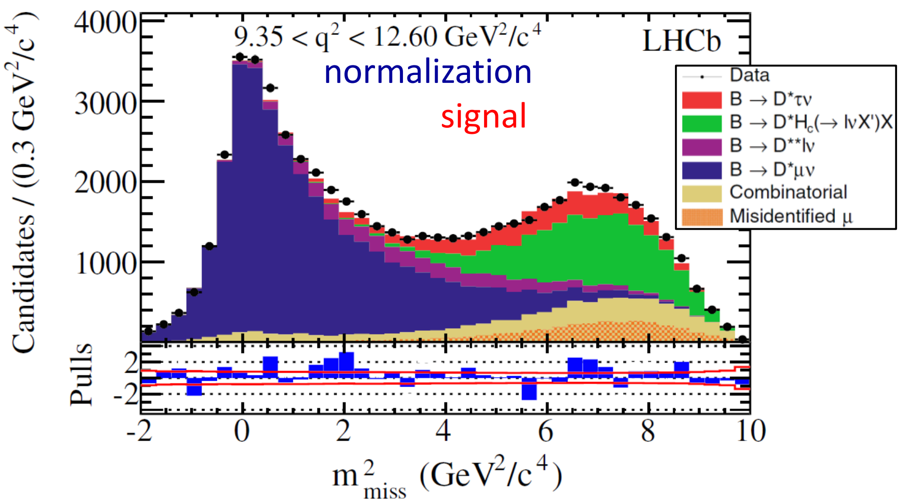
BaBar measured an excess of $B^0 \to D^{(*)}\tau^- \nu_\tau$ (**3σ away from SM!**) [Nature 546 (2017) 227]

LHCb: R(D*)
- $\overline{B}^0 \to D^{*+} \tau^- \overline{\nu}_\tau$, with $\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau$ [PRL 115 (2015) 111803]
- $B^0 \to D^{*-} \tau^+ \nu$, with $\tau^+ \to \pi^+\pi^-\pi^+(\pi^0)\overline{\nu}_\tau$ [PRL 120 (2018) 171802]

- Using $\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau$

Information from the missing mass squared $m_{miss}^2 = (P_B - P_{D*} - P_\mu)^2$ and muon energy

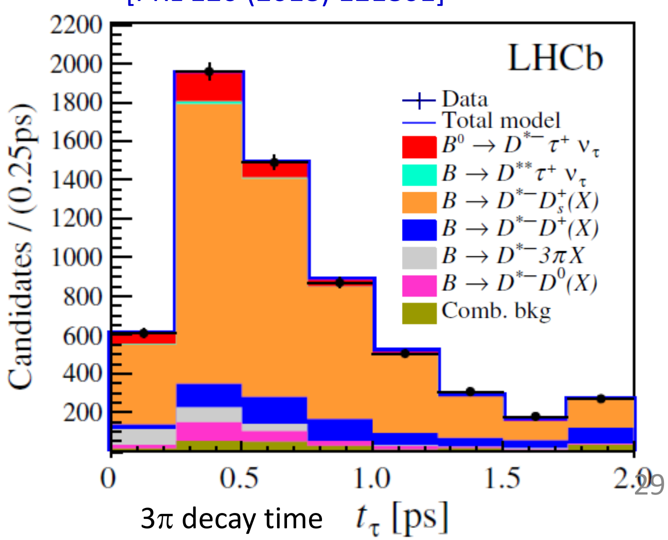
[PRL 115 (2015) 111803]



- Using $\tau^+ \to \pi^+\pi^-\pi^+\overline{\nu}_\tau$

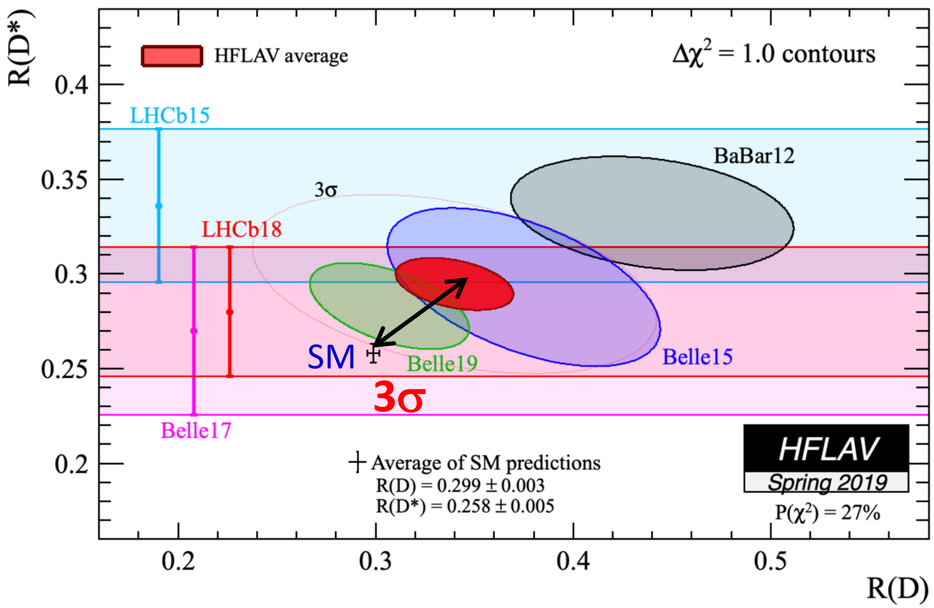
Information from the position of the pions. Normalized to $B^0 \to D^{*-}\pi^+\pi^-\pi^+$

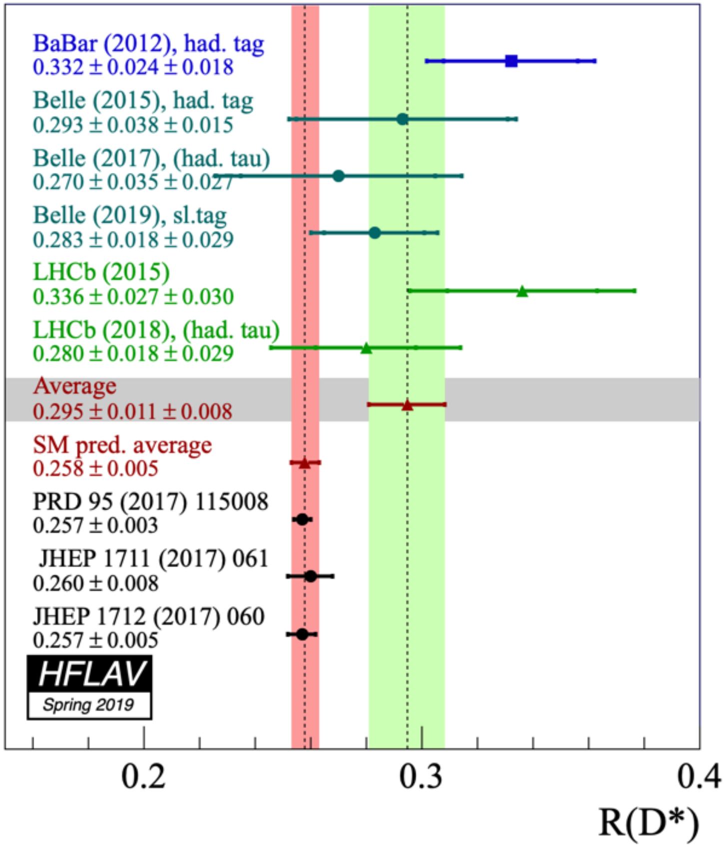
[PRL 120 (2018) 121801]



The CKM matrix

- Present global picture of R_D and R_{D^*} :





→ Average: **3σ** deviation from SM